An Incomplete-Contracts Approach to Corporate Bankruptcy*

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Abstract

This paper integrates the problem of designing corporate bankruptcy rules into a theory of optimal debt structure. We show that, in an incomplete contract framework with imperfect renegotiation, having multiple creditors increases a firm’s debt capacity while increasing its incentives to default strategically. The optimal debt contract gives creditors claims that are jointly inconsistent in case of default. Bankruptcy rules, therefore, are a necessary part of the overall financing contract, to make claims consistent and to prevent a value reducing run for the assets of the firm.

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1 Introduction

Bankruptcy law regulates the interaction between debtors and creditors when debtors default and the parties cannot work out their differences outside the courts. The law addresses two main types of conflicts: conflicts between a debtor and her creditors, and conflicts among creditors themselves. Most contributions to the large literature on bankruptcy law focus on ex-post conflicts. In particular, conflicts among creditors, the major source of complexity in modern bankruptcy law, have been analyzed from an ex-post perspective. But the design of bankruptcy law also influences initial financing and valuation of the firm. In fact, the problem is most interesting when posed in an ex-ante framework; it raises what seems like a paradox: if bankruptcy with multiple creditors is so complex, why would a firm contract with several creditors in the first place? Put differently: if conflicts of interest must be resolved ex post anyhow and these resolutions are costly, why create them ex ante? We attempt to answer these questions in an optimal contracting approach to corporate debt and bankruptcy.

The link between ex ante and ex post efficiency has been analyzed extensively in the capital structure literature. It is by now a standard result in that literature that having multiple creditors or multiple investors is a way of increasing ex ante efficiency of contracting at the cost of reducing ex post efficiency (Bolton and Scharfstein, 1996; Dewatripont and Maskin, 1995; Dewatripont and Tirole, 1994; Berglöf and von Thadden, 1994). Having multiple investors with conflicting interests tends to increase the costs of contract renegotiation, thereby increasing commitment and increasing ex ante efficiency. The latter takes the form of reduced incentives to strategically default on debt or of increased incentives to provide effort, present good investment projects, etc. In the logic of that literature, higher ex ante efficiency, and in particular reduced incentives for strategic default, increase the capacity to raise funds. From the perspective of bankruptcy law, this literature tends to suggest that it is important to create ex post inefficiencies in bankruptcy procedures so as to reduce the ex ante incentives for strategic default and thus to enhance the debt capacity of firms.

In this paper, we borrow from that literature to analyze the choice of a single versus multiple creditors and its effect on incentives for strategic default. We start from the observation that multiple creditors make contract renegotiations more difficult and emphasize a) the ex post conflicts among multiple creditors, and b) the role of bankruptcy rules in solving such con-
fects from an ex ante perspective. Contrary to the standard result of the capital structure literature, we present a model where having multiple creditors increases the capacity to raise funds while at the same time increasing, instead of decreasing, the occurrence of strategic default. Having multiple creditors allows for overleveraging of the assets of a firm by giving to each individual creditor foreclosure rights over assets that are individually feasible but jointly inconsistent. The capacity to raise funds is increased because the total repayment obligations stemming from individual foreclosure rights are higher than if the firm faced a single creditor. In other words, having multiple creditors can serve as an instrument for the firm to commit itself credibly to higher debt repayments than if it were facing a single creditor. These higher repayment obligations, however, also increase at the margin the incentives for strategic default. When the firm defaults on its debt, the sum of claims held by individual creditors exceeds the value of the firm’s assets. This is where bankruptcy rules step in. Their role is to reconcile the externality between claims of different creditors. We thus take the view, shared by many scholars of law and economics, that “bankruptcy is a situation in which existing claims are inconsistent” (Hart, 1995). In our model, the need for bankruptcy rules arises endogenously because inconsistency of the claims of creditors is not the result of chance or irrationality, but is the result of optimal contract design.

The model, therefore, takes a different perspective from the work of Bizer and De Marzo (1992), who also identify an externality between multiple creditors. In their model, this externality is eliminated in an optimal contract with one single creditor that makes borrowing from additional sources unattractive for the firm. Their work is important in that it highlights the problem of potential externalities among multiple creditors, and in that it uses this problem to shed light on inefficiencies in single creditor relationships, but is of lesser interest for the study of bankruptcy, where multiple creditors are the rule.

To provide an intuition for why two investors are better than one in our model, consider a firm negotiating with two investors to finance a project. In an incomplete contracts approach, the project generates some verifiable assets and some unverifiable cash flows in the future. With only one investor, the firm’s commitment ability is, in principle, limited by the amount of verifiable assets available for foreclosure should the firm default. However, this constraint is relaxed with two investors; the firm can promise up to the full amount of available assets to each one of the investors. When the firm
only defaults on one investor, this investor has the right to foreclose on the firm’s assets to collect her debt. If the firm defaults on both creditors, and one creditor calls the sheriff to enforce the payment, the other creditor can file for bankruptcy. In this case, the sum of the two claims will be larger than the available amount of verifiable assets and individual claims will then have to be adjusted by the court. Clearly, from a practical point of view, to make creditor claims compatible with available assets is one of the essential functions of a bankruptcy court.

When the firm has sufficient cash flows to pay off investors, two investors will extract strictly more from the firm than would one investor. Indeed, the overleveraging of the firm’s asset base forces the firm to higher repayment obligations with multiple creditors as compared to a single creditor. Such higher repayment obligations are credible because individual creditors can always exercise their individual liquidation rights in case of default on their loan. It is then in the firm’s interest to meet these higher repayment obligations in order not to forego high continuation values. These higher repayment obligations, however, increase the incentives for strategic default. Of course, when the firm has no cash flows, it is forced to default on both investors. There will, in other words, sometimes be liquidation in the good cash-flow state and always in the bad cash-flow state. Since liquidation is ex post inefficient, the optimal contract minimizes expected liquidation while ensuring that investors are repaid in expectation. The fundamental trade-off is between lowering liquidation in bankruptcy, reducing the likelihood of strategic default, and increasing the incentives to pay out cash when available.

Our results lead to the prediction that firms with large capital requirements (per unit of asset generated) should have multiple investors. These firms should also be “overleveraged”, i.e., the promised debt payments should be larger than the value of verifiable assets. This is compatible with the observation in many countries of low retrieval rates of creditors, in particular junior creditors, once firms are in bankruptcy (see Weiss (1990) and Anderson and Sundarajan (1996)). In fact, the model predicts that in bankruptcy the debtor retains some of her assets and junior creditors receive a smaller fraction of their claims than senior creditors (who are satisfied at par). As a corollary, absolute priority - the notion that creditors must be satisfied fully in bankruptcy before owners are to retain something - is violated in the present model. Again, this is consistent with the empirical literature.

Unlike most other contributions, this paper is consistent with the observa-
tion that solvent firms in real life actually enter into bankruptcy procedures. We ask how the prospect of such a procedure affects the choice of capital structure ex ante. In particular, we provide an explanation to our initial puzzle why the contracting parties introduce the ex post conflict between creditors (the overleveraging of its assets) and the design of a procedure - bankruptcy - to deal with this conflict. In the model, giving the creditors the right to trigger bankruptcy, possibly in combination with an appropriate priority structure, improves strictly upon the no-bankruptcy contract, in which liquidation claims are not coordinated after default. The notion of debtor-creditor law in the model is still rudimentary, but we believe it captures some fundamental elements of an optimal bankruptcy procedure. Bankruptcy is triggered when a creditor files to prevent his claims from being eroded through debt collection of other creditors. The procedure demands an “automatic stay” ensuring that liquidation claims are filed simultaneously. Finally, the bankruptcy court has to establish a new capital structure compatible with the value of the assets available to distribute among creditors.

An important strand of the by now large literature on bankruptcy law focuses on optimal procedural and substantial rules, taking as given pre-existing debt contracts and the decision to enter bankruptcy (see Bebchuk (1988), Aghion, Hart and Moore (1992) or Cornelli and Felli (1998)). This work rightly points out that the choice of capital structure influences what happens and what should happen in bankruptcy. Yet, it is silent on the determinants of capital structure, which is problematic as the choice of bankruptcy law, or more generally debtor-creditor law, will impact on the firm’s capital structure decision. In fact, these two issues are interrelated; even on a comparative international level for example, Rajan and Zingales (1995), White (1996), and LaPorta et al. (1998) show important correlations between financing patterns and legal rules.

Another interesting strand of research has looked at the bankruptcy problem from an ex-ante perspective. Building on the early work of Bulow and Shoven (1978), contributions such as those by Bebchuk and Picker (1996), Berkovitch, Israel, and Zender (1998), or Schwartz (1998) analyse the impact of bankruptcy on debtors’ incentives prior to bankruptcy. Cornelli and Felli (1997) have considered ex-ante incentives by creditors, and Berkovitch and

\footnote{An important exception is the work by Harris and Raviv (1995) who study the impact of different games played ex post on the ex-ante efficiency of the contract. Different from us, Harris and Raviv (1995) are only concerned with games between the debtor and one single investor.}
Israel (1999) and Povel (1996) focus on the problem of information transmission between debtor and creditor, with interesting recommendations concerning whether a code should allow debtors or creditors, or both, to trigger bankruptcy. Kordana and Posner (1999) discuss the complex voting features associated with the American Chapter 11. Like our paper, they focus on the tradeoff between reducing the cost of liquidation by lowering individual pre-bankruptcy entitlements and discouraging strategic default. Their analysis, like Berkovitch and Israel (1999) and Povel (1997) and unlike ours, is concerned with asymmetric information among investors. These ex-ante analyses are not concerned with the key question of our paper, which is the role of multiple creditors in bankruptcy.

In that respect, the contributions by Bisin and Rampini (2000), Bolton and Scharfstein (1996), and Winton (1995) are most closely related to our paper, as they all study problems of contracting and default with multiple creditors. Bisin and Rampini (2000) are interested in the ex-ante incentive effects of bankruptcy in an environment in which a debtor can borrow from several lenders. They show that a bankruptcy-like contract allows the main lender to relax the debtor’s incentive compatibility constraint, because it is a means for the main lender to commit to confiscate returns in low-return states (which is not optimal for consumption smoothing reasons, but increases the borrower’s effort incentives). Different from our model, in their model exclusive lending contracts are superior to contracts with multiple creditors, but cannot be enforced by assumption.

In the model of Bolton and Scharfstein (1996), on the other hand, multiple lending relationships are typically optimal, but their analysis does not focus on the problem of ex-post conflicts of creditors and their implications for bankruptcy. Winton (1995) approaches the problem of multiple creditors from the perspective of costly state verification, thereby generalizing the work of Townsend (1979) and Gale and Hellwig (1985). His results provide a theoretical rationale for seniority and absolute priority, and predict an ordering of monitoring activities among investors. These monitoring activities are reactions to financial distress and can therefore be interpreted as gradual bankruptcy provisions. Different from our work, in Winton (1995) the debtor borrows from several creditors by assumption.

The present paper is organized as follows. In Section 2, we describe the basic structure of the model. Section 3 discusses the benchmark case of perfect renegotiation. Section 4 develops the base case with two creditors, which is extended in Section 5. Section 5 provides a justification and more
detailed institutional analysis of the bankruptcy process. Section 6 discusses the structure of the assumed debt renegotiation. Section 7 concludes.

2 The Model

A firm can invest $I$ units of funds at date 0. The firm lives for two periods after that date. At date 1, the firm has assets in place worth $A$ which generate a cash flow $Y$. Asset value $A$ at date 1 is verifiable and deterministic, known to everybody in advance. Cash flow, $Y$, is observable, but not verifiable, and accrues to the firm’s management. The assumption that only $A$ is verifiable will play a crucial role in what follows, in particular, foreclosure on the firm’s property by the sheriff can only reach $A$, not $Y$. If the firm is not liquidated at date 1, final firm value $V_C$ is realized at date 2, where $V_C$ is a continuous random variable with cumulative density function $F(V_C)$ and support $[V_L, V_U]$. We assume that $F$ is differentiable on $(V_L, V_U)$ with density $f$, and will extend the definition of $F$ and $f$ to all of $[0, V_U]$ in the obvious way. In this paper we shall assume for simplicity that $V_C$ is non-verifiable (i.e. that management cannot credibly promise to transfer it to creditors at date 2); in ongoing related work we relax this assumption.

Short term cash flow $Y$ is realized at date 1, too, and given by

$$Y = \begin{cases} 0 & \text{with proba } 1 - q \\ Y_H & \text{with proba } q. \end{cases}$$

At date 0, the firm is run by a risk-neutral owner/manager who has no funds and raises them from external investors. Investors are risk-neutral and competitive. This implies that the firm has all the bargaining power at the financing stage, for simplicity we will assume that it has it as well at the refinancing stage. The firm is financed by $n \geq 1$ investors who each put up $I_i > 0, \sum I_i = I, i = 1, ..., n$. For simplicity, we will only compare the one and two creditor cases. Investors provide finance against the promise by management to repay $P_i, i = 1, ..., n, \text{at date } 1$. If an investor does not receive this payment at date 1, she has the right to foreclose on the firm’s assets or force the firm into bankruptcy, according to rules which we describe below.

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2Formally, we need to assume that the support of $V_C$ is $\{0\} \cup [V_L, V_U]$, where 0 has point mass 0. (so that management can always claim at date 2 that it has nothing to pay out).
From a contract-theoretic perspective, it is clear that investors’ debt collection rights must depend on the set of creditors who attempt to collect. Generally, therefore, investor \( i \) can collect the amount \( L_i(L) \), with \( i \in L \), where \( L \) is the set of all investors who want to collect. With only two outside investors, we can denote by \( D_i \leq A \) the amount investor \( i \) can foreclose if the other investor does not foreclose, and by \( C_i \) the amount if both investors want to foreclose. Obviously, the total liquidation value cannot exceed the asset value: \( C_1 + C_2 \leq A \).

Although the interactive nature of collection rights is theoretically obvious, it is less so in real world contracting, which points to an important role for bankruptcy. In fact, creditors’ collection rights under unilateral liquidation, \( D_i \), may not be consistent in the sense that their sum may exceed the value of the firm’s assets, \( A \), whereas the sum of individual collection rights when several creditors demand foreclosure can never exceed \( A \). Real-world debt contracts typically specify individual, non-interactive collection rights \( (D_i) \), which are usually governed by debt collection law, but leave much of the creditors’ rights when multilateral collection is attempted to bankruptcy law. An important practical reason for this is that “simultaneous foreclosure attempts” are practically very unlikely; there is always someone who acts first, even if others have the intention to take the same step. Therefore, the simultaneous move game which we introduce below to describe creditors’ actions is a simplification which collapses many possible sequential moves into one simultaneous move. In Section 5, we will refine the model to include the decision to trigger bankruptcy (which then creates an “automatic stay”); for now we simply assume that creditors act simultaneously and execute the \( L_i(L) \) specified in the initial contract.

Hence, we do not go into details of the bankruptcy procedure but rather describe bankruptcy as a form of “collective foreclosure”. At date 1, management is supposed to pay out \( P_i \) to the creditors. If it does not do so, it defaults and bargains over the repayment. Following the capital structure literature discussed in the Introduction, we assume that such bargaining is bilateral, i.e. that creditors are too dispersed to negotiate collectively and with one voice with the debtor. The bargaining either leads to payments, to (individual) foreclosure or to bankruptcy. Formally, we describe this sequence of events by the following extensive form game between management and creditors:

1. Nature determines \( Y \) and \( V_C \).
2. Management pays out \( r_i \in \{0, P_i\} \), \( i = 1, \ldots, n \).

3. If \( r_i = P_i \), \( i = 1, \ldots, n \), the game is over and management receives \( V_C \) at date 2.

4. If \( r_i = 0 \) for a subset of creditors \( i \in I \), management makes individual offers to each creditor \( i \in I \) to pay \( p_i \).

5. Creditors \( i \in I \) simultaneously choose to accept (\( a \)) or to (attempt to) foreclose (\( f \)).

Denoting by \( L \subseteq I \) the set of all creditors who do not accept \( p_i \), each creditor \( i \in L \) receives ("liquidates") \( L_i(L) \), and all other \( i \in I \) receive their \( p_i \). If \( L \leq A \) is the total amount of assets liquidated, the firm continues on the scale \((1 - L/A)\). This means that management obtains \((1 - L/A)V_C\) at date 2.

Note that this assumption on payouts amounts to assuming that long-term firm value is produced with constant returns to scale. Interest rates across periods are normalized to 0. In order to avoid having to consider several uninteresting cases later on, we impose some further restrictions on the parameters. First, we assume that management never wants to liquidate the firm voluntarily:

\[
V \geq A. \tag{1}
\]

Second, we assume that cash flows in the the good state are sufficiently high so as to avoid liquidity constraints in that state; specifically, we impose

\[
Y_H \geq 2A \tag{2}
\]

3 Benchmark: Perfect Renegotiation

The case of perfect renegotiation is very simple but will serve as a useful benchmark to analyze the case of bargaining frictions with two creditors. In particular, under perfect renegotiation there is no need to reconcile competing debt collection claims in case of default and therefore no need for bankruptcy.

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3We assume here (but it can be derived easily in a more general model) that default and repayment offers are never partial, so that liquidation is either full (\( L_i(L) \)) or zero.

4As for example in Hart and Moore (1998) and Harris and Raviv (1995).
rules as described in the last section. As renegotiation is perfect, we can
simplify the exposition by assuming that there is only one creditor.

As stated in Section 2, we assume that the firm cannot pledge future firm
value $V_C$ at date 1, because this value is non-verifiable. Then a debt contract
at date 0 just has to specify a repayment of $P$ (the “face value”) and the
alternative liquidation right $C$ (the “collateral”) for the creditor at date 1.

With probability $1-q$, firm cash flow is 0 and repayment is $r = 0$. In that
case, the assets are liquidated and the investor receives $C$. With probability
$q$ cash flow is $Y_H$. The firm then has the choice between paying out $P$, or
strategic default and making a payment offer to the creditor. The payment
offer $p$ must be at least equal to $C$ to be accepted by the creditor. If $p \geq C$,
the firm’s payoff is $Y_H - p + V_C$, whereas with a payment of 0 (which is the
best of all offers strictly smaller than $C$) the payoff is $Y_H + (1-C/A)V_C$. The
firm will therefore prefer to pay, and set this payment equal to $C$, whenever
$V_C - C \geq (1-C/A)V_C$, which is always satisfied by (1).

To make the contract renegotiation-proof in the good state, we can there-
fore set $P = C$. Hence, under such a contract the firm’s expected payoff will be

$$q(Y_H - C + EV_C) + (1 - q)(1 - C/A)EV_C$$
$$= qY_H + EV_C - (q + (1 - q)\frac{EV_C}{A})C$$

(3)

whereas the creditor will get $C$, either in cash or in asset value.

Since $C \leq A$, this shows immediately that credit is unavailable if $I > A$. If $I \leq A$, competition among creditors will drive $C$ down to $I$. Note
that because the coefficient in front of $C$ in (3) is strictly smaller than $-1$,
minimizing $L$ (subject to the investor’s participation constraint) is also the
efficient choice.

It is useful to summarize the above discussion in the following proposition:

**Proposition 1:** Under perfect renegotiation, there is lending if and only if
$I \leq A$. In this case, there is no strategic default on debt in equilibrium, and
the creditor gets $P = C = I$ either in cash or through liquidation.

Note that under perfect renegotiation there is no role for bankruptcy law
beyond verifying the value of the collateral and transferring it to the creditor.

### 4 Two Creditors: Optimal Contracts
We now take the same framework and assumptions as in the last section and assume two creditors instead of one. The case of two creditors is a simplified illustration of a structure in which creditors are separated and thus face coordination problems in bargaining with the firm in case of default. Debt renegotiation, therefore, is more difficult than in the case of one creditor, because bargaining must be done individually and not collectively. As noted above, each creditor \( i \) has unilateral foreclosure rights \( D_i \) and liquidation rights \( C_i \) when both demand foreclosure. To recall, unilateral foreclosure rights are the rights of a creditor if she decides to collect her debt unilaterally and the other creditor does not. Since only \( A \) can be seized in foreclosure or bankruptcy proceedings, we have \( D_i \leq A \) and \( C_1 + C_2 \leq A \).

Consider first the case in which the firm has nothing to pay out, \( Y = 0 \). Therefore, \( r_1 = r_2 = 0 \), and the creditors’ problem in stage 5 of the game played at date 1 is given by the following simple matrix game

\[
\begin{array}{c|cc}
 & f & a \\
\hline
f & 0, D_2 & 0, \infty \\
\uparrow & \downarrow & \downarrow \\
a & D_1, C_1 & C_1, C_2 \\
\end{array}
\]

Clearly, \((f, f)\) is always a Nash equilibrium of this game. It is unique if and only if \( C_1 > 0 \) and \( C_2 > 0 \), otherwise some of the other cells will be equilibria, too.

Now consider the case \( Y = Y_H \). How much management will pay out and to what extent it actually wants to prevent liquidation, depends on the parameters and on the equilibrium played by the creditors in the foreclosure game, which, in turn, is influenced by the firm’s payout offer \((p_1, p_2)\). The foreclosure game is given by the following modification of matrix (4):

\[
\begin{array}{c|cc}
 & f & a \\
\hline
f & D_1, D_2 & p_1, D_2 \\
\uparrow & \downarrow & \downarrow \\
a & p_1, p_2 & C_1, C_2 \\
\end{array}
\]

A necessary condition for \((a, a)\) to be an equilibrium (i.e. for no liquidation to take place), is that \( p_1 \geq D_1 \) and \( p_2 \geq D_2 \). Again, this equilibrium may not be unique for reasons of indifference or because the \( C_i \) are high relative to the \( p_i \). We will rule out the first (trivial) type of multiplicity by resolving indifferences in such a way that the ex-ante optimization problem has a solution, as is standard practice in agency theory. In the present context this means that creditors accept the firm’s payment whenever it is weakly greater
than their liquidation return. Under this assumption, \((a, a)\) is the unique equilibrium of (5) if and only if

\[
p_i \geq D_i, \quad i = 1, 2, \tag{6a}
\]

and

\[
p_1 \geq C_1 \text{ or } p_2 \geq C_2. \tag{7}
\]

Going back one stage in the bargaining game, which of the four cells of (5) does management want to induce when it moves at date 1?\(^5\) The continuation value for the firm’s managers is \(V_C\) as defined above. By assumption (1) and because the firm’s long-term production is constant returns to scale, management prefers paying out a given amount over keeping it in cash and having the same amount foreclosed (on average and at the margin). This immediately implies that management prefers the \((a, a)\) outcome with \(p_1 = D_1\) over \((f, a)\) and, symmetrically for the other off-diagonal. By assumption (2) and since \(D_i \leq A\), management can indeed make these payments.

Which of the two remaining cells does management prefer? Under the assumption that

\[
C_1 \leq D_1 \text{ or } C_2 \leq D_2 \tag{8}
\]

condition (7) is automatically satisfied if management sets the \(p_i\) to their lowest levels satisfying (7), \(p_i = D_i\).\(^6\) Then management’s maximum payoff with \((a, a)\) is

\[
Y_H - D_1 - D_2 + V_C. \tag{9}
\]

By the argument given above, it can achieve this payoff as a unique equilibrium outcome. Management’s payoff under \((f, f)\) is

\[
Y_H + (1 - \frac{C}{A})V_C, \tag{10}
\]

where \(C = C_1 + C_2\). Management can induce this outcome by setting \(p_1 = p_2 = 0\), if and only if \(C_1 > 0\) and \(C_2 > 0\). Comparing (9) and (10) shows that management prefers \((a, a)\) over \((f, f)\) if

\(^5\)Of course, at date 0 management prefers the \((a, a)\) outcome because liquidation is inefficient. But at date 1, its preferences are guided by the \(D_i, C_i\), and no longer by overall efficiency considerations.

\(^6\)It is easily verified that the optimal contract derived below satisfies (8).
\[
\frac{D_1 + D_2}{C} \leq \frac{V_C}{A}.
\]

In other words, management prefers to pay out if the continuation value for management \(V_C\) is higher than the threshold
\[
T = \frac{D_1 + D_2}{C_1 + C_2} A,
\]
and prefers bankruptcy otherwise.

We now turn to the contract design problem at date 0, which consists of choosing the equilibrium payments \(D_i\) (the face values) and equilibrium liquidations \(C_i\) (the collaterals). Assuming that \(C_1 > 0\) and \(C_2 > 0\) and (8) hold, the above analysis shows that management gets \((1 - C/A)V_C\) in the bad cash flow state and either \(Y_H - D_1 - D_2 + V_C\) (if (11) holds) or \(Y_H + (1 - \frac{C}{A})V_C\) (if (11) does not hold) in the good cash flow state. Letting \(D = D_1 + D_2\) and \(\theta = \text{Prob} \ (V_C \geq \frac{D}{C} A)\),

the firm’s expected surplus at date 0 is, therefore,
\[
S_0 = (1 - q)(1 - \frac{C}{A})EV_C + q\theta(Y_H - D + E[V_C \mid V_C \geq \frac{D}{C} A])
\]
\[
+ q(1 - \theta)(Y_H + (1 - \frac{C}{A})E[V_C \mid V_C < \frac{D}{C} A]).
\]

The investors’ participation constraints are
\[
(1 - q)C_i + q\theta D_i + q(1 - \theta)C_i \geq I_i, i = 1, 2,
\]
and the feasibility constraints
\[
0 \leq D_1, D_2, C_1, C_2, C \leq A
\]
must hold.

Summing (13), one gets
\[
(1 - q\theta)C + q\theta D \geq I.
\]

Hence, the contract optimization problem at date 0 can be reduced to
\[
\max_{D,C} \quad S_0 \quad \text{(16)} \\
\text{s.t.} \quad (1 - q\theta)C + q\theta D \geq I \quad \text{(17)} \\
0 \leq C \leq A \quad \text{(18)} \\
0 \leq D \leq 2A \quad \text{(19)}
\]

In order to solve this program, we can first simplify \(S_0\) to

\[
S_0 = qY_H + EV_C - (1 - q)\frac{EV_C}{A}C - q\theta D - q(1 - \theta)\frac{C}{A}E[V_C \mid V_C < \frac{D}{C}A] \quad \text{(20)}
\]

Note that (20) is very similar to the firm’s surplus in the one-creditor case, (3), the sole difference being that the efficiency loss in the good cash flow state is now split up into two terms, with probabilities \(\theta\) and \(1 - \theta\), respectively. By substituting \(D\) for \(T\) as defined in (12), dropping additive terms, and multiplying by \(-A\), problem (16)-(19) can be equivalently written as

\[
\min_{T,C} \quad [(1 - q)EV_C + q\theta T + q(1 - \theta)E[V_C \mid V_C < T]]C \quad \text{(21)} \\
\text{s.t.} \quad (A + q\theta(T - A))C \geq IA \quad \text{(22)} \\
0 \leq C \leq A \quad \text{(23)} \\
0 \leq TC \leq 2A^2 \quad \text{(24)}
\]

As this program clearly shows, the investors’ participation constraint is binding (this is obvious a priori: there is no need to leave a rent to investors ex ante). Using this to eliminate \(C\) and remembering that \(\theta = 1 - F(T)\), our problem finally becomes

\[
\min_{T} \quad \frac{(1 - q)EV_C + qT(1 - F(T)) + qF(T)E[V_C \mid V_C < T]}{A + q(T - A)(1 - F(T))} \quad \text{(25)} \\
\text{s.t.} \quad A + q(T - A)(1 - F(T)) \geq I \quad \text{(26)} \\
T \geq 0 \quad \text{(27)} \\
\frac{2A}{T} (A + q(T - A)(1 - F(T))) \geq I \quad \text{(28)}
\]

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Here, (26) is the upper constraint on $C$ (the second inequality of (18)), (27) is the lower constraint on $D$ (the first inequality in (19)), and (28) is the upper constraint on $D$ (the second inequality in (19)).

The two programs (21)-(24) and (25)-(28) are equivalent: the solutions $(T^*, C^*)$ of the former yield solutions $T^*$ of the latter and vice versa by using the participation constraint

$$(A + q(1 - F(T))(T - A))C = IA.$$  

Program (25)-(28) is highly non-linear, but it has the advantage of being a problem in only one (real) variable. On the other hand, program (21)-(24) is two-dimensional, but offers more economic insights. For the formal characterisation of the solutions and their existence the one-dimensional problem is simpler to analyse.

**Proposition 2 (Debt Capacity):** Problem (25)-(28) has a solution if

$$(1 + q - qF(2A))A \geq I. \quad (29)$$

**Proof:** Figure 1 depicts the left-hand sides of (26) and (28) as functions of $T$ which intersect at $T = 2A$. By (29), this intersection point lies above $I$, hence the constraint set of problem (25)-(28) is not empty. It clearly is compact and the objective function is continuous.\[\Box\]

Condition (29) is sufficient for existence but not necessary, as variants of Figure 1 easily show. Therefore, the left-hand side of (29) is a lower bound for the firm’s debt capacity (the largest expected gross return the firm can credibly pledge to two creditors under any contract). An investment below this threshold can be financed, any amount above it may or may not be financed, depending on the distribution of long-term returns, $F$.

Condition (29) is intuitive in several respects. Going back to the two-dimensional problem (21)-(24), the participation constraint (22) shows that the firm’s debt capacity is given by

$$\max_{T,C} (1 + q(1 - F(T))(T - A)(1 - 1))C. \quad (30)$$

The derivative of (30) with respect to $T$ is

$$(1 - F(T) - (T - A)f(T))Cq/A, \quad (31)$$

which is strictly positive for $T < V$. Because the maximand in (30) is strictly increasing in $C$, $(1 + q)A$ is an upper bound for the firm’s debt
capacity: the creditors can never get more than all assets in the bad state and cash of double that value in the good state. Condition (29) shows that this reasoning actually gives the exact debt capacity if $V \geq 2A$. In this case, by (31), increasing $C$ and $T$ all the way up to their maximum values ($A$ and $2A$, respectively) maximises the investors’ returns in (30). On the other hand, if $V < 2A$, the debt capacity cannot be as high as $(1 + q)A$ because for $T = 2A$ the incentive for strategic default for low values of $V_C$ provides a countervailing effect. In fact, in this case, if $I$ is sufficiently large (but still smaller than $(1 + q)A$) and the distribution of $V_C$ sufficiently concentrated on $[V, 2A]$, a solution to problem (25)-(28) may not exist. Then the incentive to default strategically can be so strong that investments $I > A$ cannot be financed. However, under the conditions of Proposition 2, if $V > 2A$, debt capacity is strictly greater than $I$, and some investments that could not be

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Figure 1 shows how to construct such examples. If $V < A(1 + q) - I$ (as in Figure 1), i.e. $I > A + q( V - A)$ and $F$ has sufficient mass to the left of $2A$, then the curve of $(1 + q - qF(2A))A$ will never rise above $I$. In particular, this happens if this graph peaks at $T = \bar{V}$, which will be true if $f(T) > \frac{1}{2}$ for all $T \in [V, \bar{V}]$. 

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financed with one creditor will be financed with two.

Here, the ability of the debtor to pledge his assets to each individual creditor leads to a strictly higher debt capacity than in the one creditor case. This implies in turn that when the cost of capital is higher than the value of the assets, a project will be financed only with multiple creditors and not with a single creditor because of the higher leverage provided by multiple creditors.

In fact, a simple consequence of the investors’ participation constraint (15) and Proposition 2 is that the firm will be over-leveraged with respect to its asset base whenever there is finance for projects with \( I > A \).

**Corollary (Over-Leverage):** Whenever \( I > A \) and the project is financed, the face value of debt exceeds the firm’s asset value: \( D > A \).

A further inspection of the two-dimensional problem (21)-(24) along the previous lines shows that for given \( T \), minimising bankruptcy liquidation \( C \) is desirable. However, the participation constraint (22) shows that this may come at a cost in terms of \( T \). As shown before, the derivative of the left hand side of (22) with respect to \( T \) is essentially (up to a factor \( A \)) given by (31), which is typically not monotonic in \( T \) (the countervailing effect comes from the fact that lowering \( C \) worsens the firm’s incentive to repay in the good state). However, if \( T \leq V \) the effect is unambiguous. In this case we have \( \theta = 1 \), because unilateral collection rights \( D \) (which determine payout in the good state) are sufficiently small relative to bankruptcy liquidation rights \( C \) for management never to default strategically. Therefore, the objective function of the one-dimensional problem (25) simplifies to

\[
\frac{(1 - q)EV_C + qT}{A + q(T - A)},
\]

which is strictly decreasing in \( T \) for all \( T > 0 \). In the one-dimensional problem, the parties therefore want to choose \( T \) maximally in \([0, V]\). Whether they can do so, depends on the constraints (26) and (28). An inspection of these constraints shows that (26) is slack for \( T > 2A \) and (28) for \( T < 2A \). If \( I < A \), the upper constraint on \( C \), (26), never binds; investment requirements are so low that it is always possible to satisfy the investors by liquidating less than \( A \) in bankruptcy. If \( I > A \), the constraint restricts the choice of \( T \); in fact, as Figure 1 shows, no \( T < A + (I - A)/q \) satisfies it. This implies that if \( V < A + (I - A)/q \), any solution to the contracting problem must have \( T > V \), i.e. feature strategic default with some probability.
If $2A \geq V \geq A + (I - A)/q$, the choice of $T = V$ is feasible, and its optimality depends on the right-hand derivative of the objective function of problem (25) at $T = V$ (the left-hand derivative is given by (32) and is negative, but $F$ and therefore the objective is not differentiable at $T = V$). Straightforward calculation shows that this right-hand derivative is given by

$$f(V)(V - A)((1 - q)EV_C + V) - (1 - q)(EV_C - A)$$

$$\frac{(A + q(V - A))^2}{(A + q(V - A))^2}$$ (33)

If this value is negative (which is the case if $f(V)$ is sufficiently small), it is optimal to increase $T$ beyond $V$, which again means to induce strategic default with some probability.

If $V > 2A$, the constraint on $D$, (28), becomes relevant and a similar argument applies. Going through the analysis yields the following result.

**Proposition 3 (Strategic Default):** Assume (29). Then there is no strategic default under the optimal two-creditor contract if $I > 2qA$ and $V \geq \frac{2(1 - q)A^2}{I - 2qA}$. There is always strategic default with some probability if $V < A + (I - A)/q$. In all other cases there is strategic default with positive probability if the right derivative of (25) at $T = V$, (33), is strictly negative.

**Proof:** If $I > 2qA$ and $V \geq \frac{2(1 - q)A^2}{I - 2qA}$, constraint (28) binds at the optimum and the solution is given by $T^* = \frac{2(1 - q)A^2}{I - 2qA}$ (see Figure 1). The other cases are straightforward.

Finally, it is easy to show that the constraint on $C$, (26), never binds at the optimum except for the case where the debtor must pledge the full debt capacity. Hence, the debtor retains some of the assets even after bankruptcy. This is due to the fact that continuation in the present model is always efficient and that there are constant returns to scale. Therefore, the parties have a strong ex ante incentive not to punish the debtor too hard in case of bankruptcy. This result will be modified when continuation can be inefficient or when returns to scale are not constant. However, the basic message remains valid: if the debtor has a comparative advantage using the assets, it is ex ante costly to separate her from them ex post, and, therefore, an optimal contract will aim at reducing this incidence as much as possible. This insight, simple as it is, is in sharp contrast with traditional legal reasoning that demands to satisfy creditors first in case of bankruptcy.
Proposition 4 (Deviation from Absolute Priority): If $I < (1 + q - qF(2A))A$, then the firm is not fully liquidated in bankruptcy: $C < A$.

Proof: The objective (25) is strictly decreasing for any value $T$ at which (26) binds. ■

To summarize, we have shown in this section that having multiple creditors allows to enhance the debt capacity of a firm by overleveraging its assets. This induces strategic default in equilibrium above a threshold level $T$ of $V_C$ where the debtor trades off the benefit of continuing the firm with the cost of paying out the creditors. The optimal debt contract will typically not involve full liquidation in case of default in order to optimally trade off a reduced inefficiency of liquidation after a liquidity default with an increased incentive for strategic default.

It is useful at this stage to compare our results with those of Bolton and Scharfstein (1996). They also compare optimal debt contracts with one and multiple creditors in an incomplete-contract framework. In their model, having multiple creditors reduces the incentive for strategic default whereas in our model, it increases the incentives for strategic default. The mechanisms are the following. In their model, when there are multiple creditors, complementary assets are collateralized to different creditors and there is no overleveraging possible as in our model. Because of asset complementarity, following a default creditors can jointly get a higher price when selling them to an outside buyer. This implies that the debtor must pay creditors a higher price to prevent them from choosing liquidation. This in turn dampens the incentives for strategic default. The price creditors get under liquidation thus varies endogenously with the number of creditors. In our model, the liquidation value of the firm is fixed but it is the equilibrium debt repayments that vary with the number of creditors. Having multiple creditors, each holding individual foreclosure rights, is a mechanism to credibly commit to higher repayments compared to the single creditor case. These higher repayments allow to increase the debt capacity, but also increase at the margin the incentives for strategic default and lead to observe strategic default in equilibrium, a phenomenon absent in the Bolton-Scharfstein (1996) model.
5 Bankruptcy, Debt Collection and Priority

In the base model of Section 4 we have assumed that a simultaneous attempt by creditors to collect their debt automatically triggers bankruptcy. In reality, of course, bankruptcy must be triggered by someone, and the base model is silent on this issue. In this section we generalize the base model to a model in which bankruptcy is not an automatic consequence of simultaneous debt collection, but the result of a deliberate decision by a creditor. This will at the same time provide a new rationale for bankruptcy and shed light on the role of seniority.

In order to define the more general model we must first redefine and reinterpret the model of Section 4. The main change to that model is the definition and interpretation of simultaneous foreclosure in the debt collection games (4) and (5). Instead of the collateral liquidations $C_i$ assumed in the case of $(f, f)$ there, we will now assume that simultaneous foreclosure results in an uncoordinated run for the assets, in which the first to collect his debt liquidates $D_i$, and the second the rest, $A - D_i$. Assuming that each creditor has the same probability of being first and that $D_1 + D_2 \geq A$, the payoff matrix becomes

\[ \begin{array}{c|c|c}
 & a & f \\
 a & p_1, p_2 & p_1, D_2 \\
 f & D_1, p_2 & \frac{1}{2}(A + D_1 - D_2), \frac{1}{2}(A + D_2 - D_1) \\
\end{array} \]

(34)

where $p_1 = p_2 = 0$ in the case of $Y = 0$.

In this framework, which represents a “pre-bankruptcy”, primitive state, ex-post interactions and ex-ante contracting will be as in Section 4, with the exception that $C = A$ is fixed exogenously. This simplifies the original program (21)-(24) considerably, but the resulting contract will not be optimal, as shown in Proposition 4. The reason is that the deadweight loss through complete liquidation more than outweighs the improved incentives for payout in the good state. Ex ante it is therefore optimal to reduce the threat of liquidation from $A$ to $C$.

This can be done by introducing bankruptcy into the model; and more precisely, by giving each creditor the right to trigger bankruptcy when the firm defaults on its payment towards him. Bankruptcy then means that individual debt collection is no longer allowed, and creditors receive $C_i$ instead of $D_i$. 

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In the good cash flow state, this rule will not change the firm’s payments, because these are determined by the individual claims $D_i$. In particular, a creditor who observes that another creditor pursues the firm will not trigger bankruptcy as long as $V_C \geq T^*$, for he knows that the other creditor will not liquidate in equilibrium. Yet, the rule will change the payout behaviour in the good state, because the firm knows that when defaulting it will get away with a liquidation of $C$ instead of $A$. In this case, and equally in the bad state, creditor $i$ observing the attempt to foreclose by creditor $j$ will call bankruptcy if this makes him better off than waiting, i.e. if

$$C_i > A - D_j.$$  \hfill (35)

Adding up (35) for $i = 1, 2$ yields the joint condition $C + D > 2A$, which may be satisfied by the solution to problem (21)-(24), $D^*$ and $C^*$, but need not. If $D^*$ and $C^*$ satisfy the joint condition, then an optimal contract with bankruptcy can rule out a run for the assets by either creditor by setting each creditor’s $C_i$ sufficiently high. If the joint condition is not satisfied, this will not be possible for both creditors, and at least one creditor will have an incentive to run for the assets even in the presence of a bankruptcy rule. In this case, a further contractual remedy is needed, which can be obtained by making one creditor senior.

Seniority means that the individual claim of the senior creditor has precedence over the other claim in bankruptcy. In this case, $C_i = \min(D_i, C^*)$ for the senior claim, and $C_j = C^* - C_i$ for the junior. This arrangement has the advantage of being compatible with the optimum values $D^*$ and $C^*$ derived in Section 4, but it does not necessarily eliminate runs for the assets. In fact, if $D_i > C^*$ even the senior creditor gets less in bankruptcy than when running for the assets. Yet, this flaw can easily be remedied by choosing $D_i \leq C^*$ (remember that the analysis in Section 4 has fixed only the aggregate liquidation values). Now the senior creditor has no incentive to run for his assets, not even when he observes the attempt to foreclose by the other creditor; he simply calls bankruptcy. In fact, this is what is observed in reality: if bankruptcy is triggered by a creditor, it is typically triggered by a senior creditor.\footnote{There is one small caveat to this reasoning: should the senior creditor try to foreclose individually (for which he has no strict incentive), then the junior creditor $j$ has no incentives to call bankruptcy, because $C_j < A - D_i = A - C_i$ (liquidating individually after the senior creditor is more profitable). But since this issue arises only out of indifference,}
The above discussion can be summed up in the following proposition:

**Proposition 5 (Seniority):** If $C^* + D^* > 2A$, the liquidation values $C^*$ and $D^*$ can be implemented by giving each creditor the individual right to trigger bankruptcy following default. If $C^* + D^* \leq 2A$, this arrangement must be augmented by making one creditor senior and fully collateralizing his claim: $D_i = C_i$.

### 6 Individual and collective renegotiation

The results on higher debt capacity under multiple creditors derived in Section 4 depend on the fact that creditors have unilateral foreclosure rights that they can exercise in case of default, independently of what other creditors decide. These rights should be seen as an important element of investor protection. The renegotiation procedure modeled in Section 4 emphasises the effect of these rights since renegotiation was assumed to be done on an individual basis. The ensuing prisoner’s dilemma situation forces the debtor to respect contractual claims as given by individual foreclosure rights whenever he wants to avoid strategic default. The key assumption in this approach is that creditors are too dispersed to renegotiate the debt contract collectively. Only bankruptcy brings all the contracting parties again together at one table and has the important function of reconciling their liquidation claims. This is the classical “vis attractiva” of bankruptcy.

Yet, it is theoretically conceivable and possible in practice that the debtor can unite the group of creditors, or their representatives, and extract from them joint concessions under the threat of bankruptcy. If such workouts are frictionless, the theory presented in Sections 4 and 5 collapses into the one-creditor-case discussed in Section 3. More generally, for any theory in which multiple creditors have a disciplining function for the debtor, frictionless all-inclusive negotiations in the shadow of bankruptcy present a conceptual problem. In our view, however, frictions in such negotiations can be substantial, and, in particular, increase with the number of creditors. One classical reason for these frictions is, of course, the hold up problem of the individual creditor, which is precisely the reason for institutionalised bankruptcy rules as discussed in Section 5. Another reason is the legal uncertainty ac-
companying out-of-court debt renegotiations, if individual creditors have the possibility of contesting the new arrangement in court.

How likely are courts to effectively uphold individual claims from the earlier contract? It turns out that there is no simple answer to this question, and the answer appears to change over time, at least in the legal tradition of the United States. Coercive offers have been a concern of courts for centuries. In the common law tradition the problem goes back at least to the old contract law doctrine of Foakes v. Beer, 9 App.Cas. 605 (1884). In this case a creditor had accepted a delayed payment on an instalment basis but then charged interest. The court ruled that the creditor could repudiate such an agreement if it was not supported by “consideration” (i.e., some concession from the debtor). Courts have since tried to establish what constitutes consideration. The problem has been that almost anything, e.g., if the debtor offered to pay the lower amount slightly earlier than originally agreed, could count as consideration.

Courts have also over time become concerned that hand-tying may prevent mutually beneficial renegotiation. A complete ban on renegotiation would, of course, rule out workouts, or composition, outside bankruptcy altogether. Consequently, courts in the United States have been increasingly reluctant to interfere with the freedom of contract and found consideration in the agreement of more than one creditor to accept an offer. Repudiation is now primarily limited to situations where the creditor can show that she was under duress. An example would be if the creditor signed with a gun pointed at her head or were subject to some other threat where the creditor did not have remedy. A threat to file for bankruptcy would hardly qualify (unless for the unlikely event that it could be shown that the debtor had no intention of actually filing). In fact, it seems from our reading that courts are more concerned with coercion of debtors, rather than creditors, in duress.

Courts have nevertheless tried to safeguard against coercive offers in other ways. For example, they have allowed repudiation from creditors who did not participate in the agreement. This measure was used against equity receivership, the 19th Century precursor to Chapter 11. The Trust Indenture Act of 1939 also required unanimity among affected creditors for exchange offers to be accepted (Roe, 1987). When creditors are heterogenous and widely dispersed, and not necessarily entirely predictable, obtaining consensus may

\[\text{See Katz v. Oak Industries and Kass v. Eastern Airlines for cases discussing coercion against creditors (Roe, 2000).}\]
be very difficult to obtain. Another possible ground for repudiation would be fraudulent conveyances, i.e., transfers of assets away from creditors or to one creditor at the expense of others. However, fraudulent conveyances require that the debtor “delay, hinder or defraud” a creditor. Paying one creditor ahead of another would normally not suffice if the transaction was transparent.

To summarize this discussion, the possibility of joint renegotiations raises important concerns about the value of individual foreclosure rights in allowing debtors to commit ex ante. Courts face a difficult tradeoff between respecting such rights and allowing mutually beneficial renegotiation. However, the greater the number of creditors, the more difficult are such renegotiations to achieve in the first place, and the less likely they are to go uncontested.

7 Conclusion

We have analyzed the role of bankruptcy law in an incomplete-contracts perspective where continuation of a defaulting firm is ex post efficient. If cash flow is not verifiable and only the collateral value of the firm is verifiable, then when a firm borrows from a single creditor and has all bargaining power, its debt capacity is limited to the value of its collateral. The reason is that the creditor can never expect to receive more than the collateral value in liquidation and in renegotiation. However, when a firm borrows from more than one creditor, it can increase its debt capacity by pledging its collateral value to more than one creditor by giving each the right to foreclose on its verifiable assets. This creates a commitment for the firm to pay out more in good states to prevent the exercise of individual foreclosure rights and thus helps in raising the firm’s debt capacity. Having multiple creditors thus helps to reduce the negative effects of contractual incompleteness by distinguishing between individual foreclosure rights and joint liquidation rights achieved under bankruptcy. A bankruptcy rule is necessary in order to make individual claims consistent in case of default and to prevent value reducing runs for the assets in case of default. Furthermore, depending on the parameters, it may be necessary to make one creditor senior.

In our model, all these results are derived as parts of an optimal ex-ante contract between debtor and creditors. Formally, there is, therefore, no need for a law. In practice, however, there may well be, if individuals are unable to join and write contracts specifying procedures of collective behavior. In
fact, this is the classical Rawlsian justification of legislation as a substitute for contracting in the “original position” (Rawls, 1971), an approach to law, and bankruptcy law in particular, that is wide-spread in legal thinking. The classic text of Jackson (1986), for example, when exploring the foundations of bankruptcy law, only argues that a “collective system of debt collection law” is needed, relegating the issue of private contracting to a footnote. Conceptually, our approach to the foundations of bankruptcy law does not go beyond this, we only make the hypothetical private contract explicit.

Further research is necessary to better characterize the effect of different renegotiation procedures, the role of courts in intervening in private contracts, and several other issues. The model in this paper can, however, be used to deepen our understanding of various bankruptcy laws both from the perspective of ex post and ex ante efficiency. Ultimately, this may contribute to a comprehensive comparative analysis of the effects of various bankruptcy laws across countries and across time.

10 “As such, it reflects the kind of contract that creditors would agree to if they were able to negotiate with each other before extending credit” (Jackson, 1986, p. 17).
8 References


