Downstream Concentration and Producer’s Capacity Choice*

João Vieira-Montez‡

August 2004

Abstract

This paper studies how buyers’ integration affects the capacity choice of a producer. Contrary to «conventional wisdom», we show that, under natural assumptions, integration may lead to a higher equilibrium supply level. Our result hinges on the following trade-off: for any given level of capacity, the share of the total surplus accruing to the producer is lower when concentration is high, i.e. the hold-up is more severe. Yet, this share decreases when capacity increases. This reduces the incentives to increase capacity. The rate at which this occurs is higher when concentration is low. The second effect counteracts, and may dominate, the first. When the cost of capacity is low the equilibrium supply level is always higher when downstream concentration is high.

JEL classification numbers: L10, L40.
Keywords: Buyer Integration, Capacity Choice, Hold-up.

*I am particularly thankful to Thomas von Ungern-Sternberg and Dezső Szalay for comments and discussions. I am also thankful to the participants of the EEA/ESEM 2004 and the 2004 conference of the European Association of Research in Industrial Economics. The support of a doctoral grant from Fundação para a Ciência e a Tecnologia is gratefully acknowledged.

‡Département d’Econométrie et d’Economie Politique, Université de Lausanne, Ecole des HEC, CH-1015 Lausanne, Switzerland; e-mail address: jviejram@unil.ch.
"... as one might expect, if two competing traders merge, this will worsen the (investment) incentives of the owner-manager of a firm that trades with them."\(^1\)

1 Introduction

A hold-up may arise when parties have to make investments which create more value inside a relationship than outside. Facing the impossibility to write complete contingent contracts, each party anticipates that some of the benefits from its investment will be dissipated in future bargaining. The fear of expropriation leads to inefficient investment.

Bargaining power, and consequently investment incentives, are determined by the asset ownership structure. Distinct asset ownerships thus create distinct investment incentives. These principles, introduced in Grossman–Hart (1986), Hart–Moore (1990) and Hart (1995), have been successfully applied to a large number of specific economic environments.

In this paper, we use these principles to study the following question: \textit{How does the integration of buyers of a good affect the capacity choice of the monopolist who supplies it?}

For ease of exposition we study this question in a particular setting, where a monopolist producer sells his output to a retail or manufacturing sector with varying degrees of concentration\(^2\).

Conventional wisdom seems to suggest that equilibrium production capacity will be higher when downstream concentration is low. It seems intuitive that, when concentration is high, the producer will be more exposed to expropriation, i.e. the "hold-up" is more significant.

Downstream concentration/integration may thus raise concerns of long term efficiency. In recent years, this concern has been put forward by competition authorities. Examples include the retail sector (Rewe/Meinl, Opera, Kesko/Tuko)\(^3\), the film and cable industry (Syufy, Cable Act 92)\(^4\) and, more

\(^1\)Hart–Moore (1990) p. 1148.

\(^2\)We could have alternatively considered a producer who sells to final consumers or a firm purchasing labour-complementary capital. In the former case, integration would be interpreted as the formation of consumer cooperatives. In the latter, as union formation.

\(^3\)Rewe/Meinl (EC Case No. M.1221), Opera (Conseil de la Concurrence Decision No. 3-D-11), Kesko/Tuko (EC Case No. M.784)

\(^4\)US vs. Syufy, Cable Act 92 see Congress of the United States (1992).
generally, in the European Commission’s guidelines on purchasing agreements\(^5\).

Surprisingly, the economics literature contains little formal analysis of the effect of downstream horizontal integration on producers’ investment incentives. We briefly summarize two strands of the property rights literature that can help us in answering our question.

A first topic of the literature is that asset ownership structures that leave a party in a better bargaining position (measured by it’s share of the bargaining surplus), by protecting him against expropriation, boost his investment incentives. The equilibrium investment level of this party will therefore increase (e.g. Grossman–Hart (1986), Hart–Moore (1990) and Chiu (1998)).

Hart–Moore (1990) studies similar issues to the one we address here. Based on their results, the authors argue that ”...as one might expect, if two competing traders merge, this will worsen the (investment) incentives of the owner-manager of a firm that trades with them”.

This first topic seems to support what we named conventional wisdom. In fact, for any given capacity level, we expect a negative relation between the share of surplus accruing to the producer and retail concentration (see Segal (2003) and references therein). We may thus expect retail integration to decrease the producer’s investment incentives. However, this is only part of the story.

A second topic of this literature concerns the choice of types of investment rather than the level of investment. A well known idea is that some types of technologies leave the producer in a stronger bargaining position, since they make other parties less indispensable to value creation (e.g. Holmstrom–Tirole (1991) and Segal–Whinston (2000)). This creates a strategic bias in technology choice.

A few papers study this technological bias in settings related to ours. Stole–Zwiebel (1996a,b) find that unionization and technology choices are related to one another. A firm dealing with a union has higher incentives to choose frontloaded technologies (i.e. technologies which generate a high proportion of the gains at high production levels) than a firm dealing with independent workers.

Any single worker is indispensable only to produce the last units of output. A more frontloaded technology leaves each single worker more indispensable since it’s marginal contribution to the total surplus will be higher.

\(^5\)Guidelines on the applicability of the Article 81 (2001). Purchasing agreements are expected to produce a similar effect since downstream firms coordinate their purchasing decisions.
When bargaining with independent workers, by choosing a less front-loaded technology the producer leverages its bargaining position since this makes each single worker more dispensable. When dealing with a union this strategic incentive is absent: a union is, by definition, always indispensable. Unionization may thus promote technological efficiency.

Retail integration has a similar effect on the technology choice of an upstream firm. A more concentrated retail sector provides incentives for an upstream firm to choose less frontloaded technologies (Inderst–Wey (2002)). The authors show, with some applications, that retail concentration may increase the supplier’s incentives for product and process innovation. Inderst–Wey (2003) shows how some of these results extend to a bilateral duopoly framework.

This leaves an open question: if the extent to which downstream firms are indispensable to the industry is affected by the capacity level\(^6\), our conventional wisdom may be missing an important element of the problem.

In this paper we show that, under "natural assumptions" on market revenue functions, each downstream firm becomes more indispensable as capacity increases.

Increasing capacity raises the total bargaining surplus but, because each firm becomes more indispensable, it will also erode the share of the bargaining surplus accruing to the producer (the measure of bargaining power). It thus aggravates the "hold-up". This captures the fact that bargaining power results not only of the control over a resource, but also from its relative scarcity. This "bargaining erosion" creates an additional strategic concern.

Moreover, at least at high capacity levels, the rate at which this "bargaining erosion" occurs is lower when retail concentration is high. Strategic concerns to choose a low capacity are therefore weak when retail concentration is high.

Concentration has thus two effects on producer’s incentives: it increases the "hold-up" effect but it softens the "bargaining erosion" effect.

This paper studies the relative importance of these two effects. Our main result is that, the second effect may dominate the first i.e. a higher downstream concentration does not unambiguously decrease the producer’s marginal returns to capacity. Equilibrium capacity level may well increase with retail concentration.

In our model, this will always be the case if the marginal cost of capacity is low. Only when the marginal cost of installing capacity is high will the "hold-up" effect dominate and the "conventional wisdom" go through.

\(^6\)This possibility is ruled out by assumption in Hart–Moore (1990).
At a practical level, these results should be useful to merger analyses. This work shows that signing the effect of retail concentration on the producers’ investment incentives is not as straightforward as it may seem and provides some intuition to which types of investment one should be most concerned with.

At a more theoretical level, these results provide some theoretical support to Galbraith’s (1952) theory of Countervailing Power. One of its main ideas is that, when competition fails on both sides of the market, allocative efficiency may be nurtured not by increased competition but by a process of concentration on the most competitive side. Consumers may benefit from retail concentration since prices should fall when output is increased.

Since the specific allocation of the bargaining surplus depends on the chosen solution concept, this choice is a crucial step. We chose to use the Shapley value. Since it has both axiomatic and non-cooperative foundations, the Shapley value has provided a useful benchmark for the literature\(^7\). Other solution concepts produce similar results\(^8\).

The remainder of the paper is organized as follows. Section 2 illustrates the main ideas of this paper with a simple example. The model is presented in section 3 and the analysis carried out in section 4. Section 5 takes a preliminary look at other solution concepts, and verifies that the results do not hinge on specificities of the Shapley value. In section 6, we discuss the results and their implications. Proofs are found in the appendix.

2 A simple example

Suppose a producer can, at date 0, chose capacity \(Q\) to produce either one or two units of some tradable good. The cost of a unit of capacity is 2.5, marginal costs of production are zero. There are two markets with a retailer in each market, \(i\) and \(j\). Date 1 demand is such that, in each market one unit can be sold for 6 but there is no demand for a second unit.

Suppose in addition that no long-term contracts can be signed at date 0. The producer’s profits are thus date 1 gains minus date 0 costs. The discount rate is assumed to be zero. Date 1 gains from trade will be split

\(^7\)Papers dealing with the effects of integration have often used this solution concept (e.g. Hart–Moore (1990), Stole–Zwiebel (1996a), Inderst–Wey (2003) and Segal (2003)). Noncooperative games that implement the Shapley value can be found e.g. in Gil (1989), Stole–Zwiebel (1996) and Hart–Mas-Colell (1996). For the axiomatization see e.g. Osborne–Rubinstein (1994).

\(^8\)See section 5 below.
according to each agent’s date 1 Shapley value $Sh_p(Q)$. So the producer maximizes date 1 revenue minus date 0 costs, i.e.:

$$\max_{Q \in \{1, 2\}} Sh_p(Q) - C(Q)$$

To build the intuition driving the result, we write the producer’s Shapley value as a share $\alpha(Q)$ of the industry surplus $V(Q)$, i.e. $Sh_p = \alpha \cdot V$.

Suppose the producer has chosen a capacity of one unit at date 0. Then his date 1 revenue is $Sh_p(1) = \frac{2}{3} \cdot 6$. Suppose instead the producer has chosen a capacity of two units. Then, when bargaining with retailer $i$, the threat of excluding him and sell all capacity to retailer $j$ has no value: $j$ has no use for a second unit. This weakens the producer’s bargaining position, so the share of the ex-post surplus accruing to the producer decreases. His date 1 revenue is $Sh_p(2) = \frac{1}{2} \cdot 12$. The producer’s revenue differential is:

$$\frac{\Delta Sh_p}{\Delta Q} = \alpha(1) \cdot \frac{\Delta V}{\Delta Q} + \Delta \alpha \cdot V(1) = \frac{2}{3}(12 - 6) - \frac{1}{3} \cdot 6 = 2$$

Although a capacity of two units would be efficient, ex-ante the producer chooses a capacity of one unit since the revenue differential is lower than the cost ($2 < 2.5$).

Imagine now that the two retailers merge, forming a retail monopoly. How would the producer’s capacity choice change?

Had the producer chosen a capacity of one unit his revenue would be $Sh_p'(1) = \frac{1}{2} \cdot 6$. If he chose a capacity of two units his revenue would be $Sh_p'(2) = \frac{1}{2} \cdot 12$. Since both the producer and the retailer are essential, each of them always gets half the gains from trade. In this situation, his profit maximizing capacity choice is 2 units, since the revenue differential is higher than the cost, i.e.:

$$\frac{\Delta Sh_p'}{\Delta Q} = \frac{1}{2}(12 - 6) = 3 > 2.5$$

In this particular example, retailer integration thus leads to a higher equilibrium capacity. The example is extreme since retail outlets and the tradable good exhibit perfect complementarity. However it captures the essence of the problem.

The share of date 1 surplus accruing to the producer is (weakly) higher before integration, i.e. the “hold-up” is weaker. If the producer’s share of the surplus were to remain constant while capacity increases, producer’s incentives would be unambiguously higher when facing two independent retailers.
What counteracts this effect is that the capacity choice also determines the strength of the producer’s threat to exclude a retailer, and thus the producer’s bargaining position.

Before integration, the share of surplus decreases as capacity increases \((\frac{2}{3} \text{ with 1 unit, } \frac{1}{2} \text{ with 2})\), thus increasing capacity aggravates the "hold-up". This creates a strategic incentive to keep capacity low in order to avoid a significant bargaining power erosion. We call this the "bargaining erosion" effect.

The difference in incremental revenue is:

\[
\frac{\Delta Sh_0}{\Delta Q} - \frac{\Delta Sh_p}{\Delta Q} = -\left(\frac{2}{3} - \frac{1}{2}\right)(12 - 6) + \frac{1}{3} \cdot 6 = 1 > 0
\]

The left hand side captures the increase in the "hold-up" due to integration. The right hand side of the expression the softening of the "bargaining erosion". The effect of concentration on the equilibrium capacity is determined by the interaction of these two effects. In this particular example, the second effect dominates the first one. Below, we establish these results in a much more general setting.

### 3 The Model

#### 3.1 Setup

We consider a vertical industry comprising \(n\) retailers (or manufacturing firms) and one producer. Denote the set of agents by \(\mathcal{A} = \{0, 1, \ldots, n\}\) where agent 0 is the producer. There are \(k \geq n\) identical "island markets" with one outlet in each market. Denote the set of all outlets by \(K = \{1, 2, \ldots, k\}\).

Each identical outlet generates a net revenue (gains from trade) of \(R(q)\), where \(q\) is the quantity allocated to that particular outlet. In the spirit of Hart–Moore (1990), this net revenue function may account for the possibility of retailers having access to an alternative, less valuable, competitive supply source and for any additional cost of transforming the intermediate good into a final good.

We assume that \(R(0) = 0\); is twice differentiable with \(R'(q) > 0\) and \(R''(q) < 0\) for \(0 < q < \bar{q}\); and\(^9\) \(R(q) = R(\bar{q})\) for \(q \geq \bar{q}\).

Note that each single market revenue is independent of the quantities allocated to the other markets. This might be tough of as having consumers

\(^9\)A critical assumption is that the revenue function is upper bounded. If this condition fails, we can no longer guarantee that all the results will hold.
purchasing the final product only in their "island market", i.e. retailers competing on the intermediate market but not on the final market.

Although this modelling choice is motivated by technical considerations\textsuperscript{10}, it allows us to concentrate on vertical strategic interaction while capturing interesting cases. For instance, situations where markets are non-overlapping (due to product differentiation, geography, etc.) or when consumers switch brands within stores rather than stores within brands.

The trimming of the game is the following: at date 0, the producer chooses a capacity level $Q \in R^+$, which allows him to produce a divisible homogeneous good and pays the installation costs. The cost of installing capacity is described by an increasing convex function $F(Q) = F_0 + f(Q)$ with $f'(Q) \geq 0$.

At date 1, the producer can produce up to $Q$ at some constant non-negative marginal cost\textsuperscript{11}. The allocation of his production among the retailers, as well as the distribution of gains, is determined in a multiparty bargaining game that takes place at date 1. Denote the producer's payoff of the date 1 bargaining game by $S_0(Q)$.

We are interested in a non-cooperative situation in which the producer chooses the industry capacity level at date 0, anticipating that bargaining will take place at date 1.

We thus take an "incomplete contract" approach by assuming that the terms of trade cannot be specified in advance, i.e. before capacity is chosen. This assumption is intended to capture the difficulties of contractually specifying all aspects of performance and the inability to commit not to renegotiate\textsuperscript{12}. Casual empiricism suggests that this is consistent with common practice: supply contracts in general have a shorter life than capacity investments.

\textsuperscript{10} See subsection 3.3 and also section 6 below.

\textsuperscript{11} A constant marginal costs is considered to be already accounted for in the net revenue function $R(q_i)$ since this function merely represents the gains from trade. Allowing for increasing or decreasing marginal costs would introduce additional technical complications since $R(q_i)$ would also depend on the quantities traded in the other markets.

\textsuperscript{12} We are sensitive to the fact that this assumption, while common in the literature, has been questioned on the grounds that in some sense it reflects imperfections that should perhaps be made explicit rather than assumed. Che–Hausch (1999) shows, in a bilateral setting, that when the producer's investment improves the buyer's valuation of the good (here increasing quantity increases the revenue the retailer can generate), if parties can't commit not to renegotiate, contracting doesn't perform any better than ex post negotiation.
The producer’s objective is thus:

$$\max_{Q \in \mathbb{R}^+} S_0(Q) - F(Q)$$

The objective of our analysis is to study how downstream concentration/integration affects the producer’s capacity choice.

### 3.2 Ownership structures

To study the effects of horizontal concentration we need to consider retail structures with varying degrees of concentration. The first step is to specify what a retail ownership structure is. Denote by $P(K)$ the set of all subsets of $K$, and $P(A)$ the set of all subsets of $A$ with a typical element $A$. To a retail ownership structure $Z$ corresponds a mapping $z$ from $P(A)$ to $P(K)$ where $z(A)$ is the subset of outlets controlled by $A$. We assume first that only retailers own outlets. Hence:

$$z(0) = \emptyset$$  \hspace{1cm} (1)

Second, we assume that each retailer holds exclusive property rights over some subset of outlets. Therefore, for any partition $A \cup (A \setminus A)$ of $A$, each of the outlets is owned by at most one of the subsets:

$$z(A) \cap z(A \setminus A) = \emptyset$$  \hspace{1cm} (2)

Note that it is possible for some retailers to own no outlets. Finally, the outlets owned by any subset $A'$ of $A$ must be also owned by $A$, i.e.:

$$z(A') \subseteq z(A) \ \forall A' \subseteq A$$  \hspace{1cm} (3)

**Definition 1:** To a retail ownership structure $Z$ corresponds a mapping $z$ from $P(A)$ to $P(K)$ satisfying (1), (2) and (3).

There is a family of mappings satisfying these conditions\footnote{More precisely, there exist $n^k$ distinct retail ownership structures.}. In order to compare retail ownership structures with varying degrees of concentration we use the concept of *integration*. Integration gives some retailer $i$ the control of all outlets owned by retailer $j$ in the former retail ownership structure\footnote{The reason to focus on *integration* rather than on partial transfers of assets is that by doing so we insure that asset transfer leads to a higher concentration.}. 
Definition 2: A retail ownership structure \( Z' \) is an integrated structure of the retail ownership structure \( Z \) if \( \exists i, j \in A \), with \( z(i) \neq \emptyset \) and \( z(j) \neq \emptyset \), such that for all \( A \):

\[
z'(A) = \begin{cases} 
z(A) \cup z(j) & \text{if } i \in A \\
z(A) \setminus z(j) & \text{otherwise}
\end{cases}
\]

If \( Z'' \) is an integrated structure of \( Z' \) and \( Z' \) is an integrated structure of \( Z \) then \( Z'' \) is also more concentrated than \( Z \). We can therefore establish chains on the set of ownership structures which are monotonic in concentration.

3.3 Solution concept

Using the "island market" model for the retail sector has two advantages. First, it allows us to concentrate on vertical strategic interaction. Second, it allow us to use the Shapley value as the solution concept to the date 1 bargaining game. The Shapley value describes the second stage revenue accruing to each agent. It is used on the assumption that the bargaining outcome is efficient. If one were to allow for downstream market competition this assumption would be more difficult to make.

We do not propose a particular extensive-form bargaining game that implements the Shapley value, this has been the object of previous research\(^{15}\). Stole-Zwiebel (1996a), for example, describe a game between a central player (firm/producer) and \( n \) pherepherical players (workers/retailers) that implements the Shapley value. The bargaining between the firm and any worker follows an alternating-offer bargaining game as in Binmore–Rubinstein–Wolinski (1986) with a probability of breakdown. Following breakdown, negotiations resume between the firm and the remaining workers. The equilibrium of this game is shown to be the Shapley value.

In order to introduce the Shapley value, we need to specify the gains from trade that the various partitions of agents \( A \cup (A \setminus A) \) of \( A \) can achieve on their own. In the second stage of the game, for a given retail ownership structure \( Z \), if an agreement between a subset of agents \( A \) is reached, they form a coalition \( A \) which controls \( |z(A)| \) outlets, where \( |z(A)| \) denotes the number of elements of \( z(A) \). If \( 0 \in A \), the coalition also controls the production capacity \( Q \).

\(^{15}\)See also footnote 7.
Since $R(q)$ is concave, the value coalition $A$ can generate by choosing an efficient allocation of those assets it controls at date 1 is:

$$V(Q; A; Z) = \begin{cases} |z(A)| \cdot R(\frac{Q_j}{|z(A)|}) & \text{if } 0 \in A \\ 0 & \text{otherwise} \end{cases}$$

$V(.)$ is the characteristic value function of the underlying date 1 game. Changing the outlet ownership structure will influence the game outcome because it changes the characteristic value function.

Note that, if the producer belongs to coalition $A$, $V(.)$ is increasing and concave in the number of outlets controlled by coalition $A$.

The Shapley value of agent $i$ at date 1 is:

$$Sh_i(Q; Z) = \sum_{|A| \in A} \frac{|A| - 1|!(|A| - |A|)!}{|A|!} \cdot (V(Q; A; Z) - V(Q; A \setminus i; Z))$$

To alleviate notation, note that we can write the producer’s Shapley value as\(^{16}\):

$$Sh_0(Q; Z) = \sum_{A \in A} \omega_A \cdot V(Q; A; Z)$$

The producer’s program is thus:

$$\max_{Q \in R^+} Sh_0(Q; Z) - F(Q)$$

Since $V(Q; A; Z)$ is concave in $Q$ and $F(Q)$ is assumed to be convex, the producer’s problem can be solved by first order conditions. What we are interested in is studying how the producer’s optimal capacity choice changes with integration.

A first remark is in order here. For all ownership structures, underinvestment will always occur since the producer obtains less than the industry marginal surplus, i.e.:

\(^{16}\)Where $\omega_A = \frac{(|A| - 1)!}{|A|!}$ and $V(q_0; A; Z) = 0 \forall A$. 

11
The producer underinvests in capacity because he anticipates that some of the benefits from his investment are dissipated in future bargaining. It follows that a ownership structure that yields a higher equilibrium capacity is also a more efficient one.

3.4 Difference Operators

We now introduce difference operators, previously used by Segal (2003), which will be used throughout the analysis. Define the first-order difference operator as:

$$\Delta_i V(A) = V(A) - V(A \setminus i)$$

The difference operator $\Delta_i(A)$ measures the marginal contribution of agent $i$ to a coalition $A$, for all $A \subseteq \mathcal{A}$. As it is defined, it doesn’t depend on whether coalition $A$ includes agent $i$. Although we defined the difference operator with respect to a single agent $i$, it can be applied to subsets of agents $A' \subset \mathcal{A}$.

**Definition 3**: Player $h$ is indispensable if $\Delta_{A'} V(A) = 0$ for all $A$ and $A' \subset \mathcal{A}$ whenever $h \notin A$.

The producer is always indispensable. Some retailer $i$ is indispensable if and only if $i$ is a retail monopolist.

Define also the second-order difference operator as:

$$\Delta_{ij} V(A) = \Delta_i(\Delta_j V(A))$$

It expresses player $i$’s effect on player $j$’s marginal contribution to a coalition $A$. It describes the complementarity/substitution of two agents in coalition $A$. The exact expression is developed below for clarity:

$$\Delta_{ij} V(A) = V(A \cup i \cup j) - V(A \setminus i \cup j) - V(A \setminus j \cup i) + V(A \setminus i \setminus j)$$

To alleviate notation in the remainder of the paper, when talking of a coalition $A$ we always refer to coalitions containing the producer. In the remaining cases, the characteristic value of $A$ is trivially equal to 0.
Property 1: For any two retailers $i$ and $j$, with $z(i) \neq \emptyset$ and $z(j) \neq \emptyset$:

$$
\Delta_{ij}V(A) \begin{cases} 
< 0 & \text{if } Q \in (0, |z(A \cup i \cup j)| \bar{q}) \\
= 0 & \text{if } Q \geq |z(A \cup i \cup j)| \bar{q}
\end{cases}
$$

Retailers are substitutes, so retailer $i$’s effect on retailer $j$’s marginal contribution to a coalition $A$ is negative, i.e. $\Delta_{ij}V(A) < 0$. Only when capacity is high, so that retailer $i$ has no use for those units of output sold by retailer $j$, does retailer $i$’s effect on retailer $j$’s marginal contribution become negligible.

4 Analysis

The question we are interested in is how the producer’s marginal return to investment varies with the degree of retail concentration. Since the bargaining outcome is efficient, we can write the producer’s date 1 revenue $Sh_0(Q; Z)$ as the share $\alpha(Q; Z)$ of the industry surplus $V(Q; A)$, which does not depend on the retail ownership structure. Marginal returns to investment are thus given by:

$$
\frac{\partial}{\partial Q} Sh_0(Q; Z) = \frac{\partial V(A; Q)}{\partial Q} \cdot \alpha(Q; Z) + \frac{\partial \alpha(Q; Z)}{\partial Q} \cdot V(A; Q)
$$

How do marginal returns to investment change from a retail ownership structure $Z$ to an integrated ownership structure $Z'$?

$$
\frac{\partial}{\partial Q} (Sh_0(Q; Z') - Sh_0(Q; Z)) =
$$

$$
\frac{\partial}{\partial Q} V(A; Q) \cdot \left[ \alpha(Q; Z') - \alpha(Q; Z) \right] + \left[ \frac{\partial}{\partial Q} \alpha(Q; Z') - \frac{\partial}{\partial Q} \alpha(Q; Z) \right] \cdot V(A; Q)
$$

(4)

In the next subsection we show that with integration the share of ex-post surplus accruing to the producer decreases, i.e. integration aggravates the ”hold-up”. Therefore, the first element of (4) is negative.

In subsection 4.2, we look at how the producer’s bargaining position, as measured by his equilibrium share of surplus, is affected by the level of
investment. We show that, the producer’s share decreases as capacity increases, i.e. increasing capacity aggravates the hold-up (bargaining erosion).

In subsection 4.3 we show that, at least for high capacity levels, the magnitude of the "bargaining erosion" decreases with integration, i.e. integration softens the "bargaining erosion" effect. In such a case the second element of (4) is positive. The two effects counteract one another.

Finally, in subsection 4.4 we look at the net effect of integration on investment incentives and on the equilibrium investment level.

4.1 Integration and the ”hold-up” effect

Here we study the relation between retail concentration and the share of surplus accruing to the producer. Intuition tells us that when retail concentration increases, the scope the producer has for playing one retailer off against another will be reduced. We would therefore expect the producer’s share to decrease with integration. This intuition is verified in our model.

Lemma 1 (Segal 2003): For an indispensable producer, if $Z^0$ is an integrated structure of the retail ownership structure $Z$, then:

$$Sh_0(A; Z^0) - Sh_0(A, Z) = \sum_{\{i \notin A \text{ and } j \in A\}} \omega_A \cdot \Delta^2_{ij} V(A; Z)$$

We can now establish this subsection’s main result:

Proposition 1 Integration aggravates the hold-up: If $Z'$ is an integrated retail ownership structure of $Z$, then $\alpha(Q; Z') < \alpha(Q; Z)$ for $Q \in (0, kq)$ and $\alpha(Q; Z') = \alpha(Q; Z)$ for $Q \geq kq$.

Integration increases retailers’ ability to ”hold-up” the producer because it eliminates competition among retailers. Thus it reduces the share of the bargaining surplus the producer can expect to obtain.

However, if in the second stage of the game the industry is not capacity constrained, i.e. if $Q \geq kq$, retailers have no need to compete with each other for the good and therefore downstream market structure has no effect on the producer’s revenue.

This last point, although not totally new, is interesting on its own. Consider for example the visual entertainment industry. Movie theaters and

\[17\text{ See below.}\]
cable operators are often local monopolists. The production technology is characterized by some fixed cost and some low constant fixed cost, i.e. once a film/series is produced, copies can be made at low cost and diffused on each market. For this reason, the movie theater/cable industry market structure should, a-priori, have no effect on the supply of visual entertainment.

This is a an extreme result and provides only a benchmark. Adding elements to the model could turn this result in either way. The relative importance of these elements should be taken into account.

For example, if firms are risk averse (Chae-Heidhues (1999)) downstream integration will reduce supply. Also, if producers can earn direct advertising rents (Chipty-Snyder (1999)), or if marginal costs of production are non-linear (Inderst– Wey (2002)) the producer’s bargaining position can be affected by downstream integration.

4.2 The ”bargaining erosion” effect

We now turn to the question, of how the level of capacity affects the producer’s bargaining position, as measured by his equilibrium share of surplus. We find that increasing capacity aggravates the ”hold-up”.

The intuition is as follows. When production capacity is low, if the producer and one of the retailer fail to reach an agreement, then, as long as the producer reaches an agreement with the remaining retailers, the industry revenue will not fall by a lot. The full industry surplus could be almost achieved by the producer and the remaining retailers. In such a situation, it is easy for the producer to play off one retailer against another by threatening to sell all his output through the rivals’ outlets.

As capacity increases, each retailer’s contribution becomes more fundamental to achieve the maximum industry revenue. Each retailer knows that, if the producer were to carry out the threat to sell all his output through the rivals’ outlets, this would severely decrease the industry surplus and consequently its own profits.

When capacity is high, the producer’s threat to exclude any one retailer is less credible. The choice of a high capacity has, from the producer’s perspective, the undesirable effect of eroding his bargaining position. Only in the case of a retail monopoly, will $\alpha$ be constant (and equal to one half), since both the producer and the retailer are always essential.

Also, if the industry is not capacity constrained in the second stage of the game, the share will be constant (and equal to one half) since retailers do not need to compete with each other and therefore the threat described above disappears.
Proposition 2 "Bargaining erosion" effect: for any given ownership structure $Z$ (except a retail monopolist) we have that $\frac{\partial}{\partial Q} \alpha(Q; Z) < 0$:

- a) for all "sufficiently high" $Q (< kq)$.
- b) for all $Q \in (0, kq)$ if the market revenue elasticity $\varepsilon_R(q)$ is strictly decreasing in $q$ (where $\varepsilon_R(q) = \frac{\partial R(q)}{\partial q} \cdot \frac{q}{R(q)}$).

In the case of a retail monopolist, and for all $Z$ when $Q \geq kq$, $\frac{\partial}{\partial Q} \alpha(Q; Z) = 0$.

A decreasing market revenue elasticity $\varepsilon_R(q)$ is tantamount to say that the percent increase in revenue that can be achieved with a one percent increase of input decreases as output increases. This sounds like something reasonable\textsuperscript{18}, so we should in general expect the share of the surplus accruing to the producer to decrease as capacity increases.

This "bargaining erosion" effect creates an additional strategic concern. When making his capacity choice, the producer will take into account the fact that increasing capacity erodes his bargaining position.

4.3 Integration and the "bargaining erosion" effect

We now turn to the question of how the magnitude of the "bargaining erosion effect" changes with integration.

The bargaining erosion effect arises because increasing capacity makes each retailer more essential to achieve the industry surplus. Since integration makes each retailer more essential to start off with, we may expect this strategic effect to be weaker.

In fact, we can show that at least at high capacity levels, the magnitude of the "bargaining erosion" effect decreases with integration, i.e.:

Proposition 3 Integration softens "bargaining erosion" There exists some critical value $\bar{Q}$ such that the rate at which the share of the surplus

---

\textsuperscript{18} For example, with linear pricing and zero marginal costs this is verified for both convex and linear demand functions. There exists as well a subset of convex consumer demand functions that verify this condition. This subset contains many commonly used demand functions, such as the log and inverse. An interesting exception is the constant elasticity of demand: $\varepsilon_R(q)$ is constant. However, it’s revenue is not upper bounded. Adding a positive marginal cost not only makes the revenue function upper bounded but it also guarantees that $\varepsilon_R(q)$ is strictly decreasing.
decreases as capacity increases is lower in a integrated structure for \( Q \in (\tilde{Q}, k\tilde{q}) \), i.e.:

\[
\left| \frac{\partial}{\partial Q} \alpha(Q, Z') \right| < \left| \frac{\partial}{\partial Q} \alpha(Q, Z) \right| \text{ for } Q \in (\tilde{Q}, k\tilde{q})
\]

The figure below illustrates this proposition. The rate at which \( \alpha(Q, Z) \) decreases as capacity increases is higher than the rate at which \( \alpha(Q, Z') \) decreases. The dotted line represents an alternative path for \( \alpha(Q, Z) \). The rate at which it decreases is low for low values of \( Q \), but it becomes steeper than \( \alpha(Q, Z') \) for \( Q > \tilde{Q} \).

4.4 Integration and capacity choice

To sum up, we have seen that concentration will increase expropriation (proposition 1). So the left hand side of (4) is, as expected, negative.

However, concentration can also reduce the producer’s incentive to strategically choose a low capacity level (proposition 3). So the right hand side of (4) will be positive, at least for ”sufficiently high” capacity levels.

Whether the marginal return to increased capacity raises or falls with concentration depends on the magnitude of these two effects. This subsection looks at how the trade off is solved and at how integration affects the equilibrium capacity choice.
The first result states that marginal returns always cross (perhaps more than once). So no ownership structure provides unambiguously higher investment incentives. Moreover, for "sufficiently high" $Q$ they are always higher under integration.

**Proposition 4** There exists a critical value $\tilde{Q} < (k - \max \{|z(i)|, |z(j)|\}q)$ such that $\frac{\partial}{\partial Q} Sh_0(A, Z') - \frac{\partial}{\partial Q} Sh_0(A, Z) > 0$ for $Q \in (\tilde{Q}, kq)$.

This proposition not only proves the existence of some critical capacity level after which marginal returns to capacity are higher in a integrated structure, but it also provides an upper bound to the critical value. This bound is strictly lower than the level at which any of the integrating parties becomes strictly essential\(^{19}\).

The figure below illustrates this proposition. Marginal returns to capacity are always higher in the integrated structure to the right of $\tilde{Q}$.

Recall that the cost of installing capacity is given by an increasing differentiable convex function $F(Q)$. Since $Sh(Q, Z)$ is concave it can be solved, as discussed above, by first order conditions. Solving this problem gives us the following result:

\(^{19}\)That is, the level at which, in the absence of either $i$ or $j$, increasing capacity would have no value.
Proposition 5 The equilibrium capacity $Q^*$ is strictly higher in an integrated structure, i.e. $Q^*_Z > Q^*_Z$ if the cost of capacity is low, i.e. if:

- $\frac{\partial}{\partial Q} F(\hat{Q}) \leq \frac{\partial}{\partial Q} Sh_0(\hat{Q}, Z')$
- $\arg \max Sh_0(Q, Z') - F(Q) > 0$

Equilibrium capacity increases with integration if two conditions are verified. Marginal costs are low and fixed costs are not too high (producer’s ex-ante profits must be positive). When marginal costs are low, the industry’s optimal production capacity is high, and the “bargaining erosion effect” becomes more important. Integration will then lead to an increase in equilibrium capacity.

Only when the cost of capacity is sufficiently high will the “conventional wisdom” go through, and the “hold-up effect” dominate.

The table below summarizes the effect of downstream integration on the equilibrium capacity choice $Q^*$.

<table>
<thead>
<tr>
<th>$f'(Q) = 0$</th>
<th>$F_0$ low</th>
<th>$F_0$ high</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>high</td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>

5 Other bargaining solutions

For a given game different solution concepts may be considered. This section offers a preliminary discussion of how the previous results extend to other bargaining solutions.

Since we didn’t make use of the weights of the Shapley value in any of the proofs, our results remain valid if we used instead any other random-order value.

Another solution we may want to look at is the “split the difference” rule\textsuperscript{20}. In this case, the producer’s payoff is given by:

$$S_0(Z) = V(\mathcal{A}) - \rho \sum_{i \in \mathcal{A} \setminus 0} (V(\mathcal{A}) - V(\mathcal{A}\setminus i; Z))$$

\textsuperscript{20}The producer and each single retailer bargain over the marginal contribution of the retailer. The outcome of each single negotiation is obtained using the two person Nash bargaining solution.
Where $\rho$ is the fraction each retailer obtains from his marginal contribution. It is easy to check that:

$$S_0(Z') - S_0(Z) = \rho \Delta^2_{ij} V(A; Z)$$

The second order difference operator appears here as well. Using the "split the difference" rule would therefore give us similar results.

The solution concepts we considered up to now all share a common property: pay-offs are determined a linear function of the marginal contributions of the players. We may additionally want to consider non-linear bargaining solutions.

This is of particular interest because some of the ownership effects identified in Hart–Moore (1990) using the Shapley value, may be reversed using non-linear bargaining solutions (see deMeza–Lockwood (1998) and Chiu (1998)).

We address this issue in a simple setting with two retailers, two outlets and the two distinct ownership structures: retail monopoly and "competition" (as in the introductory example). We use as a solution concept the nucleolus.

In this particular setting, the Shapley value and the nucleolus have non-cooperative foundations that can be related. Both allocations can be implemented by adaptations of the alternating offers bargaining game in a specific multilateral bargaining environment with one producer and two retailers.

The main difference stems from the fact that, when alternative trades work as threat points (inside options) we obtain the Shapley value. If alternative trades work as constraints on equilibrium payoffs (outside-options) we obtain the nucleolus.

In the case of "competition", if an agreement is reached with a single retailer, the surplus is $V(A_{-i}) = R(Q)$, and if agreement is reached with both retailers the surplus is $V(A) = 2R(Q_2)$. In this setting, the nucleolus, usually quite cumbersome to work with, takes a very simple form.

Since $V(A) \geq V(A_{-i})$ for all $Q$, the nucleolus of the producer $N_0(Q)$, is simply the value of the surplus that can be achieved if all supply is allocated to a single retailer, i.e. $N_0(Q) = R(Q)$. In the case of a retail monopoly, we are back to the simple bargaining model, i.e. the producer gets half the gains from trade and $N_0(Q) = R(Q_2)$.

---

21 Horn-Wolinski (1988) present a bargaining game whose subgame-perfect equilibrium implements the nucleolus for the case where the number of peripheral players is equal to 2. For the case of the Shapley value see Stole-Zwiebel (1996a) and subsection 3.3 above.

22 To see when one or the other case may apply see p.g. deMezza-Lockwood (1998).
For any given $Q$, the share of the surplus accruing to the producer is, once again, lower in the concentrated retail structure ($R(Q) > R(\frac{Q}{2})$). However, investment incentives are always higher in the retail monopoly structure ($\frac{\partial}{\partial Q} R(Q) < \frac{\partial}{\partial Q} R(\frac{Q}{2}) \forall Q \in (0, 2q]$). With outside option bargaining, downstream integration appears to motivate, rather than discourage, investment.

6 Conclusion

This paper studied how integration of agents who compete for an input affects the capacity choice of a monopolist supplying that input. We show that, contrary to conventional wisdom, there is no monotonic relationship between retail concentration and the producer’s level of investment.

The reason is that the producer’s bargaining position is affected by his capacity choice. While retail concentration increases the “hold-up” itself, it also gives the producer a stronger incentive to focus on increasing the industry surplus rather than on strategic concerns that can lead to a low capacity choice. As a result, the producer’s equilibrium capacity level may be higher when downstream concentration is high.

On a theoretical level these results support Galbraith’s (1952) idea of Countervailing Power in the sense that the emergence of a concentrated retail sector may improve allocative efficiency and consumer welfare. In our model it may be the case that as concentration increases, so does capacity, and consumer prices fall. Then, consumers benefit from retail concentration.

Galbraith’s informal argument is that one position of power may be neutralized by another. In our model, the mechanism at play is slightly different. Bargaining power arises from controlling a scarce resource. The producer has a strategic incentive to maintain the resource he controls relatively scarce in order to leverage his bargaining power. This effect, which distorts supply downwards, is particularly important when the retail sector exhibits a low level of concentration.

These results are of interest for other settings. For example, they cast doubt on the usual claim that unionization reduces a firm’s incentive to invest in labour-complementary capital.

We have avoided the issue of downstream competition in order to use the Shapley value as a solution concept. We conjecture that, similar results will

\[^{23}^\text{The fact that a seller enjoys a measure of monopoly power (..) means that there is an inducement to those firms from whom he buys or those to whom he sells to develop the power with which they can defend themselves against exploitation, (..). In this way the existence of market power creates an incentive to the organization of another position of power that neutralizes it.” Galbraith (1952) p. 119.}\]
hold if retailers compete for the same customers, as long as retailers have some market power. However, to rigorously address this issue we would have to work with a different set-up. This remains an open issue.

Appendix

Proof of Property 1:
Since \( |z(A \cup i \cup j)| + |z(A \setminus i \setminus j)| = |z(A \setminus i \cup j)| + |z(A \setminus j \cup i)| \) there exists \( \gamma \) such that:

\[
\gamma |z(A \cup i \cup j)| + (1 - \gamma) |z(A \setminus i \setminus j)| = |z(A \setminus i \cup j)|
\]

\[
(1 - \gamma) |z(A \cup i \cup j)| + \gamma |z(A \setminus i \setminus j)| = |z(A \setminus j \cup i)|
\]

Recall that \( V(.) \) is increasing and concave in the number of outlets a coalition \( A \) controls \( |z(A)| \). More precisely, \( V(.) \) is strictly concave in \( |z(A)| \) if \( Q \in (0, |z(A)| \bar{q}) \) and constant if \( Q > |z(A)| \bar{q} \). Property 1 follows from the concavity profile of \( V(.) \) in \( |z(A)| \).

Proof of Lemma 1:
Step 1. Integration increases \( V(A) \) for those subsets of \( \mathcal{A} \) where \( i \) is present but not \( j \), i.e. \( A \subseteq \mathcal{A} : \{i \in A \text{ and } j \notin A\} \). When \( A \) includes \( i \), this coalition controls in \( Z' \) those outlets that belonged to \( j \) in \( Z \). So:

\[
\forall A \subseteq \mathcal{A} : \{i \in A \text{ and } j \notin A\} \quad V(A; Z') - V(A; Z) =
\]

\[
V(A \cup j; Z) - V(A; Z) = \Delta_j V(A; Z)
\]

However, for those coalitions \( A \) where \( j \) is present but not \( i \), i.e. \( A \subseteq \mathcal{A} : \{j \in A \text{ and } i \notin A\} \), in \( Z' \) their characteristic value is only \( V(A \setminus j; Z) \). Thus:

\[
\forall A \subseteq \mathcal{A} : \{j \in A \text{ and } i \notin A\} \quad V(A, Z') - V(A; Z) =
\]

\[
V(A \setminus j; Z) - V(A; Z) = - \Delta_j V(A; Z)
\]

Finally, in those cases where \( \{i, j\} \subseteq A \), and in those cases where \( \{i, j\} \cap A = \emptyset \), \( V(A) \) remains unchanged. We can write down the effect of integration on the producer’s Shapley value as:
\[ Sh_0(A, Z') - Sh_0(A, Z) = \sum_{A' \subseteq A, \{i \in A \text{ and } j \notin A\}} \omega_A \Delta_j V(A; Z) - \sum_{A' \subseteq A, \{j \in A \text{ and } i \notin A\}} \omega_A \Delta_i V(A; Z) \]

**Step 2.** For all \( A' \subseteq A \) with \( \{i \in A \text{ and } j \notin A\} \), there exists \( A \subseteq A \) with \( \{j \in A' \text{ and } i \notin A\} \), such that \( A' \setminus i = A \setminus j \). So we can write:

\[
\sum_{A' \subseteq A, \{i \in A \text{ and } j \notin A\}} \omega_A \Delta_j V(A; Z) = \sum_{A' \subseteq A, \{j \in A \text{ and } i \notin A\}} V(A \setminus i \cup j; Z) - V(A \setminus i \setminus j; Z)
\]

Therefore we have:

\[
Sh_0(A, Z') - Sh_0(A, Z) = \sum_{A' \subseteq A, \{j \in A \text{ and } i \notin A\}} \omega_A (V(A \cup i \cup j; Z) - V(A \setminus i \cup j; Z) + V(A \setminus i \setminus j; Z) - V(A \setminus i \setminus j; Z)) = \sum_{A' \subseteq A, \{i \notin A \text{ and } j \notin A\}} \omega_A \cdot \Delta^2_{ij} V(A; Z)
\]

**Proof of Proposition 1:**
Divide both sides of Lemma 1 by \( V(\mathcal{A}) \) and apply Property 1 to each single element in the sum.

**Proof of Proposition 2:**

**Step 1:**
In order to follow the above intuition we start by taking the marginal contribution of each subset of retailers \( A \) to the industry \( \Delta_A(\mathcal{A}) \), and normalizing it by the industry value. The following equivalence is verified:

\[
\frac{\Delta_A(\mathcal{A}; Z)}{V(\mathcal{A})} = 1 - \frac{V(\mathcal{A} \setminus A; Z)}{V(\mathcal{A})} \quad (5)
\]

The effect of increasing capacity on the producer’s equilibrium share is given by:

\[
\frac{\partial}{\partial Q} \alpha(Q; Z) = \sum_{A \subseteq A} \omega_A \cdot \frac{\partial}{\partial Q} \left( \frac{V(A; Z)}{V(\mathcal{A})} \right) \quad (6)
\]

From (5) and (6), if the normalized marginal contribution of each subset of retailers, evaluated at some \( Q \), is strictly increasing in \( Q \), then \( \frac{\partial}{\partial Q} \alpha(Q; Z) < 0 \).
Step 2:
Derive with respect to $Q$ both sides of (5):

$$\frac{\partial}{\partial Q} \left( \frac{\Delta (A \setminus A)}{V(A)} \right) = -V(A)^{-2} \left[ \frac{\partial V(A \setminus A)}{\partial Q} \cdot V(A) - \frac{\partial V(A)}{\partial Q} \cdot V(A \setminus A) \right]$$

(7)

The expression in brackets is equal to zero for $Q \geq k_q$, and in the case of a retail monopolist. It is strictly negative (and thus $\frac{\partial}{\partial Q} \left( \frac{\Delta (A \setminus A)}{V(A)} \right) > 0$) in case a) and b). We will study them in turn below. First note that the term in brackets is strictly negative iff:

$$\frac{\partial V(A \setminus A)}{\partial Q} \cdot V(A) < \frac{\partial V(A)}{\partial Q} \cdot V(A \setminus A) \iff$$

$$\frac{1}{Q} \left[ \frac{\partial R(q)}{\partial q} \cdot |z(A \setminus A)| \cdot R(q) \right]_{q=\frac{Q}{|z(A \setminus A)|}} < \frac{1}{Q} \left[ \frac{\partial R(q)}{\partial q} \cdot |z(A)| \cdot R(q) \right]_{q=\frac{Q}{|z(A)|}} \iff$$

$$\varepsilon_R(q) \frac{\partial}{\partial q} |z(A \setminus A)| < \varepsilon_R(q) \frac{\partial}{\partial q} |z(A)|$$

Step 3: Since $|z(A \setminus A)| < |z(A)|$ for all $A \subset A$, the term in brackets is negative if the elasticity of $R(q)$ is strictly decreasing in $q$. This proofs b).

Whiteout this additional assumption on $R(q)$ we get a more limited result. $\varepsilon_R(q)$ is continuous in $q$ and:

$$\varepsilon_R(q) > 0 \forall q \in (0, \bar{q}) \text{ and } \lim_{q \to \bar{q}} \varepsilon_R(q) = 0$$

For "sufficiently high" $q$, $\varepsilon_R(q)$ is strictly decreasing in $q$. So for "sufficiently high" $Q$, for all $A \subset A$, $\frac{\partial}{\partial Q} \left( \frac{\Delta (A \setminus A)}{V(A)} \right) > 0$. This proofs the case a).

Proof of Proposition 3:

Step 1. By lemma 1, we can write the difference in the rate at which the share of the surplus decreases as capacity increases as:

$$\frac{\partial}{\partial Q} \alpha(Q; Z') - \frac{\partial}{\partial Q} \alpha(Q; Z) = \sum_{\{i \in A \text{ and } j \notin A\}} \omega_A \frac{\partial^2 V(A; Z)}{\partial Q} \frac{\Delta^2 V(A; Z)}{V(A; Z)}$$

So, any element of the sum is positive if and only if:

$$\frac{\partial}{\partial Q} \Delta^2 V(A; Z) \cdot V(A; Z) - \frac{\partial}{\partial Q} V(A; Z) \cdot \Delta^2 V(A; Z) \geq 0$$
Step 2. Since \( \Delta^2 ij V(A; Z) \leq 0 \) for all \( Q \in (0, k\bar{q}) \) (with strict inequality when \( A = A \)), the second term is always non-negative. So we focus on the first term.

\[
\frac{\partial}{\partial Q} \Delta^2 ij V(A; Z) = \frac{\partial}{\partial Q} [V(A \cup i \cup j) - V(A \setminus i \cup j) - V(A \setminus j \cup i) + V(A \setminus i \setminus j)]
\]

\[
= \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \cup i \cup j|}} - \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus i \cup j|}} - \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus j \cup i|}} + \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus i \setminus j|}}
\]

For \( k\bar{q} > Q \geq \min \{|z(A \setminus i \cup j)|; |z(A \setminus j \cup i)|\} \bar{q} \) the previous expression is always non-negative (and strictly positive for \( A = A \)). So we get:

Condition 1: for all \( Q \) “sufficiently high”:

\[
\frac{\partial}{\partial Q} \alpha(Q; Z') - \frac{\partial}{\partial Q} \alpha(Q; Z) > 0
\]

Step 3. From proposition 2 we know that at least for sufficiently high \( Q \frac{\partial}{\partial Q} \alpha(Q; Z) < 0 \). Together with condition 1 we get that there is a critical value \( \hat{Q} \), equal to minimal value satisfying both condition 1 and proposition 2, such that \( \left| \frac{\partial}{\partial Q} \alpha(Q; Z') \right| < \left| \frac{\partial}{\partial Q} \alpha(Q; Z) \right| \) \( Q \in (\bar{q}, k\bar{q}) \).

Proof of Proposition 4:

Step 1: From lemma 1, the difference in marginal returns to investment can be expressed as:

\[
\frac{\partial}{\partial Q} Sh_0(A, Z') - \frac{\partial}{\partial Q} Sh_0(A, Z) = \sum_{A:0 \in A \{i \notin A \text{ and } j \in A \}} \omega_A \frac{\partial}{\partial Q} \Delta^2 ij V(A; Z) \quad (8)
\]

For all \( A \subseteq A \):

\[
\frac{\partial}{\partial Q} \Delta^2 ij V(A; Z) =
\]

\[
\frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \cup i \cup j|}} - \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus i \cup j|}} - \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus j \cup i|}} + \frac{\partial R(q)}{\partial q} \bigg|_{q=\frac{\alpha}{|A \setminus i \setminus j|}}
\]

Step 2: For \( k\bar{q} > Q \geq \min \{|z(A \setminus i \cup j)|; |z(A \setminus j \cup i)|\} \bar{q} \) the previous expression is always non-negative (and strictly positive for \( A = A \)). So all the elements of the sum in (8) are positive for

\[
Q \in [\max \{\min \{|z(A \setminus i \cup j)|; |z(A \setminus j \cup i)|\}, k\bar{q}\}, \min \{|z(A \setminus i \cup j)|; |z(A \setminus j \cup i)|\}]
\]

25
So for $Q \in [k - \max \{|z(i)|; |z(j)|\} \tilde{q}, k\tilde{q}]$ we have:

$$
\sum_{A \in \mathcal{A}} \omega_A \frac{\partial}{\partial Q} \Delta_{ij}^2 V(A; Z) > 0
$$

**Proof of Proposition 5:**

Equilibrium capacity is given by $\frac{\partial S_0(Q^*, Z)}{\partial Q} = \frac{\partial F(Q^*)}{\partial Q}$ if $S_0(Q^*, Z) \geq F(Q^*)$. Is zero otherwise.

Since for $k\tilde{q} > Q > \tilde{Q}$ we have $\frac{\partial}{\partial Q} S_0(Q^*, Z) < \frac{\partial}{\partial Q} S_0(Q^*, Z'), Q^*(Z) < Q^*(Z')$ if:

- both $Q^*(Z)$ and $Q^*(Z')$ are higher than $\tilde{Q}$, i.e. $\frac{\partial}{\partial Q} F(Q^*) \leq \frac{\partial}{\partial Q} S_0(Q^*, Z) = \frac{\partial}{\partial Q} S_0(\tilde{Q}, Z)$.
- the producer makes positive profits, i.e. $S_0(Q^*_Z, Z') > F(Q^*_Z)$.

**References**


