Leniency Programs for Multimarket Firms: The Effect of Amnesty Plus on Cartel Formation

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Abstract

We examine the effect of the Amnesty Plus policy on firms’ incentives to engage in cartel activities. Amnesty Plus is a proactive antitrust enforcement strategy aimed at attracting amnesty applications by encouraging firms already convicted in one market to report collusive agreements in other markets. It has been heavily advertised that Amnesty Plus weakens cartel stability. We show to the contrary that Amnesty Plus does not always have this desirable effect. Only under specific conditions, Amnesty Plus deters a cartel which would have been sustainable under an antitrust policy without Amnesty Plus. Otherwise, Amnesty Plus is either neutral or even stabilizes a cartel. We also show that firms can exploit their multimarket contact to reduce the effectiveness of the Amnesty Plus policy.

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1 Introduction

Experience garnered over many years has taught antitrust authorities in the United States (US) and the European Union (EU) that companies which have been colluding in one specific product or geographic market are more likely to have engaged in cartel activities in other adjacent markets.

Due to the high diversity of businesses in multinational firms, cartel activities bear all the marks of contagion between and especially within companies. The probably most well-known example for such a cross-linked collusive pattern is the conspiracy in the markets for various vitamins. The striking feature of this complex of infringements was the central role played by Hoffmann-la Roche (HLR) and BASF, the two main vitamin producers, over the course of ten years in virtually every cartel affecting the whole extent of bulk vitamin production.\(^1\) HLR, BASF and Rhône-Poulenc instigated the first main group of cartels which consisted of price fixing agreements in the markets for vitamins A and E. The initial success of these arrangements inspired their replication in other vitamin markets. Smaller producers such as Merck, Takeda and Daiichi joined the pioneers and simultaneously colluded in a number of vitamin products. Accordingly, the European Commission (EC) stated that “the simultaneous existence of the collusive arrangements in the various vitamins was not a spontaneous or haphazard development, but was conceived and directed by the same persons at the most senior levels of the companies concerned”.\(^2\) Rhône-Poulenc’s disclosure of evidence on collusion in the markets for vitamins A and E led to the opening of an investigation. However, only BASF’s comprehensive collaboration with the US Department of Justice (DoJ) under the Amnesty Plus Program accelerated inquiries and finally led to the successful prosecution of all participants. When Rhône-Poulenc plead guilty to its vitamin conspiracies under the US Amnesty Program and applied for leniency under the 1996 EC Leniency Notice, it did not provide any information on its participation in the vitamin D3 infringement and even pursued cartel activities in other product markets such as methionine and methylglucamine.\(^3\)

In the US, convictions of global cartels in the 1990s suggest that at least a dozen firms were repeat offenders in related product industries (Connor, 2003). The DoJ has been investigating around 50 alleged international cartels in 2004, and half of them have been detected during inquiries on other markets (Hammond, 2004). With the objective of

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\(^1\)Concerned were the markets for vitamins A, E, B1, B2, B5, B6, folic acid, C, D3, H, beta carotene and carotinoids.

\(^2\)EC IP/01/1625 November 2001.

fully exploiting the multimarket contact between colluding firms, the DoJ implemented the Amnesty Plus Program in 1999 as part of its Corporate Leniency Policy. According to Hammond, “The Division’s Amnesty Plus program creates an attractive inducement for encouraging companies who are already under investigation to report the full extent of their antitrust crimes [...]” (Hammond, 2004, p.16).

Leniency programs reduce fines for cartel members that bring evidence to the antitrust authority. Amnesty refers to the complete exemption from fines. Amnesty Plus aims at attracting amnesty applications by encouraging subjects of ongoing investigations to consider whether they qualify for amnesty in other than the currently inspected markets where they engage in cartel activities. In particular, Amnesty Plus offers a firm, which currently plea-bargains an agreement for participation in one cartel, where it cannot obtain guaranteed amnesty, complete immunity in a second cartel affecting another market. Provided that the firm agrees to fully cooperate in the investigation of the conspiracy of which the DoJ was previously not aware, it is automatically granted amnesty for this second offense. Moreover, the company benefits from a substantial additional discount\(^4\), i.e. the Plus, in the calculation of its fine in any plea agreement for the initial matter under investigation.\(^5\)

Under the current EC Leniency Notice, Amnesty Plus does not exist. Although, in 2001, the Organization for Economic Co-operation and Development (OECD) recommended the inclusion of Amnesty Plus as part of the 2002 reforms of the EU Leniency Program, the EC did not seize the opportunity to follow the US example by introducing a similar policy.

The present paper studies whether and how the Amnesty Plus policy affects firms’ incentives to form a cartel.\(^6\) It seems intuitive that, following a conviction of one cartel, Amnesty Plus encourages firms to report another cartel by granting the first firm which applies for this program a substantial discount in the fine already imposed. However, we argue that, Amnesty Plus may have important consequences for cartel formation.

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\(^4\)The size of the additional discount mainly depends on three factors: The strength of the evidence provided by the cooperating company, the potential significance of the revealed case measured in terms of volume of commerce involved, geographic scope and the number of co-conspirators, and the likelihood that the DoJ would have detected the cartel absent self-reporting (Hammond, 2006).

\(^5\)As a counterpart of Amnesty Plus, the DoJ contemporaneously implemented the Penalty Plus Program. Penalty Plus increases the fines for companies that neglect to take advantage of Amnesty Plus but are nevertheless caught for a second time. The main reason why we do not include Penalty Plus in our analysis is that, whereas we want to focus on the difference between the US and the EU amnesty policy, the clause of “aggravating circumstances” in the “2006 EC Guidelines on the method of setting fines” is very similar to the US Penalty Plus.

\(^6\)In particular, we examine the possible effect of Amnesty Plus on the best collusive subgame-perfect equilibrium that can be sustained through standard trigger strategies.
in particular because it increases the firms’ incentives to report a cartel after a first detection.

We study two markets in which two identical firms play an infinitely repeated game of collusion. In each period, the firms can choose to form a cartel before interacting on the product market. Collusion generates incriminating evidence which the antitrust authority can discover with some probability. In addition, each firm can also bring this evidence to the authority. When a cartel is detected, either through an investigation or a firm’s self-reporting, each cartel member, except the first reporting firm, has to pay a fine. Amnesty Plus becomes relevant when the firms have to decide whether to report a cartel in one market after they have already been convicted in the other market.

Our main result is that Amnesty Plus may affect cartel formation in two different ways: On the one hand, Amnesty Plus may have a pro-competitive effect by dissuading the firms to create one of their cartels when they would have formed both of them in the absence of Amnesty Plus. On the other hand, Amnesty Plus may also have an anticompetitive effect as it may encourage the firms to form both cartels when they would have formed only one of them under an antitrust policy without Amnesty Plus.

We also examine whether the firms can exploit their multimarket contact by linking punishment strategies across markets. Without Amnesty Plus, the firms can always treat the markets in isolation and thus, they use multimarket trigger strategies only if this facilitates collusion. Amnesty Plus however inherently links the markets. Moreover, it is the antitrust authority which decides on the implementation of Amnesty Plus, and the firms can only try to weaken its effectiveness by adapting their strategies. In particular, we find that if the markets do not differ substantially in terms of profitability, the use of multimarket strategies, while it does not directly affect the firms’ ability to collude, lowers the pro-competitive effect of Amnesty Plus and increases its anticompetitive effect.

Surprisingly, although legal studies which mainly argue in favor of an Amnesty Plus policy in Europe are burgeoning, the existing literature contains virtually no formal economic analysis which attempts to clarify possible motives for the EC’s non-adoption of Amnesty Plus, let alone to study the potential impact of such a policy on cartel formation. We take the first step towards filling this gap in the economic theory on leniency programs.

Recent academic research such as Harrington (2008), Chen and Rey (2007), Aubert et al. (2006), Spagnolo (2004) and Motta and Polo (2003) has elaborated on the differences in conception of leniency programs and their impact on the effectiveness of antitrust enforcement. 7 This line of research mainly highlights the basic trade-offs be-

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7For an extensive overview of the economic literature on leniency programs see Spagnolo (2006).
tween destabilizing collusion and deterring cartel formation and explores whether and, if so, under which conditions, leniency programs do not deter but rather encourage the formation of a cartel. The results are embedded in a normative analysis of how the antitrust authority should design such programs to minimize their undesirable effect. Our analysis is close in purpose to this literature in that we examine how Amnesty Plus as a feature of leniency programs affects cartel stability. However, unlike in previous work, where the firms collude in one market only, we allow them to simultaneously participate in two collusive agreements.

Some studies suggest that leniency programs which not only reduce fines but offer a positive reward to whistleblowing firms can deter collusion in a more effective way. In particular, Aubert et al. (2006) find the minimal reward necessary to induce a firm to report collusion and point out that this reward may be quite large. Spagnolo (2004) shows that an optimally designed leniency program rewards the first reporting company with an amount equal to the sum of the fines paid by its former partners. On this issue, economic theory however conflicts with legal practice. Although granting positive rewards may strengthen the deterrence power of leniency programs, remunerating antitrust offenders not only raises moral concerns but may also increase the risk of negative effects in that it may further lower the expected penalty level. Hence, antitrust authorities mostly refrain from rewarding informants.\(^8\) However, it may be argued that Amnesty Plus is equivalent to granting more than 100% leniency because it not only waives the entire penalty in the second cartel but also gives a fine discount for the initial infringement (Wils, 2007). From this perspective, the justification for an Amnesty Plus policy does not seem obvious.

Another strand of literature studies the role of multimarket contact between firms in sustaining collusion when monitoring is perfect. In their seminal paper, Bernheim and Whinston (1990) build on the idea, first raised by Edwards (1955) and further developed in a finite oligopoly games context by Harrington (1987), that multimarket contact across firms may foster anticompetitive outcomes. As a benchmark, they establish an irrelevance result: with identical firms and markets and constant returns to scale technology, multimarket contact does not affect the opportunities for cooperation. However, they also identify various plausible circumstances in which strategically linking markets facilitates collusion by slackening the incentive constraints that limit the firms’ ability to sustain collusive behavior in settings of repeated interactions. Spagnolo (1999) refutes

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\(^8\)Note however that for other forms of multiagent crime, like government fraud, the US False Claim Act substantially rewards the cooperation of individual informants. Moreover, Korea has introduced a reward scheme for reporting parties in antitrust cases. However, the rewards still seem much too small to compensate for the social and economic costs whistleblowers are likely to incur.
this irrelevance result and shows that, if the firms’ static objective functions are strictly concave, multimarket contact always makes collusion viable in a set of markets even if, in its absence, it could not be sustained in any of these markets.

Relatively few papers examine the effect on collusive behavior of the interaction between multimarket contact and imperfect information. Thomas and Willig (2006) find that exploiting multimarket contact by strategically linking markets may be unprofitable. This surprising result occurs because strategic linkage may promote contagion which allows adverse shocks to spread from one market to another. Matsushima (2001) shows that efficient collusion can be achieved in the limit through the linkage of a sufficiently large number of identical markets.

We look at the interaction between Amnesty Plus and multimarket contact and its effect on the firms’ ability to collude when information is perfect. Our analysis allows to distinguish the anticompetitive effect of strategic linkage from the one of Amnesty Plus. Whereas multimarket strategies are chosen by the firms and used only if they are pro-collusive, Amnesty Plus is implemented by the antitrust authority. Hence, it links markets both when it facilitates but also when it hinders collusion.

The remainder of the paper is organized as follows. Section 2 sets up the model. Sections 3 and 4 analyze cartel formation when firms use standard trigger strategies both under a European antitrust policy without Amnesty Plus and under a US antitrust policy with Amnesty Plus. In section 5, we graphically present our main findings. In sections 6 and 7, we allow firms to use multimarket punishment. Section 8 briefly concludes. All proofs can be found in appendix A. In appendix B, we discuss how the relaxation of two important assumptions affects our results.

2 The Model

2.1 Set-up

We consider two markets \( k = 1, 2 \) in which two identical firms play an infinitely repeated game where, in each period, they can choose to form a cartel before interacting on the product market. Communication is necessary for collusion and generates hard evidence which makes it possible to establish the antitrust offense. Markets 1 and 2 may differ in profitability. In particular, market 1 is at least as profitable as market 2. Firms discount future payoffs by a common discount factor \( \delta \in [0, 1] \). We compare the firms’ cartel formation decisions under the EC Leniency Program and the US Amnesty Program whose sole difference here is that the latter allows for Amnesty Plus. Amnesty Plus
signifies that a firm which has been caught colluding in one market can get a discount in the fine already imposed by reporting the remaining cartel in the other market.

The collusive joint profit is $2\pi_k > 0$, and thus, each firm makes a cartel profit equal to $\pi_k$. Denote by $\lambda = \frac{\pi_2}{\pi_1} \in [0, 1]$ the profit ratio of the two cartels. If firms compete, they make zero profits. In case one firm unilaterally deviates from the cartel while the other continues to collude, the deviating firm earns the whole short-term cartel profit $2\pi_k$ alone, whereas the other firm gets nothing. Firms use (grim) trigger strategies. The punishment firms agreed upon starts the period following the deviation and lasts forever after.

At the time firms decide whether to enter an illegal agreement, they observe the strictness of the enforcement policy that is summarized by a conviction probability $q \in [0, 1]$ with which the Antitrust Authority (AA) opens an investigation in one market leading to the conviction of the colluding firms with certainty. Detection across markets is independent. Each convicted firm pays a market specific fine $F_k$ which is reduced under Amnesty Plus to $F_k - R_k$ in return for the disclosure of the second cartel. $R_k \in [0, F_k]$ represents the fine reduction granted to the first informant. The higher $R_k$ the more generous the Amnesty Plus policy. The firm which is eligible for Amnesty Plus is the first company which reports the second infringement and thus, it also receives amnesty in that market. If both firms simultaneously apply for Amnesty Plus, each is first with probability $\frac{1}{2}$.

To keep the analysis simple, we assume that the evidence of collusion lasts for only one period. Thus, even after a firm has deviated from a collusive agreement it is held liable for its cartel behavior and can be fined until the end of the period in which the deviation occurred. Each cartel member has the possibility to bring the incriminating evidence to the AA. The first informant is eligible for total immunity from fines under a standard Amnesty Program. In our model, the only strategic implication of this standard Amnesty Program is that, since a defecting firm must still fear conviction, a unilateral deviation is always immediately followed by the reporting of the cartel.

In practice, fines are set according to judicial principles but are often related, directly

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9To keep the model simple, we identify investigation and conviction with a single probability. However, we could introduce uncertainty with respect to the AA’s ability to prove guilty a detected cartel by substituting $qs$ for $q$ where $s$ is the probability with which the investigation succeeds. See Chen and Rey (2007) for an analysis of optimal leniency rates before any and once an investigation is opened which distinguishes the probability of launching the investigation from the probability with which it succeeds.

10The limitation period of the liability for antitrust offenses is generally a positive number of years. Article 25 of the EC Council Regulation 1/2003 states that the Commission can sue for Administrative Action until five years from the date of the infringement. Moreover, “[…]in the case of continuing or repeated infringements, time shall begin to run on the day on which the infringement ceases”.

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or indirectly, to the nature and importance of the anticompetitive behavior, and thus, to the profits from collusion. We assume that the AA sets the fine as a function of the per period collusive profits$^{11}$, $F_k = F(\pi_k)$ where $F(\cdot)$ is an increasing function. Let then $\theta_k = \frac{F_k}{\pi_k}$ denote the fine-profit ratio for market $k = 1, 2$ and suppose that $\theta_2 \geq \theta_1$. This reflects the idea that the fine rises proportionally or less than proportionally with the cartel profit, i.e. that $\theta_k = \frac{F_k}{\pi_k}$ is weakly decreasing in $\pi_k$. $^{12}$ Fine records tend to support this assumption.$^{13}$

Following a cartel conviction, we assume that the AA closely monitors the previously collusive industry and thus, firms compete and never return back to collusion in the same market.

2.2 Timing

The time structure of the game is as follows:

- $t = 0$:
  
  \textit{Stage 0}: Both firms decide in each market whether to enter a collusive agreement. If at least one firm decides not to collude in market $k$, competition takes place in this market. If this happens in both markets, the game ends for that period. If both firms choose to collude in market $k$, their communication leaves some hard evidence.

  \textit{Stage 1}: Each firm decides whether to deviate or not from the collusive agreement(s). Its rival does not observe this decision until the end of stage 2.

  \textit{Stage 2}: Each firm decides whether to report the evidence to the AA. The AA detects the cartel with probability 1 if at least one firm self-reports. The first informant gets complete immunity from fines in this market, whereas the other firm has to pay the full fine. If each cartel formed in stage 0 is reported in this stage, the game ends for this period; otherwise:

$^{11}$Since the evidence that incriminates a cartel lasts only for one period, the assumption that the AA takes the collusive profit per period and not cumulated over the whole duration of the cartel as a basis for the determination of the fine seems plausible.

$^{12}$E.g., let $F(\cdot)$ be an increasing concave function or let $F_k$ be an affine transformation of $\pi_k$: $F(\pi_k) = a + b\pi_k$ with $a \geq 0$ and $b \geq 0$. In Appendix B.1, we discuss how the relaxation of the assumption $\theta_2 \geq \theta_1$ affects our analysis.

$^{13}$E.g. Vitamin cartel: In the US, a fine equal to 127% of the collusive overcharge was imposed in the market of vitamin B2 whereas it ranged between 63% and 88% of the collusive overcharge in the more profitable vitamin C market. In the EU, the fines ranged between 63% and 88% for vitamin B2 and between 30% and 60% for vitamin C (Connor, 2005).
Stage 3: Each cartel formed in stage 0 and not reported in stage 2 is detected with probability \( q \). If the AA does not detect the cartel(s) formed in stage 0, the game ends for that period. If the AA however detects the cartel(s) formed in stage 0, the colluding firms pay the corresponding fines, and the game ends for that period. If the firms have formed both cartels in stage 0, and the AA has detected only one of them, then:

Stage 4: Each firm chooses whether to report the remaining cartel.

- \( t \geq 1 \): If both cartels have been formed but none of them has been convicted (detected or reported) in the previous period, the same time structure applies to this period. If either no cartel has been formed or the cartel(s) formed has (have) been convicted, the firms compete in both markets.\(^{14}\) If either one cartel has been formed and not convicted or both cartels have been formed but only one cartel has been convicted, then:

Stage 0: Each firm decides whether to enter a collusive agreement in the market where the cartel has gone undetected in the previous periods. If at least one firm chooses not to collude, competition takes place in this market, and the game ends for this period.

Stage 1: Each firm decides whether to deviate from the collusive agreement. Its rival does not observe this decision until the end of stage 2.

Stage 2: Each firm decides whether to report the evidence to the AA.

Stage 3: The AA detects the cartel in market \( k \) with probability \( q \). If the cartel is not detected the game ends for this period. If it is detected, the colluding firms pay the corresponding fines.

### 2.3 Individual Stability of a Cartel

We first examine under what conditions an individual cartel is sustainable if the firms interact in only one market \( k \). The firms can try to sustain repeated collusion by using trigger strategies in which they would return to competition the period following the deviation in case one of them deviates from the collusive outcome. In the presence of a standard amnesty policy where the first self-reporting firm pays no fine, deviating

\(^{14}\)We assume here that the firms can only form a cartel if this cartel has already been formed in the previous period. In Appendix B.2, we relax this assumption and discuss the strategy where the firms form cartel 1 until it is detected and then form cartel 2.
and reporting weakly dominates deviating and not reporting. A unilateral deviation is therefore always followed by an amnesty application. Hence, in the period of the deviation, the informant earns the whole cartel profit and pays no fine whereas thereafter, he gets zero profits from competition. Collusion is sustainable if the present discounted value $V_k$ of the cartel is at least as big as the gain each firm gets from a unilateral deviation in this market, that is

$$V_k \geq 2\pi_k$$

$V_k$ is the continuation value of the cartel in market $k$ and is such that:

$$V_k = q(\pi_k - F_k) + (1-q)(\pi_k + \delta V_k)$$

Solving for $V_k$ yields

$$V_k = \frac{\pi_k - qF_k}{1 - \delta(1-q)}$$

The above condition defines an **individual stability threshold** such that cartel $k$ is individually stable if and only if

$$\delta \geq \frac{\pi_k + qF_k}{2\pi_k(1-q)} = \frac{1 + q\theta_k}{2(1-q)} \equiv \tilde{\delta}(q, \theta_k)$$

A firm has no incentive to unilaterally deviate from the collusive equilibrium if $\delta \geq \tilde{\delta}(q, \theta_k)$. Both firms anticipate that collusion will be sustainable and thus, they form the cartel. However, if $\delta < \tilde{\delta}(q, \theta_k)$, the firms anticipate that, immediately after the formation of the cartel, they would both deviate and self-report. Hence, they do not form the cartel in the first place. The individual stability threshold is increasing and continuous in all its arguments. Intuitively, the higher the probability of conviction and the higher the fine-profit ratio, the more firms have to value future flows of collusive profits, and thus, the higher the $\delta$ needed to individually sustain the cartel. Note that $\tilde{\delta}(q, \theta_k) \leq 1$ if and only if $\theta_k \leq \frac{1}{q} - 2$. Otherwise, the cartel $k$ is individually unstable for any value $\delta \in [0, 1]$. Finally, the assumption $\theta_2 \geq \theta_1$ implies that $\tilde{\delta}(q, \theta_2) \leq \tilde{\delta}(q, \theta_1)$, i.e. a cartel in market 2 is equally or more difficult to sustain than a cartel in the more profitable market 1.
3 EC Leniency Program With Standard Trigger Strategies

Suppose now that the firms encounter each other in the two markets 1 and 2, and that they use standard trigger strategies to sustain collusion. Each firm plays the collusive equilibrium in market \( k = 1, 2 \) as long as the partner colludes in this market. If a firm unilaterally deviates from the collusive agreement, the other firm competes from the next period on and forever after in this market. Note that a deviation in one market triggers punishment only in this specific market. Intuitively, since punishment strategies as well as detection probabilities across markets are independent, firms treat each market in isolation, and their actions in market 1 do not influence their decisions in market 2. The condition under which a cartel is formed is therefore the same as the condition under which the cartel is individually stable. We provide a formal proof of this intuitive argument and state the following proposition:

**Proposition 1** Under the EU antitrust policy, a cartel is formed if and only if it is individually stable. More precisely,

i/ If \( \delta < \tilde{\delta}(q, \theta_1) \), no cartel is formed.

ii/ If \( \tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2) \), cartel 1 is formed whereas cartel 2 is not.

iii/ If \( \tilde{\delta}(q, \theta_2) \leq \delta \), both cartels are formed.

**Proof.** See Appendix A. ■

In what follows, we assume that \( \theta_2 \leq \frac{1}{q} - 2 \). Without this assumption, the region in case iii/ where the firms form both cartels would be empty.

4 US Amnesty Program With Standard Trigger Strategies

We now introduce an Amnesty Plus policy which allows a firm, already caught in one cartel, to benefit from a fine reduction if it is the first to report the other cartel. It has been heavily advertised that the main benefit of Amnesty Plus is its effect on cartel desistance: Amnesty Plus increases the firms’ incentives to report a cartel after the detection of another cartel. However, we argue that, above all, Amnesty Plus may have important consequences for cartel deterrence in particular because it encourages reporting after a first detection. To see these effects, we proceed by backward induction.

Suppose that the firms have formed both cartels, and the AA has detected one of them, say cartel \(-k\). The remaining cartel \( k \) survives this detection only if none of the firms
unveils the collusive evidence to the AA at the end of this period. The firms do not report cartel $k$ if and only if two conditions jointly hold. First, the fine reduction a firm gets in return for the disclosure of cartel $k$ must not exceed the discounted value of this cartel. That is

$$\delta V_k \geq R_{-k}$$

This condition defines a robustness threshold such that cartel $k$ is robust to the detection of cartel $-k$ if and only if

$$\delta \geq \frac{R_{-k}}{1 - q\theta_k + (1 - q)\frac{R_{-k}}{\pi_k}} \equiv \delta\left(q, \theta_k, R_{-k}, \frac{R_{-k}}{\pi_k}\right)$$

Note that the robustness threshold is increasing and continuous in all its arguments. In particular, it increases with the fine reduction $R_{-k}$. The more generous the Amnesty Plus policy, the higher the robustness threshold, and the more the firms find the reporting of cartel $k$ attractive. Hence, Amnesty Plus encourages firms to report a cartel once another cartel has been detected. Second, the firms will have to again form cartel $k$ at the beginning of the next period. This will happen if and only if the individual stability condition holds. Hence, cartel $k$ survives the detection of cartel $-k$ if and only if it is robust and individually stable, that is

$$V_k \geq \max\left(2\pi_k, \frac{R_{-k}}{\delta}\right)$$

If this inequality does not hold, the firms report cartel $k$. In particular, if cartel $k$ is individually unstable, the firms anticipate that they cannot form this cartel next period. Reporting cartel $k$ is then a dominant strategy for each firm. If a firm anticipates that its partner does not report cartel $k$, it gets a strictly positive fine reduction from reporting instead of zero from not reporting. Moreover, if a firm anticipates that its co-conspirator reports, it also prefers reporting since it gets Amnesty Plus with probability $\frac{1}{2}$ and avoids paying a fine with certainty. If cartel $k$ is individually stable but not robust, the Nash equilibrium in dominant strategies is to report this cartel. Again, if a firm anticipates that its partner reports cartel $k$, it also prefers to report. However, even if a firm anticipates that its partner does not report and thus that they may form the cartel again next period, it prefers to report because the fine reduction is higher than the present discounted value the firm would get from sustaining this cartel.

Let us compare the individual stability to the robustness threshold in market $k$. We
find that
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\tilde{\delta}(q, \theta_k) \geq \hat{\delta}(q, \theta_k, \frac{R_{-k}}{\pi_k}) \iff R_{-k} \leq \frac{\pi_k + qF_k}{1 - q}
\] (1)

Intuitively, if the fine reduction $R_{-k}$ is rather small, Amnesty Plus cannot induce the reporting of the cartel, and it is the individual stability and not the robustness condition which is stringent. Hence, cartel $k$ survives the detection of cartel $-k$ if and only if $\delta \geq \tilde{\delta}(q, \theta_k)$. However, if the fine reduction is large enough, the firms want to benefit from Amnesty Plus and therefore they report the cartel. In this case, it is the robustness condition which is stringent, and cartel $k$ survives the detection of cartel $-k$ if and only if $\delta \geq \hat{\delta}(q, \theta_k, \frac{R_{-k}}{\pi_k})$.

Since we can now determine the outcome in the last stage of the game after a possible detection in one of the markets, we examine the firms’ decisions in the cartel formation stage. The firms create both cartels only if they do not find the optimal unilateral deviation profitable. Hence, the joint formation of the cartels is a Nash Equilibrium if the expected present discounted value each firm gets when forming both cartels is weakly higher than the payoff from the optimal unilateral deviation, that is

\[
V_{12} \geq \text{payoff from the optimal unilateral deviation}
\]

This inequality defines a joint stability condition such that the firms form both cartels if and only if this condition is satisfied. Note that the right hand side (RHS) of the above condition does not depend on the fine reduction under Amnesty Plus since, after a unilateral deviation, at most one cartel is left, and the Amnesty Plus option is not available. The left hand side (LHS), however, is weakly increasing piecewise in $R_1$ and $R_2$ and thus, the joint stability condition becomes less stringent piecewise when the fine discounts increase. It is important to understand, that the effects of Amnesty Plus on cartel desistance and on cartel deterrence may go in opposite directions. On the one hand, Amnesty Plus strengthens the firms’ incentives to report a cartel once the other cartel has been detected. On the other hand, it may increase the expected present discounted value of the joint cartel profits as the expected fine in case of a conviction decreases. However, note that $V_{12}$ is discontinuous in the fine discounts.\[^{15}\] If $R_1$ and $R_2$ increase up to the point where reporting under Amnesty Plus gets so attractive that an individually stable cartel breaks down after the detection of the other cartel, $V_{12}$ decreases drastically. However, if the fine discounts then continue to increase, $V_{12}$ rises

\[^{15}\text{This is the reason for the “piecewise”.}\]
again. This scenario may recur if the second cartel is also individually stable and breaks down after the detection of the other cartel as the firms’ reporting incentives increase. The net effect of Amnesty Plus therefore depends on the strength of its effect, first, on the firms’ reporting incentives and, second, on $V_{12}$. It may thus be either pro- or anti-competitive. We now examine this net effect in each of the three possible constellations of cartel formation under the EU antitrust policy.

4.1 No Cartel Formed Under the EU Policy: A Neutrality Result

**Proposition 2** If both cartels are individually unstable, i.e. $\delta < \tilde{\delta}(q, \theta_1)$, Amnesty Plus is neutral: the firms form no cartel both under the EU and the US policy.

**Proof.** See Appendix A. ■

The intuition for the proof is as follows: Proceeding by backward induction, we take the creation of both cartels as given and examine the Nash Equilibrium after the AA has detected one of the cartels. Since both cartels are individually unstable, each firm has a dominant strategy in reporting the remaining cartel after the first detection. Thus, the only Nash equilibrium in the remaining market is the firms’ simultaneous denunciation of the cartel. Each firm may be first to apply for amnesty, and thereby for Amnesty Plus, with a 50% chance. Examining the joint stability condition, we find that, for any possible value of $R_{-k}$, the expected present discounted value each firm gets when forming both cartels is always smaller than the optimal unilateral deviation which takes place in both markets. This signifies that if no cartel is individually stable under the EU Leniency Program, an ever so generous Amnesty Plus policy cannot have any stabilizing effect. As a consequence, Amnesty Plus is neutral in this case.

4.2 One Cartel Formed Under the EU Policy: An Anticompetitive Effect

In Lemma 1 we give the expression for the joint stability threshold which we then use to formulate Proposition 3.

**Lemma 1** If cartel 1 survives the detection of cartel 2, but cartel 2 does not survive the detection of cartel 1, i.e. $\max \left( \tilde{\delta}(q, \theta_1), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \right) \leq \delta < \delta$
max\(\left(\delta(q, \theta_2), \tilde{\delta}(q, \theta_2, \frac{R_1}{\pi_2})\right)\), the two cartels are jointly stable if and only if

\[
\delta \geq \frac{1}{2(1 - q)^2} - \frac{1}{4(1 - q)^2} \theta_2 \left(\frac{R_1}{F_2} q(1 - q) - q(3 - q)\right) \equiv \tilde{\delta}(q, \theta_2, \frac{R_1}{F_2})
\]

**Proof.** See Appendix A. ■

**Proposition 3** If cartel 1 is individually stable whereas cartel 2 is not, i.e. \(\tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2)\), Amnesty Plus has an anticompetitive effect on cartel formation if and only if

\[
\max\left(\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2, \frac{R_1}{\pi_2})\right) \leq \delta < \tilde{\delta}(q, \theta_2)
\]

This condition defines a non-empty range of values of \(\delta\) if and only if

\[
R_2 < \frac{(1 + q\theta_2)((\pi_1 - qF_1)}{(1 - q)(1 - q\theta_2)} \quad \text{and} \quad R_1 > \frac{1 + q F_2}{1 - q} + \frac{2\pi_2}{1 - q}
\]

Amnesty Plus then encourages the firms to form the individually unstable cartel 2 which they would not have formed under the EU policy.

**Proof.** See Appendix A. ■

Proposition 3 suggests that Amnesty Plus may have an anticompetitive effect by stabilizing a cartel which would not have been sustainable under the EU Leniency Program in the presence of another cartel which is individually stable. Note that for this anticompetitive effect to potentially occur, the firms must form cartel 1 but not cartel 2 under the EU policy for a non-empty range of discount factor values. For this to be the case, the individual stability thresholds for markets 1 and 2 must differ. However, if the fines are proportional to the collusive profits, i.e. \(\theta_1 = \theta_2\), and/or if the markets are perfectly symmetric, these thresholds are identical, and Amnesty Plus cannot have any anticompetitive effect.

We sketch the proof of Proposition 3 as follows: If the individually stable cartel 1 is detected, reporting the individually unstable cartel 2 is a dominant strategy for each firm. Hence, the firms report cartel 2 and may save part of the fine already imposed. Amnesty Plus thus decreases a firm’s expected fine from a cartel conviction such that, for each firm, the joint creation of the cartels may result in an expected present discounted
value of profits larger than the sum of the individual expected present discounted cartel profits. The firms form both cartels if the payoff from the optimal unilateral deviation, which occurs in market 2 only, does not exceed the value of the joint creation of the cartels. Two conditions have to hold. First, cartel 1 must be not only individually stable but also robust to a detection of cartel 2. Hence, the robustness condition must hold for a non-empty range of values for \( \delta \) within the interval \([\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)]\). This is true if the robustness threshold for cartel 1 lies below the individual stability threshold for cartel 2. We can show that
\[
\tilde{\delta}(q, \theta_1, R_2) < \tilde{\delta}(q, \theta_2) \iff R_2 < \frac{(1 + q\theta_2)(\pi_1 - qF_1)}{(1 - q)(1 - q\theta_2)} \tag{2}
\]

Second, the cartels must be jointly stable. For the joint stability condition to hold for a non-empty range of values for \( \delta \) within the interval \([\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)]\), it is necessary and sufficient that the joint stability threshold lies below the individual stability threshold for cartel 2. We find that
\[
\tilde{\delta}(q, \theta_2, R_1, F_2) < \tilde{\delta}(q, \theta_2) \iff R_1 > \frac{1 + q}{1 - q} F_2 + \frac{2\pi_2}{1 - q} \tag{3}
\]

Combining conditions (1) and (2), three situations are possible:

i/ \( R_2 \leq \frac{\pi_1 + qF_1}{1 - q} \)

If the fine discount in market 2 is low enough, cartel 1 survives the detection of cartel 2 for all \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)]\). If inequality (3) is satisfied, and the joint stability condition holds, the firms form both cartels whereas, in the absence of Amnesty Plus, they would have formed cartel 1 alone. Otherwise, the firms collude only in market 1, and Amnesty Plus is neutral.

ii/ \( \frac{\pi_1 + qF_1}{1 - q} < R_2 < \frac{(1 + q\theta_2)(\pi_1 - qF_1)}{(1 - q)(1 - q\theta_2)} \)

The formation of the two cartels is the best collusive equilibrium if and only if both cartel 1 survives the detection of cartel 2 and the cartels are jointly stable. In this case, Amnesty Plus has an anticompetitive effect. However, if the robustness condition for cartel 1 is not satisfied, the firms cannot sustain the remaining cartel after one cartel detection, and the joint stability condition never holds. As a consequence, the firms form only cartel 1.

iii/ \( R_2 \geq \frac{(1 + q\theta_2)(\pi_1 - qF_1)}{(1 - q)(1 - q\theta_2)} \)

In this case, the fine discount under Amnesty Plus in market 2 is too high such that the robustness condition for cartel 1 cannot be satisfied. Hence, no cartel
survives the detection of the other cartel, and the joint stability condition does not hold. The firms form only the individually stable cartel 1, and Amnesty Plus is neutral.

**Corollary 1** Amnesty Plus has no anticompetitive effect on cartel formation if the fine discount a firm gets under Amnesty Plus for cartel $-k$ does not exceed the fine for the reported cartel $k$, i.e. $R_{-k} \leq F_k$.

**Proof.** Note that if $R_1 \leq F_2$ the condition $R_1 > \frac{1+q}{1-q} F_2 + \frac{2\pi q}{1-q}$ does not hold, and the joint stability condition cannot be satisfied for $\delta$ within the interval $\left[\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)\right]$. It follows from Proposition 3 that Amnesty Plus cannot have any anti-competitive effect in this case. Finally, $R_2 \leq F_1$ is always true since $F_1 \geq F_2 \geq R_2$. ■

Corollary 1 suggests that, as a simple rule to avoid any stabilizing effect of the Amnesty Plus policy, the size of the fine discount granted in one market should not exceed the fine a non successful Amnesty Plus applicant would have incurred in the other market. This rule is sufficient but not necessary. Intuitively, if $R_{-k} \leq F_k$, each firm gets a negative expected payoff from reporting cartel $k$ after the detection of cartel $-k$. This is because both firms report cartel $k$ simultaneously and thus, with probability $\frac{1}{2}$, each firm has to pay a fine which is higher than the possible fine discount. The expected present discounted value each firm gets from the joint formation of the cartels decreases and cannot exceed the payoff from the optimal unilateral deviation. Hence, if cartel 2 is individually unstable such that the firms report it after the detection of cartel 1, the two cartels can never be jointly stable. By keeping the fine reduction low, the AA therefore can avoid any anticompetitive effect of the Amnesty Plus policy.\(^{16}\)

### 4.3 Both Cartels Formed Under the EU Policy: A Pro-competitive Effect

**Proposition 4** If both cartels are individually stable, i.e. $\delta \geq \tilde{\delta}(q, \theta_2)$, Amnesty Plus has a pro-competitive effect on cartel formation if and only if at least one of the cartels is not robust and the two cartels are not jointly stable. In particular:

\(^{16}\)Consider the interaction of $n \geq 2$ firms on markets 1 and 2. We suppose that, if all the firms report a cartel simultaneously, each firm is first with probability $\frac{1}{n}$. As the first firm reporting is the only company eligible for the fine discount under Amnesty Plus, a firm’s expected payoff from reporting cartel $k$ when everyone else reports is $\frac{1}{n} R_{-k} - \frac{n-1}{n} F_k$. We have $\frac{1}{n} [R_{-k} - (n - 1) F_k] \leq 0$ if and only if $R_{-k} \leq (n - 1) F_k$. It is then straightforward that if $R_{-k} \leq (n - 1) F_k$ holds for $n = 2$, it holds a fortiori for $n > 2$.  

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If $R_2 \leq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$, Amnesty Plus has a pro-competitive effect for a non-empty range of values of $\delta$ if and only if $\frac{R_2+qF_1}{1-q} < R_1 < \frac{2\pi_2+(1+q)F_1}{1-q}$.

If $R_2 > \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$, Amnesty Plus has a pro-competitive effect for a non-empty range of values of $\delta$ for any value of $R_1 \in ]0, F_1]$.  

**Proof.** See Appendix A.  

Proposition 4 suggests that Amnesty Plus may have a pro-competitive effect by destabilizing a cartel which would have been sustainable under the EU policy. The sketch of the proof is as follows: Amnesty Plus decreases the expected present discounted value of profits each firm gets when forming both cartels if, following the detection of one individually stable cartel, the firms report the other individually stable cartel to benefit from the fine discount. $V_{12}$ may then fall below the payoff from the optimal unilateral deviation which occurs in market 2 only. As a consequence, the firms would anticipate that the cartels are not jointly stable and form only the more profitable of the cartels. To examine the exact circumstances under which the firms form both cartels, we need to find the expected discounted value each firm gets from the formation of both cartels and compare it to the payoff from the optimal deviation. Since both cartels are individually stable, each cartel survives the detection of the other cartel if and only if the robustness condition holds. Combining conditions (1) and (2), four possible situations arise:

i/ $R_1 \leq \frac{R_2+qF_1}{1-q}$ and $R_2 \leq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$

The individual stability condition in market 2 is more stringent than the robustness conditions for both cartels. Hence, each cartel survives the detection of the other cartel for all $\delta \geq \delta(q, \theta_2)$. Not surprisingly, the expected present discounted value each firm gets from forming both cartels turns out to be always weakly greater than the payoff from the optimal unilateral deviation. As a consequence, the firms form both cartels. Amnesty Plus is neutral because the firms form also both cartels under the EU policy.

ii/ $R_1 > \frac{R_2+qF_1}{1-q}$ and $R_2 \leq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$

The individual stability condition in market 1 is more stringent than the robustness condition in this market. Cartel 1 therefore always survives the detection of cartel 2. Cartel 2 however survives the detection of cartel 1 only if it is robust. If the robustness condition for cartel 2 is satisfied, the analysis is the same as in i/. Hence, the payoff from the optimal unilateral deviation does not exceed the expected present discounted profits from the joint creation of the cartels. The
firms form both cartels, and Amnesty Plus is neutral. However, if cartel 2 is not robust, the firms form both cartels if and only if the joint stability condition holds. Amnesty Plus has a pro-competitive effect in the case where this condition is not satisfied. The firms form only cartel 1 in the US whereas they would have formed both of them in the EU.

\[ \text{iii/ } R_1 < \frac{\pi_2 + qF_2}{1-q} \text{ and } R_2 > \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \]

Cartel 2 is individually stable and robust and therefore always survives the detection of cartel 1. Cartel 1 however survives the detection of cartel 2 only if it is robust. If the robustness condition for cartel 1 holds, then the analysis is the same as in i/. The firms do not find the optimal unilateral deviation profitable and create both cartels. If cartel 1 is not robust, the firms form both cartels if and only if the joint stability condition holds. Otherwise, they form only cartel 1, and Amnesty Plus has a pro-competitive effect.

\[ \text{iv/ } R_1 > \frac{\pi_2 + qF_2}{1-q} \text{ and } R_2 > \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \]

Each cartel survives the detection of the other cartel only if it is robust. If both cartels are robust, the firms form both cartels, and Amnesty Plus is neutral. If either one or none of the robustness conditions holds, the firms form both cartels if and only if the joint stability condition holds. In particular, if none of the cartels is robust, the joint stability condition anyway holds for a non-empty set of discount factor values if the conviction probability \( q \) is small enough. Otherwise, the firms form only cartel 1, and Amnesty Plus has a pro-competitive effect.

5 Discussion

We illustrate our main findings from sections 4.1, 4.2 and 4.3 by means of Figures 1 to 4. Note in particular that Figures 1 and 2 depict the results only for the case where \( F_1 > \frac{2\pi_2 + (1+q)F_2}{1-q} \). In Figure 1, we show the net effect of Amnesty Plus on cartel formation as a function of the fine discount \( R_1 \) for a given \( R_2 \leq \frac{\pi_1 + qF_1}{1-q} \) such that cartel 1, whenever it is individually stable, always survives the detection of cartel 2. Amnesty Plus is neutral for all values of \( \delta \) if \( R_1 \) is sufficiently small, i.e. \( R_1 \leq \frac{\pi_2 + qF_2}{1-q} \). Amnesty Plus has a pro-competitive effect on cartel formation for intermediate values of \( R_1 \), i.e. \( \frac{\pi_2 + qF_2}{1-q} < R_1 < \frac{2\pi_2 + (1+q)F_2}{1-q} \), and for values of \( \delta \) such that both cartels are individually but not jointly stable and such that cartel 2 is not robust to a detection of cartel 1. This region is labeled with a “+”. The firms form only cartel 1 in the US whereas they would have formed both cartels in the EU. As a measure of the size of the effect,
we use the width of the relevant interval of values for $\delta$ on the y-axis. Hence, we can
say that the pro-competitive effect increases between $\frac{\pi_2+qF_2}{1-q}$ and $R_1^*$ where it finally
reaches its maximum. Beyond $R_1^*$, this effect decreases and goes to zero as $R_1$ increases
to $\frac{2\pi_2+(1+q)F_2}{1-q}$. $R_1^*$ is determined by the intersection of the robustness threshold of cartel
2 and the joint stability threshold when cartel 1 is robust which both do not depend on $R_2$. As a consequence, the maximum size of the pro-competitive effect of Amnesty Plus
which is the difference $\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_2}) - \bar{\delta}(q, \theta_2)$ does not involve $R_2$ neither. In the region
labeled with a “+”, Amnesty Plus has an anticompetitive effect on cartel formation. This
effect occurs for higher values of $R_1$, i.e. $R_1 > \frac{2\pi_2+(1+q)F_2}{1-q}$, and for values of $\delta$ such that
cartel 1 is individually stable and robust whereas cartel 2 is not, and the two cartels
are jointly stable. The firms form both cartels in the US whereas, in the absence of
Amnesty Plus, they would have formed cartel 1 alone. The size of the anticompetitive
effect increases as $R_1$ increases from $\frac{2\pi_2+(1+q)F_2}{1-q}$ to $F_1$.

![Figure 1: Effect of Amnesty Plus if $R_2 \leq \frac{\pi_1+qF_1}{1-q}$](image)

Figure 2 depicts the net effect of Amnesty Plus for a given $R_2$ such that $\frac{\pi_1+qF_1}{1-q} <
R_2 < \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$. The only difference with respect to Figure 1 is that the robustness
threshold for cartel 1 is now above its individual stability threshold. The region where
Amnesty Plus has an anticompetitive effect may thus be truncated at the level of the
robustness threshold for cartel 1. Hence, the potential anticompetitive effect of Amnesty
Plus may be more limited while its potential pro-competitive effect remains the same.
In Figure 3 and 4, we show the net effect of Amnesty Plus for a given $R_2 \geq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$. From Proposition 3 we know that Amnesty Plus cannot have any anti-competitive effect in this case. Moreover, note that, in contrast to Figures 1 and 2, Amnesty Plus has a pro-competitive effect on cartel formation for a non-empty range of values of $\delta$ for any value of $R_1 > 0$. In Figure 3, the conviction probability $q$ is very small. In this case, the highest discount factor value for which the pro-competitive effect occurs is close to the individual stability threshold of cartel 2. Hence, the size of the potential pro-competitive effect is rather small. Note in particular that Amnesty Plus cannot have any pro-competitive effect if cartel 1 is robust to a detection of cartel 2. The pro-competitive effect only occurs if cartel 1 is not robust, and both cartels are individually but not jointly stable. The interval of discount factor values where these conditions jointly hold is never empty. The value of $R^*_1$ for which the pro-competitive effect is maximal corresponds to the intersection of the robustness threshold for cartel 2 and the joint stability threshold when both cartels are not robust. Note that, as the latter depends on both $R_1$ and $R_2$, $R^*_1$ depends here on $R_2$. 

Figure 2: Effect of Amnesty Plus if $\frac{\pi_1+qF_1}{1-q} < R_2 < \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$
Figure 3: Effect of Amnesty Plus if $R_2 \geq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$ and $q$ is very small

Figure 4 displays the effect of Amnesty Plus with $q$ not “too small”. There are two main differences with respect to Figure 3. First, the region where the pro-competitive effect of Amnesty Plus occurs is larger. Second, the pro-competitive effect may appear even if cartel 1 is robust to a detection of cartel 2. If this happens, the value of $R^*_1$ is at the intersection of the robustness threshold for cartel 2 and the joint stability threshold when cartel 1 is robust. $R^*_1$ does therefore not depend on $R_2$.

Figure 4: Effect of Amnesty Plus if $R_2 \geq \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$ and $q$ is not too small
6 EC Leniency Program With Multimarket Trigger Strategies

Suppose now that the firms try to sustain repeated collusion by consciously exploiting their multimarket contact and using multimarket trigger strategies. Each firm cooperates in market \( k = 1, 2 \) as long as its partner does. If one firm unilaterally deviates from the illegal agreement in one of the markets, the co-conspirator reacts with a reversion to competition in both markets. As a deviation in one market triggers punishment not only in the market where the deviation occurred but also in the market where the collusive agreement has been respected, the optimal deviation always takes place in both markets simultaneously. By linking the punishment across markets, firms can potentially transfer slack enforcement power from market 1 to market 2 and sustain collusion in both markets for values of \( \delta \) for which only cartel 1 would have been sustainable under single-market contact. We state the formal argument in the following proposition:

**Proposition 5** There exists a threshold \( \tilde{\pi}_2(q) < \pi_1 \) such that:

- **a-** If \( \pi_2 < \tilde{\pi}_2(q) \), i.e. the asymmetry between markets 1 and 2 is sufficiently strong, the use of multimarket trigger strategies enhances the firms’ ability to collude relative to the use of standard trigger strategies. In other words, there exists a threshold \( \tilde{\delta}(q, \theta_1, \theta_2, \lambda) \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \) such that:
  
  1. If \( \delta < \tilde{\delta}(q, \theta_1) \), no cartel is formed.
  2. If \( \tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2) \), cartel 1 is formed whereas cartel 2 is not.
  3. If \( \tilde{\delta}(q, \theta_1, \theta_2, \lambda) \leq \delta \), both cartels are formed.

- **b-** If \( \tilde{\pi}_2(q) \leq \pi_2 \leq \pi_1 \), i.e. the asymmetry between markets 1 and 2 is sufficiently weak, the use of multimarket strategies does not affect the firms’ ability to collude relative to the use of standard trigger strategies.

**Proof.** See Appendix A. ■

In Bernheim and Whinston (1990), multimarket contact does not affect collusive behavior if the markets are identical. In our model, multimarket contact turns out to be irrelevant not only for identical markets but also for markets which are not too different in terms of profitability. This finding can be interpreted as a somewhat broader version

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17 In what follows we suppose that \( \theta_k = \frac{\pi_k}{\pi} \) is strictly decreasing in \( \pi_k \). Otherwise, multimarket trigger strategies cannot have any effect on the firms’ ability to collude.
of the irrelevance result of Bernheim and Whinston (1990), and it has a straightforward explanation. In the presence of an antitrust policy, the firms' may use multimarket trigger strategies to sustain cooperation on both markets just as long as the AA does not detect one of the cartels. However, if the AA detects one of the cartels, the firms cannot use multimarket punishment thereafter, and they can sustain the remaining cartel in subsequent periods only if this cartel is individually stable. Antitrust enforcement may thus shorten the time period during which the firms use multimarket trigger strategies and therefore, it may limit the effect these strategies can have on the ease to sustain cooperation on both markets.

Nonetheless, if markets 1 and 2 are sufficiently asymmetric, the use of multimarket trigger strategies does strengthen the firms’ ability to collude. In particular, the firms form both cartels for a larger range of discount factor values.

7 US Amnesty Program With Multimarket Trigger Strategies

Let us now examine whether the firms may influence the effectiveness of the Amnesty Plus policy by using multimarket trigger strategies. To do this, we need to know how the use of multimarket strategies affects the individual stability, robustness and joint stability conditions.

Consider first the individual stability and the robustness conditions: After the detection of a cartel in one market, the firms interact only in the one remaining market. Since, without multimarket contact, the firms cannot link punishment across markets, they have to use standard trigger strategies to sustain the remaining cartel. The individual stability and the robustness conditions as well as the resulting thresholds therefore are the same as in section 4.

Second, consider the joint stability condition: The use of multimarket trigger strategies may alter the optimal unilateral deviation and thereby affect the joint stability condition. This is because the optimal unilateral deviation occurs always in the two markets with multimarket trigger strategies whereas, with standard trigger strategies, a firm may find it optimal to deviate in one market only. More precisely, if cartel 1 is individually unstable, i.e. $\delta < \tilde{\delta}(q, \theta_1)$, we have shown that the optimal unilateral deviation under both strategies is to deviate in both markets. Hence, the use of multimarket strategies does not affect the joint stability condition and thereby the neutrality of Amnesty Plus. However, if cartel 1 is individually sustainable, i.e. $\delta \geq \tilde{\delta}(q, \theta_1)$, we
have shown that the optimal unilateral deviation occurs only in market 2 when the firms use standard trigger strategies but takes place in both markets when they use multi-market trigger strategies. The joint stability condition $V_{12} \geq V_1 + 2\pi_2$ when trigger strategies are standard becomes $V_{12} \geq 2\pi_1 + 2\pi_2$ when strategies link markets. Since $V_1 \geq 2\pi_1$ whenever $\delta \geq \delta(q, \theta_1)$, the use of multimarket strategies makes the joint stability condition less stringent. Hence, the firms form both cartels for a larger range of discount factor values if they use multimarket rather than standard trigger strategies. In particular, for values of $\delta$ such that $2\pi_1 + 2\pi_2 \leq V_{12} < V_1 + 2\pi_2$ the firms form both cartels when they use multimarket strategies whereas they would have formed only cartel 1 with standard trigger strategies.

From Proposition 5 we know that, if the markets 1 and 2 are sufficiently similar in terms of profitability, multimarket trigger strategies do not affect the set of discount factor values for which the firms create only one, respectively, two cartels. However, since the use of multimarket trigger strategies may lower the joint stability threshold, the firms may anyway want to use these strategies to strengthen the anticompetitive effect of Amnesty Plus and to weaken its pro-competitive effect. If the asymmetry between the markets is strong enough, the use of multimarket strategies enlarges the region of discount factor values for which the firms form both cartels. Hence, the pro-competitive effect of Amnesty Plus may occur for a larger range of discount factor values. At the same time, however, the use of multimarket strategies makes the joint stability condition less stringent and thereby the anticompetitive effect of Amnesty Plus more likely to occur. Similarly, the set of discount factor values for which the firms form only cartel 1 and for which Amnesty Plus may have an anticompetitive effect shrinks with the use of multimarket trigger strategies. However, the joint stability condition becomes less stringent, and the occurrence of the anticompetitive effect more likely. As a consequence, if the markets differ sufficiently in terms of profitability, the net effect of the multimarket trigger strategies is ambiguous.

8 Conclusion

This paper examines the effect of the Amnesty Plus policy on the firms’ incentives to engage in cartel activities. We develop an infinitely repeated interaction framework to highlight the mechanism through which Amnesty Plus encourages, discourages or has no effect on cartel formation when firms use standard and multimarket trigger strategies. US success stories suggest that Amnesty Plus weakens cartel stability. Our analysis shows that this intuition is not always correct.
We find that Amnesty Plus may have an anticompetitive effect by stabilizing a cartel which is individually unstable in the presence of another cartel which is individually stable. If the latter cartel is detected, the firms report the former in the hope of a discount in the fine already imposed. Hence, Amnesty Plus decreases a firm’s expected fine from a cartel conviction such that, for each firm, the joint creation of the cartels may result in an expected discounted value of profits larger than the payoff from the optimal unilateral deviation. The firms would anticipate that the cartels are jointly stable and form both cartels whereas only one of them would have been created under the EU Leniency Program.

Our results also show that, Amnesty Plus may have a pro-competitive effect by destabilizing a cartel which is individually stable. Amnesty Plus decreases the expected present discounted value of profits each firm gets when forming both cartels if, following the detection of one individually stable cartel, the firms report the other individually stable cartel to benefit from the fine discount. The value of the joint creation may then fall below the payoff from the optimal unilateral deviation. The firms anticipate that the cartels are not jointly stable and form only the more profitable of the cartels whereas they would have formed both cartels under the EU policy.

We have also examined whether the firms can exploit their multimarket contact by linking punishment across markets. Amnesty Plus is implemented by the antitrust authority and inherently links the markets. We find that if the markets do not differ substantially in terms of profitability, the use of multimarket trigger strategies can partly offset the destabilizing effect of Amnesty Plus whereas it does not directly affect the firms’ ability to collude. Firms may thus want to adopt multimarket trigger strategies even if their use does not directly facilitate collusion.

Our findings suggest that an antitrust policy with Amnesty Plus may help to increase cartel deterrence insofar as its potential anticompetitive effect could be avoided. We have shown that, by setting the size of the fine discount granted in one market such that it does not exceed the fine a non successful Amnesty Plus applicant would have incurred in the other market, the antitrust authority can avoid any stabilizing effect of this policy. In view of this result, we believe that, for future research, it might be particularly fruitful to elaborate on a thorough normative analysis of how an optimal Amnesty Plus policy should be designed.
A Proofs

Proof of Proposition 1. Note first that if the firms create both cartels at the beginning of a period, the probability that the AA detects both cartels during this period is \( q^2 \), that it detects cartel 1 (cartel 2) whereas it does not detect cartel 2 (cartel 1) is \( q(1 - q) \), and that it detects none of the cartels is \( (1 - q)^2 \). These probabilities follow directly from the independence assumption on the AA’s detection technology.

i/ Assume that \( \delta < \tilde{\delta}(q, \theta_1) \). In this case, both cartels are individually unstable. The firms know that, regardless of their reporting decisions right after the detection of one cartel, they will not be able to sustain the remaining cartel in the following period. Hence, there are two possible Nash Equilibria at the end of a period where the AA detects only one of the cartels: either both firms report the remaining cartel \( k \) where each firm gets an expected payoff of \(-\frac{1}{2}F_k\) or both firms do not report cartel \( k \) where each firm gets a payoff of 0. Since we are looking for the best collusive subgame-perfect equilibrium, we focus on the Pareto superior equilibrium where the firms do not report the remaining cartel. The expected present discounted value each firm gets from the creation of both cartels is

\[
V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1 - q)(\pi_1 + \pi_2 - F_1) + q(1 - q)(\pi_1 + \pi_2 - F_2) + (1 - q)^2(\pi_1 + \pi_2 + \delta V_{12})
\]

which we rewrite as

\[
V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} \quad \leq V_1 \quad \leq V_2
\]  

(A-1)

Since \( V_1 < 2\pi_1 \) and \( V_2 < 2\pi_2 \), the optimal unilateral deviation occurs in both markets. The two cartels are then jointly stable if and only if

\[
V_{12} \geq 2\pi_1 + 2\pi_2
\]

From expression (A-1), it follows that \( V_{12} \leq V_1 + V_2 < 2\pi_1 + 2\pi_2 \). Hence, the optimal unilateral deviation results in a higher payoff than the expected present discounted value each firm gets when forming both cartels. As a consequence, the two cartels are not jointly stable, and the firms do not form both cartels. Since \( V_1 < 2\pi_1 \) and \( V_2 < 2\pi_2 \), the firms do neither form one cartel alone and thus, they form no cartel at all.
ii/ Assume that $\tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2)$. In this case, if the firms form both cartels but the AA detects cartel 2, then, if cartel 1 is not reported, the firms will again form cartel 1 in the next period. However, if the AA detects cartel 1, the firms will not form cartel 2 in the next period. According to the Pareto dominance argument stated in i/, if one cartel is discovered, the firms do not report the remaining cartel in the absence of Amnesty Plus. Hence, the expected present discounted value each firm gets from the creation of both cartels is

$$V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1 - q)(\pi_1 + \pi_2 - F_1) + q(1 - q)(\pi_1 + \pi_2 - F_2 + \delta V_1) + (1 - q)^2(\pi_1 + \pi_2 + \delta V_{12})$$

which we rewrite as

$$V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} = V_1 + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} \leq V_2 < 2\pi_2 \quad (A-2)$$

Since $V_1 \geq 2\pi_1$ and $V_2 < 2\pi_2$, the optimal unilateral deviation is to deviate in market 2 only. This deviation results in a payoff of $V_1 + 2\pi_2$ which is greater than $V_{12}$. Hence, the two cartels are not jointly stable, and the firms therefore do not form both cartels. However, since cartel 1 is individually stable whereas cartel 2 is not, it is a Nash Equilibrium to form cartel 1 alone but not to form cartel 2 without cartel 1.

iii/ Assume that $\tilde{\delta}(q, \theta_2) < \delta$. In this case, if the firms form both cartels but the AA detects one of them, they will again form the remaining cartel in the next period. Hence, the expected present discounted value each firm gets from the creation of both cartels is

$$V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1 - q)(\pi_1 + \pi_2 - F_1 + \delta V_2) + q(1 - q)(\pi_1 + \pi_2 - F_2 + \delta V_1) + (1 - q)^2(\pi_1 + \pi_2 + \delta V_{12})$$

which we rewrite as

$$V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} = V_1 + V_2 \quad (A-3)$$

The optimal deviation occurs in market 2 only (see step 1 in Proof of Proposition 4) which results in a payoff of $V_1 + 2\pi_2$. Since $V_2 \geq 2\pi_2$ this payoff is weakly
smaller than $V_{12}$. Hence, the creation of the two cartels is a Nash Equilibrium. Since $V_k > 0$ which implies that $V_{12} > V_1$ and $V_{12} > V_2$, the payoff from the joint creation of the cartels is higher than the payoffs from both the Nash Equilibrium where the firms form only cartel 1 and the Nash Equilibrium where the firms form only cartel 2. As a consequence, the firms form both cartels.

**Proof of Proposition 2.** We proceed in 3 steps. In step 1, we show that the optimal unilateral deviation occurs in both markets. In step 2, we determine the expected present discounted value $V_{12}$ a firm gets from the creation of the two cartels. We then show in step 3 that the joint stability condition can never be satisfied for any value of $\delta$.

**Step 1.** Since both cartels are individually unstable we know that $V_1 < 2\pi_1$ and $V_2 < 2\pi_2$. Hence, it must be true that $2\pi_1 + 2\pi_2 > V_1 + 2\pi_2$ and $2\pi_1 + 2\pi_2 > 2\pi_1 + V_2$. The optimal unilateral deviation therefore takes place in both markets.

**Step 2.** After the detection of one cartel, reporting the remaining cartel is a dominant strategy for each firm. This equilibrium strategy gives each firm an expected payoff of $\frac{1}{2}R_1 - \frac{1}{2}F_2$ after the detection of cartel 1 and $\frac{1}{2}R_2 - \frac{1}{2}F_1$ after the detection of cartel 2. The expected present discounted value $V_{12}$ is then

$$V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1-q)(\pi_1 + \pi_2 - F_1 + \frac{1}{2}R_1 - \frac{1}{2}F_2) + q(1-q)(\pi_1 + \pi_2 - F_2 + \frac{1}{2}R_2 - \frac{1}{2}F_1) + (1-q)^2(\pi_1 + \pi_2 + \delta V_{12})$$

which we rewrite as

$$V_{12} = \frac{\pi_1 + \pi_2 - q(F_1 + F_2)}{1 - \delta(1-q)^2} - \frac{q(1-q)}{2(1 - \delta(1-q)^2)}(F_1 + F_2 - R_1 - R_2) \quad (A-4)$$

**Step 3.** For the two cartels to be jointly stable, it is necessary and sufficient that the payoff from the optimal unilateral deviation does not exceed $V_{12}$, that is

$$V_{12} \geq 2\pi_1 + 2\pi_2$$

From equation (A-4), we know that $V_{12} \leq V_1 + V_2$ and as both cartels are individually unstable, we have $V_1 + V_2 < 2\pi_1 + 2\pi_2$. Hence, the joint stability condition never holds, and the firms do not form these cartels together. Moreover, since $V_1 < 2\pi_1$ and $V_2 < 2\pi_2$, the firms neither form one cartel alone. ■
Proof of Lemma 1. We first show that the optimal unilateral deviation takes place only in market 2 (step 1). We then determine the expected present discounted value $V_{12}$ each firm gets when forming both cartels (step 2) and derive the joint stability condition from which we easily get the joint stability threshold (step 3).

Step 1. Since cartel 1 is individually stable whereas cartel 2 is not, we have $V_1 \geq 2\pi_1$ and $V_2 < 2\pi_2$. It follows from these two inequalities that $V_1 + 2\pi_2 \geq 2\pi_1 + 2\pi_2 > V_2 + 2\pi_1$. The optimal unilateral deviation therefore occurs in market 2 only.

Step 2. After the detection of the individually stable cartel 1, it is a dominant strategy for each firm to report the individually unstable cartel 2. The expected present discounted value $V_{12}$ is:

$$V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1-q)(\pi_1 + \pi_2 - F_1 + \frac{1}{2}R_1 - \frac{1}{2}F_2) + q(1-q)(\pi_1 + \pi_2 - F_2 + \delta V_1) + (1-q)^2(\pi_1 + \pi_2 + \delta V_{12})$$

We rewrite this expression as

$$V_{12} = V_1 + \frac{\pi_2 - qF_2}{1 - \delta(1-q)^2} + \frac{q(1-q)}{2(1 - \delta(1-q)^2)}(R_1 - F_2) \quad (A-5)$$

Step 3. The formation of both cartels constitutes a Nash Equilibrium if a firm has no incentive to unilaterally deviate from the collusive agreements in both markets. For the two cartels to be jointly stable it is thus necessary and sufficient that $V_{12} \geq V_1 + 2\pi_2$. As $V_{12} \geq V_1$, the joint stability condition also implies that whenever the formation of both cartels is a Nash Equilibrium, it leads to higher profits than the Nash Equilibrium where the firms form cartel 1 only. Rewritten on $\delta$ the joint stability condition becomes

$$\delta \geq \frac{1}{2(1-q)^2} - \frac{1}{4(1-q)^2}\theta_2\left(\frac{R_1}{F_2}q(1-q) - q(3-q)\right) \equiv \delta(q, \theta_2, \frac{R_1}{F_2})$$

The formation of both cartels is a the best collusive equilibrium if and only if $\delta \geq \delta(q, \theta_2, \frac{R_1}{F_2})$. 

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**Proof of Proposition 3.** Depending on the fine discount in market 2, we mainly distinguish three situations:

i/ \( R_2 \leq \frac{\pi_1 + qF_1}{1-q} \)

As \( \tilde{\delta}(q, \theta_1) \geq \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \), cartel 1 survives a detection of cartel 2 for all \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \). The expected present discounted value \( V_{12} \) each firm gets from the creation of both cartels is given in equation (A-5). The formation of both cartels is the best collusive equilibrium if and only if \( \delta \geq \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \).

ii/ \( \frac{\pi_1 + qF_1}{1-q} < R_2 < \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \)

As \( \tilde{\delta}(q, \theta_1) < \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) < \tilde{\delta}(q, \theta_2) \), cartel 1 survives a detection of cartel 2 if and only if \( \delta \in [\tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}), \tilde{\delta}(q, \theta_2)] \). Hence, if \( \delta < \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \) the expression for \( V_{12} \) is the same as in equation (A-4). Since \( V_{12} \leq V_1 + V_2 < V_1 + 2\pi_2 \), the payoff from the optimal unilateral deviation is always strictly higher than the expected present discounted value, and the formation of both cartels is not a Nash Equilibrium. However, if \( \delta \geq \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \) the expression for \( V_{12} \) is given in equation (A-5), and the formation of both cartels is the best collusive equilibrium only if \( \delta \geq \tilde{\delta}(q, \theta_2, \frac{R_2}{\pi_2}) \).

iii/ \( R_2 \geq \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \)

Under this condition we know that \( \tilde{\delta}(q, \theta_2) \leq \tilde{\delta}(q, \theta_1, \frac{R_2}{\pi_1}) \). Cartel 1 does never survive a detection of cartel 2 for any value of \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \), and the expression for \( V_{12} \) is given in equation (A-4). Since \( V_{12} \leq V_1 + V_2 < V_1 + 2\pi_2 \), the formation of both cartels is not a Nash Equilibrium.

**Proof of Proposition 4.** We proceed in 2 steps. In step 1, we show that the optimal unilateral deviation takes place only in market 2. In step 2, we determine the exact circumstances under which the pro-competitive effect of Amnesty Plus occurs.

**Step 1.** Since both cartels are individually stable, we have \( V_1 \geq 2\pi_1 \) and \( V_2 \geq 2\pi_2 \). It follows that \( V_2 + 2\pi_1 \geq 2\pi_1 + 2\pi_2 \), and it is thus sufficient to show that

\[ V_1 + 2\pi_2 \geq V_2 + 2\pi_1 \]
which is the same as

\[ V_1 - 2\pi_1 \geq V_2 - 2\pi_2 \iff \pi_1 \left( \frac{1 - q\theta_1}{1 - \delta(1 - q)} - 2 \right) \geq \pi_2 \left( \frac{1 - q\theta_2}{1 - \delta(1 - q)} - 2 \right) \]

The above inequality holds because \( \pi_1 \geq \pi_2, \theta_1 \leq \theta_2 \) and the expressions in the brackets are positive. The optimal unilateral deviation therefore takes place only in market 2 and yields a payoff of \( V_1 + 2\pi_2 \).

**Step 2.**

i/ \( R_1 \leq \frac{\pi_2 + qF_2}{1-q} \) and \( R_2 \leq \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \)

As \( \tilde{\delta}(q, \theta_2) \geq \hat{\delta}\left( q, \theta_2, \frac{R_1}{\pi_1} \right) \) and \( \tilde{\delta}(q, \theta_2) \geq \hat{\delta}\left( q, \theta_1, \frac{R_2}{\pi_2} \right) \), each cartel survives the detection of the other cartel for all \( \delta \geq \tilde{\delta}(q, \theta_2) \). Thus, the expected present discounted value \( V_{12} \) each firm gets when forming both cartels is:

\[
V_{12} = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1-q)(\pi_1 + \pi_2 - F_1 + \delta V_2) + q(1-q)(\pi_1 + \pi_2 - F_2 + \delta V_1) + (1-q)^2(\pi_1 + \pi_2 + \delta V_{12})
\]

We can rewrite this expression as

\[
V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} = V_1 + V_2
\]

Since \( V_2 \geq 2\pi_2 \), the payoff from the unilateral optimal deviation does not exceed \( V_{12} \), and the formation of both cartels is the best collusive equilibrium. The firms create both cartels, and Amnesty Plus is neutral.

ii/ \( R_1 > \frac{\pi_2 + qF_2}{1-q} \) and \( R_2 \leq \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} \)

Cartel 1 survives the detection of cartel 2 for all \( \delta \geq \tilde{\delta}(q, \theta_2) \) whereas cartel 2 survives the detection of cartel 1 if and only if \( \delta \geq \hat{\delta}\left( q, \theta_2, \frac{R_1}{\pi_2} \right) \). If \( \delta \geq \tilde{\delta}\left( q, \theta_2, \frac{R_1}{\pi_2} \right) \), the analysis is the same as in i/ and leads to the result that the firms form both cartels, and Amnesty Plus is neutral. However, if \( \tilde{\delta}(q, \theta_2) \leq \delta < \hat{\delta}\left( q, \theta_2, \frac{R_1}{\pi_2} \right) \), the expression for \( V_{12} \) is given in equation (A-5). The firms form both cartels only if the joint stability condition holds, that is \( \delta \geq \hat{\delta}\left( q, \theta_2, \frac{R_1}{\pi_2} \right) \). From Proposition 3, we know that \( \tilde{\delta}(q, \theta_2) > \hat{\delta}\left( q, \theta_2, \frac{R_1}{\pi_2} \right) \) if \( R_1 > \frac{1+q}{1-q}F_2 + \frac{2\pi_2}{1-q} \). We thus distinguish between two subcases:
a/ $R_1 > \frac{1+q}{1-q} F_2 + \frac{2\pi_2}{1-q}$

Since $\delta(q, \theta_2) > \delta\left(q, \theta_2, \frac{F_2}{\pi_1}\right)$, we have $\delta > \delta\left(q, \theta_2, \frac{F_2}{\pi_2}\right)$ for all $\delta \in \left[\tilde{\delta}(q, \theta_2), \hat{\delta}\left(q, \theta_2, \frac{F_2}{\pi_2}\right)\right]$. The firms form both cartels for any $\delta$ in this interval.

b/ $R_1 \leq \frac{1+q}{1-q} F_2 + \frac{2\pi_2}{1-q}$

Since $\tilde{\delta}(q, \theta_2) \leq \hat{\delta}\left(q, \theta_2, \frac{F_2}{\pi_2}\right)$, the firms form both cartels if $\delta \geq \tilde{\delta}(q, \theta_2)$. In the latter case, Amnesty Plus has a pro-competitive effect on cartel formation.

iii/ $R_1 \leq \frac{\pi_2+qF_2}{1-q}$ and $R_2 > \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)}$

Cartel 2 survives the detection of cartel 1 for all $\delta \geq \tilde{\delta}(q, \theta_2)$, whereas cartel 1 survives the detection of cartel 2 if and only if $\delta \geq \hat{\delta}\left(q, \theta_1, \frac{F_2}{\pi_1}\right)$. If $\delta \geq \tilde{\delta}(q, \theta_1, \frac{F_2}{\pi_1})$, the analysis is the same as in i/ and leads to the result that the firms form both cartels. However, suppose now that $\tilde{\delta}(q, \theta_2) \leq \delta < \hat{\delta}\left(q, \theta_1, \frac{F_2}{\pi_1}\right)$. For this range of discount factor values, we derive the expression for $V_{12}$ from equation (A-5) by swapping 1 and 2, that is

$$V_{12} = V_2 + \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{q(1 - q)}{2(1 - \delta(1 - q)^2)}(R_2 - F_1) \quad (A-6)$$

The formation of both cartels is the best collusive equilibrium if and only if the cartels are jointly stable. This is equivalent to

$$\begin{align*}
A \leq 0: & \quad \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} - \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} - \frac{2\pi_2}{1 - \delta(1 - q)^2} \\
B \geq 0: & \quad + \frac{q(1 - q)}{2(1 - \delta(1 - q)^2)}(R_2 - F_1) \geq 0 \\
C \leq 0: & \quad \frac{\pi_2}{1 - \delta(1 - q)^2} - \frac{2\pi_2}{1 - \delta(1 - q)^2} - 2\pi_2.
\end{align*} \quad (A-7)$$

Note that inequality (A-7) does not depend on $R_1$. Let us show that the set of values $\delta \in \left[\tilde{\delta}(q, \theta_2), \hat{\delta}\left(q, \theta_1, \frac{F_2}{\pi_1}\right)\right]$ which satisfies inequality (A-7) is not empty if the detection probability is small enough. Setting $\delta$ to its upper bound $\hat{\delta}\left(q, \theta_1, \frac{F_2}{\pi_1}\right)$, the terms $A$ and $C$ go to 0 as $q \to 0$ whereas the term $B$ goes to $-\frac{\pi_2}{\pi_1 + \pi_2} - 2\pi_2$. This expression is strictly positive since $R_2 > \frac{(1+q\theta_2)(\pi_1-qF_1)}{(1-q)(1-q\theta_2)} > \pi_1$. Hence, we can
conclude that for $q$ sufficiently small and $\delta$ close enough to $\hat{\delta}(q, \theta_1, R_{\pi_1})$, condition (A-7) holds. Moreover, we can say that the set of values $\delta \in \left[\hat{\delta}(q, \theta_2), \hat{\delta}(q, \theta_1, R_{\pi_1})\right]$ which does not satisfy condition (A-7) is never empty whatever the value of $q > 0$.

This is so because inequality (A-7) does not hold for $\delta = \hat{\delta}(q, \theta_2)$ and thus, due to the continuity of the LHS of (A-7) with respect to $\delta$, it does not hold for $\delta$ sufficiently close to $\hat{\delta}(q, \theta_2)$. Therefore, the set over which Amnesty Plus has a pro-competitive effect within the interval $\left[\hat{\delta}(q, \theta_2), \hat{\delta}(q, \theta_1, R_{\pi_1})\right]$ is never empty.

iv/ $R_1 > \frac{\pi_2 + qF_2}{1-q}$ and $R_2 > \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)}$

We have $\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}) \geq \hat{\delta}(q, \theta_2)$ and $\hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2}) \geq \hat{\delta}(q, \theta_2)$. If $\delta \geq \max\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)$ the firms form both cartels, and Amnesty Plus is neutral. If $\hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2}) \leq \delta < \hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1})$ we get $V_{12}$ from equation (A-5), and the analysis is the same as in ii/. If $\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}) \leq \delta < \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})$ we get $V_{12}$ from equation (A-6), and the analysis from iii/ applies. If $\delta(q, \theta_2) \leq \delta < \min\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)$ the expression for $V_{12}$ is given in (A-4), and the firms form both cartels if and only if the joint stability condition holds, that is

$$
\begin{align*}
\left(\frac{\pi_1 - qF_1}{1 - \delta(1-q)^2} - \frac{\pi_1 - qF_1}{1 - \delta(1-q)}\right) + \frac{\pi_2 - qF_2}{1 - \delta(1-q)^2} - 2\pi_2
\end{align*}
$$

for $D \leq 0$ or $E \geq 0$ or $F \leq 0$

$\frac{q(1-q)}{2(1-\delta(1-q)^2)}(R_1 + R_2 - F_1 - F_2) \geq 0$ for $F \leq 0$

Let us show that the set of values $\delta \in \left[\hat{\delta}(q, \theta_2), \min\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)\right]$ which satisfies inequality (A-8) is not empty if $q$ is sufficiently small. Setting $\delta$ to its upper bound $\min\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)$, the terms $D$ and $F$ go to 0 as $q \to 0$ whereas the term $E$ goes to $1 - \min\left(\frac{\pi_2}{\pi_1 + \pi_2}, \frac{R_1}{R_{\pi_1}}, \frac{R_2}{R_{\pi_2}}, \frac{R_1}{R_{\pi_1}}, \frac{R_2}{R_{\pi_2}}\right)$, $2\pi_2$ which is strictly positive since $R_2 > \frac{(1+q\theta_2)(\pi_1 - qF_1)}{(1-q)(1-q\theta_2)} > \pi_1$ and $R_1 > \frac{\pi_2 + qF_2}{1-q} > \pi_2$. Therefore, for $q$ sufficiently small and for $\delta$ close enough to $\min\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)$ condition (A-8) holds. Moreover, the set of values $\delta \in \left[\hat{\delta}(q, \theta_2), \min\left(\hat{\delta}(q, \theta_2, \frac{R_1}{\pi_1}), \hat{\delta}(q, \theta_1, \frac{R_2}{\pi_2})\right)\right]$. 

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which does not satisfy condition (A-8) is never empty for any value of \( q > 0 \). This is because condition (A-8) does not hold for \( \delta = \hat{\delta}(q, \theta_2) \) and, due to the continuity of its LHS with respect to \( \delta \), it does not hold for \( \delta \) sufficiently close to \( \hat{\delta}(q, \theta_2) \). Therefore, the set over which Amnesty Plus has a pro-competitive effect within the interval \( \left[ \hat{\delta}(q, \theta_2), \min \left( \delta \left( q, \theta_2, \frac{\rho_2}{\pi_2} \right), \hat{\delta} \left( q, \theta_1, \frac{\rho_1}{\pi_1} \right) \right) \right] \) is never empty.

\[ \textbf{Proof of Proposition 5.} \] Note first that when firms use multimarket strategies, the optimal unilateral deviation is to deviate in both markets since punishment occurs in both markets. A firm’s payoff from such an optimal deviation is \( 2\pi_1 + 2\pi_2 \).

i/ Assume that \( \delta < \tilde{\delta}(q, \theta_1) \). From the analysis of the EU antitrust policy when firms use standard trigger strategies and, especially, from the proof of Proposition 1, we know that the expected present discounted value \( V_{12} \) each firm gets from the creation of both cartels is equal to expression (A-1):

\[
V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1-q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1-q)^2} \leq V_1 + V_2 < 2\pi_1 + 2\pi_2
\]

A unilateral deviation in both markets is therefore always profitable, and the two cartels are not jointly stable. Hence, the firms do not form both cartels. Moreover, since \( V_1 < 2\pi_1 \) and \( V_2 < 2\pi_2 \), they do not form only one of the cartels either.

ii/ Assume that \( \tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2) \). From (A-2) we get the expression for \( V_{12} \) which is:

\[
V_{12} = V_1 + \frac{\pi_2 - qF_2}{1 - \delta(1-q)^2} \leq V_2 < 2\pi_2
\]

Since cartel 1 is individually stable whereas cartel 2 is not, it is a Nash Equilibrium to form cartel 1 alone, whereas it is not a Nash Equilibrium to form cartel 2 without cartel 1. The formation of both cartels is the best collusive equilibrium if and only if \( V_{12} \geq 2\pi_1 + 2\pi_2 \). Note that this condition is less stringent than its counterpart when firms use standard trigger strategies, i.e. \( V_{12} \geq V_1 + 2\pi_2 \). The joint stability condition is therefore more easily satisfied with multimarket strategies which supports the intuition that multimarket contact is basically pro-collusive. We provide, however, a more detailed analysis of this argument: Note
that, since \( V_{12} \) is increasing in \( \delta \) over \([\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)]\), a necessary and sufficient condition for the inequality \( V_{12} \geq 2\pi_1 + 2\pi_2 \) to hold over a non-empty range of values of \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \) is that \( V_{12}(\tilde{\delta}(q, \theta_2)^-) > 2\pi_1 + 2\pi_2 \). After some algebraic manipulations, we can write the following:

\[
V_{12}(\tilde{\delta}(q, \theta_2)^-) - (2\pi_1 + 2\pi_2) = 2\pi_1 \frac{q(\theta_2 - \theta_1)}{1 - q\theta_2} - 2\pi_2 \frac{q(1 + q\theta_2)}{1 - q\theta_2 + q(1 + q\theta_2)}
\]

Therefore,

\[
V_{12}(\tilde{\delta}(q, \theta_2)^-) > 2\pi_1 + 2\pi_2 \quad \text{(A-9)}
\]

\[
\iff \frac{(\theta_2 - \theta_1)(1 - q\theta_2 + q(1 + q\theta_2))}{1 - q^2\theta_2^2} \cdot \frac{\pi_1}{\pi_2} > 1
\]

term A

We show that term A is increasing in \( \theta_2 \) over \([\theta_1, \frac{1}{q} - 2]\) by differentiating it with respect to \( \theta_2 \). Since \( \theta_2 = \frac{F(\pi_2)}{\pi_2} \) is decreasing in \( \pi_2 \), term A is also decreasing in \( \pi_2 \) over \([0, \pi_1]\). Term B, which is the same as \( \frac{1}{\lambda} \), is clearly decreasing in \( \pi_2 \). Then, the LHS of inequality (A-9) which is the multiplication of terms A and B is decreasing in \( \pi_2 \) over \([0, \pi_1]\). Moreover, it is continuous in \( \pi_2 \), goes to \( +\infty \) as \( \pi_2 \to 0 \) and takes the value 0 for \( \pi_2 = \pi_1^- \). Using the intermediate value theorem, we can say that there exists a threshold \( \bar{\pi}_2(q) \) such that inequality (A-9) is satisfied if and only if \( \pi_2 < \bar{\pi}_2(q) \). Hence, we distinguish two subcases:

- If \( \pi_2 < \bar{\pi}_2(q) \) then \( V_{12} > 2\pi_1 + 2\pi_2 \) for \( \delta = \tilde{\delta}(q, \theta_2)^- \), whereas \( V_{12} < 2\pi_1 + 2\pi_2 \) for \( \delta = \tilde{\delta}(q, \theta_1) \). Since \( V_{12} \) is continuous and increasing in \( \delta \), we can again use the intermediate value theorem to conclude that there exists a threshold \( \tilde{\delta}(q, \theta_1, \theta_2, \lambda) = \tilde{\delta} \) such that \( V_{12} < 2\pi_1 + 2\pi_2 \) for \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}] \) and \( V_{12} \geq 2\pi_1 + 2\pi_2 \) for \( \delta \in [\tilde{\delta}, \tilde{\delta}(q, \theta_2)] \). Hence, the formation of both cartels is the best collusive equilibrium for \( \delta \in [\tilde{\delta}, \tilde{\delta}(q, \theta_2)] \) but not for \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}] \).

- If \( \pi_2 \geq \bar{\pi}_2(q) \) then \( V_{12} \leq 2\pi_1 + 2\pi_2 \) for \( \delta = \tilde{\delta}(q, \theta_2)^- \). It follows that the inequality \( V_{12} < 2\pi_1 + 2\pi_2 \) holds for all \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \) which implies that forming both cartels is never a Nash Equilibrium for \( \delta \in [\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2)] \).
iii/ Assume that $\delta \geq \tilde{\delta}(q, \theta_2)$. In this case, we get the expression for $V_{12}$ from (A-3), that is:

$$V_{12} = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} = V_1 + V_2 \geq 2\pi_1 + 2\pi_2$$

From the above inequality, it is straightforward that the formation of both cartels is the best collusive equilibrium for all $\delta > \tilde{\delta}(q, \theta_2)$.

\[\]

### B Extensions

#### B.1 Relaxation of the Assumption $\theta_2 \geq \theta_1$

Suppose now that the opposite assumption $\theta_1 > \theta_2$ holds. The direct implication, albeit somewhat counterintuitive, is that cartel 2 is easier to sustain than the more profitable cartel 1, i.e. $\tilde{\delta}(q, \theta_2) < \tilde{\delta}(q, \theta_1)$.

It is straightforward that Proposition 1 remains valid, provided that we reverse the subscripts 1 and 2. Hence, under the EU antitrust policy, the firms form a cartel if and only if it is individually stable.

Proposition 2 remains true, but we have to substitute $\tilde{\delta}(q, \theta_2)$ for $\tilde{\delta}(q, \theta_1)$, or more generally, if we do not make any assumption on the size of $\theta_1$ relative to $\theta_2$, we substitute \[\min (\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2))\] for $\tilde{\delta}(q, \theta_1)$. Hence, if no cartel is individually stable, the firms do not form any of the cartels under the US antitrust policy, and Amnesty Plus is still neutral.

It is easy to show that by reversing the subscripts 1 and 2, the first part of Proposition 3 still holds. However, whereas the necessary and sufficient conditions for Amnesty Plus to have an anticompetitive effect which we provide in the second part of Proposition 3 may be satisfied under the initial assumption $\theta_2 \geq \theta_1$, the reverse is not true. One of the new conditions defining a non-empty range of values of $\delta$ for which Amnesty Plus has an anticompetitive effect would be $R_2 > \frac{1+q}{1-q}F_1 + \frac{2\pi_1}{1-q}$. Hence, since we have $R_2 \leq F_2 \leq F_1 < \frac{1+q}{1-q}F_1 + \frac{2\pi_1}{1-q}$, the latter condition cannot be satisfied, and the potential anticompetitive effect of Amnesty Plus cannot occur.

Proposition 4 remains valid, although we have to substitute $\tilde{\delta}(q, \theta_1)$ for $\tilde{\delta}(q, \theta_2)$. More generally, if we do not want to make any particular assumption on the relative size of $\theta_1$ and $\theta_2$, we substitute $\max (\tilde{\delta}(q, \theta_1), \tilde{\delta}(q, \theta_2))$ for $\tilde{\delta}(q, \theta_2)$. Hence, Amnesty Plus may still have a pro-competitive effect on cartel formation.
B.2 Unrestricted Strategy Choice

We relax the assumption that the firms can form a cartel in a period $t > 0$ only if this cartel has been formed in the previous period. The key difference with respect to our initial time structure is that, if the firms form only one cartel in some period, and the AA detects this cartel during this period, they still have to possibility to form the other cartel in the following period. More precisely, we modify the timing within a period $t \geq 1$ as follows: If no cartel has been convicted in the previous period, the time structure of the latter applies to the current period. If both cartels have been formed and convicted in the previous period, the firms compete in both markets. If either one cartel has been formed and not convicted or both cartels have been formed and only one has been convicted, the timing is the same as the one presented in stages 0 to 3 of our initial set-up.

The modification of the time structure does not affect our results under the EC Leniency Program. In particular, Proposition 1 remains valid. Neither does the modification affect the results under the US Amnesty Program with Amnesty Plus for $\delta < \tilde{\delta}(q, \theta_1)$ and $\tilde{\delta}(q, \theta_1) \leq \delta < \tilde{\delta}(q, \theta_2)$. Both the neutrality of Amnesty Plus result in Proposition 2 and the result on the potential anticompetitive effect of Amnesty Plus stated in Proposition 3 still hold.

Allowing for an unrestricted strategy choice, however, alters our results in the region where Amnesty Plus may have a pro-competitive effect, i.e. for $\delta \geq \tilde{\delta}(q, \theta_2)$. The fact that the firms may now start forming a cartel in a period $t \geq 1$ gives rise to a new equilibrium where they form cartel 1 until it is detected and then form cartel 2. This is an equilibrium because for $\delta \geq \tilde{\delta}(q, \theta_2)$, both cartels are individually stable. Amnesty Plus cannot prevent such an outcome. Each firm gets an expected payoff $V'_1$ such that

$$V'_1 = q(\pi_1 - F_1 + \delta V_2) + (1 - q)(\pi_1 + \delta V'_1)$$

The AA detects cartel 1 with probability $q$ in which case the firms form cartel 2 in the following period. With probability $(1 - q)$ the AA does not detect cartel 1, and the firms form it again in the following period. Solving the above equation for $V'_1$ we get

$$V'_1 = V_1 + q \frac{\delta}{1 - \delta(1 - q)} V_2$$

For the joint formation of both cartels to be the most profitable equilibrium, two conditions must hold: First, $V_{12} \geq V_1 + 2\pi_2$, i.e. the optimal unilateral deviation must not be profitable, second, $V_{12} \geq V'_1$, i.e. the equilibrium where the firms form both cartels
must be more profitable than the equilibrium where the firms form first cartel 1 and then only, if detected, cartel 2. We can combine these two conditions as follows:

\[ V_{12} \geq \max (V_1 + 2\pi_2, V_1^2) = \max \left( V_1 + 2\pi_2, V_1 + \frac{q}{1 - \delta(1 - q)} V_2 \right) \]

It is straightforward that the above condition is weakly more stringent than the condition \( V_{12} \geq V_1 + 2\pi_2 \). Moreover, it is strictly more stringent than the latter for at least some values of the parameters \( \delta \) and \( q \). This is because \( q\frac{\delta}{1 - \delta(1 - q)} V_2 = q\frac{\delta}{1 - \delta(1 - q)} \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} \) and \( \lim_{\delta \to 1} \left( \lim_{q \to 0} q\frac{\delta}{1 - \delta(1 - q)} \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} \right) = +\infty \). Hence, for small values of \( q \) the expression \( q\frac{\delta}{1 - \delta(1 - q)} V_2 \) can be greater than \( 2\pi_2 \) for values of \( \delta \) sufficiently close to 1.

We have shown that allowing for an unrestricted strategy choice does not affect the potential anticompetitive effect of Amnesty Plus but has an ambiguous impact on its pro-competitive effect. On the one hand, the region where the pro-competitive effect occurs may be larger because it may be more difficult to achieve the joint stability of the cartels. On the other hand, since Amnesty Plus can only deter the formation of cartel 2 as long as cartel 1 goes undetected, the pro-competitive effect of Amnesty Plus, if it occurs, is weaker relative to our previous findings.

References


