A Reappraisal of the Allocation Puzzle through the Portfolio Approach ∗

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This version: May 2010

Abstract

Paradoxically, high-investment and high-growth developing countries tend to experience capital outflows. This paper shows that this allocation puzzle can be explained simply by introducing uninsurable idiosyncratic investment risk in the neoclassical growth model. Using a sample of 67 countries between 1980 and 2003, we show that the model predicts accurately the allocation of capital flows in this sample. This is because the model accounts for two main facts: (i) TFP growth is positively correlated with capital outflows in a sample including creditor countries; (ii) the long-run level of capital per efficient unit of labor is positively correlated with capital outflows.

Key Words: Capital flows, Global imbalances, Investment risk.

JEL Class.: F36, F41, F43.

∗I would like to thank Mark Aguiar, Philippe Bacchetta, Agnès Bénassy-Quéré, Nicolas Berman, Daniel Cohen, Jean Imbs, Imen Ghattassi, Pierre-Olivier Gourinchas, Sebnem Kalemli-Özcan, Guy Laroque, Philippe Martin, Valérie Mignon, Romain Rancière and Hélène Rey for helpful comments. I also thank seminar participants at Banque de France, CREST, GREQAM, Paris School of Economics, the University of Lausanne, the University of Le Mans, the University of Evry, the European Economic Association 2009 (Barcelona), Theories and Methods of Macroeconomics (T2M) 2009 (Strasbourg), Research in International Economics and Finance 2009 (Aix-Marseille) and the North American Winter Meeting of the Econometric Society 2010 (Atlanta).

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1 Introduction

Empirical studies on capital flows to developing countries suggest that the predictions of the neoclassical growth model (Ramsey-Cass-Koopmans) are not verified in the data. This model predicts that capital should flow to countries with high growth, both to take advantage of investment opportunities and to smooth consumption. However, the opposite holds in reality: as Aizenman and Pinto (2007) and Prasad et al. (2007) show capital flows to countries with low growth and low investment. Consequently, as Gourinchas and Jeanne (2007) show, capital outflows predicted by the neoclassical model are negatively correlated with actual outflows. Providing an explanation to this puzzle is the objective of this paper. Explaining the allocation puzzle would also help understand the phenomenon of global imbalances, and especially their structure. In particular, why are surpluses originated in Asia and not in Latin America or Africa? Indeed, Asia has experienced relatively high growth rates and should have imported capital instead of exporting it.

The contribution of this paper is twofold. Theoretically, it provides a framework generating a positive correlation between growth and capital outflows and gives the conditions and channels through which such a correlation emerges. This framework involves simply an extension of the neoclassical growth model that accounts for uninsurable idiosyncratic investment risk. Empirically, using a sample of 67 countries between 1980 and 2003, we show that the model predicts accurately the allocation of capital flows in this sample. This is because the model accounts for two main facts: (i) TFP growth is positively correlated with capital outflows in a sample including creditor countries; (ii) the long-run level of capital per efficient unit of labor is positively correlated with capital outflows.

The theoretical part of this paper builds on Gourinchas and Jeanne (2007) but deviates from their approach in two important aspects. First, while they focus on the role of TFP in channelling the correlation between growth and capital outflows, we enlarge the set of potential channels by taking into account the role of capital per efficient unit of labor. Indeed, if TFP is the only source of growth, the stock of capital per efficient unit of labor should be equal across countries in the long run. However, this variable does differ substantially across countries.¹ Gourinchas and Jeanne do introduce a capital wedge (i.e. a distortion between the private and social return to capital) in order to account for these differences in capital stock, but only for robustness purposes. In our model and because of the presence of risk, it plays a crucial role.

Figures 1 and 2 illustrate and motivate this point, using our sample of developing countries. They represent the cross-country correlation of capital outflows between 1980

¹See Lucas (1990), Hsieh and Klenow (2007) and Caselli and Feyrer (2007). The volatility of the capital stock per efficient units of labor is reflected in Caselli and Feyrer by the volatility of their “naïve estimates” of the marginal product of capital and by the volatility of investment rates in Hsieh and Klenow.
and 2003 respectively with TFP growth and with the end-of-period capital per efficient unit of labor, where TFP growth and the capital per efficient unit of labor are estimated using a growth-accounting method. The correlation is positive for both variables, but it is more robust for the capital-labor ratio than for TFP growth. Actually, the correlation of capital outflows with TFP growth seems to be driven by a few outliers, which is not the case with capital per efficient unit of labor. The latter seems then to be a better candidate to explain the positive empirical correlation of growth and investment with capital outflows.

The second deviation from Gourinchas and Jeanne's approach is the introduction of uninsurable individual investment risk. As compared to the standard model, the presence of investment risk has a crucial implication: the composition of the portfolio matters. In particular, when capital and therefore risk-taking increases, capital outflows may result from a precautionary need for buffer-stock assets. However, whether such an effect actually emerges depends on the sources of growth.

The role of the different sources of growth in explaining the puzzle are studied using Angeletos and Panousi (2009)'s framework, which boils down to a Merton-Samuelson portfolio choice problem between risky capital and safe assets, including human wealth and external bonds. This framework allows to nest both the riskless approach and the approach with risk, which we call the portfolio approach. This model is extended to allow, as in Gourinchas and Jeanne (2007), for TFP growth and a capital wedge accounting for the differences in the long-run capital stock per efficient units of labor.

In this framework, the domestic capital stock is a constant fraction of total wealth in the long run, and this desired portfolio composition dominates the traditional investment and consumption smoothing effects. More specifically, if individual risk is relatively high, which is the case in developing countries, then the fraction of capital in total wealth is lower than one. This means that risk-taking through capital accumulation should be compensated by a proportional increase in non-capital wealth, which is used as a buffer stock to face that risk. Importantly, in the neoclassical growth model, non-capital wealth is composed of foreign assets, but also of human wealth, that is the discounted sum of future labor revenues. Capital outflows should then follow growth only if the increase in the capital stock is not matched by a sufficient increase in human wealth.

Consequently, since the relationship between capital and human wealth depends on the sources of growth (i.e. TFP versus capital wedge), the relationship between growth and capital outflows depends also on the sources of growth. TFP does not affect the composition of domestic assets between capital and human wealth and therefore does not affect the ratio between capital and foreign assets. As a result, an increase in the capital stock stemming from TFP growth translates into capital inflows if the country is a net debtor and into capital outflows if the country is a net creditor. On the opposite, the capital wedge affects the composition of domestic assets. For example, a country

\footnote{The data and calibration method are described fully in Section 3.}
with a lower capital wedge will have a higher capital stock. But, because capital has
decreasing marginal returns, the effect on labor income and therefore human wealth is
limited. As a consequence, capital outflows may arise in order to maintain a constant
ratio of capital to non-capital assets, even if the country is a net debtor.

This simple extension of the standard neoclassical growth model provides us with an
interpretation of the preliminary evidence given in Figures 1 and 2. First, the correlation
between capital outflows and TFP growth is conditional on the sign of the external
position. An explanation of the instability of the empirical correlation would then be
that it is driven by a few creditor countries, and should become negative if we exclude
them. Second, the correlation between capital outflows and the long-run capital stock
per efficient units of labor should be more robust to the exclusion of countries with
positive external positions.

These predictions are tested, and confirmed, on a sample of 67 developing countries
between 1980 and 2003. We then use this evidence to reinterpret the allocation puzzle,
that is the observed positive correlation of capital outflows with GDP growth and in-
vestment. We find that the capital stock per efficient units of labor is the main channel
of these correlations.

Finally, we perform a calibration analysis where the amount of capital flows pre-
dicted by the model are computed and compared to the data. While, consistently with
Gourinchas and Jeanne (2007), predicted capital flows are negatively correlated with the
actual ones in the riskless approach, the correlation becomes positive in the portfolio
approach. The portfolio approach also reproduces accurately the patterns of capital
flows across regions.

Our approach based on portfolio choice is close to Kraay and Ventura (2000). They
explain the current account dynamics by using a portfolio choice rule similar to ours:
capital is a constant fraction of total wealth. However, in their model, domestic wealth
includes only capital, whereas we take into account human wealth, which plays a crucial
role in our approach. Indeed, the way the different channels of growth relate to the
composition of domestic wealth is essential in generating our results. In the case of
TFP-driven growth, we find an effect similar to their New Rule, that is: the correlation
between investment and capital outflows depends on the sign of the net foreign asset
position. However, this effect arises because TFP affects proportionally the different
components of domestic wealth, which is not the case when investment is driven by a
lower capital wedge.

Some theoretical papers, like Mendoza et al. (2009) and Sandri (2008), explain global
imbalances by the presence of investment risk, or more generally by the weakness of
financial development in emerging countries. However, the structure of these imbalances

\[\text{See, among others, Dooley et al. (2005), Matsuyama (2007), Ju and Wei (2006, 2007), Caballero et al. (2008), Song et al. (2009).}\]
and the allocation puzzle have been overlooked. Indeed, to explain global imbalances, most researchers either consider emerging markets as a whole or take Asia (and especially China) in isolation. Aguiar and Amador (2009) are an exception. They explain why countries that grow rapidly tend to accumulate net foreign assets rather than liabilities by introducing political economy and contracting frictions. However, their approach is silent on the respective role of the factors of growth.

The remainder of the paper is organized as follows: Section 2 presents the neoclassical growth model with investment risk and its predictions; Section 3 presents the data and the calibration method; Section 4 tests the predictions of the model regarding the determinants of capital flows and reinterprets the allocation puzzle by investigating the channels of the positive correlation of capital outflows with investment and growth; and finally, Section 5 performs a calibration analysis by comparing actual and predicted flows.

2 The neoclassical growth model with idiosyncratic investment risk

In this section, we study the implications in terms of capital flows of the neoclassical growth model with investment risk. To achieve this, we use Angeletos and Panousi (2009)'s model, which generalizes the tractable Merton-Samuelson portfolio choice problem to account for human wealth. This model is extended here to account, as in Gourinchas and Jeanne (2007), for TFP growth and a capital wedge, that is a distortion between the private and social return to capital.

The model is then used to infer the amount of capital flows between an initial period and the steady state and to study the impact of the different factors of growth (TFP catch-up and the capital wedge) on these capital flows.

2.1 The household’s program

Consider a small open economy with a continuum of length 1 of infinitely-lived households, or families, indexed by $i$. Time is continuous, indexed by $t \in [0, \infty)$. Each household $i$ owns a firm which produces a homogeneous good using two factors, labor and capital, according to a Cobb-Douglas technology:

$$ Y_i^t = F(K_i^t, A_t N_i^t) = K_i^{\alpha t} (A_t N_i^t)^{1-\alpha}, 0 < \alpha < 1 $$

where $K_i^t$ is the amount of private capital invested in firm $i$ at date $t$, $N_i^t$ the amount of labor hired in the firm and $A_t$ the deterministic domestic level of technology.
Financial markets are incomplete: households can invest only in their own firm and cannot trade equity. They face an idiosyncratic investment risk, but cannot diversify away this risk through equity or any other means. The only freely traded asset, domestically and internationally, is the riskless bond $B_t$ whose return is fixed internationally to $R^*$. Denote household $i$’s net capital income by $dQ_i^t$. It is defined as follows:

$$dQ_i^t = (1 - \tau)[F(K_i^t, A_t N_i^t) - w_t N_i^t]dt - \delta K_i^t dt + \sigma K_i^t dv_i^t$$

where $w_t$ the wage rate, which is not firm-specific since the labor market is competitive. $\delta$ is the depreciation rate.

$\tau$ is a wedge on the gross capital return. Following Gourinchas and Jeanne (2006, 2007), it is introduced in order to account for the cross-country differences in capital-labor ratios that are not attributable to TFP. This wedge can be interpreted as a tax on capital income, or as the result of other distortions that would introduce a difference between social and private returns. We assume that the product of this tax $Z_t$ is distributed equally among households:

$$Z_t = \int_0^1 \tau[K_i^t A_t N_i^t]^{1-\alpha} - w_t N_i^t]di$$

The technology is exactly identical to Gourinchas and Jeanne (2007), except that time is continuous and that firms face investment risk. This risk is introduced here through a standard Wiener process $dv_i^t$. This process is iid across agents and time and satisfies $E[dv_i^t = 0]$. This risk can be interpreted as a production or a depreciation shock that affects the return on capital. It is assumed that this shock is averaged out across households, that is: $\int_0^1 dv_i^t = 0$. The parameter $\sigma$ measures the amount of individual risk. Gourinchas and Jeanne (2007)’s specification is nested when $\sigma = 0$. In the case with risk, market incompleteness is important: if households were able to diversify perfectly their investment risk, then the model would boil down to the standard neoclassical model without risk.\(^4\)

Each household is composed of $N_t$ members, and each member is endowed with 1 unit of labor which he supplies inelastically in the competitive labor market. Since, additionally, there is no aggregate risk, all aggregate variables are deterministic, including the competitive individual wage $w_t$. There is neither unemployment risk nor any other shock that would introduce an income risk besides investment risk. Therefore, labor income is deterministic. Moreover, the tax product is by assumption redistributed equally among households, and is thus also deterministic. The absence of labor income risk and, more generally, of any endowment risk is useful to make the model tractable.

\(^4\)A certain amount of contingent claims could be introduced in order to allow households to decrease their effective level of risk $\sigma$. But what is important for our results is that the final amount of risk remains positive.
Since the labor market is competitive, then $\tilde{k}_t$, the capital per efficient unit of labor $K_t/(A_t N_t)$, is constant across firms. Since, additionally, the production function has constant returns to scale, the capital income is linear in $K_t$, and we can write it as follows:

$$dQ_t = r_t K_t dt + \sigma K_t dv_t$$

where $r_t = (1-\tau) f'(\tilde{k}_t) - \delta$ is the private net return on capital, with $f(k) = F(k,1) = k^\alpha$.

As in Barro and Sala-i Martin (1995), the household maximizes the following discounted expected welfare of the family members:

$$U_t = E_t \int_t^\infty N_s \ln c_s e^{-\rho(s-t)} ds \quad (2)$$

where $\rho > 0$ is the discount rate and $c_s$ is the individual consumption of the members of household $i$ in period $t$. The population growth rate is supposed to be exogenous and equal to $n$, with $n < \rho$:

$$N_t = N_0 e^{nt}$$

We now turn to the household’s budget constraint. The evolution of the household’s financial wealth $B_t + K_t$ obeys to:

$$d(B_t + K_t) = dQ_t + [R^* B_t + N_t w_t + Z_t - C_t] dt \quad (3)$$

However, this expression of the budget constraint misses some aspects of household’s wealth. Indeed, the household’s effective wealth $\Omega_t$ includes his financial wealth, but also human wealth, that is the present discounted value of future labor income and tax product, defined as $H_t = H_t = \int_t^\infty e^{-\rho(s-t)} R^*(N_s w_s + Z_s) ds$. Therefore, $\Omega_t = K_t + B_t + H_t$. In order to include human wealth in the budget constraint, we follow Angeletos and Panousi (2009) and use the definition of human wealth to write:

$$dH_t = (R^* H_t - N_t w_t - Z_t) dt \quad (4)$$

It follows from (1), (4) and (3), that the evolution of effective wealth, in per capita terms, can be described by:

$$d\omega_t = [r_t k_t + R^* (b_t + h_t) - c_t - n \omega_t] dt + \sigma k_t dv_t \quad (5)$$

with $\omega_t = k_t + b_t + h_t$. Lower case letters, except $n$, the population’s growth rate, stand for per capita (i.e. per family member) values. For each variable $X_t (x_t)$, $x_t (x_t)$ stands for $X_t / N_t$ ($X_t / N_t$).

The household maximizes (2) subject to his budget constraint (5), which states that household’s wealth is increased by the revenues from capital, whose return $r_t + \sigma dv_t$ is risky, and from safe assets, $b_t$ and $h_t$, whose return is $R^*$, minus consumption. Population growth additionally diminishes the value of wealth per capita. Thanks to the linearity
of the budget constraint with respect to wealth, this problem boils down to a tractable Merton-Samuelson portfolio choice problem.

Kraay and Ventura (2000) and Kraay et al. (2005) also applied this portfolio choice approach to the study of capital flows, but in an AK context without human wealth, which is an important variable in this paper. Here, we use a transformation of the budget constraint introduced by Angeletos and Panousi (2009) in order to make human wealth explicitly appear, without impairing the tractability of the problem.

2.2 Technology

The country has an exogenous, deterministic productivity path \( \{A_t\}_{t=0,\ldots,\infty} \), which is bounded by the world productivity frontier:

\[
A_t \leq A_t^* = A_0^* e^{g^* t}
\]

The world productivity frontier is assumed to grow at rate \( g^* \). Following Gourinchas and Jeanne (2007), we define TFP catch-up \( \pi_t \) as the difference between domestic productivity and the productivity conditional on no technological catch-up:

\[
e^{\pi_t} = \frac{A_t}{A_0 e^{g^* t}}
\] (6)

We assume that \( \pi = \lim_{t \to \infty} \pi_t \) is well defined. Therefore, the growth rate of domestic productivity converges to \( g^* \).

2.3 Household’s behavior

In line with the Merton-Samuelson approach, it follows from the linearity of the budget constraint and the homotheticity of preferences that the capital stock and consumption choices are linear in wealth.\(^5\) Define \( \phi_t^i = k_t^i/\omega_t^i \), the fraction of effective wealth invested in private capital. For a given interest rate \( R^* \) and a given sequence of wages \( \{W_t\} \), the optimal choices of household \( i \) are given by:

\[
c_t^i = (\rho - n) \omega_t^i
\]

\[
r_t - R^* = \phi_t \sigma^2
\] (8)

\(^5\)See Kraay and Ventura (2000), Kraay et al. (2005) or Angeletos and Panousi (2009) for a full derivation.
Equation (7) shows the familiar result that consumption per capita equals the annualized value of wealth, taking into account population growth. This is a direct consequence of log utility.

Equation (8) provides a portfolio choice rule. It states that the excess return to capital \( r_t - R^* \) is equal to the risk premium. With logarithmic households, the coefficient of risk aversion is equal to one and the risk premium is the covariance between the return on capital and the return on the portfolio, which is equal here to the variance of the return to capital multiplied by the share of capital in the portfolio.

### 2.4 Steady state

The saving rule (7) and the portfolio rule \( k_t^i = \phi_t \omega_t^i \), with \( \phi_t \) defined by (8), are linear in wealth and can therefore be written in aggregate terms: \( c_t = (\rho - n) \omega_t \) and \( k_t = \phi_t \omega_t \), where \( \omega_t = \int_0^1 \omega_t^i \, di \) is the aggregate value for \( \omega_t^i \). By dividing each side by \( A_t \), they can also be written in terms of efficient units of labor, denoted \( \tilde{c}_t \), \( \tilde{k}_t \), and \( \tilde{\omega}_t \).

Capital per efficient units of labor at the firm level, \( \tilde{k}_t = K_t^i / A_t N_t^i \) is constant across firms as a result of competitive labor market. Since the labor market clears (\( \int N_t^i \, di = N_t \)), it is equal to its aggregate value \( \tilde{k}_t = K_t / A_t N_t \).

Equations (7) and (5), when expressed in aggregate terms and divided by \( A_t \), along with the no-Ponzi conditions and the equilibrium values for \( \tilde{\omega}_t \), \( r_t \) and \( \phi_t \), characterize the dynamics of \( \tilde{c}_t \) and \( \tilde{\omega}_t \). Once the paths of these variables are known, \( \tilde{k}_t = \phi_t \tilde{\omega}_t \), \( \tilde{h}_t = \int_0^\infty e^{-(R^*-(n+g^*))s+\pi_s-\sigma_t} (1-\alpha+\tau\alpha) k_t^{\alpha} \, ds \) and \( \tilde{b}_t = \tilde{\omega}_t - \tilde{k}_t - \tilde{h}_t \) can be determined.

We assume no domestic supply of bonds, so the aggregate demand for bonds \( \tilde{b}_t \) represents the country’s external position.

These equations are used to determine the steady state. We define the steady state by \( \dot{\tilde{c}} / \tilde{c} = 0 \) and \( \dot{\pi}_t = 0 \). The following proposition characterizes the steady state.\(^6\)

**Proposition 1:** The open economy steady state exists if and only if \( n + g^* < R^* < \rho + g^* \) and is defined by:

\[
(1 - \tau) f'(k^*) - \delta - R^* = \sqrt{\sigma^2(\rho + g^* - R^*)} \quad (9)
\]

\[
\tilde{k}^* = \phi^* \tilde{\omega}^* \quad (10)
\]

with \( \phi^* = \sqrt{\frac{\rho + g^* - R^*}{\sigma^2}} \), \( \tilde{\omega}^* = \tilde{k}^* + \tilde{b}^* + \tilde{h}^* \) and \( \tilde{h}^* = \frac{(1-\alpha+\alpha\tau)f'(k^*)}{R^* - g^* - n} \).

\(^6\)See the Appendix for the proof, originally provided in Angeletos and Panousi (2009). We repeat this material in order to make the paper self-contained, but also to take into account the capital wedge, which is absent in Angeletos and Panousi (2009).
The condition that the interest rate $R^*$ should be strictly lower than the growth-adjusted discount rate $\rho + g^*$ is a standard condition to insure stationarity in the presence of risk (Aiyagari, 1994; Angeletos and Panousi, 2009). The interest rate should not be too low either: $R^* > n + g^*$. Otherwise, the long-run human wealth would not be well defined.

The steady-state level of capital is defined through Equation (9) by the equality between the excess return and the steady-state risk premium. Importantly, this links the capital wedge $\tau$ to the long-run capital stock. A lower $\tau$ leads to a higher capital per efficient unit of labor $\tilde{k}^*$.

The steady-state share of capital in the portfolio is defined by Equation (10). Indeed, in steady state, the risk premium and therefore the long-run composition between safe and risky assets is defined by the equality between the return of the portfolio and the propensity to consume.

Equation (10) gives also the steady-state external demand for bonds:

$$\tilde{b}^* = \frac{1 - \phi^*}{\phi^*} \tilde{k}^* - \frac{(1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{R^* - g^* - n}$$

(11)

The first term is the amount of non-capital wealth that is needed to achieve the desired capital-to-wealth ratio $\phi^*$. This non-capital wealth is composed of safe assets, bonds and human wealth. This demand for safe assets is positive if $\phi^* < 1$ which is equivalent to $\sigma^2 > \rho + g^* - R^*$. This condition is satisfied as long as the level of risk in the country is greater than in the rest of the world.7 Assuming that the country has a relatively high level of risk is consistent with the fact that we are dealing with developing countries. We therefore maintain this assumption in the rest of the paper.

The second term represents human wealth. External bond holdings represent the need for non-capital wealth that is not satisfied by domestic human wealth.

7Formally, this argument runs as follows. The constraint that $\phi^*/(1 - \phi^*) > 0$ and equivalently that $\phi^* < 1$ can be rationalized by the a general equilibrium argument. The world as a whole is in autarky, that is $\tilde{b}^*_W = 0$. According to (10), we have:

$$\tilde{k}^*_W = \frac{\phi^*_W (1 - \alpha + \alpha \sigma_W)(\tilde{k}^*_W)^\alpha}{1 - \phi^*_W} \frac{R^* - g^* - n_W}{R^* - g^* - n}$$

Since $\tilde{k}^*_W$ is positive, $\phi^*_W/(1 - \phi^*_W)$ is necessarily positive and therefore $\phi^*_W < 1$. This is made possible by the adjustment of the world interest rate $R^*$ in order to clear the world bond market. If we assume, as in the calibration section, that $\sigma > \sigma_W$, then $\phi^* < \phi^*_W$. It is therefore consistent with the portfolio approach to assume that $\phi^* < 1$. 

10
2.5 Capital flows

Here we explicit capital outflows over a finite period \([0, T]\) as a function of the parameters, especially those at the source of growth and investment, \(\pi\) and \(\tilde{k}^*\), which is a function of \(\tau\). This is for the purpose of confronting the model with the data, following Gourinchas and Jeanne (2007).

Before deriving the level of bonds predicted by the model, some assumptions must be made. First, we abstract from unobserved future developments in productivity by assuming that all countries have the same productivity growth rate \(g^*\) after \(T\).

**Assumption 1:** \(\pi_t = \pi\) for all \(t \geq T\).

Besides, it is assumed that \(T\) is sufficiently large to be able to make the following approximation: \(\tilde{k}_t = \tilde{k}^*\) for all \(t \geq T\).

Denote by \(\Delta B^P / Y_0 = (B_T - B_0) / Y_0\) the predicted amount of capital outflows between 0 and \(T\). Under Assumption 1 and for \(T\) large, it can be written as follows:

\[
\frac{\Delta B^P}{Y_0} = e^{\pi + (n + g^*)T} \frac{\tilde{b}^*}{y_0} - \frac{\tilde{b}_0}{y_0}
\]

where \(\tilde{b}^*\) satisfies Equation (11). Notice that \(\tilde{b}^*\), according to Equation (11), is independent of \(\pi\). The effect of \(\pi\) on \(\Delta B^P / Y_0\) depends therefore only on the sign of \(\tilde{b}^*\). This gives the following proposition:

**Proposition 2:** Suppose that \(\sigma^2 > \rho - R^* + g^*\), that \(T\) is sufficiently large, that the stationarity conditions of Proposition 1 and Assumption 1 are satisfied. Consider two countries \(A\) and \(B\), identical except for their long-run productivity catch-up \(\pi\). Then there are two cases:

(i) If \(A\) and \(B\) have a positive long-run external position \((\tilde{b}^* > 0)\), then country \(A\) sends more capital outflows than country \(B\) if and only if country \(A\) catches up faster than country \(B\) towards the world technology frontier:

\[
\Delta B^A > \Delta B^B \iff \pi^A > \pi^B
\]

(ii) If \(A\) and \(B\) have a negative long-run external position \((\tilde{b}^* < 0)\), then the opposite holds:

\[
\Delta B^A > \Delta B^B \iff \pi^A < \pi^B
\]
This result is the cross-country, growth-accounting counterpart of the “New Rule” introduced by Kraay and Ventura (2000). TFP growth explains the puzzle only to the extent that countries with positive long-run external positions drive the observed positive correlation between growth and capital outflows.

To understand this, consider the ratio of external assets to capital, given by Equation (11):

\[
\frac{b^*}{k^*} = \frac{A\tilde{b}^*}{\tilde{A}k^*} = \frac{b^*}{k^*} = \frac{1 - \phi^* - (1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{(R^* - g^* - n)k^*}
\] (13)

The ratio of external assets to capital is the ratio of non-capital wealth to capital, which is constant, minus the ratio of human wealth to capital.

The level of TFP affects proportionally capital \(k^* = A\tilde{k}^*\) and human wealth \(h^* = A\tilde{h}^* = A(1 - \alpha + \alpha \tau) f(\tilde{k}^*)/(R^* - g^* - n)k^*\) because it benefits to both factors of production, capital and labor. Thus, it does not affect the composition of domestic assets \((1 - \alpha + \alpha \tau) f(\tilde{k}^*)/(R^* - g^* - n)k^*\). As a result, it does not affect the ratio between external assets and capital: An increase in \(A\) leads to an increase in \(b^* = A\tilde{k}^*k^*\) only if \(\tilde{k}^*/k^*\) is positive, through a pure portfolio growth effect.

In the following proposition, we show that the stock of capital per efficient unit of labor affects capital flows differently than TFP does, which could contribute to solve the puzzle. In our model, this variable is isomorphic to the capital wedge \(\tau\), for given risk \(\sigma\) (see Equation(9)). The capital wedge explains precisely the amount of capital per head that is not accounted for by TFP. The following proposition examines the effect of a variation in \(\tau\) on capital flows.

**Proposition 3:** Suppose that \(\sigma^2 > \rho - R^* + g^*\), that \(T\) is sufficiently large, that the stationarity conditions of Proposition 1 and Assumption 1 are satisfied. Consider two countries \(A\) and \(B\), identical except for their capital wedge \(\tau\), then there are two cases:

(i) If \(A \text{ and } B\) satisfy \(\tilde{b}^* \geq -(1 - \alpha)\frac{f(\tilde{k}^*)}{R^* - g^* - n}\), then country \(A\) sends more capital outflows than country \(B\) \((\Delta B^A > \Delta B^B)\) if and only if country \(A\) has a lower capital wedge than country \(B\) \((\tau^A < \tau^B)\):

\[
\Delta B^A > \Delta B^B \iff \tau^A < \tau^B
\]

(ii) If \(A \text{ and } B\) do not satisfy \(\tilde{b}^* \geq -(1 - \alpha)\frac{f(\tilde{k}^*)}{R^* - g^* - n}\), the opposite holds.

\[
\Delta B^A > \Delta B^B \iff \tau^A > \tau^B
\]

The proof is provided in the appendix. The way capital outflows are related to capital per efficient unit of labor (i.e. to the capital wedge) depends on a condition on \(\tilde{b}^*\), that is the long-run external position, as in the case of TFP-driven growth.
However, the condition for capital per efficient unit of labor to generate capital outflows ($\tilde{b}^* \geq 0$) is less restrictive than the condition for TFP catch-up to generate capital outflows ($\tilde{b}^* \geq -(1 - \alpha) \frac{f(\tilde{k}^*)}{R^*-\rho^*}$). In particular, a positive correlation between capital outflows and investment is compatible now with negative external positions in developing countries.

The intuition is based again on Equation (13). A decrease in the capital wedge increases capital $\tilde{k}^* = A\tilde{k}$ through the ratio of capital per efficient unit of labor $\tilde{k}^*$. Because capital has a positive effect on production, it affects also positively labor income and human wealth $\tilde{h}^* = A\tilde{h} = \frac{A(1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{(R^*-\rho^*-n)\tilde{k}^*}$. A portfolio growth effect is therefore at play, as for TFP. However, the capital wedge affects also the composition of domestic assets between capital and human wealth $\frac{(1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{(R^*-\rho^*-n)\tilde{k}^*}$. Because the capital stock has decreasing returns, the portfolio composition is tilted towards capital. In order to restore the desired ratio between non-capital and capital assets $\frac{1-\phi^*}{\phi^*}$, external bond holdings must increase. Capital outflows emerge then as the result of a portfolio composition effect.

2.6 Comparison with the riskless case

Here we compute the amount of capital outflows that the model generates in the absence of risk ($\sigma = 0$) in order to compare them with the case with risk. These capital flows will be calibrated in Section 6 in order to compare predicted flows in the riskless and portfolio approaches.

In the case without risk, the condition for stationarity is $R^* = \rho + g^*$. In that case, capital outflows, denoted by $\frac{\Delta B^R}{Y_0}$, are defined by:

$$\frac{\Delta B^R}{Y_0} = \frac{\tilde{k}^* - \tilde{k}_0}{k_0^\alpha} e^{(n+g^*)T} - (e^\pi - 1) \frac{\tilde{k}^*}{k_0^\alpha} e^{(n+g^*)T}$$

$$-e^{\pi+(n+g^*)T} \frac{(1 - \alpha + \alpha \tau) \tilde{k}^*}{k_0^\alpha} \int_0^T e^{-(\rho-n)t}(1 - e^{\pi t-\pi})dt + \left(e^{(n+g^*)T} - 1\right) \frac{\tilde{b}_0}{k_0^\alpha}$$

(14)

which is the continuous-time version of Gourinchas and Jeanne’s decomposition (see the Appendix for a complete derivation).

The first two terms reflect the effect of domestic investment on capital flows. Any investment takes place through foreign borrowing, whether it comes from convergence (that is when $\tilde{k}^* > \tilde{k}_0$) or through TFP catch up (that is when $\pi > 0$). The third term represents consumption smoothing. Following Gourinchas and Jeanne, it can be shown that this term, under some mild assumptions ($\pi < 100\%$), is negatively correlated to TFP catch-up $\pi$. The fourth term is a trend component that represents the amount of capital outflows that are necessary to maintain constant the initial external position.
According to the second and third terms, capital outflows should then be negatively correlated to TFP growth through both an investment and a consumption smoothing effect. Why are these effects absent in the portfolio approach?

Note that, in the case with risk, capital outflows in (12) can also be decomposed into similar terms:

\[
\frac{\Delta B^P}{Y_0} = \frac{1 - \phi^*}{\phi^*} \frac{\tilde{k}^* - \tilde{k}_0}{k_0^\alpha} e^{(n+g^*)T} + \frac{1 - \phi^*}{\phi^*} (e^n - 1) \frac{\tilde{k}^*}{k_0^\alpha} e^{(n+g^*)T} \\
- e^{\pi+(n+g^*)T} \frac{(1 - \alpha + \alpha \tau) \tilde{k}^\alpha}{k_0^\alpha} \int_0^T e^{-(\rho-n)t} \left( 1 - \frac{f(\tilde{k}_t)}{f(\tilde{k}^*)} e^{n_t - \pi} \right) dt + (e^{(n+g^*)T} - 1) \frac{\tilde{b}_0}{k_0^\alpha} (15)
\]

See the Appendix for the derivation. The first four terms are identical to the case without risk, except that the coefficients in front of the investment terms (first two terms) are positive, because \(\phi^* < 1\). They correspond to the need for safe assets necessary to self-insure against the additional capital stock due either to TFP catch-up (first term) or to convergence (second term). The consumption smoothing component and the trend component (respectively the third and fourth components) are identical.

Additionally, in the portfolio approach, a fifth term representing the long-run adjustment of the portfolio towards the desired composition appears. If \(\phi^* < \phi_0\), that is if the long-run desired share of capital in the portfolio is lower than the current one, then, for a given capital stock, the external position should increase in the long-run. In the riskless approach, because the composition of the portfolio is undetermined, this last term is absent. The first four terms then persist in the long run. In the portfolio approach, the investment and consumption smoothing effects may still appear in the short run, but, in the long run, the desired portfolio composition prevails and might offset these short-run effects.

3 Calibration

In order to perform our empirical analysis, we need either to measure or calibrate the parameters of the model.
3.1 Data and calibration method

We use the same sample of 69 emerging countries as in Gourinchas and Jeanne (2007). However, Jordan and Angola are removed from this sample because their working-age population does not satisfy $n < \rho$. The sample is therefore reduced to 67 countries.

The parameters which are common across countries also follow their paper: the depreciation rate $\delta$ is set to 6%, the capital share of output $\alpha$ to 0.3 and the growth rate of world productivity $g^*$ to 1.7%. Only the discount rate $\rho$ is set to a higher value of 5% (instead of 4%) in order to accommodate high growth rates of labor in the data. The time span is extended to 1980-2003 (instead of 1980-2000) in order to encompass the recent surge in capital outflows from developing countries and to shed some light on the “global imbalances” debate.

In order to determine the exogenous interest rate, we make the hypothesis that each country is too small to influence the world’s demand for bonds. We also assume that the world is composed exclusively of developed countries with zero labor growth and no distortions to the marginal capital return. The world interest rate then corresponds to the autarky steady-state interest rate with $n = 0$ and $\tau = 0$. In the portfolio approach, the amount of risk $\sigma$ in developed countries is set to 0.3, which is the amount of entrepreneurial risk commonly reported by empirical studies in the US and the Euro area (Campbell et al., 2001; Kearney and Poti, 2006). This gives $\phi^* = 13.1\%$ and $R^* = 6.55\%$. If we extend this calibration approach to the case without risk, we get $R^* = \rho + g^* = 6.7\%$, which is the long-run value of the autarky interest rate when $\sigma = 0$.

The amount of risk in emerging countries is set to $\sigma = 0.6$, in line with Koren and Tenreyro (2007), who find that the amount of both macroeconomic and idiosyncratic (sectoral) risk is roughly twice as large in developing countries as in industrial ones.

The country-specific data are the paths for output, capital, productivity and working-age population. These data come from Version 6.2 of the Penn World Tables (Heston et al., 2006). Following Gourinchas and Jeanne (2007) and Caselli (2004), the capital stock is constructed with the perpetual inventory method from time series data on real investment. The level of productivity $A_t$ is calculated as $(y_t/k_t^\alpha)^{1/(1-\alpha)}$ and the level

---

8This sample includes: Angola, Argentina, Bangladesh, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Cameroon, Chile, China, Colombia, the Republic of Congo, Costa Rica, Cyprus, Côte d’Ivoire, Dominican Republic, Ecuador, Egypt, Arab Republic, El Salvador, Ethiopia, Fiji, Gabon, Ghana, Guatemala, Hâïti, Honduras, Hong Kong, India, Indonesia, Iran, Israel, Jamaica, Jordan, Kenya, Republic of Korea, Madagascar, Malawi, Malaysia, Mali, Mauritius, Mexico, Morocco, Mozambique, Nepal, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, Singapore, South Africa, Sri Lanka, Syrian Arab Republic, Taiwan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, Uruguay and Venezuela.

9Indeed, in the portfolio approach, the world interest rate (i.e. $R^*$) is lower than $\rho + g^*$. If $\rho$ is too small, then we could have $R^* - (g^* + n)$ negative or very close to zero. In the first case, capital flows are not well defined; in the second, their variance goes to infinity.
of capital per efficient unit of labor $\tilde{k}_t$ as $(y_t/k_t)^{1/(1-\alpha)}$. The level of TFP $A_t$ and the capital per efficient unit of labor $\tilde{k}_t$ are filtered using the Hodrick-Prescott method in order to suppress business cycles. The parameter $n$ is measured as the annual growth rate of the working-age population. Under Assumption 1, the long-term catch-up $\pi$ can be measured as $\ln(A_T) - \ln(A_0) - Tg^*$. The capital wedge $\tau$ is calibrated in order to account exactly for the steady-state stock of capital per efficient unit of labor. We use the fact that, assuming that $T = 23$ is a sufficiently large number, $k^*$ is approximately equal to $\tilde{k}_T$. We thus take $\tilde{k}^* = \tilde{k}_T$. Then $\tau$ is used to adjust the private marginal return on capital to the world interest rate: $\tau = 1 - \tilde{k}_T^{-1/(1-\alpha)} R^* + \delta + \sqrt{\sigma^2(\rho + \gamma - R^*)}$. This method assigns a high capital wedge to countries with low end-of-period capital per efficient unit of labor.

Finally, actual capital flows are taken from Lane and Milesi-Ferretti (2006)'s net foreign asset data. They provide estimates for the net external position in current US dollars.\textsuperscript{10} Actual capital outflows during the period, as a share of initial output, are denoted $\frac{\Delta B}{Y_0}$.

### 3.2 Some key variables

Table 1 sums up some key parameters given by the calibration method. Consider the long-run productivity catch-up $\pi$ in column (1) of Table 1. On average, non-OECD countries have fallen behind the world frontier in terms of productivity. When looking into details, only Asian countries have caught up with the world productivity. Both Africa and Latin America fell behind.

Column (3) presents the average growth rate of capital $\frac{\Delta k}{Y_0}$. The main observation is that the capital stock decreased on average. Consistently with Gourinchas and Jeanne (2006, 2007), emerging countries were not capital-scarce but capital-abundant. Among regions, only Asia increased its capital per efficient unit of labor.

Consider now column (5). When calibrated inside the riskless framework, the average wedge $\tau$ on the net capital return is equal to 38%, which is consistent with the average wedge on gross capital return of 12% found in Gourinchas and Jeanne (2007). Africa, which has the smallest end-of-period capital level (column (4)), has therefore the highest estimated capital wedge, while Asia's estimated capital wedge is the smallest, since it has the highest end-of-period capital level. The capital wedge calibrated inside the portfolio framework is given by column (6). It is lower than the one reported in column (4).

\textsuperscript{10}These estimates are calculated using the cumulated current account data and are adjusted for valuation effects. In order to be consistent with the PPP-adjusted data used here, a PPP deflator is extracted from the Penn World Table and is used to calculate a PPP-adjusted measure of net external position.
because the risk premium accounts partially for the low levels of capital in developing countries. This leaves unchanged the regions’ ranking.

4 Determinants of capital flows

The objective of this section is to examine whether the predictions of the model are satisfied in the data. First, according to the model, the correlation between TFP catch-up and capital outflows should be negative in a sample of countries with negative external position, and positive in a sample of countries with positive external positions. Second, the correlation between capital per efficient unit of labor and capital outflows can be positive even in a sample of countries with negative external positions. These predictions are consistent with the preliminary evidence showing that the positive correlation of capital outflows with TFP catch-up is unstable and that their correlation with capital per efficient unit of labor is more robust. This section consists in examining the robustness of this preliminary evidence.

4.1 Methodology

The decomposition of capital flows in the portfolio approach (12) can be log-linearized as follows around the cross-country averages $\bar{\pi}$, $(\bar{n} + \bar{g}^\ast)\bar{T}$, $\ln(\bar{y}_0)$ and $\ln(\bar{k}^\ast)$:

$$
\Delta \frac{B_P}{Y_0} = \frac{\bar{B}}{Y_0} + \frac{\bar{B}}{Y_0} \left[ (\bar{\pi} + (\bar{n} + \bar{g}^\ast)\bar{T} - \ln(\bar{y}_0)) - (\bar{\pi} + (\bar{n} + \bar{g}^\ast)\bar{T} - \ln(\bar{y}_0)) \right] + 
\frac{\bar{B} + \kappa \bar{H}}{Y_0} \left( \ln(\bar{k}^\ast) - \ln(\bar{k}^\ast) \right) - \frac{B_0}{Y_0}
$$

(16)

where $\bar{B}/Y_0$ and $\bar{B}/Y_0$ are respectively the implied end-of-period net foreign assets and human wealth over initial output evaluated at the country averages and $\kappa = (1 - \alpha)/(1 - \alpha + \alpha \bar{T}) \approx 1$. Based on this approximation, the following structural equation can be tested in our sample:

$$
\Delta \frac{B}{Y_0}_i = \gamma_1 + \gamma_2 \pi_i + \gamma_3 \ln(\bar{k}^\ast)_i + \gamma_4 (\bar{n} + \bar{g}^\ast) T_i + \gamma_5 \ln(\bar{y}_0)_i + \gamma_6 \frac{B_0}{Y_0}_i + \epsilon_i
$$

(17)

with $i$ the country index, $\gamma_1 - \gamma_6$ the coefficients to estimate and $\epsilon_i$ an error term. According to Equation 16, we should have: $\gamma_2 = \bar{B}/Y_0$ and $\gamma_3 = \bar{B} + \bar{H} \bar{Y}_0$. Therefore, we should verify that $\gamma_2$ is sensitive to the sample average of $\bar{B}/Y_0$ and that $\gamma_3 > \gamma_2$. 

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4.2 Results

Table 2 gives the regression results for Equation (17) with the observed flows between 1980 and 2003 for the same sample as the one used for the calibration analysis.

First, consider the impact of TFP growth $\pi$ on capital outflows, given by $\gamma_2$. According to the portfolio approach, it should depend on the sign of the average net foreign asset position $\frac{E}{Y_0}$, which is sample-dependent.

Consider the results given in Table 2. When abstracting from the other correlates, the impact of $\pi$ on the observed flows is significantly positive at the 10% level if estimated on the whole sample but it turns significantly negative at the same level if restricting the sample to countries with negative external positions (respectively, columns (1) and (2)). This suggests that the coefficient is, as predicted by the portfolio approach, highly sample-dependent. The same happens when we control for the other determinants, only the coefficients are not significant (columns (3) and (4)). In this case, in line with the portfolio approach, the estimated values are not significantly different from the actual averages of $\frac{E}{Y_0}$.

This suggests that the positive correlation between capital outflows and TFP growth presented as preliminary evidence in Figure 1 was driven by a few countries with positive end-of-period external positions, among which Asia is well represented. These countries are: Botswana, China, Hong-Kong, Iran, Mauritius, Singapore, Taiwan and Venezuela. In particular, Botswana and Hong-Kong appear clearly as outliers in the figure.

Second, consider the impact of capital per efficient unit of labor $\ln(\hat{k}^*)$, which is isomorphic to the capital wedge $\tau$, on capital outflows. In the model, it depends both on the external position and on human wealth, as suggested also by Equation (16). This means that its impact should be more robustly positive than that of TFP catch-up. The results are reported in columns (3) and (4) of Table 2. The impact of $\ln(\hat{k}^*)$ is significantly positive (at the 10% level), whether we estimate it on the whole sample (column (3)) or on the sample of countries with negative end-of-period external positions (column (4)). These results imply that $\gamma_3 - \gamma_2$ are equal respectively to 0.49 and 0.77. In line with the portfolio approach, these quantities, which are estimates of $\frac{E}{Y_0}$, are greater than zero respectively at the 10% and 1% level.

These estimates of $\frac{E}{Y_0}$ imply end-of-period ratios of human wealth to output $\frac{H}{Y}$ of respectively 0.20 and 0.34. This is much lower than what the model predicts. According

---

11The actual average $\frac{E}{Y_0}$ is equal to -79% in the whole sample with a standard error of 156%. When we exclude countries with positive external positions, the actual average $\frac{E}{Y_0}$ is equal to -120% with a standard error of 74%.

12Alfaro et al. (2010) also highlight the role of these outliers in driving the correlation between growth and capital outflows, but do not provide an explanation for their behavior.

13Standard errors are equal respectively to 0.38 and 0.28.
to the model, this value should be greater than \((1 - \alpha)/R^* \simeq 10\). However, our model naturally overestimates human wealth because it assumes that household can potentially short-sell all their future income, which supposes no frictions in the bond market. In reality, many agents do not have access to financial markets. These values should then be interpreted as “effective” human wealth, that takes into account the limited access to financial markets. The discrepancy between our estimates and the model is actually an assessment of the lack of financial development in developing countries.

Note that the fact that human wealth is relatively larger in the sample with negative end-of-period external positions is consistent with the portfolio approach: according to the theory, countries with a high level of human wealth do not need to export capital and are more likely to end up with negative foreign assets.

4.3 Robustness

As we have seen, the variations in the long-run capital per efficient unit of labor \(\hat{k}^*\) across countries are crucial to explain capital flows. However, this long-run level is proxied by the end-of-period level and is then subject to measurement errors. In this section, we use the relative price of capital and its share in production as alternative proxies for the long-run capital stock \(\hat{k}^*\). We then investigate whether these proxies still explain capital flows.

To understand the role of the relative price of capital and of its share in production, we follow Caselli and Feyrer (2007). Production is defined as follows:

\[
Y_t = A_t^{1-\alpha_k} K_t^{\alpha_k} X_t^{\alpha_x} N_t^{1-\alpha_k-\alpha_x}
\]

with \(X\) denoting non-reproducible capital and \(\alpha_x\) its share in production. Non-reproducible capital is introduced in order to estimate the share of reproducible capital in production \(\alpha_k\) more accurately. Equalization between the private marginal return to capital and the risk-adjusted return to capital implies then:

\[
(1 - \tau)\frac{\hat{k}^* \alpha_k^{-1}}{P_k} = R^* + \delta + \sqrt{\sigma^2 (\rho + g^* - R^*)}
\]

where \(P_k\) is the price of investment goods in terms of final goods. In this more general setting, countries with large \(\alpha_k\) and low \(P_k\) will have a higher long-run capital ratio \(\hat{k}^*\). We can then use \(\alpha_k\) and \(P_k\) as proxies for \(\hat{k}^*\).

We estimate the relative price of capital \(P_k\) as the ratio of the investment price index to the GDP price index from the Penn World Tables 6.2 (Heston et al., 2006). We use the data of 1996, which is the year where the broadest number of countries were surveyed.
To measure $\alpha_k$, following Caselli and Feyrer (2007), we obtain the labor share $1 - \alpha_k - \alpha_x$ from Bernanke and Gurkaynak (2001) and derive the total share of capital (reproducible and non-reproducible) $\alpha_w = \alpha_k + \alpha_x$. We then use the equality between the steady-state perceived private returns on reproducible capital and on non-reproducible capital. Assuming that $\tau = 0$, the former is equal to $\alpha_k \frac{Y}{P_k} - \delta - \sqrt{\sigma^2 (\rho + g^* - R^*)}$. To infer the private return on non-reproducible capital, we must make assumptions about its risk. For simplicity, we assume that non-reproducible capital is not risky. In that case, its private return is \((\alpha_x \frac{Y}{P} + \frac{\partial P_x}{\partial t}) / P_x\), where $P_x$ is the price of non-reproducible capital in terms of final goods. In steady state, the rate of price appreciation for non-reproducible capital should be equal to $g^* + n_1 - \alpha_w \alpha_w$. Equality between returns then yields:

$$\alpha_k = \left[ \alpha_w + \left( g^* + 1 - \alpha_w \alpha_w \frac{\partial P_x}{\partial t} \right) \frac{P_x}{P_k} \right] \frac{P_k K}{P_x X + P_k K}$$

By considering Table 1, we can see how these measures relate to the capital stock across regions. Latin America benefits both from a lower price of reproducible capital $P_k$ (column (8)) and from a higher share $\alpha_k$ (column (9)) than Africa, which explains its relatively higher capital stock (column (4)). Asia has the same share as Latin America, but faces a lower price of capital, which explains why Asia has the highest capital stock.

Columns (5) and (6) of Table 2 give the determinants of capital outflows where $\ln(\tilde{k}^*)$ is replaced by $P_k$ and $\alpha_k$ respectively for the whole sample and for the sample of countries with negative net foreign assets. In both regressions, the coefficients of $P_k$ and $\alpha_k$ are significant at least at the 10% level and have the expected signs: respectively negative and positive. Interestingly, $\pi$ is significantly negative in the sample of net creditors, which is also in line with the portfolio approach.

4.4 Reinterpreting the “allocation puzzle”

In the literature, the phenomenon labelled “allocation puzzle” by Gourinchas and Jeanne (2007) covers several aspects. In its narrow definition, it refers to the positive cross-country correlation between TFP growth and capital outflows. We take it here in a broader sense, that is as the positive correlation between per-capita GDP growth and capital outflows (Prasad et al., 2007) and as the correlation between investment and capital outflows (Aizenman and Pinto, 2007). Here, we check whether the portfolio approach can account for this corpus of evidence.

Both TFP catch-up $\pi$ and capital per efficient unit of labor $\ln(\tilde{k}^*)$ are potential channels of growth and investment. However, according to the above empirical analysis and consistently with the portfolio approach, only $\ln(\tilde{k}^*)$ correlates robustly with capital outflows. This suggests that the correlation of capital outflows with investment
and growth that has been documented in the empirical literature is channelled mainly through $\ln(\hat{k}^*)$.

However, the fact that capital outflows, investment and growth are related separately to capital per efficient unit of labor is not sufficient to explain the allocation puzzle. Indeed, the positive correlation between, say, investment and capital outflows could be driven by other common factors. In that case, the allocation puzzle would not be solved. To test this, we estimate the cross-country correlation between capital outflows and investment as follows:

$$\frac{\Delta B}{Y_0} = \gamma \frac{\Delta K}{Y_0} + \nu_i$$

where $\Delta_k = \frac{K_T-K_0}{Y_0}$ refers to investment between 1980 and 2003 and $\nu_I$ is the error term. We then add $\pi$ and $\ln(\hat{k}^*)$ and ask if any of these variables soak the significance of $\gamma$. We conduct the same analysis by replacing investment $\Delta K$ by GDP growth between 1980 and 2003 $\Delta Y = \frac{Y_T-Y_0}{Y_0}$.

The results are provided in Tables 3 and 4. As column (1) of Table 3 shows, the unconditional correlation between $\frac{\Delta B}{Y_0}$ and $\frac{\Delta K}{Y_0}$ is positive and significant at the 10% level. This is the allocation puzzle. When we add $\pi$, this coefficient remains stable and significant (column (2)). This means that TFP catch-up is not at the source of the allocation puzzle for investment. However, when $\ln(\hat{k}^*)$ is added, both with and without $\pi$ (respectively columns (4) and (3)), the significance of $\gamma$ is soaked up. The same results hold when $\ln(\hat{k}^*)$ is replaced with $P_k$ and $\alpha_k$ (columns (5) and (6)). This suggests that the stock of capital per efficient unit of labor is at the origin of the positive correlation between capital outflows and investment across countries.

Column (1) of Table 4 shows that the unconditional correlation between $\frac{\Delta B}{Y_0}$ and $\frac{\Delta Y}{Y_0}$ is significantly positive at the 10% level. When we add $\pi$ and $\ln(\hat{k}^*)$, respectively in columns (2) and (3), the significance of this correlation drops. Interestingly, the coefficient of $\frac{\Delta Y}{Y_0}$ is smaller when we add both variables than when we add them separately. This suggests that both drive the allocation puzzle. However, only $\ln(\hat{k}^*)$ remains significant, indicating that it play a more important role in driving the positive correlation between growth and capital outflows. Again, the same results hold when we replace $\ln(\hat{k}^*)$ by $P_k$ and $\alpha_k$ (columns (5) and (6)).

5 Predicted flows

In this section, we adopt the same approach as Gourinchas and Jeanne (2007): we compute the amount of capital flows predicted by the model and then compare these predicted flows to the actual ones. Gourinchas and Jeanne found that the neoclassical growth model without risk failed in predicting accurately the allocation of capital flows.
between countries. However, we showed in Section 3 that, contrary to the model without risk, the predictions of the model with risk were satisfied in the data. We check here whether this is sufficient to predict the accurate direction of flows between emerging countries.

5.1 The riskless approach

In this subsection, for comparison purposes, we reproduce the results of Gourinchas and Jeanne on the performances of the neoclassical growth model without risk. Figure 3 summarizes the outcome of the riskless approach across regions. The upper panel reports the actual net capital outflows as a share of initial output $\Delta B/Y_0$: their size is $-46\%$ on average, which means that emerging countries have received net capital inflows during the period. The middle panel reports the predicted capital outflows based on equation (2.6). These estimates are constructed under the hypothesis that the productivity catch-up follows a linear trend: $\pi_t = \pi \min \{t/T, 1\}$. According to the model, non-OECD countries should have received capital inflows on average, which is the case. Moreover, average predicted flows in non-OECD countries are of the same order of magnitude as the actual ones.

However, while satisfying in terms of global trends, the picture is more contrasted when considering the direction and magnitude of flows inside the sample. According to the predictions, Africa and Latin America should have exported capital while Asia should have received capital inflows (middle panel of Figure 3). This is because Asia has benefited from high TFP catch-up during the period while the other regions have fallen behind the world frontier (see Table 1). Asia should have borrowed from the rest of the world both to take advantage of local investment opportunities and to smooth consumption in the face of rising revenues. Africa and Latin America should have exported capital to seek better returns and to smooth consumption in the face of decreasing revenues relatively to the rest of the world. In the data, the opposite happens (upper panel of the same figure).

However, if we abstract from the sign of capital flows, Latin America and Africa are correctly ranked by the model. Latin America experienced slower TFP growth than Africa and should then have exported more capital, which is consistent with the data. Therefore, Asia appears as an outlier inside the riskless approach.

On the whole, capital seems to flow in the right direction, except for Asia. The same idea emerges from Figure 4, which shows the scatter plot of actual versus predicted flows. The correlation appears as negative on the whole, which represents the allocation puzzle. However, this negative correlation is driven by a set of countries dominated by Asia, in the upper left of the graph. If we excluded these countries, the correlation would be positive. Explaining the allocation puzzle therefore entails to explain the peculiar behavior of Asian countries.
5.2 The portfolio approach

We have shown that the model without risk reproduces the allocation puzzle, and that this allocation puzzle is mainly driven by Asia. We now turn to the extension with risk, and examine whether it solves this anomaly. It appears that, contrary to the riskless approach, Asia is not an outlier in the portfolio approach. More importantly, this is not at the expense of another outlier, since Latin America and Africa are still correctly ranked. This is because the portfolio approach does not simply reverse the link between growth and capital flows. This link depends in a complex way on the different factors of growth.

The lower panel of Figure 3 reports the estimated predicted net outflows according to equation (10). The estimates are computed under the assumption that the productivity catch-up follows a linear path, as in the riskless approach. The path of capital per efficient unit of labor $\tilde{k}_t$ implied by the model is approximated by the following formula:

$$\tilde{k}_t = \tilde{k}_0 e^{\ln(\tilde{k}^*/\tilde{k}_0)(1-\lambda t)}$$

where $1 - \lambda$ is the convergence rate estimated from the data (that is $1 - \lambda = 0.3$).\(^{14}\)

Note first that the magnitude of predicted flows is above the actual ones (upper panel) by several orders of magnitude. This is a shortcoming of the portfolio approach that has been already highlighted in Kraay et al. (2005).\(^{15}\) It can also be related to the home bias in portfolio (Lewis, 1999). But this shortcoming is accentuated here by the presence of potentially huge human wealth, due to labor and productivity growth.

When abstracting from the magnitude issue, it appears that the predicted outflows in the lower panel of Figure 3 exhibit the right sign, which is negative, and, contrary to the riskless approach, the right ranking between regions. According to the model's predictions, Africa and Latin America should receive capital inflows while Asia should export capital, as in the data. Additionally, the model predicts accurately that Africa should receive more capital inflows than Latin America.

This analysis sheds some light on a neglected issue in the studies on global imbalances. Namely: why are these imbalances originated in Asia and not in other developing regions. An answer suggested here is that Asian growth is originated not only in TFP growth, but also, importantly, in a larger long-run capital per efficient unit of labor. As a consequence, growth had benefited both to capital and labor income, but this growth has been slightly tilted towards capital. Therefore, the asset demand due to a large capital accumulation has not been compensated by a matching increase in human wealth - although human wealth has grown a lot. This resulted in capital outflows. On the opposite, Africa and Latin America face large frictions on the return to capital, which

\(^{14}\)The assumed trend is a good proxy for the capital dynamics since the theory predicts that it moves smoothly from $\tilde{k}_0$ to $\tilde{k}^*$.

\(^{15}\)In their paper, the magnitude of capital flows becomes more realistic when they introduce sovereign risk.
implies that their human wealth is large relative to their capital stock. Their asset
demand is then low relative to their domestic human wealth, leading to capital inflows.

Not only does the portfolio approach rationalize the behavior of Asia relative to
other regions, but it also explains the difference between Africa and Latin America. Both regions have low capital per efficient unit of labor, but this is especially true for Africa. Human wealth is then particularly disproportionate as compared to capital in Africa. This results in larger capital inflows in Africa than in Latin America.

The portfolio approach seems therefore to be a better predictor, if not of the magnitude of flows, at least of their direction. Figure 5 sums up this idea by plotting predicted flows according to the portfolio approach against the actual ones. A positive correlation appears. Besides, this correlation is not driven by Asian countries, contrary to the riskless approach.

5.3 Sensitivity analysis

In this subsection, we perform sensitivity analysis on the portfolio approach.

5.3.1 No redistribution of the tax product

In the model, the wedge between the social and private return to capital is represented by a tax on capital income that is redistributed equally among agents through a lump-sum tax \( Z_t \). While this capital wedge is meant at rationalizing the unwillingness to accumulate capital in developing countries, it also introduces a de facto insurance means in the economy. Indeed, by redistributing a fraction of capital income, the tax transforms a risky income into a safe one. This insurance effect is another source of positive correlation between capital outflows and growth.

However, the benefits of the capital wedge do not necessarily go to the public in reality. If it results from the preemptive action of the government, it is not redistributed to households. If it arises from financial frictions, it can be in the form of a rent that goes to private agents who might not be able to diversify this source of income. The assumption that the tax product \( Z_t \) is redistributed must be then taken with a grain of salt. In order to check whether our results rely on this assumption, we assume that the tax product is not redistributed but is sunk (e.g. it is consumed by the government). This will affect exclusively the estimated human wealth, which will be lower than in the baseline when the capital wedge is positive because it does not include the tax product.

The resulting predicted capital flows are represented in Graph (a) of Figure 6 against the observed ones. Most predicted values are shifted to the right, except some very
positive values, which are shifted to the left. This is because most developing countries have a positive capital wedge, except a few countries with a high stock of capital and thus large estimated capital outflows. For the former, a lower human wealth translates into larger capital outflows and for the latter, a higher human wealth translates into smaller outflows. As a result, estimated outflows are less volatile but they are still correlated positively with the observed ones.

5.3.2 Alternative values of risk

Idiosyncratic risk is a very difficult parameter to assess, in particular in developing countries. The value that we assigned to $\sigma$ in the calibration, 0.6, is then highly questionable. In Graph (b) of Figure 6, the scatter plots of observed versus predicted flows for different values of $\sigma$ are represented. According to the graph, the higher $\sigma$, the larger the predicted capital outflows for each country. Indeed, $\sigma$ affects negatively $\phi^*$, the share of capital in the portfolio: $\sigma$ increases the need for precautionary savings for a given level of capital. As a result, the plots are shifted to the left or to the right relative to the baseline but the correlation is still positive.

The value of $\sigma$ in developed countries, 0.3, must be challenged as well. This is achieved by making the world interest rate $R^*$ vary. Indeed, the channel through which a higher risk in developed countries manifests itself is a lower world interest rate. Graph (c) of Figure 6 provides the scatter plots with different levels of interest rate. It appears that lower risk in the rest of the world (i.e. higher interest rate) has the same effect as higher risk in developing countries: it increases predicted outflows but does not alter the correlation.

In the calibration analysis, we assumed that all developing countries faced the same amount of risk. This is arguably a strength of the model since it generates heterogeneity in capital outflows without contending heterogeneity in risk. However, domestic financial development varies a lot across emerging countries and in a direction that does not seem favorable to our approach: it is believed to be stronger in Asia, which has experienced larger capital outflows. To check whether this is an issue, predicted capital flows are computed with $\sigma = 0.3$ in Asiatic countries, which is the amount of risk assigned to developed countries, and $\sigma = 0.6$ - the baseline - in the other emerging countries. Graph (d) of Figure 6 provides the scatter plot of these predicted flows against the observed values, along with the baseline. Clearly, predicted flows exhibit less excessively large values, which were composed of Asiatic countries. Still, the correlation remains positive.
5.3.3 Alternative measures of the capital wedge

In our model, the variations in capital per efficient unit of labor $\tilde{k}^*$ are accounted for by a capital wedge $\tau$. It is set in order to equalize the private marginal returns to capital across countries, thus shutting down the Lucas (1990) puzzle. However, this capital wedge is a black box: it does not say anything about the sources of the differences in capital per efficient unit of labor across countries and thus about the sources of the allocation puzzle. Besides, as in the empirical section, it might be sensitive to the calibration procedure and therefore subject to measurement errors. In this section, we use the relative price of capital and its share in production computed in the previous section as alternative measures of the capital wedge. We then investigate whether the puzzle is still solved, that is: does the model still predicts accurately the cross-country direction of flows?

Caselli and Feyrer show in their paper that allowing for differing relative price of investment $P_k$ and share of capital $\alpha_k$ across countries is sufficient to account for the cross-country differences in capital-labor ratio. Indeed, as reported in Table 1, if we calculate the capital wedge $\tau$ that would equalize the new private return to capital to the world’s interest rate, we find that the new capital wedge (column (6)) is much smaller (8% on average) than in the baseline calibration. This is in line with the findings of Caselli and Feyrer (2007).

Capital flows are then computed under the hypothesis that $\tau = 0$ but with $P_k$ and $\alpha_k$ taken into account. Graph (e) of Figure 6 plots actual capital outflows against these predicted values. As compared to the baseline, the predicted values are higher on average (the scatter plot is shifted to the right), but their correlation with observed flows is still positive.

6 Conclusion

This paper develops an extension of the traditional neoclassical growth model to risky investment that contributes to match the actual capital flows and to solve the allocation puzzle. It also provides an explanation to global imbalances and to their regional structure. Namely, it explains why capital flows to the North come from Asia and not from other regions with poor financial development as Africa and Latin America. The advantage of this approach is that it does not constitute a great departure from the textbook model and therefore allows the adoption of a development accounting approach.

The portfolio approach fares better than the riskless one in predicting capital flows to developing countries because it accounts for two main facts: (i) countries with high TFP catch-up experienced larger capital outflows, but conditional on a positive external position; (ii) countries with higher long-run capital per efficient unit of labor experienced
more capital outflows.

One further step in this line of research would consist in studying whether the portfolio approach can also account for the composition of flows. Extending the model to the possibility to trade equity, at least partially, could help predict both equity and bond holdings. We could then check whether the portfolio approach is consistent with the fact that direct foreign investment is positively correlated with growth, while securities are not, as shown by Prasad et al. (2007). Another question is the role of labor income risk, which is an important feature of developing countries, as well as investment risk. This is left for future research.

References


7 Appendix

Proof of Proposition 1

In order to prove Proposition 1, we first establish the following Lemma:

**Lemma 1:** Let $\tilde{x}_t = X_t/(A_t N_t)$ denote the value of $X_t$ in efficient labor terms at the aggregate level. For a given interest rate $R^*$, the aggregate dynamics of the economy is characterized by:

\[
\begin{align*}
\dot{\tilde{c}}_t &= (\rho - n)\tilde{\omega}_t \\
\dot{\tilde{\omega}}_t &= r_t \phi_t + R^*(1 - \phi_t) - (\rho + g^* + \dot{\pi}_t) 
\end{align*}
\]

where $r_t = r(\phi_t \tilde{\omega}_t) = (1 - \tau)\alpha(\phi_t \tilde{\omega}_t)^{\alpha-1} - \delta$ and $\phi_t$ defined by equation (8).

Equation (18) is the counterpart of Equation (7) in terms of efficient units of labor.

Equation (19) is obtained from the aggregation of the individual budget constraints (5) written in terms of efficient units of labor and where the wage clears the labor market.

Equations (18) and (19), along with the no-Ponzi conditions and the definitions of $r_t$ and $\phi_t$, characterize the dynamics of $\tilde{c}_t$ and $\tilde{\omega}_t$. Once the paths of these variables are known, $\tilde{k}_t = \phi_t \tilde{\omega}_t$, $\tilde{h}_t = \int_0^\infty e^{-(R^* - (n + g^*))s + \pi_t - \pi_t} (1 - \alpha + \tau\alpha)k^{\alpha}_{t+s}ds$ and $\tilde{b}_t = \tilde{\omega}_t - \tilde{k}_t - \tilde{h}_t$ can be determined. However, these equations are used here only to determine steady state.

We now characterize stationarity.
Since, according to Equation (18), consumption is proportional to wealth, the stationarity of consumption $\dot{c}/\bar{c} = 0$ implies the stationarity of wealth $\dot{\bar{w}}/\bar{w} = 0$. Additionally, the stationarity of catch-up $\dot{\bar{\pi}} = 0$, along with the aggregate budget constraint, implies that the long-run share of capital $\phi^*$ satisfies:
\[ r^*\phi^* + R^*(1 - \phi^*) - (\rho + g^*) = 0 \]
which is equivalent to:
\[ \frac{(r^* - R^*)^2}{\sigma^2} = \rho + g^* - R^* \]  
(20)
Therefore, for the steady state to exist, we must have $R^* \leq \rho + n$.

Since the share of capital in wealth $\phi$ is necessarily non-negative, then $r - R^*$ is also non-negative. Equation (20) therefore yields the expression of the return differential:
\[ r^* - R^* = \sqrt{\sigma^2(\rho + g^* - R^*)} \]
This equation is equivalent to Equation (9).

Equation (10) derives from the definition of $\phi$ at steady state:
\[ \phi^* = \tilde{k}^*/(\tilde{k}^* + \tilde{b}^* + \tilde{h}^*) \]  
(21)
To establish Equation (10), we must then derive the value of the steady-state share of capital $\phi^*$ and the steady-state human wealth $\tilde{h}^*$.

First, when $\sigma > 0$, we must have necessarily $R^* < \rho + n$. This can be shown by noticing that Equation (20) can be rewritten as follows:
\[ \sigma^2\phi^2 = \rho + g^* - R^* \]
Suppose that $R^* = \rho + g^*$, this would imply, when $\sigma > 0$, that $\phi^* = 0$. If wealth is stationary, this means that the stock of capital converges towards zero. Given the Cobb-Douglas specification, this implies that the return to capital would tend to infinity, which contradicts the fact that the return differential is constant, as suggested by Equation (9). Since, as shown above, $R^* \leq \rho + g^*$ for the steady state to exist, then we must have $R^* < \rho + g^*$. As a consequence, Equation (20) implies that:
\[ \phi^* = \frac{\sqrt{\rho + g^* - R^*}}{\sigma^2} \]

Second, to complete Equation (10), we have to determine $\tilde{h}$ at steady state. $\tilde{h}_t = \int_0^\infty e^{-R^*s}N_{t+s}A_{t+s}(1-\alpha + \tau \alpha)\tilde{k}_{t+s}^\alpha ds = \int_0^\infty e^{-(R^* - (n + g^*))s} e^{\pi_{t+s}} - \pi_t \tilde{k}_{t+s}^\alpha ds$. Equation (9) gives $k^*$, the steady-state value of $\tilde{k}$. We have also $\pi_t = \pi$ in the long run. Therefore,
\[ \tilde{h}^* = \frac{1 - \alpha + \tau \alpha}{R^* - (n + g^*)} \]
Replacing $\tilde{h}^*$ in Equation (21) with the steady-state $\phi^*$ yields Equation (10).
Proof of Proposition 3

Notice that the predicted capital flows must satisfy (12). According to this equation the derivative of $\Delta B_0$ with respect to $\tau$ depends only on the derivative of $\hat{b}^*$. 

Differentiating (10) with respect to $\tau$, we obtain:

$$\frac{\partial \hat{b}^*}{\partial \tau} = \frac{\partial \hat{k}^*}{\partial \tau} \left[ \frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha)\alpha \hat{k}^*(\alpha - 1)}{R^* - n - g^*} \right] - \frac{\alpha \hat{k}^*}{R^* - n - g^*}$$

In order to infer $\frac{\partial \hat{k}^*}{\partial \tau}$, we differentiate Equation (9) with respect to $\tau$ and get:

$$\frac{\partial \hat{k}^*}{\partial \tau} = \frac{\hat{k}^*}{(1 - \tau)(1 - \alpha)}$$

Replacing in $\frac{\partial \hat{b}^*}{\partial \tau}$, we can show that:

$$\frac{\partial \hat{b}^*}{\partial \tau} = \frac{1}{(1 - \tau)(1 - \alpha)} \left[ \hat{b}^* - (1 - \alpha) \frac{\hat{k}^*}{R^* - n - g^*} \right]$$

Therefore, $\frac{\partial \hat{b}^*}{\partial \tau} \leq 0$ if and only if $\hat{b}^* \geq -(1 - \alpha) \frac{\hat{k}^*}{R^* - n - g^*}$.

Derivation of Equation (14)

When $\sigma = 0$, the no-arbitrage condition $r_t = R^*$ is necessarily satisfied for all $t$. Otherwise, according to the expression of $\phi_t$ (8), the stock of capital would be infinite. Therefore, the stationarity of wealth implies:

$$R^* = \rho + g^*$$

Using (18) and (19) in per capita terms, along with the fact that $r_t = R^*$ and $R^* = \rho + g^*$, we obtain the following Euler condition:

$$\frac{\dot{c}_t}{c_t} = g^* \quad (22)$$

Therefore, $c_t = c_0 e^{\sigma t}$, and $\hat{c}_t = \hat{c}_0 e^{\sigma t} A_0 / A_t = \hat{c}_0 e^{-\pi t}$. As a consequence, we obtain at steady state: $\hat{c}^* = c_0 e^{-\pi}$. We know also that $\hat{k}_t = \hat{k}^*$ always because of the no-arbitrage
condition. The stationarity of wealth therefore implies that \( \ddot{b}^* \) is also constant in the long run and satisfies:

\[
\ddot{\omega}^* = \dot{k}^* + \ddot{b}^* + \dot{h}^*
\]

Since \( \ddot{c}^* = (\rho - n)\dot{\omega}^* \) and, as we have shown above, \( \dot{h}^* = (1 - \alpha + \tau \alpha)\dot{k}^* / (R^* - (n + g^*)) \), this equation can be rewritten as follows:

\[
\ddot{b}^* = -\dot{k}^* - \frac{(1 - \alpha + \tau \alpha)\dot{k}^*}{\rho - n} + \ddot{c}_0 e^{-\pi} + \dot{\omega}^* - \dot{\omega}_0
\]

Replacing \( \ddot{c}_0 \) using \( \ddot{c}_0 = (\rho - n)\dot{\omega}_0 \), we obtain:

\[
\ddot{b}^* = -\dot{k}^* - \frac{(1 - \alpha + \tau \alpha)\dot{k}^*}{\rho - n} + e^{-\pi} \left[ \dot{h}_0 + \dot{k}_0 + \ddot{b}_0 \right]
\]

Replacing this expression for \( \ddot{b}^* \) in Equation (12) and substituting for \( \dot{h}_0 = (1 - \alpha + \alpha \tau)\dot{k}^* / (\rho - n) - \int_{t}^{T} e^{\pi t} e^{-(\rho - n) t + \pi t} dt \), we obtain:

\[
\frac{\Delta B^R}{Y_0} = -\frac{\dot{k}^* - \dot{k}_0}{\dot{k}_0} e^{(n + g^*) T} - (e^\pi - 1) \frac{\dot{k}^*}{k_0^a} e^{(n + g^*) T}
\]

\[
- e^{\pi + (n + g^*) T} \left( \frac{1 - (\rho - n) - \int_{t}^{T} e^{-(\rho - n) t + \pi t} dt} \right) + (e^{(n + g^*) T} - 1) \frac{\ddot{b}_0}{k_0^a}
\]

Since \( 1 / (\rho - n) = \int_{t}^{T} e^{-(\rho - n) t} dt \), this expression leads to Equation (14).

**Productivity and consumption smoothing**

When \( \sigma = 0 \), under Assumption 3, the third term of Equation (2.6) can be written as follows:

\[
\frac{\Delta B^s}{Y_0} = -e^{(n + g^*) T} \left( 1 - \alpha + \alpha \tau \right) \dot{k}^* / (\rho - n) \int_{t}^{T} e^{-(\rho - n) t + \pi t} dt
\]

A sufficient condition for \( \frac{\Delta B^s}{Y_0} \) to be decreasing in \( \pi \) is that \( e^\pi - e^{\pi f(t)} \) is increasing in \( \pi \). We have:

\[
\frac{\partial [e^\pi - e^{\pi f(t)}]}{\partial \pi} = e^\pi \left[ 1 - f(t)e^{\pi(f(t) - 1)} \right]
\]

Consider the term between brackets:

\[
\frac{\partial [1 - f(t)e^{\pi(f(t) - 1)}]}{\partial f(t)} = -e^{\pi(1-f(t))}[1 + \pi f(t)]
\]
A sufficient condition for this derivative to be negative is $\pi \geq -1$. If it is the case, then for $0 \leq f(t) \leq 1$:

$$1 - f(t)e^{\pi(f(t) - 1)} \geq 0$$

which implies that $\frac{\partial[e^{\pi} - e^{\pi f(t)}]}{\partial \pi} \geq 0$. As a consequence, if $\pi \geq -1$, then $\frac{\Delta B^n}{Y_0}$ is decreasing in $\pi$.

**Derivation of Equation (15)**

In (12), we replace $\tilde{b}^*$ using (11) and $-\frac{\tilde{b}_0}{k_0}$ with $\frac{\tilde{b}_0}{k_0} \left( (e^{(n+g^*)T} - 1) \frac{\tilde{b}_0}{k_0} - e^{(n+g^*)T} \right)$. After rearranging using $\tilde{b}_0 = \frac{1 - \phi^*}{\phi^*} k_0 - \tilde{h}_0$, we get:

$$\frac{\Delta B^n}{Y_0} = 1 - \frac{\phi^*}{k_0} \left( \frac{\tilde{k}^*}{k_0} - \tilde{h}_0 \right) e^{(n+g^*)T} + \frac{1 - \phi^*}{\phi^*} (e^{\pi} - 1) \frac{\tilde{k}^*}{k_0} e^{(n+g^*)T}$$

$$-e^{\pi(n+g^*)T} (1 - \alpha + \alpha \tau) f(\tilde{k}^*) \left( \frac{1}{k_0} \left( \frac{\tilde{b}_0}{k_0} - 1 \right) \frac{\tilde{k}^*}{k_0} e^{(n+g^*)T} \right)$$

$$+ \left( e^{(n+g^*)T} - 1 \right) \frac{\tilde{b}_0}{k_0} \left( \frac{1}{\phi^*} - \frac{1}{\phi_0} \right) e^{(n+g^*)T} \frac{\tilde{k}_0}{k_0}$$

Since $1/(R^* - (n + g^*)) = \int_0^\infty e^{-(R^* - (n + g^*))t} dt$, this expression leads to Equation (15).
Table 1: Long-term capital per efficient unit of labor, capital wedge and potential determinants of capital flows

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
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<tr>
<td></td>
<td>π</td>
<td>b₀</td>
<td>Δk₀</td>
<td>k_T</td>
<td>τ♮</td>
<td>τ♯</td>
<td>P_k</td>
<td>α_k</td>
<td>Obs.</td>
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<td>Non-OECD†</td>
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<td>-31%</td>
<td>-0.65%</td>
<td>1.8</td>
<td>38%</td>
<td>28%</td>
<td>9%</td>
<td>2.61</td>
<td>35%</td>
<td>67</td>
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<td>Sub-Saharan Africa</td>
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<td>-40%</td>
<td>-1.34%</td>
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<td>55%</td>
<td>47%</td>
<td>10%</td>
<td>3.43</td>
<td>31%</td>
<td>28</td>
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<td>Latin America</td>
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<td>-35%</td>
<td>-0.75%</td>
<td>2.0</td>
<td>32%</td>
<td>20%</td>
<td>13%</td>
<td>2.13</td>
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<td>22</td>
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<td>Asia</td>
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<td>0.61%</td>
<td>2.6</td>
<td>19%</td>
<td>6%</td>
<td>2%</td>
<td>1.89</td>
<td>37%</td>
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<tr>
<td>Asia (Excluding China)</td>
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<td>-12%</td>
<td>0.8%</td>
<td>2.6</td>
<td>20%</td>
<td>6%</td>
<td>4%</td>
<td>1.91</td>
<td>38%</td>
<td>16</td>
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</table>

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

♮: calculated within the riskless approach.

♯: calculated within the portfolio approach.

‡: calculated within the portfolio approach and with the adjustment for P_k and α_k
Table 2: Determinants of observed and predicted capital outflows

<table>
<thead>
<tr>
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<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
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<tbody>
<tr>
<td>Dependent variable:</td>
<td>$\Delta \ln Y$</td>
<td>$\Delta B$</td>
<td>$\Delta B$</td>
<td>$\Delta B$</td>
<td>$\Delta B$</td>
<td>$\Delta B$</td>
</tr>
<tr>
<td>All $B_T &lt; 0$</td>
<td>$\pi$</td>
<td>0.565*</td>
<td>-0.417*</td>
<td>0.339</td>
<td>-0.380</td>
<td>0.007</td>
</tr>
<tr>
<td>ln($\tilde{k}^*$)</td>
<td>0.831*</td>
<td>0.385*</td>
<td>(0.432)</td>
<td>(0.224)</td>
<td>(0.432)</td>
<td>(0.224)</td>
</tr>
<tr>
<td>$P_k$</td>
<td>-0.288*</td>
<td>-0.208*</td>
<td>(0.146)</td>
<td>(0.110)</td>
<td>(0.146)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>5.156**</td>
<td>1.606*</td>
<td>(2.420)</td>
<td>(0.900)</td>
<td>(2.420)</td>
<td>(0.900)</td>
</tr>
<tr>
<td>$\ln(\tilde{y}_0)$</td>
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<td>1.186</td>
<td>-0.205</td>
<td>(0.735)</td>
<td>(0.257)</td>
</tr>
<tr>
<td>$(n + g^*) * T$</td>
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<td>-0.426</td>
<td>-0.252</td>
<td>-0.213***</td>
<td>(1.160)</td>
<td>(0.678)</td>
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<td>-0.928***</td>
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<td>-0.180</td>
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<td>Observations</td>
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<td>67</td>
<td>59</td>
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<td>55</td>
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<td>R-squared</td>
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<td>0.057</td>
<td>0.228</td>
<td>0.173</td>
<td>0.270</td>
<td>0.220</td>
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</table>

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Table 3: The channels of the allocation puzzle - Investment

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<th>Dependent variable: $\Delta \frac{K}{Y_0}$</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
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<td>$\Delta K$</td>
<td>0.122*</td>
<td>0.131*</td>
<td>0.066</td>
<td>0.036</td>
<td>0.066</td>
<td>0.071</td>
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<tr>
<td></td>
<td>(0.063)</td>
<td>(0.072)</td>
<td>(0.047)</td>
<td>(0.060)</td>
<td>(0.040)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-0.092</td>
<td></td>
<td></td>
<td></td>
<td>0.274</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.386)</td>
<td></td>
<td></td>
<td></td>
<td>(0.419)</td>
<td></td>
</tr>
<tr>
<td>$\ln(\tilde{k}^*)$</td>
<td></td>
<td></td>
<td>0.544***</td>
<td></td>
<td></td>
<td>0.584***</td>
</tr>
<tr>
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<td></td>
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<td>(0.182)</td>
<td></td>
<td></td>
<td>(0.202)</td>
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<tr>
<td>$P_k$</td>
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<td>-0.262*</td>
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<td>(0.137)</td>
<td>(0.140)</td>
<td></td>
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</tr>
<tr>
<td>$\alpha_k$</td>
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</tr>
<tr>
<td></td>
<td>(1.521)</td>
<td>(1.564)</td>
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<td></td>
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<tr>
<td>Constant</td>
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<td>-0.784***</td>
<td>-0.805***</td>
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<td>-1.122*</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.225)</td>
<td>(0.124)</td>
<td>(0.214)</td>
<td>(0.559)</td>
<td>(0.575)</td>
</tr>
<tr>
<td>Observations</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.091</td>
<td>0.092</td>
<td>0.167</td>
<td>0.171</td>
<td>0.182</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
### Table 4: The channels of the allocation puzzle - GDP growth

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\Delta Y}{Y_0})</td>
<td>0.175*</td>
<td>0.070</td>
<td>0.122</td>
<td>-0.004</td>
<td>0.114</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.163)</td>
<td>(0.095)</td>
<td>(0.156)</td>
<td>(0.075)</td>
<td>(0.158)</td>
</tr>
<tr>
<td>(\pi)</td>
<td>0.380</td>
<td>0.453</td>
<td>-0.126</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.596)</td>
<td>(0.573)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ln(\tilde{k}^*))</td>
<td>0.638***</td>
<td>0.643***</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.207)</td>
<td>(0.209)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_k)</td>
<td></td>
<td>-0.304**</td>
<td>-0.313**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.152)</td>
<td>(0.155)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_k)</td>
<td></td>
<td>3.173**</td>
<td>3.149*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.578)</td>
<td>(1.621)</td>
<td></td>
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<tr>
<td>Constant</td>
<td>-0.740***</td>
<td>-0.544</td>
<td>-0.869***</td>
<td>-0.636*</td>
<td>-1.118*</td>
<td>-1.143*</td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(0.404)</td>
<td>(0.146)</td>
<td>(0.354)</td>
<td>(0.584)</td>
<td>(0.641)</td>
</tr>
<tr>
<td>Observations</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.034</td>
<td>0.038</td>
<td>0.162</td>
<td>0.168</td>
<td>0.171</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Figure 1: Capital outflows against TFP catch-up - 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Outliers are Hong Kong and Botswana. China is excluded from the figure.
TFP growth is estimated as $\pi = \ln(A_T) - \ln(A_0)$, where $A_t$ is the trend of TFP, measured through a growth accounting method. See Section 3 for details.
Figure 2: Capital outflows against the long-term capital stock per efficient unit of labor - 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Outliers are Hong Kong and Botswana. China is excluded from the figure.

The long-term capital stock per efficient unit of labor is the TFP-adjusted capital/labor ratio at the end of the sample. It is measured as \( \bar{k}_t = K_t / (A_t L_t) \), where \( K_t \) is the aggregate capital stock, \( A_t \) is TFP and \( L_t \) is the labor force, and then cleaned from short-run fluctuations. See Section 3 for details.
Figure 3: Observed and predicted capital flows by region

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Observed capital flows are the observed ratio of net capital outflows to initial output, predicted capital flows in the riskless and portfolio approaches are respectively $\frac{\Delta BR}{Y_0}$ as defined by Equation (2.6) and $\frac{\Delta BP}{Y_0}$ as given by Equation (12).

The figures are unweighted country averages.

Non-OECD countries include also Korea, Mexico and Turkey.
Figure 4: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the riskless approach, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Note: China is excluded.
Figure 5: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.
Figure 6: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach, 1980-2003 - Robustness

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), Bernanke and Gürkaynak (2001), author’s calculations.