The Demand for Liquid Assets, Corporate Saving, and Global Imbalances

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Abstract

In the recent decade, capital outflows from emerging economies, in the form of a demand for liquid assets, have played a key role in the context of global imbalances. In this paper, we model the demand for liquid assets by firms in a dynamic open-economy macroeconomic model. We find that the implications of this model are very different from standard models, because the demand for foreign bonds is a complement to domestic investment rather than a substitute. We show that this complementarity is at work when an emerging economy is on its convergence path or when it has a higher TFP growth rate. This framework is consistent with global imbalances and with a number of stylized facts such as high corporate saving rates in high-growth, high-investment, emerging countries.

Key Words: Capital flows, Global imbalances, Working capital, Credit constraints.

JEL Class.: E22, F21, F41, F43.
1 Introduction

A striking feature of global capital flows in the recent decade has been the increased demand for liquid assets by emerging economies, especially emerging Asia. While the policy focus has been on the central bank accumulation of reserves, there are more fundamental underlying forces leading to global imbalances. In particular, it is interesting to notice that the increase in the demand for liquid assets has been accompanied by an increase in corporate saving in emerging Asia. Figure 1 shows the recent evolution of corporate saving for a subset of Asian countries.\textsuperscript{1} The GDP-weighted average corporate saving was 14.6\% in 2004-2008 compared to 9.8\% in the 1993-2003 period for the six countries included in Figure 1 (the simple average was 10.8\% compared to 7.3\% over the same periods). The recent period coincided with a substantial increase in foreign bond holdings. For example, holdings of US Treasury securities in these six countries increased as a proportion of GDP and went from 8.9\% of their GDP at the end of 2003 to 12.0\% in December 2008.

The objective of this paper is to propose an explanation for the link between high corporate saving and the demand for liquid assets in the context of global imbalances. We model explicitly the demand for liquid assets by firms in an infinite horizon economy with a low level of financial development. We consider both a small open economy and an asymmetric two-country framework composed of an industrial country and an emerging country. We show that, due to the lower financial development, the emerging country has a demand for liquidity that can generate net capital outflows. This demand is more likely to arise in periods of fast productivity growth.

We follow the vast literature on liquidity, where liquid assets are needed in some stages of the production process. We show that in an open economy where liquidity is used to finance working capital, the demand for foreign bonds is a complement to domestic investment. This complementarity is in sharp contrast with standard intertemporal models where capital and foreign bonds are substitutes. Consider for example an increase in domestic productivity growth. In standard models, this implies an increase in investment associated with a decline in foreign bonds through borrowing. This tends to imply a current account deficit. On the other hand, a model with liquidity demand implies an increase in foreign bonds holdings following a

\textsuperscript{1}The corporate saving data comes from Sonali et al. (2009). We are grateful to these authors for providing us with the data.
productivity shock. This means that stronger growth may lead to a current account surplus.

The model’s implications are consistent with the recent episode of global imbalances, with capital flowing from emerging Asia to the U.S. This can explain the decline in global interest rates, which is often attributed to a "saving glut". Moreover, the model is consistent with several additional facts. First, this period coincides with episodes of high growth and high investment levels in Asia. Table 1 shows that the GDP-weighted average growth rate is 8.5%, while the average investment rate is 37%. Second, the current account and growth in emerging Asia are positively correlated in the period 2004-2008. Table 1 shows that, for the six countries of Figure 1, the average correlation is 0.4, while the pooled correlation is 0.31.\(^2\) More generally, the fastest-growing countries export capital instead of attracting it, as pointed out by Lucas (1990), and more recently by Gourinchas and Jeanne (2009). Sandri (2010) also documents that episodes of growth acceleration are accompanied by net capital outflows. Third, saving is positively correlated with growth (e.g., see Attanasio et al., 2000). As we will argue, the existing literature cannot explain all these features simultaneously.

The demand for liquid assets comes from infinitely lived credit-constrained entrepreneurs who have investment projects that last two periods. Entrepreneurs need to install their capital one period before producing, so capital is a long-term asset while bonds are short-term assets. In the period where entrepreneurs install their capital, they anticipate a need for funds (working capital) to operate their firms, e.g., to hire labor. If entrepreneurs are credit constrained for their future working capital, they will need to save in liquid bonds at the same time as they invest in capital. Since bonds are used to finance inputs that are imperfect substitutes to capital, this creates a complementarity between capital and liquid assets. In contrast, if entrepreneurs are unconstrained, they can borrow their working capital and have no need for liquidity. This liquidity motive is generated by a production structure, with time-to-build and working capital, that can be naturally incorporated in a dynamic macroeconomic model.\(^3\) We assume that entrepreneurs have an investment project every other period and that at each period half the entrepreneurs have a new project.

\(^2\)This can be compared to a pooled correlation of -0.04 if we look at a larger sample of 62 emerging and developing countries over the same period 2004-2008.

\(^3\)The assumptions of time-to-build and working capital are often made in macroeconomic models. For example, see Gilchrist and Williams (2000) for multi-period investment projects and Christiano et al. (2010) for working capital to pay for the wage bill.
While our model is built to study macroeconomic questions which have hardly been addressed in the literature on liquidity, it shares many features with previous work. In particular, as in Holmstrom and Tirole (2001), the lack of pledgeability of future output is crucial to generate a demand for liquid assets.\(^4\) In a dynamic macro context, our demand for liquidity is in the spirit of Woodford (1990), where entrepreneurs receive high productivity projects on alternating dates. It is also in the spirit of Kiyotaki and Moore (2008), where entrepreneurs have a fifty percent probability of receiving a high productivity shock. Our production structure is different and does not assume productivity heterogeneity across agents. The only source of heterogeneity is the existence of two groups of entrepreneurs who start projects at alternating dates.

Our contribution is also related to a growing literature introducing credit market imperfections in open economy models.\(^5\) In particular, Song et al. (2010) model a capital outflow with firm heterogeneity specific to the Chinese economy. However, their focus is on growth and they do not introduce a demand for liquidity.

The recent literature has proposed two main explanations for the net capital outflows from emerging markets. First, emerging markets have a limited supply of financial assets (e.g., Dooley et al., 2005, Matsuyama, 2007, Ju and Wei, 2006, 2007, Caballero et al., 2008, and Aguiar and Amador, 2009). Second, net capital outflows result from precautionary saving due to idiosyncratic risk (e.g., Mendoza et al., 2009, Sandri, 2010, Angeletos and Panousi, 2010, Benhima, 2010). However, the fact that recent imbalances involve mainly liquid assets has only received limited attention. Moreover, in precautionary saving models, global imbalances are associated with a decline in investment, which is counterfactual in the case of emerging Asia. The reason is that the demand for bonds comes from a preference for safe assets as opposed to risky capital so that bonds and capital are still substitutes.\(^6\) In contrast, with a liquidity need a net capital outflow will be associated with higher productivity and higher investment. To

\(^4\)Most of the literature following Holmstrom and Tirole (2001) is cast in a microeconomic setup with two or three periods. However, Aghion et al. (2010) present a dynamic macroeconomic model where entrepreneurs hoard in the perspective of future liquidity shocks.

\(^5\)Earlier contributions include Aghion et al. (2004) and Gertler and Rogoff (1990).

\(^6\)In Mendoza et al. (2009) and especially Mendoza et al. (2007), excess saving generated by risk is diverted from domestic capital to foreign assets which leads to a decrease in investment. While Benhima (2010) shows that with investment risk growth is associated with capital outflows in the long run, Angeletos and Panousi (2010) show that financial liberalization still coincide with a decrease in investment on impact.
draw a sharp contrast with the impact of precautionary saving, we consider a model without uncertainty.

To better explain the model’s mechanism we first examine the behavior of entrepreneurs in partial equilibrium when they are either constrained or unconstrained. We show that credit-constrained entrepreneurs have a demand for liquidity and examine the properties of this demand. Then we incorporate these entrepreneurs in a dynamic small open economy and examine its dynamics and steady state. We extend the analysis to a two-country general equilibrium model, assuming that entrepreneurs in one country, the Emerging country, are constrained and those in the other country, the Industrial country, are unconstrained. We derive analytical results in a simple benchmark case and then provide numerical results in more general cases.

We show that the demand for liquidity arises whenever the emerging economy is credit constrained. When the emerging country has the same rate of impatience as the rest of the world, it is not constrained in the steady state since entrepreneurs are infinitely lived. But we show that credit constraints still emerge in three distinct situations: i) in its convergence path towards its unconstrained steady state; ii) in a steady state where TFP growth is permanently higher than in other countries; iii) with temporary increases in TFP growth. While the first two situations can be studied analytically, we use numerical simulations to examine temporary shocks. Importantly, we do not assume that the emerging country is more impatient by imposing different preferences (different discount factors). The emerging country is credit constrained because its higher growth rate makes it endogenously more impatient. We find that in all these situations, the model matches the various facts mentioned above. Indeed, when a country experiences high growth, it becomes constrained which makes capital and foreign assets complementary. This generates a positive correlation between growth, investment and capital outflows.

Although these results are derived in a stylized framework, we consider several extensions to show that the basic mechanism holds in a wider context. For example, we discuss whether it holds with additional precautionary saving due to uncertainty. Moreover, we suggest that the demand for liquid assets by entrepreneurs can be consistent with an accumulation of reserves by the central bank when there are capital controls. We also show that the demand for liquid assets can coincide with FDI inflows, thereby generating two-way capital flows.
In the next section we describe the mechanism leading to the demand for liquidity by credit-constrained entrepreneurs. Section 3 presents the small open economy model and Section 4 describes the two-country analysis. Section 5 examines various extensions and Section 6 concludes.

2 Entrepreneurs and the Demand for Liquidity

We first consider entrepreneurs in a partial equilibrium setup. This allows us to clearly understand the mechanism behind the demand for liquid assets. There are basically three ingredients in the model that are necessary to generate a demand for liquidity. First, production takes time: capital needs one installation period before it can be used in the production process. Second, a portion of the wage bill has to be paid before output is available to entrepreneurs. This generates a need for funds. The third assumption is that entrepreneurs face credit constraints. This implies that entrepreneurs are not always able to borrow all the funds needed to hire labor for production. Consequently, when they invest in capital, entrepreneurs need to keep liquid assets. The fact that liquid assets are used to finance a production factor (here, labor) that is imperfectly substitutable with capital generates a complementarity between these assets and capital.

In this section, we focus on the demand for liquidity by entrepreneurs. In particular, we study how they allocate their saving between capital and liquidity. We first describe the optimal behavior of entrepreneurs in a general setup. We then focus on a benchmark case that allows us to derive analytical results on the demand for liquidity.

2.1 The production process

Entrepreneurs are infinitely lived and maximize the present value of their utility. They have two-period production projects as it takes one period to install capital before producing. An entrepreneur starting a project at time $t$ invests $K_{t+1}$. At $t+1$, once capital is installed, he hires labor $l_{t+1}$ to produce $Y_{t+1} = K_{t+1}^\alpha (A_{t+1} l_{t+1})^{1-\alpha}$, where $A_t$ measures productivity, and pays a fraction $\kappa$ of wages $w_{t+1} l_{t+1}$. This production is available only at $t+2$. At $t+2$, the entrepreneur pays the remaining wages and gets another investment opportunity. The entrepreneur also consumes $c_t$ each period and can borrow or lend short-term bonds $B_t$ with a
gross interest rate \( r_t \).

In this setup, working capital in the form of early payment of wages (high \( \kappa \)) and credit constraints interact to generate a demand for liquidity. Entrepreneurs can use part of the proceeds from previous production to invest \( K_{t+1} \) and pay the remaining wages at \( t \). At \( t + 1 \), however, they have no income to pay \( \kappa w_{t+1}l_{t+1} \) for workers. Consequently, they have an incentive to borrow \(-B_{t+2}\). When an entrepreneur is credit-constrained, however, he will not be able to borrow the desired amount to pay for the wage bill. He will therefore have a demand for liquidity at time \( t \) in the form of a positive demand for bonds, \( B_{t+1} \). When the entrepreneur is unconstrained, there is no need for liquidity at time \( t \).

### 2.2 Optimal Behavior

Entrepreneurs maximize:

\[
\sum_{s=0}^{\infty} \beta^s \ln(c_s) \tag{1}
\]

Consider an entrepreneur who invests every other period, starting at time \( t \). Denote by \( W_t \) his initial income at time \( t \). It is made of the output from production initiated at date \( t - 2 \), \( Y_{t-1} = K_{t-1}^{\alpha}(A_{t-1}l_{t-1})^{1-\alpha} \), and of the return from bond holdings, \( r_tB_t \). Hence, \( W_t = Y_{t-1} + r_tB_t \). His budget constraint at \( t \) and \( t + 1 \) are:

\[
W_t = c_t + (1 - \kappa)w_{t-1}l_{t-1} + K_{t+1} + B_{t+1} \tag{2}
\]

\[
r_{t+1}B_{t+1} = c_{t+1} + \kappa w_{t+1}l_{t+1} + B_{t+2} \tag{3}
\]

The income of the entrepreneur at date \( t \) is allocated to consumption, \( c_{t+1} \), the remaining wages \((1 - \kappa)w_{t-1}l_{t-1}\), investment in a new project, \( K_{t+1} \), and bond holdings \( B_{t+1} \). In the following period, at \( t + 1 \), the only income is the bond return, \( r_{t+1}B_{t+1} \). This has to pay for consumption \( c_{t+1} \) and part of the wage bill \( \kappa w_{t+1}l_{t+1} \). Typically the entrepreneur will borrow, so that at the optimum \( B_{t+2} \leq 0 \).

The entrepreneur might face a credit constraint at date \( t + 1 \). Due to standard moral hazard arguments, a fraction \( 0 \leq \phi \leq 1 \) of capital has to be used as collateral for bond repayments:

\[
r_{t+2}B_{t+2} \geq -\phi K_{t+1} \tag{4}
\]

\[7\text{There could be a similar constraint at date } t, \text{ but one can show that it is never binding, precisely because of the demand for liquidity.}\]
Let $\lambda_{t+1}$ denote the multiplier associated with this constraint. The entrepreneur’s program yields the following first-order conditions:

$$
\alpha \left( \frac{K_{t+1}}{A_{t+1}l_{t+1}} \right)^{\alpha-1} = r_{t+1}r_{t+2} \left( 1 + \frac{\lambda_{t+1}c_{t+2}}{\beta} \left( 1 - \frac{\phi}{r_{t+1}r_{t+2}} \right) \right)
$$

(5)

$$
(1 - \alpha) \left( \frac{K_{t+1}}{A_{t+1}l_{t+1}} \right)^{\alpha} = \bar{w}_{t+1} \left[ \kappa r_{t+2} \left( 1 + \frac{\lambda_{t+1}c_{t+2}}{\beta} \right) + (1 - \kappa) \right]
$$

(6)

$$
\frac{c_{t+1}}{c_t} = \beta r_{t+1}
$$

(7)

$$
\frac{c_{t+2}}{c_{t+1}} = \beta r_{t+2} \left( 1 + \frac{\lambda_{t+1}c_{t+2}}{\beta} \right)
$$

(8)

The credit constraint (4) introduces three wedges in the optimal decisions. First, from equation (5), when $\lambda_{t+1} = 0$, the marginal return of capital invested at $t$ should be equal to the return of one unit invested over two periods in the bond, as capital is immobile for two periods. But when $\lambda_{t+1} > 0$, the constraint is binding at $t+1$, which implies that the entrepreneur is unable to finance the wage bill associated with the first-best capital stock. This creates a wedge between the return of capital and the bond return. Moreover, this wedge is decreasing in $\frac{\phi}{r_{t+1}r_{t+2}}$, which is the relative liquidity value of capital as compared to the bond. Second, from equation (6), when $\lambda_{t+1} = 0$, the marginal return of labor should be equal to its cost, which is given by the wage rate multiplied by $\kappa r_{t+2} + (1 - \kappa)$. The cost of the fraction $\kappa$ of wages that is paid in advance is upgraded by the interest rate because it generates an opportunity cost to the entrepreneur. When $\lambda_{t+1} > 0$, the entrepreneur has exhausted his financing capacities before hiring the first-best level of labor, which creates a wedge between the marginal productivity of labor and the wage. Finally, when $\lambda_{t+1} > 0$, it is more difficult to transfer consumption between period $t+1$ and $t+2$: there are excess saving at $t+1$, as equation (8) suggests.

2.3 A Benchmark Case

To derive simple analytical results for the constrained entrepreneur ($\lambda_{t+1} > 0$), we consider a benchmark where we make two specific assumptions: i) entrepreneurs cannot borrow: $\phi = 0$; ii) wages have to be paid entirely in advance: $\kappa = 1$. We examine the implications of relaxing these assumptions in Section 3.

With log utility, it can be shown that an entrepreneur who invests at $t$ consumes a fixed fraction of his revenue:

$$
e_t = (1 - \beta)W_t
$$

(9)
Using the Euler equation (7) at $t$, we get the following rule for consumption at $t + 1$:

$$c_{t+1} = \beta(1 - \beta)r_{t+1}W_t$$  \hspace{1cm} (10)$$

From (2) and (9), total saving at $t$ is:

$$S_{t+1} = B_{t+1} + K_{t+1} = \beta W_t$$  \hspace{1cm} (11)$$

Equation (11) states that total saving at $t$ is a constant fraction of total revenues. This equation is used to derive $B_{t+1}$. In the constrained case, we need to determine jointly $K_{t+1}$ and $B_{t+1}$. In the unconstrained case, $K_{t+1}$ is first found independently of $B_{t+1}$ and then $B_{t+1}$ can be derived from (11).

To determine whether entrepreneurs are constrained or not, it is useful to look at labor market conditions. Entrepreneurs are constrained ($\lambda_{t+1} > 0$) whenever the market wage is lower than the first best wage. Define $\tilde{w}_t = w_t/A_t$ the wage normalized by TFP and $\tilde{w}(r_{t+1}, r_{t+2}) = (1 - \alpha)[\alpha^\alpha/(r_{t+1}^\alpha r_{t+2})]^{1/\alpha}$ its first-best level. Entrepreneurs are constrained when $\tilde{w}_{t+1} < \tilde{w}_{t+1}$. In that case, the entrepreneur could make infinite profits by increasing the production scale, but is prevented by the binding credit constraint. If $\tilde{w}_{t+1} = \tilde{w}_{t+1}$, the production scale is undetermined, because of constant returns to scale. There is no reason for the entrepreneur to be constrained in that case.

2.4 The Demand for Liquidity from Constrained Entrepreneurs

When the constraint at $t + 1$ is binding, the availability of funds to finance the wage bill at $t + 1$ is limited. The fraction of saving allocated to liquidity $B_{t+1}$ therefore depends on the liquidity needs at $t + 1$, $w_{t+1}l_{t+1}$. These needs are related to the amount of capital $K_{t+1}$ invested at $t$, since $K_{t+1}$ and $l_{t+1}$ are imperfect substitutes.

Since $\phi = 0$, the first-order conditions (5) and (6) give a straightforward relationship between the liquidity needs $w_{t+1}l_{t+1}$ and capital $K_{t+1}$:

$$w_{t+1}l_{t+1} = \frac{1 - \alpha}{\alpha}r_{t+1}K_{t+1}$$  \hspace{1cm} (12)$$

To determine $K_{t+1}$ we use (3), (10), (11) with (12) to get:

$$K_{t+1} = \alpha^{\beta^2}W_t$$  \hspace{1cm} (13)$$

---

8This can be seen by combining first-order conditions (5) and (6) in the benchmark case, which yields:

$$\tilde{w}_{t+1} \left(1 + \frac{\lambda_{t+1}r_{t+2}}{r_{t+2}}\right)^{1-\alpha} = \tilde{w}(r_{t+1}, r_{t+2}).$$
Replacing in (11), we obtain:

\[ B_{t+1} = \beta(1 - \alpha \beta)W_t \]  \hspace{1cm} (14)

Moreover, since \( \phi = 0 \), \( B_{t+2} = 0 \). From (11), it is interesting to notice that the demand for liquidity \( B_{t+1} \) is proportional to the entrepreneurs saving \( S_{t+1} \).

The key implication of (13) and (14) is that the ratio between \( B_{t+1} \) and \( K_{t+1} \) is constant:

\[ \frac{B_{t+1}}{K_{t+1}} = \frac{1 - \alpha \beta}{\alpha \beta} \]  \hspace{1cm} (15)

This implies that, contrary to standard models, capital and bonds are complements, because bonds are needed to finance the wage bill, which is proportional to capital. Indeed, the bond-capital ratio is decreasing in \( \alpha \), the share of capital in the value added. The higher \( \alpha \), the lower the amount of bonds needed to finance labor. An important consequence of this result is that growth in \( K \) will naturally generate growth in \( B \), leading to so-called “global imbalances”.

The complementarity between liquidity and capital is in sharp contrast with the case where entrepreneurs are unconstrained. In the unconstrained case, capital and the demand for bonds are substitutes. Indeed, capital is determined by (5), and the demand for bonds is determined by the amount of saving that is not used for capital, just as in standard models.

### 3 A Small Open Economy Model

The entrepreneurs described above are incorporated in a small open economy model. There are two groups of entrepreneurs, with each group starting a project at alternating dates. Labor is supplied by hand-to-mouth workers. Entrepreneurs can lend or borrow at the world interest rate \( r_t \). We assume that the rest of the world has a constant productivity growth \( g^* \), a discount factor \( \beta^* \), and no financial frictions. Hence the world interest rate is constant at \( r^* = (1 + g^*)/\beta^* \).

We assume that the small open economy is defined by the benchmark, that is by \( \phi = 0 \) and \( \kappa = 1 \). Both hypotheses are justified by poor legal enforcement in emerging countries. The discount factor \( \beta \) is the same as in the rest of the world, \( \beta = \beta^* \), and the productivity growth rate is \( g_t = 1 - A_t/A_{t-1} \). After describing entrepreneurs and the labor market in this economy, we describe the dynamics and the steady state for a constant growth rate \( g \). Then, we examine examples of temporary increases in growth. It will be convenient to normalize the variables by \( A_t \) and denote \( \tilde{X}_t = X_t/A_t \).
3.1 Two Groups of Entrepreneurs

Each entrepreneur has access to a project every two periods. There are two groups of entrepreneurs, each with mass one, with overlapping projects. One group of entrepreneurs gets a project in odd periods, while the other group gets a project in even periods. The analysis of a single entrepreneur, described in the previous section, can be easily extended by slightly changing the notation. Denote by \( \tilde{B}^1_{t+1} \) and \( \tilde{B}^2_{t+1} \) the demands for bonds of entrepreneurs who are respectively in their investment and in their production periods (i.e., entrepreneurs who have started their project at time \( t \) and at time \( t + 1 \)). Then, from (14) we have:

\[
\begin{align*}
\tilde{B}^1_{t+1} &= \frac{\beta (1 - \alpha \beta)}{1 + g_{t+1}} \tilde{W}_t \\
\tilde{B}^2_{t+1} &= 0
\end{align*}
\]

and the total demand for bonds at time \( t \) is: \( \tilde{B}_{t+1} = \tilde{B}^1_{t+1} + \tilde{B}^2_{t+1} \).

The two groups of entrepreneurs never interact on the domestic labor market, as they only hire labor in their production period. Since the world interest rate \( r^* \) is given, the dynamics of the two groups can be studied independently from each other. As entrepreneurs are identical within a given category, the behavior of the aggregate economy is obtained simply by summing their policy functions.

3.2 Labor Market

3.2.1 Labor demand

In the previous section we showed that entrepreneurs are constrained when \( \tilde{w}_{t+1} < \hat{w}(r^*, r^*) = (1 - \alpha)\alpha^{\frac{a}{1-a}} / r^*^{\frac{1+a}{1-a}} \). We simply denote \( \hat{w}(r^*, r^*) \) by \( \hat{w}(r^*) \). In this case, labor demand is determined by the credit constraint. In the unconstrained case, labor demand is undetermined as long as entrepreneurs have enough funds. The maximum labor demand in this case is

\[
\tilde{l}_{t+1}(\tilde{W}_t, \tilde{w}_{t+1}) = \frac{(1 - \alpha) r^* \beta^2 \tilde{W}_t}{(1 + g_{t+1}) \tilde{w}_{t+1}}
\]

Labor demand is then described as follows:

\[
\begin{align*}
l_{t+1} &\in (0, \tilde{l}_{t+1}) \quad \text{if} \ \tilde{w}_{t+1} = \hat{w} \\
l_{t+1} &= \tilde{l}_{t+1} \quad \text{if} \ \tilde{w}_{t+1} < \hat{w}
\end{align*}
\]
3.2.2 Labor supply

Labor is supplied domestically by a continuum of hand-to-mouth workers of mass one who do not have access to the production technology and consume all their income: $c_t^w = w_t l_t$.

We assume that workers have at most 1 unit of labor to supply and that they have a reservation wage equal to $A_t w$. This gives the following labor supply equation:

$$l_t \in (0, 1) \quad \text{if } \tilde{w}_t = w$$

$$l_t = 1 \quad \text{if } \tilde{w}_t > w$$

(20)

(21)

Labor supply is infinitely elastic around $\tilde{w}_t = w$. For $\tilde{w}_t > w$, workers supply the maximum amount of labor ($l_t = 1$) and their labor supply is inelastic. These different labor supply regimes will be especially relevant when we introduce FDI.

3.2.3 Labor market equilibrium

It is useful to examine the equilibrium in the labor market as it influences the dynamics of the economy. There are three different situations for labor market equilibrium that are illustrated in Figure 2. $l^S$ represents total labor supply, while $l^D_1$, $l^D_2$, and $l^D_3$ represent labor demand for increasing levels of revenues $\tilde{W}_t$. These three states of labor demand result respectively in: (1) constrained firms with unemployment; (2) constrained firms with full employment; (3) unconstrained firms. They are illustrated by the three equilibria (1), (2), and (3).

In case (1), entrepreneurs are too poor to hire all the workforce, even at their reservation wage $w$. Therefore, the equilibrium wage is the one at which workers are indifferent between working and not working – which is precisely $\tilde{w}$ – so the equilibrium labor hired is $\bar{l}_1 < 1$. In case (2), entrepreneurs are able to hire all the workforce – so $l_2 = 1$ – but not to pay them at their marginal productivity – so $\tilde{w}_2 < \hat{w}(r^*)$. In case (3), entrepreneurs are sufficiently rich to offer the first-best wage to the workers, so $\tilde{w}_3 = \hat{w}(r^*)$.

3.3 Dynamics and Balanced Growth Path

We now examine the dynamics and the steady state of this economy for a constant growth rate $g$. We first focus on the level of income $\tilde{W}_t$, which is the state variable, and then on the level of capital $\tilde{K}_t$ and bonds $\tilde{B}_t$. We assume that the country starts with an income level, $W_0$,
below its steady state $\bar{W}$. We show that when $g = g^*$, entrepreneurs are constrained on their
collapse state path and have a demand for liquidity. But they accumulate sufficient funds over
time to become unconstrained in the long run. On the other hand, when $g > g^*$ entrepreneurs
are always constrained in the long run. We first characterize the steady with the following
proposition:

**Proposition 1.** If $w < \left( \frac{r^* \beta}{1+g} \right)^{1-\alpha} \hat{w}(r^*)$, an equilibrium where $\tilde{K}_t$, $\tilde{B}_t$, and $\tilde{W}_t$ are stationary
exists. Entrepreneurs are constrained in the stationary equilibrium if $g > g^*$ and unconstrained
if $g = g^*$. This equilibrium is characterized by the following:

(i) $\tilde{K}_t = \bar{K} = \left( \alpha \left( \frac{\beta}{1+g} \right)^2 \right)^{\frac{1}{\alpha-1}}$.

(ii) $\tilde{B}_t = \bar{B} = \frac{1-\alpha \beta}{\alpha \beta} \tilde{K}$ if $g > g^*$ and $\bar{B}$ is undetermined if $g = g^*$.

(iii) $\tilde{W}_t = \bar{W} = \bar{K}^{\alpha}$ if $g > g^*$ and $\bar{W}$ is undetermined if $g = g^*$.

The equilibrium of $\bar{B}$ and $\bar{W}$ is then unique if $g > g^*$.

We leave the proof of this proposition to the Appendix. We will instead focus on the
dynamics and illustrate this proposition graphically. Let us simply mention here that the
indeterminacy of $\bar{B}$ and $\bar{W}$ when $g = g^*$ is a typical feature of unconstrained infinite-horizon
small open economies.

The dynamics depend on whether the credit constraint is binding or not and whether
there is full employment. This corresponds to the three situations described for the labor
market. Denote by $W_1$ the threshold level of revenue where there is full employment, but
entrepreneurs are still constrained; and by $W_2 > W_1$ the threshold where entrepreneurs are
no longer constrained. This can be related to Figure 1. In case (1), $\tilde{W}_t < W_1$; in case (2),
$W_1 < \tilde{W}_t < W_2$; and in case (3), $\tilde{W}_t > W_2$. The condition for the unconstrained economy and
the values for $W_1$ and $W_2$ are derived in the Appendix.

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The case $g > g^*$ is inconsistent with the small economy assumption in the steady state. However, it is still
of interest to examine this case as we will later look at an example where the economy grows temporarily faster.
An alternative would be to consider the case $\beta < \beta^*$, which also implies that entrepreneurs are constrained
in the steady state. While this assumption is commonly used in the literature, we do not find it convincing to
explain international capital flows by differences in preferences.
Using the definition of $\tilde{W}_t$, the dynamics of firms’ revenues are described by:

$$\tilde{W}_{t+2} = \left( \frac{K_{t+1}^{1-\alpha} l_{t+1}}{1+g} \right) + r^* \tilde{B}_{t+2}$$  \hspace{1cm} (22)$$

where:

$$\tilde{K}_{t+1} = \min \left\{ \alpha \beta \left( \frac{\beta}{1+g} \right) \tilde{W}_t, \tilde{K}(r^*) \right\}$$ \hspace{1cm} (23)$$

$$l_{t+1} = \min \left\{ 1, \bar{l}(\tilde{W}_t, w) \right\}$$ \hspace{1cm} (24)$$

$$\tilde{B}_{t+2} = \max \left\{ 0, r^* \frac{\beta^2}{(1+g)^2} \tilde{W}_t - \frac{r^* \tilde{K}(r^*)}{1+g} - \frac{\tilde{w}(r^*)}{1+g} \right\}$$ \hspace{1cm} (25)$$

where $\tilde{K}(r^*) = (\alpha/r^2)^{1-\alpha}$ is the first-best efficient capital stock. This implies the following dynamics in each of the three cases:

$$\tilde{W}_{t+2} = \frac{r^2 \beta^2}{(1+g)^2} \left( \frac{\tilde{w}(r^*)}{w} \right)^{1-\alpha} \tilde{W}_t$$ \hspace{1cm} (26)$$

in case (1)

$$= \left[ \alpha \frac{\beta^2}{(1+g)^2} \tilde{W}_t \right]^\alpha$$ \hspace{1cm} (27)$$

in case (2)

$$= \frac{r^2 \beta^2}{(1+g)^2} \tilde{W}_t$$ \hspace{1cm} (28)$$

in case (3)

In cases (1) and (2), when $\tilde{W}_t < W_2$, entrepreneurs cannot reach the first-best level of capital, so that $\tilde{K}_{t+1} = \alpha \beta \left( \frac{\beta}{1+g} \right) \tilde{W}_t$ and $\tilde{B}_{t+2} = 0$. The difference between case (1) and case (2) is that, in the former, there is unemployment ($l_t = \bar{l}(\tilde{W}_t, w)$) while in the latter, all the workforce is hired ($l_t = 1$).

Finally, in case (3), when $\tilde{W}_t > W_2$, firms are sufficiently rich to achieve the first-best level of capital $\tilde{K}_{t+1} = \tilde{K}(r^*)$. Besides, $\tilde{B}_{t+2}$ is equal to $\left( r^* \frac{\beta^2}{(1+g)^2} \tilde{W}_t - \frac{r^* \tilde{K}(r^*)}{1+g} - \frac{\tilde{w}(r^*)}{1+g} \right)$, which represents the amounts of savings cumulated over two periods $\left( \frac{\beta^2}{(1+g)^2} r^* \tilde{W}_t \right)$, minus the intertemporal, growth-adjusted, costs of production $\left( \frac{r^* \tilde{K}(r^*)}{1+g} + \frac{\tilde{w}(r^*)}{1+g} \right)$. The dynamics of $\tilde{W}$ depend linearly on its past values because (i) under log utility, savings are proportional to revenues, (ii) under constant returns to scale, the return on capital is linear and, (iii) under profit maximization, the returns on capital and bonds are equalized.

Figure 3 represents the dynamics of $\tilde{W}$ when $g = g^*$. In case (1), the dynamics are linear in $\tilde{W}$, which is the result of constant returns to scale and a hyper-elastic supply of labor at
\( \dot{w} = w \). Since we assume that \( w < \dot{w}(r^*) \), entrepreneurs’ revenues are increasing along these dynamics. This is reflected in the fact that the first part of the curve (1) is above the 45-degree line. When entrepreneurs use the whole workforce, i.e., in case (2) where \( W_1 < \tilde{W}_t < W_2 \), the dynamics become concave because the marginal returns to capital are decreasing, due to a constant labor supply. The economy reaches its steady state when \( \tilde{W}_t \) reaches \( W_2 \).

To better understand the dynamics, we now turn to the evolution of capital and bonds in the convergence process. The dynamics of \( \tilde{K}_{t+1} \) are summarized by (23). \( \bar{B}_{t+1} \) is then simply the share of saving \( \beta \tilde{W}_t / (1 + g) \) that is not invested in production, while \( \bar{B}_{t+2} \) is given by (25). Figure 4 shows the evolution of these three variables as a function of \( \tilde{W}_t \). The first striking result is that \( \tilde{K}_{t+1} \) and \( \bar{B}_{t+1} \) move in the same direction when the entrepreneur is constrained (\( \tilde{W}_t < W_2 \)). This illustrates the complementarity between the two variables. This contrasts with the unconstrained case \( \tilde{W}_t \geq W_2 \), where \( \bar{B}_{t+1} \) moves independently from \( \tilde{K}_{t+1} \). The evolution of \( \bar{B}_{t+2} \) complements the analysis: \( \bar{B}_{t+2} = 0 \) when firms are constrained because they liquidate their bond holdings in \( t + 1 \), while \( \bar{B}_{t+2} \) moves independently from \( \tilde{K}_{t+1} \) in the unconstrained case. This implies that when all entrepreneurs are constrained, the domestic net foreign asset will comove with capital.

The second result from Figure 4 is that the long-run capital stock corresponds to its first-best level \( \tilde{K}(r^*) \). The reason is that the balanced growth path entails that the propensity to save \( \beta \), multiplied by the aggregate return on past saving, accommodates the growth in investment needs \( 1 + g \). This implies that the aggregate return on saving is equal to \( (1 + g) / \beta \) on the balanced growth path. When \( g = g^* \), this coincides with the world’s interest rate \( r^* \). The effect of credit constraints is then suppressed in the long run, because the opportunities of arbitrage between bonds and capital vanish. Therefore, despite being constrained during the convergence process, entrepreneurs are not constrained in the steady state.

Turning to the case \( g > g^* \), we can see that entrepreneurs are constrained in the neighborhood of \( \tilde{K} \). Figure 5 shows that \( \bar{W} < W_2 \), i.e., the constraint is binding in the steady state. This is because, when \( g > g^* \), the long-term return on domestic capital \( (1 + g) / \beta \) is higher than the world interest rate \( r^* \). This means that arbitrage opportunities are left because of the presence of binding credit constraints. The intuition for this result is that \( g \) commands the entrepreneurs’ investment needs. When \( g \) is large, entrepreneurs become constrained and the return on their saving increases relative to the world’s interest rate because they are not
able to keep up with the continuing increase in TFP, unless the return on bonds $r^*$ or their propensity to save $\beta$ increase.

This last result is important. It means that a higher growth rate overturns the classical result that entrepreneurs are eventually unconstrained. To generate credit constraints in the long run, it is therefore not necessary to assume a lower discount factor $\beta$. A higher growth rate plays the same role, since it increases the impatience rate of the economy $(1 + g)/\beta$. A demand for liquidity will therefore naturally appear in countries with high growth rates.

When entrepreneurs are constrained in the steady state, there is a simple expression for the current account and the ratio of current account to GDP is constant. Define the current account as $CA_t = B_{t+1} - B_t$. In a constrained steady state, we find:

$$\frac{CA_t}{Y_t} = \frac{(1 - \alpha\beta)\beta g}{(1 + g)^2}$$

(29)

Clearly, the current account surplus is permanently positive and increases with $g$ (as long as $g < 1$). A similar result can be found for the ratio of investment to GDP.

To summarize, we find that the economy can be constrained on its convergence path or in a steady state when $g > g^*$. In each case, there is a demand for liquidity that has significant macroeconomic implications. It implies a current account surplus generated by high corporate saving. It also coincides with high investment levels and high output growth. All these features, documented in the Introduction, are present in the context of global imbalances.

### 3.4 Experiences of Growth

Experiences of growth in emerging countries can be very different in terms of capital flows, depending on the source of growth. Here we examine two cases that lead to radically different outcomes: an economy experiencing temporarily higher TFP growth and an economy experiencing an improvement of its financial development. In the first case, there is a need for liquidity that leads to a capital outflow. In the second case, the need for liquidity is reduced, which leads to a capital inflow.

These two cases can be easily examined in our benchmark. In order to have a complete assessment of the dynamics of the economy, we need to combine the TFP-adjusted variables with the evolution of TFP, and to aggregate the two groups of entrepreneurs. We do this by
assuming that these two groups are of equal size in terms of wealth. First, we examine a TFP growth acceleration episode when \( \phi = 0 \). Second, we consider an increase in \( \phi \) from \( \phi = 0 \) to \( \phi \) large enough so the entrepreneurs are no longer constrained.

### 3.4.1 A temporary increase in \( g \)

We first consider the impact of an increase in \( g \) starting from a steady state level where \( g = g^* \). The dynamic equations (23)-(28) hold, but with a different growth rate \( g \).\(^{11}\) In terms of Figure 5, this implies that the economy is temporarily driven by the schedule characterized by \( g > g^* \). If we start from an initial steady state when \( g = g^* \), this means that we move from an unconstrained economy, starting at revenues level \( W_i \), to a constrained one where the liquidity motive becomes effective. In the figure, this is represented by the convergence from \( W_i \) towards \( \overline{W} \). When the economy goes back to its initial growth rate, the economy returns to \( W_i \).

As a numerical illustration, Figure 6 represents the effect of a 1% increase in TFP growth during 10 periods. We compare the effect of this growth acceleration on an economy with imperfect financial markets (“Constrained - Benchmark”, represented by the solid lines), whose dynamics are described by (38)-(40), to an economy with perfect financial markets (“Unconstrained - Benchmark”, represented by the dashed lines), i.e., with \( \phi \) large enough so that entrepreneurs are never constrained and with \( \kappa = 0 \). In order to make the two cases comparable, we set the initial steady state of bonds in the unconstrained model equal to that of the constrained one. We consider capital, production and wages, represented as percentages from the initial steady state; and bonds, represented as a share of initial GDP. These bonds are also decomposed into the bond demand by entrepreneurs who are at the investment stage of their project, \( B^1 \), and the bond demand by entrepreneurs who are at the production phase, \( B^2 \).

The shock occurs while the economy is in a stationary equilibrium with \( g = g^* = 0 \). During 10 periods, domestic TFP increases steadily until it reaches a level 10% higher than initially. During this period, capital, production and wages increase, whether entrepreneurs are constrained or not. When entrepreneurs face financial frictions, however, capital accumulation

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\(^{10}\)In the constrained steady state, this is not an assumption but a result stemming from the equal number of entrepreneurs in each group and the unique steady state. However, when we consider the convergence dynamics, we have to make assumptions on the initial wealth of the two groups.

\(^{11}\)The increase in \( g \) is taken as exogenous. An interesting extension of our analysis would be to consider endogenous growth changes.
is delayed. In that case, entrepreneurs can invest only after their revenues have sufficiently increased.

The main difference between the constrained and unconstrained economies lies in the reaction of capital flows: capital flows out if entrepreneurs are constrained while it flows in if they are unconstrained. In the constrained case, they have to secure liquidity ex ante, during the investment phase, in order to pay for the wage bill. In the unconstrained case, they can rely on a free access to financial markets to borrow in the production phase.\(^\text{12}\) This temporary growth period leading to capital outflows from a constrained economy is clearly consistent with recent global imbalances.

### 3.4.2 A permanent increase in $\phi$

We now consider an episode of financial liberalization, where a country suddenly increases its level of financial development measured by $\phi$. Consider the extreme case of a country that switches instantaneously from a fully constrained state ($\phi = 0$) to an unconstrained one ($\phi$ large), while it is converging to the steady state with $g = g^*$. The effect of such an experiment is straightforward and is represented in Figure 3. Assume that $\phi$ increases when revenues are at $\tilde{W}_0$. The stock of capital jumps permanently from the constrained level to its higher unconstrained level $\hat{K}(r^*)$, which generates temporary growth. Bonds on the other hand, jump permanently to a lower level, which generates capital inflows.

This experiment shows that reforms promoting financial development generate a phase of output growth with capital inflows (this outcome is typical in models with credit constraints). In this case, the demand for liquidity is not the dominant mechanism. On the contrary, domestic reforms improving the functioning of financial markets reduce or eliminate the need for liquidity, which enables a higher investment. Consequently, there is no systematic link between capital flows and growth and the relationship depends on the source of growth.

### 3.5 Calibration and Sensitivity Analysis

So far, we have used the benchmark model for its tractability. However, this benchmark model is based on extreme assumptions: $\phi = 0$ and $\kappa = 1$. Here, we relax these assumptions and

\(^{12}\)Unconstrained entrepreneurs still want to borrow in the production phase despite $\kappa = 0$ since they want to smooth consumption.
calibrate these parameters more accurately, based on the values used in the literature and on targets based on the data. We then explore the sensitivity of our results to the parameters.

3.5.1 Baseline calibration

First, to set $\kappa$ we refer to the existing literature. We found a wide range of estimates for $\kappa$: for example, Rabanal (2003) finds estimates equal to 0.20-0.25 for the US and the Euro area while Ravenna and Walsh (2006) find that $\kappa = 1$ is consistent with empirical evidence on aggregate US data. Barth and Ramey (2001), using data for trade credit from the U.S. Flow of Funds, report that over the period 1995-2000 net working capital (inventories plus trade receivables, net of trade payables) averaged an amount comparable to the investment in fixed capital, which, in our model, corresponds approximately to $\kappa = 0.5$. As a middle ground, we set $\kappa = 0.75$. As for $\phi$, we set it so that liquidity demand to GDP, $B/Y$, is equal to 40%, which is the value of gross external assets to GDP observed in our sample of six Asian countries in 2000.\textsuperscript{13} This gives $\phi = 0.2$.

Consider now the impact of a ten-period 1% increase in growth within the calibrated model, represented by the dotted lines in Figure 6 (“Constrained - Calibrated”), along with the results of the benchmark model.\textsuperscript{14} The main features of the benchmark model, that is, the high average demand for liquidity, the delay in the adjustment of capital and the increase in the demand for liquidity, are muted in the calibrated model, but are still present, even with a lower $\kappa$ and a higher $\phi$. The smaller magnitude of the demand for liquidity lies in two interconnected facts: the lower demand for liquidity by entrepreneurs who invest ($B^1$) and the higher debt capacity of entrepreneurs who pay working capital ($B^2$). The results of the benchmark case are therefore robust to a proper calibration.

3.5.2 Sensitivity

Here we examine the sensitivity of the results to different values of $\phi$ and $\kappa$. $\phi$ is set to 0.1 and 0.4, along with its baseline calibration value 0.2. $\kappa$ is set to 0.5 and 0.9, along with its baseline value 0.75. The results are represented in Figure 7. Since the effects on capital, production and wages are very similar across the different calibrations, we do not represent them.

\textsuperscript{13}The data on foreign assets is taken from Lane and Milesi-Ferretti (2007).
\textsuperscript{14}The simulations are run using Dynare (Juillard, 1996).
A net capital outflow accompanies the growth increase for all the parameter values considered in Figure 7. In each case, the proportional increase in the demand for liquidity in the investment period, $B^1$, is significant, while borrowing in the production period, $B^2$, is limited. With a higher $\phi$ it is easier to borrow in the production period (larger $B^2$) and the demand for liquidity in the investment period can be smaller. Similarly, a smaller $\kappa$ implies a smaller $B^1$. Overall, however, the proportional increase in the demand for liquidity in presence of higher growth is robust to changes in these two parameters.

4 Global imbalances

The analysis so far has been conducted by assuming that the emerging country is small, so that the interest rate is given. However, global imbalances have been taking place in a context where capital flows from emerging countries, especially China, can influence the world interest rate because of their size. We therefore extend our baseline small open economy to a two-country economy. We show that the demand for liquidity in an emerging country leads to a lower world interest rate, higher investment and output in the rest of the world, and larger global imbalances. We show that these imbalances remain as long as the demand for liquidity is effective, in particular as long as the emerging economy has a higher TFP growth.

We consider an asymmetric world composed of an Emerging country similar to the one studied earlier and an Industrial country with a high level of financial development, so that entrepreneurs are never constrained and have no need for working capital. Industrial country variables are denoted with an asterisk, so that $\kappa^* = 0$ and $\phi^*$ is large. The two countries are linked through the bond market as they can trade one-period bonds. Productivities $A_t$ and $A^*_t$ grow respectively at rate $g$ and $g^*$. Otherwise, the two countries have the same characteristics.

We first study a balanced growth path where the Emerging country grows at a permanently higher growth rate than the Industrial country. Though unrealistic, the dynamics of the growth path are informative. We can show that a permanently higher growth rate in the Emerging country generates a permanent liquidity demand and a permanent current account surplus. Second, we consider the case where both countries grow at the same rate in the long run but with $g$ temporarily larger than $g^*$. This experiment is simulated.
4.1 Balanced Growth Path

The balanced growth path with $g$ permanently higher than $g^*$ is characterized in the Appendix. Let $\tilde{K}_t = K_t / A_t$ be the Industrial capital stock normalized by Emerging TFP. Let also $\tilde{r}_t$ be the normalized interest rate: $\tilde{r}_t = r_t \left( \frac{A_t}{A_0} \right)^{(1-\alpha)(t+1/2)}$. The following Proposition characterizes a steady state where Emerging country entrepreneurs are constrained.

**Proposition 2** Assume $g > g^*$. When $t$ goes to infinity, a growth path where entrepreneurs are constrained and $\tilde{K}_t$, $\tilde{K}_t^*$, $\tilde{B}_t$, and $\tilde{r}_t$ are stationary exists and is characterized by the following:

(i) $\tilde{K}_t = \tilde{K} = \left( \frac{\alpha \beta^2}{(1+g)^2} \right)^{\frac{1}{1-\alpha}}$

(ii) $\tilde{K}_t^* = \tilde{K}_t^* = \frac{1-\alpha \beta}{\alpha \beta} \tilde{K}$

(iii) $\tilde{B}_t = \tilde{B} = \frac{1-\alpha \beta}{\alpha \beta} \tilde{K}$

(iv) $\tilde{r}_t = \tilde{r} = \left[ \alpha \left( \frac{1-\alpha \beta}{\alpha \beta} \left( \frac{\alpha \beta^2}{(1+g)^2} \right)^{\frac{1}{1-\alpha}} \right)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{1}{2}}$

Steady-state Emerging capital stock and bonds are the same as in the small open economy (see Proposition 1). Since $g > g^*$, the Emerging country is always constrained so that the liquidity demand implies that capital and bonds move in parallel. The interesting new result in the two-country economy is that the Industrial capital stock grows at the Emerging country growth rate. Moreover, Proposition 2 implies that the "imbalance" of the Industrial country, measured as $B_t^*/Y_t^*$, grows more negative over time. In other words, if the Emerging country grows permanently faster than the Industrial country, global imbalances can grow permanently.

Both countries benefit from global imbalances in the steady state. Since, Industrial entrepreneurs are unconstrained, they are the providers of liquidity to Emerging entrepreneurs. This enables a higher growth in the Emerging country. At the same time, Industrial entrepreneurs receive cheaper funding from Emerging entrepreneurs, which allows them to increase their capital stock. It actually increases at the same rate as Emerging productivity.\(^\text{15}\)

\(^\text{15}\)It can be shown that consumption in the Industrial country also grows at a higher rate than the fundamental growth rate $g^*$.\n

4.2 A Temporary Increase in $g$

A more realistic scenario is to assume that the higher growth rate in the Emerging country is temporary. Here, we simulate the impact of the same temporary increase in the domestic growth rate as in the previous section. The Emerging country’s TFP grows at a rate $g = 1\%$ for 10 periods. We compare the resulting effects when the Emerging country is constrained, as in the benchmark case, and when it is unconstrained. The results are represented in Figure 8. The reaction of the Emerging economy follows closely the reaction of the small open economy studied in the previous section. Indeed, the entrepreneurs’ liquidity motive to hold bonds dominates the arbitrage motive. This implies that the Emerging country experiences capital outflows instead of capital inflows, which translates into global imbalances: the debt level of the Industrial country has to increase.

The impact on the world interest rate differs dramatically in the constrained and unconstrained cases. In order to make the Industrial country more willing to supply bonds, the world interest rate has to decrease in the constrained case. In the unconstrained case, on the contrary, the interest rate increases as a response to the decrease in bond demand. As a result, the Industrial capital stock increases in the constrained case, while the opposite happens in the unconstrained case. In the constrained case, growth in the Emerging country is a boon for the Industrial country, because the additional resources of Emerging entrepreneurs are partly transferred to Industrial entrepreneurs. This contrasts with the standard unconstrained case, where the spillover of higher growth is negative.

5 Discussion

The model has been kept simple to illustrate the mechanism behind the demand for liquidity. But this mechanism holds in a wider context. In this section we examine four important extensions: i) uncertainty; ii) FDI; iii) capital account liberalization; iv) public debt and international reserves. While the basic mechanism may still hold in each of these extensions, they each add interesting elements to the analysis.
5.1 Uncertainty

The basic mechanism behind the demand for liquid assets arises with perfect foresight. The presence of uncertainty introduces additional mechanisms, such as precautionary saving, affecting capital flows. Fully solving the model with uncertainty has to be done numerically, but the main channels can be found from first order conditions. Assume that there is uncertainty about future TFP (revealed in \( t + 2 \)), while entrepreneurs know the productivity of current project (revealed in \( t \)). In this case, first order conditions become:

\[
\alpha \left( \frac{K_{t+1}}{A_{t+1}l_{t+1}} \right)^{\alpha - 1} = r^* \beta \left( 1 + \frac{\lambda_{t+1}}{\beta E_t} \left\{ \frac{1}{c_{t+2}} \right\} \left( 1 - \frac{\phi}{r^*} \right) \right) \tag{30}
\]

\[
(1 - \alpha) \left( \frac{K_{t+1}}{A_{t+1}l_{t+1}} \right)^\alpha = r^* w_{t+1} \left( 1 + \frac{\lambda_{t+1}}{\beta E_t} \left\{ \frac{1}{c_{t+2}} \right\} \right) \tag{31}
\]

\[
\frac{c_{t+1}}{c_t} = \beta r^* \tag{32}
\]

\[
\frac{1}{c_{t+1}} = \beta r^* E_t \left\{ \frac{1}{c_{t+2}} \right\} + \lambda_{t+1} r^* \tag{33}
\]

The impact of uncertainty is basically similar to what is found in related models (e.g. Mendoza et al., 2007). For example, equation (33) shows that consumption decisions are affected in a standard way that may generate precautionary saving. This effect would increase net capital flows and the demand for liquid assets. With risk, the steady state wealth would be strictly higher than the constrained level when \( g = g^* \). However, if \( g > g^* \), the constraint can still be binding and the entrepreneur would have a demand for liquidity. The total impact of uncertainty on liquidity demand is a quantitative question that should be analyzed in a fully calibrated model (we leave this for future research).

5.2 Foreign Direct Investment

The demand for liquid assets represents the main source of capital flows in the model. In the benchmark case (when \( \phi = 0 \)), the demand for liquidity even equals net capital flows. In reality, however, the demand for liquid assets coexists with other types of flows, because of the limited domestic supply of liquidity. A special type of flow is FDI. We can show that our model can generate FDI inflows along with outflows of bonds. Moreover, we show conditions under which net outflows can be robust to the introduction of alternative sources of financing.
that are not subject to credit frictions. One condition is that the level of development in the Emerging country is not too high.

A simple way of introducing FDI in our model is to assume that it is undertaken by unconstrained investors from the Industrial country.\textsuperscript{16} However, given the simplicity of our model this assumption may imply that unconstrained Industrial investors partially or fully crowd out Emerging entrepreneurs. To avoid this, we make two further assumptions. First, there is an increasing cost for Industrial entrepreneurs to invest in the Emerging country. This cost rules out indeterminacy for the quantity of FDI in equilibrium. Second, we assume that the Emerging country is in a situation of unemployment where workers are paid their reservation wage $w^\prime$. In Figure 2, this means that we consider equilibrium (1). FDI increases total labor demand (shifts $l_D^1$ to the right), but it has no impact on the wage rate and therefore no spillover effect to existing Emerging entrepreneurs.

More specifically, we can assume a cost $\tau$ of the iceberg type that increases with the aggregate amount of labor used, so it is not internalized by the foreign firms. Let $l^F$ be the amount of labor used by FDI and assume that $\tau = \tau(l^F)$ with $\tau(0) = 0$ and $\tau' > 0$.\textsuperscript{17} This implies the following labor demand by foreign firms:

$$l^F(\tilde{w}_{t+1}, r^*) = \left[1 - \left(\frac{\tilde{w}_{t+1}}{w(r^*, r^*)}\right)^{1-\alpha}\right]^{-1}$$

(34)

Similarly, we can write the labor demand by domestic firms as:

$$l(\tilde{w}_{t+1}, \tilde{K}_{t+1}, r^*) = \frac{(1 - \alpha)r^*}{\alpha\tilde{w}_{t+1}} \tilde{K}_{t+1}$$

(35)

where $\tilde{K}_{t+1}$ is independently defined by past capital and labor.

Now assume that Emerging opens to FDI when wages are at $w$ and that FDI is not too large so that wages do not increase. In other terms total demand at $w$ is less than one:

$$l^F(w, r^*) + l(w, \tilde{K}_{t+1}, r^*) < 1$$

(36)

In this case, Emerging entrepreneurs are not affected by FDI and keep their liquidity demand, so that both types of capital flows can coexist. As $\tilde{K}_t$ grows, however, labor demand grows and (36) will not longer hold and we are in equilibrium like (2) in Figure 2. The wage rate has

\textsuperscript{16}See Kiribaeva and Razin (2010) for a survey on different ways to model FDI.

\textsuperscript{17}This implies that the profit function for FDI is $\pi(K^F_{t+1}, l^F_{t+1}) = (1 - \tau)A_{t+1}K^F_{t+1}l^F_{t+1} - r_{t+1}K^F_{t+1} - r_{t+2}l^F_{t+1}$.
to adjust so that:

\[ l^E(\tilde{w}_{t+1}, r^*) + l(\tilde{w}_{t+1}, \tilde{K}_{t+1}, r^*) = 1 \]  

(37)

In this case, the dynamics of capital flows become more complex and depend on the details of the model.

### 5.3 Capital Account Liberalization

A demand for liquidity also changes the implications of a capital account liberalization. There is an extensive literature analyzing the implications of liberalizing international capital flows. When an economy has a low level of financial development, such a liberalization typically implies a capital inflow and an increase in investment, at least in the short run.\(^\text{18}\) In contrast, with a demand for liquidity, while there is an increase in investment there is always an initial capital outflow.

To study a capital account liberalization, we simply need to analyze the Emerging economy in autarky and then examine the convergence to its open economy steady state. For an interesting autarky equilibrium to exist, however, there must be a domestic supply of liquidity. This would not be the case in our benchmark where \(\phi = 0\). But as long as \(\phi > 0\), there is a well defined steady state in autarky. Alternatively, we could assume that there is an exogenous supply of public debt, \(B^G\), that offers the liquidity needs. This determines a steady state income level \(\tilde{W}_A\). If this supply is not too large, the Emerging economy will be constrained in autarky. For example we could have \(\tilde{W}_A = W_0\) and analyze the impact of a capital account liberalization by repeating the small open economy analysis in section 3.3. On impact, the capital stock slowly increases and is accompanied by a capital outflow. This is made possible by an increase in the return on bonds. Then entrepreneurs gradually accumulate profits. They can then invest more and increase their demand for liquidity. In a two-country model, the capital account liberalization implies an increasing current account deficit in the Industrial economy.

\(^{18}\)E.g. see Aghion et al. (2004), Aoki et al. (2009), Bacchetta (1992), or Martin and Taddei (2010). In Angeletos and Panousi (2010), a capital account liberalization implies an initial capital outflow, but is accompanied by a decline in investment.
5.4 The Role of Domestic Liquidity and International Reserves

In a closed economy, the government may alleviate the liquidity constraint by issuing liquid public debt (e.g. see Woodford, 1990). This is no longer the case in an open economy with well integrated financial markets. In this context, entrepreneurs have ample access to liquid assets in foreign countries and changes in the supply of domestic assets have little or no impact. Nevertheless, there are two potential channels through which an increase in public debt might have an impact. First, it can affect the world interest rate. This channel obviously disappears in a small open economy. Second, the increase in debt may be associated with a reduction in taxes that have real effects. Since Ricardian equivalence does not hold due to financial constraints, a decrease in taxes on entrepreneurs increases investment. However, this channel is related to tax policy rather than changes in liquidity supply.

In contrast, with limited financial integration, managing liquidity has a significant impact on investment as it affects the supply available to entrepreneurs. However, the impact of liquidity demand on net foreign assets may depend on the government’s behavior. It could actually be the same as with full capital mobility. Assume that a government issues public debt to match a demand for liquidity and uses the funds to buy foreign assets. This may lead to the same capital outflow as with full financial integration. The government simply plays a role of intermediary between the domestic financial sector and foreign borrowers. This situation actually corresponds to the recent Chinese experience (Song et al., 2010, give a similar argument). With strong capital controls in place, the central bank has been buying substantial amounts of international reserves, while at the same time it has been issuing domestic debt. In other terms, with capital controls the increase in the central bank foreign exchange reserves may simply reflect the demand for liquidity by the private sector.

A decrease in taxes in either stage of production increases the funds available to investors and leads to more investment. In terms of the demand for liquidity, a tax decline in the investment stage increases the demand for liquidity, while a tax decline in the production stage decreases the demand for liquidity. This implies that changing the tax profile (between the investment and the production stages) may affect liquidity demand without affecting investment.
6 Conclusion

In this paper, we propose a simple mechanism generating a demand for liquid assets in a dynamic small open macroeconomic model. This demand emanates from firms and is proportional to their saving. Such a demand can generate a current account surplus in fast-growing emerging economies, where firms face tighter credit constraints. In such a context, the demand for foreign bonds becomes a complement to investment. This implies that an increase in growth and in investment is accompanied by a net capital outflow, which is the opposite from the predictions of the standard intertemporal model. We show that the demand for liquidity can arise on the convergence path of an economy with an initial low level of capital. It can also occur close to a steady state, if the economy grows faster than the rest of the world (temporarily or permanently).

When we cast this mechanism in a two-country model, it gives a framework consistent with global imbalances and with all the symptoms observed in a "saving glut". Both countries benefit from these imbalances. On the one hand, the Emerging country can grow faster thanks to the liquidity provided by the unconstrained Industrial country. On the other hand, the Industrial country can build a higher capital stock thanks to the funds provided by the Emerging country. In addition to a sustained current account surplus in the Emerging economy, the model is consistent with a number of stylized facts observed in recent years. In particular, current account surpluses have been accompanied by a large level of corporate saving, a large level of investment, and rapid growth in emerging Asia. The existing literature cannot explain these facts jointly. Moreover, the model is consistent with the empirical evidence on the allocation puzzle and with the positive correlation between saving and growth. We also argue that the framework can be consistent with an increase in reserves, as is observed in China and other countries, when there are capital controls and the central bank plays the role of intermediary between the private sector and the international asset market. Moreover, we showed that the demand for liquid assets can also coincide with FDI inflows.

This paper has focused on a specific mechanism that may play an important role in some episodes. However, we have abstracted from many other factors that affect capital flows. Moreover, we have shown that even in our simple model there could be episodes of high growth accompanied by net capital inflows, as a consequence of financial deepening. This illustrates the fact that the demand for liquidity mechanism we have explored in this paper is
not always at work or not always the dominant factor. A natural extension of this research is to attempt to identify the conditions under which this mechanism can or has been relevant (besides the current global imbalance episode). The other natural extension is to introduce the basic mechanism in a more complete model. For example, the process for growth has been kept exogenous, but it could be interesting to examine the interaction between endogenous growth and the demand for liquidity. A more complete and realistic model would allow a quantitative evaluation that might prove useful in the ongoing discussion on global imbalances.
7 Appendix: Dynamics and Steady State in the Benchmark Case

7.1 Small open economy

First it is convenient to define three auxiliary variables. Define $\hat{\beta} = \left(\frac{\beta}{1+g}\right)^2$, $\hat{g} = \left(\frac{1+g}{1+g^*}\right)^2$, and $\hat{w} = \frac{w}{(1-\alpha)r}$. In order to prove the existence and unicity of the steady state, we establish the following lemma:

**Lemma 1** If $w < \hat{w}(r^*)$, the entrepreneurs’ revenues $\tilde{W}$ in the emerging country evolve according to:

\[
\tilde{W}_{t+2} = (\alpha \hat{w})^{\alpha-1} \alpha \hat{\beta} \tilde{W}_t \quad \text{if } \tilde{W}_t < W_1 \\
= \left[\alpha \hat{\beta} \tilde{W}_t\right]^\alpha \quad \text{if } W_1 \leq \tilde{W}_t < W_2 \\
= \frac{\tilde{W}_t}{\hat{g}} \quad \text{if } \tilde{W}_t \geq W_2 \\
\]

with $W_1 = \hat{g} \hat{w}$ and $W_2 = \hat{K}(r^*)^\alpha \hat{g}$.

**Proof:**

If $w < \hat{w}(r^*)$, which means that the first-best wage is higher than the reservation wage, then there is no unemployment when the firms are unconstrained. Only three situations can then exist, as represented in Figure 2 (1) Constrained firms with unemployment; (2) Constrained firms with full employment; (3) Unconstrained firms with full employment. The different dynamic equations for $W$ correspond to these different types of equilibria in the labor market.

1. In the equilibrium with unemployment, entrepreneurs are constrained, so $\tilde{K}_{t+1} = \alpha \hat{\beta} \tilde{W}_t$ and the dynamics of revenues follow:

\[
\tilde{W}_{t+2} = \left[\alpha \hat{\beta} \tilde{W}_t\right]^\alpha l_{t+1}^{1-\alpha} \quad (41)
\]

But this equation is conditional on $l_{t+1}$. In order to determine the aggregate employment level $l_{t+1}$, we use (12):

\[
l_{t+1} = \frac{\tilde{K}_{t+1}}{\alpha \hat{w}} \quad (42)
\]

Replacing $l_{t-1}$ in (41), the dynamics of $\tilde{K}$ are fully characterized:

\[
\tilde{W}_{t+2} = (\alpha \hat{w})^{\alpha-1} \alpha \hat{\beta} \tilde{W}_t \quad (43)
\]
These dynamics hold as long as $l_{t+1} < 1$, that is: $\tilde{K}_{t+1} = \alpha \beta \tilde{W}_t < \alpha \hat{w}$. Otherwise, entrepreneurs are either constrained with full employment or unconstrained. This is equivalent to $\tilde{W}_t < W_1$, with $W_1 = \tilde{g} \hat{w}$.

2. In the equilibrium with constrained firms and full employment, the dynamics of revenues obey to (41) with $l_{t+1} = 1$, which yields (39).

3. When firms are unconstrained, the dynamics of revenues must satisfy:

$$\tilde{W}_{t+2} = r^* \tilde{B}_{t+2} + \frac{r^{*2}}{\alpha(1+g)} \tilde{K}(r^*)$$

(44)

with $\tilde{B}_{t+2} = r^* \left[ \beta \tilde{W}_t - \frac{K(r^*)}{\alpha(1+g)} \right]$. Hence (40).

The first-best capital stock $\tilde{K}(r^*)$ is implementable only if it is lower than the constrained level of capital: $\tilde{K}(r^*) < \alpha \beta \tilde{W}_t$, which is equivalent to $\tilde{W}_t < W_2$, with $W_2 = \tilde{K}(r^*) \hat{g}$.\footnote{It can be checked that $W_1 < W_2$ whenever $w < \hat{w}(r^*)$.}

**Proof of Proposition 1**

We examine the different dynamic equations summarized in Lemma 1 in order to determine the steady state(s):

1. According to Lemma 1, if $\tilde{W}_t < W_1$, then the dynamics of $\tilde{W}$ follow (38). As a result, $\tilde{W}_{t+2} > \tilde{W}_t$ is equivalent to $(\alpha \hat{w})^{\alpha-1} \left[ \alpha \hat{\beta} \right] > 1$, which implies the following condition on $w$: $w < \hat{g}^{1-\alpha} \hat{w}(r^*)$.

2. Similarly, if $W_1 \leq \tilde{W}_t < W_2$, then the dynamics of $\tilde{W}$ follow (39). Consequently, $\tilde{W}_{t+2} > \tilde{W}_t$ if and only if $\tilde{W}_t < \left( \alpha \hat{\beta} \right)^{\frac{\alpha}{1-\alpha}}$.

Besides, if $g > g^*$, then $\left( \alpha \hat{\beta} \right)^{\frac{\alpha}{1-\alpha}} \in [W_1, W_2)$. In that case, there exists a unique fixed point $\bar{W} = \left( \alpha \hat{\beta} \right)^{\frac{\alpha}{1-\alpha}}$ to the dynamic equation of capital in the interval where entrepreneurs are constrained. If $g = g^*$, then $\left( \alpha \hat{\beta} \right)^{\frac{\alpha}{1-\alpha}} \notin [W_1, W_2)$. There is no fixed point in this interval.

3. Finally, if $\tilde{W}_t \geq W_2$, then any $\tilde{W}$ is stationary if $g = g^*$, since $\tilde{W}_{t+2} = \tilde{W}_t$. If $g > g^*$, then $\tilde{W}_{t+2} < \tilde{W}_t$, and there is no fixed point in this interval.
To sum up, when \( g = g^* \), any \( \tilde{W} \geq W_2 \) is a steady state. This steady state is characterized by \( \tilde{K}_{t+1} = \tilde{K}(r^*) \) and \( \tilde{B}_{t+1} = \frac{\beta}{1+g} \tilde{W} - \tilde{K}(r^*) \). For \( g > g^* \), there is a unique steady state \( \tilde{W} = \left( \frac{\alpha \beta}{\alpha + \beta} \right)^{1/\alpha} \). This steady state is characterized by \( \tilde{K}_{t+1} = \frac{\alpha \beta^2}{1+g} \tilde{W} \) and \( \tilde{B}_{t+1} = \frac{\beta(1-\alpha)\beta}{1+g} \tilde{W} \).

7.2 Two-country economy

We assume that \( 0 \leq g^* < g \), so the Emerging country grows faster than the Industrial country. In this case, when entrepreneurs are constrained, the dynamic equation for the emerging country is the following:

\[
\tilde{K}_{t+1} = \alpha \beta \tilde{K}_{t-1}^a \tag{45}
\]

On the other hand, the industrial country’s capital must satisfy:

\[
\alpha \left( \frac{A_t}{A_t^*} \right)^{-(1-\alpha)} = r_t r_{t+1} \tag{46}
\]

**Proof of Proposition 2:** We conjecture that such a stationary growth path exists and then we verify that it satisfies (i)-(iv), and that the Emerging country would indeed stay constrained under (i)-(iv).

If the emerging country is constrained, then (45) holds. The stationary solution for \( \tilde{K} \) is \( \left( \frac{\alpha \beta}{1-\alpha} \right)^{1/\alpha} \), hence (i). (iii) derives directly from the relationship of \( B_t \) and \( K_t \) when the entrepreneurs are constrained. In order to determine the stationary values of \( \tilde{r}_t \) and \( \tilde{K}_t^* \), consider the aggregate dynamics of the Industrial country:

\[
B_{t+1}^A + B_{t+1}^B + K_{t+1}^* = \beta r_t \left[ (B_t^A + B_t^B) + r_{t-1} K_{t-1}^* - \frac{(1-\beta)K_{t-1}^*}{\beta} \right] \tag{47}
\]

where \( B^A \) are bonds held by entrepreneurs who invest in \( t \) and \( B^B \) are bonds held by entrepreneurs who invested in \( t - 1 \).

Equilibrium in the international bond market yields:

\[
K_{t+1}^* - B_{t+1} = \beta r_t \left[ -B_t + r_{t-1} K_{t-1}^* - \frac{(1-\beta)K_{t-1}^*}{\beta} \right] \tag{48}
\]

Dividing by \( K_{t+1}^* \), we obtain:

\[
1 - \frac{\tilde{B}_{t+1}}{K_{t+1}^*} = \beta r_t \left[ -(1+g) \frac{\tilde{B}_t}{K_t^*} \frac{K_{t+1}^*}{K_{t+1}^*} + (1+g)^2 \frac{r_{t-1} K_{t-1}^*}{K_{t-1}^*} - (1+g) \frac{(1-\beta)K_{t-1}^*}{\beta K_{t+1}^*} \right] \tag{49}
\]

(iv) implies that \( r \) goes to zero when \( t \) goes to infinity. Using this and the fact that \( \tilde{B} \) and \( \tilde{K}^* \) are stationary, this equation yields that \( \tilde{K}^* = \tilde{B} \), hence (ii). (iv) derives directly from (46) and (ii).
In order to prove that this defines an equilibrium where the Emerging country is constrained, it is sufficient to show that $\tilde{K}$ is lower than the level of capital per efficient unit of labor that would prevail absent credit constraints with the given interest rate. This level is given by $\tilde{K}^* \frac{A_t}{X_t}$, which goes to infinity when $t$ is large. This confirms that the emerging country is constrained.
Table 1
Growth, Investment and Current Account - 2004-2008

<table>
<thead>
<tr>
<th>Country</th>
<th>Growth-Current Account Correlation</th>
<th>GDP Growth Average, %</th>
<th>Investment /GDP Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.44</td>
<td>10.8</td>
<td>0.43</td>
</tr>
<tr>
<td>India</td>
<td>0.69</td>
<td>8.5</td>
<td>0.35</td>
</tr>
<tr>
<td>Korea</td>
<td>0.42</td>
<td>4.2</td>
<td>0.30</td>
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<tr>
<td>Philippines</td>
<td>0.47</td>
<td>5.5</td>
<td>0.15</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.09</td>
<td>4.6</td>
<td>0.22</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.25</td>
<td>4.7</td>
<td>0.28</td>
</tr>
<tr>
<td>GDP-weighted</td>
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<td>0.37</td>
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<tr>
<td>Simple average</td>
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<td>0.29</td>
</tr>
<tr>
<td>Pooled correlation</td>
<td>0.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: World Bank and National Statistics Taiwan
References


Note: Include China, India, S. Korea, Philippines, Taiwan, and Thailand. Sources: Sonali et al. (2009) for corporate saving and World Bank and National Statistics Taiwan for GDP.
Figure 2: Labor market equilibrium
Figure 3: Convergence with $g = g^*$
Figure 4: Convergence with $g = g^*$
Figure 5: Steady states with $g = g^*$ and $g > g^*$
Note: $K$, $Y$ and $w$ are in percentage deviation from the initial steady state, while $B$, $B^1$ and $B^2$ are in percentage of initial output.
Figure 7: Temporary increase in productivity growth - Sensitivity analysis

Note: All variables are in percentage of initial output.
Figure 8: Temporary increase in home productivity growth - Two-country economy

Note: all variables are in percentage deviation from the initial steady state, except the industrial country’s debt $B/Y^*$, which is in percentage of output, the world interest rate $r^*$ and real exchange rate $w/w^*$, which are in levels.