Corporate Cash and Employment*

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Abstract

In the aftermath of the U.S. financial crisis, both a sharp drop in employment and a surge in corporate cash have been observed. In this paper, based on U.S. data, we document that the negative relationship between the corporate cash ratio and employment is systematic, both over time and across firms. We develop a dynamic general equilibrium model where heterogeneous firms need cash in their production process and where financial shocks are made of both credit and liquidity shocks. We show that external liquidity shocks generate a negative comovement between the cash ratio and employment. We analyze the dynamic impact of aggregate shocks and the cross-firm impact of idiosyncratic shocks. With a calibrated version of the model, the model yields a negative comovement that is close to the data.

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1 Introduction

In the aftermath of the U.S. financial crisis, both a sharp decline in employment and an accumulation of cash held by firms have been observed. While both variables are part of firms’ decisions, they are typically not considered jointly in the literature. To what extent are these two features related? Holding liquid assets facilitates the firm’s ability to pay for the wage bill. But employment and cash decisions also react to changes in firms environment, e.g., changes in credit conditions. Therefore, examining these two variables jointly sheds light on the role of financial shocks on employment, especially during the crisis. The contribution of this paper is twofold. First, it provides stylized facts on the relationship between the corporate cash position and employment. Second, it delivers an explanation to the empirical evidence by building a tractable dynamic general equilibrium framework, including both cash and employment decisions. This framework sheds a new light on the impact of financial shocks by distinguishing between liquidity and credit shocks. Liquidity shocks appear to be crucial to explain the relationship between cash and employment.

We first document a robust negative comovement between the corporate cash ratio and employment on U.S. data, which is not specific to the recent financial crisis. Using Flow-of-Funds data over the period 1980-2011, the correlation between HP-filtered employment and the share of liquid assets in total assets is $-0.41$. Moreover, using firm-level data from Compustat, the annual cross-firm correlation between employment and the cash ratio is on average $-0.29$ over the same period. Section 2 provides a detailed description of this data analysis.

To understand the optimal cash and employment decisions, we consider an infinite-horizon general equilibrium model with heterogeneous firms that need liquid funds in their production process. Liquidity is closely related to labor because firms have liquidity needs in order to finance the wage bill, which is part of working capital. We adopt a structure similar to Christiano and Eichenbaum (1995), who divide periods into two subperiods. In the first subperiod, firms use credit to install capital, while they need liquid funds to pay workers in the second subperiod. In contrast to the literature introducing working capital in macroeconomic models (see Christiano et al. 2011, for a survey), we assume that firms do not have full access to external liquidity and cannot borrow all their short-term needs. This generates a demand for cash. Liquidity that is external to the firm may take
several forms, such as credit lines, trade credits, trade receivables to customers, or late wage payments. Liquidity shocks are changes in the availability of external liquidity. We assume that firms may be hit by technology shocks, by changes in their ability to obtain long-term credit (i.e., standard credit shocks) and by liquidity shocks. These shocks can be at the aggregate or at the idiosyncratic level.

The model is designed to be tractable so that several results can be derived analytically. It suggests that liquidity shocks can explain the negative comovement between employment and the corporate cash ratio. A reduction in external liquidity generates two effects. On the one hand, lower liquidity reduces the financial opportunities of firms and depresses labor demand. On the other hand, the reduction in external liquidity makes the production process more intensive in cash to ensure that wages are fully financed. Firms assets are then tilted towards cash. Combining these two effects implies that the cash ratio increases while employment declines. This analysis points to the crucial role played by the tightening of liquidity conditions in the aftermath of the Lehman crisis. While no initial sharp reduction in credit supply was observed during the recent financial crisis, firms experienced a significant deterioration in their expected liquidity conditions. For example, Gilchrist and Zakrajsek (2012) argue that banks cut the existing corporate lines of credits during the crisis.\footnote{Ivashina and Scharfstein (2010) show that firms initially drew heavily on their credit lines, but that subsequently credit conditions tightened. Campello et al. (2011) show that some firms had their credit lines canceled and that other firms had to renegotiate their credit lines with a higher cost. More generally, credit line agreements may contain restrictive covenants that may limit the ability of borrowers to draw on their lines. See also Chari et al. (2008) or Kahle and Stulz (2013).} Also, short-term loans to business firms decreased by 9% between 2008 and 2009 (using Survey of Terms of Business Lending, maturity of less than 30 days) while the liquidity ratio sharply increased from 3.9% to 5%.

Similar to Jermann and Quadrini (2012), we derive the series of technology and financial shocks from a calibrated version of our model. We also find that financial shocks are a major source of fluctuations, especially during the recent U.S. financial crisis. However, our model gives a more subtle view of financial shocks, by disentangling the role of liquidity shocks and credit shocks. This distinction is possible through the introduction of cash holdings in our model. From the observations of the cash ratio in the data, we are able to identify liquidity shocks, along with standard credit shocks and TFP shocks. These liquidity shocks are
not significantly correlated with credit shocks over the whole dataset. Feeding the three types of shocks back in our model, we find that liquidity shocks are key to generate a negative correlation between the cash ratio and employment. Besides, liquidity shocks generate the bulk of short-term fluctuations, while credit shocks are more important in the medium-term. Idiosyncratic liquidity shocks are also important to explain the negative cross-firm correlation between cash and labor. The model is parametrized using moments distribution from firm-level data. Despite its simplicity, the model performs relatively well quantitatively to reproduce the negative cross-firm correlation between the cash ratio and employment. Our benchmark calibration gives a correlation of \(-0.13\), while it is \(-0.29\) in the data.

The optimal choice of corporate liquidity is rarely introduced in macroeconomic models, even in models with financial frictions. When it is, the focus is on investment, not labor. Liquid assets are usually held by households, typically in the form of money, to finance their consumption.\(^2\) However, firms also have liquidity needs. Papers incorporating firms’ liquidity are typically in the spirit of Holmstrom and Tirole (2011) and Woodford (1990); they include Aghion et al. (2010), Kiyotaki and Moore (2012) or Bacchetta and Benhima (2015). However, these papers do not analyze employment fluctuations.

While the link between liquidity and employment has not received much attention so far, our analysis is related to several strands of the literature. First, there is a growing literature that incorporates firms’ financial frictions in a macroeconomic context. For instance, Covas and den Haan (2011) and Jermann and Quadrini (2012) analyze corporate external finance decisions over the business cycle, such as debt and equity. However, these papers do not introduce cash. For example, in their theoretical model, Jermann and Quadrini (2012) have working capital that is fully financed by an intra-period loan. Other papers focus more closely on the relationship between financial factors and the labor market. This literature stresses the role of financial frictions influencing labor demand.\(^3\) Most of these papers provide a more detailed analysis of the labor market than we do, but they do not consider cash holdings. Our analysis focuses on the impact of liquidity conditions on labor demand.

\(^2\)There are obviously some exceptions. For example, Stockman (1981) considers a cash-in-advance constraint both for consumption and capital.

Our paper is also related to a vast theoretical literature in corporate finance on firms’ cash holdings and corporate saving. Our approach shares features with several recent papers that provide analyses at the firm level or in environments with heterogeneous firms. Some papers are particularly close to our approach as they focus on the role of financing conditions on cash decisions. Our paper differs from this literature by focusing on business cycle frequency and on employment, which plays a key role in the working capital management. In addition, we provide a general equilibrium analysis which is important in the context of employment as this is an input that is not generated by the firm (in contrast to capital). As a result, market-clearing wage fluctuations can potentially offset partial equilibrium effects. This is particularly relevant in the context of liquidity management as the wage bill affects firms’ liquidity needs. Another difference is that we make a clear distinction between liquid and less liquid assets. The recent dynamic models in the corporate finance literature consider cash a negative debt or as a residual between cash flow and investment.

Finally, our approach is consistent with the findings of the empirical literature on the determinants of corporate cash. This literature stresses in particular the precautionary motive to save cash and shows that this motive increases with cash flow uncertainty or with more uncertain access to capital markets (see for instance Almeida et al., 2004). Some papers have also analyzed the use of short-term credits, like credit lines, and their interaction with corporate cash holdings. They tend to show that cash is a substitute to credit lines, as suggested by our analysis. For instance, Campello et al. (2011) find a negative correlation between cash and credit lines.

The rest of the paper is organized as follows. Section 2 describes the negative comovement between corporate cash and employment. Section 3 presents the model and shows the basic mechanism that can lead to this negative relationship. In Section 4, we calibrate the model to analyze the dynamic impact of aggregate

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4See for example, Bolton et al. (2013), Eisfeldt and Muir (2013), Falato et al. (2013), Gao (2013) and Hugonnier et al. (2014). Some papers consider other determinants of firms’ cash holdings (Armenter and Hnatkovska, 2011; Boileau and Moyen, 2012).

5This contrasts with an older corporate finance literature, see Holmstrom and Tirole (2011).

6See, for example, Bates et al. (2009) and Almeida et al. (2013) for surveys.

7Similarly, Sufi (2009) and Lins et al. (2010) show that internal cash is used more in bad times while firms are more likely to use credit lines in good times. Acharya et al. (2013) build a model to show that firms would rather use credit lines instead of cash reserve when they face a low aggregate risk.
shocks. In Section 5, we examine the impact of idiosyncratic shocks on cross-firm correlations. Section 6 discusses various extensions and Section 7 concludes. Several results are derived in the Appendices.

2 Stylized Facts

In this section, we document the negative correlation in the U.S. between the corporate cash ratio and employment, both in aggregate terms and at the firm level. We first illustrate the aggregate correlation between corporate cash and employment over the business cycle. We use quarterly data in the non-farm non-financial corporate sector. The cash ratio, defined as the share of corporate liquidity to total assets, is built from the Flow of Funds of the United States. We define cash as the sum of private foreign deposits, checkable deposits and currency, total time and savings deposits and money market mutual fund shares. Corporate employment in logarithm is drawn from the Bureau of Labor Statistics. Figure 1 displays the HP-filtered component of employment and the cash ratio over the sample 1980q1-2011q4.

[ insert Figure 1 here ]

We observe a negative comovement between the two variables. This negative relationship is particularly striking from the Great Recession since the corporate liquidity ratio experienced a large boom from 2009 while employment has been strongly depressed. Over the whole sample, the contemporaneous correlation between employment and the cash ratio is negative \((-0.41)\) and significant at \(1\%\). We show below that this negative correlation is mostly driven by liquidity shocks in our model.

The aggregate correlations that have been documented are driven by macroeconomic shocks common to all firms. In order to capture the heterogeneity among firms, we assess the correlation between the corporate cash ratio and employment using disaggregated firm-level data from Compustat. The sample contains US non-financial firms from 1980 to 2011. We focus only on firms that are active during the whole period, which allows us to have a homogeneous panel. In addition, we

\footnote{In order to avoid any spurious correlation, we also compute the correlation when cash is divided by the one-quarter lagged value of total assets instead of its current value. The correlation is still negative \((-0.42)\) and significant. The correlation by excluding the Great Recession is lower \((-0.18)\) but significant at \(10\%\). Robustness exercises are provided in the online appendix.}
drop the 10% largest firms. This is a standard procedure (e.g., see Covas and den Haan, 2011) as the largest firms may have a specific behavior. For example, the cash holding of multinational companies might be driven by foreign tax incentives (see Foley et al., 2007). We also exclude financial and utilities firms, firms which are not incorporated in the US market and those engaged in major mergers.\footnote{Using Compustat data items, we remove firms when 6000<SIC<6999, 4900<SIC<4949, cured \neq USD and sale_fn = AB.} This is justified by the fact that part of the stock of cash holding is affected by acquisition. We also drop all firms with negative or missing values for: total assets, sales, cash and employees.\footnote{The sample is reduced to 14,563 firm-year observations. Data description and descriptive statistics are provided in the online appendix.} We use the number of employees per firm (Compustat data item #29) as our measure of employment. The corporate cash ratio is defined as the ratio between cash and short term investment (Compustat data item #1) and the book value of assets (Compustat data item #6). A firm-specific linear trend is removed from both employment and the cash ratio. Figure 2 plots the year-by-year cross-firm correlation coefficients between these two variables with their significance level.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure2}
  \caption{Year-by-year cross-firm correlation coefficients between employment and cash ratio.}
\end{figure}

Over the period, the cross-section correlation between detrended employment and cash ratio is $-0.29$ on average and it is significant at 1%.\footnote{The online appendix shows that the correlation is $-0.28\%$ (significant at 1\%) when we do not exclude the 10\% largest firms.} The negative correlation is significant in all periods and is stronger in 2007. While we only present unconditional correlations, the negative relationship between employment and cash holding is robust when we use OLS with firms-fixed effects, years-fixed effects, and standard control variables (see online appendix). In particular, this relationship is not driven only by macroeconomic shocks or by systematic differences across firms. Our model also accounts for this idiosyncratic correlation.

An important assumption of our model is that cash holding decisions are determined by wage bill financing. From our database, we observe that cash represents 18\% of their staff expenses (median value).\footnote{The series staff expense (Compustat data item #42) includes salaries, wages, pension costs, profit sharing and incentive compensation, payroll taxes and other employee benefits. To counteract the scarce availability of this variable, we extend the dataset with the 10\% largest firms. The sample now consists in 2,435 firm-year observations. The online appendix shows that the distribution of firms’ size and cash ratio is slightly affected in this alternative sample.} Moreover, in the online appendix,
we show that there is a positive relationship between past cash holding and the current amount of staff expenses. This result suggests that firms hold more cash prior to a rise in staff expenses, which is in line with our model’s assumption. This positive relationship is robust to the presence of firms-fixed effects and holds both at the firm and industry level.

In this paper, we argue that cash holdings and employment are driven by future prospects about the availability of external liquidity. There are alternative potential explanations for the negative correlation between the cash ratio and employment. First, the demand for cash can be driven by the cyclicality in the cost of cash (e.g., see Azar et al., 2014). For example, during the crisis, the flight to liquidity can partly be explained by the drop in interest rates which decreased the opportunity cost of cash. However, the negative firm-level correlation is robust to the inclusion of years-fixed effects, which indicates that it is not driven exclusively by business cycle effects like the cost of cash. A second alternative explanation emphasizes the role of unexpected shocks. For example, following a negative unexpected productivity shock, firms lay off workers, which generates more cash flow. However, using our panel of firms from Compustat, we show that the correlation coefficient remains negative and significant when we control for cash flows (see online appendix). Moreover, the correlation coefficient is still negative and significant when we use the lagged cash ratio and when we control for the size of the firm (see online appendix). These two pieces of evidence suggest that the correlation between employment and the cash ratio is not driven solely by unexpected productivity shocks.

3 A Dynamic Model of Corporate Cash Holdings

The single-good economy is inhabited by infinitely-lived heterogeneous entrepreneurs and identical households. Entrepreneurs produce, hire labor, invest, borrow, and hold cash. Households work, consume, lend to entrepreneurs and also hold cash. We abstract from financial intermediaries. Liquidity is modeled by dividing each period in two subperiods, which we refer to as beginning-of-period and end-of-period. The market for illiquid debt only opens at the beginning-of-period. Firms

the correlation between the cash ratio and employment is $-0.17$ and still significant at 1%. 

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have a liquidity need at the end-of-period as they have to pay for the wage bill.\footnote{For convenience we only consider labor as end-of-period input. In a related context, Gao (2013) considers raw material instead of labor.} This liquidity need can be covered either by external liquidity or by cash holdings. Therefore, the need for cash is affected by changes in the availability of external liquidity. We first describe the problem of entrepreneurs and then turn to their optimal behavior, focusing on optimal labor demand and cash. We characterize analytically the properties of the model in this partial equilibrium. Finally, we describe the general equilibrium model by introducing households.

3.1 Entrepreneurs

There is a continuum of entrepreneurs of length 1. Entrepreneur $i \in [0, 1]$ maximizes:

$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{it+s})$$

where $c_{it+s}$ is the consumption of entrepreneur $i$ in period $t + s$. Entrepreneur $i$ produces $Y_{it}$ out of capital $K_{it}$ and labor $l_{it}$ through the production function

$$Y_{it} = F(K_{it}, A_{it}l_{it})$$

where $F$ is a standard constant-return-to-scale production function and $A_{it}$ is total factor productivity (TFP). Capital depreciates at rate $\delta$. TFP is composed of an aggregate component and an idiosyncratic one:

$$A_{it} = A_t + \epsilon_{it}^A$$

where $A_t$ follows an AR(1) process and $\epsilon_{it}^A$ follows a Markov process, with $E(A_t) = A$ and $\int_0^1 \epsilon_{it}^A di = 0$.

Entrepreneurs enter beginning-of-period $t$ with initial income $\Omega_{it}$ and can borrow in illiquid debt $D_{it}$ to pay for their consumption, their capital, and cash $M_{it}$. Debt $D_{it}$ is illiquid in the sense that it can only be issued at beginning-of-period. We follow Jermann and Qadrini (2012) by assuming that firms benefit from a subsidy on debt, so the gross interest rate on debt is $r_t = \tau R_t$, with $0 < \tau < 1$, where $R_t$ is the before-tax interest rate.\footnote{This tax advantage of debt is also found in Hennessy and Whited (2005). It reflects the firms’ preference for debt over equity (pecking order). In our model, this pecking order is represented} Cash bears no interest. The firms’
beginning-of-period budget constraint is:

$$\Omega_{it} + D_{it} = c_{it} + K_{it} + M_{it}$$

(3)

The cash ratio $m_{it}$ is defined as the proportion of cash to total assets, i.e., $m_{it} \equiv M_{it}/(K_{it} + M_{it})$. As $D_{it}$ is never negative in equilibrium, it is never part of gross assets.\textsuperscript{15} Initial income is made of output, the remaining capital stock, and unused cash minus the gross interest rate payment on debt and the cost associated with external liquidity used in the previous subperiod:

$$\Omega_{it} = Y_{it-1} + (1 - \delta)K_{it-1} + \tilde{M}_{it-1} - r_{t-1}D_{it-1} - r_{t-1}^{L}L_{it-1}$$

(4)

where $\tilde{M}_{it-1}$ is unused cash, $L_{it-1}$ is external liquidity obtained in the previous end-of-period and $r_{t}^{L} \geq 1$ is the cost associated with it.

Liquidity shocks affect the magnitude of external liquidity $L_{it}$ available to firms. At end-of-period $t$, firms need to pay for wages out of their cash or any liquid funds they obtain in that subperiod. They face the following liquidity constraint:

$$M_{it} + L_{it} \geq w_{it}l_{it}$$

(5)

where $w_{it}$ is the wage rate. Unused cash is simply defined as $\tilde{M}_{it} = M_{it} - L_{it} - w_{it}l_{it}$. It will be equal to zero in most of our analysis. We assume that liquidity is constrained by lenders. Due to standard moral hazard arguments, a fraction $0 \leq \kappa_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used as collateral for debt repayments, i.e.,

$$r_{t}^{L}L_{it} \leq \kappa_{it}(1 - \delta)K_{it}.$$ 

(6)

We will assume that $r_{t} > r_{t}^{L}$, so that $r_{t}^{L}L_{it} = \kappa_{it}(1 - \delta)K_{it}$.\textsuperscript{16} Shocks to $\kappa_{it}$ are therefore liquidity shocks, i.e., shocks that affect the amount of external liquidity.\textsuperscript{17}

The liquidity shock $\kappa_{it}$ is assumed to be composed of an aggregate component by the fact that the firms will have a tendency to consume (which corresponds to distributing dividends) and as a consequence they will be leveraged up to the maximum level.

\textsuperscript{15}$D_{it}$ is non-negative because all firms are always constrained due to the debt subsidy and because we abstract from equity issuance. If some firms were unconstrained, they could choose a negative $D_{it}$, and thus hold both bonds and cash.

\textsuperscript{16}In reality, the interest rate on short-term liquidity is not necessarily lower than longer-term borrowing. But the borrowing period is shorter so that the actual borrowing cost is lower.

\textsuperscript{17}External liquidity could also vary with the proportion of wages that have to be paid at end-of-period.
and an idiosyncratic one:

$$\kappa_{it} = \kappa_t + \epsilon_{it}^\kappa$$

(7)

where $\kappa_t$ follows an AR(1) process and $\epsilon_{it}^\kappa$ follows a Markov process, with $E(\kappa_t) = \kappa$ and $\int_0^1 \epsilon_{i}^\kappa d\bar{t} = 0$. In our benchmark analysis, we simply assume that $\kappa_{it}$ is known at beginning-of-period $t$. Assuming that the liquidity shock is anticipated is a convenient way of capturing the perceived availability of liquidity. More generally, we can think of expected changes in the distribution of $\kappa_{it}$. In Section 6, we show that anticipated changes in the variance of $\kappa_{it}$ can have the same effect.

Finally, we assume that the entrepreneur faces a standard credit constraint at beginning-of-period $t$. A fraction $0 \leq \phi_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used as collateral for debt repayments:

$$r_t D_{it} \leq \phi_{it}(1 - \delta)K_{it}$$

(8)

In principle, the two constraints (6) and (8) could be related (e.g., as in Jermann and Quadrini, 2012). However, we specify them independently as we will estimate $\kappa_{it}$ and $\phi_{it}$ from the data.

The parameter $\phi_{it}$ is composed of an aggregate component and a firm-specific one:

$$\phi_{it} = \phi_t + \epsilon_i^\phi$$

(9)

where $\phi_t$ follows an AR(1) process with $E(\phi_t) = \phi$ and $\int_0^1 \epsilon_i^\phi d\bar{t} = 0$. In this paper, we make the distinction between a standard credit shock, $\phi_{it}$, and a liquidity shock, $\kappa_{it}$. The former can be viewed as a standard disturbance on the banking sector since it affects the long-term credit. The latter corresponds to an exogenous change in the availability of external liquid funds, which may come from different sources.

### 3.2 Optimal Cash Holding and Employment

Entrepreneurs maximize (1) subject to (3), (5) and (8). The optimization of the entrepreneur is described in details in Appendix A. We assume that shocks are anticipated so the random variables $\Phi_{it}$, $\kappa_{it}$ and $\phi_{it}$ are known at beginning-of-

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18 The presence of credit constraints at the beginning-of-period is not crucial to the main mechanisms we analyze, but it allows to study the impact of credit market shocks. Moreover, it is a convenient assumption with heterogenous firms, as it puts a limit to the size of the most productive firms.
period $t$. As cash does not yield any interest, one can also verify that (5) is always binding so that $\tilde{M}_{it} = 0$.

It is convenient to express production as a function of the capital-labor ratio $k_{it} = K_{it}/l_{it}$. We have $F(K_{it}, A_{it}l_{it}) = A_{it}l_{it}f(k_{it}/A_{it})$ where $f(k) = F(k, 1)$. The optimality conditions with respect to $l_{it}$ and $K_{it}$ imply that the capital-labor ratio is described by (see Appendix A):

$$k_{it} = A_{it}\tilde{k}(\tilde{w}_{it}, \phi_{it}, \kappa_{it})$$  \hspace{1cm} (10)

where $\tilde{w}_{it} = w_{it}/A_{it}$. As shown in the Appendix, $\tilde{k}(\cdot)$ is increasing in $\tilde{w}_{it}$, $\phi_{it}$ and $\kappa_{it}$. Indeed, a lower wage makes production less intensive in capital as opposed to labor. Besides, as capital is the collateral, lower $\phi_{it}$ and $\kappa_{it}$ reduces the collateral value of capital and thus have a negative effect on the capital-labor ratio. The effect of a reduction in TFP, $A$, is more ambiguous as it reduces both the marginal productivity of labor and capital. In the Cobb-Douglas case where $F(K, A_l) = K^a(A_l)^{1-a}$, however, we can show that overall, a lower productivity increases the capital-labor ratio when $\delta > 0$. In that case, an reduction in $A$ affects the marginal productivity of labor relatively more than the return on capital, because it does not affect the remaining stock of capital.

The cash ratio, which is a key variable in our analysis because it reflects the cash-intensity of production, can be derived from the above results. Using (5), (10), and $r_l^L L_{it} = \kappa_{it}(1-\delta)K_{it}$, we find:

$$\frac{M_{it}}{K_{it}} = \frac{1}{k_{it}} \left[ w_{it} - \kappa_{it}(1-\delta)k_{it}/r_l^L \right] = \frac{w_{it}}{k_{it}} - \kappa_{it}(1-\delta)/r_l^L$$ \hspace{1cm} (11)

The demand for cash per unit of capital is equal to the demand of cash per unit of labor, divided by the capital-labor ratio. The demand for cash per unit of labor is itself simply equal to the liquidity need per unit of labor ($w_{it}$), minus external liquidity per unit of labor ($\kappa_{it}(1-\delta)k_{it}/r_l^L$). A decrease in $\kappa_{it}$ has two effects: a direct negative effect as it diminishes the access to external finance and an indirect negative collateral effect as the capital-labor ratio decreases. These two effects both increase the cash ratio. A decrease in $\phi_{it}$ also increases the cash ratio, but only through the negative negative collateral effect. In contrast, a decrease in $A_{it}$ increases the capital-labor ratio and as a result it decreases the cash ratio. Equation (11) then implies that the cash ratio, which depends solely on $M_{it}/K_{it}$, comoves
negatively with $\kappa_{it}$ and positively with $A_{it}$.

To analyze labor demand, we will focus on cases where entrepreneurs are credit-constrained and have log utility. Appendix A shows that the credit constraint is binding whenever the wage paid by firms, $w_{it}$, is lower than the marginal return of labor, denoted $w_{it}^*$. Moreover, with log utility Appendix A shows that optimal consumption is $c_{it} = (1 - \beta)\Omega_{it}$.

In that case, it is useful to rewrite the constraint (3) using (5), (8), and $L_{it} = \kappa_{it}(1 - \delta)K_{it}$. This gives:

$$\beta\Omega_{it} + \frac{\phi_{it}(1 - \delta)K_{it}}{r_t} + \frac{\kappa_{it}(1 - \delta)K_{it}}{r_t^L} = K_{it} + w_{it}l_{it} \quad (12)$$

Equation (12) gives the budget constraint aggregated over the two subperiods. Total financing of firms, on the left-hand side, pays for inputs, on the right-hand side. Both the long-term and short-term financing conditions, represented respectively by $\phi_{it}$ and $\kappa_{it}$, affect the capacity of firms to finance labor $l_{it}$. Using (12), the optimal behavior of entrepreneurs is described in the following proposition.

**Proposition 1 (Individual policy functions)** Suppose that $u(c_{it}) = \ln(c_{it})$. If $r_t > r_t^L > 1$ and $w_t < w_{it}^*$, where $k_{it}$ is given by (10), then the liquidity constraint (5) and the credit constraints (6) and (8) are binding and the policy functions for $K_{it}$, $M_{it}$, $l_{it}$, $D_{it}$, and $\Omega_{it+1}$ satisfy:

$$l_{it} = Z_{it}\Omega_{it} \quad (13)$$

$$K_{it} = k_{it}Z_{it}\Omega_{it} \quad (14)$$

$$M_{it} = [w_t - \kappa_{it}(1 - \delta)k_{it}/r_t^L]Z_{it}\Omega_{it} \quad (15)$$

$$D_{it} = \phi_{it}(1 - \delta)k_{it}Z_{it}\Omega_{it}/r_t \quad (16)$$

$$\Omega_{it+1} = [(1 - \kappa_{it} - \phi_{it})(1 - \delta)k_{it} + A_{it}f(k_{it})]Z_{it}\Omega_{it} \quad (17)$$

where

$$Z_{it} = \frac{\beta}{[k_{it} + w_t] - (\kappa_{it}/r_t^L + \phi_{it}/r_t)(1 - \delta)k_{it}}. \quad (18)$$

**Proof.** See Appendix A. □

We call $Z_{it}$ the financial multiplier. It measures the impact of a change in income on labor demand. Notice that a decline in the financing conditions $\phi_{it}$ or
$\kappa_{it}$ implies a smaller $Z_{it}$, everything else equal. A worsening of financing conditions has thus a negative effect on inputs, including labor. However, it also decreases the capital-labor ratio as the collateral value of capital declines, which has a positive effect on labor. Under standard assumptions, the direct negative effect dominates, as shown in the following corollary.

**Corollary 1** Under the Cobb-Douglas production function, ceteris paribus, firms with lower financing conditions $\kappa_{it}$ or $\phi_{it}$ have lower employment $l_{it}$ and a higher cash ratio $m_{it}$. Moreover, a lower productivity $A_{it}$ affects negatively employment $l_{it}$ but has a negative effect on the cash ratio $m_{it}$.

**Proof.** See Appendix A. □

Corollary 1 illustrates the main mechanism in the model. An expected decrease in $\kappa_{it}$ implies a smaller amount of available liquid funds at end-of-period $t$. As a response, firms naturally increase the proportion of cash in their portfolio, as seen in (11). At the same time, they reduce their labor demand and their production, as outside funding decreases. The same occurs with a decline in $\phi_{it}$, but the increase in cash ratio is milder. This increase takes place as firms reduce their capital stock relative to labor and hence relative to their liquidity needs, because of the indirect collateral effect. On the opposite, with a decline in productivity $A_{it}$, firms increase their capital-labor ratio, which has a negative effect on their cash ratio, as their liquidity needs decline in proportion to capital. At the same time, labor declines.

The next two sections verify numerically the *ceteris paribus* result from Corollary 1 in a dynamic model where the income level $\Omega_{it}$ is endogenous and the wage rate $w_t$ is determined in the labor market. Section 4 focuses on aggregate shocks and the time-series dimension, while Section 5 focuses on the cross-firm dimension.

### 3.3 Closing the Model

The model is closed by introducing households. Since the emphasis is on firms, households are modeled in a simple way and the full description is left for Appendix B. Identical households provide an infinitely elastic supply of funds $D_t$ to firms at interest rate $R = 1/\beta$, where $\beta$ is the beginning-of-period to beginning-of-period households’ discount factor, which is the same as firms’. This is justified by a utility function linear in consumption, the absence of financial frictions for households and the fact that unlike firms, households do not benefit from a subsidy on debt.
Similarly, we assume that households’ utility is linear in cash so that their supply of cash is infinitely elastic at rate 1.

At end-of-period $t$ households also supply liquid funds $L_t$ at rate $r^L_t = 1/\psi$, where $\psi$ is the household’s discount factor between the end-of-period and the beginning-of-period. They always have sufficient cash since they receive their wages at end-of-period $t$ while they consume at beginning-of-period $t + 1$.

Finally, households have a labor supply $l^s(w_t)$ that depends positively on the wage rate. In our specification, we have $l^s(w_t) = (w_t/\bar{w})^\eta$ where $\eta > 0$ is the Frisch elasticity of labor supply and $\bar{w}$ is a positive constant (see Appendix B). The wage rate is then determined endogenously so that $l^s(w_t) = \int_0^1 l_{it} di$ where $l_{it}$ is the labor demand by firm $i$ in period $t$. According to Proposition 1, $l_{it} = l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it})$, so the equilibrium wage is defined by

$$l^s(w_t) = \int_0^1 l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it}) di,$$  \hspace{1cm} (19)

### 4 Aggregate Shocks

In this section, we focus on the time-series dimension, as described in Figure 1, of the relationship between the cash ratio and employment. For this purpose, we assume that all entrepreneurs are identical and only face aggregate shocks, so $A_{it} = \epsilon_{it} = \epsilon_i^\phi = 0$. We also assume that entrepreneurs are always constrained by setting $\tau < 1$ so that $r_t < 1/\beta$. In this context, we calibrate the model to analyze the dynamic impact and the historical behavior of productivity and financial shocks, in the spirit of Jermann and Quadrini (2012). We derive three relevant series: liquidity shocks $\Delta_t$, productivity shocks $A_t$, and standard credit shocks $\phi_t$. We show that our model reveals the presence of negative shocks on liquidity and credit over the recent period, the former contributing the most to the negative comovement between cash and labor.

#### 4.1 Equilibrium

In the absence of idiosyncratic shocks, the only potential source of heterogeneity between firms is their wealth. Since labor demand is linear in wealth, we can then write $\int_0^1 l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it}) di = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ where $\Omega_t = \int_0^1 \Omega_{it} di$. We consider a constrained equilibrium defined as follows:
Definition 1 (Constrained equilibrium under aggregate shocks only) For a given aggregate wealth $\Omega_t$ and a given realization of $A_t$, $\kappa_t$ and $\phi_t$, a constrained period-$t$ equilibrium is a level of employment $l_t$, of capital $K_t$, of cash $M_t$, of debt $D_t$, of financial multiplier $Z_t$ and of future wealth $\Omega_{t+1}$ satisfying Equations (13) to (18), where $r_t = \tau/\beta$, the wage $w_t$ clears the labor market so that $l^s(w_t) = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ with $l^s(w_t) = (w_t/\bar{w})^n$ and $k_t$ is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy $w_t < w^*_t$.

Since aggregate labor demand depends on $A_t$, $\kappa_t$, $\phi_t$ and $\Omega_t$, the equilibrium wage also depends on those variables: $w_t = w(A_t, \kappa_t, \phi_t, \Omega_t)$. For an individual firm, we saw that the credit constraint is binding whenever $w_t < w^*_t$. At the aggregate level, we can show that there exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t)$ so that $w_t < w^*_t$ is equivalent to $\Omega_t < \Omega^*$. When the wage is low, firms want to use all their resources to produce. However, because firms’ resources are limited by the credit constraints, the aggregate labour demand is low when the aggregate wealth is low, which maintains the equilibrium wage at a low level and firms are constrained in equilibrium. In this section, we focus on cases where this condition is satisfied and we discuss the case where firms are unconstrained in Section 6. The following Proposition shows under which conditions the steady state is constrained:

Proposition 2 (Constrained steady state under aggregate shocks only) The steady state is constrained if and only if $\tau < 1$.

Proof. See Appendix C. □

Individual agents and the aggregate economy will fluctuate around a constrained steady state. Intuitively, on the one hand, a wage that is lower than the marginal productivity of labor makes the credit constraint binding, as stated in Proposition 1. On the other hand, the credit constraint makes the equilibrium wage dependent on aggregate wealth. When $\tau < 1$, the net interest rate $r_t - 1$ is below the propensity to consume out of wealth $1/\beta - 1$, so firms never accumulate sufficient wealth to be able to provide an equilibrium wage equal to marginal productivity.

4.2 Calibration

Table 1 shows the calibration used for the parameters. The first five parameters are calibrated on standard values. Following Jermann and Quadrini (2012), we set
the firms' discount factor equal to $\beta = 0.9825$ and $\tau = 0.9939$, implying a subsidy on net interest debt payments of 35% and a steady-state annualized real interest rate of 4%. The Frisch parameter, $\eta$, is set to unity. We set the share of capital in production, $\alpha$, to 0.36. We assume that the cost of using liquidity, $r^L_t$, is lower than the gross interest rate, such that $r^L_t = 1.01$. The other parameters are calibrated to match two empirical targets, using aggregate data. Precisely, the model has to replicate the mean of the cash ratio and the debt to output ratio over the sample, i.e. 3.3% and 50%, respectively.\footnote{As in Section 2, the cash ratio is defined as the share of liquidity to total assets from the non-financial corporate business sector. The debt to output ratio is measured by the ratio between credit market instruments (liabilities) from the non-financial corporate business sector and the gross value added in the business sector. Data sources are available in the online appendix.} It follows that the liquidity parameter $\kappa$ is set to 0.04 and the credit parameter $\phi$ equals 0.06. Finally, we normalize $A$ to unity.

\[ \text{insert Table 1 here} \]

4.3 Liquidity, Credit and TFP Series

The theoretical framework is used to construct the three series we are interested in, namely, TFP ($A_t$), liquidity ($\kappa_t$) and credit ($\phi_t$). Let $\hat{x}_t$ denote the log-deviation of the variable $x_t$ from its deterministic trend, corresponding to HP-filtered empirical data (detailed below). For technology, we derive the Cobb-Douglas production function in loglinear terms

$$\hat{A}_t = \left[ \frac{1}{1-\alpha} \right] \hat{Y}_t - \left[ \frac{\alpha}{1-\alpha} \right] \hat{K}_t. \tag{20}$$

For the credit series, we use the loglinearized version of the credit constraint, given by Equation (8),

$$\hat{\phi}_t = \hat{D}_t - \hat{K}_t. \tag{21}$$

Finally, the liquidity series is constructed using the liquidity constraint, see Equations (5) and (6),

$$\hat{\kappa}_t = \left[ \frac{\hat{w}_t/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} \left( \hat{w}_t + \hat{l}_t \right) - \left[ \frac{\hat{M}/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} \hat{M}_t - \hat{K}_t. \tag{22}$$
All the parameters are taken from the calibration and the model’s steady-state. We use empirical data of output ($\hat{Y}_t$), measured as the gross value added in the business sector from NIPA. The wage bill ($\hat{w}_t + \hat{\ell}_t$) is measured as the hourly compensation index times hours worked in the nonfarm business sector from BLS. Debt series ($\hat{D}_t$) is measured by credit market instruments (liabilities) from the non-financial corporate business sector from Flow of Funds. Capital ($\hat{K}_t$) is measured using total capital expenditures and consumption of fixed capital of non-financial corporate business sector from Flow of Funds, as in Jermann and Quadrini (2012). Liquidity and employment are those used in Section 2. All the nominal series are deflated by the price index for gross value added in the business sector from NIPA and HP-detrended series.$^{20}$ Figure 3 plots the series of TFP, credit and liquidity, constructed from Equations (20)-(22).

[ insert Figure 3 here ]

Over the sample 1980q1 to 2011q4, the liquidity series features less persistence than the credit series and those two are more volatile than productivity. Regarding the recent period, the economy experienced a reduction in $\hat{\kappa}_t$, below its trends, which can be viewed as a shortage in external liquidity supply. This negative liquidity shock has been combined with a reduction in $\hat{\phi}_t$, interpreted as a negative credit shock. Our model predicts that the Great Recession was mostly driven by financial shocks, i.e. liquidity and credit shocks, rather than a technology shock. This latter result is in line with Jermann and Quadrini (2012) who construct a generic financial shock. As shown in Figure 1, the negative comovement between cash and employment was particularly pronounced during the financial crisis. While credit and the liquidity shocks move closely together in the Great Recession, they are not correlated over the longer sample. In order to understand the role of the three shocks on this comovement, we turn to an analysis of the impulse response functions.

4.4 Impulse Response Functions

We examine the impact of a 1 percent decrease in aggregate liquidity, technology and credit from their steady-state level. We estimate an AR(1) process on the series $\hat{\kappa}_t$, $\hat{\phi}_t$ and $\hat{A}_t$ to obtain the autoregressive parameters, such that $\rho_A = 0.76$.

$^{20}$Details on data sources are provided in the online appendix.
\( \rho_\phi = 0.97 \) and \( \rho_e = 0.41 \). The impulse response functions (IRFs) are computed by determining the equilibrium wage, \( w_t \), that clears the labor market and using in turn the policy functions (13) to (17).\(^{21}\) Figure 4 displays the IRFs to the three shocks. The solid, dotted and dashed lines correspond to a response to \( \kappa_t \), \( A_t \) and \( \phi_t \), respectively.

[ insert Figure 4 here ]

The upper panel in Figure 4 displays the responses to the cash ratio and employment to a 1 percent decrease in the three shocks. The two financial shocks generate a negative comovement between employment and the cash ratio. Considering a negative liquidity shock, i.e., a decline in \( \kappa_t \), firms have smaller external liquid funds to pay for wage bills at end-of-period. The cash ratio \( m_t \) rises through two channels. Through the direct effect firms need to compensate for the reduced access to external liquidity by relying more on internal liquidity. Through the indirect collateral effect the collateral value of capital is reduced relative to labor which reduces the scale of assets. Altogether, these two channels drive the cash ratio in the same upward direction. In the case of a negative credit shock, only the collateral motive plays a role on the cash ratio which slightly increases. The reason of this modest increase is that the credit shock does not directly affect the structure of the portfolio between internal and external liquidity. On the other hand, a reduction in financial opportunities (i.e., shortage in external liquidity and credit) lowers labor demand at beginning-of-period through the financial multiplier. Therefore, employment \( l_t \) decline. When it comes to a negative technology shock, the comovement between employment and the cash ratio is different. As explained above, a decline in productivity \( A_t \) rises the capital-labor ratio which increases in turn the scale of assets as compared to liquidity needs and generates a slight reduction in the cash ratio. Production is therefore less intensive in cash. The other effect, more standard, is to decrease employment through a tighter financial multiplier.

The lower panel in Figure 4 shows the remaining IRFs. The three recessionary shocks generate a decline in wages and therefore a reduction in liquidity needs. The response of debt is mostly driven by negative credit shocks although it evolves in the same pattern as labor in all experiments, which is in line with Covas and den Haan (2012) and Jermann and Quadrini (2012) who stress that debt is procyclical.

\(^{21}\)We check that we do have \( w_t < w_t^* \) every period.
Capital also decreases in response to the financial shocks while it increases on impact in response to the TFP shock, due to the rise in the capital-labor ratio mentioned above.

To shed further light on the dynamics of output, we compute their variance decomposition on output, as displayed in Figure 5.

\[ \text{[ insert Figure 5 here ]} \]

The three series of \( \hat{\kappa}_t, \hat{\phi}_t \) and \( \hat{A}_t \) constructed from Equations (20)-(22) are used to compute the fraction of the forecast error variance of output attributable to each type of shock. We find that liquidity shocks have a significant contribution to business cycles. More precisely, we find that on impact, the liquidity shock explains most of the variance (71%) while over a longer horizon, i.e. one year, 64% of the variance of output is explained by credit shocks. The contribution of TFP shocks peaks at 40% after two quarters.\(^{22}\) Using the identified shocks, we also compute the fraction of the correlation between the cash ratio and employment that is due to each shocks. It appears that, consistently with the IRFs, liquidity shocks explain 79% of the correlation. Episodes of drops in employments that are accompanied with a rise in cash ratio can then be mostly attributable to a liquidity shock.

\[ \text{22These results are consistent with Jermann and Quadrini (2012) who argue that financial shocks contribute to a large extent to the business cycle. One might argue that the limited contribution of TFP to output fluctuations results from our measure of TFP series constructed from Equation (20). As a robustness, we use the “utilization-adjusted quarterly-TFP series” for the U.S. business sector, produced by John Fernald and available on his website. We find that TFP shocks still explain 38% of the variance of output after two quarters.} \]

5 Cross-firms Correlations

We now assess whether the calibrated model is able to explain the cross-firm evidence of a negative correlation between cash and employment. To examine this issue, we reintroduce heterogeneous firms that are hit by idiosyncratic productivity shocks \( \epsilon^A_{it} \) and liquidity shocks \( \epsilon^\kappa_{it} \). Instead we assume for simplicity that the aggregate economy does not fluctuate by setting \( A_t = \bar{A}, \kappa_t = \kappa \). As a benchmark, we assume that credit constraints do not vary across firms and time and set \( \phi_{it} = \phi \). We relax this assumption later by assuming that firms can have different levels of credit constraints.

20
5.1 Equilibrium

As in the case with aggregate shocks only, we consider a constrained equilibrium defined as follows:

**Definition 2 (Constrained equilibrium under idiosyncratic shocks only)** For a given period-$t$ distribution of wealth, productivity and liquidity $\{\Omega_{it}, A_{it}, \kappa_{it}\}_{i \in [0,1]}$, a constrained period-$t$ equilibrium is given by the firm-specific levels of employment $l_{it}$, of capital $K_{it}$, of cash $M_{it}$, of debt $D_{it}$, of Z and of future wealth $\Omega_{it+1}$ satisfying Equations (13) to (18), where $r_t = \tau / \beta$, the wage $w_t$ clears the labor market such that (19) is satisfied with $l^*(w_t) = (w_t / \bar{w})^\beta$ and $k_t$ is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy $w_t < w^*_{it}$ for all $i \in [0, 1]$.

In our simulation exercise, we check ex post that we do have $w_t < w^*_{it}$ for all $i$.

5.2 Calibration

Beside the parameter values described in the previous section, we aim at calibrating a range for $\kappa_{it} = \kappa + \epsilon^\kappa_{it}$ and $A_{it} = A + \epsilon^A_{it}$. We assume that these shocks can take 10 equidistant possible realizations. The two shocks are assumed to follow an independent first-order Markov process with transition probability of $\frac{0.25}{9}$. More precisely, each firm has a probability of 75% to stay in the same state for $\kappa$ ($A$) and a probability of 25% to switch to one of the 9 other states, with an identical probability for each of these states. We calibrate the range for $\kappa_{it}$ and $A_{it}$ (namely, we set the minimum and maximum values) to match some distribution moments observed at the firm level. Table 1 provides the interquartile values to match, computed from the Compustat database described in Section 2. The range of the idiosyncratic liquidity and productivity shocks $\kappa_{it}$ and $A_{it}$ are set to reproduce the interquartile ratio for our two variables of interest, namely the cash ratio and employment. This implies $\kappa_{it} \in [0.01; 0.091]$ and $A_{it} \in [0.94; 1.07]$. All the other parameters are calibrated as described in Section 4.2. The numerical method to obtain the steady-state wage and distribution of firms is described in Appendix D.

5.3 Results

The upper panel of Table 2 displays firm-level moments computed from the stationary distribution. Interestingly, our stylized model provides a negative cross-firm
correlation between the cash ratio and employment, equals to \(-0.13\) under our benchmark calibration. This number is somewhat smaller than the number found in the data (\(-0.29\)).

[ insert Table 2 here ]

To understand this result, Figure 6 shows the impact of an idiosyncratic innovation of \(\kappa_{it}\) and \(A_{it}\) on the value of the labor normalized by wealth (\(l_{it}/\Omega_{it}\)) and the cash ratio (\(m_{it}\)), both weighted by the distribution probability.

[ insert Figure 6 here ]

This figure shows that, as \(\kappa_{it}\) decreases, the cash ratio is higher and labor is lower for a given \(\Omega_{it}\). Differently, firms facing a negative productivity shock adjust both labor and the cash ratio downward. Consequently, even though the two shocks predict an opposite correlation between employment and the cash ratio, our calibrated liquidity shock is strong enough to generate a reasonable negative correlation. When the amount of liquid funds is reduced, firms are able to finance less labor with the same amount of cash. To accommodate for this shock, they both accumulate more cash in order to pay for the wage bill and diminish their level of labor to limit the wage bill.

However, while the normalized labor (\(l_{it}/\Omega_{it}\)) is independent of \(\Omega_{it}\) according to Proposition 1, the level of labor \(l_{it}\) is driven by the size of the firm \(\Omega_{it}\), which depends on the history of shocks. As a consequence, the correlation between the cash ratio and labor is driven not only by \(A_{it}\) and \(\kappa_{it}\) as suggested by Figure 6, but also by \(\Omega_{it}\). The lower panel of Table 2 complements the previous figure by showing the weighted value of these variables by class of firms. While firms with a level of wealth below median have on average a substantially lower level of employment than firms with a level of wealth above median, their cash ratio is about the same on average. On the one hand, idiosyncratic innovations on liquidity (\(\kappa\)) and technology (\(A\)) affect the cash ratio and labor, as shown in Figure 6. On the other hand, they also affect firms’ wealth and therefore employment for a given level of cash. This heterogeneity of wealth generates noise that further dampens the correlation.

We can also show that the credit constraint affects the correlation between the cash ratio and employment through a multiplier effect. To do so, we consider two different states for \(\phi_i\) such that \(\phi_i = \{\phi_L, \phi_H\}\), where \(\phi_L < \phi_H\). In order to
be consistent with the calibration strategy described above, we set the value of \( \phi_{i,L} \) and \( \phi_{i,H} \) in order to match the interquartile ratio for debt to sales from our Compustat database. This strategy implies that most financially-constrained firms are those with \( \phi_i = \phi_L \) where \( \phi_L = 0.02 \), while less constrained firms have \( \phi_i = \phi_H \), where \( \phi_H = 0.10 \). Our model shows that \( \phi_L \)-type firms exhibit a less negative correlation between the cash ratio and labor than \( \phi_H \)-type firms (\(-0.11 \) and \(-0.15 \), respectively). Therefore, the simulation results reveal that the correlation between cash and labor is stronger for less financially-constrained firms. Those firms have a larger financial multiplier since they have more resources through their level of borrowing. Consequently, their labor is more sensitive to productivity and liquidity shocks, while their cash ratio is barely affected by the level of \( \phi_i \). This implies that the correlation between cash and labor is larger for a large \( \phi_i \). This is consistent with the data. Smaller firms in terms of sales or debt-to-sales ratio, that are more likely to be credit constrained, have a less negative correlation between the cash ratio and employment. For example, the 25 percent smaller firms in terms of sales have a correlation of \(-0.24 \) compared to \(-0.33 \) for the top 25 percent. The analogous correlations are \(-0.24 \) and \(-0.35 \) when we rank firms by their debt-to-sales ratio.

6 Extensions

The benchmark model has abstracted from various elements that could be relevant to the analysis. In this section we describe several extensions. First, we analyze the case where firms are not credit-constrained. Second, we discuss the impact of liquidity uncertainty with unanticipated liquidity shocks. Third, we discuss the impact of unexpected productivity shocks that provide an alternative explanation for the negative comovement between cash and employment.

6.1 Unconstrained Firms

We assumed so far that \( r_t < 1/\beta \), so that firms are always credit-constrained. This has two advantages: it enables us to examine the effect of a standard credit shock and it helps sustain an equilibrium with heterogeneous firms. It is however important to examine how this assumption affects the response of the economy to liquidity and productivity shocks. We show that in the absence of credit con-
A liquidity shock affects essentially the cash ratio while a productivity shock affects essentially labor. Cash and labor are thus more disconnected than in the benchmark constrained case.

In order to simulate the unconstrained case, we set \( r_t \) equal to \( 1/\beta \) and assume that \( \phi \) is sufficiently high so that firms never hit their credit limit. We otherwise use the same calibration as in the benchmark model. Since \( r = 1/\beta \), the level of wealth is undetermined in the steady state. For comparison purposes, we set the initial level of \( \Omega \) to the same level as in the benchmark steady state. Figure 7 shows the simulation results.

Following a negative liquidity shock, the economy experiences a decrease in employment and an increase in the cash ratio as in the benchmark. Indeed, on the one hand, firms need more cash to produce. On the other hand, as cash is costly, labor becomes less productive, so the demand for labor and the equilibrium wage decrease. Notice, however, that the effect on employment and wage is much milder when firms are unconstrained as compared to the benchmark, where firms are constrained. Indeed, as long as the cost of liquidity \( r_L \) is not too high, the liquidity shock barely affects labor productivity. Therefore, in the absence of constraint, firms do not change their labor demand dramatically. In the presence of credit constraints, the demand for labor and hence the equilibrium wage depend on firms’ resources. Since fewer external resources are available, firms have to cut on labor hiring, generating a stronger reaction of labor demand.

Consider now the effect of a negative productivity shock. While employment decreases as in the benchmark, the cash ratio remains constant. Indeed, the productivity shock has a direct negative effect on the availability of external liquidity, but it has also a negative indirect, general equilibrium effect on the wage and hence on liquidity needs. In the absence of credit constraints, the equilibrium wage is more sensitive to productivity as compared to the case with credit constraints, where labor demand and the wage depend on wealth. Since, in the latter case, the response of aggregate wealth is sluggish, then so are the responses of labor and the wage. Finally, since the wage, and hence liquidity needs, decrease more when firms are unconstrained, the increase in the cash ratio is mitigated as compared to the benchmark. Actually, the decrease in liquidity needs perfectly compensates for the decrease in external liquidity, leaving the cash ratio unchanged.
6.2 Liquidity Uncertainty

In our analysis, firms know perfectly the amount of external liquidity they can get at the end-of-period, i.e., $\kappa_{it}$ is known at beginning-of-period $t$. If instead we assume that only the distribution of $\kappa_{it}$ is known, we can analyze the impact of an increase in uncertainty in $\kappa_{it}$. Not surprisingly, an increase in liquidity uncertainty increases the demand for cash and decreases employment on average.\footnote{This hoarding behavior is reminiscent of the literature on precautionary savings initiated by Bewley (1986) and Aiyagari (1994).} In particular, if we assume that labor is set at the beginning of period, then an increase in uncertainty has the same effect as an anticipated negative liquidity shock.

To understand this result, consider the simple case where there are two possible states for $\kappa_{it}$: $\kappa^L_t = \kappa - \varpi_t$ and $\kappa^H_t = \kappa + \varpi_t$, with $\varpi_t > 0$. The magnitude of $\varpi_t$, and thus the variance of $\kappa_{it}$, is known at the beginning of period but $\kappa_{it}$ is revealed only at the end of period. When $\varpi_t$ increases, the firm increases its cash holdings. When labor is predetermined at the end-of-period, firms actually hold just enough cash to be able to finance the wage bill in the worst case where $\kappa_{it} = \kappa^L_t$. The reason is that insufficient cash would leave the firm with no revenues ($\Omega_{it+1} = 0$).\footnote{This implicitly assumes that the punishment the firms face for not honoring the contract entails both that households do not work and that money holdings are seized, leaving the firms. This also supposes that money is a perfectly pledgable asset and that households are credible enough to implement that punishment.} This prospect deters firms from putting themselves in such a situation, as the utility is logarithmic and $\log(0) = -\infty$. In the event where $\kappa_{it} = \kappa^H_t$, firms do not draw down on the whole line of credit as it is costly ($r^L_t > 1$), and they set $L_{it} = \kappa^L_t Y_{it}$. Thus, cash holdings move proportionately to $\varpi_t$ and firms behave exactly as if their anticipated liquidity shock was $\kappa^L_t$.

6.3 Unanticipated Productivity Shocks

In this paper we focus on active liquidity management by firms, i.e., the optimal choice of cash holdings $M_{it}$. However, a proportion of cash holding may come from unexpected unused cash $\tilde{M}_{it}$, which has been equal to zero so far in our analysis. This may give an alternative explanation to the negative comovement between cash and employment. Assume that productivity shocks are not known at beginning-of-period $t$ and that firms can adjust their employment within the end-of-period (i.e., employment is not predetermined as in 6.2). In that case, unused cash $\tilde{M}_{it}$ is no
longer necessarily equal to zero. For example, an unexpected decline in $A_t$ implies a lower need for liquidity and thus higher $\tilde{M}_{it}$. Thus, we would have a negative comovement between unexpected cash holding $\tilde{M}_{it}$ and labor demand. However, if the productivity shock is persistent (e.g., as in (2)) the path of productivity in subsequent periods is anticipated as in our benchmark analysis. Overall, except for the effect on impact, the dynamic effect of an unanticipated productivity shock is similar to an anticipated productivity shock.

The model therefore predicts a temporary increase in relative cash holdings. After an initial negative shock, the cash ratio is reduced to adjust for lower expected productivity. In contrast, it is sometimes argued, especially in the wake of the financial crisis, that firms keep holding cash because of low investment opportunities. For this argument to hold in our model, we should assume repeated unanticipated negative productivity shocks. Alternatively, we would need to add some adjustment costs for reducing money holdings or assume that firms’ liquidity management is totally passive, i.e., firms would not choose their optimal level of $M_{it}$.

7 Conclusion

This paper has documented a negative comovement between the corporate cash ratio and employment. Even though such a relationship may appear surprising at first sight, we show that it can be explained by liquidity shocks. These shocks make production less attractive or more difficult to finance, while they also generate a need for liquidity necessary to pay wage bills, which can be satisfied by holding more cash. Moreover, we argue that our analysis is useful in understanding the motives for firms’ cash holdings and in shedding light on the dominant shocks during the financial crisis.

Besides explaining an interesting stylized fact, the simple model developed in this paper could be extended to analyze the role of corporate liquidity in a macroeconomic environment. Several extensions could be of interest. First, instead of focusing on the business cycle frequency, the model could be used to examine longer term developments. The model would actually be consistent with the documented gradual increase in cash holdings if we assume changes in the production process that imply more end-of-period payments (e.g., with more extensive use of just-in-time technologies as reported in Gao, 2013, or with an increase in production
outsourcing). A second extension, that would lead to a richer analysis, is to introduce financial intermediaries. Third, for a better analysis of the financial crisis, it would be of interest to introduce demand shocks. Finally, the role of policy intervention would be a natural extension. The last two extensions would be related to the existing DSGE literature incorporating working capital to study monetary policy.
Appendix

A The Entrepreneur’s Problem

Entrepreneurs maximize (1) subject to (3), (4), (5), (6) and (8) and \( \dot{M}_t \geq 0 \). They also take into account the production function \( Y_{it} = F(K_{it}, A_{it}l_{it}) \). The production function has constant returns to scale so we can write \( Y_{it} = A_{it}l_{it}f(k_{it}/A_{it}) \), with \( f(k) = F(k, 1) \) and with \( k \) the capital-labor ratio \( K/l \). The Lagrangian problem is

\[
\mathcal{L}_{it} = \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) - \gamma_{is} \left[ \dot{M}_{is-1} + Y_{is-1} + (1-\delta)K_{is-1} - r_{s-1}D_{is-1} - r_{s-1}^{L}L_{is-1} + D_{is} - c_{is} - K_{is} - M_{is} \right] \\
+ \eta_{is} \left[ M_{is} + L_{is} - w_{it}l_{is} - \tilde{M}_{is} \right] \\
+ \lambda_{is} \left[ \phi_{is}(1-\delta)K_{is} - r_{s}D_{is} \right] \\
+ \nu_{is} \left[ \kappa_{is}(1-\delta)K_{is} - r_{s}^{L}L_{is} \right] \\
+ \mu_{is} \tilde{M}_{is} \}
\]

The entrepreneur’s program yields the following first-order conditions with respect to \( l_{it}, c_{it}, D_{it}, M_{it}, \tilde{M}_{it} \) and \( L_{it} \):

\[
w_t \eta_{it} = A_{it}F_{it}\beta E_t \gamma_{it+1}
\]  
\[
u'(c_{it}) = \gamma_{it}
\]  
\[
\gamma_{it} = \beta r_t E_t \gamma_{it+1} + r_t \lambda_{it}
\]  
\[
\gamma_{it} = \eta_{it}
\]  
\[
\eta_{it} = \beta E_t \gamma_{it+1} + \mu_{it}
\]  
\[
\eta_{it} = \beta r_t^L E_t \gamma_{it+1} + r_t^L \nu_{it}
\]

Studying these FOCs indicates which constraints are binding. Since \( \gamma = u'(c) > 0 \), then \( \eta > 0 \) according to (26), which implies that both budget constraints are
binding. Moreover, using (23), (25) and (26), we obtain:

$$\beta E_t \gamma_{t+1} \left( \frac{A_{it} F_{it}}{w_{it}} - r_t \right) = r_t \lambda_{it}$$

This implies that whenever $wr < AF_i$, the long-term credit constraint is binding ($\lambda > 0$). Besides, using (25), (26) and (28), we find:

$$\beta E_t \gamma_{it+1}(r_t - r_t^L) = r_t^L \nu_{it} - r_t \lambda_{it}$$

Therefore, if the long-term credit constraint is binding ($\lambda > 0$) and $r_t > r_t^L$, then the short-term credit constraint is binding too ($\nu > 0$). Finally, using (27) and (28), we find:

$$\beta E_t \gamma_{it+1}(r_t^L - 1) = \mu_{it} - r_t^L \nu_{it}$$

Therefore, if the short-term credit constraint is binding ($\nu > 0$) and $r_t^L > 1$, then the entrepreneurs hold no excess money ($\mu > 0$).

Assume now that $r_t > r_t^L > 1$ and make the guess that $\lambda > 0$ (we will determine later under which conditions the long-term credit constraint is indeed binding). Then all the constraints are binding and we can write $\bar{M} = 0$, $D = \phi(1 - \delta)K/r_t$ and $M = w_l - \kappa(1 - \delta)K/r_t^L$. We can then rewrite the objective as

$$\mathcal{L}_{it} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) + \gamma_{is}[Y_{is-1} + (1 - \delta)K_{is-1}(1 - \kappa_{is-1} - \phi_{is-1}) - c_{it} - K_{is}[1 - (1 - \delta)(\phi_{is}/r_{is} + \kappa_{is}/r_{is}^L)] - w_{is}l_{is}]$$

(29)

The optimality conditions with respect to $c_{it}$, $l_{it}$ and $K_{it}$ are:

$$\gamma_{it} = u'(c_{it})$$

(30)

$$w_{it} \gamma_{it} = A_{it} F_{it} \beta E_t \gamma_{it+1}$$

(31)

$$[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] \gamma_{it} = \beta E_t \gamma_{it+1}[F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})]$$

(32)

Combining (31) with (32), we obtain:

$$\frac{w_{it}}{A_{it}} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]F_{it}}{F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})}$$

29
\( F \) has constant returns to scale so we can write: \( F(K, Al) = Alf(K/Al) \). Therefore, \( F_K(K, Al) = f'(K/Al) \) and \( F_l(K, Al) = f(K/Al) - Kf'(K/Al)/Al \). As a consequence, \( w_t/A_{lt} = \tilde{w}(\tilde{k}_{it}, \phi_{it}, \kappa_{it}) \), with \( \tilde{k}_{it} = K_{it}/A_{it}l_{it} \) and

\[
\tilde{w}(\tilde{k}, \phi, \kappa) = \frac{[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)][f'(\tilde{k}) - \tilde{k}f'\prime(\tilde{k})]}{f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)} \tag{33}
\]

Since \( F \) is concave in both arguments, we have \( f'' < 0 \), which implies that \( \tilde{w} \) is strictly increasing in \( \tilde{k} \). If there exists a solution \( \tilde{k}(\tilde{w}_t, \phi_{it}, \kappa_{it}) \) to that equation, then this solution is unique. Finally, \( k_{it} \) is then given by \( k_{it} = A_{it}\tilde{k}(\tilde{w}_t, \phi_{it}, \kappa_{it}) \).

Note that the long-term credit constraint is binding whenever \( \tilde{w}_r < F_l \). Besides, if it is the case, then all the other constraints are binding, as \( r_t^L > 1 \) and \( r_t > r_t^L \). We thus have simply to determine when \( \tilde{w}_r < F_l \). Combing this inequality with (33), we find that this is equivalent to:

\[
f'(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi) > r_t[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)] \tag{34}
\]

\[
\Leftrightarrow \tilde{k} < (f''^{-1}((r_t[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)] - (1 - \delta)(1 - \kappa - \phi)) \]

Finally, according to (33), \( \tilde{k} \) is increasing in \( \tilde{w} \), so this inequality is satisfied for \( \tilde{w} \) lower than some \( \tilde{w}^*(\kappa, \phi) \) and thus for \( w \) lower than some \( w^*(A, \kappa, \phi) \).

In order to study how \( k \) is affected by \( \phi \), we differentiate Equation (33) with respect to it and find after rearranging

\[
\frac{\partial \tilde{k}}{\partial \phi} = \frac{-(1 - \delta)[f'(\tilde{k}) - \tilde{k}f''(\tilde{k})]}{f''(\tilde{k})[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)][f(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k}]} \left[ f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k} \right]
\]

As \( f'' < 0 \), both the numerator and denominator are negative so \( \partial \tilde{k}/\partial \phi_{it} > 0 \). Similarly, we find \( \partial \tilde{k}/\partial \kappa_{it} > 0 \). Then \( k \) is also increasing in \( \phi \) and \( \kappa \).

Differentiating Equation (33) with respect to \( \tilde{w} \), we find after rearranging

\[
\frac{\partial \tilde{k}}{\partial \tilde{w}} = \frac{[f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)]^2}{-f''(\tilde{k})[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)][f(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k}]}
\]
Note that $k = A \tilde{k}$ and $w = A \tilde{w}$ so

$$\frac{\partial k}{\partial w} = A \frac{\partial \tilde{k}}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} = \frac{\partial \tilde{k}}{\partial w} > 0$$

$$\frac{\partial k}{\partial A} = \tilde{k} + A \frac{\partial \tilde{k}}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial A} = \tilde{k} + \frac{\partial \tilde{k}}{\partial \tilde{w}} \tilde{w}$$

$$= \tilde{k} - \frac{f'(\tilde{k}) + (1-\delta)(1-\phi-\kappa)[f(\tilde{k}) - \tilde{k}f''(\tilde{k})]}{-f''(\tilde{k})[f(\tilde{k}) + (1-\delta)(1-\phi-\kappa)\tilde{k}]}$$

In the Cobb-Douglas case, we have

$$\frac{\partial k}{\partial A} = \frac{-(1-\alpha)(1-\delta)(1-\phi-\kappa)f(\tilde{k})}{-f''(\tilde{k})[f(\tilde{k}) + (1-\delta)(1-\phi-\kappa)\tilde{k}]} < 0$$

**Proof of Proposition 1** Assume that the credit constraint is binding and that $r_t > r_t^L > 1$. Then the program of the firm is described by (29) and by the FOCs (30)-(32) and by (33). We make the educated guess that there exists $c$ such that $c_{it} = (1-\chi)\Omega_{it}$. Combining our guess with (3), (5), (6), (8) and (11), we obtain

$$\chi\Omega_{it} = K_{it} + \tilde{w}_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)K_{it} = A_{it}l_{it}[\tilde{k}_{it} + \tilde{w}_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)\tilde{k}_{it}]$$

Replacing $\tilde{w}_{it}$ using (33) and rearranging, we obtain

$$\chi\Omega_{it} = A_{it}l_{it} \left[ \frac{1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it} - \kappa_{it})} \right] \Omega_{it+1}$$

As $\Omega_{it+1} = A_{it}l_{it}[f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it} - \kappa_{it})\tilde{k}_{it}]$, we have

$$\chi\Omega_{it} = \frac{1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it} - \kappa_{it})} \Omega_{it+1}$$

(35)

Using (30) and (32) under log-utility $u(c) = \log(c)$, we obtain the following Euler equation

$$\frac{1}{c_{it}}[1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] = \beta E_t \left\{ \frac{1}{c_{it+1}} \right\} [f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it} - \kappa_{it})]$$

Given that shocks are known at the beginning-of-period, $c_{it+1} = \chi\Omega_{it+1}$ is known at the beginning-of-period, so the Euler equation can be written without the ex-
pectations operator
\[
\frac{1}{c_{it}}[1 - (1 - \delta)(\kappa_{it}/r_{L}^{it} + \phi_{it}/r_{t})] = \beta \frac{1}{c_{it+1}}[f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})]
\]

Using our guess \( c_{it} = \chi \Omega_{it} \) and \( c_{it+1} = \chi \Omega_{it+1} \) to replace \( c_{it} \) and \( c_{it+1} \), we obtain
\[
\beta \Omega_{it} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_{L}^{it} + \phi_{it}/r_{t})]\Omega_{it+1}}{f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})}
\]

Combining (35) and (36) yields \( \chi = \beta \).

Combining \( c_{it} = (1 - \beta)\Omega_{it} \) with the binding constraints (3), (5) and (8), we can easily derive equations (13)-(17) in Proposition 1.

**Proof of Corollary 1** According to Equation (11), a decline in \( \kappa_{it} \) increases the cash ratio through a lower level of external liquid funds and through a lower capital-labor ratio. A decline in \( \phi_{it} \) increases the cash ratio through a lower capital-labor ratio. A decline in \( A_{it} \) decreases the cash ratio through a higher capital-labor ratio.

According to Equation (13), the effect on labor depends directly on the effect on the financial multiplier \( Z_{it} \). We can rewrite \( Z_{it} \) as follows:
\[
Z_{it} = \frac{\beta}{w_{it} + A_{it}\tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_{L}^{it} + \phi_{it}/r_{t})]}
\]

So the effect on \( Z_{it} \) depends on the effect on \( X_{it} = \tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_{L}^{it} + \phi_{it}/r_{t})] \). In the Cobb-Douglas case, we have
\[
\frac{\partial X}{\partial \phi} = (1 - \delta)f'(\tilde{k}) \frac{\alpha(1 - \alpha)(1 - \phi - \kappa)/r - (1 - \alpha)\left[1 - (1 - \delta)\left(\frac{1 - \kappa}{r_{L}} + \frac{\kappa}{r_{L}}\right)\right]}{[f''(\tilde{k})]^{2}(f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k})} < 0
\]

Similarly, we have \( \partial X/\partial \kappa < 0 \). Therefore, a decline in \( \phi \) or \( \kappa \) decreases the financial multiplier \( Z \) and hence has a negative impact on labor.

Note finally that, in the Cobb-Douglas case, \( k_{it} \) is decreasing in \( A_{it} \) as shown earlier. As a result, \( Z \) and \( l \) are increasing in \( A \).
B The Household Problem

Identical households have a linear utility $U_t$ with the discount factor $\beta$, and no financial frictions:

$$E_t U_t = E_t \sum_{s=0}^{\infty} \beta^s \left[ c_{t+s}^{hb} + \psi^{-1} \beta c_{t+s}^{he} + (1 - \beta) M_{t+s}^h - \beta \bar{w} \frac{l_{t+s}^{1+1/\eta}}{1 + 1/\eta} \right]$$  \hspace{1cm} (37)

where $c_{t+s}^{hb}$ is households’ consumption in the beginning-of-period, $c_{t+s}^{he}$ is households’ consumption in the end-of-period and $M^h_t$ are the household’s beginning-of-period money holdings. $\psi/\beta$ is the preference for end-of-period consumption relative of beginning of period consumption. As the household has a preference for the beginning-of-period, then $\psi^{-1} < 1$ so $1/\psi < 1/\beta$. the household has a preference for $t$ end-of-period over time $t+1$ beginning-of-period, $\psi^{-1} > \beta$, so $1/\psi > 1$.

Households maximize this utility subject to their beginning-of-period and end-of-period budget constraints

$$R_{t-1}D_{t-1}^h + M_t^h + c_t^h = D_t^h + r_t^M M_{t-1}^h + r_L L_{t-1} + T_t$$

where $D^h_t$ is household debt and $\tilde{M}_t^h$ are the household’s end-of-period money holdings. $r^M$ is the return of 1 unit of cash. At end-of-period, households lend part of their wage $w_t l_t$ to the firms. This lending $L_t$ yields $r_L$. $r_L$ is the equilibrium return on short-term lending. $T_t = R_{t-1}(\tau - 1)$ are taxes that finance the debt subsidy for firms.

Households’ optimization then implies that, in equilibrium, $l_t = (w_t/\bar{w})^\eta$, $R_t = 1/\beta$, $r_t^M = 1$ and $r_L = 1/\psi$, with $1 < r_L < R$. Note that, in equilibrium, households are indifferent between consuming in the beginning and the end of period. As output is available only at the beginning-of-period, consumption takes place only in the beginning-of-period.

C Equilibrium with aggregate shocks only (Proof of Proposition 2)

Before proving Proposition 2, we establish the following Lemma:
Lemma 1 There exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t)$ so that the credit constraint is binding whenever $\Omega_t < \Omega^*$. In that case the dynamics of $K_t$, $M_t$, $D_t$, $l_t$ and $\Omega_{t+1}$ follow:

$$l_t = Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (38)$$

$$K_t = k(w_t, A_t, \kappa_t, \phi_t)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (39)$$

$$M_t = (w_t - \kappa_t(1 - \delta)k(w_t)/\psi)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (40)$$

$$D_t = \phi_t(1 - \delta)k(w_t, A_t, \kappa_t, \phi_t)/r_t \quad (41)$$

$$\Omega_{t+1} = [(1-\delta)(1-\kappa_t-\phi_t)k(w_t, A_t, \kappa_t, \phi_t) + A_t f[k(w_t, A_t, \kappa_t, \phi_t)/A_t]Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t/r_t \quad (42)$$

where

$$Z(w_t, A_t, \kappa_t, \phi_t) = \frac{\beta}{[k(w_t) + w_t] - (1 - \delta)k(w_t, A_t, \kappa_t, \phi_t)(\kappa_t/\psi + \phi_t/r_t)}$$

is the financial multiplier and

$$w_t = w(A_t, \kappa_t, \phi_t, \Omega_t)$$

is the equilibrium wage so that $w(A_t, \kappa_t, \phi_t, \Omega_t)$ is the solution to $l^*(w_t) = Z(w_t, A_t, \kappa_t, \phi_t)\beta r_t \Omega_t$.

Proof. Note that, as shown earlier, the credit constraint is binding whenever $w < w^*(A, \kappa, \phi)$. Since we also have that the constrained equilibrium wage $w$ is increasing in $\Omega_t$, then there exists an increasing function $\Omega^*$ so that $w_t < w^*(A_t, \kappa_t, \phi_t)$ is equivalent to $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t)$. The rest of the Lemma derives from Proposition 1.

Using this Lemma, we can study the steady state. From Equation (42), we have that the steady-state wage must satisfy:

$$\ddot{w} + \ddot{k} - (1 - \delta)(\kappa/\psi + \phi/r)\ddot{k} = \beta[f'(\ddot{k}) + (1 - \delta)(1 - \kappa - \phi)\ddot{k}]$$

Replacing $\ddot{w}$ using (33) and rearranging:

$$1 - (1 - \delta)(\kappa/\psi + \phi/r) = \beta[f'(\ddot{k}) + (1 - \delta)(1 - \kappa - \phi)]$$

Then inequality (34) is satisfied if and only if $1/\beta > r$, which is the case if and only if $\tau < 1$. Therefore, the credit constraint is binding in the steady state ($\lambda > 0$)
whenever \( \tau < 1 \). This proves Proposition 2.

### D Numerical method

The algorithm to compute the steady-state distribution of firms is as follows:

1. We first choose a grid of wealth \( \Omega_{it} \). Our grid is a 1000-value grid over \([5, 65]\). We use the Chebychev nodes to make the grid more concentrated on low values of \( \Omega \).

2. We allocate an initial uniform and independent distribution to the values of \( \Omega_{i0}, \kappa_{i0} \) and \( A_{i0} \), and make an initial guess on the equilibrium wage \( w_0 \).

3. Given the initial distribution on \( \Omega_{it}, \kappa_{it} \) and \( A_{it} \) and the initial equilibrium wage \( w_0 \), we use Proposition 1 and the Markov Chain to compute the new distribution of \( \Omega_{it+1}, \kappa_{it+1} \) and \( A_{it+1} \). Using Proposition 1, we compute the corresponding distribution of labor demand \( l_{it+1} \). We aggregate this labor demand \( l_{it+1} = \sum_i l_{it+1} d_i \), and if \( l_{it+1} > l^s(w_t) \) (if \( l_{it+1} < l^s(w_t) \)), then we update the equilibrium wage \( w_{t+1} \) upward (downward).

4. We repeat step 3 until the equilibrium wage is reached, i.e. when aggregate labor demand is fully satisfied.
References


Table 1. Calibration Strategy

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Debt subsidy</td>
</tr>
<tr>
<td>$r$</td>
<td>Gross interest rate on bonds</td>
</tr>
<tr>
<td>$r^L$</td>
<td>Liquidity cost</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Frisch parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of output wrt capital</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Collateral share for debt</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>s.s collateral share for liquidity</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>Firm-specific collateral share for liquidity</td>
</tr>
<tr>
<td>$A$</td>
<td>Steady-state productivity shock</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Firm-specific productivity shock</td>
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</table>
Table 2. Simulated Moments

<table>
<thead>
<tr>
<th>Benchmark Calibration</th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>$m_{75%}$/$m_{25%}$</td>
<td>Interquartile ratio of $m$</td>
<td>7.60</td>
</tr>
<tr>
<td>$\ell_{75%}$/$\ell_{25%}$</td>
<td>Interquartile ratio of $\ell$</td>
<td>1.46</td>
</tr>
<tr>
<td>$corr(m, \ell)$</td>
<td>Correlation(cash ratio; labor)</td>
<td>$-0.29$</td>
</tr>
</tbody>
</table>

Average value of labor and cash ratio by class of firms

<table>
<thead>
<tr>
<th></th>
<th>$\ell$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_i$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bottom 50%</td>
<td>0.65</td>
<td>0.03</td>
</tr>
<tr>
<td>top 50%</td>
<td>1.04</td>
<td>0.03</td>
</tr>
<tr>
<td>$\kappa_i$</td>
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<td></td>
</tr>
<tr>
<td>bottom 50%</td>
<td>0.74</td>
<td>0.05</td>
</tr>
<tr>
<td>top 50%</td>
<td>0.77</td>
<td>0.01</td>
</tr>
<tr>
<td>$A_i$</td>
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<td></td>
</tr>
<tr>
<td>bottom 50%</td>
<td>0.71</td>
<td>0.02</td>
</tr>
<tr>
<td>top 50%</td>
<td>0.80</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes: In the upper panel, the empirical correlation between $m$ and $\ell$ is computed after removing the firm-specific linear trend from data. In the lower panel, all the values of labor and the cash ratio are weighted by the distribution probability.
Figure 1: Corporate Liquidity and Employment.

Note: Employment is expressed in logarithm and both variables are HP-filtered.
Figure 2: Cross-section correlation between employment and the cash ratio by year.

Note: For both variables, we remove the firm-specific linear trend. Markers with circle corresponds to correlation coefficients significant at 1%.
Figure 3: TFP, credit and external liquidity series.

Note: TFP, credit and external liquidity series are constructed from Equations (20)-(22).
Figure 4: Impulse Response Functions to liquidity, TFP and credit shocks.

Note: The solid lines correspond to the IRFs to an external liquidity shock ($\kappa$). The dotted lines correspond to the IRFs to a TFP shock ($A$). The dashed lines correspond to the IRFs to a credit shock ($\phi$).
Figure 5: Variance error decomposition on output.

Note: The black bar measures the contribution of TFP shocks ($A$) on forecast variance of output. The dark grey (light grey, resp.) measures the contribution of liquidity, $\kappa$ (credit, resp., $\delta$) shocks.
Figure 6: Value of the labor to wealth ratio \((l_i/\omega_i)\) and the cash ratio \((m_i)\).

Note: All values of \(l_i/\Omega_i\) and \(m_i\) are weighted by the distribution probability.
Note: The solid lines correspond to the benchmark model. The dashed lines correspond to the model with unconstrained firms. All IRFs are expressed in percentage deviation from the steady-state.