Abstract

The usual mechanism through which government spending can be effective in increasing GDP in a liquidity trap emphasizes the role of aggregate demand. Higher government spending generates a lower expected real interest rate through a higher real wage, which translates into higher inflation. This lower real rate increases aggregate demand. I present new evidence that casts doubt on the empirical relevance of this mechanism. In particular, liquidity traps occur exclusively in recessions, which appear to be situations where higher government spending generates less inflation than in expansion times. To rationalize this, I build a New Keynesian model with search and matching frictions in the labor market and a downward rigid nominal wage. When solved using global methods, this model generates asymmetric fluctuations of recruiting costs along the business cycle. This permits the model to generate (i) a higher government spending multiplier in recessions versus expansions and (ii) a significantly higher multiplier at the zero lower bound without relying on a counterfactually large reaction of wages and inflation, both of which are in line with empirical evidence. Decomposing the contributions of recession versus liquidity trap dynamics, I find that the latter accounts for less than half of the additional multiplier effect at the Zero Lower Bound.

Keywords: Zero lower bound, New Keynesian, Government spending multiplier, Search and Matching Frictions.

JEL Classification: E24, E31, E32, E52, E62

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1 Introduction

Six years after the American Recovery and Reinvestment Act (ARRA) has been passed, the debate about government spending multipliers is still lively among academics. The main question that is being asked is the following: can temporarily higher government spending boost the economy in a downturn? The standard way to measure the effectiveness of such a policy is to see if it is successful at raising GDP more than proportionally; in other words: to see if the government spending multiplier is large. On this subject, there is mounting empirical evidence pointing towards bigger government spending multipliers in periods of recession. Using non-linear Vector Auto-Regression methods, Bachmann & Sims (2012) and Auerbach & Gorodnichenko (2012) show that the government spending multiplier is higher when some measure of the output gap is higher than usual. There is also evidence for higher multiplier effects of government spending when the economy is in a liquidity trap (see Almunia et al. (2010) who focus on the liquidity trap episode in the Great Depression). One of the results of this literature is that an exogenous increase in government spending seems to have little if no effect on inflation in a recession. This finding is confirmed in a recent paper by Dupor & Li (2015), in which they show that the ARRA did not cause a rise in expected inflation.

Theoretically, aside from a few attempts to explain why the multiplier might be higher in a recession than in an expansion, most of the papers on this subject have been focused on episodes where the Zero Lower Bound (ZLB henceforth) is a binding constraint. The mechanism that is typically put forward in those papers is the following: by increasing inflation through higher government spending, the government can reduce the real interest rate since the nominal rate is pinned at zero. This will induce people to consume more today, generating more inflation and thus more consumption. At the end of this virtuous cycle stands a

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1Many papers have tried to estimate the multiplier effects of government purchases, among which Blanchard & Perotti (2002), Perotti (2008), Barro & Redlick (2011) and Ramey (2011).

2Owyang et al. (2013), however, using US and Canadian data along with a narrative approach find little evidence for state-dependant multipliers of government spending. In a meta-analysis of over 98 studies, Gechert & Rannenberg (2014a) find that government spending multipliers are significantly higher in recessions.

3In a recent paper, Miyamoto et al. (2015) present evidence of larger multipliers at the ZLB which are not associated with a significant increase in expected inflation. Focusing on unemployment benefits extensions in the U.S during the Great Recession, Hagedorn et al. (2013) find that these had a stimulative effect at the local level without generating an increase in measured marginal costs/inflation.

4Canzoneri et al. (2016), building on a model à la Curdia & Woodford (2010), show that counter-cyclical financial frictions can make government spending quite effective during recessions, all the more so when it is financed by debt. Focusing on the labor market, Michaillat (2014) shows that increasing public employment has a larger effect on total private employment in a recession than in an expansion. The reason is that since there is job rationing in a recession and the labor market tightness is low, public employment has a low crowding out effect on private employment in a recession.

GDP multiplier roughly three times as large as in normal times (Christiano et al. (2011)). Most of these papers use a New-Keynesian model in which prices are set as a markup over current and future expected marginal costs and labor is hired on a frictionless spot market.

However, I show that this model (even using the full, non-linear solution) cannot reproduce the observed asymmetric effects of government spending over the cycle. Given that a Zero Lower Bound episode does not always bind in a recession, but is always the consequence of a recession, it is important to understand the behavior of the economy in a recession since it will shape the dynamics of the model at the Zero Lower Bound. Furthermore, these papers rely extensively on an expected inflation channel that does not seem to be present in the data. In this paper, I provide evidence showing that a government spending shock has no positive effect on inflation both in recessions (when unemployment is relatively high) and expansions (when unemployment is relatively low). In addition, inflation reacts by less in a recession.

My objective in this paper is then to develop a model that can generate both (i) asymmetric effects of government spending over the cycle qualitatively consistent with the data and (ii) a significantly higher multiplier at the zero lower bound without relying on a large reaction of inflation. To do so, I will focus on the one building block that has been often neglected in the previous literature: the labor market. I do not need to provide a figure showing that unemployment usually rises in a recession. Surprisingly, few papers explicitly discuss the dismal state of the labor market in the mainstream literature about the impact of government spending at the ZLB. One might then wonder: is it really this important? In this paper, I argue that the answer is yes.

The reason for this can be succinctly described as follows. Empirical evidence tends to show that labor market adjustment occurs largely through the extensive margin in a recession (see van Rens (2012)). A candidate explanation for this is that there are a lot of idle resources in recessions, hiring is essentially costless in this situation. It follows that higher demand from the government in this situation and—a fortiori when the ZLB binds—is likely to be met with higher

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6In fact, it has been a binding constraint only three times in recent history: in most of developed countries during the Great Depression, in the United States and Euro Zone in the Great Recession and in Japan during the "Lost Decade(s)". Three periods which are associated with severe recessions.

7Notable exceptions include Hall (2011b), Hall (2011a) and Albertini & Poirier (2014). Hall (2011b) uses a DSGE model with a frictional labor market, heterogeneous households and (exogenous) sticky prices. He shows that an overhang following a debt-fueled housing binge coupled with the zero lower bound can generate a "long slump". Hall (2011a) develops a simple model and point to promising models that can resolve the conflict between unemployment as a result of i) frictional labor markets and ii) the level of product demand. The model I will develop in this paper belongs to the class of models that Hall (2011a) deems as promising to study unemployment dynamics at the Zero Lower Bound. Finally, Albertini & Poirier (2014) use a New Keynesian model with search and matching frictions on the labor market to study the effects of unemployment benefits extensions at the Zero Lower Bound.
employment instead of higher real wages.

I show that a New Keynesian model with search and matching frictions in the labor market and flexible real wages is consistent with this intuition. Because there is a large pool of unemployed people in a recession, firms do not have to post many vacancies to recruit in order to meet the increase in aggregate demand coming from government spending. As a result, employment (and GDP) effects of fiscal policy are going to be higher in a recession. What is crucial to capture this asymmetry is to solve the model using global solution methods. Indeed, the lower elasticity of recruiting costs to aggregate demand in recessions comes from the fact that matching frictions make the model highly non-linear. With first-order perturbation methods, the policy rules are necessarily linear and entirely miss this feature so that an increase in government spending has the same effect in recessions and expansions. In contrast, by using a global solution method, I am able to capture this asymmetry.

While the model with flexible wages does a good job on the real side of the economy, it mostly does a bad one on the nominal side. First, it fails to produce a negative relationship between the real wage and inflation: in the data, when inflation decreases, the real wage tends to increase. Second, it fails to reproduce the fact that a government spending shock has a lower effect on inflation in a recession. Third, when a large shock sends the economy at the Zero Lower Bound, the economy experiences a relatively large deflation, which is at odds with what we observed during the recent recession. Finally, while the model does deliver a multiplier of around 1.5 at ZLB, this is accompanied by a roughly one for one increase in inflation and a three-fold increase in the real wage. Both of those magnitudes are hard to reconcile with the data.

I show that these shortcomings can be tied to the assumption that real wages are perfectly flexible in this model. To the contrary, the empirical evidence that I present in this paper shows that nominal wages exhibit downward rigidity. Accordingly, I add a downward rigid nominal wage to the model with a frictional labor market, which implies that the real wage is also downward rigid.

Following a negative demand shock, inflation will decrease and the real wage will increase as a consequence. A government spending shock, generating inflation because more resources have to be used to produce more, will tend to lower the real wage. Since this latter usually accounts for a large share of marginal costs, inflation will react less than in normal times, triggering only a modest decrease in the real interest rate. As a consequence, this model is able to generate (i) a lower inflation response combined with a larger response of GDP in a recession than in an expansion and (ii) a significantly higher multiplier at the zero lower bound, and does so without relying on a large increase in inflation. Therefore, in line with the previous literature, the model says that government spending is

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8See also Schmitt-Grohé & Uribe (2012b) and the references therein.
9Daly et al. (2012) show that this is what happened during the Great Recession.
more effective at the Zero Lower Bound. However, it emphasizes another mechanism: government spending has higher multiplier effects at the ZLB not only because it generates inflation, but because it is relatively costless for firms to recruit in a deep recession. Additionally, the multiplier at ZLB conflates two effects: the fact that the recession is large and the labor market equilibrium is far from its steady state and monetary policy is not responsive anymore. Having a model that can generate both effects, I study a decomposition of the multiplier at ZLB. I find that the role of unresponsive monetary policy explains between 41-46% of the additional multiplier effect at ZLB for a multiplier in the range of 1.25-1.5.

This paper is closely related to two recent contributions. Michaillat (2014) also focuses on the labor market to understand government spending multipliers over the cycle. He shows that a model with search and matching frictions coupled with wage rigidities gives more power to fiscal policy in recessions. However, my paper differs from his in many important dimensions. First, I present empirical evidence for the effect of government spending increases in expansions and recessions that I use to discipline the model. In particular, the evidence points to the presence of downward nominal wage rigidities (henceforth DNWR). Since Michaillat (2014) assumes that real wages are rigid and depend exclusively on technology, his model is silent on the different impact of government spending on the real wage in recessions and expansions. Also, I show that wage rigidity is not necessary to generate asymmetric effects on GDP from fiscal policy over the cycle. In addition, he does not look at the impact of government spending in a liquidity trap. What I will show in this paper is that it is important to understand what happens in recessions to get reliable predictions at the ZLB. Rendahl (2014) develops a similar model, albeit with flexible prices and a constant nominal wage. He focuses on the amplification properties of labor market frictions exclusively at the ZLB in a linear setup. As such, his model is not well suited to understand the different effects of government spending in recessions versus expansions outside the ZLB. As a result, in his model all the additional multiplier effect at the ZLB comes from unresponsive monetary policy. On a technical level, none of these two papers deals with the global solution of the non-linear model with stochastic shocks as I do here.

I first describe in section 2 the usual mechanism through which one obtains a higher than normal multiplier at ZLB in the context of a simple New Keynesian model. I then show that it implies a behavior of main macroeconomic variables at odds with empirical evidence. In section 3, I develop a New Keynesian model with search and matching frictions and a downward nominal wage. I also present the calibration and solution algorithm. In section 4, I analyze the quantitative results of the model under different calibrations.
2 High multipliers in the standard New Keynesian model and empirical evidence

The standard New Keynesian model has been widely used to frame discussions about the impact of government spending policies. In contrast to the standard RBC model, it emphasizes the role of monetary policy in shaping the response of the economy to a government spending shock. When the economy is not in a recession, this makes little difference as both model predicts that the multiplier will be lower than 1. As monetary policy becomes more and more accommodative however, the New Keynesian model predicts higher multiplier effects. The ZLB is an extreme case in which monetary policy does not react to inflation. It follows that if government spending increases at the ZLB, inflation is going to decrease the real interest rate and prompt households to consume more, which will put further upward pressure on inflation. This generates a multiplier effect that is much higher than in normal times—see Eggertsson (2011) and Christiano et al. (2011). While the larger multiplier effect of government spending has received support from the data (Almunia et al. (2010) and Miyamoto et al. (2015)), this relies on a substantial rise in expected inflation that does not seem to be present in the data.

Furthermore, the standard New Keynesian model behaves very much the same in expansions and recessions. This is obviously true when the model is solved using first-order perturbations methods, as is done in the majority of cases. That being said, the standard New Keynesian model has many sources of non-linearity: (potentially) quadratic adjustment costs, a utility function that is concave in consumption and labor etc. However, in the rare cases where the model has been solved using global methods, it only displayed a minimal degree of asymmetry between expansions and recessions (see Fernández-Villaverde et al. (2015)). To confirm this, I study a global solution of a standard New Keynesian model\(^\text{10}\) using a Parameterized Expectations Algorithm as in Albertini et al. (2014).

One crucial parameter in this model is the Frisch elasticity of labor supply, as it determines the elasticity of real marginal cost (and thus, inflation) to a government spending shock. In the following simulations, I use the standard value of $\varphi = 1$ for this parameter. Additional simulations for different values as high as $\varphi = 4$ give similar results and are available upon request. I study here the impact of a government spending shock in a mild recession and a large recession with a binding Zero Lower Bound. Both arise following a discount factor shock. In each case, the expansion/boom scenario is given by the same discount factor shock of the opposite sign. In Figure 1, I plot the multiplier effects on GDP and inflation of a 1% increase in government spending.

\(^{10}\)The model is similar to the one developed in Albertini et al. (2014). See the appendix for a summary of the model equations.
Figure 1: Effects of a 1% increase in government spending, standard New Keynesian model.

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
One can see from Figure 1 that the model does not display a large degree of asymmetry between recessions and expansions when the ZLB is not a binding constraint. The reaction of GDP and inflation is only marginally lower in a recession. This comes from the fact that real marginal cost is lower after a negative aggregate demand shock. As a consequence, a given increase in government spending has a proportionally larger effect in this case. In turn, this comparatively larger reaction of marginal cost and inflation calls for a more aggressive reaction by the central bank. In equilibrium, the increase in interest rate is such that private consumption and inflation are both lower than in a boom.

When the central bank is unable to react however, this larger inflation reaction generates a decrease in the expected real interest rate which prompts a further increase in private consumption, generating a virtuous cycle. What is key at the ZLB is that the increase in government spending has a positive effect on expected inflation. In this experiment, the economy stays at the ZLB for 2 periods, private consumption is crowded in in the first period and so the GDP multiplier is (slightly) larger than 1.

To summarize, while government spending is more effective at the ZLB, it is not the case in a standard recession. In this case, a government spending shock has a marginally lower effect on inflation and GDP. To see whether this is borne out by the data, I will study a threshold VAR on post WW II U.S data.

2.1 Confronting the Standard Model with Empirical Evidence

To evaluate the effect of government spending increases in recessions and expansions, I estimate a Threshold VAR (TVAR) as in Balke (2000). I estimate the following specification:

\[ X_t = A(L)X_{t-1} + I(z_{t-1} < z^*)B(L)X_{t-1} + u_t, \]  

where \( I \) is an indicator variable that takes the value of 1 if \( z_{t-1} < z^* \) and 0 otherwise. In my baseline specification, \( z \) is the four-quarter moving-average of the unemployment rate, \( z^* \) is the median of the latter and the vector of endogenous variables is:

\[ X_t = \left[ \Delta Gt_{t-1}, Gr, Tr, Yt, Wt, Pt \right]', \]

where \( \Delta G_{t-1} \) is predicted government spending growth from quarter \( t-1 \) to \( t \) as a share of GDP, \( Gr \) is the log of real government purchases, \( Tr \) is the log of real receipts net of transfers, \( Yt \) is the log of nominal GDP divided by the GDP deflator, \( Wt \) is the log of the nominal wage divided by the GDP deflator and \( Pt \) is the log of the GDP deflator. The inclusion of government spending forecasts controls for the fact that government spending increases in any given periods have actually been anticipated one period before. This way, the government spending shock can capture the un-anticipated part of variations in government spending. As is
standard, the TVAR model is estimated by OLS and I identify the government spending shock using a Choleski decomposition with government spending ordered second after its expectation.

The model is estimated on a data set consisting of U.S quarterly time series from 1966:Q1 to 2008:Q2 which is described in the appendix. The choice of the time period follows two considerations: (1) I wish to avoid using data from 1947 to the 1950’s which exhibit large variations in prices\textsuperscript{11} and government spending (mainly due to the Korean War) that might drive the results\textsuperscript{12} and (2) the government spending forecasts are only available from 1966 onwards.

For the baseline specification, I include a constant and a linear time trend. All information criteria point to an optimal lag order of 2. I compute the responses to a government spending shock in recessions and expansions as follows: for the expansion phase, I assume that $I = 1$ and the reverse for the recession case. By construction, the impact will be the same across regimes and the impulse response may diverge as time goes on.\textsuperscript{13} It should be noted that the economy is assumed to stay in the same regime throughout the simulation period. As such, the results for the recession case should be interpreted as results for a deep recession with persistently high unemployment. To make sure that my results do not depend on this assumption, I will also study a version of this framework in which this assumption is relaxed.

One can see from Figure 2 that, consistent with much of the literature,\textsuperscript{14} government spending increases are more effective in boosting output in a recession than in an expansion. By construction, government spending has the same impact across regimes and has a statistically significant multiplier effect on output slightly below one, consistent with much of the literature. While the effect in an expansion dies out eventually, the one in recession becomes significantly different from zero for many quarters, with a point estimate of roughly 1.5.

In contrast with the standard New Keynesian model, an increase in government spending does not seem to generate an increase in (expected) inflation. The effect on the price level in a recession is both negative and lower than in an expansion. When I do not distinguish between recessions and expansions and estimate a standard SVAR over the whole sample, I still get a negative effect of

\textsuperscript{11}Another potential issue is that following the experience of World War II, private agents might have anticipated rationing measures to go along with the War.

\textsuperscript{12}Fisher & Peters (2010) also suggest to remove the Korean War from the sample when studying the impact of government spending. They cite two reasons: an excess profit tax was enacted at the same time and the increase in government spending might have been anticipated to be permanent.

\textsuperscript{13}In an earlier version of the paper, I used a smooth transmission VAR as in Auerbach & Gorodnichenko (2012) that allowed a different impact effect. However, in line with their results I did not find a significant difference on impact across regimes. Therefore, the added complication does not seem to matter much and I stick with the simpler TVAR in this paper.

\textsuperscript{14}See Gechert & Rannenberg (2014b) for a meta-regression analysis. They find that spending multipliers significantly increase by 0.6 to 0.8 units during a downturn.
Figure 2: Effects of a government spending shock on output, inflation and real wage: recessions versus expansions

Note: Error bands are 80% confidence intervals using 1000 bootstrap replications.

government spending on inflation. While it is seldom discussed, this is a rather common finding in the literature. Indeed, nearly all studies that look at the effect of government spending on the price level find a negative impact (see Fatás & Mihov (2001), Dupor & Li (2015) and Zubairy (2015)). Upon further inspection, it appears that this effect is entirely driven by what happens in the first part of the sample. A candidate explanation is that the share of government investment in public infrastructure\(^{15}\) was higher during the pre-1987 period (see Fernald (1999)). Furthermore, the returns from such an investment were arguably higher just after the second World War.

In a model with public investment that enters the production function of the private sector, an increase in government spending can potentially generate deflation through aggregate supply effects.\(^{16}\) In this kind of model, higher government spending will increase the stock of public capital which will show up as increased productivity. To investigate this, I add a measure of Total Factor Productivity\(^{17}\) in the VAR. Indeed, one can see that Total Factor Productivity increases significantly in the first part of the sample (See Figures 8, 9 and 10 in the appendix). Taking these aggregate supply effects as given, inflation still reacts less

\(^{15}\)Unfortunately, the GreenBook/SPF data does not differentiate between government consumption and investment so that I cannot study separately the impact of both.

\(^{16}\)See for example Bouakez et al. (2014).

\(^{17}\)The TFP series is the utilization-adjusted series in Fernald (2012).
in a recession than in an expansion. This is still true if I estimate the TVAR model on the post-Volcker sample (see Figure 11 in the appendix), although now it is harder to differentiate the two due to the significantly reduced sample size.

With this in mind, the positive impact of government spending on the real wage in a recession points to the presence of nominal wage rigidities. Indeed, as the nominal wage reacts only little to the rise in government spending and inflation decreases, the real wage is bound to increase. When I replace the real wage in the TVAR with the nominal wage (not reported to save space), I find that the latter one does not react to the government spending shock. The opposite happens in an expansion: as inflation increases (not significant), the real wage decreases.\(^\text{18}\)

These results are conditional on the fact that the economy is assumed to stay in the same regime during the simulation period. In the recession case, this means that the unemployment rate is higher than normal for 20 quarters, or 5 years. To relax this assumption, I impose that the economy gets out of a recession at a deterministic date. The fact that inflation decreases and the real wage increases is robust to the recession duration. Regarding GDP, as long as the recession with high unemployment lasts for more than 5 quarters, I find that a government spending increase has more effects than in an expansion —results are available upon request.

In addition, since I have the unemployment rate as a threshold variable I run the risk of capturing as a recession an expansion with structurally high unemployment. In other words, I might have some recessions with higher unemployment than in an expansion. To correct for that, I get rid of the trend component of variable \(z\) using a HP filter with a smoothing parameter of 1600. I then compute \(\tilde{z}\) as the deviation of \(z\) from its trend as my threshold variable. The results of this experiment are reported in Figure 13 in the Appendix. The response of GDP is still higher in a recession, but this effect is now less protracted. Similarly, the effects on inflation point towards i) a decrease in inflation and ii) a more similar response across regimes, with the response in a recession being still lower. The response for real wages is essentially the same.

Finally, the theory that I am going to develop in the next section points towards the important role of labor market tightness (the ratio of vacancies over unemployment) in generating asymmetric effects of government spending across recession/expansions. If I use a measure of labor market tightness\(^\text{19}\) as a threshold, then a recession is a period in which labor market tightness is unusually

\(^{18}\)A similar picture emerges if I replace government purchases in the VAR with the excess returns series computed in Fisher & Peters (2010) (see Figure 12 in the Appendix). While the effect on GDP is zero in an expansion, the effect in a recession is positive (borderline significant) in the short run and slightly negative in the long run. Now inflation reacts negatively on impact in both regimes and is not significantly different from zero afterwards. The real wage significantly decreases in both regimes.

\(^{19}\)See the Appendix for the construction of the data.
low. With this specification, I still find that output reacts more in a recession and inflation reacts less—see Figure 14 in the Appendix.

To sum up, an increase in government spending (i) does not have much effect on inflation and (ii) even less in a recession. In addition, GDP increases by more in a recession. This implies that the standard New Keynesian model cannot be considered a good laboratory to gauge the effects of government spending at the ZLB. The empirical evidence just presented also points to the presence of nominal wage rigidities. To understand the low effect of government spending on inflation, I will develop a model with search and matching frictions on the labor market and a downward rigid nominal wage. To sum up, an increase in government spending (i) does not have much effect on inflation and (ii) even less in a recession. In addition, GDP increases by more in a recession. This implies that the standard New Keynesian model cannot be considered a good laboratory to gauge the effects of government spending at the ZLB. The empirical evidence just presented also points to the presence of nominal wage rigidities. To understand the low effect of government spending on inflation, I will develop a model with search and matching frictions on the labor market and a downward rigid nominal wage.

3 A New Keynesian model with search and matching frictions on the labor market

In this section, I augment the baseline New Keynesian model with a frictional labor market along the lines of Mortensen & Pissarides (1994). The model is close to the one developed in Ravenna & Walsh (2008). Time is discrete and one period equals one quarter. There are four types of agents in this economy: consumer/households, wholesale producers, retailers and a public authority that conducts both monetary and fiscal policy. I begin with the setup of the labor market.

3.1 The Labor Market: timing and flows

The size of the labor force is normalized to one. Employment decisions are taken by wholesale firms. Specifically, let $N_{t-1}$ be the measure of employed people at the end of period $t-1$. At the beginning of period $t$, a fraction $s$ of employed workers is separated from wholesale firms. The workers that get separated immediately search for work during the period. The pool of job seekers is then

$$s_t = 1 - (1 - s)N_{t-1}.$$  

At the end of period $t$, the measure of unemployed people is given by $U_t = 1 - N_t$. To attract workers, the wholesale firms post a measure $v_t$ of vacancies. Job
seekers and vacancies are randomly matched according to the following constant returns to scale production function

\[ m_t = m \cdot s^\eta t v_{1-\eta} t, \]

where \( m \) governs the efficiency of the matching process. Let \( \theta_t \equiv \frac{v_t}{s_t} \) denote the labor market tightness. Job seekers find work with probability \( f(\theta_t) \equiv \frac{m_t}{v_t} = m \theta_{1-\eta} t \) and firms fill a vacancy at a rate \( q(\theta_t) \equiv \frac{m_t}{v_t} \). To recruit, the firms pay a constant cost of \( r \) per vacancy posted. The law of motion for the measure of employed people in the wholesale sector is given by:

\[ N_t = (1-s) N_{t-1} + v_t q_t. \] (2)

The measure of employed people in the wholesale sector at \( t \) consists of the ones that were not separated and the new matches in the current period. Therefore, the aggregate recruiting expenses incurred by wholesale firms are given by:

\[ \frac{r}{q(\theta_t)} [N_t - (1-s) N_{t-1}]. \] (3)

The household’s employment rate is given by the following law of motion:

\[ N_t = (1-s) N_{t-1} + [1 - (1-s) N_{t-1}] f(\theta_t). \] (4)

The measure of employed people today consists of those that have not been exogenously separated plus those that have been separated and managed to find a job immediately after.

### 3.2 The Representative Household

The household is assumed to be large and solves the following maximization program:

\[
\max_{C_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{j=0}^{t} \xi_j \right) \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\phi}}{1+\phi} \right\},
\]

where \( \phi \) indexes the degree of labor disutility\(^{20}\) and \( \xi_t \) is a preference shock, which follows an AR(1) process with persistence \( \rho_x \). As in Merz (1995) and Galí (2010), there is perfect insurance. The budget constraint is:

\[ P_t C_t + B_t = P_t (N_t W_t + b (1-N_t)) + R_{t-1} B_{t-1} + P_t + \frac{1}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 N_t, \]

where \( P_t \) is the price level, \( C_t \) is a Dixit-Stiglitz aggregate of different varieties produced by retailers, \( B_t \) are nominal one-period riskless bonds, \( R_t \) is the gross

\(^{20}\)With this assumption, the bargaining set will be smaller and when flexible, the real wage will fluctuate relatively less in a manner similar to Hagedorn & Manovskii (2008). This turns out to be very useful for the numerical algorithm to converge.
nominal interest rate, $W_t$ is the real wage, $b$ is the level of unemployment benefits and $P_t$ are nominal profits distributed by retailer firms, net of lump sum taxes. Finally, $\Pi_t = \frac{P_t}{P_{t-1}}$ and if $\Pi = 1$, then the price adjustment cost for monopolistic firms is rebated lump-sum to the household. This is done to prevent this term to appear in the resource constraint of this economy, which can be the source of problems when the economy reaches the Zero Lower Bound (see Braun et al. (2012)). When $\Pi = 0$, the model nests the standard case. The Lagrangian for this program is given by:

\[
L^H = E_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{j=0}^{t} \xi_j \right) \left\{ C_{t+1}^{1-\sigma} - \frac{\lambda}{1-\sigma} N_{t+1}^{1+\varphi} \right. \\
+ \lambda_t \left[ P_t (N_t W_t + b(1 - N_t)) + R_{t-1} B_{t-1} + P_t + 12 \left( \frac{\Pi_t}{\Pi} - 1 \right) N_t - P_t C_t - B_t \right] \\
+ \left. V_{N,t} \left[ (1 - s) N_{t-1} + [1 - (1 - s) N_{t-1}] f(\theta_t) - N_t \right] \right\}.
\]

The Lagrange multiplier on the budget constraint is the marginal utility of private consumption. The Lagrange multiplier on the law of motion of employment $V_{N,t}$ gives the value for the household of having one more employed worker. The first order conditions with respect to $C_t$ and $B_t$ is:

\[
1 = R_t E_t \left\{ \frac{\Lambda_{t+1}}{\Pi_{t+1}} \right\}
\]

where $\Lambda_{t+1} = \beta \xi_t \left[ \frac{C_{t+1}}{C_t} \right]^{-\varphi}$ is the stochastic discount factor which the household uses to discount future consumption. From this equation it is clear that a preference shock will have the effect to increase to returns on savings, prompting household members to consume less today.

### 3.3 The wholesale producer

There is a continuum of mass 1 of wholesale producers, indexed by $i$. They post vacancies to attract new workers, who become immediately productive. They produce output according to the following constant returns to scale production function:

\[
Y^w_t(i) = N_t(i),
\]

where the $w$ superscript stands for wholesale. The output of the wholesale firm is sold to the retailers in a competitive market\(^{21}\) at a price $P^w_t$—I drop the index $i$.
for the price since the firms are atomistic and take the price as given. The same goes for the real wage. The profit of the wholesale firm is then:

\[ \frac{P^w_t}{P_t} N_t(i) - \frac{W_t}{P_t} N_t(i) - r \cdot v_t(i), \] (6)

where \( P_t \) is the welfare relevant price index of the final good and \( W_t \) is the nominal wage. The wholesale firm maximizes its profits by choosing the number of people it wants to employ and the number of vacancies it has to post to do so, subject to the constraint (2). The Lagrangian of this problem then is:

\[ L^F = \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^{t} \xi_j \right) \beta^t C_t^{-\sigma} \left\{ \left( \frac{P^w_t - W_t}{P_t} \right) N_t(i) - r \cdot v_t(i) \right. \]

\[ \left. - V_{f,t}(i) \left[ N_t(i) - (1-s)N_{t-1}(i) - q(\theta_t) v_t(i) \right] \right\}, \]

where I have substituted for the production function. Combining the first order conditions with respect to the measure of employment and vacancies, I get the following expression:

\[ \frac{P^w_t}{P_t} = \frac{W_t}{P_t} + \frac{r}{q(\theta_t)} - (1-s)E_t \frac{r}{q(\theta_{t+1})}. \] (7)

In equilibrium, the marginal revenue product by one more worker should be equal to the cost incurred by the firm from having this additional worker. This includes the recruitment cost (since a vacancy is filled at a rate \( q(\theta_t) \), the firm has to post \( \frac{r}{q(\theta_t)} \) of them to attract one worker) and the real wage payment. To this, we must add the benefit (the negative cost) of having one less worker to recruit tomorrow. Since all the wholesale producers are identical, they will all hire the same measure of workers and produce the same quantity of output

\[ Y^w_t(i) \equiv Y^w_t \quad \forall i \]

which they sell at price \( P^w_t \) to the retailers.

### 3.4 Wage setting

Once the government spending and preference shocks are realized, job seekers and firms meet and bargain over the real wage, which cannot go below a floor overemploy. This can also be the case when the production function exhibits decreasing returns to scale (see Cahuc et al. (2008)). With perfect competition and constant returns to scale, all workers are the same so that the marginal wage is equal to the average wage.

\[ ^{22}\text{In practice, we can reduce the problem of directly choosing the number of people to employ, the measure of vacancies necessary to achieve this level given by (3). However, it is useful to state the maximization problem with respect to both } N_t(i) \text{ and } v_t(i) \text{ to derive the value for the firm of an additional worker, which will be used later to derive the bargained wage.} \]
level $W$ that is taken as exogenous by both participants.\textsuperscript{23} Let $\tilde{V}_{N_t}$ and $V_{J_t}$ denote, respectively, the value of employment for the household and the value of a job for a firm. The first one is equal to the Lagrangian multiplier in front of the employment transition equation in the consumer program, scaled by the marginal utility of private consumption $C_t^{-\sigma}$. Likewise, $V_{J_t}$ is equal to the Lagrangian multiplier in front of the employment transition equation in the firm program. The real wage (denoted by $\tilde{W}$) is the one that maximizes the log of the joint surplus of the representative firm and household, taking into account that the agreed upon real wage outcome is bounded from below, i.e.

$$\tilde{W}_t = \arg \max_{W \geq W} \log \left( \tilde{V}_{N_t}^{1-\mu} V_{J_t}^{\mu} \right) ,$$

where $\mu$ is the bargaining power of the firm. In this setup, the presence of downward wage rigidity will imply that both participants anticipate that the constraint might be binding in the next period. As a consequence, the expression for the bargained wage will be different from the standard flexible wage and is given by:

$$W_t = W_t^{\text{flex}} + \lambda_t^w \tilde{V}_{N_t} \frac{r}{q(\theta_t)} + (1-s)(1-\mu)\mathbb{E}_t D(\lambda_{t+1}^w, \theta_{t+1})$$  \hspace{1cm} (8)

$$D(\lambda_{t+1}^w, \theta_{t+1}) \equiv \Lambda_{t+1}(1-f(\theta_{t+1}))(\frac{r}{\theta_{t+1}} \frac{1}{1-\mu \frac{\gamma}{\theta_{t+1}}})$$ \hspace{1cm} (9)

$$W_t^{\text{flex}} = \mu \left( b + \chi \frac{N_t^q}{C_t^{-\sigma}} \right) + (1-\mu) \left( \frac{P_t^w}{P_t} + (1-s)\mathbb{E}_t \Lambda_{t+1} r \cdot \theta_{t+1} \right) ,$$ \hspace{1cm} (10)

where $b$ is the replacement rate of unemployment benefits and $\lambda_{t+1}^w \geq 0$ is the multiplier on the downward rigidity constraint, which further implies that

either $W_t > \tilde{W}_t$ & $\lambda_t^w = 0$ or $W_t = \tilde{W}_t$ & $\lambda_t^w > 0$.

Note that from equation (8), the negotiated wage nests the usual flexible solution if $\lambda_{t+1}^w = \lambda_{t+1}^w = 0$. This happens if the rigidity constraint is so loose (for instance, $\tilde{W}_t = 0$) that it is never binding in equilibrium. On the other hand, if the constraint is occasionaly binding, then the two wages will be different. Even if $\lambda_{t+1}^w = 0$ so that the constraint is not binding today, the mere possibility that it might be binding tomorrow creates a wedge between the flexible and rigid wage.\textsuperscript{24} Following Schmitt-Grohé & Uribe (2012b), I allow the nominal wage to be potentially downward rigid.\textsuperscript{25} Formally, nominal wages are required to satisfy the following condition:

$$W_t \geq \gamma W_{t-1}, \quad \gamma \geq 0.$$

\textsuperscript{23}The wage of incumbents and new workers will be the same in this framework, since all the workers are identical.

\textsuperscript{24} From equation (9), the difference can be positive or negative. This effect is not quantitatively strong however: for all the $\xi_t$ grid points on which the model is solved, the maximum difference between the two wages is equal to 0.2% of the steady state wage (which is the same in both settings).

\textsuperscript{25}The prevalence of downward rigid nominal wages has been documented by many studies. See Dickens et al. (2007) for a survey of this literature. More recent contributions include Barattieri et al. (2014) and Schmitt-Grohé & Uribe (2015).
Straightforward algebraic manipulations imply that real wages are downward rigid as a consequence:

$$W_t \geq \frac{\gamma W_{t-1}}{\Pi_t}, \quad \gamma \geq 0. \quad (11)$$

Following Hall (2005), I verify that the realized real wage always lies in the bargaining set for this model. When \(\gamma = 0\), the model has a flexible real wage.

### 3.5 The retailers

There is a large number of retailers, indexed by \(j\). They buy intermediate goods from the wholesale producers, which they use to transform into final goods with a one to one technology:

$$Y_t(j) = N_t.$$

They each sell a different variety of the final good. They know that they face a demand for their variety given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t,$$

where \(\epsilon\) is the elasticity of substitution across the different varieties, \(P_t(j)\) is the price of the variety produced by retailer \(j\) and \(Y_t\) is a CES aggregate of all the varieties. The retailer pays a quadratic cost to adjust his price as in Rotemberg (1982). He buys \(Y_{tw}^t\) of intermediate goods at a price \(P_{tw}^t\). Therefore, the real marginal cost of producing one more unit of the final good is equal to \(MC_t = \frac{P_{tw}^t}{P_t}\). Anticipating the results, I assume a symmetric equilibrium for the retailers: each one will produce the same amount using the same quantity of intermediate good. Therefore, his real profit is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^{t} \xi_j \right) \beta^t C_t^{-\sigma} P_t(j) \frac{\Pi_t}{P_t}. \quad (12)$$

Since they are owned by the households, the retailers’ problem is to choose a price that maximizes the present discount value of real profits given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^{t} \xi_j \right) \beta^t C_t^{-\sigma} \frac{P_t(j)}{P_t}. \quad (13)$$

The first order condition with respect to \(P_t\) then gives the standard New Keynesian Phillips Curve:

$$\phi \frac{\Pi_t}{\Pi} \left( \frac{\Pi_t}{\Pi} - 1 \right) = 1 - \epsilon + \epsilon MC_t + \phi \mathbb{E}_t \left\{ \Lambda_{t+1} \frac{Y_{t+1}}{Y_t} \frac{\Pi_t}{\Pi} \left( \frac{\Pi_t}{\Pi} - 1 \right) \right\}, \quad (13)$$

where \(\Pi\) is the steady state inflation rate.
3.6 Fiscal and Monetary Policy

The government finances an exogenous stream of expenses $G_t$ plus unemployment benefits by levying non-distortionary, lump-sum taxes. Government expenses follow an $AR(1)$ process with persistence $\rho_g$. In contrast to Michaillat (2014), government spending does not take the form of public employees. While public employees do represent a sizable share of government spending in the data, I am interested here—as is most of the literature on the effects of government spending—in the effects on aggregate output of the purchase of goods by the government. In fact, public employment did not represent a large share of the American Recovery and Reinvestment Act of 2009, if anything at all.\(^{26}\) The budget constraint of the government then is:

$$T_t + \frac{B_t}{P_t} = G_t + b(1 - N_t) + \frac{R_{t-1}}{P_t} B_{t-1}$$

The Monetary Authority sets the gross nominal interest rate according to:

$$R_t = \max \left\{ 1, \Pi_t \left( \frac{\Pi_t}{\Pi} \right)^{\phi_{\pi}} \right\}.$$ \hfill (14)

For simplicity, the nominal interest rate does not react to its past value.

3.7 Equilibrium

Substituting the definition of real profits in the household’s budget constraint and combining the result with the government budget constraint, one gets the resource constraint of this economy:

$$Y_t \left[ 1 - (1 - 1) \frac{\phi}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 \right] = C_t + G_t + \frac{r}{\phi(\beta_t)} [N_t - (1 - s)N_{t-1}] - b(1 - N_t).$$ \hfill (15)

3.8 Model Solution and Calibration

The model requires a large shock on the discount factor to drive the economy to the ZLB. Therefore, I do not rely on log-linear approximations around a deterministic steady state, as is usually done. Being based on a Taylor expansion of the first order conditions, these approximations are only valid in a small neighborhood of the steady state. In fact, it has been shown that the usual discount factor

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\(^{26}\)With spending reversals on the state level, one can even argue that the net effect of ARRA on public jobs might be negative.
shock takes the economy too far from the steady state for those approximations to remain valid (see Braun et al. (2012)). Christiano & Fisher (2000) argue that a special case of projection methods, the Parameterized Expectations Algorithm (PEA) is the most efficient one to approximate models with occasionally binding constraints. Accordingly, I solve the model globally using this algorithm, as in Albertini et al. (2014).

This algorithm consists in approximating the expectations functions of the model by a simple polynomial function of the state variables. Beginning with a first guess of the coefficients relating the expectations functions to the polynomials, I can compute the policy rules relating the endogenous variables to the state variables. Importantly, since there are two occasionally binding constraints, I estimate four different policy rules, i.e. one for each possible case. As a consequence, this algorithm is very well suited to take into account the eventual kinks in the policy rules. Using these along with the transition equations of the state variables, I can compute the expectations using a Gauss-Hermite quadrature. I then regress those expectations on the state variables to update the value of the coefficients in front of the polynomials. I iterate on these coefficients until the difference at successive iterations is small enough. I explain the algorithm in greater detail in the Appendix D.

I now move to the calibration of the model. The model is calibrated at quarterly frequency. The elasticity of substitution across goods is equal to $\epsilon = 6$, which yields a markup of 20%. I set $\beta = 0.994$ to get an annual real interest rate of 2.5% as in Fernández-Villaverde et al. (2015). I set $\sigma = 1$ as in Christiano et al. (2011). I choose the price rigidity parameter so that in the linear New Keynesian Phillips Curve the elasticity of inflation to the real marginal cost is consistent with an average price duration of four quarters. This yields $\phi = 60$. Now moving to the labor market, I set $\eta = 0.5$ for the elasticity of the matching function with respect to unemployment (see Pissarides & Petrongolo (2001)). The exogenous separation rate is set to $s = 0.11$ as in Krause et al. (2008). The labor disutility parameter is set to $\varphi = 1$ as in Fernández-Villaverde et al. (2015). Following Michaillat (2014), steady state unemployment is set to 6.4%, which yields an employment level of $N = 0.936$. The matching efficiency parameter is set so that, at steady state, $q(\theta) = 0.7$ (see Ravenna & Walsh (2008)). This yields $m = 0.657$. I set a standard value of $b = 0.4$ for the replacement rate. The bargaining power of the household is set to $\mu = 0.5$ so that the Hosios (1990) condition holds. Finally, $\chi = 0.657$ is set so as to balance the steady state wage equation. I set the vacancy posting cost parameter so that total recruitment costs $r \cdot v$ amount to 1% of steady state output as in Christiano et al. (2013). This ensures that the presence of recruitment costs in the resource constraint will not matter much quantitatively. The share of government spending with respect to output is set to the conventional value of 20%. To prevent the adjustment costs to play a big role in the resource constraint, I assume $1 = 1$ in my baseline calibration. The calibrated parameters are summarized in Table 1 (in the Appendix).

\footnote{It also turns out that the model is easier to solve with this assumption.}
The more volatile and persistent the shocks are, the harder it is to get an accurate solution for the model. Based on these considerations, I set $\sigma_x = \sigma_\xi = 0.002$. As in Christiano et al. (2011) I set $\rho_g = \rho_\xi = 0.8$. Finally, following Schmitt-Grohé & Uribe (2012a) and Schmitt-Grohé & Uribe (2012b) I set $\gamma = 0.99$ in the model with downward nominal wage rigidity. For the model with flexible wage, I set $\gamma = 0$.

### 3.9 Price and quantities adjustment in expansions/recessions

In his paper, Michaillat (2014) argues that when the economy is in a recession, labor market tightness is low and reacts less when the government increases public employment. As a consequence, the crowding out effect on private labor demand is less than in an expansion, when labor market tightness is high and reacts a lot more. Since in his framework the real wage is entirely exogenous and given by aggregate productivity, the latter is not amenable to study the implications inflation dynamics.

In the present setup, the real wage can (and will) react differently in expansions and recessions. Since the real wage is an important component of real marginal cost and firms set prices as a markup over the latter, inflation will react differently over the cycle. Given that arguably inflation plays a large role at the Zero Lower Bound, this is a desirable feature. To study how prices and quantities adjust respectively when the labor market is depressed or booming, I plot the ergodic distribution\(^{28}\) of unemployment, inflation and the real wage with respect to labor market tightness in Figure 3.

I first focus on the model with a flexible real wage, *i.e.* the left panels. Generally, a low tightness corresponds to parts of the state space where demand for final goods is low, therefore labor demand is low. This explains the negative relationship between unemployment and labor market tightness. Additionally, the later is more convex than linear as employment prospects gradually worsen as labor market tightness declines. This feature comes from the matching frictions embedded in the model. Indeed, from equation 4 one can see that given $N_{t-1}$, $1 - N_t$ is a convex function of $\theta_t$. When labor market tightness is low, so is $f(\theta_t)$ and it is much harder for job seekers to actually find a job. On the firm side, the presence of a large pool of job seekers means that it is easier to recruit them and $q(\theta_t)$ is high. It follows that variations in labor demand will trigger an adjustment along the employment margin when labor market tightness is lower than its steady state value.

\(^{28}\)I also compute the residuals of the Euler equations for the whole simulation period. For a simulation length of 1 million periods, all three residuals are of the order of $10^{-6}$ on average. Therefore, despite the presence of multiple kinks the algorithm is able to represent rather accurately the dynamics of the model.
Figure 3: Inflation, Employment and the Real Wage in Good/Bad Times, based on one million simulated data points.

Notes: Inflation is reported in annualized terms. Unemployment is in percent of the labor force. The real wage is in percent deviation from its steady state value.
As quantities adjust relatively quickly to variations in labor demand during a recession, prices adjust relatively less. Indeed, it is apparent from the inflation and real wage distributions that they become less sensitive to labor market conditions when the latter are dire, i.e. labor market tightness is low.

The same goes for the model with downward wage rigidity, with an additional twist. On the one hand, this rigidity means that the real wage will be too high in a recession, depressing profits and hiring activities of firms. In a RBC model with a frictional labor market, this would make the recession worse. Here, the downward rigidity of wages also means that marginal costs and inflation will fall by less in a deep recession. As a consequence, this will mitigate the vicious feedback loop that turns deflation into a higher real interest rate at the ZLB.\(^\text{29}\) Those two effects are of a similar magnitude, so that the distribution of employment outcomes are not very different in the two settings.

To finish with this subsection, the absence of a deflationary spiral makes it an appealing framework to work with, since we did not really observe a large fall in inflation during the current Great Recession. This has been dubbed the "missing deflation" puzzle.\(^\text{30}\) For this reason and the fact that the empirical findings described earlier pointed towards the presence of nominal wage rigidities, I will mainly focus on the model that features such frictions. In particular, I will analyze (i) the type of recession this model generates and (ii) whether fiscal policy has asymmetric effects over the cycle.\(^\text{31}\) It is however useful to briefly look first at the model with flexible wages to understand the specific contribution of search and matching frictions on the labor market.

### 4 Government Spending Effects in Recessions vs Expansions

In the model I have developed, fluctuations are driven by two aggregate demand shocks: a preference and a government spending shock. In this section, I will use the preference shock to create a recession or an expansion. Once the economy is in one of these two states, I will implement a government spending shock and gauge the potentially different effects in these two scenarios. To make sure

\(^{29}\)I thank Tommaso Monacelli for suggesting this idea.

\(^{30}\)See Gordon (2013), Del-Negro et al. (2014) and Coibion & Gorodnichenko (2015), among others.

\(^{31}\)I should say upfront that the goal is not to match perfectly the impulse responses estimated earlier on U.S. data. To do that, I would need to add additional ingredients such as consumption habits, private capital with adjustment costs etc. These would add endogenous state variables and thus make the model much harder to solve globally. Rather, I read the empirical evidence as providing a role for labor market frictions and wage rigidities in the propagation of government spending shocks. My main goal is then to study how much these features contribute to the now well-documented state-dependent effects of fiscal policy.
that the effects can be clearly discernible on the figures, I calibrate the size of the government spending shock to match the size of the spending component of the ARRA, which was around 1.58% of GDP.\footnote{See Albertini et al. (2014) for more details.}

4.1 A First Quantitative Exercise with Flexible Wages

In this subsection, I assume $\gamma = 0$. This will allow me to (i) highlight the role of the non-linear algorithm in generating asymmetric responses in recessions vs expansions and (ii) motivate the inclusion of a downward rigid nominal wage. To generate a recession, I take the highest level of the shock $\xi_t$, conditional on the fact that the ZLB is not binding. For the expansions scenario, I take the opposite of this shock.

To gauge the effects of government spending in recessions vs expansions, I proceed as follows. For each scenario, I plot the difference between the simulated path of the economy with and without the government spending shock, scaled by the initial variation in government spending. As such, the responses depicted in Figure 4 can be interpreted as dynamic multipliers. It is clear that this model, when solved globally, can generate a larger effect of government spending in a recession. Initially, labor market tightness —and thus, recruitment costs—rise less in a recession.\footnote{Note that the government spending shock has a large effect on labor market tightness, whether in expansion or recession. This is due to the fact that households experience disutility from labor market activities. Similar to the analysis in Ljungqvist & Sargent (2015), the presence of disutility diminishes the size of the fundamental surplus that can be allocated to vacancy creation. As such, an increase in government spending will generate a comparatively large increase in vacancies and labor market tightness.} A large pool of unemployed people means that a vacancy is filled more easily. As a consequence, employment and GDP react more after a government spending shock in a recession. The underlying mechanism is similar to the one presented in Michaillat (2014). Here, I show that this mechanism holds even without assuming an exogenous rigid wage. The fact that employment is comparatively higher in a recession (conditional on the government spending shock) implies that consumption decreases by less in a recession. As a result, contrary to the empirical evidence presented before, inflation reacts by more in a recession.

Because of its simplicity, the model is not able to produce a hump-shaped response for the main variables as in the empirical responses shown before. Indeed, with a separation rate of $s = 0.11$, unemployment dynamics are not sufficient to yield such a result.\footnote{With a lower separation rate, the model would exhibit a higher degree of persistence, but the algorithm fails to converge for such parameter values.} Consequently, as in the standard New Keynesian model, the maximum response is reached on impact and the economy goes back monotonically to its steady state. This caveat notwithstanding, one can see from Figure 4
Figure 4: Impulse Responses to a Government Spending Shock in Expansions/Recessions with flexible wage.

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
that employment—and thus, GDP—reacts more in a recession because it is easier to meet the additional demand coming from higher government purchases.

Consider now what happens with a larger shock that sends the economy at the ZLB. In this setup nothing prevents the real wage to adjust downward so that after a negative preference shock sends the economy at the ZLB, a deflationary spiral breaks out. Likewise, when government spending increases while the economy is at the ZLB, the impact multiplier effect on inflation is close to one and the real wage registers a three-fold increase. Both of these features can be seen in Figure 15 (in the Appendix). In the next subsection, I show how the introduction of a downward rigid wage can mitigate these shortcomings.

### 4.2 Simulations with a Downward Rigid Wage

I first analyze a standard recession when only the DNWR constraint is binding.\(^{35}\) In Figure 5, I plot the results of the first experiment. I first concentrate on the responses of the economy without an increase in government spending.

Regarding GDP, the recession is larger in magnitude than the expansion. This reflects the fact that adjustment operates more through quantities in recessions. Since there is a fall in inflation in a recession, the real wage will be higher than the desired real wage and firms will cut down on vacancy posting. This exacerbates the fall in employment in a recession. In an expansion, an increase in inflation will play an opposite role so that firm’s profits are higher and they post more vacancies.

The flip side of this feature is that inflation reacts more in an expansion than in a recession. In the latter scenario, the high level of the real wage prevents marginal costs from falling too much, which dampens the fall in inflation. In an expansion, this effect is altogether absent. On this front, the predictions of the model line up well with the empirical evidence showing that recessions are usually more severe than booms are expansionary. The muted decline in inflation is also a desirable feature in light of the recent debate about the “missing deflation” puzzle.

I now turn to the effects of government spending in an expansion. This induces a higher demand for final goods, which is met by higher employment through increased vacancy posting. The increase in vacancies is however limited as a high labor market tightness makes it harder to recruit workers. In addition, the real wage increases and puts upward pressure on the real marginal cost. This generates inflation and calls for the Central Bank to increase its nominal rate. The

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\(^{35}\)For my baseline calibration, I need a bigger shock to make the ZLB constraint binding than for the DNWR one (see the policy rules in Figure 16 in the appendix). This means that I can disentangle the effects of the two constraints.
Figure 5: Recession/Expansions with and without Government Spending Shocks with downward rigid wage

Note: Both variables are expressed as percentage deviations from their steady state value.
resulting increase in the real interest rate depresses private consumption, which decreases labor demand. In the end, the multiplier effect is lower than 1 and rather small. More precisely, taking the difference on impact between an expansion with and without government spending, scaling it to the initial increase in government spending gives a GDP multiplier of 0.31 on impact. The reason why it is much smaller than the one obtained with the simple New Keynesian model is because employment is now a state variable that only adjusts partially in the short run.

With this in mind, I now analyze the effects of government spending in a typical recession. One can see that increasing government spending has a higher multiplier effect in a recession. Computing the latter yields a value of 0.51 on impact, i.e. 65% higher than in the expansion scenario. To make sure that the difference between the two does not depend too much on the fact that the recession is comparatively larger than the expansion, I augment the size of the shock that generates an expansion so that it is of the same magnitude. In this experiment, the difference between the multipliers is now of 67% in favor of the multiplier in a recession. Basically, as the expansion is stronger, the economy is over-heating and the adjustment occurs even more through prices rather than quantities.

The interpretation goes as follows. The same mechanisms that have been described for the model with flexible wage apply here. Additionally, since the DNWR constraint is binding on impact, more inflation leads to a lower real wage. In equilibrium, this dampens the response of inflation. As a result, while the multiplier effect on GDP is higher in a recession, the effect on inflation is actually lower. This can be seen clearly in Figure 6, where I again plot the difference between the path with and without an increase in government spending for each scenario.

4.3 A Liquidity Trap Scenario

I now study the effects of government spending when the preference shock is large enough to send the economy in a liquidity trap. In this case, both the DNWR and ZLB constraints are going to be binding. For the sake of comparison, I still plot the simulations during the boom period in Figure 7.

The preference shock that generates a deep recession sends the economy at the ZLB for 3 periods. I verify that the increase in government spending does not affect the duration of the liquidity trap so that the multiplier that I compute is not biased upwards. Without the increase in government spending, GDP troughs at $-3.1\%$. With the increase in government spending, the trough is at $-2.73\%$.

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36This requires a shock that is 18% higher.
37I could potentially engineer a deeper recession, but then labor market tightness becomes negative and approximations error will increase. For these reasons, I stick with this calibration.
Figure 6: Impulse Responses to a Government Spending Shock in Expansions/Recessions with downward rigid wage.

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
Figure 7: ZLB/Boom with and without Government Spending Shocks with downward rigid wage

Note: Both variables are expressed as percentage deviations from their steady state value.
Consistent with a large body of literature, if I compute the difference between those two numbers and scale it by the increase in government spending, I obtain a multiplier larger than 1. In this case, I obtain 1.25.

When the economy reaches the ZLB, the recession is a lot worse than the associated expansion. The recession is characterized by a rather small deflation that puts upwards pressure on the real wage, thereby limiting the deflationary spiral. Indeed, note that the reaction of inflation is rather symmetric between the boom and ZLB scenarios.

The fact that government spending has more effects in a liquidity trap is nothing new. But usually this rests on an inflationary spiral that does not seem present in the data. In this model however, because wages are downward rigid and firms can recruit more easily in a recession, inflation reacts slightly less with the increase in government spending compared to the expansion case. As a consequence, the evidence that a fiscal package did not generate a burst of inflation does not necessarily constitute evidence for its inefficiency.

Now that I have a framework that can yield both (i) a higher multiplier in a recession and (ii) an even bigger one at ZLB, I can address the following question: How much of the multiplier effect at ZLB has to do with the fact that the nominal interest rate cannot go below zero? To answer this question, I keep the same magnitude of the shock, but I simulate the economy without taking into account the ZLB. So the recession will have a binding wage constraint and (possibly) a negative net interest rate.

What I find is that indeed, the net nominal interest is negative and troughs at -6.8% in annualized terms. As a result, the recession is dampened with an GDP decline of only -2.1%. Still, higher government spending further mitigates the fall in GDP and the resulting multiplier effect is 0.8621. Since the multiplier effect at ZLB is 1.25, the difference with an expansion multiplier of 0.31 is equal to .94. This means that the dynamics of the model without the ZLB constraint explain roughly 59% of the multiplier effect. If I increase the size of the shock so that the trough in GDP is the same with and without the ZLB constraint, I get a multiplier effect of 1.31 without ZLB. This comes mostly from the part that the required shock is pretty large. In their paper, Miyamoto et al. (2015) find an impact multiplier at ZLB of 1.5. If I increase the size of the shock so that I match their estimate, I find that the dynamics of the model without the ZLB constraint still explain 54% of the multiplier effect.

In the Appendix, I consider a related framework in which wage rigidity comes from a credible bargaining setup as in Christiano et al. (2013), which in turn builds on Hall & Milgrom (2008). I show that in this case the multiplier is still higher at the ZLB, with inflation reacting comparatively less than in an expansion. Also, to show that search and matching frictions are essential for this, I solve a version of the model without recruiting costs $r = 0$ and with a separation rate of $s = 1$. In this case, I show that a downward rigid wage without search and matching frictions does not generate meaningful asymmetries.
A known shortcoming of the New Keynesian model is that it predicts a sizable deflation after a negative aggregate demand shock that sends the economy in a liquidity trap. There is ongoing research that tries to show why such a deflation did not materialize during the current Great Recession (See Gordon (2013), DelNegro et al. (2014) and Coibion & Gorodnichenko (2015)). To the extent that real wages will be higher with deflation in the model with downward nominal wage rigidity, it should dampen the fall in inflation by mitigating the fall in marginal cost. I find that it is indeed the case. I calibrate the preference shock so that it generates the same trough in GDP with and without downward wage rigidity. In both cases, the economy stays in a liquidity trap for 3 periods and GDP troughs at −3.1%. The trough in inflation in the model with flexible wages is −2.61% after such a shock. For a comparable recession, the model with downward wage rigidity predicts a maximum fall in inflation of −1.5%.

5 Conclusion

In this paper I have focused on how the setup of the labor market plays a role in the transmission mechanisms of government spending in a recession which is possibly big enough to generate a liquidity trap. I have developed a model that accounts well for the relative behavior of labor market variables after a government spending shock in recessions and expansions. I have also shown that when one takes into account the fact that a liquidity trap is always associated with an unemployment crisis, higher government spending can be efficient at stimulating GDP. This comes not mostly from the fact that government spending is inflationary, but also from the fact that recruiting additional workers is essentially costless in a severe recession.

All in all, government spending does not seem to be the right tool to generate inflation in a liquidity trap. As a consequence, the virtuous cycle on inflation and the real interest rate does not play an important role. The government spending multiplier is indeed unusually large in a liquidity trap, but for reasons that are different than the ones that have typically been pushed forward. In fact, a substantial part of the multiplier effects of government spending at the ZLB has nothing to do with the fact that monetary policy is unresponsive.


A Simple New Keynesian Model

The simple New Keynesian model that I use in section 2 is a special case of the one developed in Albertini et al. (2014), albeit with only non-productive government spending. It features Rotemberg-type quadratic price adjustment costs. The production function has constant returns to scale and the household has a standard utility function

\[ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi N_t^{1+\varphi}. \]

I only report the equilibrium conditions here and refer the interested reader to the aforementioned paper for more details.

A.1 Summary of the model

The model can be summarized by the following set of equations:

\[ N_t = C_t + G_t + N_t \frac{\phi}{2} (\Pi_t / \Pi)^2 \]  
\[ 1 = R_t \mathcal{E}^1_{t+1} \]  
\[ \epsilon \cdot MC_t = \epsilon - 1 + \phi (\Pi_t / \Pi - 1) (\Pi_t / \Pi) - \phi \mathcal{E}^2_{t+1} \]  
\[ MC_t = \chi N_t^{\varphi} C_t^{\sigma} \]  
\[ R_t = \max \left\{ 1, \frac{\Pi_t \Pi}{\beta} \left( \frac{\Pi_{t+1}}{\Pi_t} \right)^{\phi \Pi_t} \right\}. \]

where the expectation functions are given by:

\[ \mathcal{E}^1_{t+1} = \mathbb{E}_t \frac{\Lambda_t \Pi_{t+1}}{\Pi_{t+1}} \]  
\[ \mathcal{E}^2_{t+1} = \mathbb{E}_t \left\{ \Lambda_t \frac{Y_{t+1} \Pi_{t+1}}{Y_t} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \right\} \]

where

\[ \Lambda_t \Pi_{t+1} = \beta^\varphi \frac{C_t^{\frac{1+\varphi}{\sigma}}}{C_t}. \]

B Data

In what follows I describe the data series used for the STVAR estimation. I indicate when data starts later than 1947.
• The data for output is the Seasonally Adjusted Annual Rate Gross Domestic Product in Quarterly Billions of Chained 2005 Dollars from BEA.

• The data for government spending is nominal government spending (BEA table 3.1) divided by the GDP deflator.

• The data for taxes is nominal taxes net of transfers divided by the GDP Deflator.

• The data for the nominal wage is compensation per hour in the nonfarm business sector at quarterly frequency, seasonally adjusted. Available from 1951 onwards. Fred series COMPNFB. The real wage is nominal wage divided by the Producer Price Index for All Commodities, also from Fred database : series PPIACO. Data has been seasonally adjusted using the program remst.m from Weron (2006).

• To construct a measure from labor market tightness I need data on vacancies and the unemployment rate.
  – To construct a series for vacancies I use data from Foroni et al. (2015), which combines the Help Wanted Index up to 1994 and Barnichon & Figura (2010)’s data afterwards.
  – The unemployment rate for 16 and over is taken from BLS, series ID LNS14000000.
C Empirical Results

Figure 8: Effects of a Government Spending Shock. SVAR specification. Sample 1966:Q4-2008:Q2
Figure 9: Effects of a Government Spending Shock. SVAR specification. Sample 1966:Q4-1987:Q1

![Graphs showing the effects of a government spending shock on various economic indicators.](image)
Figure 10: Effects of a Government Spending Shock. SVAR specification. Sample 1987:Q3-2008:Q2
Figure 11: Effects of a Government Spending Shock. TVAR specification. Sample 1982:Q1-2008:Q4
Figure 12: Effects of a Government Spending Shock using Fisher-Peters excess return series
Figure 13: Effects of a Government Spending Shock using cyclical component of unemployment as threshold
Figure 14: Effects of a Government Spending Shock using labor market tightness as threshold

D Computational Details

D.1 The Bargaining Problem

The wage bargaining problem with the downward rigidity constraint can be represented by the following Lagrangian:

\[ \mathcal{L}_t \equiv (1 - \mu) \log(\tilde{V}_{N,t}) + \mu \log(V_{J,t}) + \lambda_i^{\text{pr}} (\mathcal{W}_t - \mathcal{W}_J) \]  

(24)

Taking the derivative with respect to \( \mathcal{W}_t \) and multiplying by \( \tilde{V}_{N,t} \cdot \frac{r}{q(\theta_t)} \), I get:

\[ (1 - \mu) \frac{r}{q(\theta_t)} = \left[ \mu - \lambda_i^{\text{pr}} \frac{r}{q(\theta_t)} \right] \tilde{V}_{N,t} \]

(25)

\[ \Leftrightarrow \tilde{V}_{N,t} = (1 - \mu) \frac{r}{\mu - \lambda_i^{\text{pr}} \frac{r}{q(\theta_t)}} \]
Using this equation alongside the household’s first order condition with respect to \( N_t \), I can thus express \( \tilde{V}_{N,t} \) as

\[
\tilde{V}_{N,t} = W_t - b - \chi \frac{N_t^q}{\lambda_t} + (1 - s) E_t \Lambda_{t,t+1} \tilde{V}_{N,t+1} \\
= W_t - b - \chi \frac{N_t^q}{\lambda_t} + (1 - s) E_t \Lambda_{t,t+1} (1 - f(\theta_{t+1})) (1 - \mu) \frac{r}{q(\theta_{t+1})} \frac{r}{\mu - \lambda_t^w q(\theta_{t+1})}
\]

(26)

Likewise, from the firm’s FOC with respect to \( N_t \), I can express \( \frac{r}{q(\theta_t)} \) as

\[
\frac{r}{q(\theta_t)} = MC_t - W_t + (1 - s) E_t \Lambda_{t,t+1} \frac{r}{q(\theta_{t+1})}
\]

(27)

Using equations (26) and (27) to substitute for \( \tilde{V}_{N,t} \) and \( \frac{r}{q(\theta_t)} \) in equation (25), after some algebra I get:

\[
W_t = \mu \left( b + \chi \frac{N_t^q}{\lambda_t} \right) + (1 - \mu) MC_t + (1 - s) (1 - \mu) E_t \Lambda_{t,t+1} \frac{r}{q(\theta_{t+1})} \\
+ \lambda_t^w \frac{r}{q(\theta_t)} \tilde{V}_{N,t} + (1 - s) (1 - \mu) E_t \Lambda_{t,t+1} (1 - f(\theta_{t+1})) \frac{r}{q(\theta_{t+1})} \left\{ \frac{\lambda_t^w}{1 - \mu q(\theta_{t+1})} \right\} \\
= \mu \left( b + \chi \frac{N_t^q}{\lambda_t} \right) + (1 - \mu) MC_t + (1 - s) (1 - \mu) E_t \Lambda_{t,t+1} r \cdot f(\theta_{t+1}) \\
+ \lambda_t^w \frac{r}{q(\theta_t)} \tilde{V}_{N,t} + (1 - s) (1 - \mu) E_t \Lambda_{t,t+1} (1 - f(\theta_{t+1})) \frac{r}{q(\theta_{t+1})} \left\{ \frac{\lambda_t^w}{1 - \mu q(\theta_{t+1})} \right\} \\
\equiv \mathcal{W}_t^\text{flex} + \lambda_t^w \frac{r}{q(\theta_t)} \tilde{V}_{N,t} \\
+(1 - s) (1 - \mu) E_t \Lambda_{t,t+1} (1 - f(\theta_{t+1})) \frac{r}{q(\theta_{t+1})} \left\{ \frac{\lambda_t^w}{1 - \mu q(\theta_{t+1})} \right\}
\]

Alternatively, if I regroup all \( t + 1 \) terms, I get:

\[
\mathcal{W}_t = \mu \left( b + \chi \frac{N_t^q}{\lambda_t} \right) + r \frac{\lambda_t^w}{q(\theta_t)} \tilde{V}_{N,t} \\
+ (1 - \mu) \left( MC_t + (1 - s) E_t \Lambda_{t,t+1} \left( 1 - \frac{\mu (1 - f(\theta_{t+1}))}{\mu - r \frac{\lambda_t^w}{q(\theta_{t+1})}} \right) \right).
\]
While the comparison with the flexible wage is less clear with this expression, it has only one expectation term to approximate, so I use this equation when computing the equilibrium.

D.2 Summary of the model

The model can be summarized by the following set of equations:

$$N_t = (1 - s)N_{t-1} + \left[1 - (1 - s)N_{t-1}\right]f(\theta_t)$$

$$N_t = C_t + G_t + \left[N_t - (1 - s)N_{t-1}\right] \frac{r}{q(\theta_t)} + N_t \frac{\phi}{2}(\Pi_t/\Pi)^2$$

$$1 = R_t \mathcal{E}_{t+1}^1$$

$$\epsilon \cdot MC_t = \epsilon - 1 + \phi(\Pi_t/\Pi - 1)(\Pi_t/\Pi) - \phi \mathcal{E}_{t+1}^2$$

$$MC_t = \mathcal{W}_t + \frac{r}{q(\theta_t)} - (1 - s)\mathcal{E}_{t+1}^3$$

$$R_t = \max \left\{ 1, \frac{\Pi}{\beta} \left( \frac{\Pi_t}{\Pi} \right)^\phi \right\}.$$

$$\mathcal{W}_t = \mu \left( b + \chi \frac{N_t}{C_t} \right) + r \frac{\lambda^p \tilde{V}_{N_t}}{q(\theta_t)} + (1 - \mu) \left( MC_t + (1 - s)\mathcal{E}_{t+1}^4 \right),$$

where the expectation functions are given by:

$$\mathcal{E}_{t+1}^1 = \mathbb{E}_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$$

$$\mathcal{E}_{t+1}^2 = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{Y_{t+1}}{\Pi} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \right\}$$

$$\mathcal{E}_{t+1}^3 = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{r}{q(\theta_{t+1})} \right\}$$

$$\mathcal{E}_{t+1}^4 = \mathbb{E}_t \Lambda_{t,t+1} \left( 1 - \frac{\mu(1 - f(\theta_{t+1}))}{\mu - r \frac{\lambda^p \tilde{V}_{N_{t+1}}}{q(\theta_{t+1})}} \right),$$

where

$$\Lambda_{t,t+1} = \beta \mathcal{E}_{t+1} \left[ \frac{C_{t+1}}{C_t} \right]^{-\sigma}.$$
$s_t \in S_t$ let $C_i : s_t \mapsto C_i(s_t)$ be the function that returns the Chebychev polynomial of order $i \in \mathbb{N}$ evaluated at the point $s_t$. I first build a linearly spaced grid of the state variables centered on the steady state value for each one. I then evaluate $C_i(\cdot)$ at each point of the grid. For a given polynomial degree $p$ (I choose $p = 3$ in the simulations) and for each grid point, I construct a modified grid which is composed of the products of $C_i(\cdot)$ evaluated at different grid points, with the restriction that the product should be of degree less or equal than $p$. For example, I take the first grid point for each state variable, I evaluate each one with $C_1(\cdot), \ldots, C_p(\cdot)$, keeping only the products of degree less or equal than $p$. This gives me the first line of the final grid. For the second line, I take the same points but the last one is the second grid point of the last state variable, and so and so on. I end up with a grid $\tilde{S} \in \mathbb{R}^{(p+1)s}$, with $s$ being the number of state variables.

The expectation functions are approximated by a simple function of the Chebychev polynomials of the state variables, namely:

$$E^i_{t+1} = \tilde{s}_t \cdot \Xi^i, \quad i = \{1, 2, 3\},$$

where $\Xi^i$ has no time subscript since it is a time-invariant "policy rule". Let $\Xi$ denote $[\Xi^1, \Xi^2, \Xi^3]$. This is the object on which I will iterate until convergence. The endogenous variables will also be expressed as a function of the Chebychev polynomials of the states. It is sufficient to approximate the policy rules for two of the endogenous variables: the labor market tightness $\theta_t$ and the inflation rate $\Pi_t$. Given expectations and the states variables, all the other endogenous variables can be computed. Let $\Omega$ be the set of coefficients that relates $\theta_t$ and $\Pi_t$ to the Chebychev polynomials of the state variables. The algorithm then works as follows:

1. Choose a value for the learning parameter $\zeta \in [0, 1]$ and the stopping criterion, $\epsilon$.
2. Start with an initial guess for $\Xi$, say $\Xi_0$. As a first guess, I evaluate the expectations functions at steady state.
3. For each point of the grid on state values, compute the value of the expectations. Given a first guess for $\Omega_0$, compute $\Omega$ using a Newton algorithm.
4. Using $\Omega$ and the law of motion of the state variables, reevaluate the expectations functions using a Gauss-Hermite quadrature.
5. Regress these new expectations on the grid of state variables, which gives $\Xi_1$.
6. Compute $\hat{\Xi}_1 = \zeta \Xi_1 + (1 - \zeta) \Xi_0$
7. If $\left\| \frac{\hat{\Xi}_1 - \Xi_0}{\Xi_0} \right\| < \epsilon$ then stop. Else return to step 2, using $\hat{\Xi}_1$ and the last solution for $\Omega$ as guesses.

---

*I actually compute four policy rules: one for each case given the two occasionally binding constraints.*
## E Calibration

<table>
<thead>
<tr>
<th>Steady-State Target</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual steady state inflation</td>
<td>( \pi )</td>
<td>0.02</td>
<td>Standard</td>
</tr>
<tr>
<td>Steady state job filling rate</td>
<td>( q )</td>
<td>0.7</td>
<td>Ravenna &amp; Walsh (2008)</td>
</tr>
<tr>
<td>Steady state unemployment</td>
<td>( u )</td>
<td>0.064</td>
<td>Michaillat (2014) (JOLTS 2001-2011)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
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<td>Match annual interest rate of 2.5%</td>
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<tr>
<td>Price adjustment</td>
<td>( \psi )</td>
<td>60</td>
<td>Match Calvo probability of 0.75</td>
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<tr>
<td>Matching efficiency</td>
<td>( m )</td>
<td>0.657</td>
<td>Steady State wage equation</td>
</tr>
<tr>
<td>Elast. of subst. between goods</td>
<td>( \epsilon )</td>
<td>6</td>
<td>Markup of 20%</td>
</tr>
<tr>
<td>Recruiting cost</td>
<td>( r )</td>
<td>0.0636</td>
<td>1% of steady state output</td>
</tr>
<tr>
<td>Government spending share</td>
<td>( g )</td>
<td>0.2</td>
<td>Average post WW II in USA</td>
</tr>
<tr>
<td>Response to inflation</td>
<td>( \phi_\pi )</td>
<td>1.5</td>
<td>Standard</td>
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<td>Matches/seekers elasticity</td>
<td>( \eta )</td>
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<td>Pissarides &amp; Petrongolo (2001)</td>
</tr>
<tr>
<td>Firm bargaining power</td>
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<td>0.5</td>
<td>Hosios (1990)</td>
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<td>Separation rate</td>
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<td>Krause et al. (2008)</td>
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<tr>
<td>Replacement rate</td>
<td>( b )</td>
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<td>Ravenna &amp; Walsh (2008)</td>
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<tr>
<td>Risk aversion coefficient</td>
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<td>Christiano et al. (2011)</td>
</tr>
<tr>
<td>Labor disutility</td>
<td>( \varphi )</td>
<td>1</td>
<td>Fernández-Villaverde et al. (2015)</td>
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F Simulations

Figure 15: ZLB/Boom with and without Government Spending Shocks with flexible wage. Both variables are scaled by their respective steady state value.
E1 Policy rules

Figure 16: Policy rules for the model with downward rigid nominal wages as a function of preference shock
G Robustness Analysis

In the previous subsection, I have highlighted the role of search and matching frictions and a downward rigid wage jointly. But it could well be that the assumption of a downward rigid wage—independently from search and matching frictions—is sufficient to get the same result. To really pinpoint the role of a downward rigid wage, I will study a simplified version of the model in which recruitment costs play no role and the level of employment is no longer a state variable. Formally, I assume that the vacancy posting cost parameter is equal to $r = 0$. I also assume that the separation rate is equal to $s = 1$. Both of these assumptions imply that the real wage is equal to the reservation wage of the wholesale firm. It also implies that the level of employment is entirely determined by the level of labor market tightness. I report in the appendix (see Figure 19) the response of a one standard shock to government spending when the economy is sent at the ZLB for 4 periods.
What stands out of Figures 18 and 19 is the fact that the effects of government spending are not that different in an expansion versus in a recession. Absent vacancy posting costs and employment duration within the firm, the incentives to post vacancies are roughly the same whether the economy is in a recession or not. This result is consistent with the analysis in Michaillat & Saez (2016). They show that a matching market without recruiting cost is isomorphic to a disequilibrium market. In this setup, the multiplier does not depend on the level of unemployment.\footnote{I thank an anonymous referee for suggesting this reference.} These responses do not accord well with the empirical evidence that I presented and therefore the simulations at the ZLB for this model should be taken with a grain of salt. It turns out that the results of these simulations are mainly driven by the assumption of downward wage rigidity and still generate a low reaction of inflation.

In addition, even though it seems to be borne out by a large body of empirical evidence, the assumption of downward rigid nominal wage has some shortcomings. First and foremost, it is imposed ad hoc and does not come from first principles. It is not sure then why the agents will come to choose this type of wage agreement instead of every other one for which the real wage stays in the bargaining set. Since the real wage plays an important role in determining real marginal costs, it could be that the particular assumptions on wage rigidity might play an important role.

Accordingly, I consider a model where the real wage is rigid not because of an ad-hoc constraint, but because of how firms and workers negotiate. Specifically, I consider the simple model developed in Christiano et al. (2013), which in turn builds on Hall & Milgrom (2008). The firms and workers negotiate following the alternating offer bargaining scheme developed in Rubinstein (1982). As in Hall & Milgrom (2008), the inertia of real wages comes from the fact that the costs of bargaining are relatively insensitive to business cycle fluctuations. There is no presumption in this setup about whether real wages are more rigid going downward or upward.

Because my focus is again on the non-linearity inherent to the matching setup between job seekers and vacancies, I solve the model using the PEA algorithm. I use the same calibration as Christiano et al. (2013), with one slight modification. To avoid having one more state variable for the dispersion of prices, I assume that firms face price adjustment costs as in Rotemberg (1982) instead of the Calvo (1983) framework employed in their paper. I set $\phi = 96$ for the price rigidity parameter, again to target a Calvo parameter of 0.8 with the value for the elasticity of substitution across goods taken by Christiano et al. (2013). I show the policy rules for the main variables of interest in Figure 20 (in the appendix) as a function of the preference shock.

The main takeaway from Figure 20 is the fact that the policy rules for labor market tightness, real marginal cost and thus, inflation are non-linear. Therefore,
as in the model with downward nominal wage rigidity, whether the increase in government spending will occur in a boom or in a recession it will have different effects on those variables. Another feature to be noted is that the kink coming from the ZLB does not seem to impact their respective shape. As a result, the effect of government spending on those variables at the ZLB will be roughly the same as if it were only a big recession. It is clear in Figure 21, where I show the multiplier effects of government spending in a recession (after a negative aggregate demand shock that is not big enough to send the economy to the ZLB) and in a boom (following a preference shock of opposite sign that generates higher output, employment, inflation etc.).

Labor market tightness and real marginal costs react much less in a recession, which generates a lower rise in inflation. Since the Taylor principle is active, lower inflation calls for a lower increase in the nominal rate and thus consumption and GDP rise more than in an expansion. This increase is modest, to say the least, especially compared to the one obtained with my baseline model. This comes from the fact that the bargaining setup in Christiano et al. (2013) makes real marginal costs more sluggish. As a result, inflation reacts very little to a government spending shock and the crowding out effect on private consumption is minimal. In the expansion case, this generates an impact multiplier on GDP of approximately 0.8. Since private consumption necessarily decreases after a government spending shock outside the ZLB, there is not much room for a substantially higher multiplier in a recession.

In Figure 22, I show the same picture but this time the recessionary preference shock is big enough to send the economy at the ZLB for one period. In this case, government spending has even less effect on labor market tightness, real marginal cost and inflation. Thus the crowding out effect on private consumption is nil and the multiplier effect on GDP is just equal to 1, i.e. the increase in government spending. I have tried with more flexible prices so that the economy will stay longer at the ZLB (up to 6 quarters for $\phi = 40$). Even in this case, the multiplier effect of government spending at the ZLB does not exceed one. What remains true also is the fact that a government spending shock in a deep recession (and a fortiori in a liquidity trap) generates little if no inflation.
G.1 Downward rigid wages without labor market frictions

Figure 18: Spending multiplier in the model with downward rigid wages without labor market frictions in a recession

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
Figure 19: Spending multiplier in the model with downward rigid wages without labor market frictions in a liquidity trap

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
G.2 Model with Credible Bargaining

Figure 20: Policy Rules for the Model with Credible Bargaining.
Figure 21: Multipliers effects of Government Spending in Boom and Recession, Model with Credible Bargaining.

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.
Figure 22: Multipliers effects of Government Spending in Boom and ZLB, Model with Credible Bargaining.

Notes: The black line represents the difference between the path of GDP/inflation with i) only a positive preference shock and ii) a preference and government spending shock. This difference is then scaled by the initial increase in government spending, so that the response is the multiplier effect of government spending. The red dashed line represents the same but with a negative preference shock.