Booms and Busts with dispersed information

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June 2014

[1] I would like to thank Daniel Andrei, Georges-Marios Angeletos, Philippe Bacchetta, Isabella Blengini, Fabrice Collard, Luisa Lambertini, Francesco Lippi, Baptiste Massenot, Aude Pommeret, seminar participants at the University of Lausanne, EPFL, the University of Cergy-Pontoise, the University of Zurich and the University of Bern, and participants to the CIRPÉE-DEEP-TSE Macroeconomics Workshop in Toulouse for helpful comments. Helena Texeira provided useful research assistance for this project.

Abstract

This paper shows that dispersed information generates booms and busts in economic activity. Boom-and-bust dynamics appear when firms are initially over-optimistic about demand due to a noisy news. Consequently, they overproduce, which generates a boom. This depresses their mark-ups, which, to firms, signals low demand and overturns their expectations, generating a bust. This emphasizes a novel role for imperfect common knowledge: dispersed information makes firms ignorant about their competitors' actions, which makes them confuse high noise-driven supply with low fundamental demand. Boom-and-bust episodes are more dramatic when the aggregate noise shocks are more unlikely and when the degree of strategic substitutability in quantity-setting is stronger.

Keywords: Imperfect Common Knowledge, Expectations, Recessions.
JEL Classification Numbers: E32, D83, D52.

Boom-and-bust episodes are a recurring feature in economic history. Boom periods where new projects employ large resources are followed by downturns where few resources are used. Famous recent episodes include the 2001 dotcom bubble or the recent housing boom and the subsequent subprime crisis. Before that, the Dutch Tulip Mania in the 17th century and the boom in railroad construction that preceded the recession of 1873 in the US are well-known historical examples. The fact that economic activity can turn from heedless optimism to dire pessimism is also a cornerstone in economic theory. Keynes (1936) argued that "animal spirits" play a fundamental role in the economy, while Pigou (1927) advanced the idea that business cycles may be the consequence of "waves of optimism and pessimism".

Few models produce recessions that feed into initial expansions. None, to our knowledge, generates booms and busts that are due to reversals of expectations from excessive optimism to excessive pessimism. We show in this paper that such successions of optimism and pessimism waves may arise as the result of imperfect common knowledge.

Our model focuses on the difficulties faced by firms to correctly forecast the state of demand when deciding on their supply level. We illustrate the mechanism in a standard two-period Dixit-Stiglitz model with imperfect competition. Busts originate in the preceding booms as the result of an initial over-optimistic news about demand. When a positive aggregate noise shock occurs, i.e. when the news is on average *excessively optimistic* about the state of demand, firms over-produce, which generates a boom in the first period. This however also depresses prices, hence lowers profits and mark-ups, which, to firms, signals low demand. This makes their new expectations *excessively pessimistic*, generating a bust in the second period. Importantly, expectations do not simply revert to the true value of demand but *undershoot* it. This is because firms confuse high supply due to an aggregate noise shock with low fundamental demand. Dispersed information plays a crucial and novel role here: it makes firms ignorant about their competitors' actions, and thus about aggregate supply, generating the confusion.

In the model, the role of firms' performances in shaping their expectations and generating downturns is key. This approach is supported by panel (a) of Figure 1 which represent firms' unit profits and mark-ups in the US, around the NBER recessions.It appears that profits and mark-ups peaked several quarters before the onset of many recessions. In particular, the 2001 and 2008 recessions are both preceded by a gradual decrease in profits and mark-ups. Our setup is consistent with these facts as firms form their expectations based on their profits (equivalently, their markups), which contributes to turn a boom into a bust. The recession of 1990 shows the same features. The picture is less clear for the recessions of the 1970's and the 1980's, which were induced either by the oil price or by monetary policy. The recessions of 1953, 1958 and 1960 are also preceded by a gradual decrease in profits and mark-ups. Noticeably, several of these episodes were characterized by excessive optimism about demand.[1] Panel (b) of Figure 1 represents real profits in the past quarter along with the expectations, taken from the survey of professional forecasters, of current and future profits. Whatever the horizon, expected profits track closely the past realized profits. Panel (c) shows unit profits along with the "Anxious index" (the perceived probability of a fall in GDP in the next quarter). Unit profits are strongly negatively correlated with the anxious index.[2] This is suggestive that expectations are a consistent channel through which reversals in profits lead to reversals in economic activity.

To explain drops in expectations after an optimism wave, it is crucial that both depressed demand and excessive optimism on the firms' part generate low prices. This stems first from strategic substitutability. Indeed, strategic substitutability between firms implies that an increase in supply decreases the mark-up, while an increase in demand increases it. Competition, concave utility, fixed inputs are some realistic

[1]Between 2000 and 2006, the number of vacant houses all year round rose by 20% while construction spending increased by 45%, feeding the real estate bubble that lead to the subprime crisis. During the dotcom bubble, many companies such as Pets.com, Webvan and Boo.com went bankrupt because they failed to build a customer base. Similarly, the 1960 recession coincided with a drop in domestic car demand, which shifted to foreign cars; the recession of 1958 can also be explained by the reduction in foreign demand, due to a world-wide recession. A last supporting example is the recession of 1953. According to the Council of Economic Advisors (1954): "Production and sales gradually fell out of balance in the early months of 1953. [...] The reason was partly that, while demand was high, business firms had apparently expected it to be higher still."

[2]The correlation is -0.56 and significant at 1%. As a comparison, the correlation with GDP is much lower: -0.14 and significant at 10% only.

sources of strategic substitutability, and we show that a standard parametrization implies a high degree of strategic substitutability.[3] Second, the fundamental demand shock generates an increase in demand whereas the noise shock generates an increase in supply. We show that noisy news on technology do not generate boom-and-bust dynamics because both the noise and the fundamental increase supply and affect firms' prices in the same way, thus generating no confusion.

This approach provides several insights. First, the degree of strategic substituability between firms determines the severity of busts following booms, as strategic substitutability worsens the signals that result from over-production. This is consistent with the industry evidence in Hoberg and Phillips (2010). They show that boom and bust dynamics are more likely to arise in competitive industries. Second, the less frequent noise shocks are, the more severe are the boom-bust cycles. This is because firms believe more easily that negative signals arise from actual low demand when noise shock are less likely. Using data from the Survey of Professional Forecasters to identify the volatility of noise shocks relative to fundamental shocks, we find that busts are substantial as a result of the low volatility of noise shocks. Third, we show that temporary aggregate demand shocks which firms mistakenly interpret as a permanent shock can play the role of the initial aggregate noise shock. In that case, the dynamics start with an increase in credit, which is consistent with several boom-and-bust episodes. Finally, we show that our mechanism is robust to adding more dynamics and that boom and busts can last several periods, inasmuch as there are lags in information processing.

Close to our approach, the news shocks literature relates optimism and pessimism waves to aggregate signals about current or future productivity.[4] Depressions nevertheless do not breed into past exuberance. Waves of optimism fade out progressively as agents learn about the true state. They do not generate excessive pessimism.

[3]This might be surprising given that the New-Keynesian literature emphasizes the role of strategic complementarities among firms. However, as emphasized by Angeletos and La'O (2009), parameters that yield strategic complementarities in price-setting typically generate strategic substitutabilities in quantity-setting.

[4]See, among others: Beaudry and Portier (2006), Jaimovich and Rebelo (2009), Blanchard et al. (2009) and Lorenzoni (2009).

Indeed, this literature usually focuses on the boom, not on the bust. In particular, the main challenge has been to explain how positive news about the future could generate a boom in a full-fledged DSGE model.[5] Notable exceptions are Beaudry and Portier (2004), Christiano et al. (2008) and Lambertini et al. (2011), where busts arise due to the cumulated economic imbalances when a positive news is revealed to be false. In our setup, on the opposite, busts are driven by the agents' expectational errors.

This paper relates also to imperfect information models with dispersed information, which date back to Lucas (1972) and Frydman and Phelps (1984).[6] In particular, Hellwig and Venkateswaran (2009), Lorenzoni (2009), Angeletos and La'O (2009), Graham and Wright (2010), Gaballo (2013) and Amador and Weill (2012) study models where agents decide quantities and receive market signals.[7] In these models, as in ours, a market imperfection prevents agents from learning the relevant information from the market. Here, we assume that the labor market opens before the goods market and that transactions take place in nominal terms. Firms then observe only the nominal wage, which only partially reveals the fundamental, when deciding their labor hiring (and hence their production), and observe their nominal price (and hence their mark-up) only at the end of period. Moreover, in these models, as in our approach, the response of output to fundamental shocks is muted, precisely because information is only partially revealed by market prices. Noise shocks however generate short-lived booms that fade out in time but no boom-and-bust dynamics. This is essentially because endogenous signals are mostly contemporaneous, which does not leave room for booms and busts to arise. As we show in our dynamic extension, lags in - endogenous - information are essential for lengthy booms-and-busts

[5]Burnside et al. (2011) and Adam and Marcet (2011) focus more specifically on the housing market and on the build-up of optimism waves.

[6]See also, among others, Woodford (2001), Sims (2003), Shin and Amato (2003), Ball et al. (2005), Bacchetta and Wincoop (2006), Lorenzoni (2009), Amador and Weill (2012), Mackowiak and Wiederholt (2009), Hellwig and Venkateswaran (2009), Angeletos and La'O (2009) and Graham and Wright (2010). This literature is surveyed in Hellwig (2006) and Lorenzoni (2011).

[7]Townsend (1983), Sargent (1991) and Pearlman and Sargent (2005) are earlier contributions. See also Nimark (2011) and Rondina and Walker (2012) for recent methodological advances on the topic.

to appear.

Section 1 presents the set up, a standard Dixit-Stiglitz monetary model with imperfect competition. To convey the intuition of the mechanism, we first present a simplified version of the model in Section 2 where transactions take place in labor terms and not in monetary terms. As a result, the wage, which is normalized to one, does not convey any information at all. The model is then simpler to solve but the fundamental mechanism is present. Section 3 then presents the full monetary version, where the nominal wage gives additional but partial information about the fundamental. In Section 4, we examine some extensions of the model. Section 5 concludes.

1 A two-period Dixit-Stiglitz model

We consider a two-period general equilibrium monetary model with imperfect competition à la Dixit-Stiglitz. There is one representative household who consumes a continuum of differentiated goods indexed by $i \in [0,1]$ and supplies labor on a competitive market. Each good is produced by a monopolistic firm using labor. Aggregate demand is affected by a preference shock.

1.1 Preferences and technology

There is a representative household with the following utility function:

$$U = U_1 + \beta U_2 \tag{1}$$

where $0 < \beta < 1$ is the discount factor and U_t is period-t utility:

$$U_t = \Psi \frac{Q_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\eta}}{(1+\eta)} \tag{2}$$

$Q = \left(\int_0^1 (Q_i)^{1-\rho} di \right)^{\frac{1}{1-\rho}}$ is the consumption basket composed of the differentiated goods Q_i, $i \in [0,1]$, and L is labor. $\rho \in (0,1)$ is the inverse of the elasticity of substitution between goods. $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution. $\eta > 0$ is the inverse of the Frisch elasticity of labor supply. Ψ determines the preference of the household for consumption relative to leisure.

Money is the numéraire. The consumer maximizes her utility under the following budget constraint, expressed in nominal terms:

$$\int_0^1 P_{it}Q_{it}di + M_t + B_t = W_tL_t + \int_0^1 \Pi_{it}di + M_{t-1} + r_{t-1}B_{t-1} + T_t \tag{3}$$

where P_{it} is the nominal price of good i, T_t are the nominal transfers from the government, M_t are money holdings, B_t are bond holdings, r_{t-1} is the nominal return on bond holdings, W_tL_t is the nominal labor income and Π_{it} are the nominal profits distributed to the household by firm i.

Money is created by the government and supplied to households through transfers T, following $M_t - M_{t-1} = T_t$. Bonds are in zero supply, so $B_t = 0$ in equilibrium. The only role played by bonds in this economy is to make money a dominated asset.

Finally, the household faces a cash-in-advance constraint, $\int_0^1 P_{it}Q_{it}di \leq M_{t-1}+T_t$. Because money yields no interest, this constraint holds with equality. Solving for the price index and combining with the government budget constraint, we obtain the quantity equation:

$$P_tQ_t = M_t \tag{4}$$

where $P = \left(\int_0^1 (P_i)^{\frac{-(1-\rho)}{\rho}} di\right)^{\frac{-\rho}{1-\rho}}$ is the general price index.

There is a $[0,1]$ continuum of firms who produce differentiated goods. The production function of each firm $i \in [0,1]$ involves labor with a constant return to scale technology:

$$Q_{it} = AL_{it} \tag{5}$$

where A is the level of productivity. Firm i's profits are therefore:

$$\Pi_{it} = P_{it}Q_{it} - W_tQ_{it}/A \tag{6}$$

Firm i's unit profit is therefore $P_{it} - W_t/A$ and its mark-up is AP_{it}/W_t.

1.2 Shocks, timing and information

At the beginning of period 1, the economy is hit by a shock on the preference parameter Ψ. This represents a permanent "demand shock" for the differentiated goods.We assume that $\psi = \log(\Psi)$ follows a normal distribution with mean zero and standard error σ_ψ. We assume that ψ is directly observed by households, but not by firms. The dynamics of the model is then determined by the inability of firms to correctly forecast ψ.

Similarly, firms are hit by a permanent shock on A, the level of productivity. However, as firms observe their level of productivity, productivity shocks are not subject to imperfect information issues in our model. It could still be possible though that productivity is not perfectly measured by managers within the firm, making informational issues relevant for productivity.[8] We consider this case in an extension of the model. For now, as productivity does not play any role, we normalize productivity A to 1.

The money supply is set by the government to $M_t = \bar{M}\exp(m_t)$, where m_t is a monetary shock. m_t follows a normal distribution with mean zero and standard error σ_m. Since we assume that the household receives money transfers in the beginning of period, so she observes m_t directly but the firm does not. Monetary shocks do not play a fundamental role in the model. They merely make nominal wages imperfect signals of the fundamental shock ψ.[9]

At this stage, we can define Ω_{it}, the set of information available to firm i when making its time-t production decision. At the beginning of period 1, firm i receives an exogenous news about ψ that incorporates both an aggregate and an idiosyncratic error:

$$\psi_i = \psi + \theta + \lambda_i \tag{7}$$

where θ and λ_i are both normal with mean zero and respective standard errors σ_θ and σ_λ. θ is an aggregate noise shock, while λ_i is an idiosyncratic noise shock that cancels

[8]For example, think about estimating the marginal cost as opposed to the average cost of production.

[9]Monetary shocks play a similar role in Amador and Weill (2012), where they are called "velocity" shocks.

out at the aggregate level: $\int_0^1 \lambda_i di = 0$. In section 4, we lay down an extension of the model where this initial news is an endogenous signal and θ and λ_i are temporary demand shocks. We assume that, besides ψ and m_t, the household observes θ and λ_i.[10]

The other signals observed by firms when making their production decisions depend on the market assumptions. We consider an environment where prices are fully flexible, but where markets are incomplete. First, each period, the labor market opens before the goods market. Second, transactions are made in terms of money and wages are not contingent. As a result, labor hirings and nominal wages are determined first, before firms can observe nominal prices. This has two important consequences. On the one hand, it makes quantities predetermined with regards to nominal prices. In other words, quantities are contingent on the news ψ_i and on the nominal wage W_t, but not on the relevant variable for their output decisions, which is the mark-up P_{it}/W_t. On the other hand, nominal prices incorporate new information that the firms can use when setting their next period supply.

Therefore, the information set of firm i at the beginning of period 1 is $\Omega_{i1} = \{\psi_i, W_1\}$. At the beginning of period 2, firms have observed the price of their good during period 1, so $\Omega_{i2} = \{\psi_i, W_1, P_{i1}, W_2\}$.

Importantly, we assume that the aggregate supply is not part of their information set. The idea behind this restrictive information structure is that firms pay attention to their local interactions and limited attention to public releases of aggregate information. Firms do collect public information (the nominal wage for example), but only if they are confronted to this information during their economic interactions. Aggregate supply is not part of their information set because they trade an individual good.[11]

In order to save notations, we denote by $E_{it}(y)$ the expected value of variable y conditional on Ω_t^i.

[10] This is without loss of generality given that the household observes all quantities and prices in all markets, and therefore can infer θ and λ_i using her information on ψ and m_t.

[11] We could add noisy aggregate quantities to the information sets to represent imperfect attention to aggregate supply, but we prefer to represent imperfect information in a parsimonious way by assuming that agents simply do not observe aggregate quantities.

1.3 Equilibrium

Before characterizing the equilibrium, we derive the household's and firms' decisions. the household sets its demand for goods, money and abor supply to maximize her utility (1) subject to her budget constraint (3) and the cash-in-advance constraint (4), given full information about the shocks hitting the economy and given the nominal prices and wages. Each firm i sets its supply Q_{it} monopolistically to maximize its profits (6) subject to the demand schedule for good i and given its information set Ω_{it}.

1.3.1 Household's decisions

The consumer's maximization program yields the following demand for each variety:

$$Q_{it} = Q_t \left(\frac{P_{it}}{P_t} \right)^{\frac{-1}{\rho}}$$

In logarithmic terms, this equation writes:

$$p_i - p = \rho \left[q - q_i \right] \tag{8}$$

where lower-case letters denote the log value of the variable and where time subscripts are dropped. Everything else equal, aggregate income increases the demand for good i, which increases p_i, and the more so as the elasticity of substitution between goods $1/\rho$ is low (ρ is large).

The consumer's maximization program yields the following demand for goods:

$$Q^{\gamma} L^{\eta} = \frac{\Psi W}{P}$$

In logs, and after using $Q = L$, this yields:

$$w - p = \sigma q - \psi \tag{9}$$

where $\sigma = \gamma + \eta$. $1/\sigma$ is the macro elasticity of labor supply to the real wage. When

σ is high, this elasticity is low and the wage reacts strongly to changes in supply q.

The money market clears, so $P_t Q_t = \bar{M}\exp(m_t)$. In logs, this gives:

$$p + q = m \tag{10}$$

The constant term is neglected for simplicity.

1.3.2 Optimal supply by firms

Optimal supply by firm i must be such that prices satisfy:

$$E_i\left(\frac{P_{it}}{W_t}\right) = \frac{1}{(1-\rho)}$$

This simply means that the mark-up P_{it}/W_t must be equal to $1/(1-\rho)$ in expectations. Since shocks are log-normal, this equation can be written in logs:

$$E_i(p_i) - w = 0$$

where the constant term has been discarded. It is useful to define the normalized supply $\hat{q}_i = \sigma q_i$. By using the individual and aggregate demand equations (8) and (9), the optimal - normalized - individual supply can be written as a function of expected aggregate supply and the expected fundamental shock:

$$\hat{q}_i = \tilde{\sigma} E_i(\psi) - (\tilde{\sigma} - 1)\, E_i(\hat{q}) \tag{11}$$

where $\tilde{\sigma} = \sigma/\rho$. In order to decide its optimal supply $\hat{q}_i$, firm i has two variables to infer: the fundamental shock ψ, but also the aggregate supply $\hat{q}$. Indeed, firms exert an externality on each other due to their interaction on the labor market and to competition. Aggregate supply has two opposite effects on individual profits. First, it increases the real wage $w - p$, and the more so as σ is large, that is as the macro elasticity of labor supply is low. Second, it increases the real price $p_i - p$ of good i, but the less so as the elasticity of substitution $1/\rho$ is large. Therefore, profits are adversely affected by aggregate supply when both σ and $1/\rho$ are large. For $\tilde{\sigma} < 1$, the positive effect through the individual price counteracts the negative effect through the real wage and quantity-setting features strategic complementarity. For $\tilde{\sigma} > 1$, the opposite holds so quantity-setting features strategic substitutability.

In the remainder of the paper, we make the assumption that quantity-setting features strategic substitutability:

Assumption 1 (Strategic substitutability) $\tilde{\sigma} > 1$.

As we will see later, this assumption is strongly satisfied with a standard parametrization.

1.4 Anatomy of booms

Equilibrium prices, quantities and wages in period 1 and 2 are characterized by Equations (8)-(11) given the realization of shocks ψ, θ, $(\lambda_i)_{i\in[0,1]}$, m_1 and m_2, with $q = \int_0^1 q_i di$ and $p = \int_0^1 p_i di$.

In order to characterize the anatomy of booms, we make the following definition:

Definition 1 (Boom patterns) *Consider q_1 and q_2, the respective values of output in period 1 and 2. We define the following boom patterns for q:*

- *Long-lived boom:* $0 < q_1 \leq q_2$.
- *Short-lived boom:* $0 \leq q_2 < q_1$.
- *Boom-and-bust:* $q_1 > 0$ *and* $q_2 < 0$.

In a long-lived boom, production increases between period 1 and period 2, contrary to a short-lived boom and a boom-and-bust, where production increases first and then decreases. However, in a short-lived boom, production does not decrease as much as it has increased in the first period, whereas it does in a boom-and-bust episode. Therefore, a boom-and-bust episode is not simply characterized by a decrease in output following an increase in output. Output has to be lower than its initial value.[12]

1.5 Perfect information / perfect markets outcome

Before solving the model with imperfect information, consider the perfect information outcome. If firms were all able to observe ψ directly, then they would set:

$$\hat{q}_{it} = \hat{q}_t = \psi \tag{12}$$

and the equilibrium mark-up would satisfy:

$$p_{it} - w_t = 0 \tag{13}$$

Note that the perfect information outcome would also arise if markets were complete, that is if firms were able to specify wages in terms of their individual good. Indeed, the wage in terms of good i $w - p_i$ is equal to minus the mark-up $p_i - w$. Hence, for firm i it is equivalent to set the wage in terms of good i and the mark-up. The firm can then satisfy its optimality condition $p_i - w = 0$, and therefore $\hat{q}_i = \psi$. In practice, firms adjust their labor demand until the real wage corresponds to the desired mark-up. In the model, wages reveal the relevant information to firms only partially because markets are incomplete, as wages are specified in nominal terms.

2 A useful simplification

The main mechanism of the model comes from the fact that the firm cannot observe the mark-up $p_i - w$, which is the relevant price for deciding its production level. It observes it partly through the nominal wage w, but the nominal wage is itself affected by nominal shocks, so it is only a noisy signal of the mark-up.

[12] Our definition of booms and busts is in a sense stronger than in some other papers. For example, in Angeletos and La'O (2009) and Christiano et al. (2008), "boom-and-bust" cycles correspond to our short-lived booms. Our definition is closer to the "Pigou cycles" in Beaudry and Portier (2004).

In this section, we consider a simplified version of the model where transactions are specified in terms of labor, not of money. The labor market thus does not convey any information to firms, since the wage is equal to one. This has implications on the information set of firms since firms do not even observe w, the wage in terms of money. In this case, we have a simpler problem where the wage does not convey any information but where quantities are still determined ahead of the mark-up, which is at the core of the model's mechanism. Now quantity setting must satisfy $E_i(p_i - w) = 0$ and Equation (11) is still valid. The only difference with the full monetary model is that the information sets are now $\Omega_{i1} = \{\psi_i\}$ and $\Omega_{i2} = \{\psi_i, p_{i1} - w_1\}$. This will help us grasp the intuition before turning to the full-fledge model.

2.1 First period production

In the first period, firms' aggregate supply under-reacts to the fundamental demand shock and over-reacts to the aggregate noise shock, which is standard under imperfect information. As a result, firm's mark-up, which is observed by firms at the end of period, is positively affected by the fundamental shock and negatively by the noise shock.

Indeed, as firms receive the news $\psi_i = \psi + \theta + \lambda_i$ at the beginning of period 1, they extract information from this news according to the following standard formula:

$$E_{i1}(\psi) = k_\psi \psi_i = k_\psi(\psi + \theta + \lambda_i) \tag{14}$$

where k_ψ is the standard bayesian weight $k_\psi = \sigma_\psi^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$.

On the other hand, firm i's supply follows (11). In order to derive supply as a function of the shocks, we use the method of undetermined coefficients. We establish the following (see proof in the Appendix):

$$\hat{q}_{i1} = K_\psi \psi_i \tag{15}$$

where $K_\psi = \tilde{\sigma}\sigma_\psi^2/[\tilde{\sigma}(\sigma_\psi^2 + \sigma_\theta^2) + \sigma_\lambda^2]$. We have $0 < K_\psi < 1$, and, under Assumption

1, $K_\psi > k_\psi$. At the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = K_\psi(\psi + \theta) \tag{16}$$

Since $K_\psi < 1$, the aggregate supply under imperfect information, as compared with the equilibrium supply under perfect information (12), reacts less to the fundamental shock ψ, because information is noisy. On the opposite, aggregate supply over-reacts to the aggregate noise shock θ, because firms cannot distinguish it from the fundamental.

Moreover, as there is strategic substitutability in the economy, we have $K_\psi > k_\psi$, which means that agents over-react to their private signal ψ_i. Each firm i expects that the other firms combine their private signal with zero, the unconditional expectation of ψ, which is public information, to set their individual supply. Because of strategic substitutability, firms under-react to any public information, as public information is common to all firms and thus generates negative externalities. A contrario, firms over-react to any private information because private information is exempt from negative externalities. A positive noise shock θ therefore generates a boom that is due both to imperfect information per se but also to the over-reaction to private signals. This implication of strategic interactions is in line with the literature on imperfect common knowledge and is not specific to our paper (see for example Angeletos and Pavan (2007)). Here, as we will show soon, imperfect common knowledge plays an additional role, which is to produce confusion between demand and supply. This effect plays through the new signal gathered by firms at the end of period 1, their mark-up.

2.2 New signal

The new signal received by firms is their mark-up:

$$p_{i1} - w_1 = \psi - \frac{\tilde{\sigma} - 1}{\tilde{\sigma}}\hat{q}_1 - \frac{1}{\tilde{\sigma}}\hat{q}_{i1} \tag{17}$$

This equation comes from the combination of individual demand (8) and aggregate demand (9). The mark-up is affected directly by the fundamental shock ψ since a demand shock lowers the real wage. It is also affected by the individual and aggregate supply $\hat{q}_{i1}$ and $\hat{q}_1$. The individual supply decreases the mark-up because it decreases the relative price of good i. Importantly, aggregate supply has a negative effect on the mark-up under Assumption 1.

The firm can use $p_{i1} - w_1$ to extract information on ψ by combining it with its other signal ψ_i. The firm knows individual supply $\hat{q}_{i1} = K_\psi \psi_i$, but ignores $\hat{q}_1$ because of dispersed information. Therefore, it can "filter" the mark-up from the influence of $\hat{q}_{i1}$ but not from the influence of $\hat{q}_1$. The "filtered" mark-up writes as follows:

$$p_{i1} - w_1 + \frac{1}{\tilde{\sigma}}\hat{q}_{i1} = \psi - \frac{\tilde{\sigma} - 1}{\tilde{\sigma}} q_1 = \left(1 - \frac{\tilde{\sigma} - 1}{\tilde{\sigma}} K_\psi\right)\psi - \frac{\tilde{\sigma} - 1}{\tilde{\sigma}} K_\psi \theta$$

Note that the fundamental shock ψ has a positive effect on the filtered mark-up. Indeed, as $K_\psi < 1$, aggregate supply does not fully respond to the demand shock, so aggregate demand is in excess of supply, which stimulates the mark-up. On the opposite, the noise shock θ has a negative effect on the filtered mark-up under Assumption 1. In this case, aggregate supply is in excess of demand, which depresses the mark-up. Assumption 1 ensures that supply does not generate its own demand through a "market potential" effect. Therefore, as a result of strategic substitutability, a positive shock on θ makes the filtered mark-up a *negative* signal of ψ.

Dispersed information is crucial here. If firms received the same information, then they would be able to infer q_1 even without observing it directly, because they would be able to infer what the other firms do. They could then filter their mark-up from the influence of others' supply and infer ψ. In short, even if firms observed low mark-ups following a positive noise shock $\theta > 0$, they would be able to put these mark-ups in perspective with the high aggregate supply $\hat{q}_1$. As a result, low mark-ups would not be perceived as a negative signal on ψ, but simply as the result of high supply.[13]

[13]Formally, if information was common ($\lambda_i = 0$), it would be straightforward to infer ψ by combining $p_{i1} - w_1$ and $\hat{q}_1 = K_\psi \psi_i$. Of course this result is trivial since there are as many shocks

This yields the following lemma:

Lemma 1 *The information set available at the beginning of period 2* $\Omega_{i2} = \{\psi_i, p_{i1} - w_1\}$ *is equivalent to two independent signals of* ψ*, a public signal* s *and a private signal* x_i*, defined as follows:*

$$s = \psi - \omega_\theta \theta$$

$$x_i = \psi + \omega_\lambda \lambda_i$$

with $\omega_\theta = (\tilde{\sigma} - 1)K_\psi / [\tilde{\sigma} - (\tilde{\sigma} - 1)K_\psi]$ *and* $\omega_\lambda = \omega_\theta/(1 + \omega_\theta)$*. Under Assumption 1,* $\omega_\theta > 0$*. Besides,* ω_θ *is increasing in* $\tilde{\sigma}$*.*

Proof. s is obtained simply by normalizing the filtered mark-up. x_i is obtained by combining s with ψ_i. As x_i and s are independent linear combinations of ψ_i and $p_{1i} - w_1$, the information set $\{x_i, s\}$ is equivalent to $\{\psi_i, p_{1i} - w_1\}$. ■

As suggested, Assumption 1 implies that θ generates a negative signal on ψ, as $\omega_\theta > 0$. Since the degree of strategic substitutability makes the mark-up react more negatively to aggregate supply, s reacts more negatively to θ when $\tilde{\sigma}$ is higher.

2.3 Second period

Using the above discussion, we characterize the following patterns:

Proposition 1 (Boom-busts - Simple model) *Following a positive fundamental shock* ψ*, output experiences a long-lived boom. Under Assumption 1, following a positive noise shock* θ*, output experiences a boom-and-bust.*

Consider first expectations. As firms receive two independent signals of ψ, solving for $E_{i2}(\psi)$ is straightforward (see proof in the Appendix):

$$E_{i2}(\psi) = f_x x_i + f_s s = (f_x + f_s)\psi - f_s \omega_\theta \theta + f_x \omega_\lambda \lambda_i \tag{18}$$

to identify (ψ and θ) as signals ($p_{i1} - w_1$ and ψ_i). However, even if $p_{i1} - w_1$ was observed with noise, θ would not affect the filtered signal negatively. For example, suppose that firms observe $p_{i1} - w_1 + z$, where z follows a normal distribution with mean zero and standard error σ_z. Firms can still use $\hat{q}_1$ to clean the mark-up from θ: $p_{i1} - w_1 + \frac{1}{\tilde{\sigma}}\hat{q}_1 + \frac{\tilde{\sigma}-1}{\tilde{\sigma}}\hat{q}_1 = \psi + z$.

with $0 < f_x < 1$, $0 < f_s < 1$ and $k_\psi < f_x + f_s < 1$. f_s is decreasing in $\tilde{\sigma}$.

As $f_x + f_s > k_\psi$, following a fundamental shock ψ, the forecast of ψ becomes closer to the fundamental in the second period as firms gather more information. On the opposite, the effect of a noise shock θ on the forecast of ψ turns from positive in the first period ($k_\psi > 0$) to negative in the second period ($-f_s\omega_\theta < 0$). In other words, following a positive θ shock, firms observe lower mark-ups than expected, because of an excessive aggregate supply. They revise their forecasts of ψ downwards, because low mark-ups can also signal a low ψ, that is low demand. Equation (18) states that this updating more than reverses the initial positive forecast.

Whereas the effect of θ on the second-period forecast is clearly negative when $\tilde{\sigma} > 1$, the marginal impact of the strategic substitutability parameter $\tilde{\sigma}$ is not straightforward. While $\tilde{\sigma}$ has a positive effect on the reaction of the filtered mark-up s to noise ω_θ, it has a negative impact on the weight firms put on this public signal f_s. Indeed, as the public signal becomes more reactive to θ, it becomes a poorer signal of ψ, so firms rely less on it to infer ψ. However, in the limit case where σ_θ^2 goes to zero, the first effect dominates, as suggested by the following corollary (see proof in the Appendix):

Corollary 1 *As σ_θ^2 goes to zero, $-f_s\omega_\theta$ goes to $-(\tilde{\sigma}-1)\sigma_\psi^2/(\sigma_\psi^2+\sigma_\lambda^2)$.*

This implies that, following a noise-driven boom, expectations can be arbitrarily low as aggregate noise shocks are unlikely and strategic substitutability is strong ($\tilde{\sigma}$ is large). Indeed, when the noise shock is unlikely, firms put a large weight f_s on the public signal s, so the effect of $\tilde{\sigma}$ on ω_θ dominates.

The effect of θ on output derives naturally from its effect on expectations. In period 2, as in period 1, the supply by firm i follows Equation (11). In order to determine the optimal supply by a firm, we use the method of undetermined coefficients again and derive the following (see proof in the Appendix):

$$\hat{q}_{i2} = F_x x_i + F_s s \tag{19}$$

with $0 < F_x < 1$, $0 < F_s < 1$ and $K_\psi < F_x + F_s < 1$. Under Assumption 1, we also have $F_x > f_x$ and $F_s < f_s$. At the aggregate level, firms produce the following quantities:

$$\hat{q}_2 = [F_x + F_s]\psi - F_s \omega_\theta \theta \tag{20}$$

In period 2, as $F_x + F_s > K_\psi$, following a fundamental shock ψ, output gets closer to its first-best value. On the opposite, the effect of the aggregate noise shock θ on aggregate supply becomes negative through the public signal s.

As in period 1, firms over-react to their private signal (here x_i) and under-react to their public signal (here s), as the economy features strategic substitutability. Whereas this property magnifies the initial boom, it mitigates the subsequent bust. This, however, is not a crucial feature of our model. It hinges on the assumption that the mark-up is not affected by any additional noise, due for example to a temporary aggregate or idiosyncratic demand shock. In the case of idiosyncratic noise, the bust could be magnified. This nevertheless does not change the main result of the model, which is the succession of booms and busts.

As the effect of strategic substitutability on the forecast is ambiguous, its effect on output is also ambiguous. In the case where σ_θ^2 goes to zero, we can nevertheless derive some results, summarized by the following corollary (see proof in the Appendix):

Corollary 2 *As σ_θ^2 goes to zero, F_s goes to f_s, so $-F_s\omega_\theta$ goes to $-(\tilde{\sigma}-1)\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$.*

As σ_θ^2 goes to zero, the extent of imperfect information becomes smaller, and the strategic component of optimal supply disappears, which implies that F_s goes to f_s. The optimal supply becomes closer to its certainty-equivalent counterpart where the public noise is not under-weighted, that is $\hat{q}_{i2} = E_{i2}(\psi)$. Applying Corollary 1, we obtain that the response of aggregate quantities to the aggregate noise shock can become arbitrarily large as the degree of strategic substitutability $\tilde{\sigma}$ increases.

2.4 Calibration

We implement a numerical analysis, where the baseline parameters are set as described in Table 1. The preference parameters γ, η and ρ, which determine whether

Assumption 1 is satisfied or not, are crucial. First, micro studies report values for the elasticity of substitution between goods that are of the order of 6-7, so we set $1/\rho$ to 7.[14] Second, γ, the inverse of the elasticity of intertemporal substitution, is commonly admitted to be around 1, so we set $\gamma = 1$.[15] Finally, Mulligan (1999) suggests that labor supply elasticities can easily be as large as 2, which suggests a value of 1/2 for η. We set $\eta = 0$ as a conservative benchmark. As a result, $\sigma = 1$ and $\tilde{\sigma} = 7$, which strongly satisfies Assumption 1.[16] We later examine alternative parametrizations.

Qualitatively, the precise values of the shocks' standard errors do not matter, as $\tilde{\sigma} > 1$ is enough to generate booms and busts. However, we need to set reasonable values in order to assess quantitatively the evolution of output. In order to roughly quantify σ_ψ, σ_θ and σ_λ, we proxy the expected output $E(q)$ and its expectational error $q - E(q)$, as well as the expected profits $E(\pi)$ and their expectational error $\pi - E(\pi)$ using a VAR(1) on data taken from the Survey of Professional Forecasters. We identify the standard errors by using the variance-covariance matrix of the residuals of the VAR.[17] The results are shown in Table 1. Importantly, the standard error of the aggregate noise shock is relatively small which, combined with a large $\tilde{\sigma}$, generates strong busts according to Corollary 2.

Consider the dashed lines in Figure 2, which represent the results for the simplified version of the model. The left panels show the effect of a unitary shock on fundamental demand ψ on the average forecast, supply, mark-up and wage in period 1 and 2. In the baseline calibration, the first-best response of both the forecast and supply is 1, and the first-best response of the mark-up is 0. As expected, this shock does not fully translate to $E_1(\psi)$ and $\hat{q}_1$, which respond positively but remain below

[14]See Ruhl (2008) and Imbs and Mejean (2009).

[15]See Attanasio and Weber (1993) and Vissing-Jörgensen and Attanasio (2003).

[16]This is not inconsistent with the existence of strategic complementarities in price-setting in New-Keynesian models since, as emphasized by Angeletos and La'O (2009), parameters that yield strategic complementarity in price-setting typically generate strategic substitutability in quantity-setting. Note that Angeletos and La'O (2009) have a model with quantity-setting, and yet their parametrization generates strategic complementarities. This is because they have a different interpretation of ρ. In their approach, this parameter governs "trade linkages" while in ours it governs the degree of imperfect competition.

[17]The detailed methodology is described in the Appendix.

1. As a result of excess demand, the mark-up increases in period 1. In period 2, the forecast and supply get closer to 1, as firms receive further positive information through their mark-up. As a result, the mark-up gets closer to 0. Between period 1 and period 2, output switches from 80% of its capacity to 94%.

Consider now the right panels of the figure, which represent the effect of an aggregate noise shock $\theta = 1$. In the first period, $\theta = 1$ is observationally equivalent to a shock on the fundamental, so firms respond to it by increasing output by 59%. However, the response of output is now above its first-best value, which is zero. Goods are then in excess supply, which implies that the response of the mark-up is negative. Crucially, as explained above, the fact that the mark-up reacts differently after a fundamental and a noise shock explains why the forecasts can be reversed after a noise shock. In the second period, the forecast and supply turn negative in the second period, as expected. More specifically, output drops by 74% below its capacity.

Of course, because the volatility of the noise shock is small, it represents only 5% of the total variance of aggregate output. However, a more realistic way of thinking about the volatility of noise shocks is the frequency with which they arise. Boom-and-bust episodes are then all the sharper as they happen infrequently.

3 Full-fledged model

In this section, we solve the full monetary version of the model, that is with prices specified in nominal terms, which means that nominal wages are now part of the firms' information set. We show that the forecasts of ψ become more precise when firms are able to observe the nominal wage, but they are still imperfect because of monetary shocks. The dynamics still features boom-and-bust cycles.

3.1 Labor market and information structure

At the beginning of period, the household establishes competitively with firms the amount of labor that she will provide against a pre-specified nominal wage w_t.

Firms therefore observe w_t at the same time as they decide how much they produce. The information structure of the model is modified. At the beginning of period 1, firms still receive the exogenous signal ψ_i, so $\Omega_{i1} = \{\psi_i, w_1\}$. In period 1, they observe their nominal price p_{i1}, so $\Omega_{i2} = \{\psi_i, w_1, p_{i1}, w_2\}$, which is equivalent to $\Omega_{i2} = \{\psi_i, p_{i1} - w_1, w_1, w_2\}$.

As a result of household's optimization, the nominal wage partly reveals the preference shock:

$$w_t = \left(1 - \frac{1}{\sigma}\right) \hat{q}_t - \psi + m_t$$

This equation is obtained from combining Equations (9) and (10). The nominal wage depends positively on the monetary shock m_t and negatively on the preference shock ψ. The level of aggregate production $\hat{q}_t$ has an ambiguous effect on the nominal wage because it stimulates the real wage through labor demand on the one hand while it creates nominal deflation on the other. If the macro elasticity of labor supply $1/\sigma$ is low, then the first effect dominates and $\hat{q}_t$ has a positive effect on w_t. Since the firm does not observe m_t nor $\hat{q}_t$ directly, the nominal wage will not perfectly reveal ψ.

Notice that the case with a unitary elasticity of labor, that is with $\sigma = 1$, is simpler since in that case w_t does not depend on $\hat{q}_t$:

$$w_t = -\psi + m_t$$

We solve this special case analytically and show that the main message of the simple version of the model is not affected. We then consider a numerical analysis to assess booms and busts quantitatively and to discuss the role of σ.

3.2 Analytical results with $\sigma = 1$

In the first period, firms can use two signals: $\psi_i = \psi + \theta + \lambda_i$ and $w_1 = -\psi + m_1$. The resulting supply is affected as follows (see proof in the Appendix):

$$\hat{q}_{i1} = K^*_\psi \psi_i - K^*_w w_1 \tag{21}$$

where $0 < K_\psi^* < 1$, $0 < K_w^* < 1$.[18] Besides, we have $K_\psi^* < K_\psi$, and $K_\psi < K_\psi^* + K_w^* < 1$. At the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = [K_\psi^* + K_w^*]\psi + K_\psi^*(\theta + \lambda_i) - K_w^* m_1 \tag{22}$$

As in the simple version of the model, firms under-react to the fundamental shock ψ and over-react to the noise shock θ, since $K_\psi^* + K_w^* < 1$ and $K_\psi^* > 0$. But because they receive an additional signal on ψ, they react more to ψ and less to θ, as $K_\psi^* + K_w^* > K_\psi$ and $K_\psi^* < K_\psi$.

At the end of period 1, firms' mark-ups constitute a new signal. As Equation (17) is still valid, we can derive the following lemma:

Lemma 2 *The information set available at the beginning of period 2* $\Omega_{i2} = \{\psi_i, p_{i1} - w_1, w_1, w_2\}$ *is equivalent to four independent signals of* ψ*: two monetary signals* $-w_1 = \psi - m_1$ *and* $-w_2 = \psi - m_2$ *and two real signal, a public signal* s^* *and a private signal* x_i^**, defined as follows:*

$$s^* = \psi - \omega_\theta^* \theta$$

$$x_i^* = \psi + \omega_\lambda^* \lambda_i$$

with $\omega_\theta^* = (\tilde{\sigma} - 1)K_\psi^* / [\tilde{\sigma} - (\tilde{\sigma} - 1)K_\psi^*]$ *and* $\omega_\lambda^* = \omega_\theta^* / (1 + \omega_\theta^*)$*. Under Assumption 1,* $\omega_\theta^* > 0$ *and* $\omega_\theta^* < \omega_\theta$*,* $\omega_\lambda^* < \omega_\lambda$*. Besides,* ω_θ^* *is increasing in* $\tilde{\sigma}$*.*

Proof. Here again s^* is obtained simply by normalizing the filtered mark-up and x_i^* is obtained by combining s^* with ψ_i. $\omega_\theta^* < \omega_\theta$ follows from $K_\psi^* < K_\psi$ and $\omega_\lambda^* < \omega_\lambda$ follows from $\omega_\theta^* < \omega_\theta$. ■

Since $\omega_\theta^* < \omega_\theta$ and $\omega_\lambda^* < \omega_\lambda$, x_i^* and s^* are more precise signals of ψ than x_i and s. Besides, firms can use two additional signals of ψ, the nominal signals $-w_1 = \psi - m_1$ and $-w_2 = \psi - m_2$. In the monetary version of the model, firms have more precise information on ψ. However, we still have that θ affects negatively the public signal of ψ. Supply in period 2 is therefore only affected quantitatively (see proof in the

[18] See the Appendix for the precise values of K_ψ^* and K_w^*.

Appendix):

$$\hat{q}_{i2} = F_x^* x_i^* + F_s^* s^* - F_w^*(w_1 + w_2) \tag{23}$$

where $0 < F_w^* < 1$, $0 < F_x^* < 1$ and $0 < F_s^* < 1$.[19] At the aggregate level, firms produce the following quantities:

$$\hat{q}_2 = [F_x^* + F_s^* + 2F_w^*]\psi - F_s^* \omega_\theta^* \theta - F_w^*(m_1 + m_2) \tag{24}$$

As in the simple model, the contribution of θ to the public signal s^* is negative under strategic substitutability, so the effect of θ on q_{i2} is still negative. Therefore, as in the simple version of the model, we can derive a proposition on boom patterns:

Proposition 2 (Boom-busts - Full-fledged model) *Following a positive fundamental shock ψ, output experiences a long-lived boom. Under Assumption 1, following a positive noise shock θ, output experiences a boom-and-bust.*

Again, the bust generated by θ can still be potentially large, as implied by the following lemma (see proof in the Appendix):

Corollary 3 *As σ_θ^2 goes to zero, $-F_s^* \omega_\theta^*$ goes to $-(\tilde{\sigma} - 1)\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2 + \sigma_\psi^2 \sigma_\lambda^2/\sigma_m^2)$.*

Therefore, following a noise-driven boom, the bust can be arbitrarily large as aggregate noise shock are unlikely and strategic substitutability is strong.

3.3 Calibration

We now perform a numerical analysis in order to assess the boom patterns quantitatively and to examine the role of σ. For that, we need first to assess σ_m, which determines the informativeness of nominal signals. We proceed in the same way as for the other standard errors: we proxy the expected price $E(p)$ and its expectational error $p - E(p)$ through the residuals of a VAR(1) on data from the Survey of Professional Forecasters and we identify the standard errors by using the variance-

[19] See the Appendix for the precise values of F_x^*, F_s^* and F_w^*.

covariance matrix of the residuals of the VAR.[20] We find $\sigma_m = 0.12$. The results are represented in Figure 2.

We can compare the baseline monetary model, with $\sigma_m = 0.12$, to the simple model, which is equivalent to the monetary model with σ_m going to infinity. The results are qualitatively similar but are quantitatively different. In the monetary model, the forecasts, quantities and mark-ups are closer to their first-best values than in the simple model. This is because the nominal wages provide additional information to firms about the underlying shocks. As a result, the boom-and-bust dynamics arising from an aggregate noise shock $\theta = 1$ is milder. However, as stated in Corollary 3, boom-and-bust episodes are still large when noise shocks are relatively unlikely, which is the case in our calibration. In the first period, output rises by 81% above capacity and then drops by 72% below capacity.

The macro elasticity of labor $1/\sigma$ is an important parameter because it determines the informational content of nominal wages. This parameter depends in general on the structure of the economy. There is a lot of disagreement in the literature on its precise value.[21] Richer models find estimates of σ that vary between 0.3 and 3.[22] Our baseline parametrization falls within this range. In Figure 2, we represent the case $\sigma = 0.3$ as a robustness check. It represents the lower bound for admissible values of σ, which should go against strong booms and busts. First, this decreases $\tilde{\sigma}$ and therefore could limit the magnitude of the bust. Second, a low σ makes the wage a better signal of the fundamental shock. Indeed, on the one hand, a positive shock on ψ generates a direct downward pressure on wages. On the other, the increase in labor demand generates an upward pressure, which mitigates this signalling effect, except if the wage is not too reactive (σ is low). Even with this conservative calibration, the bust is still sharp. Output still drops by 57% below capacity in the bust period.

[20] The detailed methodology is described in the Appendix.

[21] In the New Keynesian literature, σ has been the subject of a lot of of attention because it corresponds to the degree of "real rigidities", that is, the elasticity of the marginal cost to the output gap. See Woodford (2003) for a discussion on this parameter.

[22] For example, this value is equal to 0.33 in Dotsey and King (2006), to 0.34 in Smets and Wouters (2007) to 2.25 in the baseline parametrization used by Chari et al. (2000) and to 3 in Galí (2009).

4 Discussion and extensions

In this section we address several limitations of the benchmark model. First, we allow for noisy news about productivity shocks and show that only noisy news on demand generate booms and busts. Second, we add more dynamics and discuss how lengthy boom-and-bust episodes can appear. Finally, we propose an extension that endogeneizes the initial signal and that accounts for the fact that booms and busts are typically accompanied by a surge in credit.

4.1 Introducing productivity shocks

In this section we extend the model to accommodate productivity shocks. This is done for two purposes: first, we demonstrate that noise shocks about supply do not tend to generate a boom and bust cycle but rather a short-lived boom, contrary to noise shocks about demand; second, we show that noise shocks on demand still generate booms and busts in this more general setting.

4.1.1 Optimal supply and information

We assume now that the level of productivity A is a stochastic variable. It represents a permanent "supply shock". We assume that $a = \log(A)$ follows a normal distribution with mean zero and standard error σ_a.

We assume that the firm cannot observe a directly. Instead, firm i receives a news about a that incorporates both an aggregate and an idiosyncratic error:

$$a_i = a + v + u_i \tag{25}$$

where v and u^i are both normal with mean zero and respective variance σ_v^2 and σ_u^2. v is an aggregate noise shock, while u_i is an idiosyncratic noise shock that cancels out at the aggregate level: $\int_0^1 u^i di = 0$.

The manager of firm i sets quantities in order to satisfy:

$$E_i(p_i - w) = -E_i(a)$$

Using the individual and aggregate demand equations, the optimal individual supply can be written as a function of expected aggregate supply and expected shocks:

$$\hat{q}_i = \tilde{\sigma}[E_i(\psi) + (\eta + 1)E_i(a)] - (\tilde{\sigma} - 1)\, E_i(\hat{q}) \tag{26}$$

We first consider the case with productivity shocks only within the simple version of the model, where the relevant information sets are $\Omega_{i1} = \{a_i\}$ and $\Omega_{i1} = \{a_i, p_{i1} - w_1\}$. We then consider the more general case with both productivity and demand shocks within the full monetary version, where the relevant information sets are $\Omega_{i1} = \{\psi_i, a_i, w_1\}$ and $\Omega_{i2} = \{\psi_i, a_i, p_{i1} - w_1, w_1, w_2\}$.

4.1.2 Productivity shocks only

To illustrate the effect of noise shocks on productivity, we consider the simple case with only productivity shocks. We also first illustrate the mechanism within our baseline calibration with $\eta = 0$. In that case, the optimal supply by firm i in period 1 follows:

$$\hat{q}_{i1} = \tilde{\sigma} E_{i1}(a) - (\tilde{\sigma} - 1) E_{i1}(\hat{q}_1) \tag{27}$$

In the first period, the firm's problem with productivity shocks is strictly similar to the firm's problem with demand shocks solved in Section 3. We can therefore easily derive the following:

$$\hat{q}_{i1} = K_a a_i \tag{28}$$

with $K_a = \tilde{\sigma}\sigma_a^2/[\tilde{\sigma}(\sigma_a^2 + \sigma_v^2) + \sigma_u^2]$. As a consequence, at the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = K_a(a + v) \tag{29}$$

Because firms do not observe productivity, the new signal received by firms is not their mark-up $p_i - w + a$, but their real price $p_i - w$, which they can filter from the influence of their own supply:

$$p_{i1} - w_1 + \frac{1}{\tilde{\sigma}}\hat{q}_{i1} = -\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\hat{q}_1 = -\frac{\tilde{\sigma}-1}{\tilde{\sigma}}K_a(a+v)$$

Importantly, in the case of productivity shocks, both the fundamental and the noise shocks are supply shocks, so they both affect the real price negatively, as $K_a > 0$. As a consequence, contrary to demand shocks, a positive noise shock about productivity does not generates a negative signal on the fundamental through prices. It thus does not generate boom-bust cycles.

In the general case where $\eta \geq 0$, the informational content of the real price is affected:

$$p_{i1} - w_1 + \frac{1}{\tilde{\sigma}}\hat{q}_{i1} = -\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}(1+\eta)K_a - \eta\right)a - \frac{\tilde{\sigma}-1}{\tilde{\sigma}}(1+\eta)K_a v \tag{30}$$

If η is large enough, then a productivity shock increases the real price. This is because the real wage decreases as the household has to exert less effort to produce a given quantity of output. In that case, boom and bust episodes can still appear following a positive noise shock v. It is therefore important to calibrate the model in order to assess properly the dynamics. We do this in a more general setting with both demand and productivity shocks.

4.1.3 Productivity and demand shocks

We now consider the full-fledged model with both demand and productivity shocks. We first consider the baseline calibration of Table 1 where $\gamma = 1$ and $\eta = 0$, and then turn to the more realistic case where $\eta = 0.5$. The standard errors σ_a, σ_v and σ_u are set in order to match the variance-covariance matrix of $E(a)$ and $a - E(a)$, using a VAR(1) on data taken from the Survey of Professional Forecasters.[23] We find $\sigma_a = 0.016$, $\sigma_v = 0.036$ and $\sigma_u = 0.021$. The fact that the volatility of the aggregate noise on productivity is larger than the volatility of fundamental productivity is consistent with the empirical findings of the literature on news and

[23]The detailed methodology is described in the Appendix.

noise.[24] σ_m is set to 0.12 as before. The results are represented in Figure 3.

Consider first the baseline calibration. In the first period, the effect of the demand shock and its noise does not change as compared to the case with only demand shocks, because the demand and productivity signals are independent. However, the real price observed at the end of period 1 is now affected by both productivity and demand shocks, as well as by their respective noises. In particular, a negative real price might be driven by a positive productivity shock, and could therefore be perceived as good news by managers. However, this effect seems negligible, as the reactions of output to θ is very close to the case with only demand shocks. This is because the initial news on a is not very precise, so managers do not act upon that news in the first period, which makes the price a poor signal of productivity. The real price is then used by firms to update their expectations of ψ but not their expectations of a.

In the baseline calibration where $\eta = 0$, the productivity and its noise are observationally equivalent and generate the same dynamics. Because the real price is a poor signal of a, managers do not improve their knowledge of a between period 1 and period 2, so a has the same effect in period 1 and period 2.[25] With $\eta = 0.5$, the boom following an increase in a is more long-lived while the boom following an increase in v is more short-lived, because the real price is affected by a and v in different ways, so observing the real price helps firms better disentangle a from v in period 2. Boom-and-bust episodes still only appear after a shock on θ.[26]

4.2 More dynamics

A caveat to our analysis is that boom-and-bust episodes actually last more than 2 periods. However, we show in this extension that lags in the processing of endogenous

[24] See for example Blanchard et al. (2009).

[25] Of course, if managers have alternative sources of information and learn about a, then the economy would experience a long-lived boom following a shock on a and a short-lived boom following a shock on v.

[26] Notice that the fundamental productivity a is better identified with $\eta = 0.5$. This is because the wage is negatively affected by productivity as the household has to work less to produce a given level of output.

information naturally generates longer booms and busts.

We consider the full-fledged version of the model with the baseline calibration. There is an infinite number of periods, starting from $t = 1$, where the permanent demand shock ψ is realized. Firms observe the news ψ_i at $t = 1$, but they observe the mark-up $p_{it-1} - w_{t-1}$ and the nominal wage $w_t = m_t - \psi$ only at $t + T$, where T represents the informational lag.

Figure 4 represents the behavior of output and expectations for different values of T when the economy experiences a positive noise shock $\theta = 1$. There is a boom stage that lasts T periods and a bust stage that lasts T periods. As the firms initially receive positive news, their output increases. For T periods, firms do not receive any additional information because of informational lags, so output remains high. At $T + 1$, when the firms first observe a negative mark-up, their expectations on ψ drop below their fundamental level, which is 0. This ends the boom stage and starts the bust stage. Since past output was constant, firms do not receive any new information through mark-ups. However, they do observe different realizations of the nominal wage, so their expectations improve and get closer to 0, but remain well below 0. At $2T + 1$, firms observe positive mark-ups and can infer ψ with certainty, which ends the bust stage.

4.3 Credit and endogenous initial signal

We now introduce credit in order to account for the typical surge in credit that characterizes booms and busts. To do so, we introduce a non-produced traded good X, in fixed supply $\bar{X}$ in the country but in infinite supply from the rest of the world. Households can exchange good X with the rest of the world and save or borrow vis-à-vis the rest of the world. Strategic substitutability still affects the nontradable sector where firms produce differentiated goods as described in Section 1. In this extension, we also endogeneize noise shocks by introducing an initial period 0, where temporary aggregate and idiosyncratic demand shocks can appear. These temporary demand shocks generate noise because firms cannot distinguish them from the permanent demand shock. Therefore, an aggregate temporary demand shock will generate credit among households and in the same time mislead firms about the true value of the permanent shock.

We introduce a demand for a traded good by amending the model of Section 1 in the following way. The specification of the utility (2) becomes for $t = 1, 2$:

$$U_t = \Psi \log \left(Q_t^{\mu} X_t^{1-\mu} \right) - L_t \tag{31}$$

with $0 < \mu < 1$. μ is the share of nontradable goods in consumption. When $\mu = 1$, the utility function boils down to (2) where $\gamma = 1$ and $\eta = 0$. Q now refers to the consumption of non-traded goods. In the initial period $t = 0$, utility is now:

$$U_0 = \Psi\Theta \log \left(Q_t^{\mu} X_t^{1-\mu} \right) - L_t \tag{32}$$

with $Q_0 = \left(\int_0^1 \Lambda_i Q_i^{1-\rho} di \right)^{\frac{1}{1-\rho}}$. $\theta = \log(\Theta)$ is a temporary aggregate demand shock and $\lambda_i = \log(\Lambda_i)$ is a temporary idiosyncratic demand shock for good i, where θ and λ_i have the same characteristics as described in Section 1. The household now maximizes $U = U_0 + \beta U_1 + \beta^2 U_2$ subject to the budget constraints:

$$\int_0^1 P_{it} Q_{it} di + M_t + P_t^x X_t + r P_t^x D_{t-1} = W_t L_t + \int_0^1 \Pi_{it} di + M_{t-1} + T_t + P_t^x \bar{X} + P_t^x D_t$$

for $t = 0, 1, 2$. P_t^x is the price of good X in nominal terms and D_t is international borrowing in terms of tradable goods, which yields interest $r = 1/\beta$. Households now can trade intertemporally with the rest of the world through D. We assume that they start with no international debt so $D_{-1} = 0$.

The cash-in-advance constraint and the government budget constraint are the same as before, which yields (10).

The aggregate and individual demands remain as described in Equations (8) and (9) in periods 1 and 2, except that we have $\gamma = 1$ and $\eta = 0$. In period 0, they are additionally affected by the aggregate and individual transitory shocks θ and λ_i:

$$q_{i0} = q_0 - \frac{1}{\rho} \left[p_{i0} - p_0 - \lambda_{i0} \right]$$

$$w_0 - p_0 = q_0 - \psi - \theta$$

Ex ante, firms observe nominal wages $w_0 = m_0 - \psi - \theta$, so they produce $q_i = q = -hw_0$, where $h = \sigma_\psi^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_m^2)$. As a result, in period 0, they observe mark-ups:

$$p_{i0} - w_0 = \psi + \theta + \lambda_i + hw_0$$

which, combined with w_0 gives the initial signal ψ_i:

$$\psi_i = p_{i0} - w_0 - hw_0 = \psi + \theta + \lambda_i$$

Period-0 mark-ups are therefore an imperfect signal of the permanent demand shock ψ. This signal is perturbed by the aggregate and idiosyncratic demand shocks θ and λ_i. This gives an economic significance to the initial signal ψ_i in the baseline model. Regarding the dynamics of the non-traded good in period 1 and 2, the only difference with the baseline model with $\gamma = 1$ and $\eta = 0$ is that agents start period 1 with an additional nominal signal w_0.

We derive the following Proposition (see proof in the Appendix):

Proposition 3 (Boom-busts and capital flows) *Under Assumption 1, a positive aggregate transitory demand shock θ generates capital inflows in period 0 and a boom-and-bust in period 1 and 2. A positive aggregate permanent shock ψ generates no capital flows and a long-lived boom in period 1 and 2.*

A temporary demand shock θ generates a demand boom in period 0 which makes households increase their borrowing. This same temporary demand boom makes firms mistakenly interpret it as a permanent boom, making them over-optimistic, which triggers a boom-and-bust dynamics in the non-tradable sector. A permanent increase in demand does not generate a boom-and-bust dynamics since firms are confirmed in their beliefs. At the same time, households do not borrow as the shock is permanent.

5 Conclusion

This paper has shown that, in a model where agents have imperfect common knowledge and learn from prices, fundamental shocks lead to long-lived booms while noise shocks lead to boom-and-busts. These dynamics are rooted in dispersed information and the fact that, due to market timing and to the monetary nature of transactions, endogenous signals do not communicate enough information. Besides, the way fundamental shocks and their corresponding noise correlate with the endogenous signals shapes the response of output to news.

While the focus of the literature is on the channel from information to equilibrium variables, this paper shows in a simple model that studying the way equilibrium variables affect information generates new insights. In more general frameworks, this approach is subject to technical difficulties, that are now reduced thanks to the contributions of Nimark (2011) and Rondina and Walker (2012). We believe that these recent methodological advances open an avenue for future research in that direction.

Finally, on the empirical front, while the literature has investigated the extent of imperfect information (Mankiw et al., 2003; Pesaran and Weale, 2006; Coibion and Gorodnichenko, 2012), our analysis shows that the source of information is important as well. The nature of the signals agents learn from could be as important as the precision of those signals. Future empirical research should document what variables agents observe and extract information from.

References

Adam, Klaus and Albert Marcet (2011), Booms and busts in asset prices, CEP Discussion Papers dp1059, Centre for Economic Performance, LSE.

Amador, Manuel and Pierre-Olivier Weill (2012), 'Learning from private and public observations of others'actions', *Journal of Economic Theory* **147**(3), 910–940.

Angeletos, George-Marios and Alessandro Pavan (2007), 'Efficient use of information and social value of information', *Econometrica* **75**(4), 1103–1142.

Angeletos, George-Marios and Jennifer La'O (2009), Noisy business cycles, Working Paper 14982, National Bureau of Economic Research.

Attanasio, Orazio P and Guglielmo Weber (1993), 'Consumption growth, the interest rate and aggregation', *Review of Economic Studies* **60**(3), 631–49.

Bacchetta, Philippe and Eric Van Wincoop (2006), 'Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?', *American Economic Review* **96**(3), 552–576.

Ball, Laurence, N. Gregory Mankiw and Ricardo Reis (2005), 'Monetary policy for inattentive economies', *Journal of Monetary Economics* **52**(4), 703–725.

Beaudry, Paul and Franck Portier (2004), 'An exploration into pigou's theory of cycles', *Journal of Monetary Economics* **51**(6), 1183–1216.

Beaudry, Paul and Franck Portier (2006), 'Stock prices, news, and economic fluctuations', *American Economic Review* **96**(4), 1293–1307.

Blanchard, Olivier J., Jean-Paul L'Huillier and Guido Lorenzoni (2009), News, noise, and fluctuations: An empirical exploration, NBER Working Papers 15015, National Bureau of Economic Research, Inc.

Burnside, Craig, Martin Eichenbaum and Sergio Rebelo (2011), Understanding booms and busts in housing markets, NBER Working Papers 16734, National Bureau of Economic Research, Inc.

Chari, V. V., Patrick J. Kehoe and Ellen R. McGrattan (2000), 'Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem?', *Econometrica* **68**(5), 1151–1180.

Christiano, Lawrence, Cosmin Ilut, Roberto Motto and Massimo Rostagno (2008), Monetary policy and stock market boom-bust cycles, Working Paper Series 955, European Central Bank.

Coibion, Olivier and Yuriy Gorodnichenko (2012), 'What can survey forecasts tell us about information rigidities?', *Journal of Political Economy* **120**(1), pp. 116–159.

Council of Economic Advisors (1954), Economic report of the president, 1954, Technical report, Federal Reserve Archive.

Dotsey, Michael and Robert G. King (2006), 'Pricing, production, and persistence', *Journal of the European Economic Association* **4**(5), pp. 893–928.

Frydman, Roman and Edmund S. Phelps (1984), Cambridge University Press.

Gaballo, Gaetano (2013), Price dispersion and private uncertainty, mimeo.

Galí, J. (2009), *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press.

Graham, Liam and Stephen Wright (2010), 'Information, heterogeneity and market incompleteness', *Journal of Monetary Economics* **57**(2), 164 – 174.

Hellwig, Christian (2006), Monetary business cycle models: Imperfect information (review article, march 2006), UCLA Economics Online Papers 377, UCLA Department of Economics.

Hellwig, Christian and Venky Venkateswaran (2009), 'Setting the right prices for the wrong reasons', *Journal of Monetary Economics* **56, Supplement**(0), S57 – S77.

Hoberg, Gerard and Gordon Phillips (2010), 'Real and financial industry booms and busts', *The Journal of Finance* **65**(1), 45–86.

Imbs, Jean and Isabelle Mejean (2009), Elasticity optimism, CEPR Discussion Papers 7177, C.E.P.R. Discussion Papers.

Jaimovich, Nir and Sergio Rebelo (2009), 'Can news about the future drive the business cycle?', *American Economic Review* **99**(4), 1097–1118.

Keynes, J.M. (1936), *General Theory Of Employment , Interest And Money*, Atlantic Publishers & Distributors (P) Limited.

Lambertini, Luisa, Caterina Mendicino and Maria Teresa Punzi (2011), Leaning against boom-bust cycles in credit and housing prices, Working Papers 201101, Center for Fiscal Policy, Swiss Federal Institute of Technology Lausanne.

Lorenzoni, Guido (2009), 'A theory of demand shocks', *American Economic Review* **99**(5), 2050–84.

Lorenzoni, Guido (2011), 'News and Aggregate Demand Shocks', *Annual Review of Economics* **3**(1), 537–557.

Lucas, Robert Jr. (1972), 'Expectations and the neutrality of money', *Journal of Economic Theory* **4**(2), 103–124.

Mackowiak, Bartosz and Mirko Wiederholt (2009), 'Optimal sticky prices under rational inattention', *American Economic Review* **99**(3), 769–803.

Mankiw, N. Gregory, Ricardo Reis and Justin Wolfers (2003), Disagreement about inflation expectations, Working Paper 9796, National Bureau of Economic Research.

Mulligan, Casey (1999), Substition over time: Another look at life-cycle labor supply, *in* 'NBER Macroeconomics Annual 1998, volume 13', National Bureau of Economic Research, Inc, pp. 75–152.

Nimark, Kristoffer (2011), Dynamic higher order expectations, Technical report, Department of Economics and Business, Universitat Pompeu Fabra.

Pearlman, Joseph G. and Thomas J. Sargent (2005), 'Knowing the forecasts of others', *Review of Economic Dynamics* **8**(2), 480–497.

Pesaran, M. Hashem and Martin Weale (2006), *Survey Expectations*, Vol. 1 of *Handbook of Economic Forecasting*, chapter 14, pp. 715–776.

Pigou, A.C. (1927), *Industrial fluctuations*, Macmillan and co., limited.

Rondina, Giacomo and Todd Walker (2012), Information equilibria in dynamic economies with dispersed information, mimeo, University of San Diego.

Ruhl, Kim J. (2008), Solving the Elasticity Puzzle in International Economics, mimeo.

Sargent, Thomas J. (1991), 'Equilibrium with signal extraction from endogenous variables', *Journal of Economic Dynamics and Control* **15**(2), 245 – 273.

Shin, Hyun Song and Jeffery D. Amato (2003), Public and Private Information in Monetary Policy Models, Computing in Economics and Finance 2003 38, Society for Computational Economics.

Sims, Christopher (2003), 'Implications of rational inattention', *Journal of Monetary Economics* **50**(3), 665–690.

Smets, Frank and Rafael Wouters (2007), 'Shocks and frictions in us business cycles: A bayesian dsge approach', *American Economic Review* **97**(3), 586–606.

Townsend, Robert M (1983), 'Forecasting the forecasts of others', *Journal of Political Economy* **91**(4), 546–88.

Vissing-Jörgensen, Annette and Orazio P. Attanasio (2003), 'Stock-market participation, intertemporal substitution, and risk-aversion', *American Economic Review* **93**(2), 383–391.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Woodford, Michael (2001), Imperfect common knowledge and the effects of monetary policy, NBER Working Papers 8673, National Bureau of Economic Research, Inc.

A Proofs

Denote by ξ_i a gaussian vector of shocks of size N, where the n first elements are aggregate shocks and the $N-n$ last elements are idiosyncratic shocks, and S_i a vector of signals of size K such that there exists a (N, K) matrix H such that:

$$S_i = H'\xi_i \tag{33}$$

We denote by $\tilde{H}$ the (N, K) matrix such that all the n first lines are equal to the n first lines of H and the $N-n$ last lines are equal to zero. Let P be a (N, K) matrix such that:

$$E(\xi_i|S_i) = PS_i \tag{34}$$

The following lemma will be useful in proving several propositions:

Lemma 3 *Consider the following equation:*

$$\hat{q}_i = [\tilde{\sigma}X'E(\xi_i|S_i) - (\tilde{\sigma}-1)E(\hat{q}|S_i)] \tag{35}$$

where X is a vector of size N. Then, if $I + (\tilde{\sigma}-1)\tilde{H}'P$ is invertible, we have:

$$\hat{q}_i = AS_i$$

where A is a size-K row vector such that:

$$A = \tilde{\sigma}X'P[I + (\tilde{\sigma}-1)\tilde{H}'P]^{-1} \tag{36}$$

Proof of Lemma 3.

We use the method of undetermined coefficients to solve for A. We first form the educated guess that there exist a size-K row vector A such that

$$\hat{q}_i = AS_i \tag{37}$$

then, using Equation (33), we obtain

$$\hat{q}_i = AH'\xi_i$$

Hence, aggregating across firms, taking expectations and using Equation (34):

$$E(\hat{q}|S_i) = A\tilde{H}'E(\xi_i|S_i) = A\tilde{H}'PS_i$$

Replacing in Equation (35):

$$\hat{q}_i = [\tilde{\sigma}X'PS_i - (\tilde{\sigma}-1)A\tilde{H}'PS_i] = [\tilde{\sigma}X'P - (\tilde{\sigma}-1)A\tilde{H}'P]S_i$$

Using the guess, we can write:

$$A = \tilde{\sigma}X'P - (\tilde{\sigma}-1)A\tilde{H}'P$$

If $I + (\tilde{\sigma}-1)\tilde{H}'P$ is invertible, we can solve for A and obtain (36).

■

Derivation of Equations (15) and (16).

According to Equation (11), $\hat{q}_{i1}$ follows (35) with $S_i = \psi_i$, $\xi_i = \begin{pmatrix}\psi & \theta & \lambda_i\end{pmatrix}'$ and $X = \begin{pmatrix}1 & 0 & 0\end{pmatrix}'$. Besides, S_i follows (33) with $H = \begin{pmatrix}1 & 1 & 1\end{pmatrix}'$ and $\tilde{H} = \begin{pmatrix}1 & 1 & 0\end{pmatrix}'$; and $E(\xi_i|S_i)$ follows (34) with $P = \begin{pmatrix}k_\psi & \bar{k}_\psi & 1-k_\psi-\bar{k}_\psi\end{pmatrix}'$ with $k_\psi = \sigma_\psi^2/(\sigma_\psi^2+\sigma_\theta^2+\sigma_\lambda^2)$ and $\bar{k}_\psi = \sigma_\theta^2/(\sigma_\psi^2+\sigma_\theta^2+\sigma_\lambda^2)$. Therefore, applying Lemma 3, we obtain:

$$K_\psi = A = \frac{\tilde{\sigma}k_\psi}{1+(\tilde{\sigma}-1)(k_\psi+\bar{k}_\psi)} = k_\psi\left(1+\frac{(\tilde{\sigma}-1)(1-k_\psi-\bar{k}_\psi)}{1+(\tilde{\sigma}-1)(k_\psi+\bar{k}_\psi)}\right)$$

We have: $K_\psi = \tilde{\sigma}k_\psi/[\tilde{\sigma}k_\psi+\tilde{\sigma}\bar{k}_\psi+(1-k_\psi-\bar{k}_\psi)]$, which implier $0 < K_\psi < 1$ as $k_\psi+\bar{k}_\psi < 1$. Besides, we have $k_\psi+\bar{k}_\psi < 1$ and under Assumption 1 $\tilde{\sigma} > 1$, so $K_\psi > k_\psi$.

■

Derivation of Equation (18).

The standard signal extraction formula gives us that $E_{i2}(\psi) = f_x x_i + f_s s$ with

$$f_x = \frac{(\omega_\lambda \sigma_\lambda)^{-2}}{(\sigma_\psi)^{-2} + (\omega_\theta \sigma_\theta)^{-2} + (\omega_\lambda \sigma_\lambda)^{-2}} = \frac{(1+\omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2}{(1+\omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2 + \sigma_\psi^2 \sigma_\lambda^2 + \omega_\theta^2 \sigma_\theta^2 \sigma_\lambda^2}$$

$$f_s = \frac{(\omega_\theta \sigma_\theta)^{-2}}{(\sigma_\psi)^{-2} + (\omega_\theta \sigma_\theta)^{-2} + (\omega_\lambda \sigma_\lambda)^{-2}} = \frac{\sigma_\psi^2 \sigma_\lambda^2}{(1+\omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2 + \sigma_\psi^2 \sigma_\lambda^2 + \omega_\theta^2 \sigma_\theta^2 \sigma_\lambda^2}$$

where we used $\omega_\lambda = \omega_\theta/(1+\omega_\theta)$. Obviously, $0 < f_x < 1$, $0 < f_s < 1$ and $f_x + f_s < 1$. Besides, we can show that $f_x + f_s > k_\psi$. Indeed, using the definitions of f_x, f_s and k_ψ, we can show that this is equivalent to: $[(\sigma_\theta^2 + \sigma_\lambda^2)(1+\omega_\theta) - \sigma_\lambda^2 \omega_\theta]^2 > 0$, which is always the case. Finally, we can show that f_s is decreasing in K_ψ as $\omega_\theta < 1$. Since ω_θ is increasing in $\tilde{\sigma}$, then f_s is decreasing in $\tilde{\sigma}$.

■

Proof of Corollary 1.

First, we examine the behavior of the coefficient ω_θ as σ_θ goes to zero. From the definition of K_ψ, we derive:

$$K_\psi = k_\psi \left(1 + \frac{(\tilde{\sigma}-1)(1-k_\psi-\bar{k}_\psi)}{1+(\tilde{\sigma}-1)(k_\psi+\bar{k}_\psi)}\right)$$

with $k_\psi = \sigma_\psi^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$ and $\bar{k}_\psi = \sigma_\theta^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$.

When σ_θ goes to zero, k_ψ goes to $k_\psi = \sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$ and $\bar{k}_\psi$ goes to zero. As a result, K_ψ goes to $\tilde{\sigma}\sigma_\psi^2/(\tilde{\sigma}\sigma_\psi^2 + \sigma_\lambda^2)$. Hence, using the definition of ω_θ given in Lemma 1, we can show that ω_θ goes to $(\tilde{\sigma}-1)\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$.

Using the definition of f_s given in the derivation of Equation (18), it is straightforward to see that f_s goes to 1 as σ_θ goes to zero. As a consequence, $-f_s\omega_\theta$ goes to $-(\tilde{\sigma}-1)\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$.

■

Derivation of Equations (19) and (20).

According to Equation (11), $\hat{q}_{i2}$ follows (35) with $S_i = \begin{pmatrix} s & x_i \end{pmatrix}'$,

$$\xi_i = \begin{pmatrix} \psi \\ -\omega_\theta \theta \\ \omega_\lambda \lambda_i \end{pmatrix}$$

and $X = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}'$. Besides, S_i follows (33) with

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\tilde{H} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and $E(\xi_i|S_i)$ follows (34) with

$$P = \begin{pmatrix} f_s & f_x \\ 1 - f_s & -f_x \\ -f_s & 1 - f_x \end{pmatrix}$$

with f_s and f_x defined as in the derivation of Equation (18). Therefore, applying Lemma 3, we obtain:

$$\begin{pmatrix} F_s \\ F_x \end{pmatrix} = A' = \begin{pmatrix} \frac{f_s}{1+(\tilde{\sigma}-1)f_x} \\ \frac{\tilde{\sigma} f_x}{1+(\tilde{\sigma}-1)f_x} \end{pmatrix} = \begin{pmatrix} f_s \left(1 - \frac{(\tilde{\sigma}-1)f_x}{1+(\tilde{\sigma}-1)f_x}\right) \\ f_x \left(1 + \frac{(\tilde{\sigma}-1)(1-f_x)}{1+(\tilde{\sigma}-1)f_x}\right) \end{pmatrix}$$

We have $0 < f_x < 1$ and according to Assumption 1, we have $\tilde{\sigma} > 1$, so $F_x > f_x$ and $F_s < f_s$. Besides, one can show that $F_x + F_s > K_\psi$. Indeed, using the definitions of F_x, F_s, K_ψ and ω_λ, we can show that this is equivalent to: $[(\tilde{\sigma}\sigma_\theta^2 + \sigma_\lambda^2)(1+\omega_\theta) - \sigma_\lambda^2 \omega_\theta]^2 > 0$, which is always the case. Finally, we have $F_x + F_s = (f_s + \tilde{\sigma} f_x)/(1 - f_x + \tilde{\sigma} f_x)$. Since

$f_x + f_s < 1$, then $F_x + F_s < 1$.

■

Proof of Corollary 2.

Using the definition of f_s given in the derivation of Equation (18), it is straightforward to see that f_s goes to 1 and f_x goes to zero as σ_θ goes to zero. As a result, following the definition of F_s given in the derivation of Equations (19) and (20), we show that F_s goes to f_s as σ_θ goes to zero.

Hence, as σ_θ goes to zero, $-F_s\omega_\theta$ goes to $-f_s\omega_\theta$ which, according to Corollary 1, goes to $-(\tilde{\sigma} - 1)\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$.

■

Derivation of Equations (21), (22), (23) and (24) and Proof of Corollary 3.

We proceed respectively as for Equations (15), (16), (19) and (20) and for Corollary 2. See the Online Appendix for details.

■

Proof of Proposition 3.

The proof proceeds in two steps.

Capital flows First, we show that a positive temporary demand shock generates an increase in the consumption of tradable goods in period 0 relative to period 1 and 2. Households have then to borrow in period 0 and reimburse their debt in period 1 and 2. On the opposite, with a positive permanent demand shock, households consume the same amount during the three periods and do not borrow. See the Online Appendix for details.

Period-1 and period-2 production We proceed as for Equations (16) and (20). See the Online Appendix for details.

■

B Calibration

To match σ_ψ, σ_θ and σ_λ we proxy $E(q)$, $q - E(q)$, $E(\pi)$ and $\pi - E(\pi)$ as the residuals of a VAR using US data from the Survey of Professional Forecasters. We use after-tax corporate profits ($CPROF$) and the real GDP ($RGDP$) from 1968:4 to 2014:1 at a quarterly frequency. We perform a VAR(1) on the following vector of variables $(CPROF6_t - CPROF1_t, CPROF_{t+5} - CPROF6_t, RGDP6_t - RGDP1_t, RGDP1_{t+5} - RGDP6_t)'$ where $CPROF1_t$ and $RGDP1_t$ are the logs of respectively the historical value of corporate profits and GDP in $t-1$. $CPROF6_t$ and $RGDP6_t$ are the corresponding projection values for quarter $t+4$.

The residuals of the VAR

$$\begin{pmatrix} CPROF6_t - CPROF1_t \\ CPROF1_{t+5} - CPROF6_t \\ RGDP6_t - RGDP1_t \\ RGDP1_{t+5} - RGDP6_t \end{pmatrix} - \begin{pmatrix} CPROF\widehat{6_t - C}PROF1_t \\ CPROF1_{t+5} - CPROF6_t \\ RGDP6_t - RGDP1_t \\ RGDP1_{t+5} - RGDP6_t \end{pmatrix}$$

are used as proxies for

$$\begin{pmatrix} E(\pi) \\ \pi - E(\pi) \\ E(q) \\ q - E(q) \end{pmatrix}$$

As we use past expectations in the VAR, $E(\pi)$, $\pi - E(\pi)$, $E(q)$ and $q - E(q)$ are driven by the new signal ψ_i, so they correspond to the first period values in the model. σ_ψ, σ_θ and σ_λ are then set to match $V(E_1(\pi_1))$, $V(\pi_1 - E_1(\pi_1))$ and $V(q_1 - E(q_1))$ in the simplified model.

To match σ_m, we extend the VAR to include the nominal price ($PGDP$) from the same dataset in the same way in order to proxy $E(p)$ and $p - E(p)$. σ_m is then set to match $V(E(p_1))$ in the full-fledged model, taking σ_ψ, σ_θ and σ_λ as determined above.

Finally, to match σ_a, σ_v and σ_u, we measure productivity by $PROD = RGDP -$

EMP where EMP is the log of nonfarm payroll employment. Now the sample has to be restricted to 2003:4-2014:1 due to the availability of the employment variable. The residuals of the VAR

$$\begin{pmatrix} PROD6_t - PROD1_t \\ PROD1_{t+5} - PROD6_t \end{pmatrix} - \begin{pmatrix} PROD\widehat{6_t - P}ROD1_t \\ PROD1_{t+5} - PROD6_t \end{pmatrix}$$

are used as proxies for

$$\begin{pmatrix} E(a) \\ a - E(a) \end{pmatrix}$$

σ_a, σ_v and σ_u are then set to match $V(E_1(a))$, $V(E_1(a))$ and $V_1(a - E(a))$.

Table 1: Baseline calibration for the numerical analysis

Parameter	Value
γ	1
η	0
$1/\rho$	7
σ_ψ	0.057
σ_θ	0.011
σ_λ	0.070

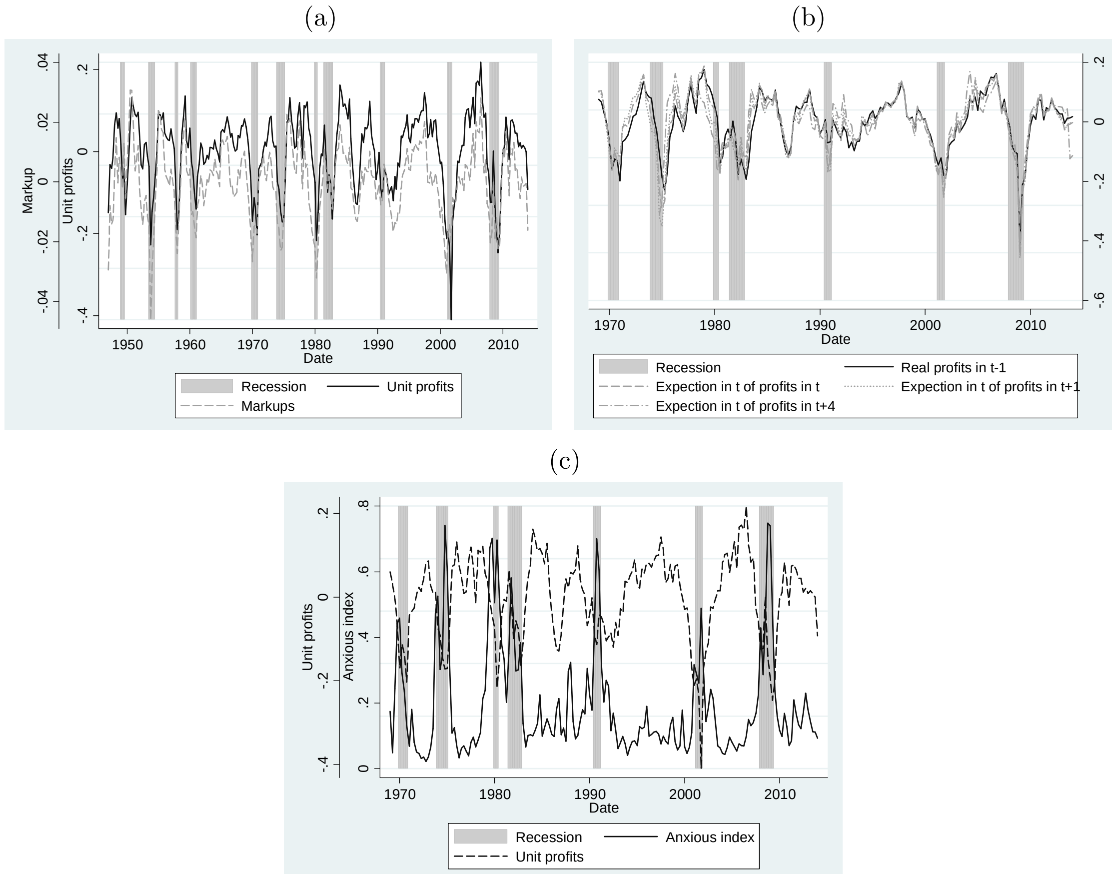

Figure 1: Stylized facts - Profits, mark-ups and recessions

Source: NBER, Federal Reserve Bank of Saint-Louis, Federal Reserve Bank of Philadelphia (Survey of Professional Forecasters), author's calculations. Unit profits are unit profits of nonfinancial corporations. The mark-up is defined as the inverse of the labor share of nonfinancial corporations. Real profits are corporate profits after tax (Corporate Profits with Inventory Valuation Adjustment (IVA) and Capital Consumption Adjustment (CCAdj)) divided by the implicit GDP deflator. Expectations are constructed using real profits in quarter $t-1$ and expected growth in nominal profits and the GDP deflator. The Anxious index is the expected probability of a fall in real GDP in the following quarter. The series are in quarterly frequency, in logs and detrended using the Hodrick-Prescott filter (with a smoothing parameter of 1600), except the Anxious index which is in level.

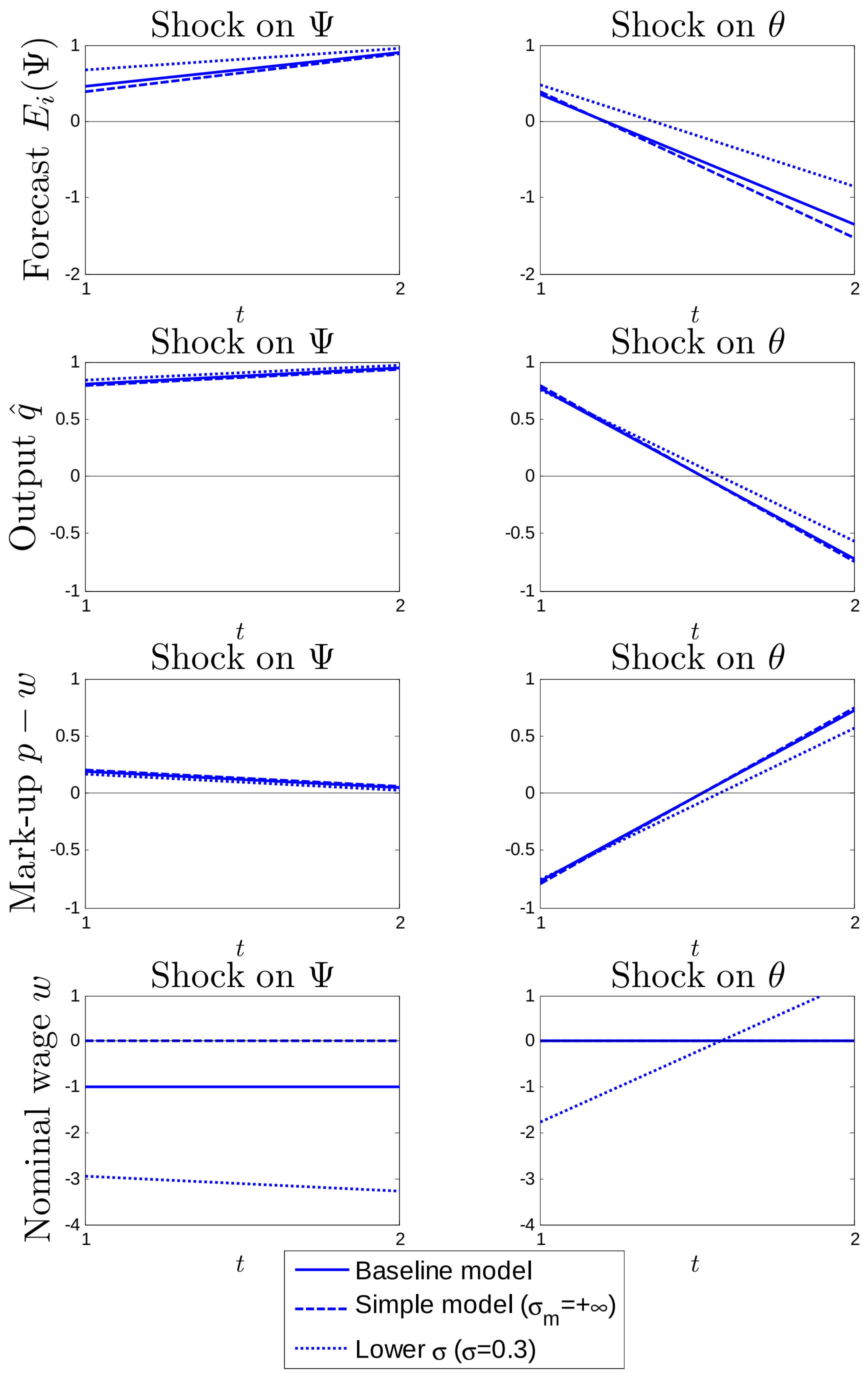

Figure 2: Impulse responses

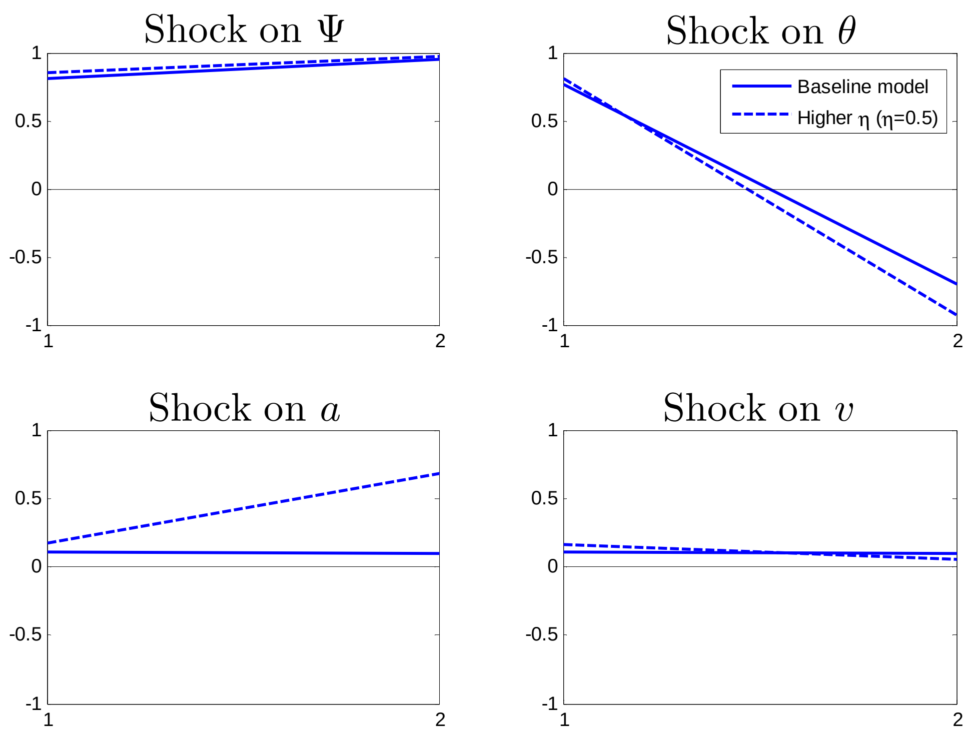

Figure 3: Impulse responses - With productivity shocks

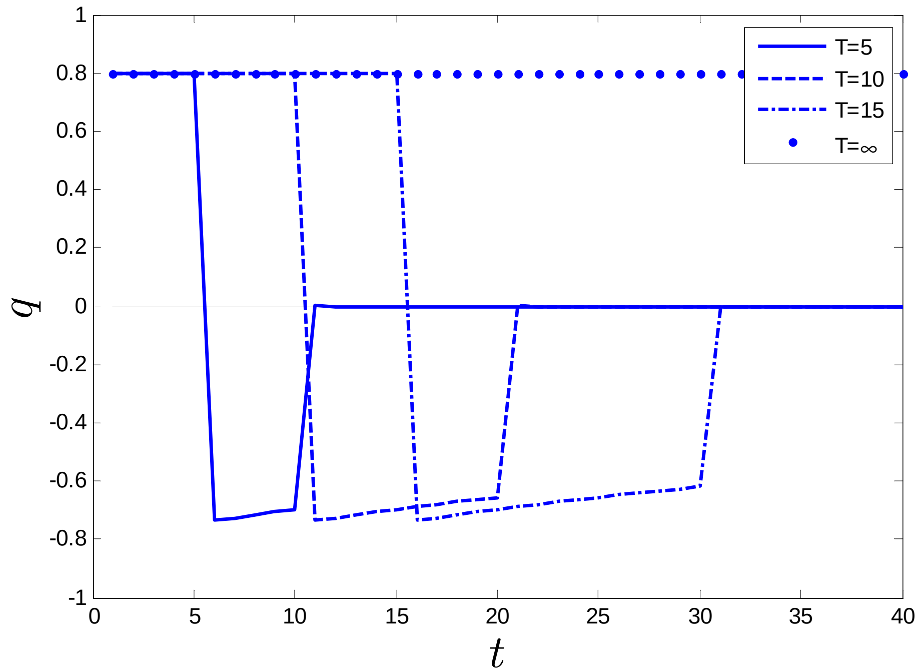

Figure 4: Impulse responses - Dynamic extension