BOHMIAN-TYPE QFT AND MALAMENT NO GO THEOREM

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- a) Single particles species, this is the disjoint union of the *n*-particle configuration space

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b) for several particles species, there is the Cartesian product of several copies of $\Gamma Q = \bigcup_{n=0}^{\infty} Q^{[n]}$, one for each species. One obtains a configuration space which is an union of sectors $Q^{[n]}$ where $n = (n_1, ..., n_\ell)$ is the ℓ -tuple of the particles number for a certain species of particles.

E.g.:

 $\mathsf{QED} \to \mathcal{Q}$ is a product of 3 copies of

$$\Gamma \mathcal{Q} = \bigcup_{n=0}^{\infty} \mathcal{Q}^{[n]}$$

Particles species involved: electrons, positrons and photons;

In this case, a configuration space specifies number and position of all e^- , e^+ , γ .

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In this case we could permute the particles without changing their world lines

NB: in the usual NRBM we cannot permute particles since they are labelled: if there is a permutation, it will entails a different trajectory for the particle;



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As a consequence, we can consider Ψ_t as a function on configuration space;

Here, as in NRBM, Ψ_t has a double role: it guides the particles' motion and determines the statistical distribution of the positions.

Particles have positions at any given time in a real physical space

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Possible configurations of particles have a representation in a position's space $\Gamma Q^{[n]}$;

• Ψ_t is a vector in an appropriate Fock space;

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- These events are the novel element respect standard bohmian trajectories, but trivially they are essential to explain processes involving particle creation and annihilation;
- NB: these events are intrinsically stochastic, so the evolution of a physical system in this theory will not be deterministic, in opposition to the non relativistic regime (here we will call these random events "jumps").





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- Where to jump;
- How to move between the jumps ——> deterministic evolution;
- The state vector evolves according to the Schrödinger equation;
- NB: generally speaking, world lines follow classical bohmian trajectories interrupted randomly by stochastic jumps which correspond to particle creation/annihilation events.

What do we mean when we say that a certain configuration follows classical bohmian trajectories?

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- The law of motion for Q_t depends on the state vector and on its Hamiltonian

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the "continuous" path of a world line is governed by a first-order differential equation which is very close to the bohmian guidance equation:

$$\frac{dQ_t}{dt} = v^{\Psi_t} = \operatorname{Re} \frac{\overline{\Psi_t} (Q_t)(\dot{q}\Psi_t)Q_t}{\overline{\Psi_t}(Q_t)\Psi_t(Q_t)}$$

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- □ In BQFT, as in standard QFT, Hamiltonian is a sum of terms:

$$H_{tot} = H_0 + H_{int}$$

It corresponds to a deterministic motion

given by a velocity field in configuration space

It defines the deterministic path of the world lines between the jumps;

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$$\sigma = \sigma(q', q, t) = \sigma^{\Psi(t)}(q', q)$$

 \Box σ is a transition from a configuration q to a configuration q' at time t;

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$$\sigma = \sigma(q',q,t) = \sigma^{\Psi(t)}(q',q)$$

- \Box σ is a transition from a configuration q to a configuration q' at time t;
- □ H_{tot} appears in the Schrödinger equation for Ψ , thus it becomes clear how the law of motion for the configuration of particles depends on Ψ and H_{tot} .

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- The jump rate, which depends on the configuration, on the state vector and on H_{tot} , is given by

$$\sigma(q',q,t) = \frac{2}{\hbar} \frac{(\mathrm{Im}\overline{\Psi(q)}\langle q|H_{int}|q'\rangle\Psi(q'))^{+}}{\overline{\Psi(q)}\Psi(q)}$$

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To sum up, the total Hamiltonian gives a "deterministic" motion with velocity v randomly interrupted by jumps with rate σ after each of which the deterministic motion is resumed (and again interrupted).

- $\hfill\square$ The possible jumps are very restricted and they can change the particles' number only by ± 1
- 1) appearance of a particle (e.g. emission)
- 2) disappearance of a particle (e.g. absorption)
- 3) replacement of one particle by two particles (creation)
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- \Box What is stochastic about Q_t ?
- a) The times at which jumps occur;
- b) The destination of the jumps;

The probabilities for a) and b) are governed by the wave function
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If $Q(t_0)$ is chosen at random with distribution $|\Psi(t_0)|^2$, then at every later time $t > t_0 Q(t)$ is distributed with density $|\Psi(t)|^2$

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- This process is equivariant, and it establishes the empirical equivalence between BQFT and standard QFT;
- NB: equivariance in BQFT is differently understood respect NRBM in virtue of the stochastic nature of the events which are observed at the QFT level.

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- □ Markovian processes involve transition probabilities from one configuration to another → this is completely adherent to the stochasticity of jumps rates for creation/annihilation events;

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- Markovian processes involve transition probabilities from one configuration to another → this is completely adherent to the stochasticity of jumps rates for creation/annihilation events;
- These probabilities are characterized by a linear operator called forward generator L_t
- $\hfill\square$ The distribution ρ_t of Q_t evolves according to

$$\frac{\partial \rho_t}{\partial t} = L_t \rho_t.$$

Generalization of equivariance from deterministic to Markovian processes:

given the transition probabilities, the $|\Psi|^2$ - distribution is equivariant iff for all times t and t' with t' > t the distribution Q_t with distribution $|\Psi|^2$ evolves into a configuration $Q_{t'}$ with distribution $|\Psi_{t'}|^2$.

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To close the circle: "A jump process is a Markov process on configuration space for which the only motion that occurs is via jumps".

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- Main claim: there cannot be a relativistic quantum theory of localizable particles;
- □ All talk about particles has to be understood as talk about fields → particles are defined as field's excitations → fields are the beables of the theory;
- Malament's aim: to show the physical impossibility to have a quantum filed theory with a particle ontology;
- The notion of "particle" entails some peculiar features among which its localizability: this result is concerned with this property

The theorem is about position measurement of a (supposed) localized particle in Minkowski spacetime and shows how we cannot localize a particle in this space, no matter where we perform the measurement;

NB: in SQM particles are not localized but localizable

- The arena: Minkowski spacetime;
- Let \mathcal{M} be a Minkowski spacetime, and let S be a family of parallel spacelike hyperplanes that cover \mathcal{M} ;
- Let us take a spatial set Δ to be any bounded open set within some particular S_i;
- \square NB: for the following argument it is no important how large is Δ ;

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- Let \mathcal{M} be a Minkowski spacetime, and let S be a family of parallel spacelike hyperplanes that cover \mathcal{M} ;
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- \square NB: for the following argument it is no important how large is Δ_i ;
- Definition of a quantum state:
- 1) \mathcal{H} Hilbert space, the rays of which represent the pure state of our system;
- 2) An assignment to each spatial set Δ of a projection operator P_{Δ} on \mathcal{H} ;
- 3) A unitary $\mathbf{a} \mapsto U(\mathbf{a})$ representation in \mathcal{H} of the translation group in $\mathcal{M}_{\mathbf{i}}$

- □ Here P_{Δ} means the event that the particle would be found in Δ if a particular detection experiment were performed;
- □ Conditions on the structure $(\mathcal{H}, \Delta \mapsto P_{\Delta}, \mathbf{a} \mapsto U(\mathbf{a}))$

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- 1) Translation Covariance Condition:

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where $P_{\Delta+a}$ is a result one obtains translating Δ by a_i ;

This condition means that the statistics of a measurement does not change with spatial translations: We can suppose to conduct the experiment not at its original site, but translated in another place, which is displaced from the first by **a**: $P_{\Delta+a}$ represents the event that if a particular experiment were performed, the particle would be found in $\Delta + a$.

2) Energy Condition:

For all future directed timelike vectors \mathbf{a} in \mathcal{M} , if $H(\mathbf{a})$ is the unique selfadjoint hamiltonian operator satisfying

$$U(t,\mathbf{a}) = e^{-iH(\mathbf{a})}$$

then the spectrum of $H(\mathbf{a})$ is bounded below;

This condition says that a particle has a ground energy state;

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3) Localizability Condition:

If Δ_1, Δ_2 are disjoint spatial sets of a single S, then

$$P_{\Delta_1}P_{\Delta_2} = P_{\Delta_2}P_{\Delta_1} = 0$$

this condition rules out the "infinite speed" possibility for a particle: a particle cannot be detected in two disjoints (spacelike related) spatial sets at the same given time;

4) Locality Condition:

If Δ_1, Δ_2 are spatial sets (not necessarily on the same hyperplane) that are spacelike related, then

$$P_{\Delta_1}P_{\Delta_2} = P_{\Delta_2}P_{\Delta_1}$$

Here it is imposed the "stamp of relativity", the independence of outcomes holds in time \rightarrow the statistical independence of position measurements is propagated and preserved in time!

- Claim: Malament's theorem
- If the structure $(\mathcal{H}, \Delta \mapsto P_{\Delta}, \mathbf{a} \mapsto U(\mathbf{a}))$ satisfies condition (1)-(4), then

 $P_{\Delta} = 0$, in every Δ

Conclusion: any candidate of a particle relativistic quantum theory that satisfies these conditions must predict that the probability to find the particle in any Δ is 0. Since this conclusion is not acceptable, the claim assumes the status of a no-go theorem for a particle ontology.

- Two options on the table:
- Malament's theorem shows how it is physically impossible to have a consistent theory with a particles ontology;
- 2) Bohmian QFT, a quantum field theory of particles which is empirically adequate since it recovers all the set of predictions of standard QFT;

- Two options on the table:
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- Questions:
- Could we apply Malament's result to the structure of Bohmian QFT?
- If it were not the case, which kind of arguments do we have to dismiss this theorem?

- Malament's theorem is based on the usual SQM formalism: the description of a physical system is provided only in terms of the wave function;
- The theorem is concerned with measurements of position operator, it is not about particles

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here is valid the same argument against SQM

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BM and BQFT have a richer and clearer ontological structure: we assume that our world is *made of particles*, our primitive variables, which have definite positions at any given time;

- \Box The "primitiveness" of PO \Rightarrow two different aspects:
- a) primitive variables are "irreducible" to others notions, they are not inferred o derived from other entities;
- explanatory function of the primitive variables: they are primitive because every physical object or phenomenon must be connected and explained in terms of PO;

- Realism about operators, two quotations:
- The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory at the most fundamental level. [...] Does not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these more fundamental concepts? (Bell, 1981)

Realism about operators, two quotations:

- The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory at the most fundamental level. [...] Does not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these more fundamental concepts? (Bell, 1981)
- Here are some words which, however legitimate and necessary in application, have no place in a formulation with any pretension to physical precision: system; apparatus; environment; microscopic, macroscopic; reversible, irreversible; observable; information; measurement. [...] The notions of "microscopic" and "macroscopic" defy precise definition. [...] Einstein said that it is theory which decides what is "observable". I think he was right. [...]"observation" is a complicated and theory-laden business. Then that notion should not appear in the formulation of fundamental theory. (Bell, 1990)

 Malament's argument excludes non local interactions, while BQFT incorporate non locality in its dynamical equation for the primitive variables

as in the non relativistic regime, the velocity of particles depends on the entire configuration

The point here is that within the bohmian framework one is not trying to "make the standard QFT a bohmian one", rather one is trying to built up a new physical theory with different features respect standard QFT;

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- The point here is that within the bohmian framework one is not trying to "make the standard QFT a bohmian one", rather one is trying to built up a new physical theory with different features respect standard QFT;
- □ In conclusion we have a different structure for the definition and the description of a quantum system than $(\mathcal{H}, \Delta \mapsto P_{\Delta}, \mathbf{a} \mapsto U(\mathbf{a}))$

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a description which includes physical assumptions about the primitive ontology;

Conclusions

- BQFT is a counterexample to Malament's theorem, so it is possible to have a theory with a particle ontology;
- BQFT is however an effective theory: we have to introduce cut offs (UV and IR) in order to have well defined Hamiltonians;

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- BQFT is however an effective theory: we have to introduce cut offs (UV and IR) in order to have well defined Hamiltonians;
- □ Here effective means only that this theory is not strictly speaking a fundamental theory → e.g. gravitational phenomena are not explained (this is also the case of any QFT)

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Effectiveness has not to be regarded as a negative feature: it says only that a certain theory has limits in its applications: we cannot say anything under certain physical energies!

Conclusions

 Lorentz invariance: BQFT requires a preferred reference frame, therefore it is not Lorentz invariant

However, the theory is empirically equivalent to a Lorentz-invariant theory

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Experimentally there are no possibilities for an observer to determine which frame is the preferred one, so the BQFT's empirical predictions are Lorentz invariant

BQFT is equivalent to a fully Lorentz-invariant theory.

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- □ Figure 1 and figure 2 (R. Tumulka) have been taken from DGTZ 2004.