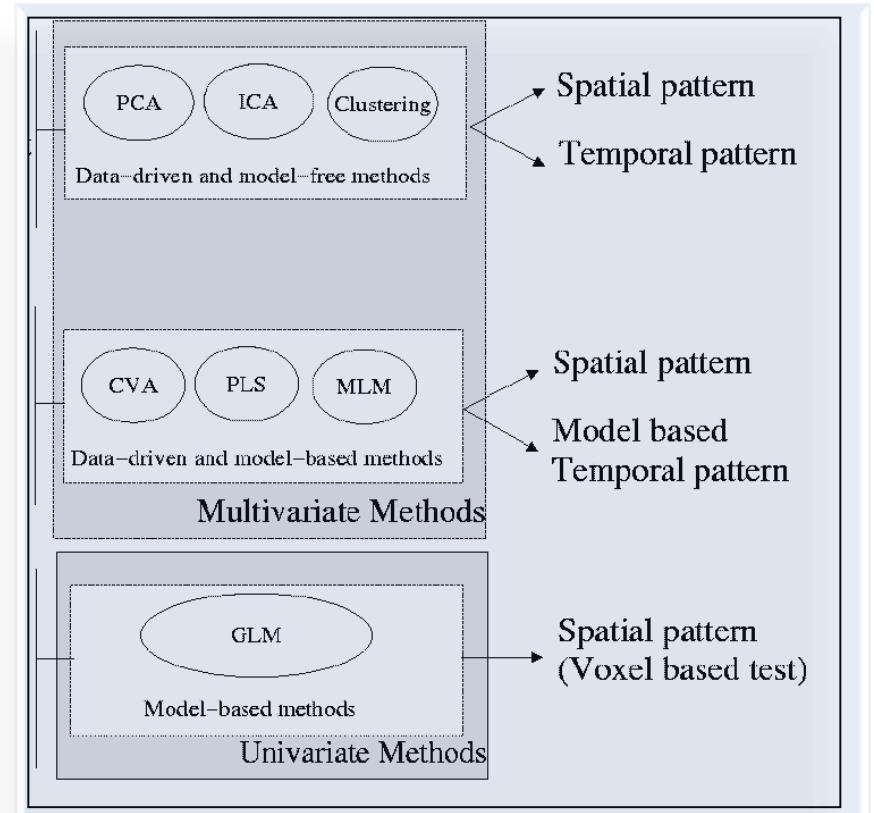
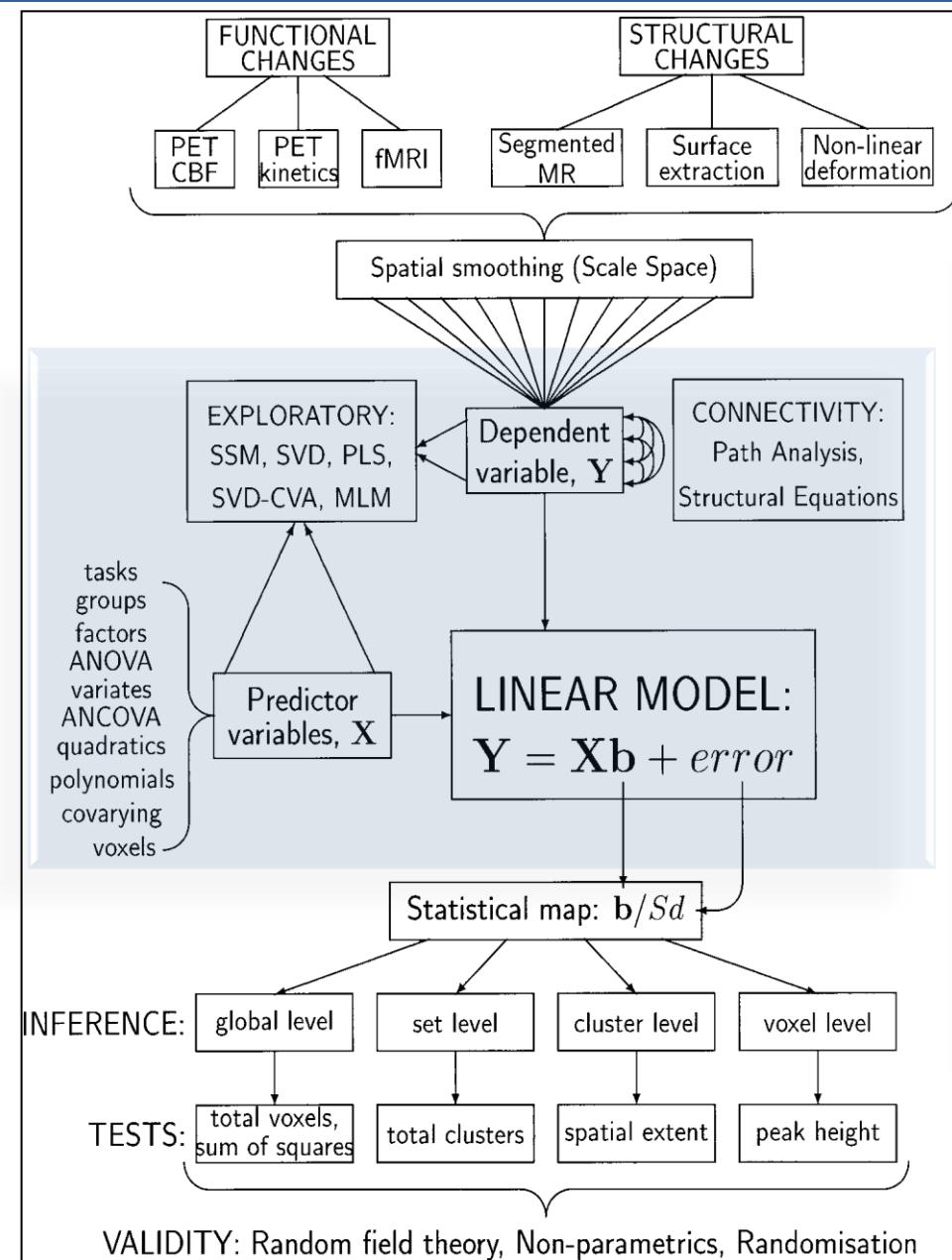


# Multivariate Methods

## Some applications

Ferath Kherif

# Multivariate methods



Kherif et al, 2003

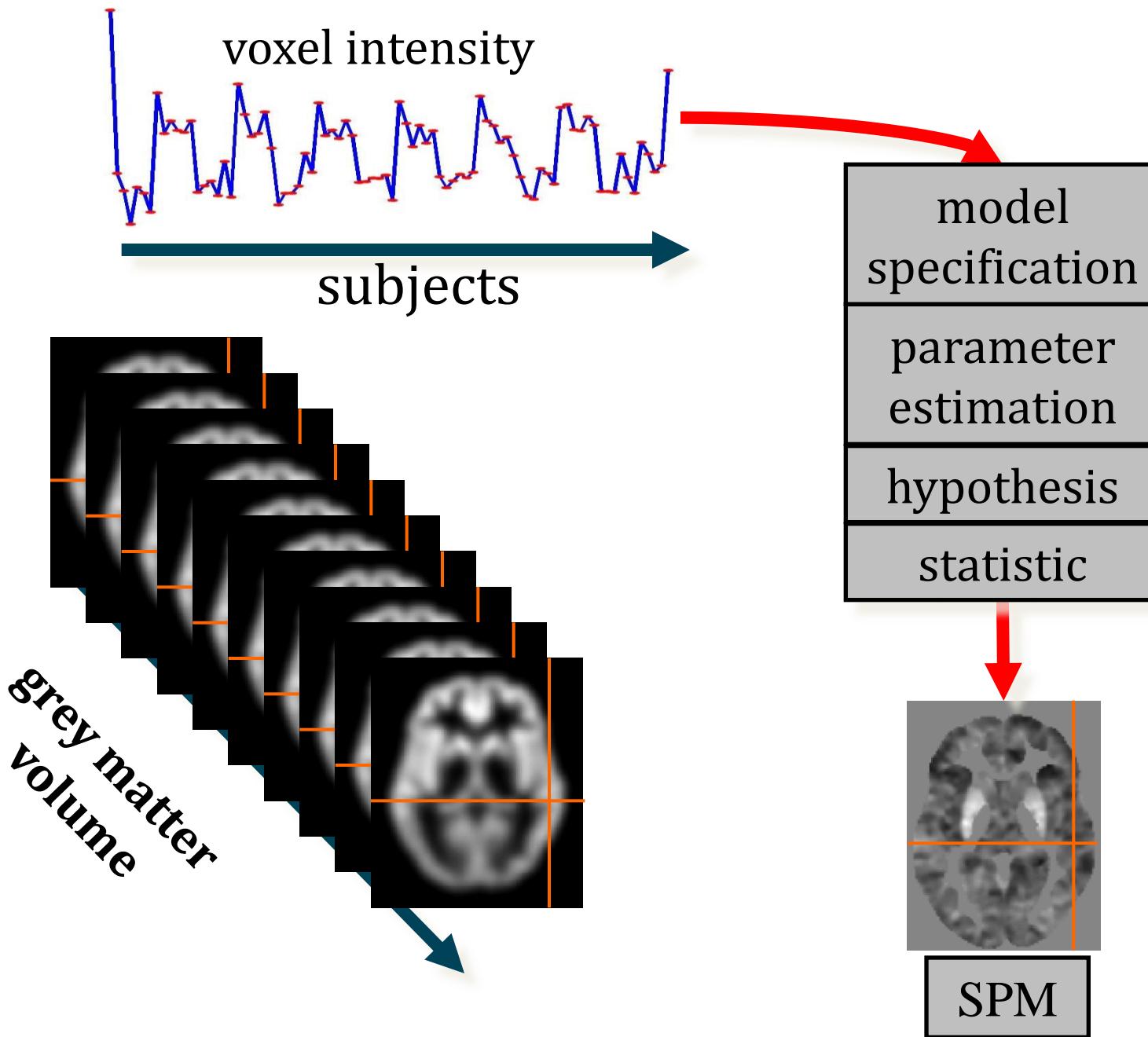
From K Worsley, 2001

- **Encoding model : generative model**  
relates context (independent variable) to  
brain observation(dependent variable).
$$X \longrightarrow Y$$
- **Decoding model : recognition model**  
relates brain outcome (independent  
variable) to context (dependent variable).
$$Y \longrightarrow X$$
- **Integration model : calibration model**  
relates brain activity to other exogenous  
variables.
$$Y \longleftrightarrow X$$

# Multivariate methods

- **Encoding model : generative model**  
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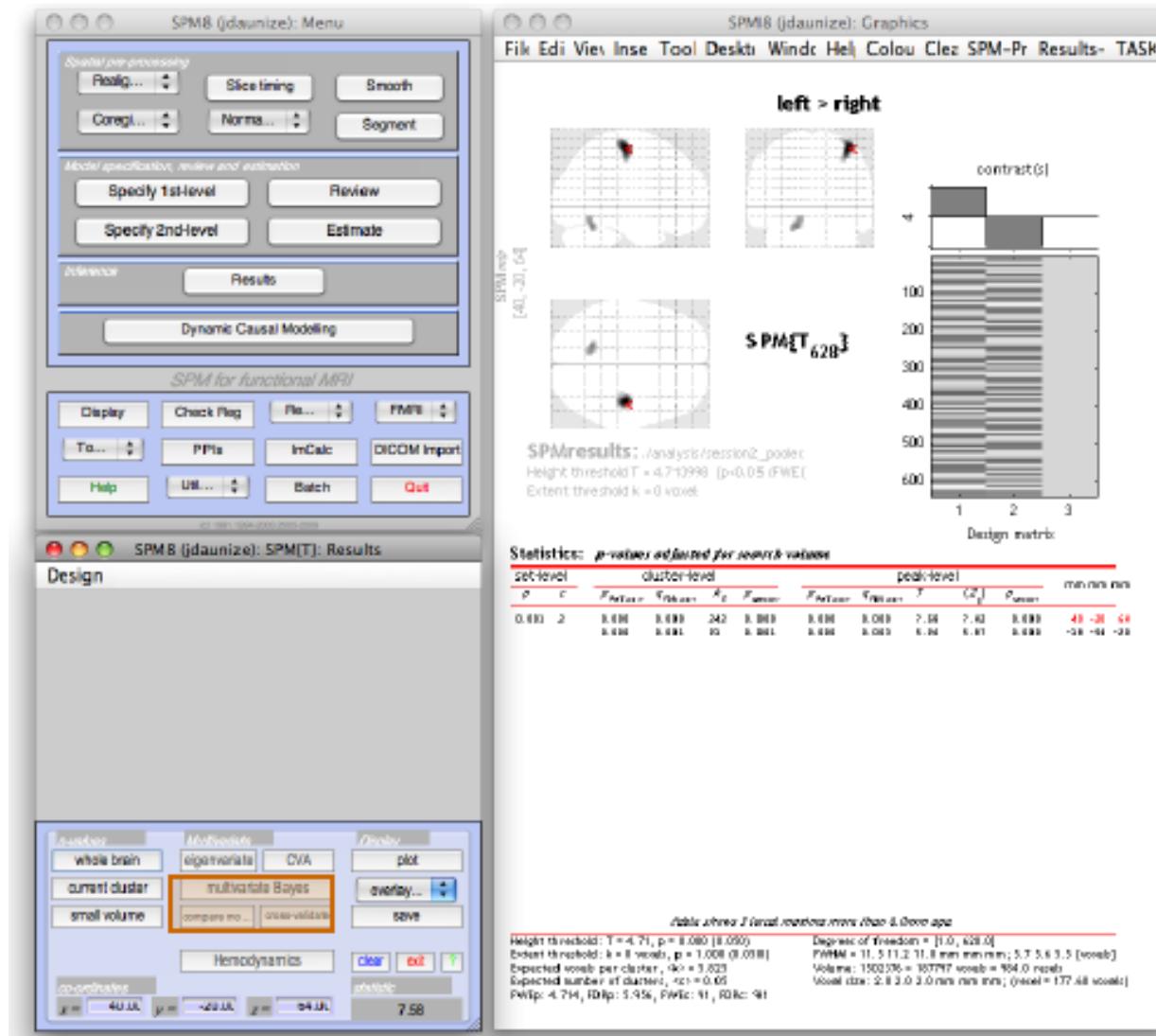
# Voxel-Based Morphometry - SPM



# Multivariate methods

- **Encoding model : generative model**  
relates context (independent variable) to  
brain activity (dependent variable).  $X \longrightarrow Y$
- **Decoding model : recognition model**  
relates brain activity (independent  
variable) to context (dependent variable).  $Y \longrightarrow X$
- **Integration model : calibration model**  
relates brain activity to context.  $Y \longleftrightarrow X$

# Multivariate methods



- target:

$$TX = Xc$$

design matrix

contrast

- confounds:

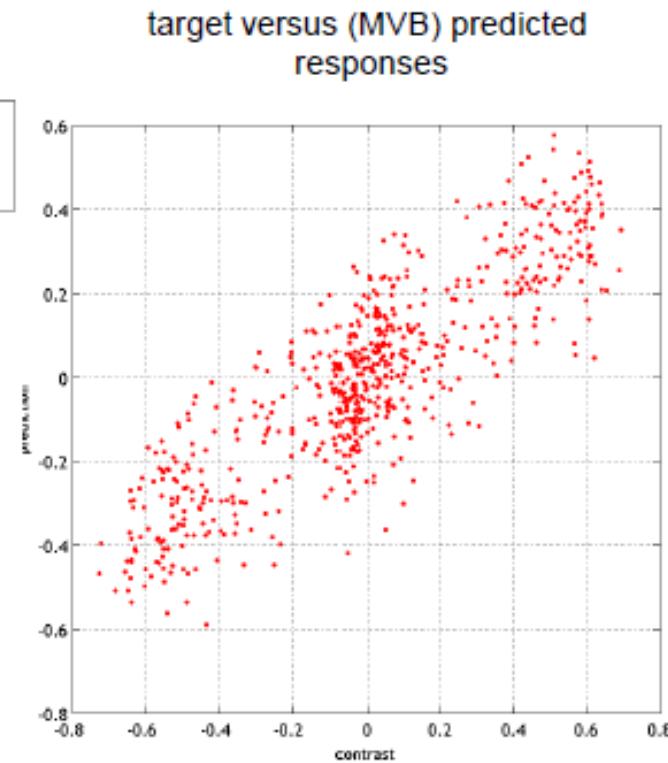
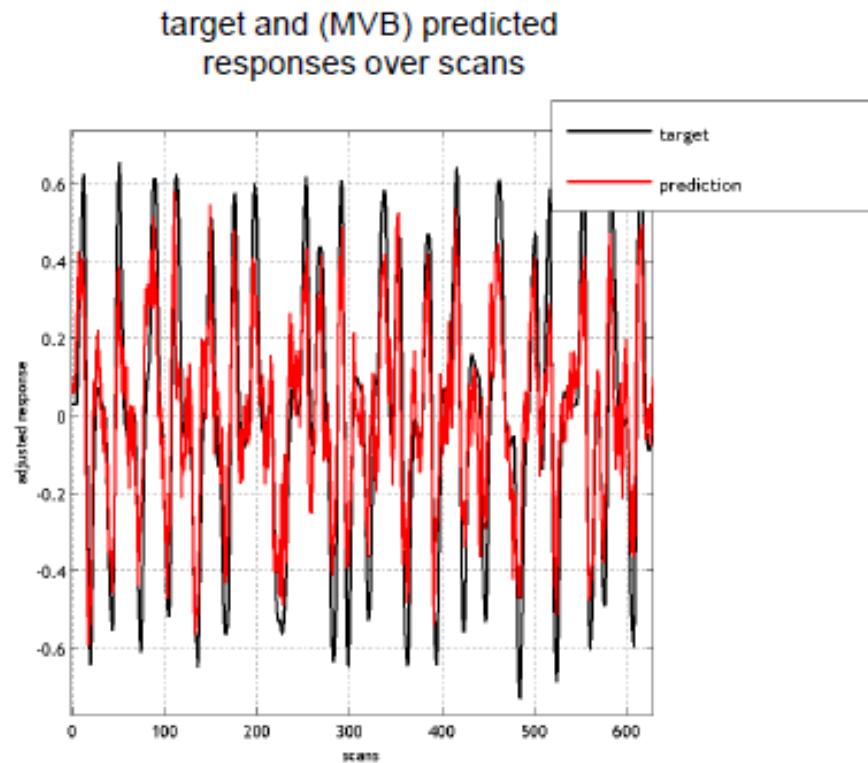
$$G = \mathbf{X}(I - cc^T)$$

$$\Rightarrow Gc = 0$$

## Example

predicted responses from left & right motor cortices

- MVB-based predictions closely match the observed responses. But crucially, they don't perfectly match them. Perfect match would indicate overfitting.

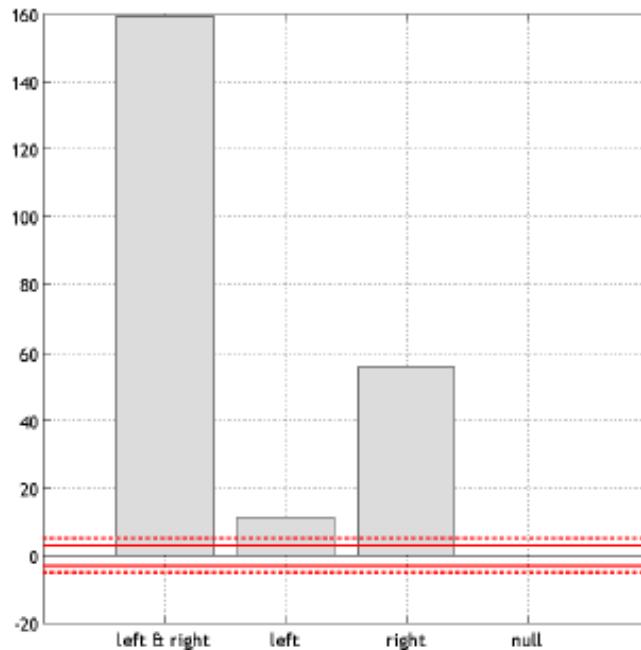


## Example

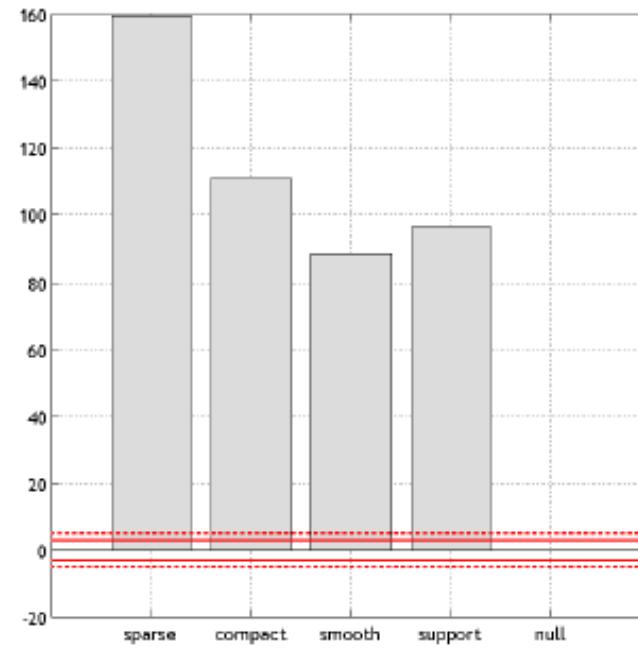
### model comparison illustration

- The best model corresponds to a sparse representation of motion ; as one would expect from functional segregation.

log-evidence of X-Y sparse mappings:  
effect of lateralization



log-evidence of X-Y bilateral mappings:  
effect of spatial deployment



# Multivariate methods

- **Encoding model : generative model**  
relates context (independent variable) to  
brain observation(dependent variable).

$$X \longrightarrow Y$$

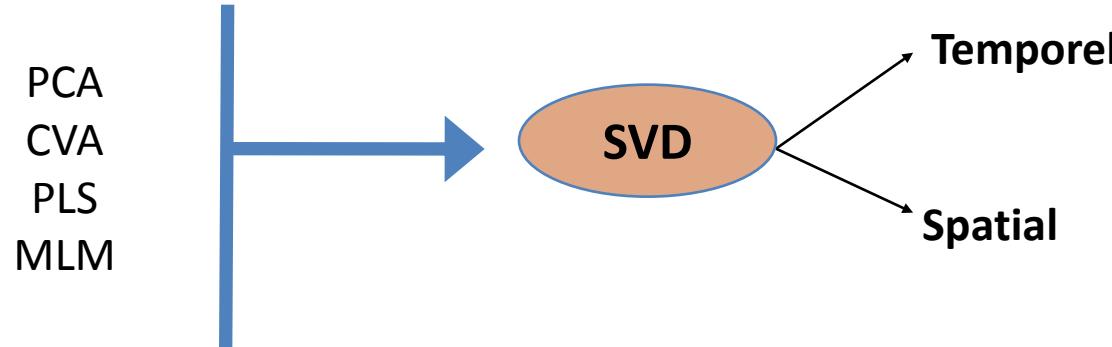
- **Decoding model : recognition model**  
relates brain outcome (independent  
variable) to context (dependent variable).

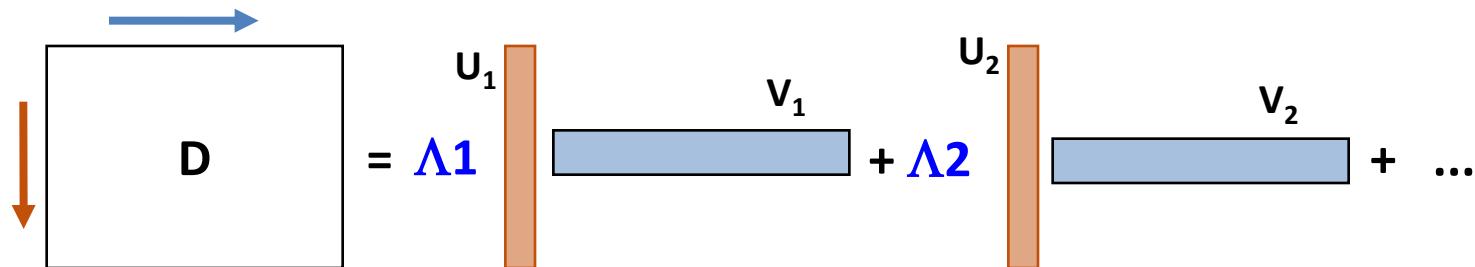
$$Y \longrightarrow X$$

- **Integration model : calibration model**  
relates brain activity to other exogenous  
variables.

$$Y \longleftrightarrow X$$

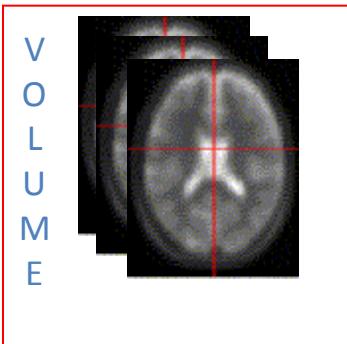
# Singular value decomposition : SVD





A diagram illustrating the Singular Value Decomposition (SVD) of a matrix  $D$ . A large white rectangle labeled  $D$  has a blue arrow pointing to its top-right corner and an orange arrow pointing to its bottom-left corner. To the right of the matrix, the equation  $D = \Lambda 1 \begin{matrix} U_1 \\ V_1 \end{matrix} + \Lambda 2 \begin{matrix} U_2 \\ V_2 \end{matrix} + \dots$  is shown. The term  $\Lambda 1$  is in blue. The matrices  $U_1$  and  $V_1$  are represented by vertical orange bars;  $U_1$  is positioned above  $V_1$ . Similarly,  $U_2$  and  $V_2$  are represented by vertical orange bars positioned above  $V_2$ .

# Multivariate methods

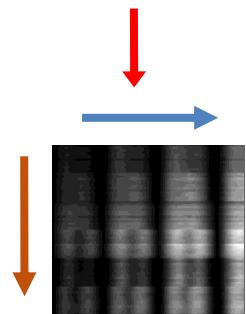


- PCA :  $\mathbf{Y}$  (time x voxels)

PLS :  $\mathbf{X}'\mathbf{Y}$  (param x voxels)

CVA :  $(\mathbf{X}'\mathbf{X})^{-1/2}\mathbf{X}'\mathbf{Y}(\mathbf{R})^{-1/2}$  (param x voxels)

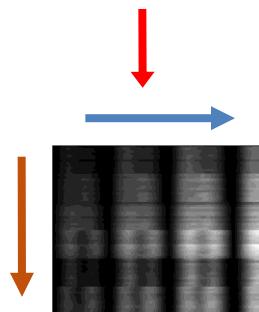
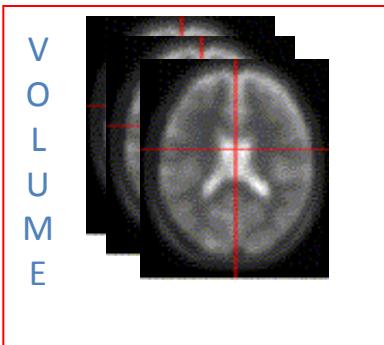
MLM :  $(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1/2}\mathbf{X}'\mathbf{Y}(\mathbf{R}^*)^{-1/2}$  (param x voxels)



$$\begin{aligned}
 & \xrightarrow{\hspace{2cm}} \\
 & = \Lambda_1 \begin{matrix} \mathbf{U}_1 \\ \end{matrix} + \Lambda_2 \begin{matrix} \mathbf{U}_2 \\ \end{matrix} + \dots
 \end{aligned}$$

$\mathbf{U}_1$        $\mathbf{v}_1$        $\mathbf{U}_2$        $\mathbf{v}_2$

# Multivariate methods

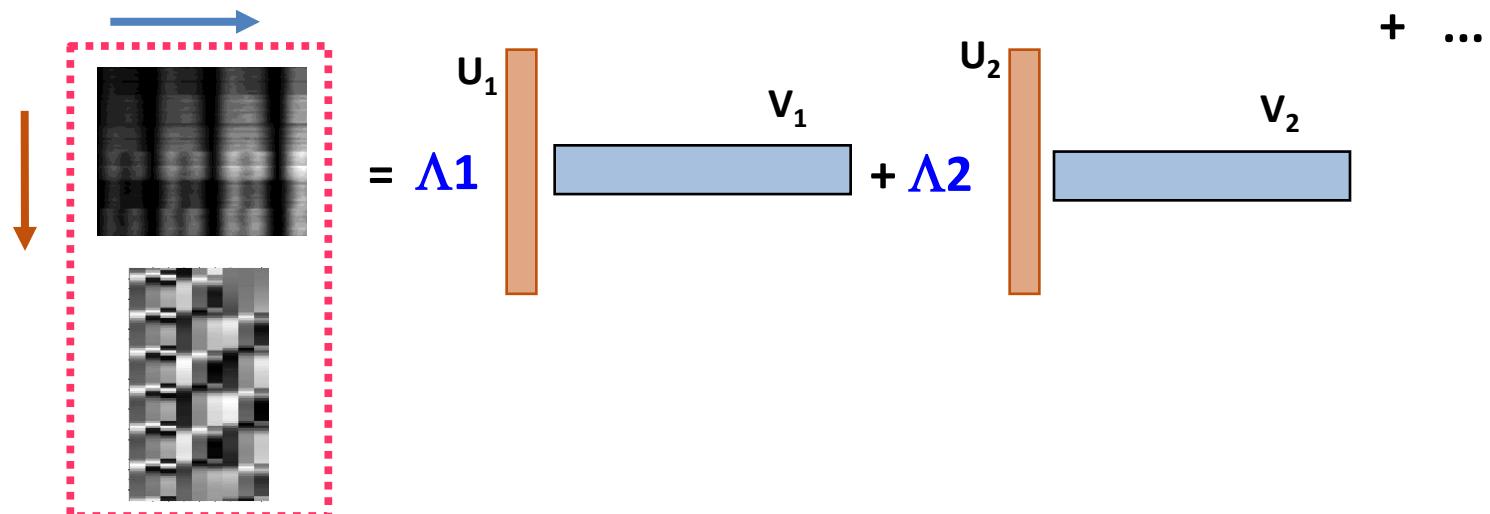


PCA :  $\mathbf{Y}$  (times x voxels)

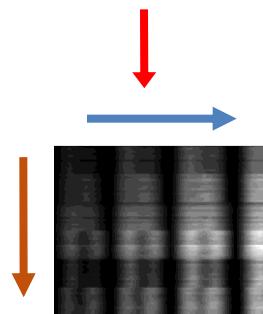
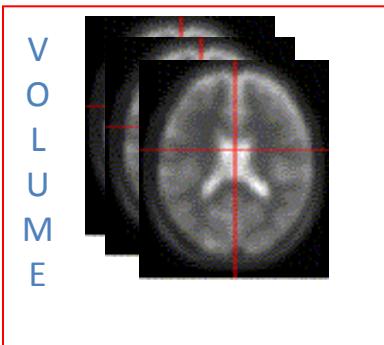
• PLS :  $\mathbf{X}'\mathbf{Y}$  (param x voxels)

CVA :  $(\mathbf{X}'\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R})^{-1/2}$  (param x voxels)

MLM :  $(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R}^*)^{-1/2}$  (param x voxels)



# Multivariate methods



PCA :  $\mathbf{Y}$  (times x voxels)

PLS :  $\mathbf{X}'\mathbf{Y}$  (param x voxels)

•CVA :  $(\mathbf{X}'\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R})^{-1/2}$  (param x voxels)

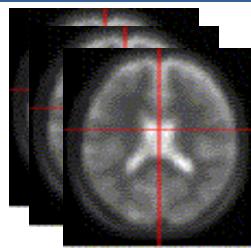
MLM :  $(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R}^*)^{-1/2}$  (param x voxels)

$$\mathbf{U}_1 \quad \mathbf{V}_1 = \Lambda_1 + \mathbf{U}_2 \quad \mathbf{V}_2 + \dots$$

$\mathbf{R}$ , voxels x voxels residual matrix

# Multivariate methods

V  
O  
L  
U  
M  
E

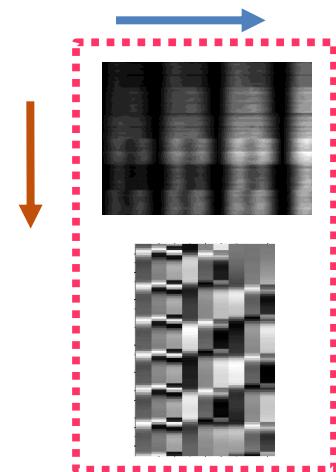
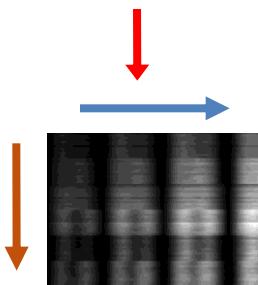


PCA :  $\mathbf{Y}$  (times x voxels)

PLS :  $\mathbf{X}'\mathbf{Y}$  (param x voxels)

CVA :  $(\mathbf{X}'\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R})^{-1/2}$  (param x voxels)

• MLM :  $(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1/2} \mathbf{X}'\mathbf{Y} (\mathbf{R}^*)^{-1/2}$  (param x voxels)



$$= \Lambda_1 \begin{matrix} \mathbf{U}_1 \\ \vdots \end{matrix} + \Lambda_2 \begin{matrix} \mathbf{U}_2 \\ \vdots \end{matrix} + \dots$$

Empirical (PEB) priors on voxel-weights

$$\begin{aligned} \mathbf{W}\mathbf{X} &= \mathbf{R}\mathbf{Y}\beta + \zeta \\ \text{cov}(\beta) &= \mathbf{U}\Sigma^2\mathbf{U}^T \end{aligned}$$

empirical priors



Null:  $\mathbf{U} = \emptyset$

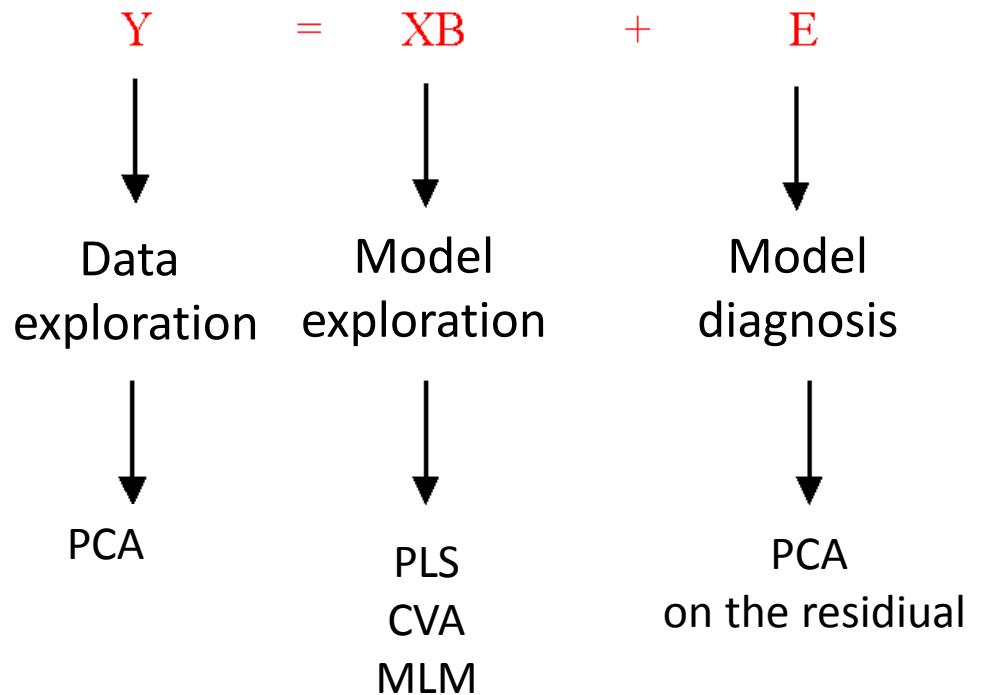
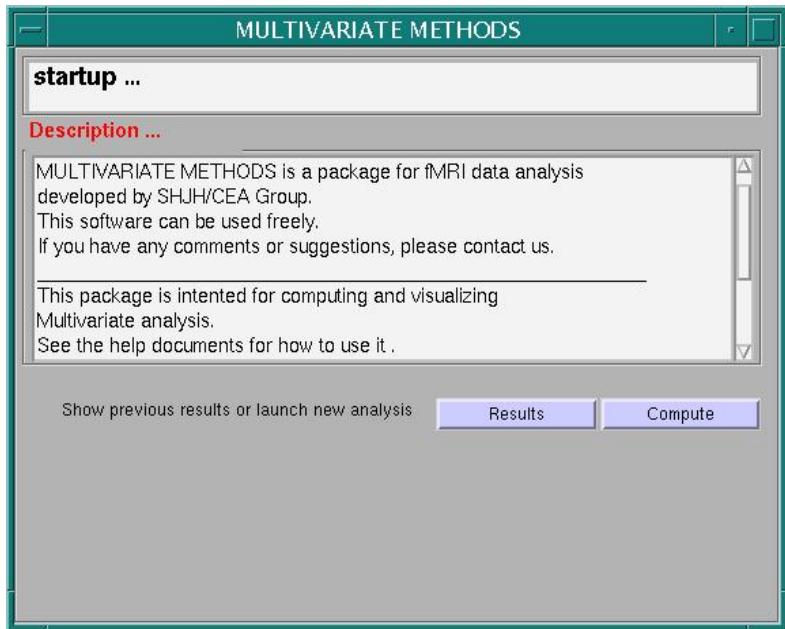
Spatial vectors:  $\mathbf{U} = \mathbf{I}$

Smooth vectors:  $\mathbf{U}(\vec{x}_i, \vec{x}_j) = \exp(-\frac{1}{2}(\vec{x}_i - \vec{x}_j)^2 \sigma^{-2})$

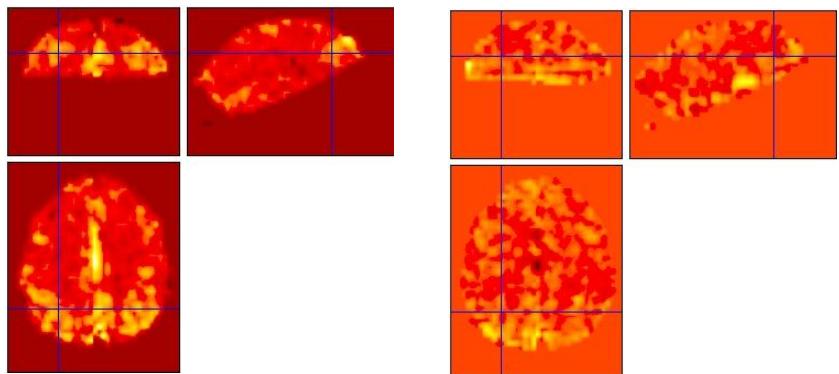
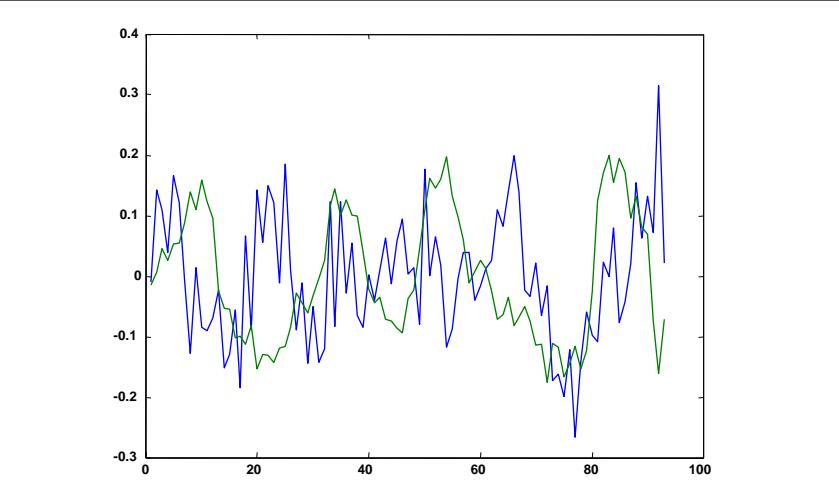
Singular vectors:  $\mathbf{U}\mathbf{D}\mathbf{V}^T = \mathbf{R}\mathbf{Y}^T$

Support vectors:  $\mathbf{U} = \mathbf{R}\mathbf{Y}^T$

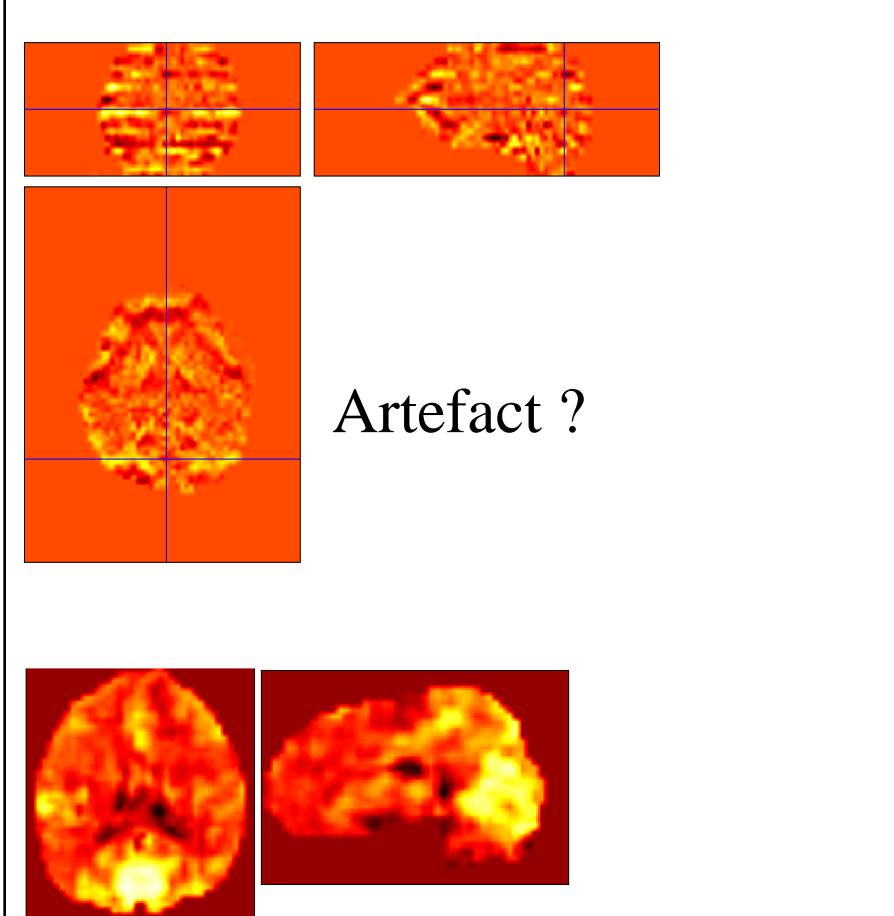
# Multivariate methods



# Multivariate methods

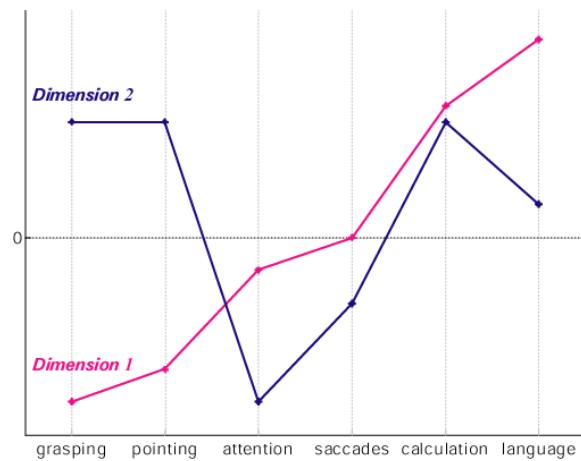
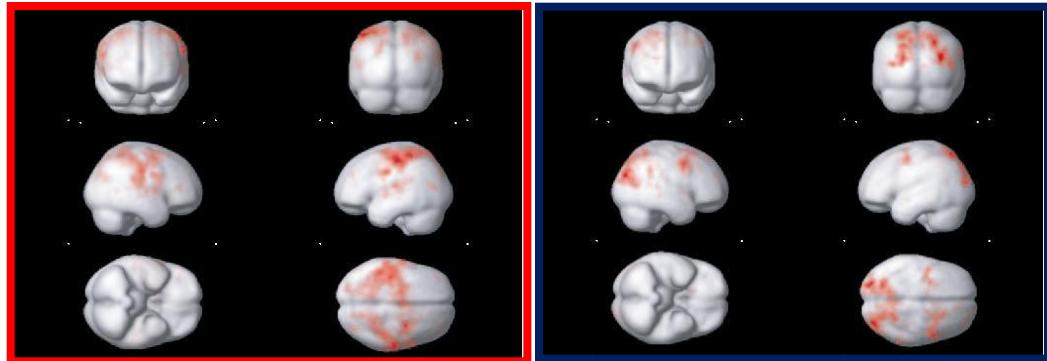
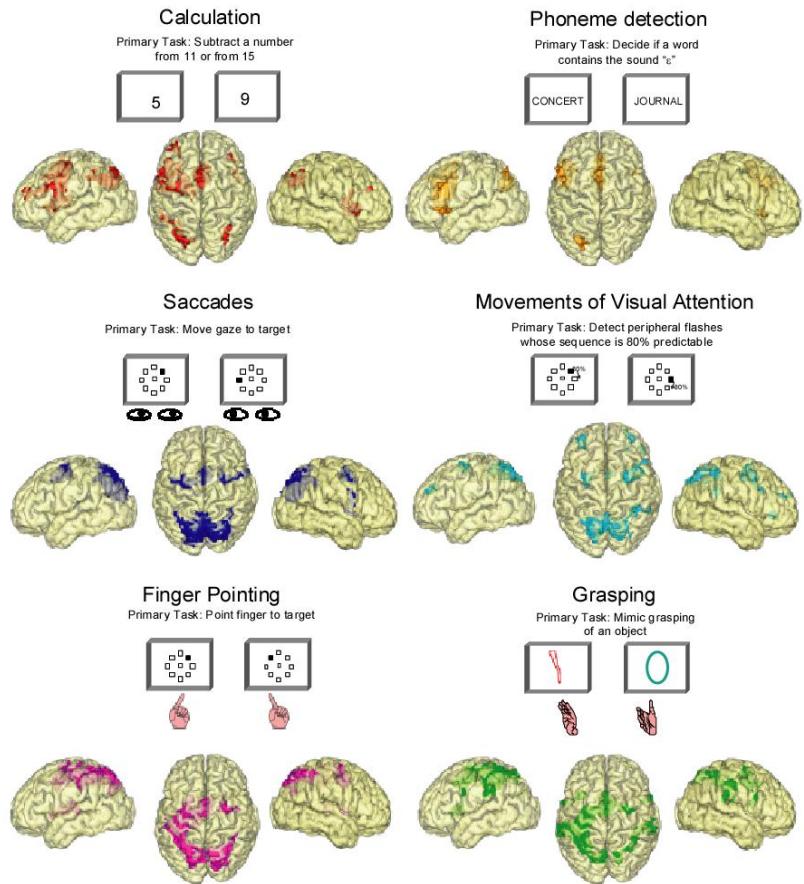


activation



Activation, resting state?

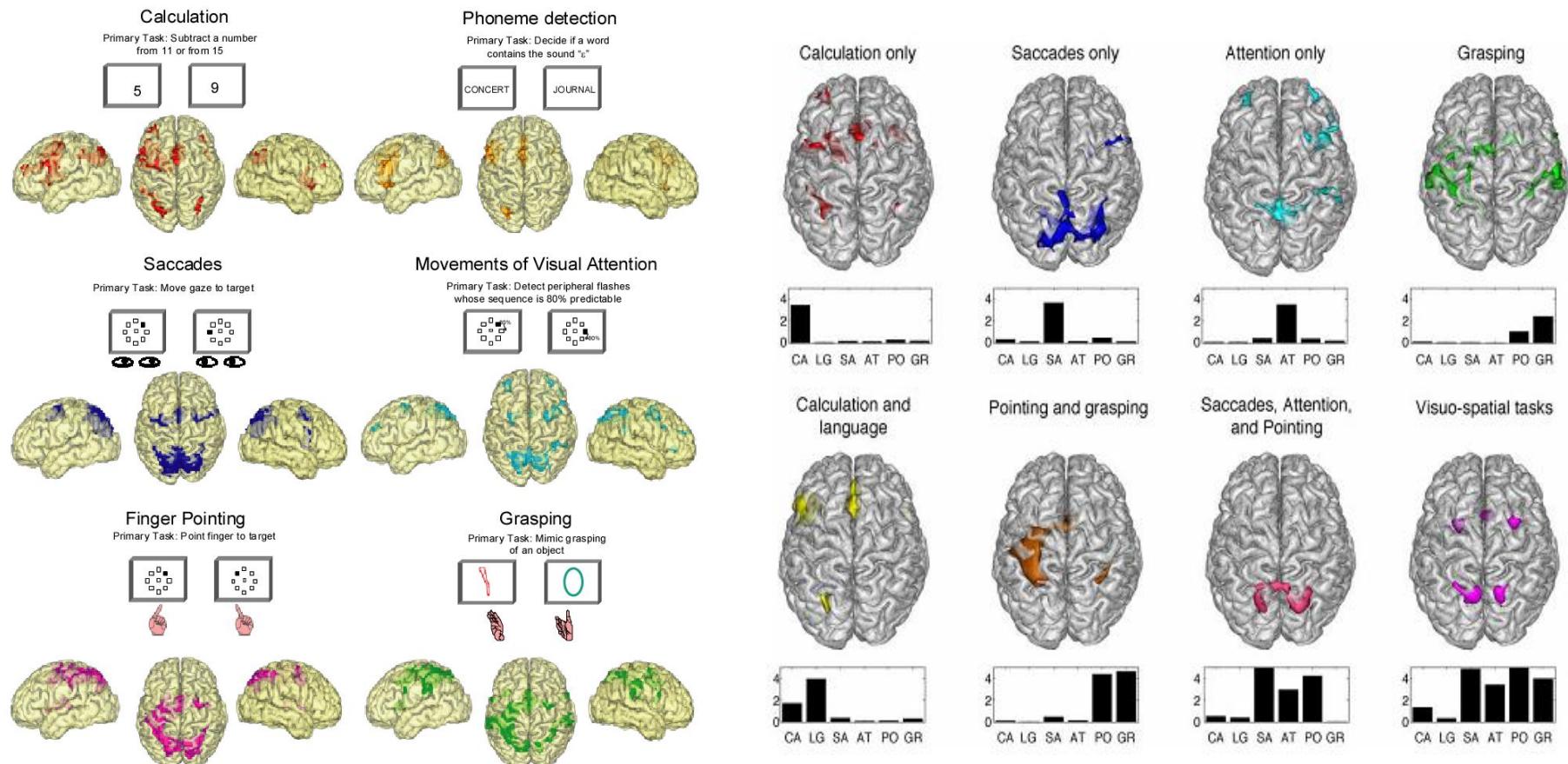
# Multivariate methods



SPM analyses

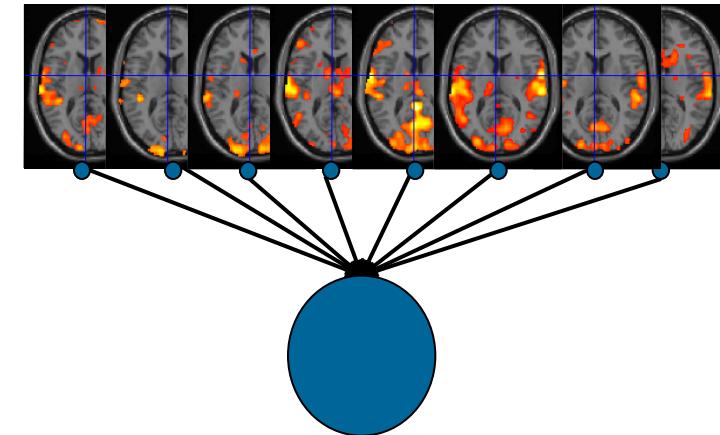
Networks

# Multivariate methods



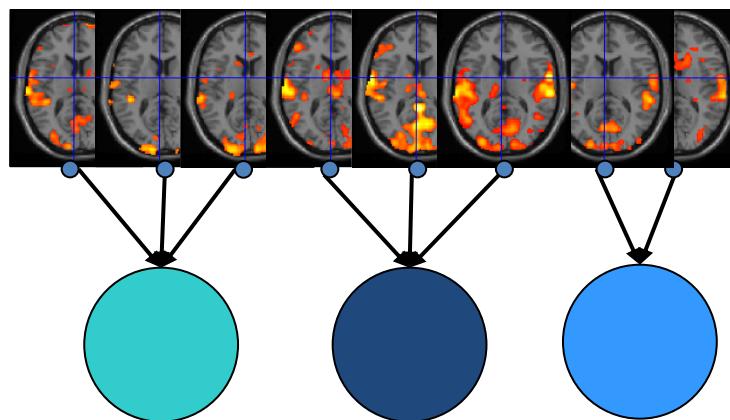
SPM analyses

Spatial clustering



## Typical Group Analysis

Assumes that subjects are drawn from the same and unique population.



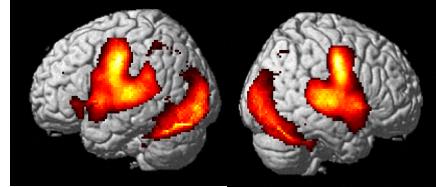
## Individual difference Models:

Assume that subjects are drawn from a multimodal population.

- 1) *Apriori* comparisons: to look for expected differences.
- 2) Unbiased classification: to reveal hidden (unexpected) differences

## Variability in language system

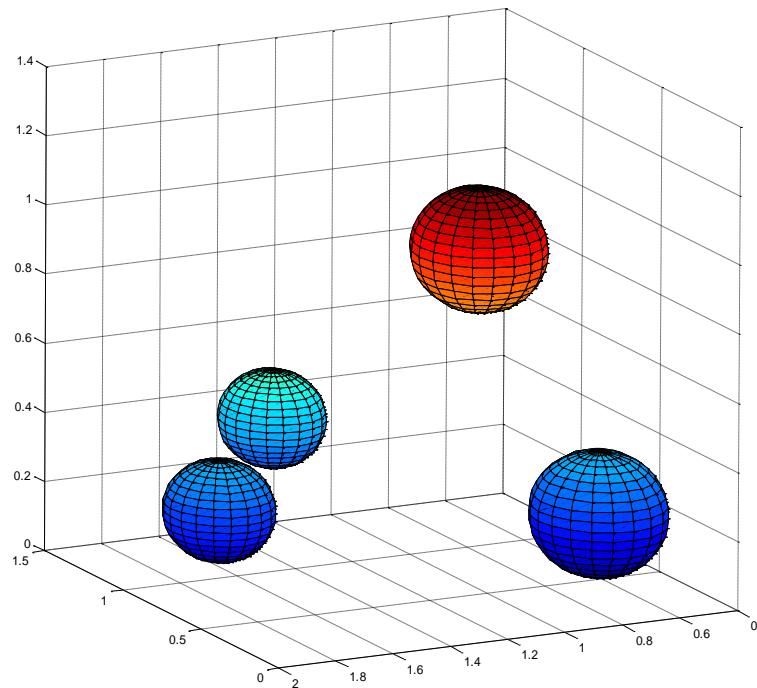
Common to all 4 groups



Pars Orbitalis (Group 4 > others)

Dorsal Premotor (Group 3 > others);

Pars Opercularis (Group 2 > others);



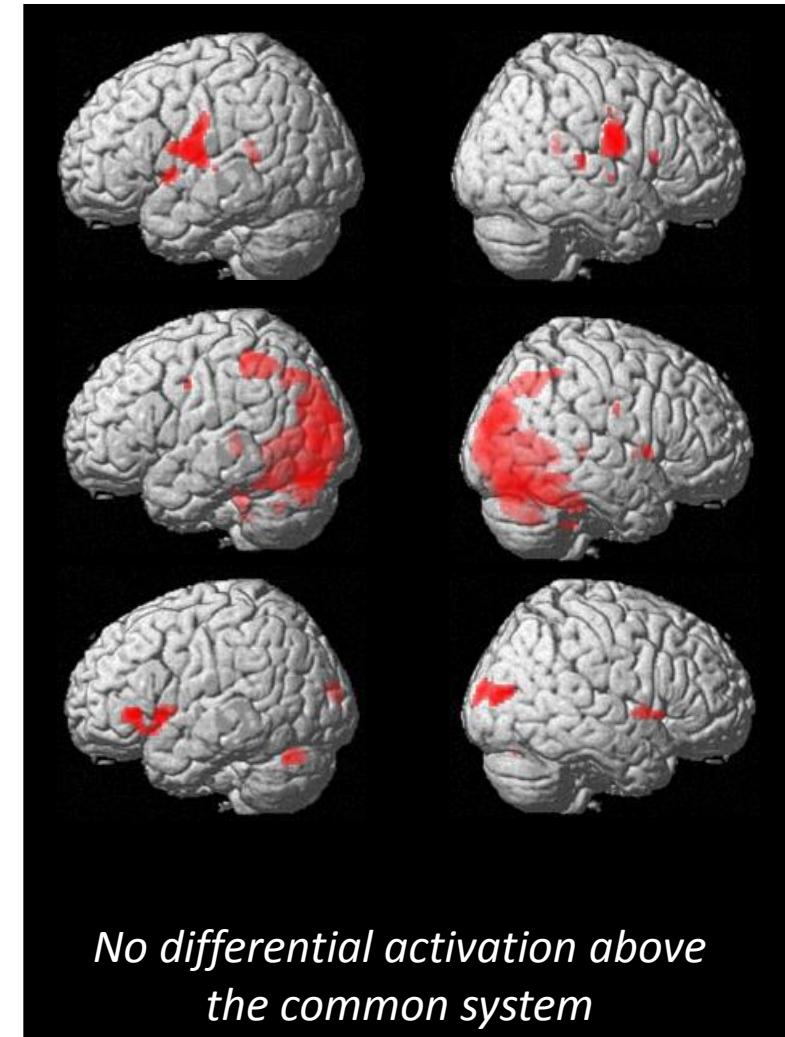
Group 2  
>others

Group 3  
>others

Group 4  
>others

Group 1  
> others

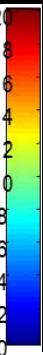
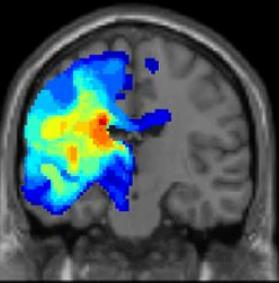
Group Differences



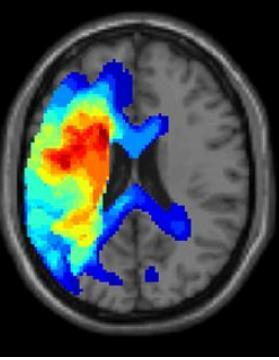
The healthy controls (HC) varied in term of age, gender and handedness

The patients were divided in two groups (PL, PC) depending on whether their lesion damaged an area that was activated in the healthy control group.

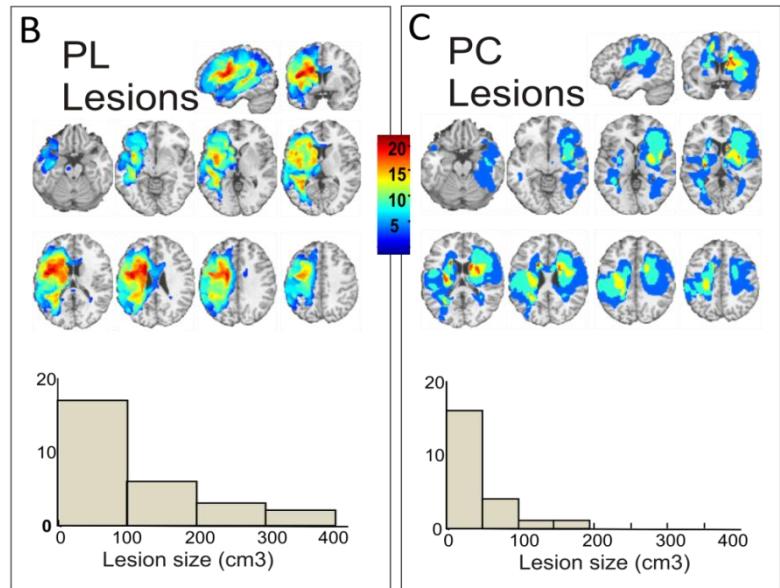
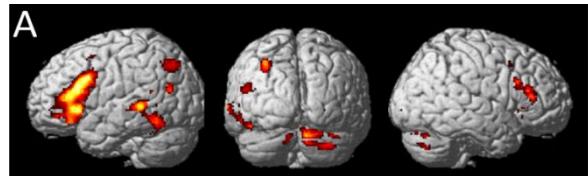
HC : semantic > perceptual



PL : Lesion  
Overlap  
Map



- 80 healthy controls (HC)
- The 50 patients were divided in two groups :
- PC : 22 Patients Controls with no damage to the “semantic areas”
- PL : 28 patients with Lesions in the “semantic areas” :
  - frontal (n=11),
  - posterior temporal (n=7),
  - subcortical (n= 5)
  - or premotor areas (n=5).



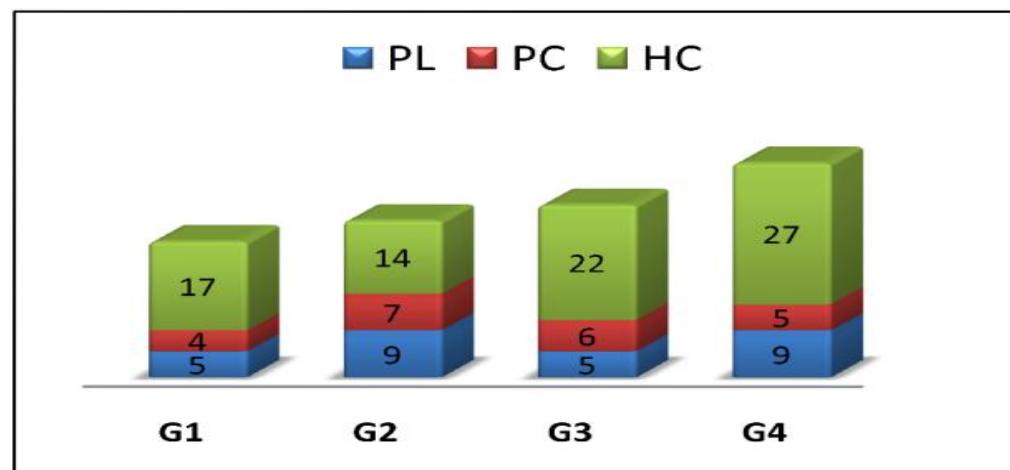
Remarkably, the subgroups had an equivalent mix of patients and controls and an equivalent mix of PL-patients and PC-patients.

GMM-group 1: 5 PL ; 4 PC; 17 HC .

GMM-group 2: 9 PL; 7 PC; 14 HC.

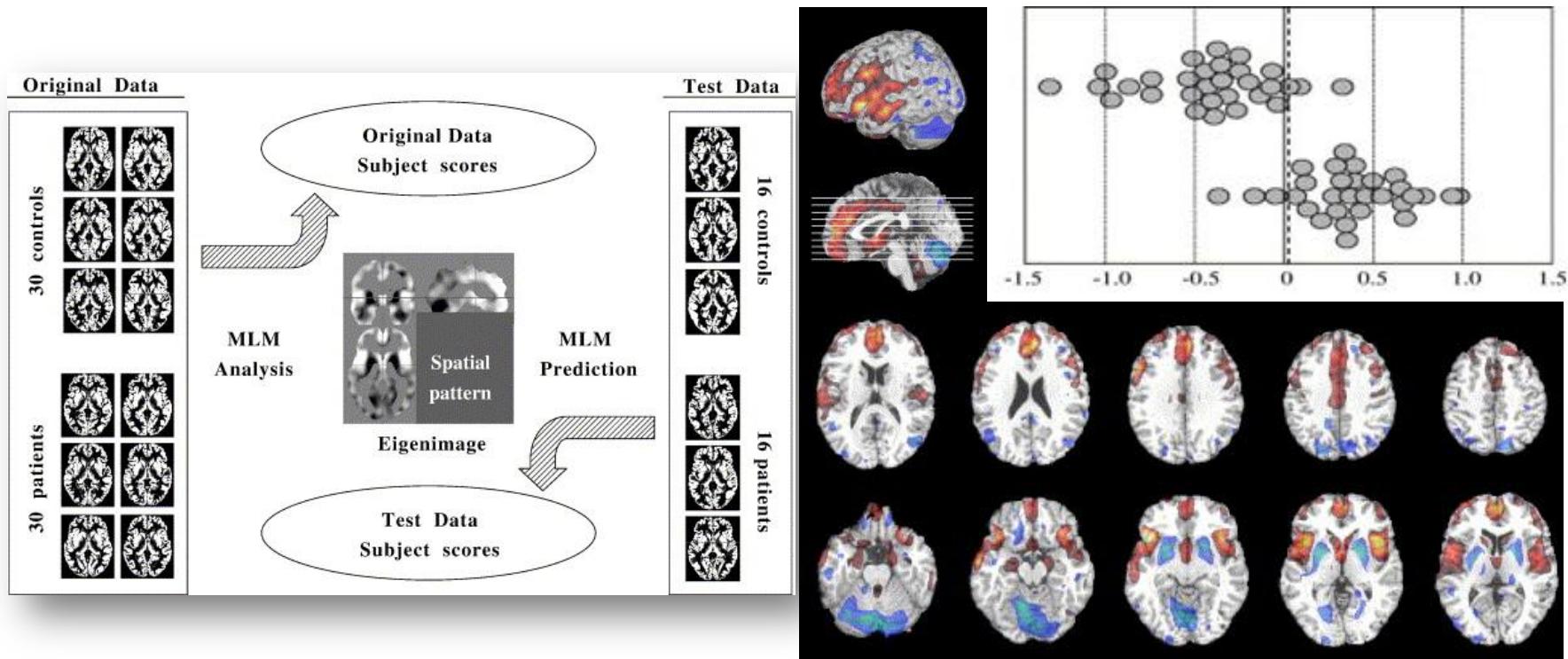
GMM-group 3: 5 PL; 6 PC; 22 HC.

GMM-group 4: 9 PL; 5 PC; 27 HC.



# VBM Schizophrenia

Multivariate VBM successfully differentiates schizophrenia patients from healthy controls



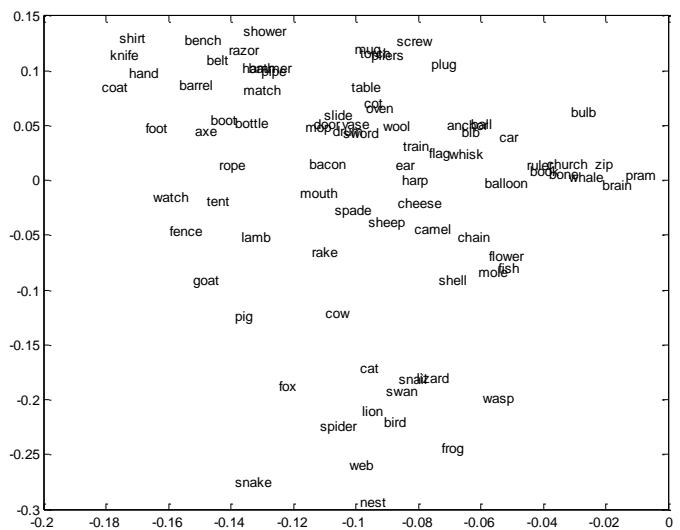
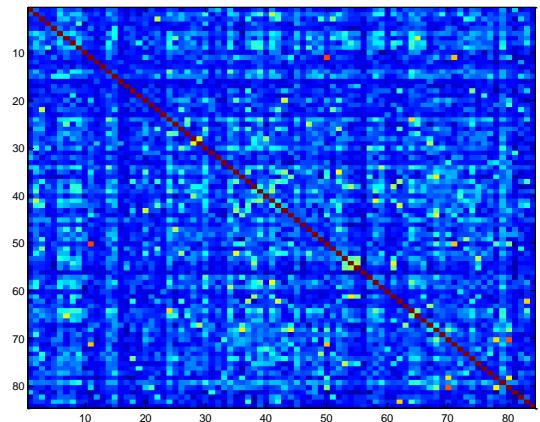
## Question: how Concepts are represented in the brain

### Comparing words and picture allow to answer this question

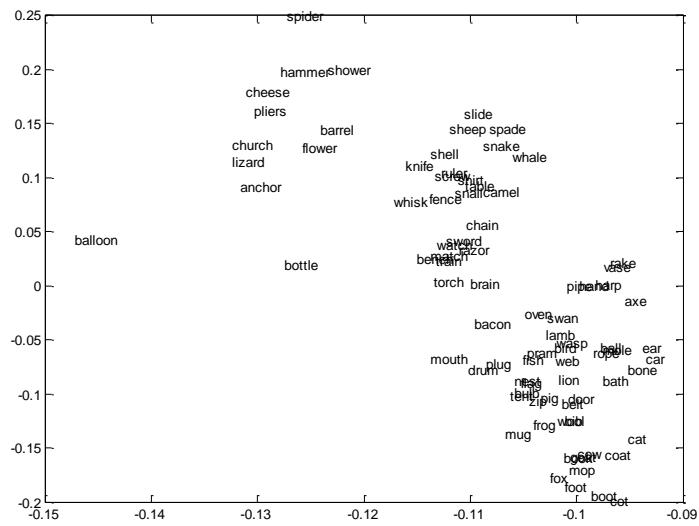
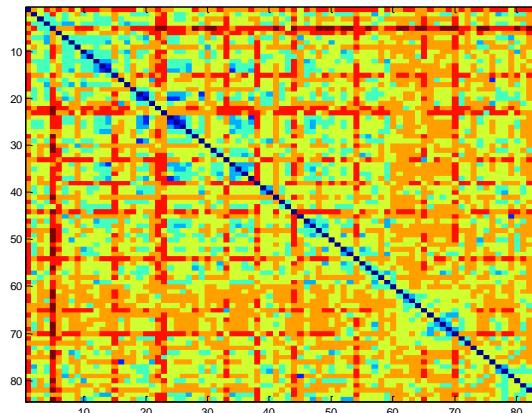
'anchor' 'bench' 'bulb' 'cot' 'flower' 'knife' 'mug'  
'axe' 'bib' 'camel' 'cow' 'foot' 'lamb' 'nest'  
'bacon' 'bird' 'car' 'door' 'fox' 'lion' 'oven'  
'ball' 'bone' 'cat' 'drum' 'frog' 'lizard' 'pig'  
'balloon' 'book' 'chain' 'ear' 'goat' 'match' 'pipe'  
'barrel' 'boot' 'cheese' 'fence' 'hammer' 'mole' 'pliers'  
'bath' 'bottle' 'church' 'fish' 'hand' 'mop' 'plug'  
'belt' 'brain' 'coat' 'flag' 'harp' 'mouth' 'pram'

'rake' 'shower' 'table'  
'razor' 'slide' 'tent'  
'rope' 'snail' 'torch'  
'ruler' 'snake' 'train'  
'screw' 'spade' 'vase'  
'sheep' 'spider' 'wasp'  
'shell' 'swan' 'watch'  
'shirt' 'sword' 'web'

# Semantic association LSA

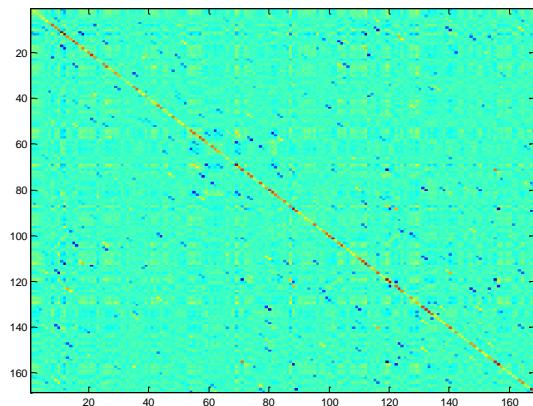


# Orthographic (spelling) distance



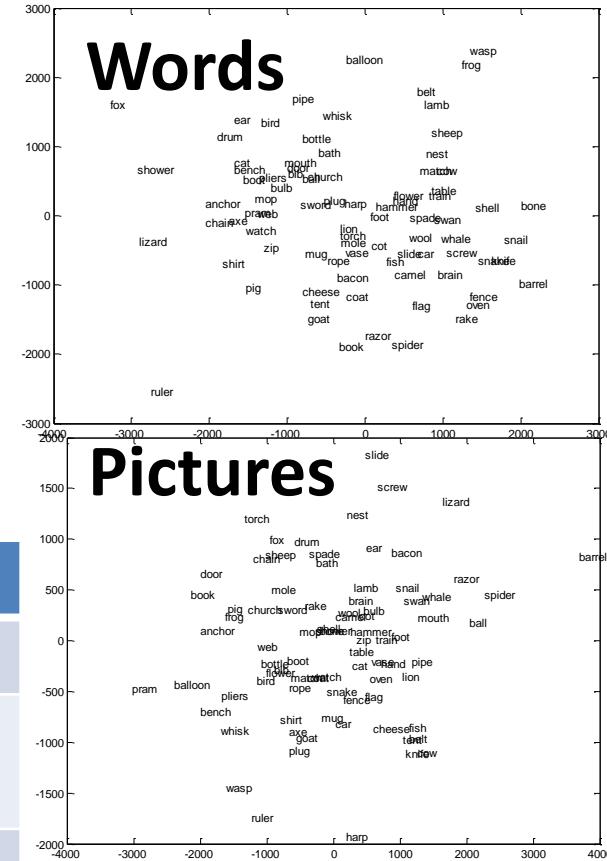
# Similarity based on brain activation

For each subject : Estimate the contrast image corresponding to one word or picture  
 With our dataset 84 images for the word set and 84 images for the picture set.



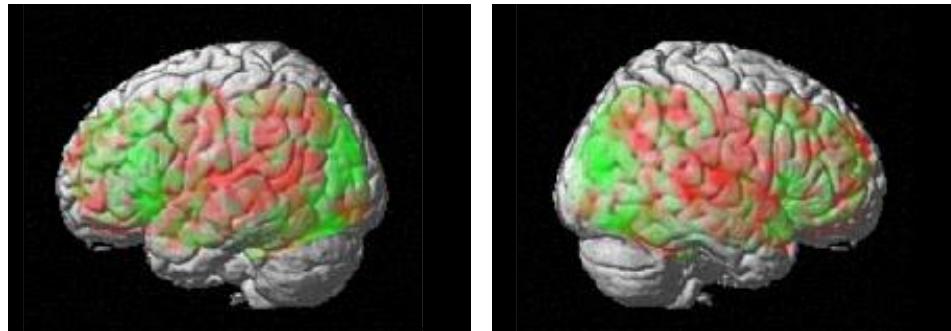
Correlation between the similarity measures

	Phi Words	Phi Pictures
Sim Semantic	0.4	0.4
Sim Orthographic	0.4	0.2
Phi words		0.49

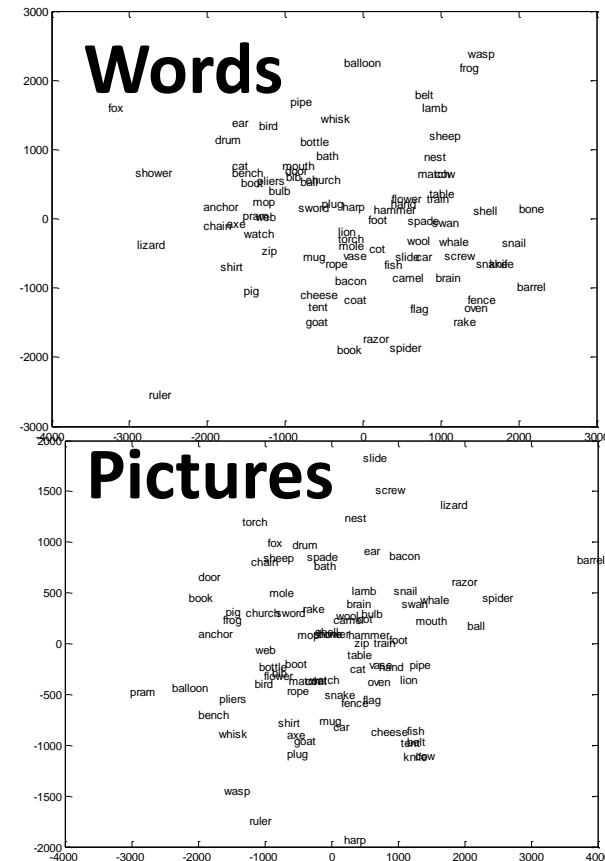


# Similarity based on brain activation

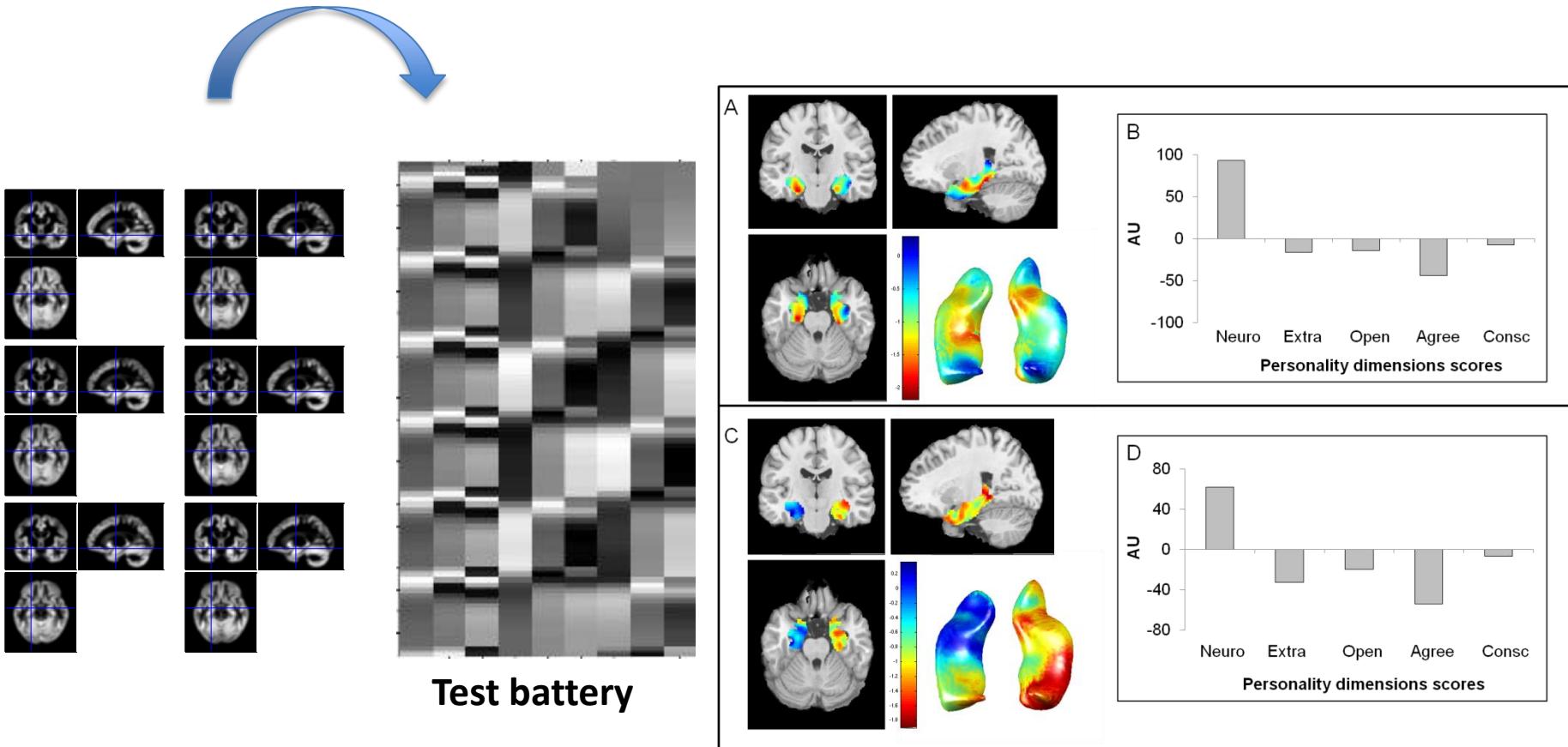
For each subject : Estimate the contrast image corresponding to one word or picture  
With our dataset 84 images for the word set and 84 images for the picture set.



- The predictive MM model achieved 64% of accuracy for detecting the concept
  - And 90 % accuracy for detecting the modality



# Combining clinical and structural information

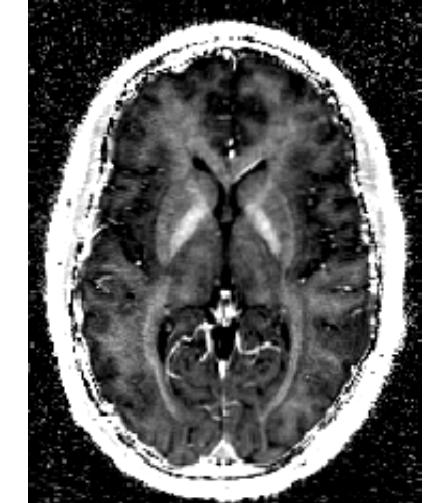
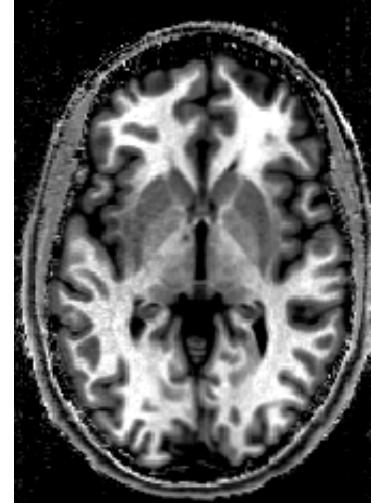
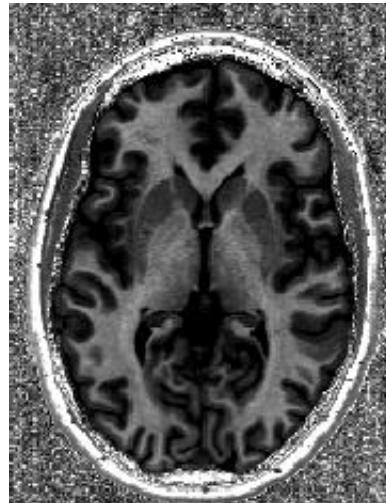
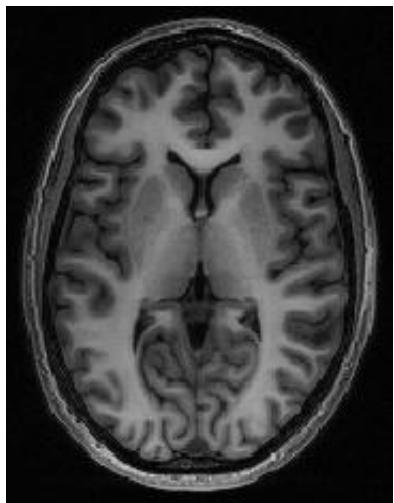
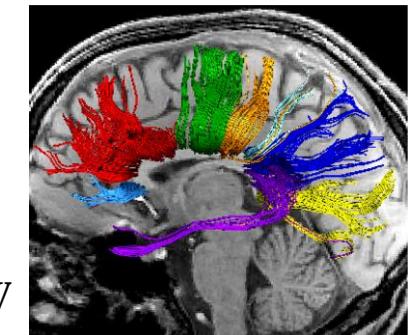


**Test battery**

**Zufferey et al in preparation**

# Multi-spectral structural imaging

- Clinically feasible (off-the-shelf sequences, 18min. duration )
  - Combination of T1, MT & PD sequences – resolution 1mm<sup>3</sup>
  - Quantitative maps
  - Post hoc – T1, MT, R2\* synthetic maps



T1w

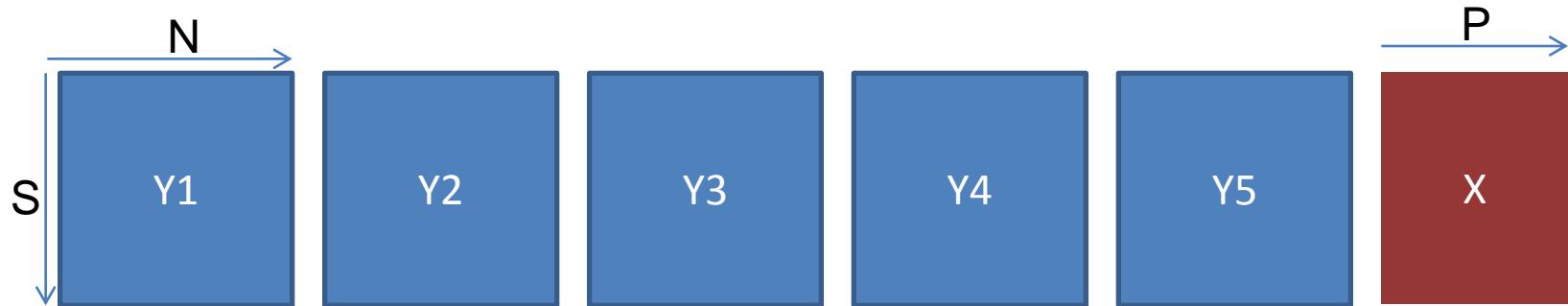
T1

MT

R2\*

# possible MV extensions

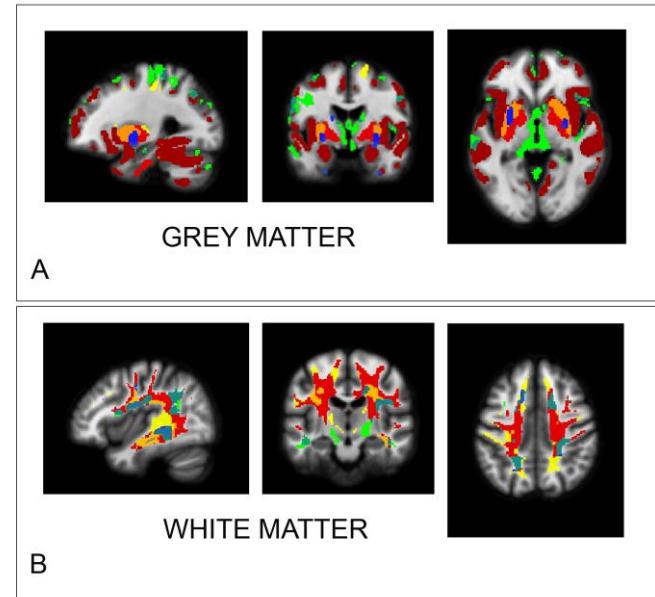
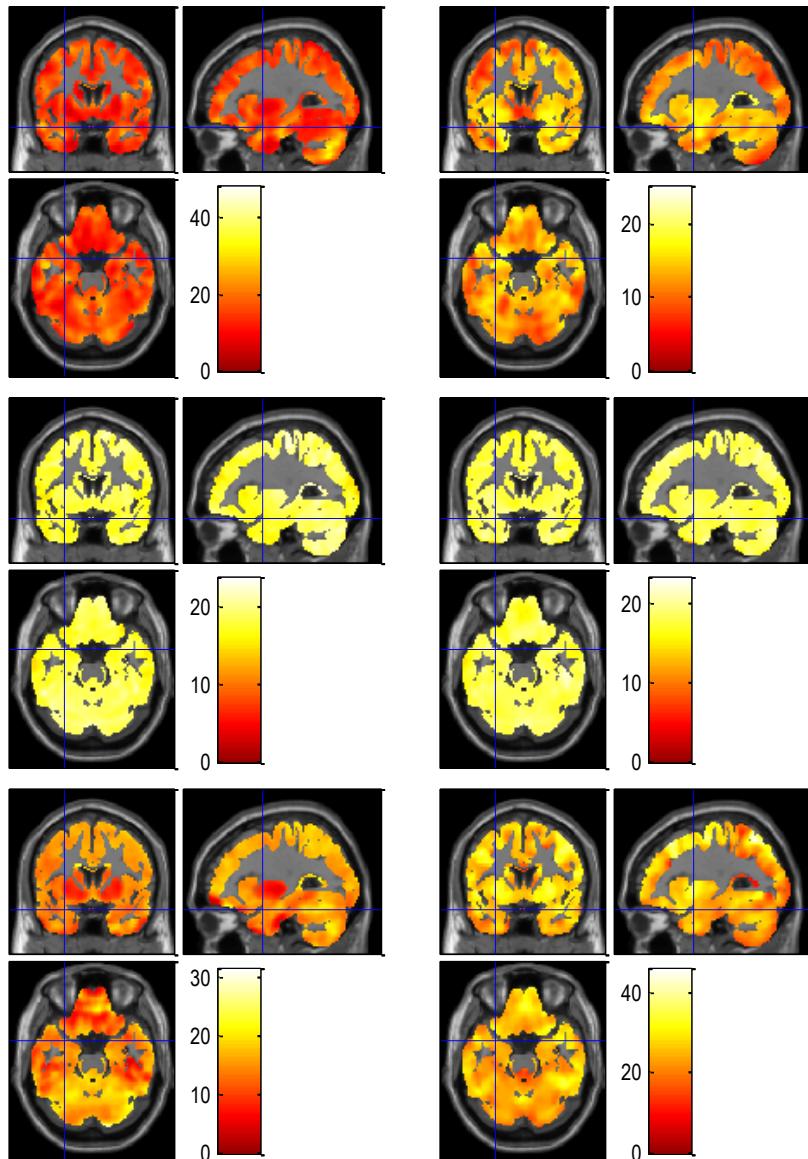
- Voxelwise multivariate
- Whole brain Multivariate



- Voxelwise covariate
- Causal model (SEM)

# Voxelwise multivariate

- |   |   |
|---|---|
| <ul style="list-style-type: none"><li>• Single variable at each voxel</li><li>• Univariate ANOVA</li><li>• Separate analyses</li><li>• sphericity assumption (?).</li><li>• Can be powerfull ()</li></ul> | <ul style="list-style-type: none"><li>• Multiple variables at each voxel</li><li>• Multivariate <math>T^2</math> : MANOVA profile analyses</li><li>• sphericity assumption (not required).</li><li>• More powerfull (n should be big)</li></ul> |
|---|---|



## Fingerprint of tissue properties

	FA	MD	MT	R2*	VBM
1	-	-	-	-	-
2	+	-	-	-	-
3	+	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-
6	+	-	-	-	-
7	+	-	-	-	-

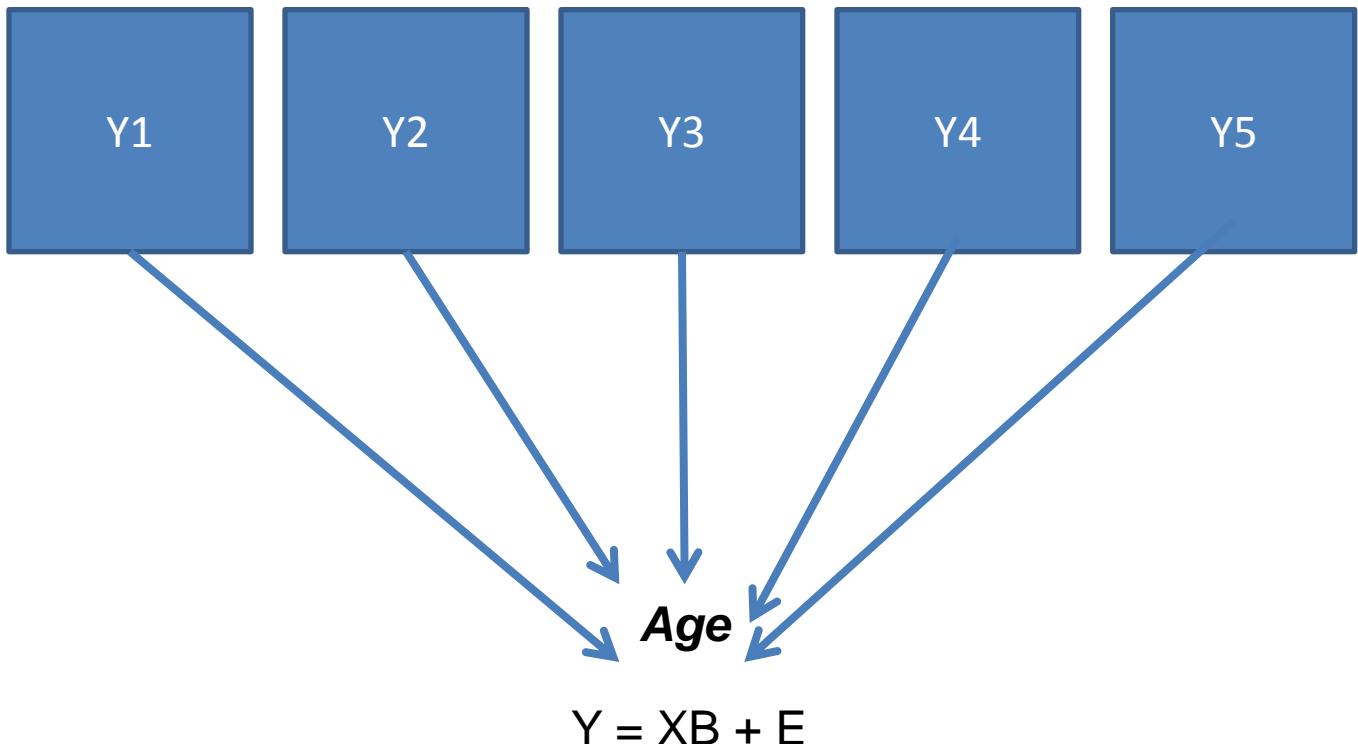
  

	FA	MD	MT	R1	R2*
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	+	-	-	-
5	-	-	-	-	-
6	-	-	-	-	-

# Whole brain Multivariate

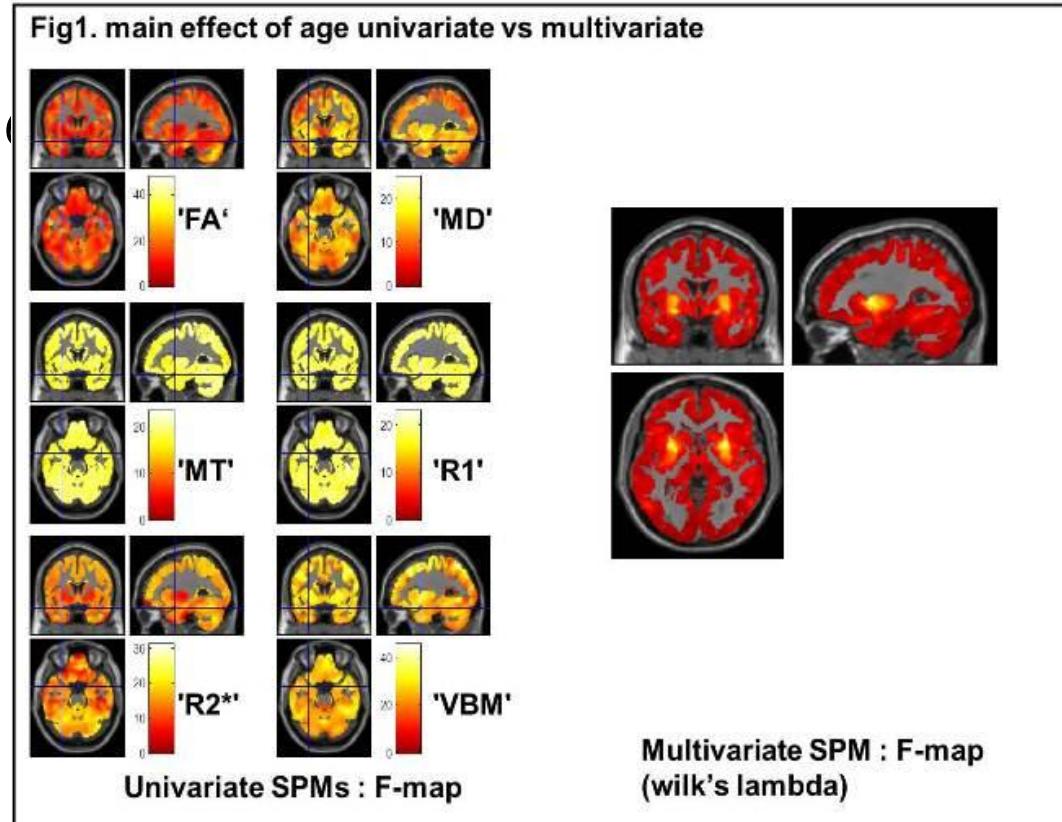
## Predicting age using multiple sources

20	26.0701
24	43.6065
24	27.6333
25	30.7243
25	33.3435
25	36.2153
26	33.0484
27	39.9625
27	45.7347
28	36.5503
29	38.6941
30	40.2426
30	36.8966
32	35.1314
34	28.7709
35	40.5643
38	44.8832
39	34.9653
45	38.6204
54	38.9066
61	44.1481
66	48.5588
69	45.8561
75	44.3738
77	47.3703
87	48.0909



# Multivariate inference – age prediction

- Voxel-based MANOVA/MANCOVA using parameter maps as dependent variables
  - advantage - error Type I protection and better sensitivity
  - output - multivariate F-statistic (Wilks' lambda)



# Multivariate inference – age prediction

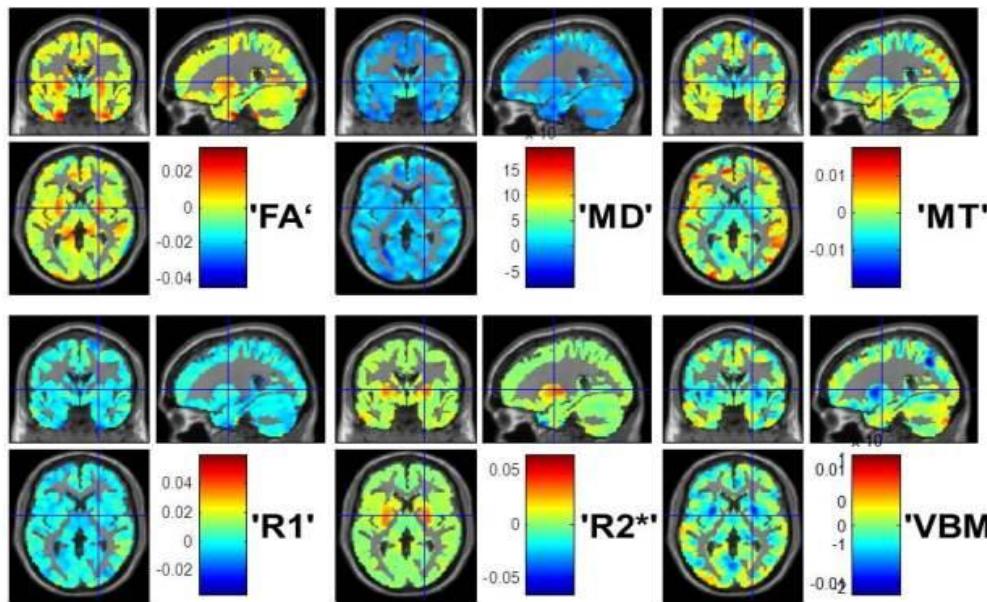
- Multivariate calibration analysis using partial least square regression (PLSR)

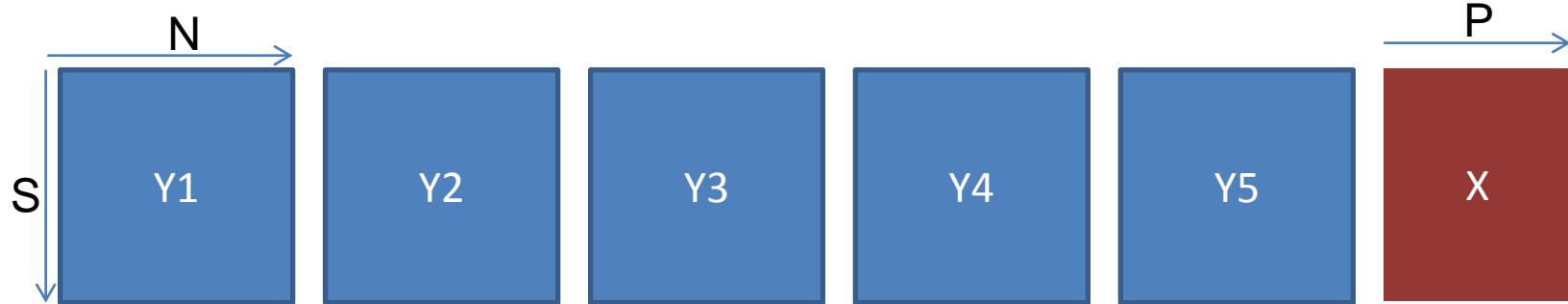
- leave-one-out cross-validation - Mean Error in prediction:

12.75y - FA, 11.8y - MD, 12.4y - MT, 12.6y - R1, 9.7y - R2\* and 9.4y - VBM.

COMBINED Mean Error in prediction - 7.8 years

**F Fig3. Voxel weight for age prediction.**





Voxelwise covariate : ajustement

$$Y_1 = X\beta + Y_2\delta + \epsilon$$

Causal inference (SEM)

