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## Etude des modèles de Whittle markoviens probabilisés

**Thèse de doctorat**

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# A study on probabilized markovian Whittle's models

## Abstract

The probabilized markovian Whittle's model is a first order simultaneous autoregressive model of spatial field which expresses simultaneously every variable of the field as a random weighted mean of the adjacent variables of the field, weakened by a multiplicative coefficient  $\rho$ , and added by an error term (which is a spatially independent gaussian homoscedastic variable, not directly measurable). In our case, the weighted mean is an arithmetic mean which is random because of two conditions : (a) two variables are adjacent (in the graph sense) with a probability  $1 - p$  if the distance which separates them is lower than a certain threshold, (b) there is no adjacency for distances over this threshold. These conditions determine a model of adjacency (or model of connexity) of the spatial field. A probabilized markovian Whittle's model on the conditions where  $p = 0$  gives a classical Whittle's model which is more familiar in geography, econometrics, ecology, sociology, etc., and which  $\rho$  is the autoregression coefficient. Our model is thus a probabilized form at the field connexity level of the classical Whittle's model form, bringing an innovative description of the spatial autocorrelation.

We begin by describing our spatial model by showing the complexity effects introduced by the connexity model on the pattern of variances and the spatial correlation of the field. We study then the estimation problem for the autoregression coefficient  $\rho$  for which beforehand we make a detailed analysis of its information in the Fisher's and Kullback-Leibler's sense. We show that an efficient unbiased estimator of the coefficient  $\rho$  possesses an efficiency which varies according to the parameter  $p$ , generally in a not monotonous way, and to the adjacency network structure. In case the field connexity is not observed, we show that a bad specification of the maximum likelihood estimator of  $\rho$  can bias this one according to  $p$ . We propose in this context other ways to estimate  $\rho$ .

To finish, we study the power of significance tests of  $\rho$  for which test statistics are classical variants of the Moran's  $I$  (Cliff-Ord test) and the maximal Moran's  $I$  (inspired by the Kooijman method). We observe the power variation according to the parameter  $p$  and the coefficient  $\rho$ , showing by this way the duality of the spatial autocorrelation between intensity and connectivity in the context of autoregressive models.