

Logic is Tripartite — A defence of non-bivalence*

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Abstract: Sentences about contingent future pose a problem for logical theory. I analyze the non-bivalent view according to which sentences about a future event are neither true nor false as long as the event is contingent. Major implications are that the definition of the connectors cannot be truth-functional and the concept of truth cannot be Tarskian. Many-valued logics are not an adequate tool to formalize the position, but a theory of truth-value gaps is. The ontology of the case asks for two basic logical values: truth and falsity, implying a tripartition of sentences (true, false, neither) and algebraic values (designated, anti-designated, neither) rather than a reduction à la Suszko.

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1 The case of *future contingents*

The following considerations have been nourished by the enterprise to formulate the contingency of the future. More precisely, they result from the project of giving an adequate formulation to the position which rejects the universal validity of the principle of bivalence:

BIV Every proposition is either true or false,

while maintaining the *Law of the Excluded Middle*:

LEM $\models p \vee \neg p$

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The first text known to us that expresses this position is Aristotle's *Peri Hermeneias* (also *De Interpretatione*, *De Int* in the following), especially chapter 9 containing the famous example of the sea battle. Lukasiewicz had been inspired by this text to develop his three-valued logic L_3 .¹

For Lukasiewicz, it is evident that Aristotle limits the principle of bivalence to propositions about the past, the present and the necessary. It is different for the propositions referring to future contingent events, says Aristotle (*De Int* 18a33), like for "Tomorrow there will be a sea-battle", and Lukasiewicz takes him to mean that these are neither true nor false. He is thus in line with an interpretation of Aristotle that is called "traditional" or "standard" and that I will call "anti-bivalent" or "non-bivalent". We may also characterize the corresponding position by these terms, independently of the question whether it is to be found in Aristotle or not. In the following, I will point out some implications of 1) the anti-bivalent position and 2) the anti-bivalent reading of Aristotle.

One problem is that a logic with three or more truth values cannot maintain LEM, as I will show. Now, LEM is both defended by Aristotle in the sea battle chapter² and it corresponds to our intuitions concerning future contingents: It must be true that tomorrow there is going to be a sea-battle or there is not going to be one — even if it is still undetermined whether there will be one or not and if the simple propositions ("There will be one", "There will be none") are neither true nor false. But there is a different way of negating bivalence and thus an alternative to the failing many-valued approach to formulate the contingency of the future. A logic of truth-value gaps, as proposed by van Fraassen, seems to be the adequate way to formalize the non-bivalent position.

Another problem has been pointed out by those who contested that it is possible at all to maintain the *excluded third* without also affirming bivalence. The two principles are considered equivalent by many authors. If these were right, any non-bivalent position that affirms the *excluded third* would be inconsistent. We have to realize indeed that bivalence is implied by LEM, not of it alone however, but only if another principle is added that links the proposition p and the truth of ' p '. This is achieved by the T-schema that Tarski used in his "semantic conception of truth".³

We are thus confronted with the incompatibility of the following theses:

1. \neg BIV (for Non-Bivalence): Sentences about future contingents are neither true nor false. So BIV is not universally valid.
2. LEM: The *excluded middle* is a tautology: $\models p \vee \neg p$

¹Lukasiewicz 1930 [19], pp. 51ff.

²Aristotle, *De Int* 19a31–38; [36], tr. Ackrill, p. 30.

³Tarski 1972 [35], p. 162; 1993 [36], p. 103; in his text of 1952 "The Semantic Conception of Truth", Tarski identifies this conception with "the classical Aristotelian conception of truth" that he finds in *Metaphysics* [3], Gamma 1011b26 ([34], p. 14).

3. T: The definition of truth implies biconditionals like “ $V‘p’$ iff p ” (“The sentence ‘ p ’ is true iff p ”).

Because of the inconsistency of this set of positions, non-bivalence concerning future contingents is incompatible with a Tarski-style concept of truth. We will see that at least van Fraassen and the ancient authors do indeed not adhere to it. Before considering this, it seems adequate to resume some basic definitions and distinctions.

2 The distinction of *excluded middle* and *bivalence*

The confusion of the two principles dates back far. Only in 1920 did Łukasiewicz differentiate them, establishing the expression “bivalence”.⁴ Still nowadays, not all authors distinguish them or if they do, they often take them to be equivalent.⁵ So let us compare the two principles, written in a formalized way but without quantifiers and signs of tautology in order to point out the differences:

BIV $V‘p’ \text{ w } F‘p’$

LEM $p \vee \neg p$

Three differences are obvious to the eye:

1. Bivalence uses the notions of truth and falsity whereas LEM is free of predicates.
2. The *excluded middle* uses negation whereas *bivalence* is free of it.
3. Bivalence is a disjunction (exclusive ‘or’), LEM, on the other hand, is an adjunction (inclusive ‘or’).

Frequently, bivalence has been identified with the conjunction of LEM and another principle of the same type, i.e. non-contradiction:

NC $\neg(p \wedge \neg p)$

LEM and NC together form a principle similar to LEM, but with the adjunction replaced by a disjunction, i.e. by an exclusive ‘or’, symbolized by ‘w’. I will call it ‘complement-principle’, because it expresses the fact that the contradictories p and $\neg p$ are mutually exclusive while covering the whole logical space (there is no third in between them).

⁴Łukasiewicz 1930 [19], p. 64; this article refers to a conference given in June 1920. The text has been first published in Polish, also in 1920. Tarski made use of the principle, see e.g. 1952 [34], p. 26.

⁵Cf., for example, von Kutschera [18], p. XI. or Malinowski [22], p. 7.

Comp $p \text{ w } \neg p$

If we compare this principle to bivalence, the third difference vanished. But the second one seems not to be essential either, for bivalence may be expressed by using negation:

BIV \neg $V'p' \text{ w } V'\neg p'$

The two versions of bivalence are only equivalent if the relation “ $F'p'$ iff $V'\neg p'$ ” holds. But this is no problem, because the definition of negation “ $\neg p'$ is true iff ‘ p' ’ is false” is adequate to the case of future contingents, no matter whether a bivalent or a non-bivalent position is defended: It is false that tomorrow there will be a sea-battle iff it is true that it will not happen. If it is neither true nor false that there will be one, then it is not false, but also neither true nor false to say that it will not happen.

To conclude, the two principles do not essentially differ in their use of connectors. It is rather that BIV, using the truth-predicate, is a semantic principle while LEM, not referring to the truth or falsity of the propositions it is composed of, is purely syntactic.

3 The relation between *excluded middle* and *bivalence*

The difference is clear enough such that it is not possible to treat the two principles as identical. One could, however, use them interchangeably if they were equivalent. Since BIV uses predicates that do not occur in the *excluded middle*, a link is needed in order to establish equivalence between the two. This link is given by the schema T that, according to Tarski, must be implied by any definition of truth:⁶

T $V'p'$ iff p .

The example is well-known: “Snow is white” is true iff snow is white. “Snow is white” is false iff snow is not white.⁷

It had been remarked before, e.g. by Martha Kneale, that we need a definition of truth in order to establish the equivalence of *excluded middle* and *bivalence*. She did not propose a valid derivation, however:

“Given the definitions of truth which we have quoted [Aristotle’s passage in his *Metaphysics*, see below], the principles [*excluded middle* and

⁶According to Tarski, the T-schema in itself is not a definition of truth. But by substituting any sentence for p , we obtain what he calls a “partial definition”. The general definition of truth would be the conjunction of all these partial definitions; in this sense, the T-schema is implied by the definition of truth (Tarski 1952 [34], p. 16; 1972 [35], p. 162; 1993 [36], p. 105).

⁷Tarski 1993 [36], p. 103; see also Tarski 1972 [35], pp. 162–71.

bivalence] are, however, obviously equivalent; for if ‘It is true that P’ is equivalent to ‘P’, ‘P or not-P’ is plainly equivalent to ‘It is true that P or it is false that P’.”⁸

The alleged equivalence is not evident, however. We are not entitled to just replace ‘p’ by ‘it is true that p’ and ‘not-p’ by ‘it is false that p’ in a thesis. If we replace ‘p’ by ‘it is true that p’, we have to replace it in the whole formula, thus obtaining ‘it is true that p or it is not true that p’, which is compatible with non-bivalence: if p is not true, it may be neither true nor false. What is still missing is the transition from ‘it is not true that p’ to ‘it is true that not-p’, which would in turn presuppose bivalence.

A valid derivation of BIV from LEM and T has been proposed by Susan Haack. She construes it as follows:⁹

1	$V'p' \leftrightarrow p$	T
2	$p \vee \neg p$	<i>excluded third</i>
3(3)	p	assumption
4(3)	$V'p'$	Def. of ‘ \leftrightarrow ’ and MPP (1,3)
5(5)	$\neg p$	assumption
6(5)	$V'\neg p'$	$\neg p/p(1)$; Def. of ‘ \leftrightarrow ’ and MPP(1,5)
7(3)	$V'p' \vee V'\neg p'$	introduction of adjunction (4)
8(5)	$V'p' \vee V'\neg p'$	introduction of adjunction (6)
9	$V'p' \vee V'\neg p'$	elimin. of assumptions (2,3,5,7,8)
10	$V'p' \vee F'p'$	Def. of ‘F’ (9)

We only have to add NC in the premises in order to arrive at a disjunctive conclusion. The derivation shows that we obtain the principle of bivalence from the *excluded middle* together with the schema T, which is in turn implied by the definition of truth according to Tarski.

The consequence of this is threefold. First of all, defenders of a non-bivalent position concerning future contingents must reject a Tarski-style concept of truth implying the biconditionals T. The non-bivalent understanding of the contingency of the future thus implies that truth is undefinable.

Secondly, if the definition is contained in a text that, like Aristotle’s *Peri Hermeneias*, affirms LEM while explicitly restricting bivalence, the text is contradictory.

Thirdly, it becomes clear that Tarski’s semantic conception of truth is less innocent, logically and ontologically, than it seems at first sight. For unless we are willing to give up LEM, bivalence follows. Tarski’s conception of truth is a bivalent one.

⁸Kneale [17], pp. 46–48.

⁹Haack [15], pp. 67–68. The symbols are different from hers and instead of her Tp, I write $V'p'$.

4 The meaning of ‘or’

The anti-bivalent position has been harshly criticized, even ridiculed. It seems that the reproaches, often expressed in a manner of moral or intellectual indignation, derive from a certain comprehension of the inclusive ‘or’ or adjunction (also called disjunction or alternation). The meaning of that connector is usually expressed by a truth table or the following matrix:

∨	true	false
true	true	true
false	true	false

This definition of adjunction is truth-functional. According to the principle of truth-functionality, the truth value of a compound sentence depends exclusively on the truth values of its components. Thus, ‘ $p \vee q$ ’ is true if and only if at least one of the two sentences, p or q , is true.

Now, how is it possible that the formula ‘ $p \vee \neg p$ ’ be true? According to the truth-functional definition, at least one of the two: p or $\neg p$, would have to be true. In bivalent logic the formula is a tautology because either p is true and with it also the complex sentence, or p is false, hence non- p is true, which again renders true the compound phrase. According to the non-bivalent position however, if p refers to a future contingent event, it is neither true nor false; the same holds for non- p . So neither p nor $\neg p$ is true, hence ‘ $p \vee \neg p$ ’ cannot be true either. Claiming the truth of the *Excluded Middle* thus amounts to a violation of the usual truth-functional way of understanding ‘or’ (and the other connectors).

Cicero fiercely attacked those who combine non-bivalence and LEM, a position that he attributed to the Epicureans:

“[...] unless perhaps we wish to follow the Epicureans, who say that such statements are neither true nor false, or, when they are embarrassed to say that, say what is even more shameless, that disjunctions from opposites (e.g.: ‘Either Philoctetes will be wounded or he will not be’) are true, but of the statements included in them neither is true. [38] What amazing presumption and pitiful ignorance of logical discourse!”¹⁰

Quine did not show any more comprehension:

“[...] has brought Professor Weiss to the desperate extremity of entertaining Aristotle’s fantasy that ‘It is true that p or q ’ is an insufficient condition for ‘It is true that p or it is true that q ’.”¹¹

Hintikka calls this position not desperate but absurd:

¹⁰Cicero, *De fato* 16, [9], p. 37f.; tr. Sharples 1991.

¹¹Quine 1953 [28], p. 65; 1966 [29], p. 21.

“According to their [Colin Strang’s and Martha Kneale’s] view, the point of Aristotle’s discussion is to assert the truth of the disjunction (1) [‘p or not-p’] even when p is a sentence dealing with an individual future event but to deny that either p or not-p should therefore be true. Even if we disregard the intrinsic absurdity of this alleged doctrine of Aristotle’s, which has provoked the deserved ridicule of Cicero and W.V.Quine [...]”¹²

The quotations illustrate the deeply entrenched truth-functional understanding of ‘or’, nowadays as well as in Antiquity. The non-bivalent position indeed necessitates a different, non-truth-functional understanding of the connectors. This will get clearer when we consider the failure of many-valued logics to express the non-bivalent position concerning contingent future.

5 The inadequacy of many-valued logics

Three-valued truth-functional systems do not contain the *excluded middle*. This is easy to see if the third ‘value’, next to truth and falsity, is attributed to propositions that are neither true nor false. If of two sentences neither is true nor false, then the adjunction of the two is equally neither true nor false. In case of truth-functionality, this definition holds for no matter what sentences, so it also holds for two contradictories: if neither p nor $\neg p$ is true, ‘ $p \vee \neg p$ ’ cannot be true either. Assigning 1 to truth, 0 to falsity and $1/2$ to the third value, this can be expressed by the following matrix:

$q \setminus p$	1	$1/2$	0	$p \vee \neg p$
1	1	1	1	1
$1/2$	1	$1/2$	$1/2$	$1/2$
0	1	$1/2$	0	1

LEM is not true for all valuations. It seems that truth-functional logics are incapable to express the non-bivalent view of future contingents.

It is no solution either to claim that the value $1/2$ is sufficient to make LEM a law, i.e. to consider not only 1 but also $1/2$ as a designated value. For this would mean that, contrary to our assumptions, the algebraic value $1/2$ implies truth. Stated otherwise, LEM is only as true as one of its components. If the third value does not imply truth for a simple sentence, it does not for the compound formula either.

In the special case of propositions about future contingents, a three-valued system only makes sense if the third value is not designated. For if a simple proposition has the value $1/2$, it may become false. As long as we exclude that a proposition changes its truth value from true to false or vice versa, the

¹²Hintikka [16], p. 163.

formalization of future contingency by a three-valued system with a designated intermediary value is inadequate.

Antique philosophers proposed to understand the affair and Aristotle's position differently by introducing the distinction of a definite and an indefinite distribution of truth and falsity on the contradictory propositions.¹³ This led to some problems of interpretation in itself, but it can be and it was understood as the distinction of definite and indefinite truth (or falsity).¹⁴ That in turn motivates the consideration of a four-valued system. At first sight, the situation resembles that of the three-valued case, $2/3$ standing for 'being indefinitely true' and $1/3$ for 'being indefinitely false'. Again, LEM does not have the value 1 if p , and hence $\neg p$, have values in between 0 and 1:

$q \setminus p$	1	$2/3$	$1/3$	0	$p \vee \neg p$
1	1	1	1	1	1
$2/3$	1	$2/3$	$2/3$	$2/3$	$2/3$
$1/3$	1	$2/3$	$1/3$	$1/3$	$2/3$
0	1	$2/3$	$1/3$	0	1

If the values of the simple phrases are *indefinite*, the *excluded middle* only has the value $2/3$. Now, while in the three-valued case, $1/2$ is as distant from 0 as from 1, $2/3$ is definitely 'nearer' to truth than to falsity.

There are two possibilities: either a sentence that is 'indefinitely true' is simply true, then $2/3$ is a designated value, or it is not and $2/3$ is not designated. In the latter case, that is, if a proposition that is indefinitely true is not true *tout court*, LEM is not necessarily true either, so the system does not adequately render the non-bivalent position concerning future contingents.

If, on the other hand, an 'indefinitely true' proposition is true *tout court*, we face the distinction of two kinds of truth (and two kinds of falsity). In this case, although algebraic values in between 0 and 1 are added, no new *truth* values are added. We remain within the bivalent frame, just distinguishing two kinds of each of the two classic truth values. If a tautology is a formula that receives truth (or a designated value) for any valuation, then ' $p \vee \neg p$ ' is indeed a tautology in such an algebraically four-valued system.

However, in this case we abandon the non-bivalent interpretation of Aristotle in favour of a bivalent reading of his text and we quit the systematic non-bivalent position, adopting a bivalent one that allows for degrees, levels or intensities of being true.¹⁵ A system of logic is logically, and not just algebraically many-valued, only if it uses more than the two classical truth values, i.e. only if some of its algebraic values do not imply truth or falsity.

¹³Ammonius and Boethius continue what has already been established by peripatetic commentators whose texts are lost. Cf. Seel 2001 [31], pp. 28ff.

¹⁴Mignucci 2001 [24], p. 252; Seel 2001 [31], p. 240.

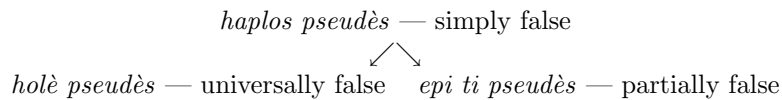
¹⁵The question has been raised whether definite truth and falsity are not identical with necessity and impossibility (Seel 2001 [31], pp. 234–246).

This means that at least one algebraic value has to be neither designated nor anti-designated. An example may illustrate this.

6 Logical many-valuedness

There has been a series of attempts to formalize parts of Aristotle’s theories by applying many-valued semantics. In *Analytica Priora* II, 2–4, Aristotle distinguishes *epi ti alèthès* — *epi ti pseudès* on the one hand and *holè alèthès* — *holè pseudès* on the other.¹⁶ Suppose that Wilma says “All swans are white”. Now there are black swans, although not many. So Wilma is wrong. Suppose further that Fred says “All swans are blue”. He is also wrong, but admittedly in a different way. If there is no naturally blue swan at all, then he is wrong about every swan. Somehow, Fred is more wrong than Wilma. This can be expressed by saying that some SaP (universal affirmative) sentences are *epi ti pseudès*, i.e. false about something and others are *holè pseudès*, i.e. completely false or false in every case.

Using the distinction of fundamental and derivative truth-values,¹⁷ we may state that the fundamental value of falsity, or simple falsity, is distinguished into the derivative values of complete and partial falsity:



In this case it is clear that what is partially false is false and what is partially true is true. The value 1/3 is anti-designated, 2/3 is designated. The distinction of the truth-values just adds information to being true or false; it does not add any new truth values.

To resume: Either an algebraically many-valued system uses more than two truth-values; then it does not sustain the *excluded middle* and it is not adequate as a logic of future contingents.

Or the *excluded middle* is sustained, but this requires that the different algebraic values denote just kinds of truth and falsity and we do not deal with more than two truth values. The non-bivalent position concerning future contingents assumes that some propositions are neither true nor false, i.e. have neither of the two fundamental truth values.

7 The universal two-valuedness of logic

It has been argued by Suszko that every many-valued logic is indeed a bivalent logic, the main argument being that each set of algebraic values may

¹⁶Offenberger 1990 [25], pp. 43ff., 64–83.

¹⁷Offenberger 1990 [25], pp.43ff. He uses the terminology of A. Sinowjew/H. Wessel, *Logische Sprachregeln*, 1975.

be regrouped into two sets, one containing all and only designated values, the other all and only non-designated ones, and that these two sets would correspond to truth and falsity.¹⁸

Suszko is undoubtedly right in making the distinction between algebraic and logical values. And it is correct that, as soon as many-valued logics are considered, the set of designated values replaces the truth-value 1 in defining consequence and tautology. It is further acceptable to say that the set of designated values contains the values that denote truth. As I have argued in the last chapter, it is important whether a value is designated, i.e. implying truth or not.

It seems to me mistaken, however, to identify falsity with non-designatedness. Consider the case of a four-valued logic in which only the algebraic value 1 is designated; then $2/3$ is not designated, so it should, if we follow Suszko's line of thought, express falsehood, which is unacceptable. Why should a sentence with the algebraic value $2/3$, rather on the side of truth, we might say, be considered false?

There is a special reason to reject the identification of falsity with undesignatedness in the case of future contingents. If ever a proposition is not yet true, but will become true, it is certainly not false. Otherwise it would change its value from false to true.

Lukasiewicz once used the notion of "refutedness", being the contrary of "designatedness". Instead, we could also speak of designated and anti-designated values, characterizing the ones implying truth and the others implying falsehood respectively. If there are some in between, these are undesignated, but also non-anti-designated or non-refuted. There are three sets of algebraic truth values, the set of designated ones (expressing truth), the set of refuted ones (expressing falsehood) and, perhaps empty, the set of neither designated nor refuted ones (expressing neither truth nor falsity). In this sense, every logic is threefold.

This is analogous to the fact that any logic creates three sets of formulas: tautologies, contradictions and those true or false only for some valuations. If we are looking for a universal trait of every logic, it is again rather that it is tripartite. Neither logical falsity nor simple falsity is identical with non-truth, as Suszko's reduction suggests.

Béziau follows the idea of an analogy between algebra and logic. There was a development from concrete to abstract to universal algebra. It is not proven that this will also work in a similar way for logics, but even it does, we may doubt that the reduction to bivalence is a necessary ingredient of Universal Logic, nor is it clear what the analogy to this would be in algebra. In any case, universalization means abstraction, not reduction.

One of the abstract features of any logic is that it divides its formula into three sets: tautologies (having designated values), contradictions (having

¹⁸Suszko [33], p. 378.

refuted values) and the contingently true or false. Secondly, some logics use three sets of algebraic ‘truth’-values: next to the designated and the refuted ones those that are neither. In these two respects logic is essentially tripartite, if we presuppose that there are exactly two truth-values.

8 The definition of bivalence

The attempt to formulate future contingency by a many-valued logic, as Lukasiewicz tried to do, definitely failed. If it were the only way of negating bivalence, the formulation of future contingency with a non-bivalent logic would be impossible.

This is the moment to give a still preciser definition of the principle of bivalence, for in the presented form it does not clearly show the two possibilities of being negated. In fact, the principle contains two theses:¹⁹

1. There are exactly two truth values: true and false.
2. Every proposition has exactly one of it.

Béziau has cast the same sense in a somewhat more mathematical vocabulary:²⁰

Ba The set of truth-values is limited to two values.

Baa These truth-values are ‘true’ and ‘false’.

Bf The evaluation relation is a function.

Bt The evaluation relation is a total relation.

Bf and Bt together express the above second condition. The differentiation is clear and correct, but the case of paraconsistency is not of interest here, so we do not need it.

Ba and Baa differentiate what I expressed by the first clause. Let us check whether this distinction is valuable for present purposes. To separate Baa from Ba makes sense only if the two truth values could be different from truth and falsity. And indeed, remembering the above discussion it may be questioned whether the two values are true and false and not rather true and not-true, or designated and non-designated — or refuted. Concerning the latter proposal, it seems clear to me that we do not understand truth by way of designatedness, but vice versa. As we have seen with the four-valued systems, it is rather the answer to the question whether a certain algebraic value implies truth or falsity *tout court* that decides about its designatedness.

Ba, on the other hand, allows for the introduction of a different and somewhat metaphysical line of thought. It excludes the existence of more than two

¹⁹Groneberg 2002 [14], p. 243.

²⁰Béziau 2004 [7].

truth-values, but also the case that there is only one. We may indeed consider whether there is not just one essential predicate, i.e. truth, while falsity is derivative, being something like the privation or negation of truth. This rather Parmenideian discourse confronts us with the question whether falsity is reducible to truth, or symmetrically and arbitrarily either one may be defined by the other — as in the case of existential and universal quantification or possibility and necessity — or whether falsity is not an irreducible property next to truth. Now if falsity is irreducible, it cannot be identified with non-designatedness, for the latter is the simple negation of designatedness or truth. Suszko's proposal thus rather expresses a fundamentally monovalent point of view that accepts only one fundamental value: designatedness. If, on the contrary, there are two independent basic truth values, and only in that case, the set of algebraic values is possibly tripartite.

By integrating Béziau's distinction, we arrive at the following definition of bivalence:

Val2 There are exactly two irreducible truth-values

T&F The truth values are 'true' and 'false'.

Min \forall Each proposition has at least one of it.

Max \forall Each proposition has at most one of it.

The 'irreducibility' in *Val2* means that bivalence is not reducible to monovalence. In this sense, Suszko's reduction leads to a non-bivalent logic with just one fundamental logical value that is truth, whereas the non-bivalent position concerning future contingents conforms to the first two clauses, just negating the third, claiming that not every proposition has a truth value.

For present purposes, the twofold definition will be sufficient:

1. There are exactly two truth values: true and false.
2. Every proposition has exactly one of it.

9 The non-bivalent position adequate for *future contingents*

Whereas Lukasiewicz always mechanically negated the first part when he negated bivalence, the only chance to escape the traps of many-valued logics is to not touch the first part, but to negate the second, which leads to a logic of truth-value gaps. Bas van Fraassen presented such a theory, the system 'S' of supervaluations.²¹ It is based on elementary bivalent logic and presupposes bivalent valuations. In the case of a TV-gap all possible valuations have to be considered:

²¹Van Fraassen 1966 [11], pp. 481–95; 1969 [12], pp. 67–91.

A sentence is true in S if it is true for all bivalent valuations.

It is false in S, if it is false for every valuation.

In the other cases, it has no truth value in S.

	p	q	$\neg p$	$p \vee q$	$p \wedge q$	$p \vee \neg p$	$p \wedge \neg p$
bivalent	1	1	0	1	1	1	0
	1	0	0	1	0	1	0
	0	1	1	1	0	1	0
	0	0	1	0	0	1	0
S	–	–	–	–	–	1	0

Although based on bivalent valuations and truth-functional definitions of the connectors, S is non-bivalent and non truth-functional.

Van Fraassen also realized the problem of the derivability of bivalence from T and LEM. Therefore, he proposes another way to conceive truth — the only way, it seems, that avoids a decision in favor of bivalence.²² We only need to replace the biconditional “‘p’ is true iff p” by the double conclusion:

“p; hence: It is true that p
it is true that p; hence: p”²³

For van Fraassen, everything depends on that distinction between conditional and conclusion.²⁴ In his logic, $p \models q$ (p, hence q) does not imply $\models p \rightarrow q$. The *theorem of deduction* is not valid. Richmond Thomason has given a temporal example for this, combining supervaluations with a tensed logic that Arthur Prior had developed.²⁵ The possible valuations are valuations in alternative future histories (h, g). A sentence about the future is true iff it will be true in every future history. Further, be U necessity in the sense of inevitability: Up is true iff p is true for any future development. Now, on the one hand Fs: “the sea-battle will take place” has UFs as consequence, for in this logic, Fs is true iff s is true in every future. In this case, UFs is also true. On the other hand, the conditional “if the sea-battle will occur, it will occur inevitably” is not necessarily true:

$Fp \models UFs$, without $\models Fp \rightarrow UFs$

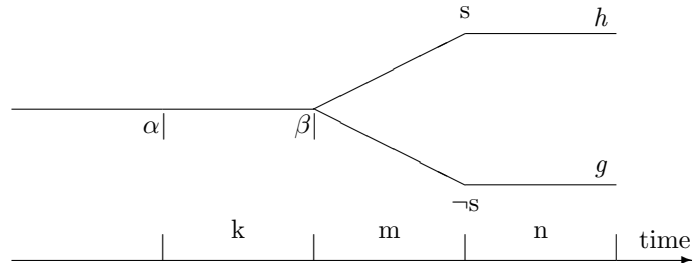
The following model shows why:

²²Van Fraassen 1966 [11], pp. 481–95; Groneberg 2001 [13], pp. 252–54; 2002 [14], pp. 247–49.

²³Van Fraassen 1966 [11], p. 494.

²⁴Van Fraassen 1966 [11], pp. 493–95.

²⁵The system that Prior named ‘O’ after Ockham; Thomason [37], pp. 275f.



Consider truth at point α : there are two histories bifurcating into the future at β such that $V_\alpha 'F(k+m)s'$ is not defined. But there is a truth relative to histories h and g : $V_\alpha^h 'F(k+m)s' = 1$ and $V_\alpha^g 'F(k+m)s' = 0$. Since the sea-battle happens in h , the prediction will be true in h ; since it will not happen in g , it will be false there. In this case, UFs is false at α .

The conditional ' $Fs \rightarrow UFs$ ' is not true at α since it is false in h : $V_\alpha^h 'Fs \rightarrow UFs'$ is false, because Fs is true in alternative h (the sea-battle will happen) but UFs is not (it is not inevitable).

10 The Aristotelian truth

If we follow van Fraassen's proposal, we must discard Tarski's conception of truth. This does not mean, however, to give up the original idea that Tarski apparently found in Aristotle. I'll terminate by presenting this original position that had been maintained, rather explicitly, by Aristotle's commentators of quite different languages.

Tarski stated that his concept of truth had been inspired²⁶ by the following passage in Aristotle's *Metaphysics*:

“[...] for to say that that which is is not or that which is not is, is a falsehood; and to say that that which is is and that which is not is not, is true [...]”²⁷

Surprisingly, he does not refer to the seabattle chapter that contains a formulation much closer to his and that might be read as a definition of truth:²⁸

“For if it is true to say that it is white or is not white, it is necessary for it to be white or not white; and if it is white or is not white, then

²⁶Tarski 1952 [34], pp. 14–15; 1972 [35], p. 162, Fn2; 1993 [36], p. 102.

²⁷Aristotle, *Metaphysics* [3], Gamma 1011b26; tr. Ackrill; Tarski (1972 [35], p. 162, Fn2) uses Tricot's translation: “Dire de l'Être qu'il n'est pas, ou du Non-Être qu'il est, c'est le faux; dire de l'Être qu'il est, et du Non-Être qu'il n'est pas, c'est le vrai.”

²⁸In this respect, we have to correct the opinion of the Kneales who remark ([17], p. 46) that Aristotle's *Organon*, of which *De Int* is a part, contains no definition of truth.

it was true to say or deny this. If it is not the case, it is false, if it is false, it is not the case.”²⁹

Frequently, the content of this passage is called principle of correspondence, and it is formalized as a biconditional.³⁰ The first sentence seems to say the following:

$$\square ((V'p' \rightarrow p) \wedge (V'\neg p' \rightarrow \neg p) \wedge (p \rightarrow V'p') \wedge (\neg p \rightarrow V'\neg p'))$$

This may be simplified to

$$\square ((V'p' \leftrightarrow p) \wedge (V'\neg p' \leftrightarrow \neg p))$$

where the second part is redundant, being a consequence of the first (by substitution), such that we obtain

$$\square (V'p' \leftrightarrow p),$$

i.e. schema T.

Formalized this way, Aristotle’s position becomes contradictory if read in a non-bivalent way,³¹ for he endorses the *excluded third*, which, together with T, implies bivalence. In order to maintain a consistent non-bivalent reading of Aristotle, we have to apply van Fraassen’s proposal, which means not to formalize the passage as a biconditional, but as a double conclusion.

Now, logical consequence is often expressed by a grammatical conditional.³² We have to be careful in this respect, less when we translate texts — be they ancient or recent — but when we formalize them. The transition from the linguistic to the logical level poses roughly the same problems for most natural occidental languages.

But not only does Aristotle’s quoted text allow for a formalization by consequences, it even asks for it. For in the *Prior Analytics*, Aristotle also presents his syllogisms in the form of grammatical conditionals:

“Ei gar to A kata pantos tou B kai to B kata pantos tou Γ, anangkè to A kata pantos tou Γ katègoreisthai.”³³

The “*ei gar . . . anangkè . . .*” corresponds to “for if . . . , then necessarily . . .”. This form is identical to the one he uses in the characterisation of truth in *Peri Hermeneias* (the translation of which is given above):

“Ei gar alèthes eipein hoti leukon è ou leukon estin, anangkè eimai leukon è ou leukon, [and a further conclusion, this time without necessity:] kai ei esti leukon è ou leukon, alèthes èn phanai è apophanai.”³⁴

²⁹ *De Int* 18a39–b4 [4], tr. Ackrill.

³⁰ Seel formalized it just this way recently ([31], pp. 10 and 15).

³¹ Groneberg 2002 [14], pp. 245ff.

³² Cf. Austin 1952 [5], Rose 1968 [30] or Corcoran 1982 [10].

³³ Aristotle, *Analyticon Proteron*, 25b37–39 [2].

³⁴ *De Int* 18a39 [4].

If it is evident that in his syllogistics Aristotle discusses conclusions, we have to realize that he regularly uses grammatical conditionals, often accompanied by a term for necessity, in order to express logical consequences.

In his formalization of Aristotle's syllogistics of 1951, Lukasiewicz insisted on not formalizing them as conclusions, but as logical conditionals, his reason being the grammatical form. That position was criticized in the following and we may assume that the dispute is settled in favour of a reading of the syllogistics as representing inferences.³⁵

By the way, it seems that Lukasiewicz had not realized the contradiction in Aristotle's texts if his conception of truth is rendered in a biconditional way and if at the same time, chapter 9 of *Peri Hermeneias* is read as expressing non-bivalence.

While the latter text is rather difficult and lends itself, in abstraction of other Aristotelian writings, to a conditional as well as a conclusive reading, the ambiguity is completely absent in his commentators. The following passages are taken from the few still existing commentaries of *De interpretatione* written in between Aristotle's time and the 12th century. Still in Greek, Ammonius (ca. 440–520) comments

hoti tē alētheia tōn logōn heptai hē hyparxis tōn pragmatōn kai tō pseudei hē anhyparxia (In Int 140, 32–34)

that the existence of the things *follows upon* the truth of the sentences and non-existence upon their falsity (tr. Blank in Blank/Kretzmann [8], p. 103; identical in Seel [31], p. 97)

Shortly afterwards, Ammonius uses the expression *akolouthein*, once again expressing consequence:

legō de hoti ou monon tois logois alētheousin akolouthein anangkē tēn hyparxin tōn pragmatōn, alla kai tē hyparxei tēn alētheian tōn logōn (141, 8–10)

I mean that it is not only necessary that the existence of the things *follow upon* the sentences being true, but also the truth of the sentences upon the existence <of the things>. (tr. Blank in Blank/Kretzmann [8], p. 104; identical in Seel [31], p. 99)

Of Boethius (480–524), who did probably not know Ammonius' commentary, we have two commentaries in Latin. He expresses himself in a quite similar way:

³⁵Patzig (1968 [26]) followed Lukasiewicz; Austin (1952 [5]) and Prior (in Rose 1968 [30], p. 25) criticized him. The case is decided, in my mind, by the remarks of Aristotle himself (*Analyticon Proteron*, [2], chap. I, 24a10ff.) and by the arguments of Corcoran (1982 [10], pp. 105–107) and Smiley (1973 [32]).

[...] ait enim hanc esse rerum *consequentiam*, ut rem subsistentem propositionis veritas *consequatur*, veritatem propositionis rei, de qua loquitur propositio, essentia *comitetur*. (I 109, 25–28)

[...] for he [Aristotle] says that it is a *consequence* of the state of affairs that the truth of the proposition *follows along* with the occurrence of the state of affairs, that the being of the state of affairs of which the proposition speaks *accompanies* the truth of the proposition. (tr. Kretzmann in Blank/Kretzmann [8], p. 133)

In his second commentary he says:

Si enim dixerit, quoniam nix alba est, et hoc verum, veritate propositionis *sequitur* necessitas rei [...] Amplius quoque propositiones rerum necessitates *sequuntur* (II 206, 11–18)

For if anyone were to say that snow is white, and this is true, then the necessity of the state of affairs *follows* the truth of the proposition. [...] Furthermore, propositions also *follow* the necessities of the things ([8], p. 158)

The same consequential terminology is used in Arabic. Of al-Fârâbî (870–950) and Averroes (1126–1198), we have complete commentaries. Averroes' *middle commentary* is a close paraphrase of the Aristotelian text:

Now the nature of the existent thing *follows upon* [*ittabâ*] the true sentence, and the true sentence *follows upon* it. (tr. Butterworth [6], 30, p. 142)

Al-Fârâbî comments after having quoted Aristotle:

For it *follows from* the truth of a sentence that the state of affairs <described in it> exists; and it *follows from* the falsity of a sentence that the state of affairs does not exist. This <is also true> conversely: from the existence of a state of affairs *follows* of necessity that it is true for anyone to say that it exists; and from the non-existence *follows that* it is false for anyone to say that it exists (85.12–14, tr. Zimmermann [1], p. 79)

These authors are not arbitrary but representative examples for a careful rendering of Aristotle's non-bivalent position. The conclusive characterization of truth avoids the trap of inconsistency that is menacing the non-bivalent position. The implications of the position are nevertheless important: it means to give up the truth-functional understanding of the connectors, the Tarskian understanding of truth and the deduction theorem.

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