The Role of Money in Double Coincidence Environments*

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Abstract

This paper studies the role of money in asymmetric double coincidence of real wants environments where in each meeting each agent is a consumer of the other agent's production. Traders who meet at random finance their purchases through current production, sale of divisible money, or both. It is shown that in the absence of valued money if traders have asymmetric tastes for each other's good, they produce and exchange socially inefficient quantities. With valued money, however, traders exchange efficient quantities if the asymmetry of tastes is not too large. It is also shown that terms of trades in the monetary economy are strictly better than those in the corresponding barter economy, that the Friedman rule holds, and that the allocation of resources in the monetary economy converges to the allocation in the barter economy as the growth rate of the money supply is increased.

Keywords: Money, Double Coincidence, Bargaining, Search.

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The need for trades to be ‘‘balanced’’ (satisfy the quid pro quo) at each
exchange ...  conflicts with the potential for exploiting all the gains
from trade.  The role of money is to attenuate this conflict.
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Ostroy and Starr (1990, p. 26)

1 Introduction

With no doubt, models of bilateral trades have sharply improved our understanding of the nature of money. The first wave of models developed by Ostroy (1973), Starr (1972), and Ostroy and Starr (1974) focus on the exchange process when bilateral trades are subject to a *quid pro quo* (or bilateral budget balance) restriction and when prices are given and assumed to clear markets.[1] In a second wave of models, Kiyotaki and Wright (1989, 1991, 1993) use random matching to represent the time-consuming trading process, to see what frictions can make monetary exchange an equilibrium, and to determine endogenously which objects serve as media of exchange.

In both frameworks the authors establish the welfare-improving role of money. The reason is similar: In a barter economy to fulfill the quid pro quo requirement, goods received must be paid for by a opposite delivery of goods of equal value. In a monetary economy, however, this link is broken because agents can finance their purchases through monetary transfers. In Kiyotaki and Wright (1989, 1991, 1993) money improves welfare by increasing the *frequency* of trades: with money, production and exchange take place in double and single coincidence meetings, without money, in double coincidence meetings only. In Ostroy and Starr's framework, the benefit of money is that it saves on organizational complexity (Ostroy and Starr 1974) or, alternatively, that it saves trading time (Starr 1976).

In this paper we focus on a different welfare-improving role of money which has been largely neglected in the literature. The benefit of money arises because it affects the quantities that are produced, exchanged, and consumed relative to what traders would exchange in the same situation in a barter economy. To study this role, we build a search model with perfectly divisible money, divisible goods, and idiosyncratic preferences. Agents, endowed with money and production opportunities, have the choice to finance their consumption through current production, sale of money, or both.[2] Terms of trades are endogenous and are determined in bilateral bargaining.[3]

[1] Quid pro quo requires that in any trade the value of the goods delivered by a trader must equal the value of the goods he receives. The origin of the quid pro quo restriction is that trades must be based on mutually agreements: a trade is neither a theft nor a gift. Quid pro quo is more restrictive than an intertemporal budget constraint on the entire sequence of trades.

[2] This choice, with the exception of Engineer and Shi (1998) and Laing et al. (2000), has not been available in previous search models of money. Rather, in these models buyers exchange money for consumption goods and sellers can only barter with other sellers.

[3] This is a natural extension of what Ostroy and Starr had in mind when they imposed the quid pro quo restriction in bilateral trade. See Starr (1989, p. 113) who argues that "a bilateral trading procedure should make sense in terms of representing optimizing behavior of the agent at each stage of the trading process as well as at

To study this neglected role of money we deliberately exclude the frequency effect of money as it appears in Kiyotaki and Wright (1991, 1993) by considering a *double coincidence of real wants* environments where in each bilateral meeting each agent is a consumer of the other agent's production. Consequently, money cannot speed up trades, rather it allows agents to trade *socially efficient* quantities in the sense that they equal the solution to a social planner's problem.

The key result of the model is that in a monetary economy the terms of trade in all matches are strictly better relative to the terms of trade that the traders would attain in the corresponding barter economy. To explain why money improves the terms of trade, consider the bargaining between two agents where each agent gets utility from consuming his partner's production good. Without money, they bargain over the quantities that are produced, consumed, and exchanged. The exchanged quantities simultaneously determine the total surplus of the match and how the traders split this surplus. Because agents cannot determine independently the size of the surplus and its split, and because each agent only cares about its own surplus, traders with asymmetric preferences attain a bargaining solution that does not maximize the total surplus of the match.

With money, agents bargain over the quantities and over a monetary transfer. Money improves the terms of trades because it transforms a bargaining game without transferable utility into a game with transferable utility. We, therefore, consider *utility transfer* as an important function of money.[4] In order to fulfill this function (i) money must be divisible and (ii) it must be equally valued by all agents[5]. Consequently, a monetary transfer does not affect the size of the total surplus of a match because changes in the payoffs cancel each other. This allows traders to *separate* the decisions of how much to produce and how to split the resulting total surplus. In contrast, real production is an *imperfect* device to transfer utility because marginal utility for the consumer and marginal cost of the producer vary with the quantity produced and exchanged and, in general, do not coincide. Consequently, an exchange of real commodities does not affect the payoffs symmetrically and, therefore, affects the total surplus of the match.

A necessary condition for barter trades to be inefficient and for monetary exchange to improve the terms of trade are asymmetric matches. We introduce asymmetric preferences by assuming that when two agents, say i and j, meet, preferences for each other's production good are represented by two random variables ε_i and ε_j where ε_i measures how much agent i likes agent j's good and ε_j measures how much j likes i's good. If $\varepsilon_i > \varepsilon_j$, agent i values agent j's output more

the endpoint. This results in a strategic foundation for the budget balance or quid pro quo constraint." In search models of money, strategic bargaining has been introduced by Shi (1995) and Trejos and Wright (1995).

[4] A bargaining game is a game with transferable utility if there is a device that allows players to simultaneously decrease their own utility payoff and increase the one of their partner by the same amount. A two-person bargaining game with transferable utility can be fully characterized by the disagreement payoffs of the two players and by the total transferable wealth available to the players (Myerson, 1991, pp.384-385). Note that money transfers *indirect* utility because money does not generate direct utility in our model: it is a claim to future consumption.

[5] Because agents have idiosyncratic preferences, money is the only good to be identically valued by all agents. This characteristic is related to Williamson and Wright's (1994) finding according to which the benefit of money arises because, contrary to real commodities, its quality can be recognized by everyone.

than j values i's output when the same quantities are exchanged.

In a barter economy, efficiency is attained in symmetric matches only, i.e., if $\varepsilon_i = \varepsilon_j$. In asymmetric matches, however, agents produce and exchange inefficient quantities and the degree of inefficiency increases in the degree of asymmetry. To see why bartering between asymmetric traders is inefficient assume that $\varepsilon_i > \varepsilon_j$. From a social point of view, agent j should produce a larger quantity than agent i. Thus, efficiency requires that while agent i receives a lot and produces little, agent j receives little and produces a lot. In a decentralized environment, without compensation, agent j will not agree to such an arrangement: the quid pro quo restriction is not fulfilled. Rather, the bargaining will result in inefficient quantities produced and exchanged.[6]

In contrast to the barter economy, in a monetary economy efficiency is also attained in asymmetric matches if the asymmetry, which we measure by some distance function $\mathcal{D}(\varepsilon_i, \varepsilon_j)$, is smaller than the real value of money holdings. The intuition behind this result is that, in contrast to the barter economy, agent j will agree to produce and exchange efficient quantities if agent i is able to compensate him by transferring claims to future consumption (money). If i's constraint on money holding is not binding, he is able to do so and i and j produce and exchange efficient quantities. If not, the bargaining will result in inefficient quantities produced and exchanged.

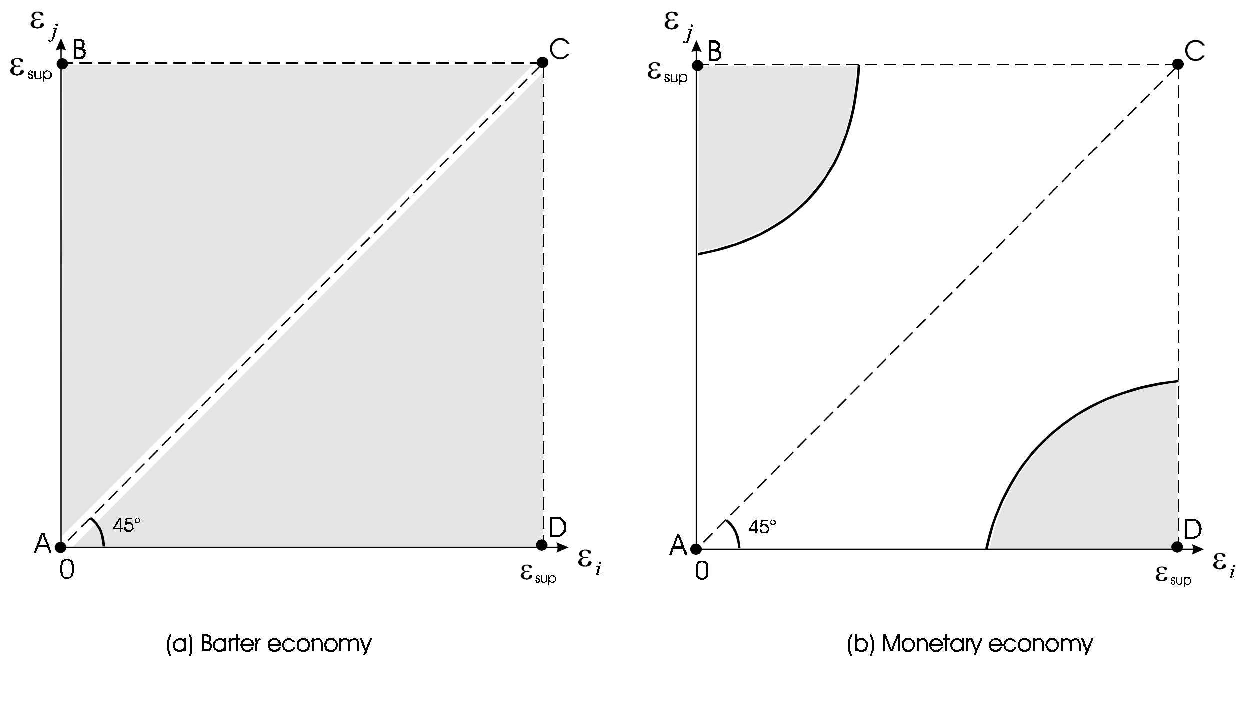


Figure 1: Efficient and inefficient trades with and without money.

Figure 1 displays regions with efficient and inefficient trades with and without money for all

[6] This argument is clear in the absence of double-coincidence of wants. Then $\varepsilon_i > 0$ and $\varepsilon_j = 0$ and efficiency requires that agent j produces a positive quantity and receives nothing.

match types, i.e., realizations of the random variables ε_i and ε_j. With money, the quantities exchanged are inefficient (efficient) for all $\varepsilon = (\varepsilon_i, \varepsilon_j)$ in the shaded (nonshaded) region — See Figure 1b. Without money, they are efficient on the 45-degree line and inefficient for all other ε — See Figure 1a. It is evident that the fraction of efficient trades is larger in the monetary economy. Moreover, the degree of inefficiency (welfare loss) in any of the matches in the grey area of the monetary economy is strictly smaller than the degree of inefficiency of the corresponding match in the barter economy.

We also show that lowering the inflation rate increases the real value of money, which reduces the set of matches with inefficient exchanges. In Figure 1b the shaded region of inefficient trades shrinks. Moreover, we find that the Friedman rule holds, i.e., when the gross growth rate of money approaches the discount factor, the set of inefficient trades vanishes. In contrast, the allocation of resources in the monetary economy converges gradually to the allocation in the barter economy as the expansion of the money supply is increased.

Figure 1 is also useful for a comparison of our model to the previous search and money models. In this literature there are only four types of meeting represented by the corners of Figure 1, which are denoted by A, B, C, and D. In B and D we have single coincidence meetings, in C symmetric double coincidence meetings, and A no coincidence meetings. In this literature, traders attain the efficient outcome when they barter (point D), and they exchanged inefficient quantities when they trade money for goods (point B and C). This suggests — and Figure 1 shows that this is not true — that money introduces inefficiencies into the bargaining that are not present in a barter economy.

1.1 Literature

The papers most closely related to our work are Engineer and Shi (1998, 2000) and Laing et al. (2000). In Engineer and Shi (1998, 2000) money can also be welfare improving in a double coincidence of wants environment. Engineer and Shi (1998) consider a random matching model with indivisible goods and indivisible money. There are low and high valuation goods and divisible service sidepayments that allow agents to bargain about the amount of these services that each agent produces for the other agent in asymmetric matches. By assumption marginal cost of these sidepayments is larger than their marginal utility. Consequently, production of sidepayments is a social waste. The welfare improving role of money comes from its ability to reduce the production of these sidepayments.

In contrast to Engineer and Shi (1998), we consider a model with divisible goods and divisible money and with a continuum of match types. There are no sidepayments, rather matched agent bargain about the amount of goods and money to be exchanged. There are more differences. First, in contrast to their paper, we have a unique monetary equilibrium (and a unique barter equilibrium). Second, the barter economy is generically inefficient. In their model, the barter economy is inefficient iff the bargaining power of the seller is either large or small. Third, in their model, inefficiency arises because agents barter too much. In our model, in all barter trades, while the production of the more valued good is too low, production of the less valued good is too large. Fourth, increasing the real stock of money unambiguously improves welfare.

Engineer and Shi (2000) extends their previous analysis by considering a divisible goods model where agents have symmetric preferences for each other's goods. Inefficient bartering is a result of asymmetric bargaining weights. Unlike Engineer and Shi (2000), we consider symmetric bargaining between agents that have asymmetric preferences for each other's good and the bargaining is described by a strategic game rather than an axiomatic solution. Moreover, money is perfectly divisible because the indivisibility of money introduces its own inefficiencies into the bargaining (Berentsen and Rocheteau 2000).

Laing et al. (2000) consider a divisible money and divisible goods model in a monopolistic competition setup. They distinguish firms and households, and prices are posted by firms. As in our model, in double coincidence meetings agents have the choice to finance their consumption either through money, real commodities, or both. In contrast to our model, in equilibrium agents only use one method to finance their purchases.[7]

The paper is organized as follows. In Section 2 we present the environment. In Section 3 we derive and characterize the barter equilibrium and in Section 4 the monetary equilibrium. Section 5 contains a discussion of the results and Section 6 concludes.

[7] In their pure monetary equilibrium agents always pay with money. In their mixed trading equilibrium, they pay with goods in double coincidence meetings and with money in single coincidence meetings.

2 The environment

We consider a random-matching model with divisible goods and divisible money along the lines of Shi (1999). The economy is populated with a large number of households each consisting of a continuum of members of measure one that regard the household's utility as the common objective. In the market, household members attempt to exchange money or their production good for consumption goods. In this attempt household members follow the strategy that has been given to them by their households. In some sense, household members behave as automaton that just execute household strategies.[8]

Although households differ in their preferences and production opportunities, they all consume and produce the same quantities so that each household and each good can be treated symmetrically. In the following we refer to an arbitrary household as household h. Decision variables of this household are denoted by lower-case variables. Capital-case variables denote other households' variables, which are taken as given by the representative household h. Furthermore, variables corresponding to the next period are indexed by $+1$.

2.1 Technology and preferences

The economy consists of a continuum of infinitely-lived households and a continuum of goods which are represented by points on the same circle of circumference 2 denoted by $\mathcal{C}$. Each household is composed of a continuum of members who share the same technology and have the same preferences. Households are specialized in both consumption and production. Household $h \in \mathcal{C}$ has the technology to produce good h. Producing q units of a good yields disutility $c[q] = q$. Goods cannot be stored and production is instantaneous. Household members derive utility from consuming *all* goods other than their production good. The most preferred good of household h is h^* chosen at random from $\mathcal{C}$. If we draw at random a commodity k from $\mathcal{C}$, the length l of the arc between k and h^* is uniformly distributed on $[0, 1]$. The function mapping the distance between the good that is consumed and the most preferred good, l, and the quantity consumed, q, into utility is continuous in both arguments, non-increasing in l, and increasing in q. We adopt the following function:[9]

$$\mathcal{U}(l, q) = \varepsilon(l)\, u(q).$$

The function $\varepsilon(l)$ is decreasing and twice differentiable and satisfies $\varepsilon(0) = \varepsilon_{\text{sup}}$ and $\varepsilon(1) = 0$. The utility associated with the consumption of q units of the most preferred good is $\varepsilon_{\text{sup}} u(q)$

[8] By adopting Shi's (1997, 1998, 1999) device, we avoid nondegenerate distributions of money holdings that are difficult to control. With this device, idiosyncratic risks of household members are smoothed out within each household. This facilitates the analysis because we can focus on a representative household.

[9] The analysis would also hold for any utility function satisfying $\frac{\partial U(l,q)}{\partial q} > 0$, $\frac{\partial U(l,q)}{\partial l} < 0$, $\frac{\partial^2 U(l,q)}{\partial q^2} < 0$, and $\frac{\partial^2 U(l,q)}{\partial q \partial l} < 0$.

whereas the utility of consumption of the worst good is zero. Furthermore, we assume that u is increasing and twice differentiable, and satisfies $u[0] = 0$, $u'' < 0$, and $u'[0] = \infty$. The probability that ε is less than $x \in [0, \varepsilon_{\text{sup}}]$ for a good chosen at random is equal to:

$$\mathbb{P}[\varepsilon(l) \leq x] = \mathbb{P}\left[l \geq \varepsilon^{-1}(x)\right] = 1 - \varepsilon^{-1}(x) \equiv F(x),$$

where $F(.)$ is a cumulative distribution with support $[0, \varepsilon^{\text{sup}}]$ and density f.

Finally, the utility of a household in one period is the sum of the consumption utilities of its members minus their disutilities of production. The discount factor is $\beta \in (0, 1)$.

2.2 Matching and Money

Time is discrete. In each period, household members meet pairwise and at random. We normalize the length of time of a period such that in each period each household member meets another member. Our assumptions about technology and preferences imply that in *each* match there is a double-coincidence of real wants, i.e., each household is a consumer of the other household's production good.[10] In general, however, agents' preferences for other agents' goods in a match are not symmetric. For an agent i matched with an agent j the type of the match is given by the pair $(\varepsilon_i, \varepsilon_j) \in [0, \varepsilon_{\text{sup}}] \times [0, \varepsilon_{\text{sup}}]$. From the point of view of agent i's partner, the match type is $(\varepsilon_j, \varepsilon_i)$. Generically, $(\varepsilon_i, \varepsilon_j) \neq (\varepsilon_j, \varepsilon_i)$. In the following, we adopt the following notation. For all $\varepsilon = (\varepsilon_i, \varepsilon_j)$, $\varepsilon' = (\varepsilon_j, \varepsilon_i)$. Thus, if ε describes the match type from the point of view of agent i, ε' described the match type from the point of view of agent j. For any individual, the distribution of the matches can be described by the measure μ defined as follows:

$$\forall A \subseteq [0, \varepsilon_{\text{sup}}] \times [0, \varepsilon_{\text{sup}}], \qquad \mu(A) = \int_0^{\varepsilon_{\text{sup}}} \int_0^{\varepsilon_{\text{sup}}} \mathbf{1}_A(x, y) f(x) f(y)\, dx dy,$$

where $\mathbf{1}_A(x, y)$ is an indicator function that is equal to one if $(x, y) \in A$ and zero otherwise.

In the monetary economy, in addition to the consumption goods, there is also an intrinsically worthless, storable, and fully divisible object called fiat money. At the beginning of each period, each household has m units of money. The households divide evenly the money stock among its members so that each member holds m units of money in a match. After this, household members are matched and carry out their exchanges according to the prescribed strategies. Within a period, a member of the household cannot transfer money balances to another member of the same household. Thereafter, members bring back their receipts of money and each agent consumes the goods he has bought. At the end of a period, the household receives money transfer τ, which is perceived as lump-sum and can be negative, and then carries the stock m_{+1} to $t + 1$.

[10] In fact, there are single-coincidence meetings (along the segments $]AB]$ and $]AD]$ in Figure 1). The measure of single-coincidence meetings, however, is zero.

2.3 Terms of trades

The terms of trades are determined through bargaining games with alternating offers (Rubinstein, 1982). Two important features of the bargaining are that (i) bargaining strategies are determined at the household level but are carried out by household members, and (ii) household members observe the match type but cannot observe the marginal value of money and the level of money holdings of their trading partners. Because of (ii) households' strategies depend on the match type and on the distribution of their potential bargaining partners' characteristics. In equilibrium, this distribution is degenerated: all households have the same marginal utility of money and hold the same quantity of money. Because of (i) any strategy aiming to reveal ones true characteristics (if different than the average) is ineffective because the bargaining partner commits to the bargaining strategy chosen by its household.[11]

In the following we consider the bargaining between agent i of household h and agent j from any other household. The bargaining proceeds as follows. Each period is divided into an infinite number of subperiods of length Δ. If, in a given subperiod, it is agent i's turn to make an offer and agent j rejects the offer, in the following subperiod it is agent j's turn to make a counteroffer. If an offer is refused, the negotiation breaks down with probability $\delta\Delta$ ($\delta > 0$). The possibility of an exogenous break-down of the negotiation gives an incentive to traders to agree immediately.[12]

In the alternating offer game, offers and counteroffers converge to the same limiting proposal when Δ goes to zero. Consequently, the first-mover advantage vanishes when Δ goes to zero.[13] Because of this and because, as we will see, it facilitates the derivation of the dynamic equation describing the marginal utility of money, we let members of household h make the first offer in all meetings. In equilibrium all households have the same characteristics: as a consequence, first offers of h are always accepted. Moreover, because the length of time between two consecutive offers is infinitesimal, these first offers are exactly the counteroffers that would have been made by h's partners.

In the following, we will consider a barter economy and a monetary economy. In a barter economy, agent i and j bargain over the quantities q_ε^b and q_ε^s that are produced, consumed, and exchanged where agent j (i) produces commodity $q_\varepsilon^b(q_\varepsilon^s)$ and consumes commodity $q_\varepsilon^s(q_\varepsilon^b)$. The subscript ε indicates that the bargaining solution will depend on the type of match. In a monetary economy, they also bargain over a monetary transfer x_ε where $x_\varepsilon > 0$ means that agent i transfers x_ε units of money to agent j.

[11]This pricing procedure is different to the one assumed by Rauch (2000) where the bargaining strategy is determined at the level of the household's members and where there is no private information.

[12]This breakdown may be explained by the weariness of players when the agreement is postponed (Muthoo, 1999, p.73). Instead of a break-down of the bargaining, cost of delaying an agreement could arise from the fact that households discount the future (Shi 1998). Although formally equivalent, this last interpretation would not be completely rigourous in a discrete time model.

[13]This argument is standard in the bargaining literature. See, for example, Muthoo (1999, chapter 3), Osborne and Rubinstein (1990, chaper 3).

3 Barter economy

In order to capture the benefit of money we compare the allocation of resources of the barter economy with the one of the monetary economy. In this section we present the barter equilibrium where money has no value.

3.1 The offers

Consider the bargaining between agents i and j. Agent i, following the strategy prescribed by household h, proposes the terms of trade $\left(q_\varepsilon^b, q_\varepsilon^s\right)$ if the match type is $\varepsilon = (\varepsilon_i, \varepsilon_j)$, i.e., if his valuation for the good produced by j is $\varepsilon_i u\left(q_\varepsilon^b\right)$ and if j's valuation for the good he produces is $\varepsilon_j u\left(q_\varepsilon^s\right)$. According to his offer, q_ε^b is the quantity he consumes and agent j produces, and q_ε^s is the quantity he produces and j consumes. If it is agent j's turn to make an offer, j proposes the terms of trade $\left(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s\right)$ where the index ε' indicates that, from the point of view of j, the match type is $\varepsilon' = (\varepsilon_j, \varepsilon_i)$. According to j's offer, $Q_{\varepsilon'}^s$ is the quantity i consumes and j produces, and $Q_{\varepsilon'}^b$ is the quantity i produces and j consumes.

Denote $D_{\varepsilon'}$ the expected surplus of agent j if he rejects the offer: it is taken as given by the household of agent i. Then, any optimal offer $\left(q_\varepsilon^b, q_\varepsilon^s\right)$ must satisfy:

$$\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b = D_{\varepsilon'} \tag{1}$$

According to (1), agent i's offer makes agent j just indifferent between accepting or rejecting the offer. The expected surplus of agent j equals

$$D_{\varepsilon'} = (1 - \delta\Delta)\left[\varepsilon_j u\left(Q_{\varepsilon'}^b\right) - Q_{\varepsilon'}^s\right] \tag{2}$$

If agent j rejects the offer, negotiations break down with probability $\delta\Delta$. With probability $(1 - \delta\Delta)$ there is no breakdown and agent j makes the counteroffer $\left(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s\right)$ after a period of time of length Δ.

3.2 The program of the household

A household's trading strategy consists of the terms of trade $\left(q_\varepsilon^b, q_\varepsilon^s\right)$ for each $\varepsilon \in E$, and an acceptance rule that specifies whether an offer is acceptable or not. According to (1), household members make offers that will not be refused. For each period, the household chooses $\left(q_\varepsilon^b, q_\varepsilon^s\right)_{\varepsilon \in E}$ to solve the following dynamic programming problem:

$$\begin{aligned} V &= \max_{q_\varepsilon^b, q_\varepsilon^s} \left\{ \int_E \varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s \, d\mu\left(\varepsilon_i, \varepsilon_j\right) + \beta V \right\} \\ & \quad \text{s.t. to (1),} \end{aligned} \tag{3}$$

where V is the lifetime discounted utility of the household h. The right-hand side of equation (3) has the following interpretation. For each match type ε, a member of household h makes the first proposal $(q^b_\varepsilon, q^s_\varepsilon)$ that is immediately accepted by his partner. The net utility in an ε-meeting is then $\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon$. Note that there is no state variable linking the present to the future. Consequently, for each period and each match type, households just maximize current utility of consumption net of current disutility of production. Substituting q^b_ε by its expression given by (1) and differentiating V with respect to q^s_ε yields the following first-order condition:

$$\varepsilon_i u'\left(q^b_\varepsilon\right) = \frac{1}{\varepsilon_j u'\left(q^s_\varepsilon\right)} \qquad \forall \varepsilon \in E \tag{4}$$

According to (4), the terms of trade $(q^b_\varepsilon, q^s_\varepsilon)$ equalize marginal utility of consumption, $\varepsilon_i u'\left(q^b_\varepsilon\right)$, to marginal disutility of production, $\frac{1}{\varepsilon_j u'(q^s_\varepsilon)}$. To see this, note that equation (1) implies that $\frac{\partial q^s_\varepsilon}{\partial q^b_\varepsilon} = \frac{1}{\varepsilon_j u'(q^s_\varepsilon)}$. Thus, to buy one additional unit of consumption, agent i must produce $\frac{1}{\varepsilon_j u'(q^s_\varepsilon)}$ units of good for agent j, which costs $\frac{1}{\varepsilon_j u'(q^s_\varepsilon)}$ in terms of utility.

3.3 The barter equilibrium

In equilibrium, all households have the same characteristics. Consequently, they make the same offer if it is their turn to make a proposal: $\left(q^b_\varepsilon, q^s_\varepsilon\right) = \left(Q^b_\varepsilon, Q^s_\varepsilon\right)$ for all ε. Equations (1) and (2) yield:

$$\varepsilon_j u\left(q^s_\varepsilon\right) - q^b_\varepsilon = (1 - \delta\Delta)\left[\varepsilon_j u\left(q^b_{\varepsilon'}\right) - q^s_{\varepsilon'}\right] \tag{5}$$

Definition 1 *A steady state barter equilibrium is a set of offers* $\left\{\left(q^b_\varepsilon, q^s_\varepsilon\right)\right\}_{\varepsilon \in E}$ *satisfying equations (4) and (5).*

We want to compare the outcome of the decentralized economy to the allocation that a social planner would choose. The social planner treats all household symmetrically and consequently maximizes the utility of a representative household. In the Appendix we show that welfare is maximized if, for each match type $\varepsilon \in E$, the planner chooses the terms of trade $(q^b_\varepsilon, q^s_\varepsilon)$ that maximize the total surplus of the match. For each match type, total surplus equals $\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon + \varepsilon_j u\left(q^s_\varepsilon\right) - q^b_\varepsilon$. Thus, total surplus is maximized if q^b_ε and q^s_ε satisfy

$$\varepsilon_i u'\left(q^b_\varepsilon\right) = \varepsilon_j u'\left(q^s_\varepsilon\right) = 1 \qquad \forall \varepsilon \in E \tag{6}$$

Equation (6) simply states that total surplus in a match is maximized if, for each good, marginal utility of consumption equals marginal disutility of production. Denote efficient quantities by q^{b*}_ε and q^{s*}_ε, respectively. In what follows we focus our attention on the limiting case when the length of time between two consecutive offers approaches zero, i.e., $\Delta \to 0$. Then, offers $(q^b_\varepsilon, q^s_\varepsilon)$ and counteroffers $(q^s_{\varepsilon'}, q^b_{\varepsilon'})$ coincide and the lifetime discounted utilities of all households are equal.

Proposition 1 *Assume that $\Delta \to 0$. Then, there is a unique barter equilibrium where in all asymmetric matches the quantities exchanged are inefficient and in all symmetric matches they are efficient. Furthermore, in an ε-meeting, the quantities traded $\left(q_\varepsilon^b, q_\varepsilon^s\right)$ satisfy (4) and*

$$\frac{\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b}{\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s} = \frac{1}{\varepsilon_i u'\left(q_\varepsilon^b\right)} \tag{7}$$

Proposition 1 establishes the existence and uniqueness of the barter equilibrium.[14] In this barter equilibrium, traders exchange socially inefficient quantities when they have asymmetric tastes for each other's goods and efficient quantities when the have symmetric tastes. The reason for the observed inefficiencies is that in a barter economy the quantities produced and consumed determine simultaneously the size and the split of the total surplus of the match. Because households only care about their own surplus the traders attain a bargaining solution that does not maximize the size of the surplus of the match.

To see this, consider a match between agent i and agent j and assume that $\varepsilon_i > \varepsilon_j$. From a social point of view, agent j should produce a larger quantity than agent i. Thus, efficiency requires that while agent i receives a lot and produces little, agent j receives little and produces a lot. In a decentralized environment, without compensation, agent j will not agree to such an arrangement: the *quid pro quo* requirement is not fulfilled. Rather, the bargaining will result in inefficient quantities produced and exchanged.

Note that if households could credibly commit to exchange efficient quantities $\left(q_\varepsilon^{b*}, q_\varepsilon^{s*}\right)$ in each meeting, each household would be better off relative to the decentralized outcome. Thus, from the perspective of the households, the outcome is Pareto inefficient.[15]

Corollary 1 *Consider an $(\varepsilon_i, \varepsilon_j)$-meeting in a barter economy with $\varepsilon_i > \varepsilon_j$. Then, $q_\varepsilon^b < q_\varepsilon^{b*}$ and $q_\varepsilon^s > q_\varepsilon^{s*}$. Furthermore, agent i receives more than half of the total surplus of the match.*

Corollary 1 establishes that if agent i likes agent j's good relatively more, i.e., if $\varepsilon_i > \varepsilon_j$, agent i gets less and agent j more than the efficient quantity, respectively. Moreover, agent i receives more than half of the total surplus. The analysis here complements the results in Engineer and Shi (2000). In their model, while there is a symmetric coincidence of real wants, there are asymmetric bargaining weights in all matches. This asymmetry generates socially inefficient terms of trades. Agents that have less (more) bargaining weight than their bargaining partner in a match receive less (more) than the socially efficient quantity. In our model the agent that is more eager to get the other agent's good, gets less than the efficient quantity.

[14] Note that the solution $\left(q_\varepsilon^b, q_\varepsilon^s\right)$ of equations (4) and (7) also maximizes the symmetric Nash product, i.e., $\left(q_\varepsilon^b, q_\varepsilon^s\right) = \arg\max \left[\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s\right] \left[\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b\right]$.

[15] One way to correct this inefficiency is to endow households with perfect knowledge of all past transactions. With this knowledge there are punishment strategies that implement the socially efficient outcome. See Kocherlakota (1998).

Finally, we want to emphasize that the barter economy displays a *double coincidence of real wants* in all matches. In each match, each household is a consumer of the other household's production, the traded quantities yield a positive surplus, and the exchange is quid pro quo. Nevertheless, in the Section 4 we show that a monetary equilibrium exists and that money improves welfare. In fact, we show that the terms of trade between two agents are always better in the monetary equilibrium compared to the terms of trades when the two agent are in the same situation in the barter economy.

4 Monetary economy

4.1 The offers

In the monetary economy each agent holds m units of money when matched. We again restrict our attention to meetings between member i of household h and some agent j from another household. Assume that, in a given subperiod, it is agent i's turn to make an offer. Agent i, following the strategy prescribed by his household, proposes the terms of trade $\left(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon\right)$ if the match type is $\varepsilon = (\varepsilon_i, \varepsilon_j)$. As in the barter economy, q_ε^b is the quantity of goods produced by agent j and consumed by agent i and q_ε^s is the quantity of goods delivered by agent i. x_ε is the quantity of money exchanged. If $x_\varepsilon > 0$, agent i delivers x_ε units of money to agent j and if $x_\varepsilon < 0$, he receives x_ε units of money. If it is agent j's turn to make an offer, he proposes the terms of trade $\left(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s, X_{\varepsilon'}\right)$. From his point of view, the match type is $\varepsilon' = (\varepsilon_j, \varepsilon_i)$.

If $V(m)$ denotes the lifetime expected utility of household h endowed with m units of money, the marginal value of money is $\omega = \beta V_m(m_{+1})$ where V_m is the derivative of V with respect to money holdings m. Furthermore, denote Ω the marginal value of money of other households (including j). If agent j accepts the offer $\left(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon\right)$ and if $x_\varepsilon > 0$, the acquired money balances x_ε will add to j's household money balances at the beginning of the next period, whose value today is Ωx_ε. Agent j accepts the offer if $\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b + x_\varepsilon \Omega \geq D_{\varepsilon'}$, where $D_{\varepsilon'}$ denotes the expected surplus of agent j if he rejects the offer. Thus, any optimal offer by agent i satisfies:

$$\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b + x_\varepsilon \Omega = D_{\varepsilon'} \tag{8}$$

The only difference between barter offer (1) and monetary offer (8) is that the latter includes the monetary transfer x_ε. As in Section 3, on can derive an explicit expression for $D_{\varepsilon'}$. We find,

$$D_{\varepsilon'} = (1 - \delta\Delta)\left[\varepsilon_j u\left(Q_{\varepsilon'}^b\right) - Q_{\varepsilon'}^s - X_{\varepsilon'}\Omega\right]. \tag{9}$$

where $\left(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s, X_{\varepsilon'}\right)$ is agent j's proposal when it is his turn to make an offer. Note that $X_{\varepsilon'}$ is a monetary transfer from j to i.

4.2 The program of the household

When the household determines the terms of trades, it is subject to two sets of constraints. First, household members cannot spend more money than what they have:

$$x_\varepsilon \leq m \qquad \forall \varepsilon \in E \tag{10}$$

Second, household members cannot ask for more money than what their bargaining partner holds:

$$-x_\varepsilon \leq M \qquad \forall \varepsilon \in E \tag{11}$$

A household's trading strategy consists of the terms of trade $\left(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon\right)$ for each $\varepsilon \in E$, and an acceptance rule for each offer $\left(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s, X_{\varepsilon'}\right)$ by another household. Again, agent i from household

h makes the first offer which is immediately accepted by j. For each period, the household chooses $\left\{m_{+1}, \left(q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon\right)_{\varepsilon\in E}\right\}$ to solve the following dynamic programming problem:

$$V(m) = \max_{q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon, m_{+1}} \left\{ \int_E \varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right) + \beta V(m_{+1}) \right\} \tag{12}$$

subject to the constraints (8), (10), (11), and

$$m_{+1} - m = \tau - \int_E x_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right) \tag{13}$$

The variables taken as given in the above problem are the state variable m and other households' choices (the capital case variables). The integral in equation (12) aggregates the net utilities in all meetings. Equation (13) specifies the law of motion of the household's money balances. The first term on the right-hand side specifies the additional currency the household receives each period. We assume that the supply of money increases at rate $(\gamma - 1)$ which implies that next period's money supply is $M_{+1} = \gamma M$.

Denote λ_ε the multipliers associated with constraints (10). The multipliers associated with constraints (11) will be denoted by π_ε. Then, the program of the household can be rewritten as follows

$$\begin{aligned} V(m) = \max_{q^b_\varepsilon, q^s_\varepsilon, m_{+1}} & \left\{ \int_E \varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right) + \int_E \lambda_\varepsilon\left(m - x_\varepsilon\right) d\mu\left(\varepsilon_i, \varepsilon_j\right) + \right. \\ & \left. + \int_E \pi_\varepsilon\left(M + x_\varepsilon\right) d\mu\left(\varepsilon_i, \varepsilon_j\right) + \beta V(m_{+1}) \right\} \end{aligned}$$

where m_{+1} is given by equations (13). Furthermore, according to (8), x_ε can be expressed as a function of q^s_ε and q^b_ε. The first-order conditions and the envelope condition are as follows:

$$\varepsilon_i u'\left(q^b_\varepsilon\right) = \frac{\lambda_\varepsilon - \pi_\varepsilon + \omega}{\Omega} \qquad \forall \varepsilon \in E \tag{14}$$

$$\varepsilon_j u'\left(q^s_\varepsilon\right) = \frac{\Omega}{\omega + \lambda_\varepsilon - \pi_\varepsilon} \qquad \forall \varepsilon \in E \tag{15}$$

$$\lambda_\varepsilon\left(m - x_\varepsilon\right) = 0 \qquad \forall \varepsilon \in E \tag{16}$$

$$\pi_\varepsilon\left(M + x_\varepsilon\right) = 0 \qquad \forall \varepsilon \in E \tag{17}$$

$$\frac{\omega_{-1}}{\beta} = \int_E \omega + \lambda_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right) \tag{18}$$

Let us first interpret the optimal choices of q^b_ε and q^s_ε. For each match type, the household has two decisions. First, how much to consume. Second, how to finance this consumption because the household can either finance current consumption through sale of money, production, or both. The first decision implies the equalization of the marginal utility of consumption to the marginal cost of financing this consumption. The second decision is an *arbitrage* condition. This condition requires that the value of the amount of money that the household must pay is equal to the marginal cost of producing additional goods to finance this purchase.

To see the relationship between the two choices note that the first-order conditions (14) and (15) imply the following relation:

$$\varepsilon_i u'\left(q_\varepsilon^b\right) = \frac{\lambda_\varepsilon - \pi_\varepsilon + \omega}{\Omega} = \frac{1}{\varepsilon_j u'\left(q_\varepsilon^s\right)} \qquad \forall \varepsilon \in E \tag{19}$$

The left-hand side of (19) is the marginal utility of consumption. The right-hand side is the marginal cost of financing the additional consumption through production (see equation (8)). The middle term is the value of the amount of money that the household must pay to finance this additional consumption. The arbitrage condition is given by the equality on the right-hand side of (19). It simply states that a household when purchasing an additional unit of consumption good uses the payment instrument that costs less.

To interpret the middle term of (19), note that the Kuhn-Tucker conditions (16) and (17) imply that λ_ε and π_ε cannot be both positive at the same time. Suppose, first, that agent i's money holdings are binding, i.e., $\lambda_\varepsilon > 0$ and $\pi_\varepsilon = 0$. Then, to buy another unit of a good, the household must give up $\frac{1}{\Omega}$ units of money (see equation (8)). Increasing the monetary payment has two costs to the household. He gives up the future value of money ω and he faces a tighter constraint (10). Note that the household is willing to produce at a higher marginal cost to finance this consumption when constraint (10) is binding. Together, ω and λ_ε measure the marginal cost of obtaining a larger quantity of goods in exchange for money. Suppose, now, that j's constraint on money holdings is binding from th point of view of household i, i.e., $\lambda_\varepsilon = 0$ and $\pi_\varepsilon > 0$. Because j's constraint is binding his marginal value of money is larger than Ω and, therefore, agent i has to pay less to obtain this additional consumption. The shadow price π_ε can be interpreted as a discount that agent j is willing to sacrifice to obtain additional money if his constraint is binding.

Equations (16) and (17) are the Kuhn-Tucker conditions associated with the multipliers λ_ε and π_ε, respectively. Finally, equation (18) describes the evolution of the marginal value of money. It states that the marginal value of money today, $\frac{\omega_{-1}}{\beta}$, equals the marginal benefit of money in all meetings, $\int_E \omega + \lambda_\varepsilon d\mu\left(\varepsilon_i, \varepsilon_j\right)$. This integral has the following interpretation: In a ε-meeting the value of an additional unit of money is equal to $\omega + \lambda_\varepsilon$, which is simply ω if the agent is not cash constraint. Note that the value of money is larger in a meeting where the agent is cash constraint because an additional unit of money would allow him to avoid inefficient production.

4.3 The equilibrium

In equilibrium, all households have the same characteristics, which implies that the values of the different variables of the household h equal the values of the same variables of all other households. Consequently, capital-case variables and lower-case variables are equal: $\omega = \Omega$,

$m = M$, and $\left(q^b_{\varepsilon'}, q^s_{\varepsilon'}, x_{\varepsilon'}\right) = \left(Q^b_{\varepsilon'}, Q^s_{\varepsilon'}, X_{\varepsilon'}\right)$ for all ε. Then, equations (8) and (9) yield:

$$\varepsilon_j u\left(q^s_\varepsilon\right) - q^b_\varepsilon + x_\varepsilon \omega = (1 - \delta\Delta)\left[\varepsilon_j u\left(q^b_{\varepsilon'}\right) - q^s_{\varepsilon'} - x_{\varepsilon'}\omega\right] \tag{20}$$

Definition 2 *A monetary steady state equilibrium is a* $\omega > 0$ *and a set* $\left\{\left(q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon, \lambda_\varepsilon, \pi_\varepsilon\right)\right\}_{\varepsilon \in E}$ *such that equations (14), (15), (16), (17), (18), and (20) hold.*

We again want to compare the outcome of the decentralized economy to the allocation that a social planner would choose in order to maximize social welfare. As in the barter economy, welfare is maximized if in each meeting q^b_ε and q^s_ε satisfy $\varepsilon_i u'\left(q^b_\varepsilon\right) = \varepsilon_j u'\left(q^s_\varepsilon\right) = 1$. Lemma 1 characterizes the terms of trade in the monetary equilibrium when the length of time between two consecutive offers approaches zero.

Lemma 1 *Assume that* $\Delta \to 0$ *and consider an* $(\varepsilon_i, \varepsilon_j)-$*meeting. Then, there are two cases:*

Case a. *One of the traders' constraints on money holdings is binding. If agent i's constraint on money holding is binding,* $q^b_\varepsilon < q^{b*}_\varepsilon$, $q^s_\varepsilon > q^{s*}_\varepsilon$, $x_\varepsilon = m$, *and agent i receives more than one half of the total surplus of the match. The exchanged quantities* q^b_ε *and* q^s_ε *satisfy equations (19) and*

$$\frac{1}{\varepsilon_i u'\left(q^b_\varepsilon\right)} = \frac{\varepsilon_j u\left(q^s_\varepsilon\right) - q^b_\varepsilon + m\omega}{\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon - m\omega}. \tag{21}$$

Case b. *No trader is constrained by his money holdings. Then,* $q^b_\varepsilon = q^{b*}_\varepsilon$, $q^s_\varepsilon = q^{s*}_\varepsilon$, *and* x_ε *satisfies the following splitting rule:*

$$\frac{\varepsilon_j u\left(q^{s*}_\varepsilon\right) - q^{b*}_\varepsilon + x_\varepsilon\omega}{\varepsilon_i u\left(q^{b*}_\varepsilon\right) - q^{s*}_\varepsilon - x_\varepsilon\omega} = 1. \tag{22}$$

According to case b of Lemma 1, if no agent is constrained by his money holdings, the matched agents produce, exchange, and consume efficient quantities and they split the total surplus of the match evenly. If one of the agents' constraint on money holdings is binding (case a of Lemma 1), the terms of trade are inefficient and the agent whose constraint is binding receives more than half of the total surplus. Note that in Lemma 1 we do not mention the case when agent j's constraint on money holding is binding because by symmetry we would have $q^b_\varepsilon > q^{b*}_\varepsilon$, $q^s_\varepsilon < q^{s*}_\varepsilon$, $x_\varepsilon = -m$.[16]

To see why, in contrast to the barter equilibrium, in matches with asymmetric tastes efficient trades can occur, consider a match between agents i and j and assume that $\varepsilon_i > \varepsilon_j$. Because for a given quantity of goods i's marginal utility of consumption is higher than j's one, then efficiency requires that agent j produces a larger quantity than agent i. Agent j will agree to such an arrangement if agent i is able to compensate him by transferring sufficient claims to

[16] Note that the triplet $\left(q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon\right)$ given in Lemma 1 is also solution of the following program:

$$\max_{q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon} \left[\varepsilon_i u(q^b_\varepsilon) - q^s_\varepsilon - x_\varepsilon\omega\right]\left[\varepsilon_j u(q^s_\varepsilon) - q^b_\varepsilon + x_\varepsilon\omega\right] \text{ s.t. } -m \le x_\varepsilon \le m$$

future consumption (money). If i's constraint on money holding is not binding, he is able to do so and i and j exchange efficient quantities. If not, the bargaining will result in inefficient quantities produced and exchanged.

Money, in contrast to real production, is a perfect device to transfer utility because for all household an additional unit of money has the same real value ω.[17] Real production on the other hand is an imperfect mean to transfer utility because marginal utility for the consumer and marginal cost of the producer vary with the quantity produced and exchanged. We want to emphasize the importance of money being equally valued for efficiency. If money were not equally valued, according to (14), inefficient quantities would be produced and exchanged even if constraints on money holdings were not binding ($\lambda_\varepsilon = \pi_\varepsilon = 0$). For example, if $\omega > \Omega$, which means that household h values money more than other households, household h would receive less and produce more than the efficient quantities.

In the following we want to characterize the set of match types that generate inefficient trades. To derive this set, we introduce a particular distance function $\mathcal{D}(\varepsilon_i, \varepsilon_j)$, which measures the asymmetry in preferences in a match. It is defined as follows:

$$\mathcal{D}(\varepsilon_i, \varepsilon_j) = \left| \frac{\varphi(\varepsilon_i) - \varphi(\varepsilon_j)}{2} \right|$$

where $\varphi(\varepsilon) = \varepsilon u\left(u'^{-1}\left(\frac{1}{\varepsilon}\right)\right) + u'^{-1}\left(\frac{1}{\varepsilon}\right)$.[18]

Proposition 2 *There is a $\overline{\gamma} > \beta$ such that if $\gamma \in (\beta, \overline{\gamma})$, a unique monetary equilibrium exists. It has the following properties:*

(i) An $(\varepsilon_i, \varepsilon_j)$-trade is inefficient if and only if $\mathcal{D}(\varepsilon_i, \varepsilon_j) > m\omega$,

(ii) The measure of inefficient trades is decreasing in $m\omega$ and tends to zero as $\gamma \rightarrow \beta$.

Proposition 2 establishes the existence of a unique monetary equilibrium with valued fiat money if the gross growth rate of the money supply γ is not too high. According to (8), the amount of money that must be spent to buy one additional unit of a good is $\frac{1}{\omega}$. Hence, the price level in this economy is $p = \frac{1}{\omega}$ and the real stock of money is $\frac{m}{p} = m\omega$. A monetary equilibrium is characterized by a unique stationary level of the real stock of money, $m\omega$, which depends negatively on the growth rate of the money supply $(\gamma - 1)$. As a consequence, money is neutral but not super-neutral.

In contrast to the barter equilibrium, in the monetary equilibrium asymmetric tastes for each other's goods do not always generate inefficient trades. Whether a trade is inefficient depends on the degree of asymmetry of the match as measured by the distance $\mathcal{D}$. Efficient quantities are

[17] All households have the same marginal value of money because the distribution of money is degenerate. In models with a nondegenerate distribution of money (e.g., Berentsen (2000), Rocheteau (2000), and Zhou (1999)), the marginal value of money is not equal for all agents.

[18] Note that $\mathcal{D}(\varepsilon_i, \varepsilon_j)$ satisfies the three properties of a distance. (i) $\mathcal{D}(\varepsilon_i, \varepsilon_j) \geq 0$ for all $(\varepsilon_i, \varepsilon_j)$. Furthermore, $\mathcal{D}(\varepsilon_i, \varepsilon_j) = 0$ only when $\varepsilon_i = \varepsilon_j$. (ii) $\mathcal{D}(\varepsilon_i, \varepsilon_j) = \mathcal{D}(\varepsilon_j, \varepsilon_i)$. (iii) $\mathcal{D}(\varepsilon_i, \varepsilon_j) \leq \mathcal{D}(\varepsilon_i, \varepsilon_k) + \mathcal{D}(\varepsilon_k, \varepsilon_j)$.

exchanged in a match if the distance in preferences is less than the real value of money $m\omega$. This result implies two things. First, it is the degree of asymmetry for each other's goods that matters for efficiency. Second, if the real stock of money is large, a randomly chosen match type is less likely to generate inefficient terms of trades. Or, in other words, the measure of inefficient trades decreases with the real value of money.

Inflation is costly in this model because it generates a misallocation of resources. A higher rate of the money supply increases the misallocation because it reduces the set of meetings where agents produce and exchange efficient quantities. When the gross growth rate of the money supply approaches the upper-bound $\overline{\gamma}$, the allocation of resources in the monetary economy converges to the (mis)allocation of the corresponding barter economy. In contrast, when the gross growth rate of the money supply approaches the discounting factor, almost all trades are efficient, i.e., the Friedman's rule holds.

5 Discussion

In the following we want to develop more intuition for the results presented in the previous Sections. To do so, we use the fact that the solution of Rubinstein's bargaining game between the households of agents i and j maximizes the symmetric Nash product of the traders' surpluses S_i and S_j, respectively (See footnotes 14 and 17). In what follows we assume, without loss in generality, that $\varepsilon_i > \varepsilon_j$. Bargaining solutions and figures are drawn for a single $\varepsilon \in E$ only.

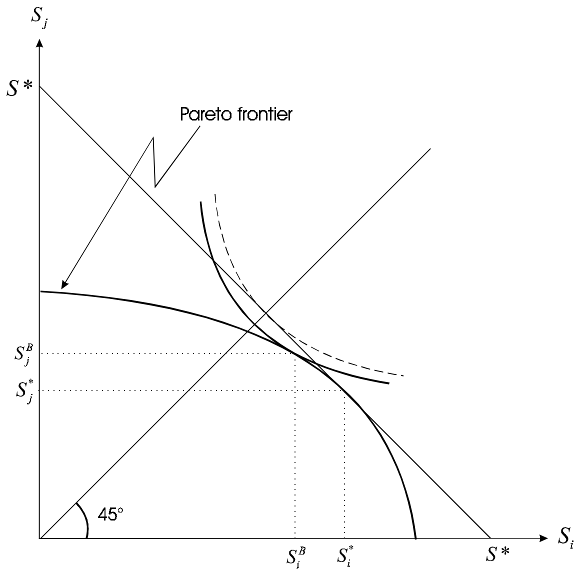


Figure 2: Socially Inefficient Bartering

Consider, first, the barter economy where the surpluses of agents i and j are $S_i = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s$ and $S_j = \varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b$, respectively. The Pareto frontier of the bargaining set is the concave curve in Figure 2.[19] Each point on this curve uniquely determines what quantities of goods i and j produce, exchange, and consume. The bargaining solution is determined by the tangency point between the Pareto frontier and a Nash product curve (the convex curves in Figure 2). The surpluses of the players at this point are denoted by $\left(S_i^B, S_j^B\right)$. Note that $\left(S_i^B, S_j^B\right)$ is located to the right of the 45° line, which means that i gets more than half of the surplus (see Corollary 1).

The S^*S^* line represents every possible split (S_i, S_j) of the maximal total surplus of the match $S^* = S_i^* + S_j^*$ where $S_i^* = \varepsilon_i u(q_\varepsilon^{b*}) - q_\varepsilon^{s*}$ and $S_j^* = \varepsilon_j u(q_\varepsilon^{s*}) - q_\varepsilon^{b*}$. Consequently, the S^*S^* line is the Pareto frontier of the same bargaining game with transferable utility. A bargaining game is a game with transferable utility if there is a device that allows players to decrease their own payoff and increase the one of their partner by the same amount. A two-person bargaining game with transferable utility is fully characterized by the disagreement payoffs of the two players and by the total transferable wealth (S^*) available to the players i and j.

[19] See the appendix for a formal derivation of the Pareto frontier and some of the other technical details.

Bartering is socially inefficient because $\left(S_i^B, S_j^B\right) \notin S^*S^*$, which means that the terms of trades do not maximize the total surplus of the match. Note that $\varepsilon_i > \varepsilon_j$ implies $S_i^* > S_j^*$ and that if the asymmetry in preferences is large, S_j^* may be negative. Moreover, at $\left(S_i^*, S_j^*\right)$ the slope of the Pareto frontier is -1. Hence, $\left(S_i^*, S_j^*\right)$ is at the tangency point between the Pareto frontier and line S^*S^*.

In the barter economy agents have no device to transfer utility, which implies that $\left(S_i^*, S_j^*\right)$ is the only feasible split of the maximal total surplus S^*. If, however, the traders had a device to transfer utility, they could attain any point on the S^*S^* line by producing and consuming efficient quantities and, then, split total utility S^* through a transfer of utility. Note that the Nash product curve (see the doted curve in Figure 2) has a tangency at the S^*S^* line where this line crosses the 45° line. Consequently, if agents could transfer utility, they would produce and exchange efficient quantities and, then exchange utility to attain $S_i = S_j = S^*/2$.

We, next, discuss how the Pareto frontier is transformed when money is introduced into the economy. The nature of the transformation is displayed in Figure 3. In the monetary economy, the surpluses of agents i and j are $S_i^M = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega$ and $S_j^M = \varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega$, respectively. In this Figure, the Pareto frontier of the barter economy is the dotted concave curve and the Pareto frontier of the monetary economy is the solid concave curve. There is again the S^*S^* line that represents every possible split (S_i, S_j) of the maximal total surplus of the match.

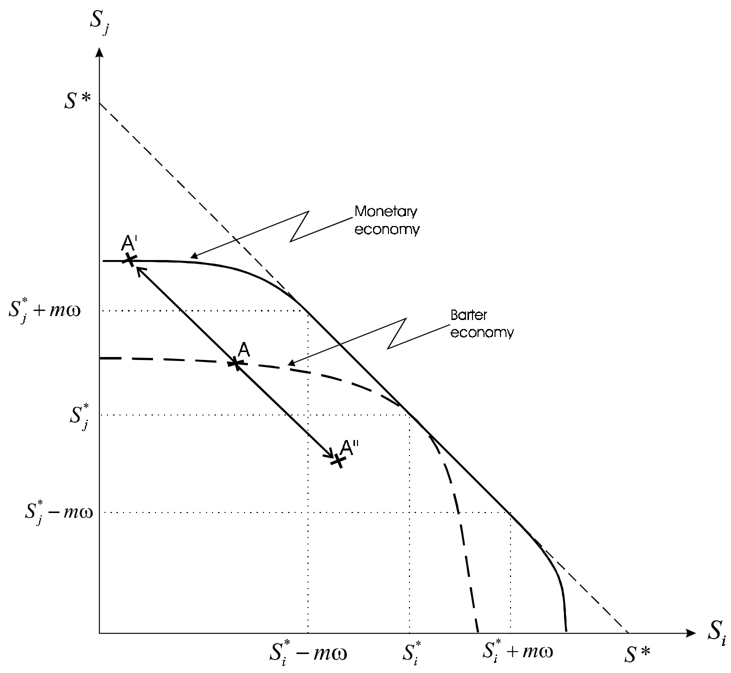


Figure 3: Pareto frontier with and without money.

To see how the transformation works, consider point A and the associated segment $[A', A'']$. Point A lies on the Pareto frontier of the barter economy and it uniquely determines what quantities of goods i and j produce and exchange. The segment $[A'A'']$ represents the set of surpluses that i and j can reach by transferring money without changing these quantities. At A' (A'') player j (i) receives m units of money so that $A' = (S_i - m\omega, S_j + m\omega)$ $(A'' = (S_i + m\omega, S_j - m\omega))$. Note that the slope of the segment $[A'A'']$ is -1 because households have the same constant marginal value of money.

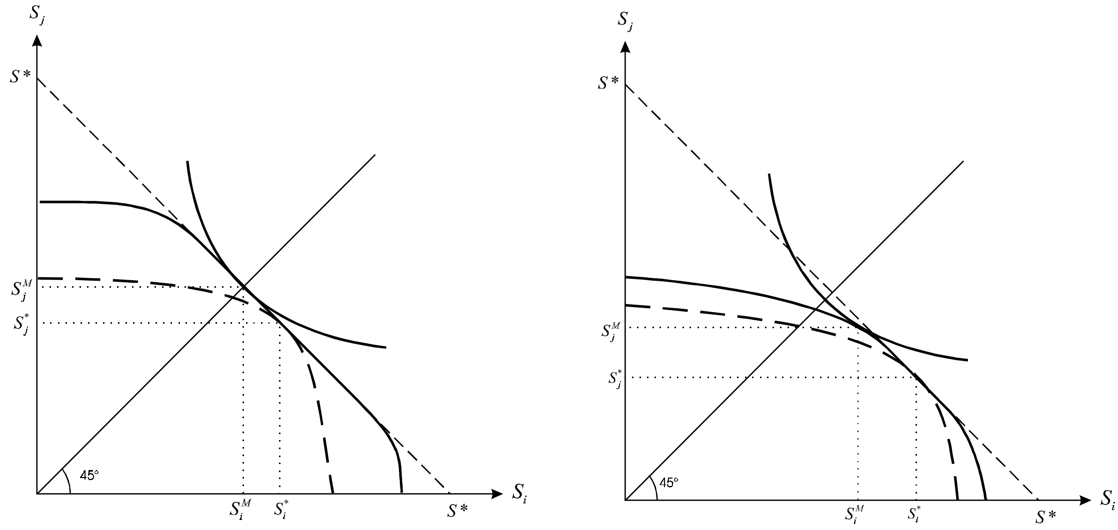
As drawn in Figure 3, only point A' lies on the Pareto frontier of the bargaining set in the monetary economy. This is true for any point on the Pareto frontier of the barter economy that satisfies $S_i < S_i^*$. To the contrast, if $S_i > S_i^*$, $(S_i + m\omega, S_j - m\omega)$ lies on the Pareto frontier of the monetary economy. If $S_i = S_i^*$, the whole segment $[A'A'']$ lies on the Pareto frontier of the bargaining set in the monetary economy. Moreover, the whole segment lies on the S^*S^* line.

If agents had a device to transfer utility, they could attain all points on the line S^*S^*. In the monetary economy, money is such a device, in the sense that agents can attain all points on the S^*S^* line satisfying $S_i \in [S_i^* - m\omega; S_i^* + m\omega]$. It fulfills this role by allowing agents to trade claims to future consumption. However, money is not a perfect device to transfer utility because the Pareto frontier in the monetary economy does not fully coincide with the S^*S^* line. The larger the real value of money, the better money can fulfill its role to transfer utility.

The set of agreements that can be reached in the monetary economy contains the set of arrangements that can be reached in the barter economy. Thus, with valued money the total surplus in a match cannot be lower than the total surplus attained in the corresponding barter economy. Consequently, welfare in a monetary economy must be at least as large as in the corresponding barter economy. Figure 3 illustrates this fact. Money does not enlarge the set of feasible transactions; traders can exchange any quantity of goods, with or without money.[20] The benefit of money arises because it allows production and exchange of goods that would not satisfy the *quid pro quo* requirement without a transfer of money.

In the monetary economy traded quantities are efficient if the asymmetry in preferences measured by the distance D is lower than the real value of money. Figure 4 displays an asymmetric match where efficient quantities are exchanged (a) and an asymmetric match where inefficient quantities are traded (b). The dotted curves are the Pareto frontiers of the bargaining set in the barter economy. The real value of money $(m\omega)$ is larger in the figure 4 (a) than in figure 4 (b) because the segment of the Pareto frontier that lies on the S^*S^* line is larger in the left figure.

[20] For a similar argument consider Ostroy (1973).



(a) Socially efficient trade

(b) Socially inefficient trade

Figure 4: Efficient and inefficient trades in the monetary economy.

In Figure 5 we show how the distance $\mathcal{D}(\varepsilon_i, \varepsilon_j)$ can be graphically represented in the previous figures. To do this, we use the fact that $\mathcal{D}(\varepsilon_i, \varepsilon_j)$ is equal to $\left|\frac{S_i^* - S_j^*}{2}\right|$. Thus, $\mathcal{D}(\varepsilon_i, \varepsilon_j)$ is the absolute value of the difference of the surpluses when agents produce efficient quantities divided by two. The point E^M represents the surpluses of agents when efficient quantities are traded and when agents split the total surplus equally.

Point E^B is the efficient allocation when agents produce efficient quantities and do not exchange money. To reach point E^M from E^B, agent i has to give up $\mathcal{D}(\varepsilon_i, \varepsilon_j)$ units of real money, which means that j receives this amount. Thus, if $\mathcal{D}(\varepsilon_i, \varepsilon_j) \leq m\omega$, the traders can attain E^M because agent i's real value of money holdings is sufficiently large. If, to the contrast, $\mathcal{D}(\varepsilon_i, \varepsilon_j) > m\omega$, the traders cannot attain allocation E^M because agent i has not enough money to compensate j.

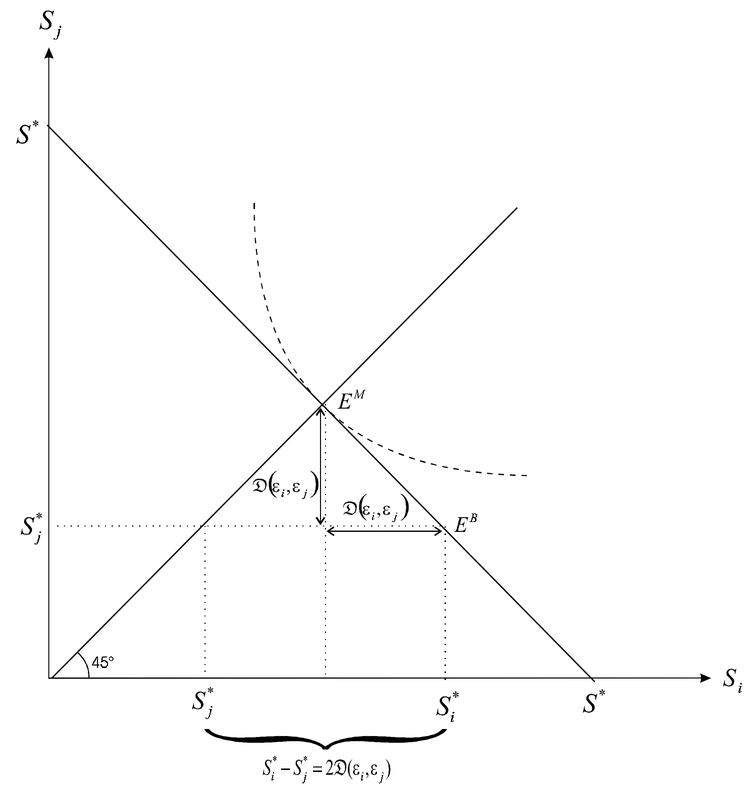


Figure 5: Frontier between efficient and nonefficient trades.

6 Conclusion

Most models that have captured the role of money as a medium of exchange have either emphasized its ability to increase the volume of trades or its ability to solve an information problem. In this paper we have focused on a complementary role by addressing the following question: Can money improve welfare in a double coincidence of real wants environment where in each bilateral meeting each agent is a consumer of the other agent's production?

Our answer is positive: If individuals have asymmetric preferences for each other's goods, money generates a strictly better allocation of resources. To study this question, we have considered a random-matching model with divisible money and divisible real commodities, where agents have the choice to finance current consumption either with money, real production, or both. In contrast to previous search models of money, in our environment money neither increases the frequency of trades, nor does it solve an information problem. Rather, the welfare improving role of money lies on its ability to improve the terms of trade in each bilateral meeting relative to the terms of trade attained in the same meetings in a barter economy.

We have found that if in a match the real value of money is sufficiently large, the traders produce, consume, and exchange socially efficient quantities and they use money to split the total surplus of the match evenly. We have also shown that agents produce and consume the same (socially efficient) quantities and attain the same surpluses as if they had a device to transfer utility. Thus, money is a *device* to transfer utility. The benefit of money arises because it allows agents to separate the decisions of how much to produce and exchange and how to split the resulting total surplus.

Although money is welfare improving, there is always a fraction of meetings where traders produce and exchange inefficient quantities. Inefficient trades occur if the degree of asymmetry in a match measured by some distance is larger than the real quantity of money which is decreasing in the inflation rate. Consequently, inflation is costly in our model because it generates a misallocation of resources. A higher expansion of the money supply increases the misallocation because it reduces the set of meetings where agents produce and exchange socially efficient quantities. When the gross growth rate of the money supply approaches some upper-bound, the allocation of resources in the monetary economy converges to the (mis)allocation of the corresponding barter economy. In contrast, when the gross growth rate of the money supply approaches the discount factor, almost all trades are efficient, i.e., the Friedman's rule holds.

There are several ways to extend our analysis. An interesting extension is to consider nondegenerate distributions of money holdings. With a nondegenerate distribution of money holdings, the marginal value of money will be different among individuals and this could affect the role of money as a device to transfer utility. One could also consider a dual-currency or a two countries and two monies version of the model to study the determination of exchange rates and to compare the welfare property of the model relative to the one currency model. Finally, one could add some search externalities (by endogenizing the search intensity of traders for instance) to see how changes in monetary policy affect these externalities.

Appendix

A1. Welfare In the paper we compare the outcome of the decentralized economy with the outcome that a planner would choose in order to maximize welfare W. The social planner treats all household symmetrically and, consequently, maximizes the expected lifetime utility of a representative household W subject to the constraint that $(q^b_\varepsilon, q^s_\varepsilon)$ must be equal to $(q^s_{\varepsilon'}, q^b_{\varepsilon'})$.

$$\max_{q^b_\varepsilon, q^s_\varepsilon} W = \frac{\int_E \varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right)}{1-\beta} \text{ s.t. } q^b_\varepsilon = q^s_{\varepsilon'} \text{ and } q^s_\varepsilon = q^b_{\varepsilon'} \tag{23}$$

The problem can be reformulated as follows:

$$\max_{q^b_\varepsilon, q^s_\varepsilon} W = \frac{\int_{E'} \varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon + \varepsilon_j u\left(q^s_\varepsilon\right) - q^b_\varepsilon \, d\mu\left(\varepsilon_i, \varepsilon_j\right)}{1-\beta}$$

where $E' = \{(\varepsilon_i, \varepsilon_j) \in E | \, \varepsilon_i \geq \varepsilon_j\}$. The first-order conditions are

$$\begin{aligned} \varepsilon_i u'\left(q^b_\varepsilon\right) &= 1 \qquad \forall \varepsilon \in E \\ \varepsilon_j u'\left(q^s_\varepsilon\right) &= 1 \qquad \forall \varepsilon \in E. \end{aligned}$$

A2. Proof of Proposition 1. The demonstration proceeds in two parts.

Part 1: Determination of the terms of trade.

Relabelling equation (5) yields:

$$\varepsilon_i u\left(q^s_{\varepsilon'}\right) - q^b_{\varepsilon'} = (1-\delta\Delta)\left[\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon\right] \tag{24}$$

For any Δ, the terms of trade $(q^b_\varepsilon, q^s_\varepsilon)$ and $(q^b_{\varepsilon'}, q^s_{\varepsilon'})$ are determined simultaneously and satisfy (4), (5), (24), and again (4) where $(\varepsilon_i, \varepsilon_j)$ is replaced by $(\varepsilon_j, \varepsilon_i)$. In the following, we consider the solution when $\Delta \to 0$. First, equations (4), (5) and (24) imply that $\lim_{\Delta\to 0} q^b_\varepsilon = \lim_{\Delta\to 0} q^s_{\varepsilon'}$ and $\lim_{\Delta\to 0} q^s_\varepsilon = \lim_{\Delta\to 0} q^b_{\varepsilon'}$. Second, rearrange equations (5) and (24) to get:

$$\begin{aligned} \left[\varepsilon_j u\left(q^s_\varepsilon\right) - \varepsilon_j u\left(q^b_{\varepsilon'}\right)\right] - \left(q^b_\varepsilon - q^s_{\varepsilon'}\right) &= -\delta\Delta\left[\varepsilon_j u\left(q^b_{\varepsilon'}\right) - q^s_{\varepsilon'}\right] \\ \left[\varepsilon_i u\left(q^s_{\varepsilon'}\right) - \varepsilon_i u\left(q^b_\varepsilon\right)\right] - \left(q^b_{\varepsilon'} - q^s_\varepsilon\right) &= -\delta\Delta\left[\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon\right] \end{aligned}$$

Divide the two last equations, use equation (4), and take the limit $\Delta \to 0$ to get (7).[21]

Part 2: Existence and Uniqueness.

Equation (4) defines a negative relationship between q^b_ε and q^s_ε. Furthermore, according to (4), $\lim_{q^s_\varepsilon\to 0} q^b_\varepsilon = +\infty$ and $\lim_{q^s_\varepsilon\to+\infty} q^b_\varepsilon = 0$. Equation (7) defines a positive relationship between q^b_ε and q^s_ε. Furthermore, according to (7), $\lim_{q^s_\varepsilon\to 0} q^b_\varepsilon = 0$ and $\lim_{q^s_\varepsilon\to+\infty} q^b_\varepsilon = +\infty$. Hence, the terms of trade $\left(q^b_\varepsilon, q^s_\varepsilon\right)$ are the unique solution of equations (4) and (7). Accordingly, the barter equilibrium exists and is unique.■

[21] For more details and a related demonstration, see Osborne and Rubinstein (1990, Section 4.4.) and Muthoo (1999).

A3. Proof of Corollary 1. Note, first, that $\varepsilon_i > \varepsilon_j$ implies that $q_\varepsilon^{b*} > q_\varepsilon^{s*}$. Assume, next, that $q_\varepsilon^b \geq q_\varepsilon^{b*}$. This implies that $\varepsilon_i u'\left(q_\varepsilon^b\right) \leq \varepsilon_i u'\left(q_\varepsilon^{b*}\right) = 1$. Equation (4) implies $q_\varepsilon^{s*} \geq q_\varepsilon^s$ and, therefore,

$$\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s \geq \varepsilon_i u\left(q_\varepsilon^{b*}\right) - q_\varepsilon^{s*} > \varepsilon_j u\left(q_\varepsilon^{s*}\right) - q_\varepsilon^{b*} \geq \varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b \tag{25}$$

According to (7), $\varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^b \geq \varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s$, which contradicts (25). Thus, if $\varepsilon_i > \varepsilon_j$ we must have $q_\varepsilon^b < q_\varepsilon^{b*}$ and by (4) $q_\varepsilon^s > q_\varepsilon^{s*}$.

To see how the traders split the total surplus of the match, manipulate equation (7) to get:

$$\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s = \frac{\varepsilon_i u'\left(q_\varepsilon^b\right)}{1+\varepsilon_i u'\left(q_\varepsilon^b\right)}\left\{\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^b + \varepsilon_j u\left(q_\varepsilon^s\right) - q_\varepsilon^s\right\} \tag{26}$$

Note, first, that $q_\varepsilon^b < q_\varepsilon^{b*}$ implies $\varepsilon_i u'\left(q_\varepsilon^b\right) > \varepsilon_i u'\left(q_\varepsilon^{b*}\right) = 1$. This and equation (26) imply that i's fraction of the total surplus is $\frac{\varepsilon_i u'\left(q_\varepsilon^b\right)}{1+\varepsilon_i u'\left(q_\varepsilon^b\right)} > \frac{1}{2}$.■

A4. Proof of Lemma 1. By relabelling equation (20), we get:

$$\varepsilon_i u\left(q_{\varepsilon'}^s\right) - q_{\varepsilon'}^b + x_{\varepsilon'}\omega = (1-\delta\Delta)\left[\varepsilon_i u\left(q_\varepsilon^b\right) - q_\varepsilon^s - x_\varepsilon\omega\right] \tag{27}$$

For any Δ and a given $(q_{\varepsilon'}^b, q_{\varepsilon'}^s, x_{\varepsilon'})$, the offers $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ and multipliers $(\lambda_\varepsilon, \pi_\varepsilon)$ are determined by equations (14)-(17), and (20). Moreover, for any Δ and for a given $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, the offers $(q_{\varepsilon'}^b, q_{\varepsilon'}^s, x_{\varepsilon'})$ and multipliers $(\lambda_{\varepsilon'}, \pi_{\varepsilon'})$, are determined by equation (27) and by equations (14)-(17) where $(\varepsilon_i, \varepsilon_j)$ is replaced by $(\varepsilon_j, \varepsilon_i)$.

Note, first, that equations (19), (20), and (27) imply that $\lim_{\Delta\to 0} q_\varepsilon^b = \lim_{\Delta\to 0} q_{\varepsilon'}^s$, $\lim_{\Delta\to 0} q_\varepsilon^s = \lim_{\Delta\to 0} q_{\varepsilon'}^b$, and $\lim_{\Delta\to 0} x_\varepsilon = \lim_{\Delta\to 0} -x_{\varepsilon'}$. Moreover, λ_ε and $\pi_{\varepsilon'}$ are either both positive or both equal to zero. To see this, suppose a contrario that $\lambda_\varepsilon > 0$ and $\pi_{\varepsilon'} = 0$. From (19),

$$\begin{aligned}
\pi_{\varepsilon'} &= 0 \Rightarrow q_{\varepsilon'}^b \geq q_{\varepsilon'}^{b*} = q_\varepsilon^{s*} \text{ and } q_{\varepsilon'}^s \leq q_{\varepsilon'}^{s*} = q_\varepsilon^{b*} \\
\lambda_\varepsilon &> 0 \Rightarrow q_\varepsilon^b > q_\varepsilon^{b*} \text{ and } q_\varepsilon^s < q_\varepsilon^{s*}
\end{aligned}$$

Furthermore, from (27),

$$\lim_{\Delta\to 0} \varepsilon_i u\left(q_{\varepsilon'}^s\right) - \varepsilon_i u\left(q_\varepsilon^b\right) + q_\varepsilon^s - q_{\varepsilon'}^b = \lim_{\Delta\to 0}\left[-\left(x_\varepsilon + x_{\varepsilon'}\right)\omega\right]$$

The left-hand side is negative. Hence, the right-hand side must be negative too, which implies that $x_{\varepsilon'} > m$. This is impossible.

Note, next, that equations (16) and (17) imply that λ_ε and π_ε cannot be positive at the same time. Consequently, there are two cases:

Case (a): $\lambda_\varepsilon > 0$ and $\pi_\varepsilon = 0$ or $\lambda_\varepsilon = 0$ and $\pi_\varepsilon > 0$

Let us consider, first, $\lambda_\varepsilon > 0$ and $\pi_\varepsilon = 0$. This implies that the constraint on money holdings of agent i is binding and $x_\varepsilon = m$. It also implies $\lim_{\Delta\to 0} x_{\varepsilon'} = -m$, $\lim_{\Delta\to 0} \lambda_{\varepsilon'} = 0$, and $\lim_{\Delta\to 0} \pi_{\varepsilon'} = \frac{\omega\lambda_\varepsilon}{\lambda_\varepsilon+\omega}$. To see this, note that when Δ approaches zero (14) and (15) imply that

$$\frac{\omega}{\omega-\pi'_\varepsilon} = \frac{\lambda_\varepsilon+\omega}{\omega}.$$

If $\pi_\varepsilon = 0$, the first-order conditions (14) and (15) equal

$$\varepsilon_i u'\left(q^b_\varepsilon\right) = \frac{1}{\omega}\left(\lambda_\varepsilon+\omega\right) \quad (28)$$

$$\varepsilon_j u'\left(q^s_\varepsilon\right) = \frac{\omega}{\omega+\lambda_\varepsilon} \quad (29)$$

Equations (28) and (29) imply that if agent i's constraint is binding, $\varepsilon_i u'\left(q^b_\varepsilon\right) > 1 > \varepsilon_j u'\left(q^s_\varepsilon\right)$, that is, the quantity q^b_ε produced and consumed is inefficiently low and the quantity q^s_ε produced and consumed is inefficiently large.

By rearranging (20) and (27), and by dividing these two equations we get

$$\frac{\left[\varepsilon_j u\left(q^s_\varepsilon\right) - \varepsilon_j u\left(q^b_{\varepsilon'}\right)\right] - \left(q^b_\varepsilon - q^s_{\varepsilon'}\right) + (x_\varepsilon + x_{\varepsilon'})\omega}{\left[\varepsilon_i u\left(q^s_{\varepsilon'}\right) - \varepsilon_i u\left(q^b_\varepsilon\right)\right] - \left(q^b_{\varepsilon'} - q^s_\varepsilon\right) + (x_\varepsilon + x_{\varepsilon'})\omega} = \frac{\varepsilon_j u\left(q^b_{\varepsilon'}\right) - q^s_{\varepsilon'} - x_{\varepsilon'}\omega}{\varepsilon_i u\left(q^b_\varepsilon\right) - q^s_\varepsilon - x_\varepsilon\omega} \quad (30)$$

Take the limit as $\Delta \to 0$ and use (19) to get equation (21). If agent i is constrained by his money holding, he receives more than one half of the total surplus of the match. To see this, consider the following equation, which is derived from equation (21):

$$\varepsilon_j u(q^s_\varepsilon) - q^b_\varepsilon + m\omega = \frac{\omega}{2\omega+\lambda_\varepsilon}\left[\varepsilon_i u(q^b_\varepsilon) - q^b_\varepsilon + \varepsilon_j u(q^s_\varepsilon) - q^s_\varepsilon\right] \quad (31)$$

According to (31), agent j's fraction of the total surplus of the match is endogenous and depends on the ratio of the shadow price λ_ε to the marginal utility of money ω. If $\lambda_\varepsilon > 0$, $\frac{\omega}{2\omega+\lambda_\varepsilon} < \frac{1}{2}$ and, therefore, $\varepsilon_j u(q^s_\varepsilon) - q^b_\varepsilon + m\omega < \varepsilon_i u(q^b_\varepsilon) - q^s_\varepsilon - m\omega$.

Next, consider $\lambda_\varepsilon = 0$ and $\pi_\varepsilon > 0$. The constraint on money holdings of agent j is binding: $x_\varepsilon = -m$. This implies $\lim_{\Delta\to 0} x_{\varepsilon'} = m$, $\lim_{\Delta\to 0} \lambda_{\varepsilon'} = \frac{\omega\pi_\varepsilon}{\pi_\varepsilon+\omega}$ and $\lim_{\Delta\to 0} \pi_{\varepsilon'} = 0$. Hence, we have $\left(q^b_\varepsilon, q^s_\varepsilon\right) = \left(q^s_{\varepsilon'}, q^b_{\varepsilon'}\right)$ where $\left(q^b_{\varepsilon'}, q^s_{\varepsilon'}\right)$ satisfies (19) and (21).

Case (b): $\lambda_\varepsilon = 0$ and $\pi_\varepsilon = 0$

This implies $\lim_{\Delta\to 0} \lambda_{\varepsilon'} = \lim_{\Delta\to 0} \pi_{\varepsilon'} = 0$. The first-order conditions (14) and (15) imply that $q^b_\varepsilon = q^{*b}_\varepsilon$ and $q^s_\varepsilon = q^{*s}_\varepsilon$. They trade efficient quantities. Equations (20) and (27) imply that if $\Delta \to 0$, we get an equation analogous to (30), and then (22): Traders split the total surplus of the match equally.■

A5. Proof of Proposition 2 The demonstration proceeds in five steps.

1st step : Determination of the set I.

Define $I' \subseteq E$ the set of $(\varepsilon_i, \varepsilon_j)$ such that $\lambda_\varepsilon > 0$. Note that $\lambda_\varepsilon > 0$ implies that the constraint on agent $i's$ money holdings is binding and $\varepsilon_i > \varepsilon_j$. It also implies that agents i and j exchange inefficient quantities and do not split the total surplus of the match equally. Hence, $\lambda_\varepsilon > 0$ if (22) is satisfied for a value of $x_\varepsilon > m$. Thus,

$$1 > \frac{\varepsilon_j u\left(q_\varepsilon^{s*}\right) - q_\varepsilon^{b*} + m\omega}{\varepsilon_i u\left(q_\varepsilon^{b*}\right) - q_\varepsilon^{s*} - m\omega}. \tag{32}$$

From (6) we have $q_\varepsilon^{b*} = u'^{-1}\left(\frac{1}{\varepsilon_i}\right)$ and $q_\varepsilon^{s*} = u'^{-1}\left(\frac{1}{\varepsilon_j}\right)$. Consequently, condition (32) can be rewritten as

$$\varphi\left(\varepsilon_i\right) - \varphi\left(\varepsilon_j\right) > 2m\omega. \tag{33}$$

Condition (33) is equivalent to $\mathcal{D}(\varepsilon_i, \varepsilon_j) > m\omega$.

Denote $\widetilde{\varepsilon} = (\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_j)$ the value of ε such that (33) is satisfied with equality:

$$\varphi\left(\widetilde{\varepsilon}_i\right) - \varphi\left(\widetilde{\varepsilon}_j\right) = 2m\omega. \tag{34}$$

Equation (34) implicitly defines a function $\mathcal{F}$ such that $\widetilde{\varepsilon}_j = \mathcal{F}\left(\widetilde{\varepsilon}_i, m\omega\right)$. One can show that $\mathcal{F}_1\left(\widetilde{\varepsilon}_i, m\omega\right) = \frac{\partial \widetilde{\varepsilon}_j}{\partial \widetilde{\varepsilon}_i} > 0$ and $\mathcal{F}_2\left(\widetilde{\varepsilon}_i, m\omega\right) = \frac{\partial \widetilde{\varepsilon}_j}{\partial m\omega} < 0$. Furthermore, for all $m\omega > 0$ $\mathcal{F}\left(\varepsilon_{\text{sup}}; m\omega\right) < \varepsilon_{\text{sup}}$ and there exists $\widehat{\varepsilon}(m\omega) > 0$ such that $\mathcal{F}\left(\widehat{\varepsilon}; m\omega\right) = 0$ (see Figure 5). Consequently, the set I' is defined as follows:

$$\begin{aligned} I' &= \left\{(\varepsilon_i, \varepsilon_j) \in E \,|\varphi\left(\varepsilon_i\right) - \varphi\left(\varepsilon_j\right) > 2m\omega\right\} \\ &= \left\{(\varepsilon_i, \varepsilon_j) \in E \,|\varepsilon_j < \mathcal{F}\left(\varepsilon_i\right)\right\} \end{aligned}$$

By symmetry, we can define the set I'' of ε such that $\pi_\varepsilon > 0$.

$$I'' = \left\{(\varepsilon_i, \varepsilon_j) \in E \,\middle|\, (\varepsilon_j, \varepsilon_i) \in I'\right\}$$

Then, $I = I' \cup I''$. See Figure 5.

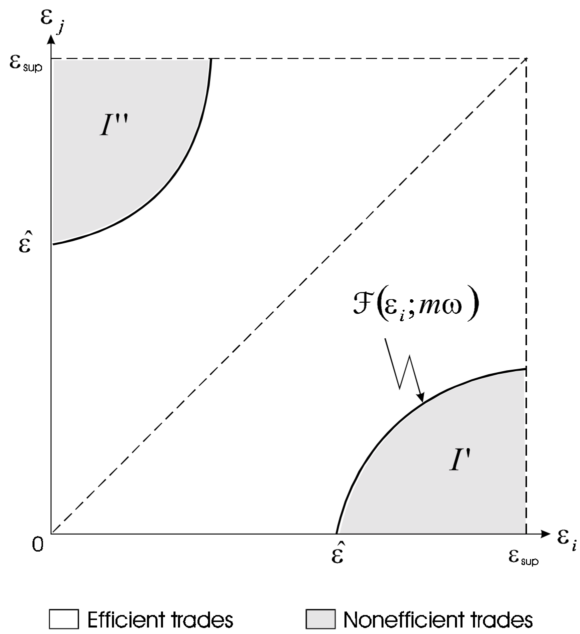


Figure 5: Set of inefficiencies.

2nd step : The measure of inefficient trades.

In order to measure the set of inefficient trades, it is useful to introduce the indicator function $\mathbf{1}_A(x)$ which is equal to one if $x \in A$ and 0 otherwise. Furthermore, because of the symmetry of the sets I' and I'' we have $\mu(I) = 2\mu(I')$.

Let us calculate $\mu(I')$. By definition,

$$\mu(I') = \int \mathbf{1}_{[0,\mathcal{F}(\varepsilon_i;m\omega)[}(\varepsilon_j)\, d\mu(\varepsilon_i,\varepsilon_j) = \int_{[0,\varepsilon_{\sup}]} \left(\int_{[0,\varepsilon_{\sup}]} \mathbf{1}_{[0,\mathcal{F}(\varepsilon_i;m\omega)[}(\varepsilon_j)\, dF(\varepsilon_j) \right) dF(\varepsilon_i)$$

This integral indicates that we measure the set of points such that $\varepsilon_j < \mathcal{F}(\varepsilon_i)$ with the measure μ. The integral $\int_{[0,\varepsilon_{\sup}]} \mathbf{1}_{[0,\mathcal{F}(\varepsilon_i;m\omega)[}(\varepsilon_j)\, dF(\varepsilon_j) = \int_0^{\mathcal{F}(\varepsilon_i;m\omega)} dF(\varepsilon_j)$ is equal to $F[\mathcal{F}(\varepsilon_i;m\omega)]$ for all $\varepsilon_i > \widehat{\varepsilon}$ and zero otherwise. Hence,

$$\mu(I') = \int_{[\widehat{\varepsilon},\varepsilon_{\sup}]} F[\mathcal{F}(\varepsilon_i;m\omega)]\; dF(\varepsilon_i)$$

By differentiating with respect to $m\omega$ we get:

$$\frac{\partial\mu(I')}{\partial m\omega} = \int_{[\widehat{\varepsilon},\varepsilon_{\sup}]} f[\mathcal{F}(\varepsilon_i;m\omega)] \times \mathcal{F}_2(\varepsilon_i;m\omega)\; dF(\varepsilon_i) - F[\mathcal{F}(\widehat{\varepsilon};m\omega)]\, \frac{\partial\widehat{\varepsilon}}{\partial m\omega}$$

Because $F[\mathcal{F}(\widehat{\varepsilon};m\omega)] = 0$, we have:

$$\frac{\partial\mu(I')}{\partial m\omega} = \int_{[\widehat{\varepsilon},\varepsilon_{\sup}]} f[\mathcal{F}(\varepsilon_i;m\omega)] \times \mathcal{F}_2(\varepsilon_i;m\omega)\; dF(\varepsilon_i) < 0$$

Consequently, the set of inefficiencies which is measured by $\mu(I) = 2\mu(I')$ is increasing in $m\omega$.

3rd step: Determination of $m\omega$.

The quantity $m\omega$ is determined by the envelop condition (18) which can be rewritten as follows:

$$\frac{\omega_{-1}}{\beta} = \int_{I'} \lambda_\varepsilon d\mu(\varepsilon_i,\varepsilon_j) + \omega$$

By dividing by $\frac{\omega}{\beta}$, we get:

$$\frac{\omega_{-1}}{\omega} = \beta\left\{\int_{I'} \frac{\lambda_\varepsilon}{\omega} d\mu(\varepsilon_i,\varepsilon_j) + 1\right\} \tag{35}$$

The rate of growth of money supply is $\gamma - 1$. Thus,

$$\frac{m}{m_{-1}} = \gamma$$

In the steady-state, $q^b_\varepsilon = q^b_{\varepsilon,-1}$ and $q^s_\varepsilon = q^s_{\varepsilon,-1}$ for all ε. As it will demonstrated in the following, q^b_ε and q^s_ε depend on $m\omega$: this implies that $m\omega$ must be constant in the steady-state, and therefore:

$$\frac{\omega_{-1}}{\omega} = \gamma$$

Hence, eq. (35) gives:

$$\gamma = \beta \left\{ \int_{I'} \frac{\lambda_\varepsilon}{\omega} d\mu(\varepsilon_i, \varepsilon_j) + 1 \right\} \tag{36}$$

From (28) and (29), it can be verified that q_ε^b is decreasing in $\frac{\lambda_\varepsilon}{\omega}$ whereas q_ε^s is increasing with $\frac{\lambda_\varepsilon}{\omega}$. When $\lambda_\varepsilon > 0$ $\varepsilon_i u'(q_\varepsilon^b) > 1$ and $\varepsilon_j u'(q_\varepsilon^s) < 1$. Hence, LHS of (31) is increasing in $\frac{\lambda_\varepsilon}{\omega}$ whereas RHS of (31) is decreasing in $\frac{\lambda_\varepsilon}{\omega}$. Consequently, equation (31) determines a unique value for $\frac{\lambda_\varepsilon}{\omega}$. Moreover,

$$\frac{\partial \frac{\lambda_\varepsilon}{\omega}}{\partial m\omega} < 0, \qquad \frac{\partial q_\varepsilon^b}{\partial m\omega} > 0, \qquad \frac{\partial q_\varepsilon^s}{\partial m\omega} < 0$$

Equation (31) implicitly defines a function $\psi(\varepsilon; m\omega) = \lambda_\varepsilon/\omega$ for all $\varepsilon \in I'$ with $\frac{\partial \psi}{\partial m\omega} < 0$. Consequently, equation (36) can be rewritten as:

$$\gamma = \beta \left\{ \int_{I'} \psi(\varepsilon; m\omega) d\mu(\varepsilon_i, \varepsilon_j) + 1 \right\} \tag{37}$$

From (37), $m\omega$ is entirely characterized by the following equation:

$$\Psi(m\omega) = \frac{\gamma}{\beta} - 1, \tag{38}$$

where $\Psi(m\omega) = \int_{I'} \psi(\varepsilon; m\omega) d\mu(\varepsilon_i, \varepsilon_j)$. By differentiating (38) with respect to $m\omega$, we find $\Psi' < 0$ as long as $\mu(I') > 0$. This implies that if there is a $m\omega > 0$ satisfying (38) such that $\mu(I') > 0$ then it is unique.

4th step : Existence of a unique monetary equilibrium.

For existence and uniqueness we need to show that there is unique $x > 0$ satisfying $\Psi(x) = \frac{\gamma}{\beta} - 1$. To do this we study the properties of $\Psi(x)$. Let us first derive $\Psi(0)$. If $m\omega = 0$, equations (14) and (31) are equivalent to (26) and $q_\varepsilon^s = \widehat{q}_\varepsilon^s$ and $q_\varepsilon^b = \widehat{q}_\varepsilon^b$ where $\widehat{q}_\varepsilon^s$ and $\widehat{q}_\varepsilon^b$ are the quantities exchanged in the barter economy. Furthermore,

$$\frac{\lambda_\varepsilon}{\omega} = \varepsilon_i u'\left(\widehat{q}_\varepsilon^b\right) - 1$$

Then,

$$\Psi(0) = \int_E \varepsilon_i u'\left(\widehat{q}_\varepsilon^b\right) - 1 d\mu(\varepsilon_i, \varepsilon_j) > 0$$

Second, there is a critical value $\overline{x} > 0$ such that for all $x \geq \overline{x}$, $\Psi(x) = 0$ and for all $x < \overline{x}$, $\Psi(x) > 0$. To see this, note that equation (32) implies that $I' = \emptyset$ when $m\omega$ is large enough. Finally, from step 3 $\Psi'(x) < 0$ if $x < \overline{x}$.

We conclude that equation (38) determines a unique positive value of $m\omega$ if $\gamma \in (\beta, \overline{\gamma})$ where $\overline{\gamma} = \beta \{\Psi(0) + 1\}$.

5th step : The limit case $\gamma \to \beta$.

From (38), $\Psi(m\omega) \to 0$, that is

$$\lim_{\gamma \to \beta} \int_{I'} \frac{\lambda_\varepsilon}{\omega} d\mu(\varepsilon_i, \varepsilon_j) = 0$$

This implies that $\lim_{\gamma \to \beta} \mu(I') = 0$: in the limit traders exchange efficient quantities in all matches.■

A6. Derivation of Pareto frontier of the bargaining set Surpluses of agents i and j are:

$$S_i = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega, \quad (39)$$

$$S_j = \varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega \quad (40)$$

The Pareto frontier of the bargaining set is the set of pairs (S_i, S_j) such that it is not possible to increase S_i without decreasing S_j. In a barter economy $\omega = 0$ whereas in a monetary economy $\omega > 0$. Formally, the Pareto frontier is determined by the following program:

$$\max_{q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon} S_i = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega$$

$$\text{s.t. } \varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega \geq S_j \text{ and } -m \leq x_\varepsilon \leq m$$

Denote μ_ε, λ_ε, and π_ε the multipliers associated with the first, second, and third inequality, respectively.

- **The Pareto frontier in the barter economy**

The Lagrangian of this program is:

$$\mathcal{L} = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s + \mu_\varepsilon \left[\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b - S_j\right]$$

The first order conditions with respect to q_ε^b and q_ε^s are:

$$\varepsilon_i u'(q_\varepsilon^b) = \mu_\varepsilon, \quad (41)$$

$$\varepsilon_j u'(q_\varepsilon^s) = \frac{1}{\mu_\varepsilon} \quad (42)$$

From (41) and (42) we have:

$$\varepsilon_i \varepsilon_j u'(q_\varepsilon^b) u'(q_\varepsilon^s) = 1 \quad (43)$$

Equations (39), (40), and (43) define a negative relationship between S_i and S_j, which is the Pareto frontier denoted by $\mathcal{P}^B$. One can show that the slope of $\mathcal{P}^B$ is negative, i.e., $\frac{dS_j}{dS_i} = \frac{-1}{\varepsilon_i u'(q_\varepsilon^b)} < 0$. Moreover, because S_i is an increasing function of q_ε^b, $\mathcal{P}^B$ is concave, i.e., $\frac{d^2 S_j}{dS_i^2} < 0$.

- **The Pareto frontier in the monetary economy**

The Pareto frontier in the monetary economy is denoted by $\mathcal{P}^M$. The Lagrangian has the following expression:

$$\mathcal{L} = \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega + \mu_\varepsilon \left[\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega - S_j\right] - \lambda_\varepsilon (x_\varepsilon - m) + \pi_\varepsilon (x_\varepsilon + m)$$

The first order conditions with respect to q_ε^b, q_ε^s and x_ε are:

$$\begin{aligned} \varepsilon_i u'(q_\varepsilon^b) &= \mu_\varepsilon, & (44)\\ \varepsilon_j u'(q_\varepsilon^s) &= \frac{1}{\mu_\varepsilon}, & (45)\\ \omega(\mu_\varepsilon - 1) &= \lambda_\varepsilon - \pi_\varepsilon & (46) \end{aligned}$$

Again, from (44) and (45) we find that

$$\varepsilon_i \varepsilon_j u'(q_\varepsilon^b) u'(q_\varepsilon^s) = 1$$

There are three cases to distinguish: (i) $\lambda_\varepsilon = \pi_\varepsilon = 0$, (ii) $\lambda_\varepsilon > 0$ and $\pi_\varepsilon = 0$, and (iii) $\lambda_\varepsilon = 0$ and $\pi_\varepsilon > 0$

Case (i): $\lambda_\varepsilon = \pi_\varepsilon = 0$

The constraint on money holdings of both players are not binding. Then, from (46) we have $\mu_\varepsilon = 1$. From (44) and (45) $q_\varepsilon^b = q_\varepsilon^{b*}$ and $q_\varepsilon^s = q_\varepsilon^{s*}$. As a consequence, according to (39) and (40):

$$S_i = S_i^* - x_\varepsilon \omega,\ S_j = S_j^* + x_\varepsilon \omega, \text{ and } S_i + S_j = S^*.$$

When the constraints on money holdings are not bindings the points of the Pareto frontier lie on the S^*S^* line. Moreover, the constraint $-m \leq x_\varepsilon \leq m$ can be rewritten as follows:

$$-m\omega + S_i^* \leq S_i \leq m\omega + S_i^*$$

Case (ii) : $\lambda_\varepsilon > 0$ and $\pi_\varepsilon = 0$

We have $x_\varepsilon = m$ and, from (46), $\mu_\varepsilon = 1 + \frac{\lambda_\varepsilon}{\omega}$. Hence, one can see that (44) and (45) are exactly the first order conditions of the program of the household. Furthermore, (39) and (40) yield:

$$\begin{aligned} S_i + m\omega &= \varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s,\\ S_j - m\omega &= \varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b \end{aligned}$$

This implies that $(S_i + m\omega, S_j - m\omega) \in \mathcal{P}^B$. Furthermore, the condition $\lambda_\varepsilon > 0$ implies $\mu_\varepsilon > 1$ and $q_\varepsilon^b < q_\varepsilon^{b*}$ and $q_\varepsilon^s > q_\varepsilon^{s*}$. As a consequence, $S_i + m\omega < S_i^*$.

Case (iii) : $\lambda_\varepsilon = 0$ and $\pi_\varepsilon > 0$.

The reasoning is similar to case (ii). We have $x_\varepsilon = -m$ and $(S_i - m\omega, S_j + m\omega) \in \mathcal{P}^B$. This occurs when $S_i - m\omega > S_i^*$.

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