

# Biases of the Ordinary Least Squares and Instrumental Variables Estimators of the Intergenerational Earnings Correlation: Revisited in the Light of Panel Data

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## Abstract

The OLS estimator of the intergenerational earnings correlation is biased towards zero, while the instrumental variables estimator is biased upwards. The first of these results arises because of measurement error, while the latter rests on the presumption that the education of the parent family is an invalid instrument. We propose a panel data framework for quantifying the asymptotic biases of these estimators as well as a mis-specification test for the IV estimator. Using US data we estimate the bias of the OLS estimator to be in the order of 10% to 20%, and that of IV to vary between 40% and 60%. Supporting evidence that the IV estimator is inconsistent, in face of the specification test, is however limited, and results are shown to be sensitive to the assumptions underlying the serial correlation in transitory incomes.

Keywords: Intergenerational mobility, measurement error, panel data, biased estimation.

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## 1. Introduction

The last decade has witnessed a renewed interest in questions pertaining to the distribution of income at one point in time, and also in the extent to which children inherit the economic status of their parents. In the empirical intergenerational mobility literature, researchers have had to deal with the challenging question of estimating the correlation between life-time incomes of parents and children using short time series, and at times a unique observation, on family resources. It has been immediately recognized in early contributions to the literature (Bowles, 1972; Atkinson et al., 1983) that the use of annual income measures to proxy permanent incomes produced an errors in variables problem, and as a result, the ordinary least squares estimator of the intergenerational correlation was biased towards zero.

As a response to this problem, researchers working with panel data have averaged the resources of the parent family (i.e. the explanatory variable) over typically three to five years in order to reduce the bias resulting from the estimation of the standard Galtonian regression of income transmission. Illustrations of averaging methods can be found in Behrman and Taubman (1990), Solon (1992), Zimmerman (1992), Björklund and Jantti (1997), Mulligan (2000) and others. A problem remaining though, is that not all data sets provide repeated observations on the parent family's resources. A typical example in this context is the National Child Development Study (see for instance Dearden et al. 1997 for a discussion). When no time series variation is available on the parent family's resources, it is typical then to instrument the unique measurement on this variable using the education of the family head. A standard argument formalized by Solon (1992) is that when the parent family head's education features as an explanatory variable in a model of the determinants of the child's income, the instrumental variables estimator of the intergenerational correlation is biased away from zero.

Thus it has become a known result that the ordinary least squares and instrumental variables estimators bracket the intergenerational correlation<sup>1</sup>. There have been several efforts in quantifying the bias of the ordinary least squares estimator since the work of Bowles (1972), and estimates provided by Zimmerman (1992) and Abul Naga (2000a) suggest that the extent of the resulting bias is perhaps in the order of 30% or more. Little is known however about the degree to which the instrumental variables methodology over-estimates the intergenerational correlation.

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<sup>1</sup>This result is often emphasized in the literature. See for instance Björklund and Jantti (1997) and Dearden et al. (1997).

ational correlation. A key quantity in evaluating the bias of the ordinary least squares estimator is the variance ratio of permanent to total income. A remaining question then is what comparable parameters (for which, prior knowledge of their magnitudes is required) play a similar role in the correction of the bias of the instrumental variables estimator.

There are two reasons why we feel that this exercise may be of direct relevance to researchers and policy makers. Firstly, having an order of magnitude about the bias of the instrumental variables estimator provides another route to refining our knowledge on the extent of income continuities across generations. More importantly though, if it is found that the bias of the instrumental variables estimator is small, or negligible <sup>2</sup>, earlier results which may have been read to exaggerate the underlying intensity of income inheritance in the population, may be re-appraised in a different light.

In order to evaluate the biases of the ordinary least squares and instrumental variables estimators we use panel data to derive consistent estimators for economic relations observed subject to measurement error. As shown by Griliches and Hausman (1986), Hsiao and Taylor (1991) and others, a wide range of errors in variables models are identifiable in the panel data context. As a by-product of our discussion therefore, we also propose a consistent estimator of the intergenerational correlation when repeated measurements are available on the parent family's income. Likewise, we suggest a test for evaluating the claim that the instrumental variables estimator is upwardly inconsistent.

In order to analyse the large sample biases of the ordinary least squares [OLS] and instrumental variables [IV] estimator in a joint framework, we express these as functions of the structural parameters of a model of income transmission. We then inquire about the nature of the data required in order to recover these unknown structural parameters from the biases of the two estimators. It turns out that the two fundamental quantities required in order to perform this exercise are (1) the correlation between education and permanent income and (2) the variance ratio of permanent to total income. In turn, we propose estimators for these quantities when panel data are available to the researcher. The proposed consistent estimator of the intergenerational correlation and test for the instrumental variables estimator also arise from our study of the identification of the structural model of income transmission.

We then use the Panel Study of Income Dynamics, a longitudinal survey from

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<sup>2</sup>Solon (1992), referring to related empirical evidence from the United States, entertains this hypothesis.

the United States, in order to evaluate numerically the biases of the OLS and IV estimators. Our results suggest that the ...rst of these under-estimates the intergenerational correlation by about 10% to 20%, while the latter over-states by about 40% to 60%.

In the next section we examine the identi...cation of a structural model of income transmission initially suggested by Solon (1992), where the OLS estimator is biased towards zero, and IV is biased away from zero. In section 3 we present estimates of the biases resulting from these two methods using our US data. Section 4 concludes the paper with a summary of the main points.

## 2. Biases of OLS and IV

In this section we study the biases of the ordinary least squares and instrumental variables estimator of the intergenerational correlation. We show that without repeated measurements on parental income, these biases cannot generally be quanti...ed. With panel data however, it is shown that the process of income transmission is identi...ed; hence we may readily derive formulas to evaluate numerically the biases of these estimators.

The framework we consider is a Galtonian regression of the child's life-time income,  $\log(I_c)$  on that of her/his parents',  $\log(I_p)$ , where  $\log(\cdot)$  is the logarithmic function. Let  $\hat{c}_i = \log(I_c)_i - E(\log(I_c))$  and  $\hat{p}_i = \log(I_p)_i - E(\log(I_p))$ :  $\hat{c}_i$  and  $\hat{p}_i$  are the logarithms of the life-time incomes of children and parents, measured in deviation from their respective means, which we shall denote below in more simple terms as permanent incomes. The Galtonian regression model may be written as:

$$\hat{c}_i = \beta \hat{p}_i + u_i \quad (1)$$

where  $i = 1; \dots; n$  indexes data on family  $i$ , and it is assumed that  $E(u_i | \hat{p}_i) = 0$  for all  $i$ . In contrast with the theoretical model (1), it is assumed that the researcher observes annual data  $y_{it}$  and  $x_{it}$  which are taken as noisy measurements on the permanent incomes  $\hat{c}_i$  and  $\hat{p}_i$ .

$$y_{it} = \hat{c}_i + \hat{A}_{it} \quad (2)$$

$$x_{it} = \hat{p}_i + \hat{B}_{it} \quad (3)$$

Below we shall assume that all data are measured in deviation from their cross-sectional means. The researcher regresses  $y_{it}$  on  $x_{it}$  by ordinary least squares to

obtain an estimator  $\hat{\Delta}_{OLS}$ :

$$\hat{\Delta}_{OLS} = \frac{\sum_i y_{it} x_{it}}{\sum_i x_{it}^2} \quad (4)$$

Because  $x_{it}$  is a noisy measurement on  $\hat{p}_i$ , the above estimator is biased towards zero. Define  $e^a$  as the education of the parent household head and  $e = e^a - E(e^a)$ , as education measured in deviation from its mean. An alternative estimation strategy for the Galtonian model (1) consists in instrumenting  $x_{it}$  using  $e_i$ . This produces the following instrumental variables estimator:

$$\hat{\Delta}_{IV} = \frac{\sum_i e_i y_{it}}{\sum_i e_i x_{it}} \quad (5)$$

In order to formalize the argument that the instrumental variables estimator may be upwardly biased, we shall assume, following Solon (1992), that parental education features as an explanatory variable in a structural model of the determinants of the child's income:

$$\hat{c}_i = \pm_0 \hat{p}_i + \pm_1 e_i + v_i \quad (6)$$

where  $v_i$  is taken to be uncorrelated with  $e_i$  and  $\hat{p}_i$ . Since, from (1),  $\bar{c} = E(\hat{c}_i) = E(\hat{p}_i)$ , it also follows from (6) that  $E(\hat{c}_i) = \pm_0 E(\hat{p}_i) + \pm_1 E(e_i)$ , i.e. that

$$\bar{c} = \pm_0 + \pm_1 \frac{E(e_i)}{E(\hat{p}_i)} \quad (7)$$

Solon shows that if parental education is a determinant of the child's income,  $\hat{\Delta}_{IV}$  and  $\hat{\Delta}_{OLS}$  respectively provide upper and lower bound estimates of the intergenerational correlation  $\bar{c}$ , in the sense that  $\text{plim}(\hat{\Delta}_{IV}) > \bar{c} > \text{plim}(\hat{\Delta}_{OLS})$ ; where  $\text{plim}(\cdot)$  is the probability limit operator. Probability limit formulas for these two estimators are given in Solon(1992). Here we shall write them in slightly different forms, as functions of the parameters of the structural model (6):

$$\text{plim}(\hat{\Delta}_{OLS}) = \frac{\pm_0 E(\hat{p}_i^2) + \pm_1 E(\hat{p}_i e_i)}{E(\hat{p}_i^2) + E(e_i^2)} \quad (8a)$$

$$\text{plim}(\hat{\Delta}_{IV}) = \pm_0 + \pm_1 \frac{E(e_i^2)}{E(\hat{p}_i e_i)} \quad (9)$$

In deriving the above relations it is further assumed that  $\hat{A}_{it}$ , the transitory income component pertaining to  $y_{it}$  in (2), is uncorrelated with all other variables. By defining  $\rho = E(\hat{p}_i^2) / [E(\hat{p}_i^2) + E(e_i^2)]$  as the variance ratio of permanent to total

income, also known as the signal to total variance ratio, we may use (7) in order to obtain a more familiar expression for  $p \lim(\hat{\alpha}_{OLS})$ :

$$p \lim(\hat{\alpha}_{OLS}) = \alpha \quad (8b)$$

from which it comes out more clearly that  $\hat{\alpha}_{OLS}$  is asymptotically biased downwards, as the variance ratio  $\lambda$  is smaller than one. The instrumental variables estimator on the other hand is biased away from zero provided  $\alpha_1 > 0$  and education is positively correlated with permanent income.

Solving the above system (8a) and (9), it may be verified that  $\alpha_0$  and  $\alpha_1$ , the parameters of (6), relate to the population second moments in the following manner

$$\alpha_0 = \frac{p \lim(\hat{\alpha}_{OLS}) - \lambda \frac{1}{2} p \lim(\hat{\alpha}_{IV})}{1 - \lambda^2} \quad (10)$$

$$\alpha_1 = \frac{p \lim(\hat{\alpha}_{IV}) - p \lim(\hat{\alpha}_{OLS})}{1 - \lambda^2} \quad (11)$$

where  $\lambda = \frac{E(\epsilon_p \epsilon_e)}{E(\epsilon_p^2)E(\epsilon_e^2)^{1/2}}$  is the correlation coefficient between permanent income and education. That is, because a single measurement  $x_{it}$  will not identify the variance of permanent income, it is not possible to evaluate  $\lambda$  or  $\frac{1}{2}$ . From this observation it also follows that under such circumstances it is not possible either to identify the structural model (6), nor is it possible to evaluate the large sample biases of the OLS and IV estimators.

In a panel data environment however, the researcher will usually possess repeated measurements on family income. Let  $x_{it}$  and  $x_{is}$  be two such measurements for periods  $t$  and  $s$ . Zimmerman (1992) assumes that measurement error exhibits an auto-regressive structure, so that the correlation between  $\epsilon_{it}$  and  $\epsilon_{is}$  diminishes as the time distance between periods  $t$  and  $s$  increases. On the other hand, empirical research on the covariance structure of earnings (see for instance MaCurdy, 1982; Abowd and Card, 1989 and Schluter, 1998) suggests that errors of measurement exhibit a moving average structure. That is, passed a certain time period – most probably two years – changes in annual earnings are serially uncorrelated. Accordingly, we may use here the identifying assumption

$$E(\epsilon_{it} \epsilon_{is}) = 0 \quad |t - s| > q \quad (12a)$$

in order to estimate the variance of permanent income, and hence, the signal to total variance ratio  $\lambda$ . Clearly, if  $q = 0$  this amounts to assuming that transitory

variations in income are uncorrelated over time <sup>3</sup>. More generally however, if  $x_{it}$  and  $x_{is}$  are measurements taken at least  $q + 1$  periods apart, transitory variations in earnings have zero correlation, in which case we may use the covariance between any two such measurements in order to estimate the variance of permanent income:

$$E(x_{it}x_{is}) = E(\hat{\sigma}_p^2) \quad |t - s| > q \quad (12b)$$

The above equation is an equivalent formulation to the moment condition (12a).

With this extra condition we may note that it is now possible to identify the structural model (6) for the determinants of the child's income. In particular, let  $m(x; y)$  denote the sample covariance between variables  $x$  and  $y$ ; and let

$$\hat{\sigma}_p^2 = \frac{m(x_{it}x_{is})}{m(x_{it}^2)} \quad (13)$$

$$\hat{\eta} = \frac{m(x_{it}e_i)}{[m(x_{it}x_{is})m(e_i^2)]^{1/2}} \quad (14)$$

denote estimators for  $\sigma_p^2$  and  $\eta$ .  $\hat{\sigma}_p^2$  is consistent for  $\sigma_p^2$  under condition (12) <sup>4</sup>, while  $\hat{\eta}$  is consistent under the additional assumption that transitory income is uncorrelated with educational attainment <sup>5</sup>. Note finally that the estimator  $\hat{\sigma}_p^2$  consistently estimates  $\text{plim}(\hat{\sigma}_p^2)$  so that we may consistently estimate  $\pm_0$  and  $\pm_1$  using the following expressions:

$$\hat{\pm}_0 = \frac{\hat{\sigma}_{OLS}^2 \hat{\eta}^2 \Delta_{IV}}{1 - \hat{\eta}^2} \quad (15)$$

$$\hat{\pm}_1 = \frac{h_{IV} \hat{\sigma}_{OLS}^2 \hat{\eta}^2 m(x_{it}e_i) = m(e_i^2)}{1 - \hat{\eta}^2} \quad (16)$$

We may note that (16) provides a direct test for the null hypothesis that  $\hat{\sigma}_{IV}^2$  is consistent, against the alternative that it is upwardly inconsistent. If  $\pm_1$  is

<sup>3</sup>This would also imply that lags and leads of income may be used to instrument  $x_{it}$  in the estimation of (1). See Abul Naga and Krishnakumar (2000) for a discussion.

<sup>4</sup>We may note that other consistent estimators for  $\sigma_p^2$  are available, such as  $\hat{\sigma}_p^{2+} = 2m(x_{it}x_{is})/[m(x_{it}^2) + m(x_{is}^2)]$ . In panels that stretch over a long enough period of time several covariance terms may also be used in order to estimate the variance of  $\hat{\sigma}_p^2$ .

<sup>5</sup>This statement follows from the fact that under the assumption  $E(e_{it}e_i) = 0$ ; we have that  $E(x_{it}e_i) = E(\hat{\sigma}_{pi} + e_{it})e_i = E(\hat{\sigma}_{pi}e_i)$ .

estimated to be statistically not different from zero, it may be concluded that  $\text{plim}(\hat{\alpha}_{IV}) = \text{plim}(\hat{\alpha}_{OLS}) = \alpha$ ; i.e. that the instrumental variables estimator is consistent. Under the alternative, it is taken that  $\alpha_1 > 0$ ; that is, that  $\hat{\alpha}_{IV}$  systematically over-estimates the intergenerational correlation. By going back to the structural model (6) we obtain an alternative intuition for this test. The argument is as follows: if  $\alpha_1 = 0$ ; this implies that parental education is not a determinant of the child's income, and hence the instrumental variables estimator achieves consistency. An evaluation of the hypothesis that  $\hat{\alpha}_{IV}$  is consistent is therefore undertaken here by treating (6) as an extended regression of (1), where it is maintained, under the null hypothesis, that  $\alpha_1 = 0$ .

Substituting (15) and (16) into (7) entails the following estimator for  $\alpha$ :

$$b = \hat{\alpha}_{OLS} \quad (17)$$

which is a feasible form for the consistent adjusted least squares estimator (Meijer and Wansbeek, 2000)<sup>6</sup>.

In our empirical applications we shall evaluate the large sample biases of the OLS and IV estimators in percentage terms, i.e. in the form  $\text{bias}_{OLS} = \text{plim}(\hat{\alpha}_{OLS}) - \alpha$  and  $\text{bias}_{IV} = \text{plim}(\hat{\alpha}_{IV}) - \alpha$ . In the sample these may be evaluated by the quantities (13) and

$$b = \frac{\hat{\alpha}_{IV}}{\hat{\alpha}_{OLS}} \quad (18)$$

In the section that follows we use data from the Panel Study of Income Dynamics in order to estimate the biases of these two estimators of the intergenerational correlation parameter  $\alpha$ .

### 3. Evidence from the PSID

In order to quantify the biases of the ordinary least squares and instrumental variables estimators of the intergenerational correlation, we look at income continuities in a US sample of parents and children. Our data are extracted from the SRC ...le, the random sample, of the University of Michigan's Panel Study of Income Dynamics (PSID). A full account of the PSID, its history and main data ...les, can be found in Hill (1993).

We have data on the incomes of parents over the four-year period 1967-70, and the resources of children are observed in 1987 and, four years later, in 1991. We

<sup>6</sup>See Abul Naga (2000b) for an alternative derivation of the estimator  $b$ .

have 526 such parent and child pairs. The income concept taken here is the total labour income of the household head (measured in 1967 dollars – the year prior to which the survey was started). Parents and children are at least 25 years of age when their earnings are observed, and we have selected one child per parent family. The average age of the family head was 42.5 years for parents in 1967, and 35.2 years for children in 1987. Finally, as earnings are bound to vary over the life cycle in a non-random way, we have run prior regressions of the logarithm of labour income on age and age squared of the household head in each given year, and have chosen to work with the residuals from these initial regressions in the results reported below.

Our baseline regression is that of the child's 1987 earnings on those of the parent family in 1967. For this regression we initially consider two estimators: OLS and IV, which are intended to bracket the true value for the intergenerational correlation  $\bar{\rho}$ . In our sample these take on respectively the values 0.263 and 0.473, reported in the top line of table 1. It may also be noted, in anticipation of our test results, that the standard error of the OLS estimator is 0.045, and that of the latter is twice this magnitude<sup>7</sup>. Under the assumption that the transitory component of income is uncorrelated over time, we use the covariance between 1967 and 1968 labour income in order to estimate the variance of permanent income. On the basis of (12), this implies that  $q = 0$ , so that in table 1 we denote the above assumption of absence of serial correlation as an MA(0) scenario. The parameter  $\rho$  is then estimated as the sample covariance between 1967 and 1968 earnings, divided by the sample variance of 1967 earnings (see equation 13). In our data this produces an estimate of 0.90 for the signal to total variance ratio. This is suggestive of a rather small bias for the OLS estimator in comparison to conclusions reached by other authors in this respect. Bowles (1972) for instance reports estimates for  $\rho$  ranging between 0.70 and 0.83 using various income concepts, Zimmerman (1992) estimates this ratio to be 0.73 for wages and 0.66 for earnings, while Abul Naga (2000a) estimates this quantity to be in the range of 0.57 for family incomes and 0.62 for earnings.

Our estimate of  $\rho$  in turn can be used to correct the bias of the OLS estimator using (17). Given the correction factor underlying the MA(0) model findings, the estimator  $b$  implies a value of 0.29 for the intergenerational correlation  $\bar{\rho}$ . Using

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<sup>7</sup>Note that the standard error of the OLS estimator is a function of  $\rho$  (cf. Meijer and Wansbeek, 2000 and also Abul Naga, 2000b). As we move down the table, the OLS estimate of  $\bar{\rho}$  does take on the same value. However, its standard error changes with the estimate of  $\rho$ . This effect is however an order of magnitude of  $10^{-4}$ , and accordingly is not reported here.

(18), we evaluate the large sample bias of the instrumental variables estimator. The estimate of  $\rho$  corresponding to the MA(0) scenario is equal to 1.617, implying  $\hat{\rho}_{IV}^{\Delta}$  over-estimates the intergenerational correlation by 62%. The sample covariance between 1967 and 1968 earnings also provides the estimate of the variance of  $\hat{\rho}$ , used in the estimation of the correlation  $\frac{1}{2}$  between education and permanent income (cf. equation 14). The estimate of  $\frac{1}{2}$  corresponding to the first line of table 1 is equal to 0.51. We are not aware of other estimates of this parameter based on micro data. The simple correlation between education and age-adjusted annual earnings, taking a value of 0.48 in our 1967 data, would typically provide a lower bound estimate for  $\frac{1}{2}$ ; since the presence of an earnings transitory variance component would inflate the denominator of such an expression.

The parameters  $\pm_0$  and  $\pm_1$  of the structural model (6) for the determinants of the child's permanent income are estimated using (15) and (16). As  $\hat{\pm}_0$  and  $\hat{\pm}_1$  are complicated functions of the sample moments  $m(x_{it}x_{is})$ ,  $m(x_{it}e_i)$ , etc., the delta method is used here in order to derive expressions for their standard errors (see the appendix of the paper for further details).  $\pm_0$  is estimated at 0.23 with a standard error of 0.07 while  $\pm_1$ , the effect of the parents' education on the child's permanent income, is estimated at 0.04 with a standard error of 0.02. It may be noted from this second finding that  $\hat{\pm}_1$  is statistically different from zero – a point we shall take up again below.

Panel data not only provide sufficient information in order to identify a general class of errors in variables models, but they also allow the researcher to relax some assumptions regarding the serial correlation underlying measurement errors. In the next line of table 1, the MA(1) estimates, we allow the correlation between transitory incomes to be non-null in any two consecutive years, but we assume that it is zero for data observed two or more years apart. The MA(1) estimates will therefore be consistent under the assumption that the MA(0) specification is correct. The reverse however is not true, since then the estimate of  $\rho$  would then be inflated in the numerator by the covariance between transitory earnings components of years  $t$  and  $t + 1$ . The covariance between 1967 and 1969 earnings is therefore used in the second line of the table in order to estimate the numerator of  $\rho$ : The corresponding estimate of the variance ratio falls from 0.90 to 0.84, as the auto-correlation between earnings taken two years apart drops. In turn, this implies that  $\hat{\rho}$  revises the estimate of  $\rho$  upwards to 0.31. The large sample bias of the instrumental variables estimator is likewise revised downward to an over-estimate of the intergenerational correlation by 52% (instead of 62% for the earlier findings).

This same exercise is further repeated in the last line of table 1, under the assumption that errors of measurement follow an MA(2) process. The estimate of  $\rho$  further drops to 0.79 when the covariance between 1967 and 1970 earnings is used to evaluate the numerator of (13), bringing it more in line with other available estimates. As a result,  $b$  re-scales the 0.26 OLS estimate of  $\bar{r}$  by a factor of 1.27 to arrive at an estimate of 0.34 for the intergenerational correlation. Again, the percentage bias implied by the instrumental variables estimator appears smaller, and  $\sigma^2$  is now estimated at 1.41, instead of 1.52 under the MA(1) assumption, and 1.62 when errors of measurement were taken to be uncorrelated over time.

Of particular relevance to our discussion is a test of the assumption that  $\hat{\Delta}_{IV}$  is inconsistent and biased away from zero. Inspecting (11), it may be noted that  $\pm_1$  equals zero either if  $\text{plim}(\hat{\Delta}_{IV}) = \text{plim}(\hat{\Delta}_{OLS}) = \rho = \bar{r}$ ; or if the correlation between education and permanent income is zero (a very unlikely assumption given many years of research on human capital). Assuming a positive correlation between education and permanent income, a rejection of the null hypothesis that  $\pm_1 = 0$  would imply that the instrumental variables estimator is inconsistent. It may be inferred from the results of table 1 that the  $t_j$  statistic for  $\pm_1$  is equal to 2.05 under the MA(0) scenario, to 1.89 for the MA(1) scenario, and 1.55 for the MA(2) formulation.

If the alternative to the null is the assumption that  $\pm_1 > 0$  – i.e. that  $\hat{\Delta}_{IV}$  is upwardly inconsistent, then the critical size of the test at the 5% level is 1.64 rather than 1.96. On such grounds, we may conclude that there is evidence supporting the postulate that the instrumental variables estimator is biased away from zero when we maintain the assumption that errors of measurement are uncorrelated. The null hypothesis is still rejected at the 5% level (but not at the 2.5% level) for the MA(1) formulation. For the MA(2) formulation however, the  $t_j$  statistic taking a value of 1.55, the assumption that  $\hat{\Delta}_{IV}$  is consistent is no longer rejected at the 5% significance level. This latter result may at first seem rather surprising given the 41% bias imputed to the instrumental variables estimator under the MA(2) scenario. However, a heuristic account for this finding may be provided by observing that the standard error of  $\hat{\Delta}_{IV}$  is quite large. By this it is meant that under the null hypothesis that  $\hat{\Delta}_{IV}$  is consistent, a 95% (90%) confidence interval for  $\bar{r}$  obtained from this estimator covers values ranging between 0.28 (0.32) and 0.66 (0.63)<sup>8</sup>. In particular, the implied value for the intergenerational correlation in the third line of table 1,  $b = 0.335$ , is within these confidence bounds.

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<sup>8</sup>Under the alternative hypothesis, both the large sample mean and variance of  $\hat{\Delta}_{IV}$  are incorrectly estimated using the results of table 1)

It has been noted by Reville (1995) that estimates of the intergenerational correlation rise as children age. In table 2 we have undertaken the same estimations as those of table 1, with the difference that children's earnings are observed four years later, in 1991. It may be observed now that the bounds provided by OLS and IV are respectively 0.33 and 0.60, as opposed to 0.26 and 0.47 in the results of table 1. The b estimates of the intergenerational correlation are also higher than the corresponding ones of table 1. It is also again the case that b takes on increasing values as we move down the table from the MA(0) to the MA(2) specification. This latter result is also a consequence of the fact that the same parental data (1967 to 1970 earnings) have been used in the estimations of tables 1 and 2. It is also for this reason that  $\hat{\rho}_0$  and  $\hat{\rho}_1$  take on identical values in the results of tables 1 and 2.

If those who are expected to experience the largest earnings growth rates are individuals who undertake more important educational investments, we have an explanation as to why estimates of  $\hat{\rho}$  rise as children age<sup>9</sup>. A similar explanation can be used to account for the pattern observed in table 2 that estimates of  $\hat{\rho}_0$  and  $\hat{\rho}_1$  also increase as children grow older. In particular, the t-statistics pertaining to  $\hat{\rho}_1$ , the coefficient on parental education, increase, leading now to a rejection of the hypothesis that  $\hat{\rho}_{IV}$  is consistent under all three moving average specifications considered. The t-statistics pertaining to  $\hat{\rho}_1$  for the MA(0) to MA(2) are respectively 2.78, 2.56 and 2.16, in comparison to 2.05, 1.89 and 1.55 for the results of table 1.

#### 4. Conclusions

By providing repeated measurements on error-ridden variables, panel data allow the researcher to identify a wide range of errors in variables models, of which the Galtonian model of income transmission is an example. We have used this result in the present paper in order to quantify the large sample biases of the ordinary least squares and instrumental variables estimators of the intergenerational correlation, and to provide consistent estimators for the structural parameters of the Galtonian model. As a by-product of our discussion we have proposed a test to investigate the null hypothesis that the instrumental variables estimator is consistent, against the alternative that it is inconsistent and biased away from zero.

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<sup>9</sup>Solon (1999) for instance postulates that measurement error in the early years of employment is 'mean-reverting' and accordingly biases estimates of the intergenerational correlation downwards.

We have estimated the intergenerational correlation of earnings at a value of 0.26 using the method of ordinary least squares, and 0.47 using the method of instrumental variables. These estimates rise to 0.33 and 0.60 when the earnings of children are observed four years later. Depending on the assumptions retained regarding the serial correlation in errors of measurement, our estimate of the signal to total variance ratio (the large sample bias of OLS) takes values ranging between 0.79 and 0.90. The estimate of this variance ratio drops as observed earnings are allowed to correlate over longer periods of time, suggesting that the bias of the OLS estimator is larger, and that of the instrumental variables estimator is smaller than when one assumes the presence of no serial correlation between transitory income components. The large sample bias of the instrumental variables estimator is estimated at 62% when errors of measurement are taken to be uncorrelated over time, but drops to 41% under an MA(2) specification.

We note finally that the hypothesis that the instrumental variables estimator is consistent is rejected under MA(0) and MA(1) specifications for errors of measurement, but that the t-value of the test is only 1.55 under an MA(2) scenario. A re-estimation of the MA(2) specification when children are four years older however entails a t-value of 2.16 for this same test, allowing us thus to reject the assumption that the instrumental variables estimator is consistent.

With the exception of the work of Zimmerman (1992), previous studies in the literature which have used multiple measurements on parental income have often been too brief, if not silent, in their discussion of the serial correlation in measurement errors. One conclusion which emerges from our results, is that the identification of the Galtonian model ought not be discussed separately from questions pertaining to the serial correlation underlying transitory incomes. For this reason, we also feel that further research is necessary before we can validate or reject the claim that the instrumental variables estimator systematically over-estimates the intergenerational correlation. Such research may adopt a more general ARMA formulation for the transitory income component, enabling thus the data-analyst to nest within this framework both the auto-regressive and moving average error structures. Also, longitudinal surveys from other countries ought to be examined in order to draw a more complete picture concerning the biases of the ordinary least squares and instrumental variables estimators of the intergenerational earnings correlation.

## 5. Appendix

In this appendix we briefly discuss the estimation of the covariance matrix of  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$  by the delta method.

Using the definitions of  $\hat{\alpha}_{OLS}$ ,  $\hat{\alpha}_{IV}$ ,  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$ ; we can write  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$  as follows:

$$\hat{\alpha}_0 = \frac{m(x_{it}y_{it})}{m(x_{it}x_{is})} - \frac{m(x_{it}e_{it})}{m(x_{it}x_{is})} \frac{m(y_{it}e_{it})m(x_{it}x_{is}) - m(x_{it}y_{it})m(x_{it}e_{it})}{m(e_{it}^2)m(x_{it}x_{is}) - m^2(x_{it}e_{it})} \quad (A1)$$

$$\hat{\alpha}_1 = \frac{m(y_{it}e_{it})m(x_{it}x_{is}) - m(x_{it}y_{it})m(x_{it}e_{it})}{m(e_{it}^2)m(x_{it}x_{is}) - m^2(x_{it}e_{it})} \quad (A2)$$

where  $j_t - s_t > q$  for the MA(q) scenario pertaining to the serial correlation in measurement errors.

Define  $\mu$  as the following  $5 \times 1$  vector of sample moments:

$$\mu = \begin{bmatrix} m(y_{it}e_{it}) \\ m(x_{it}x_{is}) \\ m(x_{it}y_{it}) \\ m(x_{it}e_{it}) \\ m(e_{it}^2) \end{bmatrix}$$

The covariance matrix  $V(\mu)$  is estimated using fourth order sample moments of the type  $\sum_{i=1}^n y_{it}^2 e_{it}^2 = n$ ;  $\sum_{i=1}^n y_{it} e_{it} x_{it} x_{is} = n$  etc. Denote this  $5 \times 5$  covariance matrix as  $\hat{V}$ .

Write  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$  as functions  $g_0(\mu)$  and  $g_1(\mu)$  and define  $J$  as the  $2 \times 5$  Jacobian matrix with elements  $J_{kl} = \frac{\partial g_k(\mu)}{\partial \mu_l}$  where  $k = 0, 1$  and  $l = 1, \dots, 5$ . We estimate the covariance matrix of  $\hat{\alpha} = [\hat{\alpha}_0 \quad \hat{\alpha}_1]'$  by the delta method using the following expression:

$$\text{cov}(\hat{\alpha}) = J \hat{V} J' \quad (A3)$$

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**Table 1 : estimation results**

	OLS	IV	$\rho$	$\lambda$	$\gamma$	$\delta_0$	$\delta_1$	$b$
MA(0)	0.263 (0.045)	0.473 (0.096)	0.507	0.899	1.617	0.230 (0.072)	0.039 (0.019)	0.292 (0.050)
MA(1)			0.524	0.844	1.517	0.251 (0.075)	0.036 (0.019)	0.312 (0.053)
MA(2)			0.543	0.786	1.412	0.277 (0.090)	0.031 (0.020)	0.335 (0.058)

Notes

- 1 The income concept is total labour income of the household head, measured in 1967 dollars.
- 2 The parents' earnings data used pertain to the four-year period 1967-70. The child's earnings pertain to 1987.
- 3 Standard errors appear inside parentheses, n=526.

**Table 2 : further estimation results when children are four years older**

	OLS	IV	$\rho$	$\lambda$	$\gamma$	$\delta_0$	$\delta_1$	$b$
MA(0)	0.332 (0.050)	0.602 (0.108)	0.507	0.899	1.634	0.288 (0.061)	0.050 (0.018)	0.369 (0.056)
MA(1)			0.524	0.844	1.532	0.314 (0.066)	0.046 (0.018)	0.393 (0.060)
MA(2)			0.543	0.786	1.427	0.347 (0.073)	0.041 (0.019)	0.422 (0.064)

Notes

- 1 The income concept is total labour income of the household head, measured in 1967 dollars.
- 2 The parents' earnings data used pertain to the four-year period 1967-70. The child's earnings pertain to 1991.
- 3 Standard errors appear inside parentheses, n=526.