Social Welfare Orderings:

A Life-Cycle Perspective

Ramses H. Abul Naga*

DEEP, Université de Lausanne, CH-1015 Lausanne, Switzerland.

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Abstract

Consider the problem of measuring long-run household welfare and investigating welfare orderings from cross-section data. Life-cycle theories emphasize that consumption is allocated intertemporally on the basis of a long-term concept of resources, which differs from household income. Expenditure is also subject to transitoriness because diaries on spending are

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kept for a period of two to four weeks. Via joint modelling of household income and expenditure, we provide bounds for the deficit curve of life-cycle incomes using two distinct predictors of this unobservable. A third predictor generates a deficit curve also lying within these bounds.

Keywords: Social welfare orderings, life-cycle income, household income and expenditure, deficit curves.

JEL codes: C_2, C_3, D_6, I_3

1. Introduction

The measurement of household welfare, variations in levels of living across regions, and over time, are important means through which a government assesses its economic policy. Specific groups such as the poor may be the primary focus of such analyses. More generally however, policies which affect the population at large, for example tax reforms, may call for a complete analysis of the distribution of household welfare as a whole.

There has been tremendous progress over the last fifty years in formulating empirical models for the analysis of household welfare. Advances have been made in providing flexible representations of preferences, controlling for variations in household demographic structure and analyzing limitations on choice induced by quantity constraints (see for instance Deaton and Muellbauer, 1980; Pollak and Wales, 1992 and Slesnick, 1998). Parallel developments in the literature on the measurement of inequality following the work of Atkinson (1970), have emphasized the possibility of obtaining orderings of distributions with limited conditions on the form of the underlying social valuation function (for example Foster and Shorrocks, 1988; Davies and Hoy, 1995 and the survey of Cowell, 2000).

One remaining practical problem however which has received limited attention in recent years is that families may plan their consumption over longer time horizons than those covering the typical household cross-sectional survey. While the typical expenditure survey records purchases of commodities over perhaps two to four weeks (the former being the case in the British Family Expenditure Survey, and the latter for its Swiss counterpart), life-cycle theories emphasize that decision-makers adopt a broader time frame and concept of resources on the basis of which consumption is planned[1]. Not withstanding the problem that within the same survey income sources and types of expenditures may be recalled over different time horizons, the shortness of the time period over which data are gathered introduces measurement error. This in turn implies that observed income and expenditure will tend to inflate the true level of inequality underlying these variables. Thus Muellbauer (1983) writes:

> "All this makes comparisons between surveys a very dangerous affair. Even, for the same survey, comparisons of inequality and poverty between regions and socio-economic groups are subject to the danger that differences in transitoriness become confused with more fundamental differences."

If these *more fundamental differences* may be traced back to the distribu-

[1] While the life-cycle is often considered as the duration of adult or working life, in the context of consumption decisions Friedman (1957) suggests that a 3 year time span may be a reasonable approximation.

tion of life-cycle resources, then we may summarize the understanding reached on this problem some twenty years ago (Attfield, 1976; Musgrove, 1979 and Muellbauer, 1983) along the following lines: the variance of the logarithm of income or expenditure over-estimates the underlying variance of *true* income or expenditure. Likewise, the variance of fitted income or expenditure (via regression on household composition variables, housing status and access to other durables and productive assets) under-estimates the underlying relevant variance because of unobserved heterogeneity in the data. Muellbauer however noted that if the same unobserved variance component was present in both observed income and expenditure, joint estimation of a two-equation system for these two variables would enable the researcher to obtain a precise estimate of the variance of the relevant true income over which households plan their consumption.

There are several reasons why a fresh look at this problem may be beneficial today. It may be argued that the variance of logarithms is mostly sensitive to large incomes, and therefore, that it mainly records changes in the tails of the distribution. More importantly though, the variance of logarithms is but one among many inequality measures. Its use is in a sense arbitrary to the extent that other distributionally sensitive inequality measures are available, which could induce a different ordering of income distributions [2]. In this sense, attempting to tackle this earlier problem within an ordinal approach to the ranking of distributions, the aim of the present paper, would be of value in that it would remove some arbitrariness involved with the choice of a summary measure such as the variance of logarithms. It should be emphasized in all fairness though that the cardinal ranking of income distributions was certainly the best one could do at the time of the earlier contributions of Attfield, Musgrove and Muellbauer and that many

[2] More precisely, such a situation may occur in presence of intersecting Lorenz curves.

results on welfare and poverty orderings have only began to appear in later years (Shorrocks, 1983; Atkinson, 1987; and Foster and Shorrocks, 1988).

In the present paper therefore we follow earlier work on the joint modelling of household income and expenditure, and show that within-model predictors of life-cycle income can be used to derive welfare orderings for these unobserved resources. A discussion of this prediction problem in a somewhat less general form appears in Abul Naga and Burgess (1997), but the welfare orderings these predictors induce in relation to the distribution of unobserved life-cycle incomes has not been the focus of their paper. It is shown here that within a suitably defined class, a predictor of life-cycle income based on household income, expenditure, or a weighted sum of these two, will entail a distribution dominated in a social welfare sense (or alternatively, second order stochastic dominance) by the distribution of unobserved life-cycle incomes. Likewise, a predictor constructed from household socio-demographic (right-hand side) variables will induce a distribution which welfare dominates the distribution of unobserved life-cycle incomes. These results mirror Muellbauer's earlier conclusions for the variance of logarithms, and are essentially based on the same explanation that predictors obtained from left-hand variables (i.e. income and expenditure) overstate inequality of unobserved resources, while those based on right-hand variables abstract from unobserved variance components.

While at the level of generality associated with ordinal welfare analysis we are not able to recover the deficit curve of unobserved life-cycle incomes, we propose a third deficit curve, constructed from a weighted sum of the two predictors discussed above, which may be jointly used with these in applied work. The deficit curve obtained from this third predictor is shown, as in the case of the distribution of life-cycle incomes, to lie between the bounds set by the deficit curves based on respectively left-hand and right-hand variables. This third deficit curve is shown however to differ from the deficit curve pertaining to life-cycle incomes.

It should be noted finally that the deficit curve and generalized Lorenz curve (Shorrocks, 1983) are two equivalent tools used for the purpose of investigating welfare orderings. The results stated in this paper can therefore equivalently be cast within the framework of the generalized Lorenz curve [3]. Likewise, it will come out more clearly below that our proposed framework is equally applicable in the context of investigating inequality orderings. Our emphasis here on welfare stems from the fact that inequality orderings only translate into welfare orderings in the context of examining distributions with identical means (in practice, a

[3] See also Davidson and Duclos (2000) for other graphical devices used for the investigation of second order stochastic dominance.

scenario which may be approximately relevant when comparing some tax / subsidy problems which induce small allocational distortions).

Section 2 below discusses a simple two period consumption allocation model with the purpose of distinguishing observed income from life-cycle income, and highlighting that consumption expenditure is a function of the latter; that is, an unobserved variable. In section 3 we present the empirical framework used for the joint modelling of household income and expenditure. Section 4 introduces the class of predictors used to draw inferences about the distribution of life-cycle incomes. Section 5 discusses the distributional orderings generated by these predictors, in relation to one another, and also in relation to the distribution of life-cycle incomes. Section 6 contains applications on Swiss data drawn from two household expenditure surveys, while section 7 contains concluding comments.

2. Consumption and life-cycle income

In this section we consider a resource allocation problem for a household assumed to live for two periods: today, for which the data analyst observes information from a cross-section survey, and tomorrow, the unknown future. Though highly simplified, the example helps to motivate the discussion that follows in section 3 on the joint modelling of income and consumption expenditure. More importantly, understanding the assumptions underlying a linear model of income and expenditure (see below) helps to identify potential drawbacks of the proposed approach, and to draw an agenda for the elaboration of more realistic empirical frameworks for future research.

Assume then that a household maximizes a utility function $v(C_1, C_2)$, which in accordance with life-cycle theories, is taken to be homothetic. Let A_t denote assets at period t. Using the terminal condition $A_2 = 0$, resources are allocated subject to the following budget constraint:

$$C_1 + \frac{1}{1+r} C_2 = A_0(1+r) + m_1 + \frac{1}{1+r} m_2 \tag{1}$$

The interest rate r is taken to be a constant (known) quantity, A_0 is the initial stock of assets and m denotes labor income. Though m_2 is unknown in period 1, it is assumed that households have point expectations about future quantities (Deaton and Muellbauer, 1980 ch. 4).

Because preferences are taken to be homothetic, first order conditions produces the result that consumption expansion paths are straight lines emanating from the origin. In other terms,

$$C_1 = \beta_C(r). \left[A_0(1+r) + m_1 + \frac{m_2}{1+r}\right] \qquad (2)$$

Leaving aside problems of measurement error for now, in theory, if a researcher seeks to measure long run welfare, period 1 consumption expenditure (which s/he may observe on the basis of a cross-section survey) contains all the relevant information concerning the household's lifetime resources.

Introducing labor income uncertainty into this story leaves the above account unchanged if the utility function $\upsilon(C_1, C_2)$ is taken to be quadratic. Under such a formulation, the marginal utility of income is linear so that (2) would hold with expected future labor income $E(m_2)$ replacing m_2[4]. The assumption of quadratic utility in fact provides a formal justification for Friedman's (1957) formulation of the Permanent Income Hypothesis, which is the setup used by Attfield, Musgrove and Muellbauer in their respective papers.

While homothetic preferences provide a convenient way of discussing intertemporal allocations of consumption at a macro level, demographic composition variables are likely to play an important role in explaining expenditure at the micro level. We may for instance wish to make the marginal utility of consumption a

[4] See Abul Naga and Bolzani (2000).

function of demographic variables, in an effort to understand how families smooth their consumption over the life cycle. Such variables may be incorporated in our setup by generalizing preferences to a quasi-homothetic structure. Define η as life-cycle income:

$$\eta = A_0(1+r) + m_1 + \frac{m_2}{1+r} \tag{3}$$

With quasi-homothetic preferences, household consumption functions remain linear, with the difference now that they possess non-zero intercepts:

$$C_1 - \theta_1 = \beta_C(r)\left[\eta - \theta_1 - \frac{\theta_2}{1+r}\right] \tag{4a}$$

where θ_t is subsistence expenditure at time t; a function of household characteristics. Let $\alpha(r) = \theta_1[1-\beta_C(r)] - \theta_2\beta_C(r)/[1+r]$ denote this intercept, a function of current and future (unobserved) household needs. Then we may rewrite (4a) more compactly as

$$C_1 = \alpha(r) + \beta_C(r)\eta \tag{4b}$$

Noting that the interest rate is constant in the cross-section, if the above intercept can be approximated by a set of demographic controls D_C we may write

$$\alpha = d_{1C}\delta_{1C} + \cdots + d_{lC}\delta_{lC} + error \tag{5}$$

Noting more generally that for reasons of transitoriness discussed in the introduction, consumption expenditure is unlikely to be measured without error, we can then write

$$C_1 = \beta_C \eta + D_C \delta_C + e_C \tag{6}$$

Because period 1 outlay, $m_1 + A_o(1 + r)$, also contains information about the households' lifetime resources, we may write a similar equation to (6) for period 1 income:

$$I_1 = \eta + D_I \delta_I + e_I \tag{7}$$

where e_I and e_C are respectively disturbance terms pertaining to the income and expenditure equations. Setting $\delta_I = 0$ in (7), we would have the Friedman and Kuznets (1945) decomposition of observed income into a long run component η and a random term e_I, taken to be uncorrelated with η. Adding demographic variables to this set-up is a means of controlling for the fact that, say professionals and manual workers, have different income-age profiles. The system (6) and (7) is the same as that in Muellbauer (1983), with the difference here that income and expenditure are taken in levels, whereas in the former they are modeled in logarithms. In the present set-up however it is important for us to maintain these in their original scale, so that the predictors of life-cycle income we derive be also measured in the same money units as income and expenditure. This will come out more clearly in section 4, when we examine more closely the prediction problem. Prior to this, however, we turn to the empirical framework used to estimate the above system, (6) and (7).

3. An empirical framework

In this section we complete the system (6) and (7) with the introduction of an equation for the determinants of η. In what follows we shall often be working with a system of equations. In due course therefore, a vector notation will be introduced. Let y_1 denote household income ($y_1 = I_1$), y_2 denote consumption expenditure ($y_2 = C_1$), and u_j denote the disturbance term associated with y_j. For a given household i, we would write with this new notation

$$y_{i1} = \eta_i + D_{i1}\delta_1 + u_{i1} \tag{8a}$$

$$y_{i2} \quad = \quad \beta_C \eta_i + D_{i2}\delta_2 + u_{i2} \tag{8b}$$

where η_i is an unobserved random variable. It is assumed throughout that $E[u_{ij}|\eta_i] = 0$ for all i and $j = 1, 2$. The fact that the above model is not identifiable in its present form dates back to the work of Friedman (1957). The argument is as follows: replacing η by y_1 in (8b), that is proxying life-cycle income by the measure of income reported in a cross-sectional survey, produces an errors-in-variables problem. Zellner (1970) and Jöreskog and Goldberger (1975) identify the above model by assuming the existence of a set Z of determinants of life-cycle income:

$$\eta_i = \gamma' Z_i + \varepsilon_i \tag{9}$$

where ε is a disturbance term assumed to be uncorrelated with Z. In the context of our work, Z may contain variables such as the educational attainment and health status of family members engaged in employment. Ownership of financial and productive assets and other durables such as housing may also be included in a such a set. To achieve identification, all Z variables must be uncorrelated with all three disturbances of the system, that is ε, u_1 and u_2. This may initially appear to be a strong requirement. However, only the existence of a single Z variable satisfying these orthogonality restrictions is required; typically, the educational attainment of the household head may perform this role. The validity of the orthogonality assumptions for other Z variables may then be tested by means of exogeneity tests for over-identifying variables, described in chapter 5 of Godfrey (1988).

Substituting (9) for η in (8), we obtain the following reduced form for the income and expenditure system:

$$y_{i1} = \gamma' Z_i + \delta_1' D_{i1} + \varepsilon_i + u_{i1} \tag{10a}$$

$$y_{i2} = \beta_C \gamma' Z_i + \delta_2' D_{i2} + \beta\varepsilon_i + u_{i2} \tag{10b}$$

Define $Y_i = [y_{i1}, y_{i2}]'$, $\beta = [1, \beta_C]'$ and $U_i' = [u_{i1}, u_{i2}]'$. Likewise define $D_i = \begin{bmatrix} D_{i1}' & 0 \\ 0 & D_{i2}' \end{bmatrix}$ and $\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$. The following vector notation will be useful for the discussion of the prediction of life-cycle income η in the section below:

$$Y_i = \beta\gamma' Z_i + D_i\delta + \beta\varepsilon + U_i \tag{11}$$

The variance of ε is denoted as $\sigma_{\varepsilon\varepsilon}$, the 2×2 covariance matrix of U_i is written as Ω, and the matrix Σ_Y denotes $E(Y_i Y_i')$.

4. Prediction

Consider the problem of drawing inferences about household permanent income after estimation. This problem has been studied by Abul Naga and Burgess (1997). We may essentially predict permanent income η using (1) Y (indicators), (2) Z (determinants) and (3) a combination of Y and Z, which we shall write as the vector $W = [Y' \quad Z']'$. Given the linearity assumption underlying our framework, it is natural to focus our discussion on the class of linear predictors. Also, to the extent that observations are collected from random samples, it should be the case that data on family j ought to be uninformative about the life-cycle income of family i. For this reason, below we shall, as in section 2, suppress the subscript i.

It is perhaps simplest to consider first a predictor of η using Z variables, which we shall denote as $\widehat{\eta_z}$. This takes the form:

$$\widehat{\eta_z} = E(\eta \mid Z) = \gamma' Z \tag{12}$$

This is nothing other than the regression of η on Z, with the interpretation that $\widehat{\eta_z}$ is centered around the mean of η. For predictors based on Y variables, that is household income and expenditure, we may define a class C_Y:

$$C_Y = \{\widehat{\eta_Y} = b'(Y - D\delta) \mid b'\beta = 1\} \tag{13}$$

with the property that members of this class are also centered around $\gamma' Z$. To see that this is the case, note that using the constraint $b'\beta = 1$, we have from (11)

$$b'(Y - D\delta) = \gamma' Z + \varepsilon + b'U \tag{14}$$

Taking expectations, we see that the class C_Y defines unbiased predictors in the sense that

$$E[\widehat{\eta_Y} \mid Z] = E[b'(\beta\gamma' Z + \beta\varepsilon + U)|Z] = \gamma' Z \tag{15}$$

More generally, we may define predictors of life-cycle income which are based on Y and Z variables. Predictors chosen in the class C_W:

$$C_W = \left\{\widehat{\eta_W} = \tau\widehat{\eta_Y} + (1-\tau)\widehat{\eta_z}\right\} \qquad 0 < \tau < 1 \tag{16}$$

will also be centered around $\gamma' Z$, the mean of the unobservable η. As a result, the three predictors will satisfy the property

$$E\left[\widehat{\eta_Y} \mid Z\right] = E\left[\widehat{\eta_Z} \mid Z\right] = E\left[\widehat{\eta_W} \mid Z\right] \tag{17}$$

However, they will be shown in the section below to convey different pictures about the underlying level of inequality in the distribution of life-cycle incomes. Furthermore, applying the law of iterated expectations, we have a result that will be used in section 6:

$$E_Z[E(\widehat{\eta_Y} \mid Z)] = E_Z[E(\widehat{\eta_Z} \mid Z)] = E_Z[E(\widehat{\eta_W} \mid Z)] = E[\eta] \tag{18}$$

It is convenient at this stage to select specific members of the classes C_Y and C_W to be used in the empirical section of our paper. Define $\Sigma_Y = E(YY')$, and consider the problem

$$\min E[b'(Y - D\delta) - \eta]^2 - \mu(b'\beta - 1) \tag{19}$$

The solution to the above problem defines the optimal Mean-Square Error (MSE) predictor in the class C_Y:

$$\eta_Y^* = \left(\beta' \Sigma_Y^{-1} \beta\right)^{-1} \beta' \Sigma_Y^{-1} (Y - D\delta) \tag{20a}$$

where an asterisk appears as a superscript to denote mean-square error optimality. It is possible to show with some algebra that the above formula may more simply be written as

$$\eta_Y^* = \kappa \cdot \left[\frac{1}{\omega_{11}}(y_1 - \delta_1' D_1) + \frac{\beta_C}{\omega_{22}}(y_2 - \delta_2' D_2)\right] \tag{20b}$$

for $\kappa = (\beta' \Omega^{-1} \beta)^{-1}$ and Ω being the covariance matrix of U. Other things equal then, the variable y_j is assigned a larger weight the smaller the related measurement error variance ω_{jj} is.

As comes out from (12), $\widehat{\eta_Z}$ is a regression function, and accordingly we have that $\eta_Z^* = \widehat{\eta_Z} = \gamma' Z$. The minimum MSE predictor in the class C_W will also be a weighted sum of the predictors η_Z^* and η_Y^*:

$$\eta_W^* = \tau^o \eta_Y^* + (1 - \tau^o) \eta_Z^* \tag{21}$$

The weights dictated by τ^o

$$\tau^o = \frac{MSE(\eta^*_Z)}{MSE(\eta^*_Z) + MSE(\eta^*_Y)} \tag{22}$$

are chosen so as to give η^*_Y more weight in the prediction of η, the larger is the mean-square error of η^*_Z (see the appendix to Abul Naga and Burgess, 1997).

5. The three deficit curves

In this section we turn to the question of undertaking distributional analysis for life-cycle incomes. A central concept for ordering distributions in terms of well-being is the integral of the cumulative distribution function

$$\Phi(s;\eta) = \int_0^s F(\eta)d\eta \tag{23}$$

also known as the deficit curve. Consider the problem of comparing two distributions F^A and F^B over an income range $[0, S]$. Then, any social welfare function which exhibits a preference for higher incomes and social aversion to inequality will prefer F^A to F^B provided the deficit curve Φ^A lies below Φ^B everywhere in the income range $[0, S]$ [5].

Below it is shown that deficit curves for two of the three predictors, namely $\widehat{\eta_Y}$ and $\widehat{\eta_Z}$, provide a useful ordering in relation to the deficit curve of the unobserved variable η. More specifically, we show that the deficit curve of η lies between the deficit curves of the two predictors $\widehat{\eta_Y}$ and $\widehat{\eta_Z}$. From a practical point of view, this result shows that we are not able to recover the deficit curve of an unobserved variable, but that the proposed framework allows us to set bounds, or limits, on this curve.

While η is unobserved, we note that we have three deficit curves – one for each of $\widehat{\eta_Z}$, $\widehat{\eta_Y}$, and $\widehat{\eta_W}$, which can be put to good use. Observe that for an upper bound income s^+,

$$\Phi(s^+; \eta) = s^+ - E(\eta) \tag{24}$$

Given the property (18), it follows that the deficit curves of $\widehat{\eta_Z}$, $\widehat{\eta_Y}$, and $\widehat{\eta_W}$ all intersect at s^+. Our next result is obtained by writing $\widehat{\eta_Y}$ as a mean-preserving spread of η, and the latter as a mean-preserving spread of $\widehat{\eta_Z}$.

Substituting η for $\gamma' Z + \varepsilon$ in (14), we have that for any member of the class

[5] See Foster and Shorrocks (1988). As stated earlier in the introduction, the results presented in this section may be stated alternatively in relation to the generalized Lorenz curve, or any geometric concept used to investigate second order stochastic dominance.

of predictors C_Y

$$\widehat{\eta_Y} = \eta + b'U \tag{25}$$

with $E(U|Z) = 0$. The class C_Y thus generates predictors with the same overall mean as η, but with extra noise arising from the disturbances in observed income and expenditure. Accordingly, $\widehat{\eta_Y}$ will overstate the level of inequality underlying η. Conversely, as these variables have equal means, social welfare as measured by $\widehat{\eta_Y}$ is always less than the level of well-being underlying the distribution of η. Geometrically, the deficit curve of $\widehat{\eta_Y}$ lies above the deficit curve of η, until the point s^+, the highest income level, where the curves $\Phi(s;\widehat{\eta_Y})$ and $\Phi(s;\eta)$ intersect [6]. Likewise, re-writing (9),

$$\eta = \widehat{\eta_Z} + \varepsilon \tag{26}$$

so that $\Phi(s;\widehat{\eta_Z})$ will lie everywhere below $\Phi(s;\eta)$, up to the point s^+ where the two curves will meet.

Recall that the class C_W is constructed by mixing predictors from the class C_Y with $\widehat{\eta_Z}$. As such, any specific member of C_W, written as $\tau\widehat{\eta_Y} + (1-\tau)\widehat{\eta_Z}$, will add noise to η (with weight τ) via the presence of transitoriness in observed income and expenditure. But it is also the case that a given $\widehat{\eta_W}$ removes a share

[6] This is in fact the exact problem studied by Rothschild and Stiglitz (1970) in the context of ranking risky returns using expected utility analysis.

$(1-\tau)\varepsilon$ of the unobserved heterogeneity underlying η. More formally,

$$\widehat{\eta_W} = \eta + \tau b'U - (1-\tau)\varepsilon \tag{27}$$

As a result, therefore, the deficit curves $\Phi(s;\widehat{\eta_W})$ and $\Phi(s,\eta)$ cannot be shown to satisfy any general ordering.

With values of τ close to 1, the predictors $\widehat{\eta_W}$ and $\widehat{\eta_Y}$ will convey similar information about η. Conversely, with τ approaching zero, $\widehat{\eta_W}$ will rely essentially on the information contained in Z in order to predict life-cycle income.

By letting $\succeq$ denote social welfare dominance of F^A over F^B, we may write our final result, which orders $\widehat{\eta_Y}$, $\widehat{\eta_Z}$ and $\widehat{\eta_W}$ in terms of social welfare. Once again, this result exploits the property that $\widehat{\eta_W}$ is a weighted sum of $\widehat{\eta_Y}$ and $\widehat{\eta_Z}$, and that $\widehat{\eta_Y}$ is dominated by $\widehat{\eta_Z}$ in a social welfare sense:

$$\widehat{\eta_W} = \tau\widehat{\eta_Y} + (1-\tau)\widehat{\eta_Z} \succeq \tau\widehat{\eta_Y} + (1-\tau)\widehat{\eta_Y} = \widehat{\eta_Y} \tag{28}$$

$$\widehat{\eta_Z} = \tau\widehat{\eta_Z} + (1-\tau)\widehat{\eta_Z} \succeq \tau\widehat{\eta_Y} + (1-\tau)\widehat{\eta_Z} = \widehat{\eta_W} \tag{29}$$

To summarize then, we have shown that the deficit curves pertaining to $\widehat{\eta_Y}$ and $\widehat{\eta_Y}$ provide, respectively, upper and lower bounds, to the deficit curve of η (equs.

25 and 26), and that they also bound the deficit curve of $\widehat{\eta_W}$ (equs. 28 and 29). However, $\widehat{\eta_W}$ cannot be generally ordered in relation to η (equation 27).

6. An application

The Swiss recession of the 1990s may provide a useful example in applied welfare analysis for the need to distinguish between the distribution of household expenditure (or income) on the one hand, and that of life-cycle resources. In 1990 Switzerland was coming to the end of a growth cycle, and by 1998 was barely emerging from the recession that followed. Unfortunately, the Swiss Federal Statistical Office did not conduct the family expenditure survey between these two years. The 1990 survey sampled approximately 2000 households, while the figure rose to about 9000 in the latter year.

An unfortunate feature of the 1990 survey was that a household head in retirement was not asked to report any information concerning her or his educational attainment. As this variable was used to predict life-cycle income (see the appendix and table A1) we had to drop families in retirement from our data. There were only a handful of cases where the household head was still in employment passed the age of 65. We have also deleted these data points. A similar selection rule was adopted for our 1998 data, leaving us with 1654 families from the 1990 survey and 7948 data points from the latter data set. Resources were measured in 1990 francs; the 1998 data having been deflated using the national Consumer Price Index.

Table 1 provides summary statistics for the distribution of household income and expenditure pertaining to the two survey years. Of considerable importance is the apparently stagnant level of average consumption expenditure over the recession years, and the dramatic increase in inequality as measured by the coefficient of variation (the standard deviation divided by the mean). Average family consumption expenditure increased from 58280 to 58940 francs over the recession period; while the coefficient of variation rose by about 44% starting from a value of 0.45 and finishing at 0.65. Mean household income however increased by 10% over this period. Income inequality also increased: the coefficient of variation has recorded a more moderate (yet substantial) rise of 19%.

These distributional changes are indeed substantial. The stagnation in average expenditure in comparison to 1.25% annual growth in mean income certainly deserves some explanation. If the recession had not been anticipated, consumers would have had to cut down their expenditure levels in response to the news brought by the recession concerning their projected income growth. Alternatively, if consumers are prudent (more on this below), the cutting down of consumption may be an optimal response to the perceived increase in income risk during times of recession. The widening of expenditure inequality in comparison to income is not easily explained. One possible account however may have to do with a change in data quality across the two surveys. The 1990 survey was essentially a consumption expenditure survey; whereas in 1998 the income data were gathered in much more detail, distinguishing its many sources [7] (rents, private and public transfers, capital income etc.) The less dramatic recorded increase in income inequality may thus reflect a reduction in measurement error.

For a given sample, the three predictors discussed in section 4 can be used to construct deficit curves for the distribution of life-cycle incomes. It was shown in section 5 that any member of the class C_Y, together with $\widehat{\eta_Z} = \gamma' Z = \eta_Z^*$ will bracket the deficit curve of η. Furthermore, they also bracket the deficit curve $\Phi(s; \widehat{\eta_W})$, for predictors $\widehat{\eta_W}$ members of C_W, constructed as a weighted combination of $\widehat{\eta_Y}$ and $\widehat{\eta_Z}$. In table 2 we use the 1990 survey to construct the deficit curves pertaining to η_Y^* and η_Z^* and η_W^* [8]. These curves are plotted in figure 1, where we omit the top income decile in order to make the differences between these more visible. The top curve, $\Phi(s; \eta_Y^*)$ understates social welfare because

[7] This added emphasis on income is also reflected in the change in name of the survey, initially known as the *Enquête sur la Consommation des Ménages*, and rebaptised in 1998 as the *Enquête sur la Consommation et le Revenu des Ménages*. See Office Fédéral de la Statistique (1992,1999).

[8] See equations (20) and (21) as well as the appendix for further detail

measurement error in income and consumption expenditure inflate the level of inequality underlying η. The bottom curve (for η_Z^*) depicts a more egalitarian scenario than the amount of inequality underlying η, because it abstracts from the unobserved heterogeneity component ε pertaining to this variable.

The predictor η_W^* in the context of the 1990 data is a function $\eta_W^* = 0.50\eta_Y^* + 0.50\eta_Z^*$ (see the appendix). It generates the middle deficit curve in figure 1. While this is not identical to the deficit curve of η, by combining information contained in Y and Z variables, it certainly goes some way beyond the practice of using household expenditure to proxy life-cycle resources. To illustrate how household expenditure and the predictor η_W^* may convey different stories about the evolution of social welfare, we plot in figure 2 the 1990 and 1998 deficit curves for household expenditure, and in figure 3 the respective curves constructed from the predictor η_W^*. It may be noted in both cases that the deficit curves do not intersect for the bottom 9 income groups. The Anderson (1996) test of second order stochastic dominance, reported in the last column of each of tables 3 and 4, indicates that everywhere with the exception of the top group, these differences are statistically different from zero at the 5% significance level. The difference however between the two approaches is that when using consumption expenditure, the 1990 curve is initially below the 1998 curve, whereas the predictor η_W^* produces the reverse scenario.. Thus, if we restrict our focus to the bottom groups, we would be led to conclude on the basis of the documented changes in the distribution of household expenditure that poverty was higher in 1998. However, we would reverse our conclusions when working with the predictor η_W^*.

Clearly, one difference between the two approaches in the present application is that household expenditure is not adjusted for family size, whereas in the case of η_W^* (and all members of the class C_W) the Y variables are translated about the demographic controls $D\delta$ [9]. The main difference however between the two approaches is that η_W^* also uses infirmation on income, alongside household expenditure and Z variables. As discussed above, average income has grown over the recession period whereas average consumption has remained more or less constant. The 10% increase in average income over this eight-year period is certainly one reason why η_W^* depicts a more favorable state of affairs (at least up to the ninth group) for 1998 in comparison to 1990. Changes in the relative weights allocated by η_W^* to Y and Z variables, giving more emphasis to household income (again, see the appendix) is also part of the explanation for this opposite finding.

[9] Silber (1998) uses the Gini index to compare inequality of (1) per capita household expenditures between individuals, (2) standardized consumption (using equivalence scales) over equivalent adults and (3) household expenditures unadjusted for family size between households, in the Swiss context. He finds that inequality is highest under (1) and lowest under (2). Inequality is intermediate under (3), the standard used in table 3 of our paper,

7. Conclusions

Because families allocate their consumption intertemporally over time horizons typically exceeding those covered by cross-sectional expenditure surveys, the distribution of household expenditure (or income) fails to provide an accurate account of the distribution of long-run welfare. Unfortunately unobservable in cross-section data, life-cycle income is the pertinent variable for this purpose. Homothetic preferences generate expenditure functions proportional to life-cycle income. On such basis it may be tempting to argue that expenditure data convey all the required information about life-cycle incomes. The same argument could be made in the context of linear in life-cycle income expenditure functions, generated by quasi-homothetic preference structures.

Such claims however run into a practical problem, namely that household expenditure is unlikely to be measured free of error. This is all the more true when keeping in mind that often families are asked to keep a diary of their spending over a two to four weeks period, and that these data are then converted to annual equivalents. In earlier contributions to the literature, it was suggested to jointly model household income and expenditure, from which it was shown possible to consistently estimate the variance of long-run income. In the present paper we have taken a fresh look at this problem in the light of recent advances pertaining to the ordinal analysis of income distributions. The purpose of our paper was to show that this same framework proposed earlier could provide bounds for the deficit curve of the distribution of life-cycle incomes. An upper bound to this deficit curve was obtained by constructing predictors of life-cycle income using household income and expenditure chosen from a suitably defined class of predictors. The lower bound was constructed from a predictor of life-cycle income using determinants of this variable such as the educational attainment of the family head. Finally, a weighted sum of these two predictors was used to construct a third deficit curve, also contained within these bounds.

The framework we have proposed also finds applications in the context of the ordinal analysis of poverty and inequality pertaining to life-cycle incomes. That our methodology is applicable in the context of poverty was implicit from our discussion in section 6. To see that this is the case, note that a wide range of poverty measures may be expressed as social welfare indices applied to a restricted range of the distribution of the appropriate income standard. As such then, by defining a poverty line (or alternatively, a range of such thresholds) we may conduct an ordinal analysis of poverty pertaining to life-cycle incomes by inspecting the various deficit curves over the relevant range of the distribution of resources.

The relation between social welfare and inequality indices has been studied by Dagum (1990) among others. From an ordinal perspective, Shorrocks shows that the generalized Lorenz curve, a re-scaling of the traditional Lorenz curve, can be used to investigate welfare orderings when the distributions of interest have different means. The ordinal analysis of inequality may thus be seen in this perspective as a specific case of the problem studied by Shorrocks, in which incomes are normalized by the mean of the relevant distribution. In related work, Abul Naga (2001), we therefore exploit this result in order to construct Lorenz curves for life-cycle incomes using the same predictors proposed here.

Though the purpose of this paper was to investigate the possibility of undertaking ordinal analyses for life-cycle incomes, rather than to model household consumption, it is clear that relaxing many of the assumptions underlying our empirical framework could add realism to our setup. Here we discuss three limitations of our framework, but a more complete account of the empirical challenges confronting the general life-cycle framework can be found in Deaton(1997, ch. 6) and Browning and Crossley (2001).

An important identifying assumption underlying our empirical framework is that household income and consumption only correlate via their joint dependence on life-cycle income. This means that measurement errors are not allowed to correlate. If measurement error captures the effect of uncertainty about the future, this zero correlation assumption means that we are ruling out that consumption may be driven in part by transitory changes in income, as documented by Hall and Mishkin (1982). At another extreme, households on low income may be cash-constrained so that their consumption expenditure at a given point in time is equal to their money income (on this, see Deaton, 1997 pp. 363-372). Credit constraints are thus done away with, and this is a second important limitation of our framework.

It is also quite plausible to assume that the marginal utility of consumption increases at low expenditure levels. The convexity of the marginal utility function in presence of uncertainty gives rise to a precautionary motive for saving, which is absent from our framework. As mentioned in section 2, the introduction of labor income uncertainty in our framework poses no major problems, provided the consumption function remains linear in (expected) life-time resources. However, the linearity of the marginal utility of consumption excludes precautionary behavior. In absence of credit constraints, allowing for some form of prudence on behalf of households could certainly add flexibility to our framework.

Panel data providing repeated measurements on family income and expenditure are increasingly becoming available. Future research which addresses the question dealt with in this paper in a panel data environment may help to deal with some of the limitations of our framework. It is certainly the case that more data on income and expenditure at the household level enable us to construct more precise predictors of life-cycle income. Note however that with longitudinal data it may be possible to add more structure to the error vector U. For instance, we may allow consumption to depend on an transitory income, which may enter separately from measurement error per se (on this, see Bhalla, 1979 who works with Indian panel data). Alternatively, with repeated measurements on a system of two equations, under reasonable identifying assumptions we could add in the expenditure equation another time-invariant component alongside ε, taken to capture an element of prudence in consumption behavior. By allowing the researcher to control more adequately for time-invariant unobserved heterogeneity, panel data joint modelling of income and expenditure would certainly constitute an important step forward.

8. Appendix

Separate models were estimated for the 1990 and 1998 waves of the Swiss FES, in order to construct the predictors η_Y^*, η_Z^* and η_W^* defined in section 4. The same variables were used for the two years. We have thus made the definitions of variables as close as possible in the two years. Some differences were easy to deal with. For instance, the 1990 survey contained data on the number of children under age 10, whereas in the 1998 survey this same variable pertained to all children, without any age limit. It was often possible however to go back to the individual level data in order to construct comparable variables. The fact however that there existed no data on the education level of household heads in retirement, in the 1990 survey, meant that we have excluded this socio-economic group from the two samples used in this application. We have also dropped the remaining few observations on families with a head above the age of 65 engaging in paid employment.

The set of Z variables used in the estimations included the following characteristics of the household head: educational attainment, sex, logarithm of age and a dummy for Swiss citizenship (see table A1). We also had a dummy *GER AREA* for residence in the German speaking part of Switzerland. The orthogonality test for the Z variables, SARG TEST, is distributed as a χ^2 variate (with 4 degrees of freedom in the present context). The test takes a value of 9.36 for the 1990 data and 7.23 for the 1998 data. Given the critical value of 9.49 at the 5% level, we may take these variables as being plausible instruments. D variables used as controls included household size, the number of children under the age of 10 and dummies for marital status and home ownership. We also added dummies for independent employment, employment in agriculture, and part-time work. These same D variables were used for both equations; i.e. $D_{i1} = D_{i2} = \bar{D}_i$.

The predictor η_Y^* is then a weighted sum $b_1'(y_{i1} - \delta_1'\bar{D}_i) + b_2'(y_{i2} - \delta_2'\bar{D}_i)$. The weights b_1 and b_2 (the line entries *INC* and *CONS* and column entry *YPRED*) are given in table A1. Thus, for 1990 η_Y^* is constructed as

$$\eta_Y^* = 0.50 \cdot (y_{i1} - \delta_1'\bar{D}_i) + 0.59 \cdot (y_{i2} - \delta_2'\bar{D}_i)$$

while for the 1998 data the formula is

$$\eta_Y^* = 0.61 \cdot (y_{i1} - \delta_1'\bar{D}_i) + 0.52 \cdot (y_{i2} - \delta_2'\bar{D}_i)$$

The coefficients γ on the Z variables for the predictor η_Z^* are given in the next column of the table. Finally, for $\eta_W^* = \tau^o\eta_Y^* + (1 - \tau^o)\eta_Z^*$, the value of τ^o is given under the line entry τ and column entry *WPRED*. Accordingly, we construct η_W^* for the 1990 data as

$$\eta_W^* = 0.50 \cdot \eta_Y^* + 0.50 \cdot \eta_Z^*$$

and for the 1998 survey as

$$\eta_W^* = 0.69 \cdot \eta_Y^* + 0.31 \cdot \eta_Z^*$$

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TABLE 1: Summary Statistics

	CONS90	CONS98	INC90	INC98
mean	58.28	58.94	61.02	67.53
std. dev.	26.24	38.32	31.77	42.06
coef. var.	0.45	0.65	0.52	0.62

Notes: *n*=1654 for the 1990 survey ; *n*=7948 for the 1998 survey. *CONS* denotes household annual consumption expenditure and *INC* is household income. Resources are measured in 1990 Swiss Francs X 10^3

TABLE 2: The Three Deficit Curves for 1990

S	ZPRED	WPRED	YPRED
26.91	0.00	1.41	2.69
34.31	0.20	2.81	4.72
38.06	0.61	4.00	6.15
40.60	1.24	5.04	7.33
42.97	2.16	6.14	8.51
46.01	3.80	7.77	10.19
52.10	7.79	11.61	13.99
57.96	12.38	16.02	18.20
66.19	19.79	22.85	24.74
281.16	223.62	223.62	223.62

Notes: n=1654, resources are measured in 1990 Swiss Francs X 10^3 . ZPRED is the predictor $\boldsymbol{h}^*_Z$, WPRED is the predictor $\boldsymbol{h}^*_W$ and YPRED is the predictor $\boldsymbol{h}^*_Y$

TABLE 3: Consumption Deficit Curves

S	DEF_{90}	DEF_{98}	TEST
28.29	1.08	1.52	-3.73
35.47	1.88	2.55	-3.91
41.28	3.13	4.04	-4.14
46.63	4.81	5.95	-4.26
52.21	7.15	8.50	-4.21
58.05	10.25	11.74	-3.98
65.30	14.89	16.46	-3.60
75.42	22.50	24.05	-3.02
92.70	37.36	38.70	-2.19
1210.80	1105.40	1099.60	**1.19**

Note: *TEST* denotes Anderson's test of second order stochastic dominance.

TABLE 4: WPRED (h^{*}_{w}) Deficit Curves

S	DEF_{90}	DEF_{98}	TEST
28.84	2.00	1.33	5.73
32.84	2.75	1.89	5.75
36.00	3.63	2.67	5.52
39.33	4.84	3.82	5.02
42.78	6.40	5.37	4.42
46.81	8.60	7.59	3.78
51.80	11.83	10.83	3.25
58.26	16.68	15.68	2.82
68.77	25.55	24.63	2.24
344.56	286.25	286.88	**-0.46**

Note: *TEST* denotes Anderson's test of second order stochastic dominance.

TABLE A1: Prediction of unobserved life-cycle income

	1990 data			1998 data		
	YPRED	*ZPRED*	*WPRED*	*YPRED*	*ZPRED*	*WPRED*
INC	0.50			0.61		
CONS	0. 59			0.52		
EDU		5.70			6.41	
SWISS		-2.99			4.07	
GER AREA		0.96			-0.47	
SEX		3.94			0.03	
ln-AGE		9.46			11.43	
t^o			0.50			0.69

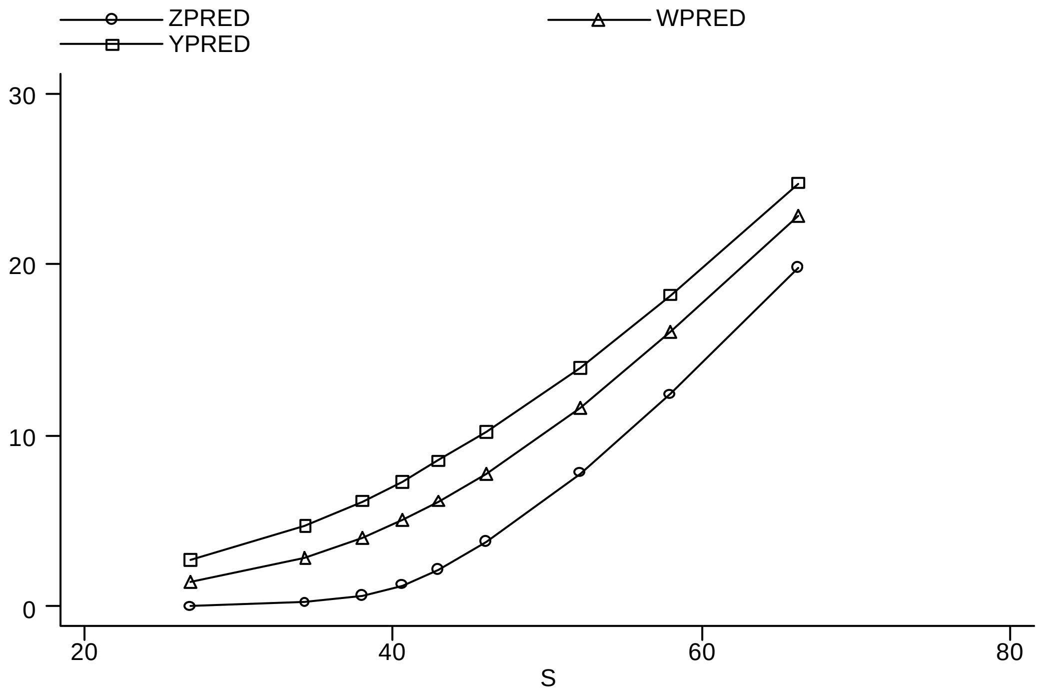

Figure 1 : The Three Deficit Curves for 1990

Notes: Resources are measured in 1990 francs X 10^3. The last observation has been deleted from the graph (in order to make differences more visible).

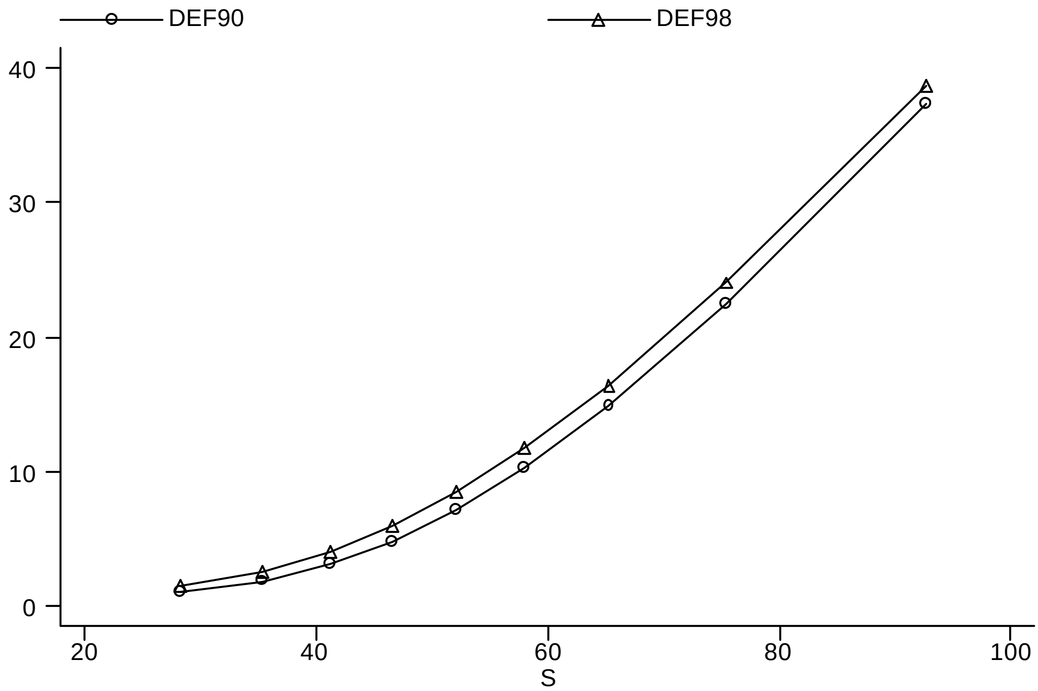

Figure 2 : Consumption Deficit Curves

Notes: Resources are measured in 1990 francs X 10^3. The last observation has been deleted from the graph (in order to make differences more visible).

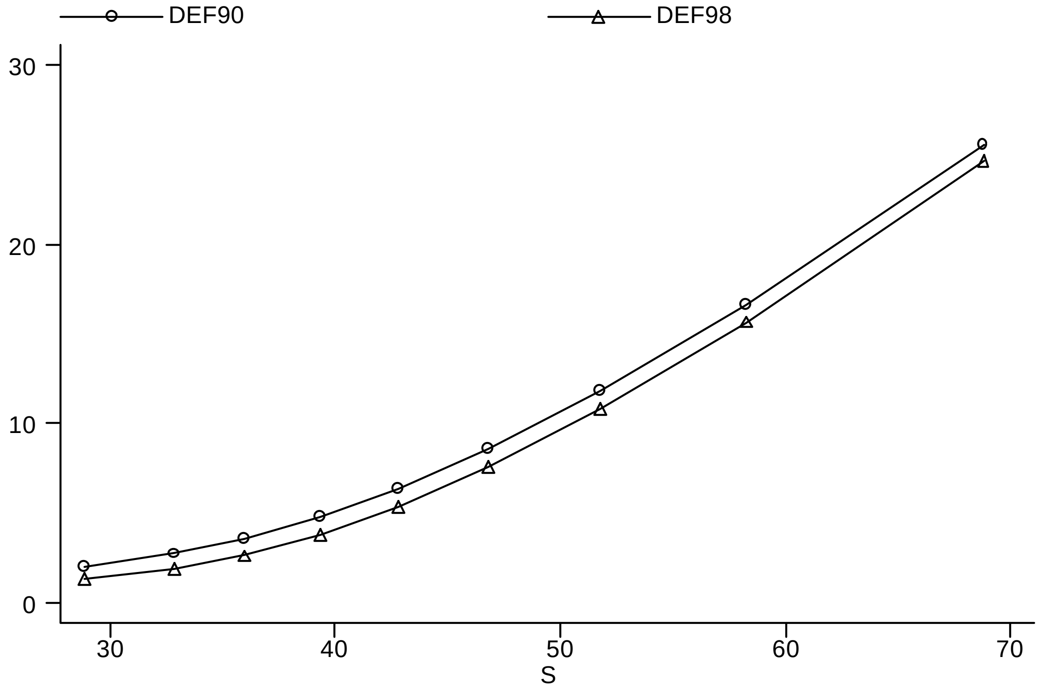

Figure 3 : WPRED Deficit Curves

Notes: Resources are measured in 1990 francs X 10^3. The last observation has been deleted from the graph (in order to make differences more visible).