When are Retail Stores Preferable to Auctions?

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Abstract: Although auctions have many desirable properties, they have two undesirable features from buyers' perspective: Auctions impose waiting costs on buyers, which leads to "false trading." Sometimes, buyers pass up other valuable opportunities while waiting for the results of the auction. Other times, buyers make undesired duplicate purchases. As a result, the seller will prefer running a retail store, where the seller commits to sell at a given price, to running an auction. We show that stores are optimal if the good is perishable and/or becomes obsolete quickly. Stores are also preferred when the market is thin and when alternatives for the good being sold are easy to find. Auctions are preferred when the good is storable, when it does not become obsolete too quickly, when the market is thick and when no substitutes are available for the good being sold. These predictions are consistent with a number of observed phenomena.

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When are Retail Stores Preferable to Auctions?

It is commonly alleged that an auction should be run whenever the seller wants to maximize the proceeds from selling a good that potential buyers value differently. In practice, however, most goods are actually sold using store structures, in which the seller simply posts a price and the first buyer that agrees to pay that price gets the good. How can this dominance of stores be explained?

Although advantageous at first sight, auction schemes have two undesirable features that restrict their use in many practical situations. The first is the *waiting cost* they impose on bidders: in order to command a price premium above the reservation price, any form of auction requires several bids to accumulate before it is actually run. As a result, there is usually a certain waiting period between the time an auction is announced (when the good would actually be available for sale) and the time the winner is determined (when the good becomes available to the buyer).

The second problem with auctions is related to the *uncertainty* they impose on bidders. To see this, consider a buyer seeking to purchase a bottle of milk for his next breakfast that bids in a milk auction. Until the auction is closed, he does not know whether he made the winning bid or not. If he sees another opportunity to purchase milk, let it be a store or an auction, he faces the following dilemma. If he does not purchase the milk there, he runs a significant risk of not having milk the next morning. On the other hand, if he does purchase milk in the store or chooses to bid on the second auction, he takes the risk of ending up with two bottles of milk. Thus, auctions can potentially give rise to a significant amount of false trading: some buyers that would agree to pay more than the market clearing price do not get the item, whereas buyers that wouldn't agree to pay as much as the clearing price get it because they are locked in by the auction scheme. This risk of false trading has important consequences for the way auctions are designed. Since bidders will not wish to submit bids too long before the auction closes, sellers will typically choose to run auctions that close at a pre-specified *time* and not after a certain *number of bids* has been received.

Intuitively, one would expect waiting costs and the risk of false trading to make the auction a less appealing selling institution than the store. This paper presents a formal model confirming this intuition. We model bidders' waiting costs and the risk of false trading by assuming that bidders having to wait for the auction to close will engage in a search process to purchase the good elsewhere. When such a purchase opportunity is found, a bidder that was originally planning to participate in the auction drops out. With less bidders participating in the auction, the seller's expected revenue falls. As a result, the costs imposed on bidders by the auction get *internalized* by the seller and he is more likely to sell the good using a store structure rather than an auction.

Our results show that the optimal choice of the store as opposed to the auction structure is driven by three factors: (1) the extent to which the good being sold is perishable and/or the seller has a strong time preference, (2) the thickness of the market, and (3) the size of bidders' waiting costs and the extent to which alternatives to purchase the good elsewhere are available. More specifically, the following results can be established:

1. If the good is perishable, becomes obsolete quickly and/or the seller has a strong time preference, then the store is the preferred structure. This effect arises because perishability and discounting make it more costly for the seller to wait for additional bidders before closing the auction. As a result, the optimal auction has a small expected number of bidders, and the store may dominate.

2. If the market is thick in the sense that the rate of bidder arrivals is high, the auction is the preferred structure. This effect arises because when the rate of bidder arrivals rises, the gain in expected selling price per unit time the seller waits before closing the auction is increased, making it worth to wait for a large number of bidders.

3. If bidders' waiting costs are high and/or alternatives for the good are easy to find, then the store is the preferred structure. This effect arises because if bidders defect quickly, the auction is unable to achieve the level of bidder participation required to make the expected selling price rise significantly above the reservation price.

Our results therefore demonstrate that the thickness of a market can be a critical factor for the choice between store and auction. Furthermore, they provide a rationale for the conventional wisdom that goods that are "unique" (in the sense of having no close substitutes) and durable (not perishable) should be sold through auctions, while perishable goods or goods with close substitutes for which finding an alternative is easy should be sold through a store structure.

Although a number of papers deal with the optimal design of selling institutions, none has considered the issues addressed in this paper. Most of the literature is concerned with the optimal design of stores *or* auctions considered in isolation, and not with the optimal choice *between* these two structures. The literature on stores has mainly analyzed the question of how an optimal price or a price-quantity schedule should be set. For example, Spence (1977) develops an optimal quantity discount scheme for a monopolist facing customers with different valuations for different quantities, while Lazear (1986) analyzes the optimal time path of prices when the seller faces uncertainty about the market for the item being sold. On the other hand, starting with the pioneering work of Vickrey (1961), the auction design literature has focused on the choice among different auctioning schemes (Myerson (1981), Riley and Samuelson (1981), Bulow and Roberts (1989)).

Of the few papers comparing stores and auctions, none has considered bidders' waiting costs and the problems associated with false trading as explanations for the dominance of stores. For instance, Wang (1993) considers the optimal choice between store and auction from the perspective of the explicit costs of running these two institutions. He shows that the advantage of running auctions periodically instead of selling goods using a store arises from the fact that the storage costs incurred by the auction are typically lower than the displaying costs incurred by a store. He demonstrates that auctions tend to dominate stores when the dispersion of buyers' valuations is high. The intuition for this result is that an auction allows the seller to extract the second-order statistic of bidders' valuations, whereas the store does not. This feature is more valuable, the more dispersed bidders' valuations are. However, since his analysis completely ignores the waiting costs auctions impose on bidders, his results tend to be biased in favor of the auction.

De Vany (1987) compares stores, time-based auctions (which close after a fixed time has elapsed) and thickness-based auctions (which close once a certain number of bids has been received) from the perspective of transactions costs. He shows that the higher expected selling price in the auction structures can be offset by their higher transactions costs, which, in his model, are captured by the seller's holding cost and the buyers' inspection and waiting cost. As a result of these higher transactions costs, the store typically dominates the two auctioning schemes when the prior distribution of buyer valuations is not too diffuse. When dispersion is high, however, the auction tends to be the preferred structure. Although De Vany computes bidders' waiting cost explicitly in his analysis, he does not allow bidders facing high waiting costs to search for alternatives and defect from the auction. As a result, his results are also biased towards the auction.

The remainder of the paper is organized as follows. Section 1 describes the store. Section 2 considers the properties of the optimal auction. Section 3 compares the two selling institutions and derives conditions under which each of those forms dominates. Section 4 concludes.

1 The Store

Consider a risk-neutral seller trying to sell a single unit of an indivisible good using a store. He announces a listing price p for the good; the first customer that agrees to pay the listed price gets the good.

Potential customers arrive according to a Poisson process with intensity λ, implying that the customer interarrival time is exponentially distributed with parameter λ. Each potential buyer i has a valuation v_i for the good, which is random, independent of other buyers' valuations and has distribution function F and density f with support $[\underline{v}, \overline{v}]$. The customer buys the good if $v_i > p$. If his valuation is below v_i, the good remains available for sale to another customer.

Let $\alpha > 0$ denote the discount rate for this problem, which can be interpreted either as the seller's time preference, as the extent to which the item being sold is perishable, or as the speed with which it becomes obsolete.[1] Let $0 \leq R < \overline{v}$ denote the seller's reservation utility if the good goes unsold. In the spirit of dynamic programming, R can be viewed as summarizing all the future of the problem (except for the effect of discounting, which is contained in $e^{-\alpha t}$); it could represent either the utility the seller derives from consuming the good himself, or the expected revenue from a subsequent sale.

The store owner's problem is to set a price schedule $\{p_t\}$ that maximizes the expected revenue from the sale, Π_S. Given p_t and the distribution of buyers' valuations, for any customer walking into the store at time t, the good will be sold with probability $1 - F(p_t)$. Thus, the store's expected revenue is $p_t(1 - F(p_t))$. With probability $F(p_t)$, the good will not be sold, and the store owner receives the reservation utility R. Formally, the store owner's problem is to

$$\max_{\{p_t\}} \Pi_S = \int_0^\infty e^{-\alpha t} \lambda e^{-\lambda t} \left(p_t(1 - F(p_t)) + F(p_t)R\right) dt \tag{1}$$

where $\lambda e^{-\lambda t}$ is the density of interarrival times and $e^{-\alpha t}$ captures the effect of perishability and discounting. With R given, one can do no better than maximizing $p_t(1-F(p_t)) + F(p_t)R$ time by time, yielding the first-order condition

$$1 - F(p_t) - p_t f(p_t) + f(p_t)R = 0 \tag{2}$$

[1] In the first interpretation, all amounts received by the seller are simply discounted at a rate α. In the second and third interpretation, bidders' valuations for the good would be decaying exponentially through time, with a time-dependent distribution $F(v, t)$ rescaled so as to lie on the support $[e^{-\alpha t}\underline{v}, e^{-\alpha t}\overline{v}]$ (the same would apply to the reservation utility R). Both of these cases can be handled with the formulation used below and yield similar conclusions.

which immediately implies that p_t is a constant independent of time, p^*, satisfying the usual hazard rate formula,

$$p^* = R + \frac{1 - F(p^*)}{f(p^*)} \tag{3}$$

Using this solution, the optimal Π_S is defined implicitly as

$$\Pi_S = \int_0^\infty \lambda e^{-(\alpha+\lambda)t} \left(p^*(1 - F(p^*)) + F(p^*)R\right) dt \tag{4}$$

Integrating,

$$\Pi_S = \frac{\lambda}{\alpha + \lambda} \left(p^*(1 - F(p^*)) + F(p^*)R\right) \tag{5}$$

Note that since $p^* = R + \frac{1-F(p^*)}{f(p^*)} > R$, $p^*(1 - F(p^*)) + F(p^*)R > R$ and therefore $\Pi_S > \frac{\lambda}{\alpha+\lambda}R$. As a result, there exists some $\alpha^* > 0$ such that the seller derives more than his reservation utility from selling the good whenever $\alpha < \alpha^*$. This result, which will be important below, is summarized in the following proposition:

Proposition 1: Let $p^* = R + \frac{1-F(p^*)}{f(p^*)}$. Then, for all $\alpha < \alpha^* = \lambda \frac{(p^*-R)(1-F(p^*))}{R}$, the seller's expected utility from the sale, Π_S, is strictly greater than his reservation utility R.

Proof: Immediate by solving the condition $\Pi_S = \frac{\lambda}{\alpha+\lambda} \left(p^*(1 - F(p^*)) + F(p^*)R\right) > R$ for α.

Proposition 1 has the obvious implication that if the discount rate is very high (either because the seller has a very high time preference, the good is very perishable or becomes obsolete quickly), then the seller will not even attempt to sell the good and prefer to consume it himself (trivially, attempting a sale is always optimal if $R = 0$).

2 The Auction

2.1 The Model

Suppose now that the seller decides to sell the good using an auction. He announces a reservation price p and the auction's closing time T. Bidders that arrive before T are allowed to submit bids for the good. At time T, the highest bidder gets the good at a price equal to the second-highest bid or the reservation price p, whichever is greater.[2]

[2]Given the equivalence theorem for independent private value auctions (Myerson (1981), Riley and Samuelson (1981)), this assumption is innocuous for the optimality of the auction as opposed to the store in the setting considered here, but simplifies the exposition as bidders bid their true valuation.

In order to be able to compare store and auction, the same arrival process and the same valuation distribution as in the case of the store are assumed. However, one cannot simply assume that all the bidders that show up end up actually bidding in the auction. Because of the auction's distinctive property of closing only at time T, bidders that arrive before T must wait before they know whether they get the good. As a consequence of the direct cost of waiting and the potential cost of false trading, bidders do not submit their bids as soon as they arrive. Rather, they search for opportunities to buy the good elsewhere and come back to bid in the auction only if they have found no suitable alternative by time T.

Suppose that alternative purchase opportunities are idiosyncratic to each bidder and arrive exponentially at rate μ for each individual bidder engaged in the search process. Assuming such a distribution for the time until a bidder finds an outside option can be considered as the limiting case of a situation in which bidders visit a certain number of places per unit time until the auction closes and there is a certain probability that they find a suitable good in each of these places (note that the good need not be strictly identical to the one being auctioned, as some bidders will choose to purchase an inferior alternative because they do not want to wait).

In practical applications, the value of μ will depend positively on two factors, each having its own economic interpretation: on how important immediacy of the purchase is to bidders (with high direct waiting costs leading them to search for alternatives more intensively or to decide to purchase goods that are inferior to the one being auctioned), and on the extent to which substitutes for the good being auctioned exist (which would typically be high for commodity products and low for original artwork). Note that if many substitutes exist, then the probability that bidders find an alternative by time T is high and so would be the risk of false trading if bidders chose to submit their bids as soon as they arrive (i.e., before searching for alternatives).

The number of bidders that participate in the auction at time T, N, equals the number of bidder arrivals minus the number of bidders that have found an alternative by time T. Thus, as shown in the Appendix, N follows a Poisson distribution with parameter $\tilde{\lambda}(T) = \frac{\lambda}{\mu}(1 - \exp(-\mu T))$, and the probability of having N bidders at time T is given by

$$q_N(T) = e^{-\tilde{\lambda}(T)} \frac{\tilde{\lambda}(T)^N}{N!} \tag{6}$$

To gain some intuition for this result, recall that the expected number of bidders in the auction at time T is given by $\tilde{\lambda}(T)$. We have

$$\frac{\partial \tilde{\lambda}(T)}{\partial T} = \lambda e^{-\mu T} > 0 \tag{7}$$

and

$$\frac{\partial^2 \tilde{\lambda}(T)}{\partial T^2} = -\lambda \mu e^{-\mu T} < 0 \tag{8}$$

Therefore, as the seller waits longer before closing the auction, the expected number of bidders does increase, but at a decreasing rate because some of the bidders find an alternative and drop out from the auction (note that for all $T > 0$, $\frac{\partial \tilde{\lambda}(T)}{\partial \mu} \frac{\lambda}{\mu^2} e^{-\mu T} \left(1 + \mu T - e^{\mu T}\right) < 0$). As will be shown shortly, it is this phenomenon which potentially makes the auction inferior to the store as a selling institution and explains the dominance of store structures in practice. Note that if bidders do not have outside options ($\mu \to 0$), $\tilde{\lambda}(T) \to \lambda T$, the special case analyzed by Wang (1993) in which the auction dominates the store in most cases.

The seller's utility from auctioning the good at time T, $\Pi_A(T)$, is equal to the present value of the expectation over N of the revenue from an auction with N bidders, π_N:

$$\Pi_A(T) = e^{-\alpha T} E(\pi_N) \tag{9}$$

where α again denotes the discount rate. The seller's problem is to select an auctioning time T and a reservation price p so as to maximize this expression.

In order to determine the value of π_N, let R again denote the reservation utility derived by the seller if the good goes unsold at time T. In the spirit of dynamic programming, the value of R can be considered as given and equal to that for the store, as a seller unable to sell his good in either structure – store or auction – will subsequently use the optimal selling strategy. At time T, a sale will occur if the highest bid is above the reservation price, p. If N bids are received at time T, then the probability of a sale equals $1 - F^N(p)$. Conditional on a sale occurring, the revenue from the auction equals the second-highest-order statistic of bidders' valuation if it lies above the reservation price p, and the reservation price if it does not. Using the density function of the second-highest-order statistic, $N(N-1)(1-F)F^{N-2}f$, the probability that the highest-order statistic lies above the reservation price but the second-highest does not is given by[3]

$$\begin{aligned}
\phi(p) &= (1 - F^N(p)) - \int_p^{\overline{v}} N(N-1)(1-F(y))F^{N-2}(y) f(y) dy \\
&= (1 - F^N(p)) - (1 - F^N(p) - NF^{N-1}(p)(1 - F(p))) \\
&= NF^{N-1}(p)(1 - F(p))
\end{aligned} \tag{10}$$

[3] This can also be seen by considering the joint density of the highest-order statistic x_1 and the second-highest-order statistic x_2, $N(N-1)F^{N-2}(x_2)f(x_2)f(x_1)$. Then, $\text{Prob}(x_1 > p, x_2 < p) = \int_p^{\overline{v}} \left(\int_0^p N(N-1)F^{N-2}(x_2)f(x_2)dx_2\right) f(x_1)dx_1 = \int_p^{\overline{v}} NF^{N-1}(p)f(x_1)dx_1 = NF^{N-1}(p)(1-F(p))$.

With probability $F^N(p)$, the good goes unsold, and the seller receives the reservation utility R. Therefore, the expected revenue from the auction conditional on receiving N bids is given by

$$\pi_N = \int_p^{\overline{v}} yN(N-1)(1-F(y))F^{N-2}(y)f(y)dy + pNF^{N-1}(p)(1-F(p)) + F^N(p)R \quad (11)$$

Using the fact that $NF^{N-1}(p)(1-F(p)) = -\int_p^{\overline{v}} NF^{N-1}(y)(1-F(y)-yf(y))dy - \int_p^{\infty} yN(N-1)F^{N-2}(y)(1-F(y))f(y)dy$ and $F^N(p) = 1 - \int_p^{\overline{v}} NF^{N-1}(y)f(y)dy$, this expression can be rewritten as

$$\begin{aligned}\pi_N &= \int_p^{\overline{v}} NF^{N-1}(y)\left(yf(y) - (1-F(y))\right)dy + F^N(p)R \\ &= R + \int_p^{\overline{v}} NF^{N-1}(y)(J(y)-R)f(y)dy \qquad (12)\end{aligned}$$

where $J(y) \equiv y - \frac{1-F(y)}{f(y)}$ denotes the Bulow and Roberts (1989) marginal revenue function. Equation (12) is the standard result that the expected utility from an auction is equal to the expectation of the maximum of the highest bidder's marginal revenue and reservation utility (see Bulow and Klemperer (1996)). Note that because it is the conditional expectation of an order statistic, π_N is an increasing and concave function of N.

Taking the first-order condition on (12), the optimal reservation price p^* solves $J(p^*) = R$, or, using the definition of J,

$$p^* = R + \frac{1-F(p^*)}{f(p^*)} \quad (13)$$

Note that consistent with the Riley and Samuelson (1981) result, the optimal reservation price p^* does not depend on the number of bidders participating in the auction and is identical to the store's posted price.

Let $\pi(\tilde{\lambda}) = E(\pi_N)$. Since $\tilde{\lambda} = \tilde{\lambda}(T)$, the seller's expected utility from an auction taking place at time T can be written as

$$\Pi_A(T) = e^{-\alpha T}\pi(\tilde{\lambda}(T)) = e^{-\alpha T}\sum_{N=0}^{\infty} e^{-\tilde{\lambda}(T)}\frac{\tilde{\lambda}(T)^N}{N!}\pi_N \quad (14)$$

Using the expression for π_N, one can rewrite the above in an alternate form convenient to derive some of the results below, namely

$$\begin{aligned}\Pi_A(T) &= e^{-\alpha T}\sum_{N=0}^{\infty} e^{-\tilde{\lambda}(T)}\frac{\tilde{\lambda}(T)^N}{N!}\left(\int_p^{\overline{v}} NF^{N-1}(y)(J(y)-R)f(y)dy + R\right) \\ &= e^{-\alpha T}\left(R + \tilde{\lambda}(T)\int_p^{\overline{v}} e^{-\tilde{\lambda}(T)(1-F(y))}(J(y)-R)f(y)dy\right) \qquad (15)\end{aligned}$$

2.2 Properties of the Optimal Auction

Using expressions (14) and (15), the properties of the optimal auction can now be determined. To start with, one can establish that the properties of π_N, which is an increasing and concave function of N, carry over to π as a function of $\tilde{\lambda}$.

Proposition 2: $\pi(\tilde{\lambda}) = E(\pi_N)$ is increasing and concave in $\tilde{\lambda}$.

Proof: We first establish that π is increasing in $\tilde{\lambda}$. Taking the first derivative of π with respect to $\tilde{\lambda}$ yields

$$\frac{\partial \pi}{\partial \tilde{\lambda}} = e^{-\tilde{\lambda}}\left(\sum_{N=1}^{\infty}\pi_N\frac{\tilde{\lambda}^{N-1}}{(N-1)!} - \sum_{N=0}^{\infty}\pi_N\frac{\tilde{\lambda}^N}{N!}\right) = e^{-\tilde{\lambda}}\left(\sum_{N=1}^{\infty}\pi_N\left(\frac{\tilde{\lambda}^{N-1}}{(N-1)!} - \frac{\tilde{\lambda}^N}{N!}\right) - \pi_0\right) \qquad (16)$$

Grouping terms of like powers of $\tilde{\lambda}$ then yields

$$\frac{\partial \pi}{\partial \tilde{\lambda}} = e^{-\tilde{\lambda}}\sum_{N=0}^{\infty}\frac{\tilde{\lambda}^N}{N!}\left(\pi_{N+1} - \pi_N\right) > 0 \qquad (17)$$

To establish concavity, differentiate π one more time to obtain

$$\begin{aligned}\frac{\partial^2 \pi}{\partial \tilde{\lambda}^2} &= e^{-\tilde{\lambda}}\left(\left(\left(\sum_{N=2}^{\infty}\left(\frac{\tilde{\lambda}^{N-2}}{(N-2)!} - \frac{\tilde{\lambda}^{N-1}}{(N-1)!}\right) - \pi_1\right) - \left(\sum_{N=1}^{\infty}\pi_N\left(\frac{\tilde{\lambda}^{N-1}}{(N-1)!} - \frac{\tilde{\lambda}^N}{N!}\right) - \pi_0\right)\right)\right. \\ &= e^{-\tilde{\lambda}}\left(\pi_0 + (\tilde{\lambda}-2)\pi_1 + \sum_{N=2}^{\infty}\pi_N\left(\frac{\tilde{\lambda}^{N-2}}{(N-2)!} - 2\frac{\tilde{\lambda}^{N-1}}{(N-1)!} + \frac{\tilde{\lambda}^N}{N!}\right)\right) \qquad (18)\end{aligned}$$

Grouping terms of like powers of $\tilde{\lambda}$ then yields

$$\frac{\partial^2 \pi}{\partial \tilde{\lambda}^2} = e^{-\tilde{\lambda}}\sum_{N=0}^{\infty}\frac{\tilde{\lambda}^N}{N!}\left(\pi_{N+2} - 2\pi_{N+1} + \pi_N\right) < 0 \qquad (19)$$

Since $\tilde{\lambda}(T)$ is strictly increasing and concave in T, proposition 2 implies that the expected (undiscounted) revenue from the auction will be strictly increasing and concave in the auction's closing time T as well: $\frac{\partial \pi}{\partial T} = \frac{\partial \pi}{\partial \tilde{\lambda}} \frac{\partial \tilde{\lambda}}{\partial T} > 0$, $\frac{\partial^2 \pi}{\partial T^2} = \frac{\partial^2 \pi}{\partial \tilde{\lambda}^2} \left(\frac{\partial \tilde{\lambda}}{\partial T} \right)^2 + \frac{\partial \pi}{\partial \tilde{\lambda}} \frac{\partial^2 \tilde{\lambda}}{\partial T^2} < 0$. This property is caused by the nature of π_N as the conditional expectation of an order statistic.

Therefore, choosing the auction's optimal closing time T involves a tradeoff for the seller: on the one hand, a higher T increases the expected number of bidders participating in the auction, $\tilde{\lambda}(T)$ and therefore the expected revenue from the sale π. On the other hand, as the closing time T is increased, discounting reduces the present value of the proceeds. The optimal closing time T^* is the one that balances these two effects.

As in the case of the store, if the discount rate is very high, the seller may even decide to consume the good on his own rather than trying to auction it. More specifically, one can establish the following result:

Proposition 3: If the discount rate is sufficiently high, so that $\alpha \geq \lambda \frac{\int_p^{\overline{v}} (J(y)-R)f(y)dy}{R}$, then the seller prefers not to auction the good.

Proof: To establish this result, it suffices to show that if the above condition is met, $\partial \Pi_A / \partial T \leq 0$ at $T = 0$ and is nonincreasing thereafter. Using (14), one has

$$\frac{\partial \Pi_A}{\partial T} = e^{-\alpha T} \frac{\partial \pi}{\partial \tilde{\lambda}} \frac{\partial \tilde{\lambda}}{\partial T} - \alpha e^{-\alpha T} \pi = e^{-\alpha T} \left(\frac{\partial \pi}{\partial \tilde{\lambda}} \frac{\partial \tilde{\lambda}}{\partial T} - \alpha \pi \right) \tag{20}$$

Using (15) yields

$$\frac{\partial \pi}{\partial \tilde{\lambda}} = \int_p^{\overline{v}} e^{-\tilde{\lambda}(T)(1-F(y))} (J(y) - R) \left(1 - \tilde{\lambda}(T)(1 - F(y)) \right) f(y) dy \tag{21}$$

Noting that $\tilde{\lambda} = 0$ for $T = 0$, evaluating this expression at $T = 0$ yields

$$\left. \frac{\partial \pi}{\partial \tilde{\lambda}} \right|_{\tilde{\lambda}=0} = \int_p^{\overline{v}} (J(y) - R) f(y) dy \tag{22}$$

Moreover, as $T \to 0$, $\partial \tilde{\lambda} / \partial T = \lambda$ and $\Pi_A(0) = R$, so the condition for the seller to prefer no sale reads

$$\left. \frac{\partial \Pi_A}{\partial T} \right|_{T=0} = \lambda \int_p^{\overline{v}} (J(y) - R) f(y) dy - \alpha R \leq 0 \tag{23}$$

Solving for α then establishes the condition for optimality of no sale. In order to establish that no other T can yield a higher expected revenue, it suffices to show that Π_A is concave in T. Note that

$$\frac{\partial^2 \Pi_A}{\partial T^2} = e^{-\alpha T} \left(\frac{\partial^2 \pi}{\partial \tilde{\lambda}^2} \frac{\partial \tilde{\lambda}}{\partial T} + \frac{\partial \pi}{\partial \tilde{\lambda}} \left(\frac{\partial^2 \tilde{\lambda}}{\partial T^2} - \alpha \frac{\partial \tilde{\lambda}}{\partial T} \right) \right) - \alpha \frac{\partial \Pi_A}{\partial T} \tag{24}$$

Using the concavity of π and $\tilde{\lambda}$, the term in parentheses is negative. But this implies that at any point where $\frac{\partial \Pi_A}{\partial T} = 0$, Π_A is concave in T, implying that all T such that $\frac{\partial \Pi_A}{\partial T} = 0$ must be maxima. As Π_A is (weakly) decreasing at $T = 0$, it cannot reach a maximum for some $T > 0$ without first reaching a minimum. As there can be no such minima, Π_A can have no maxima other than $T^* = 0$ either, implying that $T^* = 0$ (meaning that the seller just gets his reservation utility) is the unique optimal "selling" strategy when $\alpha \geq \lambda \frac{\int_p^{\overline{v}} (J(y)-R) f(y) dy}{R}$.

The economic intuition behind proposition 3 is straightforward: if the discount rate lies above the instantaneous proportional gain from auctioning the good (which is equal to the product of the arrival rate λ and the proportional premium of expected marginal utility over the reservation price, $\frac{\int_p^{\overline{v}} (J(y)-R) f(y) dy}{R}$), it is not worth running an auction, and the seller prefers consuming the good himself (trivially, a sale is again always optimal if $R = 0$). If the discount rate lies below that critical value, it is worth for the seller to wait some positive time T, hoping that at least one bidder will show up for the auction. Therefore, the seller chooses to close the auction after a fixed interval $T > 0$. Conversely, as the rate of bidder arrivals λ increases, the average number of bidders participating in the auction and the expected gain from waiting for additional bidders increases, and the seller is more likely to attempt a sale.

It is worth noting that the conditions in propositions 1 and 3 are in fact equivalent. Indeed, using the definition of J, the condition of proposition 3 can be rewritten as

$$\begin{aligned} \alpha &> \lambda \frac{\int_{p^*}^{\overline{v}} \left(y - \frac{1-F(y)}{f(y)} - R \right) f(y) dy}{R} \\ &= \lambda \frac{\int_p^{\overline{v}} (y-R) f(y) dy - (\overline{v} - p) + yF(y)\Big|_p^{\overline{v}} - \int_p^{\overline{v}} y f(y) dy}{R} \\ &= \lambda \frac{(p^* - R)(1 - F(p^*))}{R} = \alpha^* \end{aligned} \tag{25}$$

Thus, whenever $\alpha \geq \lambda \frac{(p^*-R)(1-F(p^*))}{R}$, not selling the good is the optimal strategy, and neither store nor auction are run. We now show that if the above condition for optimality of no sale is not met, an auction with positive T will be optimal. Moreover, the auction's optimal closing time T^* is unique:

Proposition 4: Whenever $0 < \alpha < \lambda \frac{(p^*-R)(1-F(p^*))}{R}$, there exists a unique $T^* > 0$ maximizing (14).

Proof: Recall that whenever $\alpha < \lambda \frac{(p^*-R)(1-F(p^*))}{R}$, Π_A is increasing in T at $T = 0$, and that $\Pi_A(0) = R > 0$. Therefore, the existence of an optimal $T^* > 0$ can be established by noting that $\pi(\tilde{\lambda}(T))$ is bounded, implying that

$$\lim_{T\to\infty}\Pi_A(T)=\lim_{T\to\infty}e^{-\alpha T}\pi(\tilde{\lambda}(T))=0<\Pi_A(0) \tag{26}$$

To establish uniqueness, it suffices to show that Π_A is concave in T for all T such that $\partial\Pi_A/\partial T=0$, a result that was already derived in the proof of proposition 3.

The results in propositions 3 and 4 arise because of the tradeoff between waiting for more bidders to show up (which increases the expected revenue from the sale) and the effect of discounting (which reduces its present value). Note that in the special case in which $\alpha\to 0$, it is optimal for the seller to wait a very long time before closing the auction, and the expected payoff from the auction is $\overline{v}$.

As mentioned above, if the discount rate is sufficiently high, the seller will prefer consuming the good on his own and will run neither store nor auction. In what follows, we therefore assume that $\alpha<\lambda\frac{(p^*-R)(1-F(p^*))}{R}$, so that the seller does attempt a sale and the question of which structure is preferable (i.e., store or auction) becomes relevant.

Our next results deals with the effect of changes in the discount rate α, the rate of bidder arrivals λ and the availability of outside options μ on the value of the auction.

Proposition 5: $\Pi_A(T^*)$ is strictly decreasing in the discount rate α and the availability of outside options μ and strictly increasing in market thickness λ.

Proof: Note that for all $T>0$, $\frac{\partial\Pi_A}{\partial\alpha}=-Te^{-\alpha T}\pi(\tilde{\lambda})<0$, $\frac{\partial\Pi}{\partial\mu}=e^{-\alpha T}\frac{\partial\pi(\tilde{\lambda})}{\partial\tilde{\lambda}}\frac{\partial\tilde{\lambda}}{\partial\mu}<0$ and $\frac{\partial\Pi_A}{\partial\lambda}=e^{-\alpha T}\frac{\partial\pi}{\partial\tilde{\lambda}}\frac{\partial\tilde{\lambda}}{\partial\lambda}>0$. Therefore, the claimed relationships must hold as well for the optimal T^*.

The next three results describe how the properties of the optimal auction depend on the discount rate α, the rate of bidder arrivals λ and the availability of outside options μ.

Proposition 6: The auction's optimal closing time T^* is a decreasing function of the discount rate α.

Proof: Since

$$\frac{dT^*}{d\alpha}=-\left.\frac{\partial^2\Pi_A/\partial T\partial\alpha}{\partial^2\Pi_A/\partial T^2}\right|_{\frac{\partial\Pi_A}{\partial T}=0} \tag{27}$$

and we have shown in proposition 4 that $\partial^2\Pi_A/\partial T^2<0$ whenever $\partial\Pi_A/\partial T=0$, the sign of

$dT^*/d\alpha$ will be equal to that of $\partial^2\Pi_A/\partial T\partial\alpha$. Using (20) yields

$$\left.\frac{\partial^2\Pi_A}{\partial T\partial\alpha}\right|_{\frac{\partial\Pi_A}{\partial T}=0} = \left.\left(-Te^{-\alpha T}\left(\frac{\partial\pi}{\partial\tilde{\lambda}}\frac{\partial\tilde{\lambda}}{\partial T} - \alpha\pi\right) - e^{-\alpha T}\pi\right)\right|_{\frac{\partial\Pi_A}{\partial T}=0} = -e^{-\alpha T}\pi < 0 \qquad (28)$$

establishing the result.

Since $\tilde{\lambda}(T)$ is strictly increasing in T, proposition 6 implies that as the seller becomes more impatient and/or the good more perishable, the expected number of bidders participating in the auction, $\tilde{\lambda}$, falls.

When varying λ and μ, the optimal expected number of bidders chosen by the seller, $\tilde{\lambda}(T^*)$ is a more appropriate description of the seller's auctioning strategy than the auction's closing time T^*, which by itself does not say much about how the auction will look like in terms of bidder participation. Rather than the auction's closing time T, the next two results therefore consider the effect of λ and μ on the optimal $\tilde{\lambda}(T^*)$.

Proposition 7: The expected number of bidders in the optimal auction, $\tilde{\lambda}(T^*)$, is an increasing function of the bidder arrival rate λ. Moreover, $\left.\frac{\partial\tilde{\lambda}}{\partial T}\right|_{T=T^*}$ is increasing in λ.

Proof: Rewrite the first-order condition (20) as

$$\frac{\partial\pi/\partial\tilde{\lambda}}{\pi}\frac{\partial\tilde{\lambda}}{\partial T} = \alpha \qquad (29)$$

Next, observe that by concavity of π, $\Phi(\tilde{\lambda}) = \frac{\partial\pi/\partial\tilde{\lambda}}{\pi}$ is a strictly decreasing function of $\tilde{\lambda}$. Using the fact that $\frac{\partial\tilde{\lambda}}{\partial T} = \lambda e^{-\mu T} = \lambda - \tilde{\lambda}\mu$, (29) can be rewritten as

$$\Phi(\tilde{\lambda})(\lambda - \tilde{\lambda}\mu) = \alpha \qquad (30)$$

Suppose now that λ is increased, and hold all other parameters constant. Then, $\lambda - \tilde{\lambda}\mu$ increases, so for the first-order condition (30) to hold, $\Phi(\tilde{\lambda})$ must fall, implying that $\tilde{\lambda}$ must rise, establishing the first result. To establish the second result, note that since $\Phi(\tilde{\lambda})$ has decreased in the new optimum, $\frac{\partial\tilde{\lambda}}{\partial T} = \lambda e^{-\mu T} = \lambda - \tilde{\lambda}\mu$ must have increased.

Proposition 8: The expected number of bidders in the optimal auction, $\tilde{\lambda}(T^*)$, is a decreasing function of the rate of arrival of outside options μ. Moreover, $\left.\frac{\partial\tilde{\lambda}}{\partial T}\right|_{T=T^*}$ is decreasing in μ.

Proof: As in the proof of proposition 6, rewrite the first-order condition as $\Phi(\tilde{\lambda})(\lambda-\tilde{\lambda}\mu) = \alpha$. Holding all other parameters constant, increase μ. Then, $\lambda - \tilde{\lambda}\mu$ falls, so for the first-order condition to hold, $\Phi(\tilde{\lambda})$ must increase, implying that $\tilde{\lambda}$ must fall, establishing the first result. To establish the second result, note that since $\Phi(\tilde{\lambda})$ has increased in the new optimum, $\frac{\partial\tilde{\lambda}}{\partial T}=\lambda e^{-\mu T}=\lambda-\tilde{\lambda}\mu$ must have decreased.

The results on the properties of the optimal auction derived in this section make intuitive sense: if α is low, so that waiting is not too costly, then the seller waits a long time before closing the auction. If λ is high, so that the market is thick, then the seller increases the expected number of bidders in the optimal auction, $\tilde{\lambda}$. Finally, if μ is high, so that bidders' waiting costs are high or alternatives for the good are easy to find, the expected number of bidders in the optimal auction is reduced.

These results imply that if α or μ are high or λ is low, the expected number of bidders in the auction $\tilde{\lambda}$ will be low. With a low $\tilde{\lambda}$, the probability of having more than one bidder show up in the auction will be low as well, and the auction will look more a store in the limit. Therefore, one could expect the store to dominate the auction for high α and μ and for low λ. The next section shows that this is indeed the case.

3 The Optimal Market Structure

We now turn to the question of which market structure – store or auction – will be optimal for the seller. We first present a general condition under which the auction can dominate the store. In a second step, we show how the optimality of the store or the auction depends on the seller's time preference and the extent to which the good is perishable (α), the thickness of the market (λ) and the extent of waiting costs and availability of outside options (μ).

Proposition 9: The auction is the preferred structure if and only if there exists some $\tilde{\lambda}\in[0,\lambda/\mu]$ (for $\mu\neq 0$) or some $\tilde{\lambda}\geq 0$ (for $\mu=0$) such that

$$\left(1-\tilde{\lambda}\frac{\mu}{\lambda}\right)^{\alpha/\mu}\pi(\tilde{\lambda})-\frac{\lambda}{\alpha+\lambda}\left(p^*(1-F(p^*))+RF(p^*)\right)>0 \qquad (\mu\neq 0)$$
$$e^{-\alpha\tilde{\lambda}/\lambda}\pi(\tilde{\lambda})-\frac{\lambda}{\alpha+\lambda}\left(p^*(1-F(p^*))+RF(p^*)\right)>0 \qquad (\mu=0)$$

Proof: The condition for the auction to dominate is that

$$e^{-\alpha T}\pi(\tilde{\lambda})>\frac{\lambda}{\alpha+\lambda}\left(p^*(1-F(p^*))+RF(p^*)\right) \tag{31}$$

Using the fact that $\tilde{\lambda}=\frac{\lambda}{\mu}(1-e^{-\mu T})$ for $\mu\neq 0$ and $\tilde{\lambda}=\lambda T$ for $\mu=0$, we have $T=-\frac{\ln(1-\tilde{\lambda}\frac{\mu}{\lambda})}{\mu}$ for $\mu\neq 0$ and $T=\frac{\tilde{\lambda}}{\lambda}$ for $\mu=0$. Thus, $e^{-\alpha T}=\left(1-\tilde{\lambda}\frac{\mu}{\lambda}\right)^{\alpha/\mu}$ for $\mu\neq 0$ and $e^{-\alpha\tilde{\lambda}/\lambda}$ for $\mu=0$. Substituting into (31), the result follows.

The intuition for proposition 9 is the following: for the auction to be able to dominate the store, two things are required:

1. First, a sufficient number of bidders $\tilde{\lambda}$ must be able to accumulate in order for $\pi(\tilde{\lambda})$ to rise significantly above the expected payoff from the store. The maximum achievable number of bidders in the auction, $\lim_{T\to\infty} = \frac{\lambda}{\mu}$, depends positively on market thickness and negatively on waiting costs and false trading.

2. Second, this accumulation must occur sufficiently quickly so that the payoff from the auction does not get discounted too heavily. As is apparent from the expressions $T = -\frac{\ln(1-\tilde{\lambda}\frac{\mu}{\lambda})}{\mu}$, this accumulation will occur faster, the higher λ and the lower μ.

Note that a high λ and a low μ positively affect both factors required for the auction to dominate the store. This is best illustrated using a numerical example. Suppose that bidders' valuations are uniformly distributed on the interval $[0, 1]$. Then, the reservation price for both store and auction is given by $p^* = \frac{1+R}{2}$, and one has

$$\pi(\tilde{\lambda}) = R + \int_{\frac{1+R}{2}}^{1} e^{-\tilde{\lambda}(1-y)} 2\left(y - \frac{1+R}{2}\right) dy = 1 - 2\frac{1 - e^{-\tilde{\lambda}\frac{1-R}{2}}}{\tilde{\lambda}} \tag{32}$$

Therefore, the condition for the auction to dominate is that there exists a $\tilde{\lambda}$ such that

$$\left(1 - \tilde{\lambda}\frac{\mu}{\lambda}\right)^{\alpha/\mu} \left(1 - 2\frac{1 - e^{-\tilde{\lambda}\frac{1-R}{2}}}{\tilde{\lambda}}\right) - \frac{\lambda}{\alpha + \lambda}\left(\frac{1+R}{2}\right)^2 > 0 \tag{33}$$

The expected revenue from the auction as a function of $\tilde{\lambda}$ and the expected revenue of the store are depicted in Figure 1 for three different situations. In the base case with $\lambda = 5$ and $\mu = 1$ (top panel), the auction (dotted line) slightly dominates the store (solid line) if a $\tilde{\lambda}$ around 2 is selected. If the drop-out rate μ is increased to 2.5 (middle panel), then the auction no longer dominates the store: no $\tilde{\lambda}$ can be found such that (31) is satisfied. Finally, if the arrival rate λ is increased to 10 (lower panel), the auction dominates the store more clearly than in the base case.

This numerical example suggests that a high market thickness and low availability of substitutes should make dominance of the auction more likely. The analysis that follows shows that this intuition is correct. The next result deals with market thickness and demonstrates that if the rate of bidder arrivals is sufficiently high, then the auction will be the preferred structure.

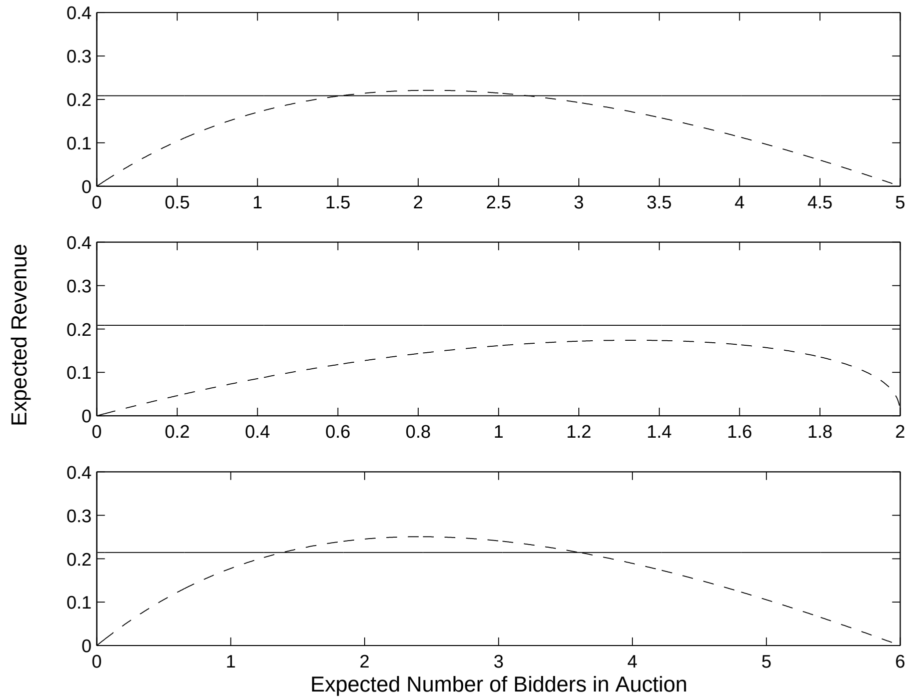

Figure 1: The choice between store (solid line) and auction (dotted line) depending on market thickness λ and waiting costs μ (value of the parameters: $\alpha = 1$, $R = 0$, $\{\lambda, \mu\} \in \{\{5, 1\}, \{5, 2.5\}, \{10, 1\}\}$.

Proposition 10: For all parameter constellations (α, μ) such that selling is optimal ($\alpha < \lambda\frac{(p^*-R)(1-F(p^*))}{R}$), there exists a rate of bidder arrivals λ_0 such that auctioning is optimal if $\lambda > \lambda_0$.

Proof: Auctioning will be optimal if

$$\Pi_A(T) = e^{-\alpha T}\pi(\tilde{\lambda}(T)) > \Pi_S = \frac{\lambda}{\alpha + \lambda}\pi_1 \tag{34}$$

Using the definition of π, a sufficient condition for the auction to dominate is that

$$\sum_{N=0}^{\infty} e^{-\tilde{\lambda}(T)} \frac{\tilde{\lambda}(T)^N}{N!} \left(\pi_N - e^{\alpha T}\pi_1\right) > 0 \tag{35}$$

or

$$\sum_{N=2}^{\infty} \frac{\tilde{\lambda}(T)^N}{N!} \left(\pi_N - e^{\alpha T}\pi_1\right) + \tilde{\lambda}(T)(1 - e^{\alpha T})\pi_1 + (\pi_0 - e^{\alpha T}\pi_1) > 0 \tag{36}$$

Now, choose T small enough such that $\pi_2 - e^{\alpha T}\pi_1 > 0$, i.e. $T < \frac{\ln(\pi_2/\pi_1)}{\alpha}$ (this assumption is innocuous, as if the auction dominates for that "arbitrarily" chosen T, it will necessarily do so for the optimal T^* as well), and consider the limit as $\lambda \to \infty$, which for constant T implies $\tilde{\lambda}(T) \to \infty$. The summation term contains only positive terms by construction and therefore tends to plus infinity in polynomial progression, whereas the second term tends to minus infinity linearly and the third term is constant. Therefore, for constant T, the whole expression tends to plus infinity as $\lambda \to \infty$, and the auction will be preferred to the store for large enough λ.

The intuition for proposition 10 is straightforward. If the rate of bidder arrivals is high, many bidders will accumulate even if the auction's closing time is small, and it is worth for the seller to run an auction rather than a store because of the higher expected selling price. This result is consistent with a number of empirical observations. For example, the market for fresh fish and the Amsterdam fresh flower market, which are characterized by high thickness (with all interested bidders showing up every morning at the announced time to bid), are typically run using auctions. An interesting implication of proposition 8 is that the optimal way to sell the same good, with the same distribution of bidder valuations, can be either a store or an auction depending on the thickness of the market. This has been observed on the California real-estate market, where houses, which had historically been sold using posted prices (i.e., a "store"), were commonly auctioned as the market became thick in the late 1990's.

Turning to bidders' waiting cost and false trading, the next result shows that if bidders' waiting costs are high or alternatives are easy to find, the seller will always prefer a store.

Proposition 11: For all parameter constellations (α, λ) such that selling is optimal ($\alpha < \lambda \frac{(p^*-R)(1-F(p^*))}{R}$), there exists a rate of arrival of outside options μ_0 such that the seller always prefers the store to the auction if $\mu > \mu_0$.

Proof: Since $\alpha < \lambda \frac{(p^*-R)(1-F(p^*))}{R}$, we know that the seller's expected utility from running the store is $\Pi_S > R$, so all we need to show is that for μ high enough, he will achieve at most R using an auction. Note that for all T, $\lim_{\mu\to\infty} \tilde{\lambda}(T) = 0$, so it follows from (15) that for all $T \geq 0$, $\lim_{\mu\to\infty} \Pi_A(T) = e^{-\alpha T} R \leq R$, and the store is therefore strictly preferred to the auction if μ is sufficiently high. The uniqueness of the critical μ_0 follows from the fact that Π_A is strictly decreasing in μ for all $T > 0$ while Π_S does not depend on μ.

The result in Proposition 11 does tell us that waiting costs and false trading play a critical role in the choice between store and auction, but it does not allow a precise characterization of when an auction or a store will be preferred. The next proposition puts a much tighter bound on the parameter constellations under which auctions can arise as optimal selling institutions.

Proposition 12: Let $g(\tilde{\lambda}) \equiv \frac{\int_{p^*}^{\overline{v}} e^{-\tilde{\lambda}(1-F(y))}(J(y)-R)f(y)dy}{\int_{p^*}^{\overline{v}}(J(y)-R)f(y)dy} < 1$. An auction such that $\tilde{\lambda} g(\tilde{\lambda}) < \frac{\lambda}{\alpha+\lambda} - \frac{\alpha}{\alpha+\lambda}\frac{R}{(p^*-R)(1-F(p^*))}$ cannot dominate a store.

Proof: The necessary and sufficient condition for the store to dominate is that

$$e^{-\alpha T}\left(R+\tilde{\lambda}(T)\int_{p^*}^{\overline{v}} e^{-\tilde{\lambda}(T)(1-F(y))}(J(y)-R)f(y)dy\right) < \frac{\lambda}{\alpha+\lambda}\left(p^*(1-F(p^*))+RF(p^*)\right) \quad (37)$$

Therefore, a sufficient condition for the store to be preferred is that

$$\tilde{\lambda}(T)\int_{p^*}^{\overline{v}} e^{-\tilde{\lambda}(T)(1-F(y))}(J(y)-R)f(y)dy < \frac{\lambda}{\alpha+\lambda}\left((p^*-R)(1-F(p^*))-\alpha R\right) \quad (38)$$

Using the definition of $g(\tilde{\lambda})$ and the fact that $\int_p^{\overline{v}}(J(y)-R)f(y)dy=(p^*-R)(1-F(p^*))$, the result follows.

Proposition 12 implies that for the auction to be able to dominate the store, a minimum expected number of bidders $\tilde{\lambda}$ is required in the auction. This is because the benefit of using an auction as opposed to a store stems from the fact that a price higher than the reservation price p^* can be extracted. This, however, can only occur if the average number of bidders is sufficiently high.

Proposition 12 is best illustrated using a numerical example. Suppose again that that bidders' valuations are uniformly distributed on the interval $[0,1]$. Using the fact that $p^*=\frac{1+R}{2}$, one has

$$g(\tilde{\lambda})=\frac{\int_{\frac{1+R}{2}}^{1} e^{-\tilde{\lambda}(1-y)}\left(y-\frac{1+R}{2}\right)dy}{\int_{\frac{1+R}{2}}^{1}\left(y-\frac{1+R}{2}\right)dy}=\frac{\frac{1}{\tilde{\lambda}^2}\left(e^{-\tilde{\lambda}\frac{1-R}{2}}-1\right)+\frac{1}{\tilde{\lambda}}\left(\frac{1-R}{2}\right)}{\frac{1}{2}\left(\frac{1-R}{2}\right)^2} \quad (39)$$

Therefore, the store is guaranteed to dominate the auction for all $\tilde{\lambda}$ such that

$$\frac{\frac{1}{\tilde{\lambda}}\left(e^{-\tilde{\lambda}\frac{1-R}{2}}-1\right)+\frac{1-R}{2}}{\frac{1}{2}\left(\frac{1-R}{2}\right)^2} < \frac{\lambda}{\alpha+\lambda}-\frac{\alpha}{\alpha+\lambda}\frac{R}{\left(\frac{1-R}{2}\right)^2} \quad (40)$$

Both $\tilde{\lambda}g(\tilde{\lambda})$ and $\frac{\lambda}{\alpha+\lambda}-\frac{\alpha}{\alpha+\lambda}\frac{R}{\left(\frac{1-R}{2}\right)^2}$ are depicted in Figure 2 for $\lambda=5$ and $\lambda=10$. Note that the minimum expected number of bidders in the auction $\tilde{\lambda}$ below which the store will be the preferred structure is greater than 1 and becomes larger, the larger the arrival rate λ. This is because the store becomes more profitable as λ is increased.

The result of proposition 12 is related to proposition 9 but somewhat weaker, as the effect of the time required for bidders to accumulate on the value of the sale is not taken explicitly into account. Nevertheless, it is a very convenient starting point to derive results that allow to rule out the dominance of the auction in many cases. The reason is that the rate μ at which bidders

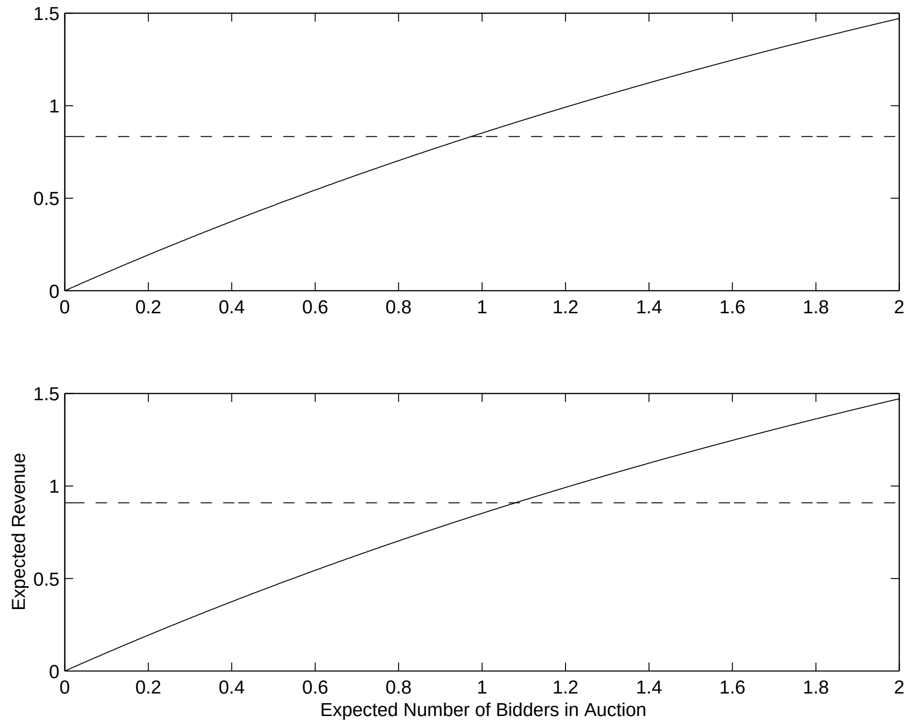

Figure 2: Minimum value of $\tilde{\lambda}$ below which the store is always preferred to the auction (value of the parameters: $\alpha = 1$, $R = 0$, $\lambda \in \{5, 10\}$.

find alternatives constrains the maximum expected number of bidders $\tilde{\lambda}$ that the seller can achieve in an auction. Therefore, knowing that there exists a critical $\tilde{\lambda}$ below which the store will dominate the auction, and since $\tilde{\lambda}$ is a strictly decreasing function of μ, a corresponding value for μ above which it is *certain* that the store will be preferred to the auction can be determined:

Proposition 13: Whenever $\mu > \frac{\frac{\lambda}{\alpha}(\alpha+\lambda)}{\frac{\lambda}{\alpha} - \frac{R}{(p^*-R)(1-F(p^*))}}$, the store dominates the auction.

Proof: Noting that $\tilde{\lambda} = \frac{\lambda}{\mu}(1 - e^{-\mu T}) < \frac{\lambda}{\mu}$ and using the result of Proposition 12 and the fact that $g(\tilde{\lambda}) < 1$, the store can be guaranteed to dominate whenever

$$\frac{\lambda}{\mu} < \frac{\lambda}{\alpha + \lambda} - \frac{\alpha}{\alpha + \lambda} \frac{R}{(p^* - R)(1 - F(p^*))} \tag{41}$$

or

$$\mu > \frac{\lambda(\alpha + \lambda)}{\lambda - \frac{\alpha R}{(p^*-R)(1-F(p^*))}} = \frac{\frac{\lambda}{\alpha}(\alpha + \lambda)}{\frac{\lambda}{\alpha} - \frac{R}{(p^*-R)(1-F(p^*))}} \tag{42}$$

Proposition 13 confirms the basic intuition that running an auction, which requires a large number of bidders to work well, becomes less interesting than the store if bidders are not ready to wait because they can find opportunities to purchase the good elsewhere. It also implies that regardless of the distribution of bidders' valuations, if both the discount rate α and the bidder arrival rate λ are reduced proportionally, the value of μ above which the store is guaranteed to dominate the auction falls by the same proportion.

To understand the basic intuition behind proposition 13, is instructive to consider a limiting case in which the discount rate is very low. In such a setting, ignoring outside options would lead to the recommendation that an auction should be run. However, using (41), a sufficient condition for the store to dominate the auction when $\alpha \to 0$ is that $\mu > \lambda$. The reason is that if this condition is satisfied, the average time $1/\mu$ until bidders find an alternative to purchase the good elsewhere is lower than the average bidder interarrival time, $1/\lambda$. As a result, the probability of having two bidders if an auction is run is very low and the seller prefers to sell the good using a store.

The result in proposition 13 is consistent with empirical evidence. "Commodity" goods, such as groceries, are seldom sold through auctions because of the close availability of many substitutes and the size of waiting costs (immediacy is important to buyers). On the other hand, "unique" items such as original artwork, for which substitutes are difficult to find and immediacy low, are commonly sold through auctions. Note that once again, two goods with identical distributions of bidder valuations can be sold optimally either using a store or an auction, depending on the size of waiting costs and whether alternative purchase opportunities are available to the bidders or not.

The next result shows that if the discount rate is sufficiently low, the auction will be the preferred structure.

Proposition 14: Suppose that $\lambda \gg \mu$. There exists some $\alpha \geq 0$ so that the auction dominates the store.

Proof: Note that when $\alpha \to 0$, the payoff from the store is $\Pi_S = \pi_1$, whereas the payoff from the auction is $\sum_{N=0}^{\infty} e^{-\tilde{\lambda}(T)} \frac{\tilde{\lambda}(T)^N}{N!} \pi_N$. Also, since $\lambda \gg \mu$, $\tilde{\lambda}$ can be made sufficiently large so that $\pi(\tilde{\lambda}) > \pi_1$ by setting a large auction closing time T. Therefore, using arguments similar to those in the proof of proposition 8, one can establish that the auction dominates for low α.

Note that a value of $\tilde{\lambda}$ strictly (and possibly significantly) above 1 must be achievable for the auction to dominate for low α. The reason is that because of the concavity of $\pi(\tilde{\lambda})$, Jensen's inequality implies that $\pi(1) < \pi_1$, and the auction cannot dominate the store if $\tilde{\lambda} < 1$. As a result, the condition $\lambda > \mu$ would not be sufficient to guarantee that the auction could dominate the store, and $\lambda \gg \mu$ is required.

Proposition 14 implies that other things equal, goods that are perishable or become obsolete quickly should be sold using stores rather than auctions, while goods whose value does not fall through time should be sold using auctions. This prediction is consistent with the empirical evidence. Artwork, for example, is hardly perishable, and the auction is therefore the preferred structure. On the other hand, vegetables are typically sold using stores (as mentioned above, fresh fish and flowers are sold through auctions because of the thickness of the market).

4 Summary and Conclusion

Auctions have the undesirable feature that they impose waiting costs on bidders and give rise to false trading. As a result, the seller will often prefer running a retail store to running an auction. Three factors are shown to play a critical role in the seller's choice between store and auction: (1) the perishability of the good and discounting, (2) the thickness of the market, and (3) the extent of buyers' waiting costs and the availability of alternatives. More specifically, the following can be established:

1. If the seller's time preference, the perishability of the good being sold or the speed at which it becomes obsolete are high, then the store tends to be the preferred structure.

2. If the market is very thick in the sense that many bidders arrive per unit time, the auction tends to be preferred to the store. This prediction consistent with the widespread use of auctions in the California real estate market in the late 1990's.

3. High bidders' waiting costs and the availability of alternatives to purchase an identical or similar good elsewhere tend to favor the store. This result is consistent with a number of real-life phenomena, such as the fact that goods that have close substitutes (such as groceries) are almost always sold through store structures, while goods that are "unique" in the sense of having no close substitutes (of which artwork is a classical example) are often sold using auctions. Waiting costs and the risk of false trading provide an explanation for the extreme dominance of store structures in practice.

An important implication of these results is that identical goods (having the same distribution of buyers' valuations and the same perishability) may be sold differently depending on the market environment. In an environment in which arrivals are frequent and alternatives difficult to find, the good will be sold using an auction. However, the store may be the optimal structure to sell the *same* good if the arrival rate is low and alternatives are hard to find.

The development of markets on the Internet provides an interesting application of this analysis. At first sight, the Internet would seem to favor auctions because it allows several arrival streams to be pooled together, at least for goods that can be shipped. On the other hand, Internet technology makes searching for alternatives easier, which tends to favor stores. This factor can explain why stores have become more and more common on the Internet in the last few years, with even originally all-auction electronic marketplaces such as ebay now allowing sellers to run retail stores rather than auctions.

Appendix

In this appendix, we derive the distribution of the number of bidders participating in the auction at time T. To do so, the auction is best viewed as a queuing system in which customers arrive at a rate λ and each customer in the system leaves at a rate μ. Let $q_N(t)$ denote the probability that there are N bidders in the system at time t. These probabilities must satisfy the following system of Chapman-Kolmogorov differential equations:

$$\frac{dq_0(t)}{dt} = -\lambda q_0(t) + \mu q_1(t), \qquad (N = 0) \tag{43}$$

$$\frac{dq_N(t)}{dt} = -(\lambda + N\mu)q_N(t) + (N+1)\mu q_{N+1}(t) + \lambda q_{N-1}(t), \qquad (N > 0) \tag{44}$$

The intuition for this system of differential equations is as follows: if the system is currently empty ($N = 0$), there is a probability λ per unit time that a bidder will show up, reducing the probability that the system remains empty by λ. On the other hand, if there is one bidder in the system ($N = 1$), there is a probability μ per unit time that a bidder will leave it and bring it to state 0. Together, these factors imply (43). More generally, if there are currently N bidders in the system, there is a probability λ per unit time that a bidder will arrive and bring it to state $N + 1$, and a probability $N\mu$ that one of the bidders will leave it and bring it to state $N - 1$. On the other hand, if the system is in state $N - 1$, there is a probability λ that one bidder will arrive and bring the system to state N. Finally, if there are $N + 1$ bidders in the system, there is a probability $(N + 1)\mu$ that one of the bidders will depart. This then implies (44).

To solve this system, multiply the equation for N by z^N and sum over all N to obtain

$$\sum_{N=1}^{\infty} z^N \frac{dq_N(t)}{dt} + \mu(z-1) \sum_{N=1}^{\infty} N z^{N-1} q_N(t) = \lambda(z-1) \sum_{N=0}^{\infty} z^N q_N(t) \tag{45}$$

Defining $Q(z,t) = \sum_{N=0}^{\infty} z^N q_N(t)$, this equation can be rewritten as

$$\frac{\partial Q}{\partial t} + \mu(z-1)\frac{\partial Q}{\partial z} = \lambda(z-1)Q \tag{46}$$

This equation can then be solved with the initial condition that there are no bidders at time 0, $q_0(0) = 1$, yielding

$$Q(z,t) = \exp\left(\frac{\lambda}{\mu}(z-1)(1-e^{-\mu t})\right) \tag{47}$$

To determine the state probabilities at time t, $q_N(t)$, take a Taylor series expansion of Q around $z = 0$, holding t constant. The probability of state N will be proportional to the coefficients of

z^N in that Taylor series. Noting that

$$\frac{\partial^N Q}{\partial z^N} = \exp\left(\frac{\lambda}{\mu}(z-1)(1-e^{-\mu t})\right)\left(\frac{\lambda}{\mu}(1-e^{-\mu t})\right)^N \tag{48}$$

the state probabilities are given by

$$q_N(t) = \gamma \frac{1}{N!}\frac{\partial^N Q}{\partial z^N}\bigg|_{z=0} = \gamma \frac{1}{N!}\exp\left(-\frac{\lambda}{\mu}(1-e^{-\mu t})\right)\left(\frac{\lambda}{\mu}(1-e^{-\mu t})\right)^N \tag{49}$$

with γ a scaling constant ensuring that the probabilities sum to 1. Noting that

$$\sum_{N=0}^{\infty} \frac{1}{N!}\exp\left(-\frac{\lambda}{\mu}(1-e^{-\mu t})\right)\left(\frac{\lambda}{\mu}(1-e^{-\mu t})\right)^N = 1 \tag{50}$$

one has $\gamma = 1$ and the probability of having N bidders at time t is given by

$$q_N(t) = \frac{1}{N!}\exp\left(-\frac{\lambda}{\mu}(1-e^{-\mu t})\right)\left(\frac{\lambda}{\mu}(1-e^{-\mu t})\right)^N \tag{51}$$

Defining $\tilde{\lambda}(t) = \frac{\lambda}{\mu}(1-e^{-\mu t})$, the state probabilities (51) can be recognized as those of a Poisson distribution with parameter $\tilde{\lambda}(t)$,

$$q_N(t) = e^{-\tilde{\lambda}(t)}\frac{\tilde{\lambda}^N}{N!} \tag{52}$$

which is the result used in the text.

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