Decentralizing the Stochastic Growth Model

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Abstract

The objective of this paper is to propose a number of alternative decentralized interpretations of representative agent style stochastic growth economies and to explore their implications for the generality of this model construct. Under our first interpretation, firms exist forever and undertake all multiperiod investment decisions while consumer-worker-investors only own financial claims to the firm's output. This contrasts with the more standard decentralization approach where firms exist on a period-by-period basis and consumer-worker-investors have direct title to the economy's capital stock. Under our second interpretation shareholders hire a manager who undertakes the firm's investment decisions in conformity with his incentive contract. The time series properties of the shareholder-manager economy are seen to replicate those of the analogous representative agent economy if and only if the manager's contract assumes a specific form. This suggests the time series properties of an economy where incentive contracts such as stock option plans are pervasive will differ from those of more standard real business cycle models.

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1. Introduction

The one good optimal growth model remains the fundamental paradigm underlying all dynamic macroeconomic analysis. First analyzed by Cass (1965) and Koopmans (1965) in a certainty setting and subsequently generalized to uncertainty by Brock and Mirman (1972), Mirman and Zilcha (1975) and others, this model also forms the basis for modern growth theory and much of asset pricing theory. Kydland and Prescott (1982) and numerous successors have explored the model's potential for explaining business cycle regularities.

Many of the business cycle applications use a version of the model which, in its central planning formation, can be expressed by:

(1)
$$\max_{\{c_t, i_t, n_t\}} E(\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t))$$
$$\text{s.t. } c_t + i_t \leq f(k_t, n_t)\lambda_t$$
$$k_{t+1} = (1-\Omega)k_t + i_t$$
$$k_0 \text{ given.}$$

where c_t, i_t, k_t and n_t, represent, respectively, per capital consumption, investment, capital stock and labor service provided in period t, u(,) the period utility function of an infinitely lived representative agent, β his subjective discount factor, Ω the capital stock depreciation factor, and λ the stationary random technology shock.

The use of a central planning model to characterize market economies is legitimized by the fact that, under standard assumptions, the unique Pareto optimal allocation proposed by the central planner must coincide with the outcome of an economy in competitive general equilibrium, whatever the specific interpretation given to the decentralizing framework.

The decentralization scheme is usually defined along the lines first proposed by Prescott and

Mehra (1980) and Brock (1982) (PMB in what follows). Under the PMB interpretation, physical capital is owned by infinitely lived consumer-worker-investors and rented, on a period by period basis, to one-period-lived firms. This interpretation has proved extremely convenient because it permits a single class of agents to confront an intertemporal decision problem. This property has made it the preferred modeling choice even when the first welfare theorem does not apply. In this situation, however, the specifics of the decentralization scheme are not as innocuous. In addition, less standard modeling features are now frequently incorporated into the basic construct and it is far from clear that the PMB interpretation is always economically consistent with these enrichments. Many authors, for instance, use the PMB interpretation in conjunction with restrictions on the ability to adjust the stock of capital. But assuming costs of adjusting the capital stock when capital goods are supposed to be exchanged every period between consumer-worker-investors and the firm is not fully convincing. Danthine and Donaldson (1994, 2002) introduce multiperiod debt and labor contracts. These are clearly inconsistent with a set-up which postulates firms that will not exist in the subsequent period when these commitments are to be fulfilled. For the same reason, discussions on the impact of alternative financial decisions, issues of retained earnings and bankruptcy are precluded in such a context.

Even more damaging, the current device of having consumer-worker-investors own both physical capital and shares often presumes that, in equilibrium, these two assets will pay identical returns. As a result, the variability of the returns to equity and capital must coincide. With capital stock displaying little quarter-by-quarter variation, this coincidence prevents this class of models from ever satisfactorily explaining equity return variation.

The traditional decentralization interpretation thus appears to limit severely the types of issues that can be addressed with the stochastic growth model as a base. The goal of the present

note is to discuss alternative decentralized interpretations of this model with a view towards giving more content to the firm. In our models, firms, like their real-world counterparts, are long-lived and undertake the long-term investments. Investors own financial assets only, and they do not own physical capital directly. In Section 3, we describe the natural extension where it is assumed that firms and consumers interact in the presence of complete Arrow-Debreu securities markets. This interpretation – used by Rouwenhorst (1995) and Jermann (1998), among others - is straightforward and flexible as it can accommodate a large number of extensions. It does not particularly enhance the realism of the model, however. For this reason, Section 4 demonstrates that the stochastic growth model can also be decentralized with “meaningful” firms although only an equity security is being traded . This more operational scheme has been put to use in Danthine and Donaldson (1994, 2001). A closely related set-up is used by Altug and Labadie (1994, Chapter 4). Section 5 goes one step further in allowing shareholders to delegate the firm’s investment and hiring decisions to a class of professional firm managers. We show, however, that if this decentralized alternative is to generate the same equilibrium behavior as (1), substantial restrictions must be imposed on the form of the manager’s compensation function.
We start by briefly reviewing, in Section 2, the PMB approach to decentralizing dynamic general equilibrium models. Section 6 offers concluding comments to the paper.

2. The traditional approach to decentralizing the stochastic growth model

In this section we present the decentralized interpretation of problem (1) as first proposed by Prescott-Mehra (1980) and Brock (1982). We follow Brock (1982) who is somewhat more general than Prescott and Mehra (1980) while remaining faithful to the spirit of their story.[1] The

[1] Prescott and Mehra (1980) maintain a constant returns to scale assumption and thus, under their

idea is to assume that the representative consumer-worker undertakes all savings in the form of physical capital, while the firm is content to rent, on a period by period basis and after observing the period realization of the technology shock, both labor and capital at competitively determined rates. The consumer also collects the residual profit after the firm pays the factors of production.

In this view, firms are assumed to exist for only one period and to be recursively reorganized over and over again. The firm's problem is thus static and hence especially simple: after viewing the shock λ_t, it chooses capital and labor so as to maximize profits, d_t, on a period by period basis; i.e.,

(2) $$\max_{k_t^f, n_t^f} d_t = \max_{k_t^f, n_t^f} f(k_t^f, n_t^f)\lambda_t - w(K_t, \lambda_t)n_t^f - r(K_t, \lambda_t)k_t^f,$$

where the superscript "f" denotes firm-related quantities. The first-order conditions for problem (2) are the obvious:

(3) $$f_1(k_t^f, n_t^f)\lambda_t = r(K_t, \lambda_t), \text{ and}$$

(4) $$f_2(k_t^f, n_t^f)\lambda_t = w(K_t, \lambda_t).$$

where $w_t = w(K_t, \lambda_t)$, and $r_t = r(K_t, \lambda_t)$ are, respectively, the competitively determined wage and capital rental rates. We anticipate the fact that these quantities are both functions of the economy-wide state variables: capital stock K_t, and the shock to technology λ_t. This latter variable is assumed to follow an exogenous stationary stochastic process with continuous transition density $dF(\lambda_{t+1}; \lambda_t)$. The stochastic process on these state variables is assumed known to all the economy's participants (rational expectations).

The representative consumer-worker-investor in turn solves

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interpretation, there are no profits and no equity share claim to them.

$$\max_{\{i_t\},\{z_t\}} \; E\left(\sum_{t=0}^{\infty} \beta^t \, u(c_t, 1-n_t)\right)$$

$$\text{s.t.} \;\; c_t + i_t + \; q_t^e \, z_{t+1} \le \; n_t \, w_t + r_t \, k_t + z_t(q_t^e \; + d_t),$$

$$k_{t+1} = (1\text{-}\,\Omega\,)k_t + i_t$$

$$z_t \le \; 1$$

where $q_t^e = q^e(K_t, \; \lambda_t)$ is the price of the equity security and z_t represents the agent's fractional holdings of the single equity share.

Under quite general assumptions[2], the necessary and sufficient first-order conditions for the consumer-worker-investor are given by:

(5) $n_t : u_2(c_t, 1-n_t) = \; u_1(c_t, 1-n_t)w(K_t, ë_t),$

(6) $i_t,\!: \;\; u_1(c_t, 1-n_t) = \; â\int u_1(c_{t+1}, 1-n_{t+1})[r(K_{t+1}, ë_{t+1}) + (1-Ù)] \; dF\,(ë_{t+1}; ë_t)$, and

(7) $z_t,\!: \;\; u_1(c_t, 1-n_t)q^e(K_t, ë_t) = \; â\int u_1(c_{t+1}, 1-n_{t+1})[q^e(K_{t+1}, ë_{t+1}) + d(K_{t+1}, ë_{t+1})] \, dF\,(ë_{t+1}; ë_t)$.

Brock (1982) proves the existence of continuous pricing functions $r\;(K_t, \lambda_t)$, $w\,(K_t, \lambda_t)$, and $q^e\,(K_t, \lambda_t)$ for which (3), (4), (5), (6), and (7) are satisfied (agent optimization) and

(8) (i) $k_t^f = k_t = K_t,$
(ii) $i_t = I_t$

(9) $n_t^f = n_t$

(10) $z_t, = 1$

[2] In particular, throughout this paper, we will assume (i) u( ) and f( ) are strictly increasing, concave, and continuously differentiable, (ii) there exists a $\overline{K}$ such that $f(K,1)\lambda \le K$ for $K > \overline{K}$ and $\lambda \le \lambda_{max}$, and (iii) $dF(\lambda_{t+1},\lambda_t)$ is continuous and for any continuous $g(K,\lambda)$, $\int g(K,\lambda')dF(\lambda';\lambda)$ is continuous as a function of λ.

(market clearing). Substituting the expressions for $r(K_t, \lambda_t)$, $w(K_t, \lambda_t)$ (viz., equations (3) and (4)) into equations (5) and (6) gives

$$(11) \qquad u_2(c_t, 1-n_t) = u_1(c_t, 1-n_t)\, f_2(k_t, n_t)\, \lambda_t \text{ and}$$

$$(12) \qquad u_1(c_t, 1-n_t) = \beta \int u_1(c_{t+1}, 1-n_{t+1})[f_1(k_{t+1}, n_{t+1})\lambda_{t+1} + (1-\delta)]\, dF(\lambda_{t+1}; \lambda_t).$$

Equations (11) and (12) are the first-order necessary and sufficient conditions for the familiar central planning problem (1).

The unattractive aspect of Brock's(1982) decentralization scheme (shared also by Prescott and Mehra(1980)) is the passive role assigned to the firm: it solves no intertemporal problem. This property is highly unrealistic: it is typically the firm that determines its intertemporal investment plan while investors participate only in the residual claims market. As we have noted, it is also severely limiting from a modeling perspective because it precludes the introduction of several plausible real-world features that have the potential to significantly alter the model's performance.

We now suggest three alternative procedures for decentralizing the one good stochastic growth model, all of which admit the presence of infinitely lived firms that undertake their own investment decisions. We start by the natural extension where a full set of Arrow-Debreu securities is available for trading.

3. Decentralizing with a Complete Set of Arrow-Debreu State Contingent Claims

Let $q_t(s_{t+j})$ denote the period t price of a state contingent claim which pays one unit of consumption if state s_{t+j} is observed in period t+j, where $s_{t+j} \in S_{t+j}$, the set of all possible such states, and nothing otherwise. As before, the states are economy-wide capital stock-shock pairs

(K_t, λ_t) but we use this representation at the moment for economy of notation. The expression $z_t(s_{t+j})$ denotes the quantity of the state contingent claim which pays in state s_{t+j} held by the consumer-worker-investor at the start of period t. In general, lower case variables represent agent specific quantities while upper case variables denote economy wide magnitudes. There is a continuum of consumer-worker investors indexed by [0, 1]. With this notation in hand a consumer-worker-investor is assumed to solve the following problem:

(13) $$V(z_0(s_0); K_0, \lambda_0) = \max_{\{i_t, z_t(s_{t+j})\}} E\left(\sum_{t=0}^{\infty} \hat{a}^t \, u(c_t, 1-n_t)\right)$$

s.t.

(14) $$c(s_t) + \sum_{j=1}^{\infty} \int_{s_{t+j}} q_{t+1}(s_{t+j}) z_{t+1}(s_{t+j}) \le \sum_{j=1}^{\infty} \int_{s_{t+j}} q_t(s_{t+j}) z_t(s_{t+j}) + z_t(s_t) + w_t(s_t) n(s_t)$$

It is assumed, at this stage, that the consumer-worker-investor's problem is well defined and that at each period t he takes as given the set of state contingent claims prices $\{q_t(s_{t+j})\}_{j=1,\, s_{t+j} \in S_{t+j}}^{\infty}$, and the wage rate $w_t(s_t)$. The term $z_t(s_t)$ represents the number of claims, acquired in period t-1, which pay off in the current state $s_t = (K_t, \lambda_t)$, while the term $w_t(s_t)n_t$, defines the agent's total period-t wage income; $c(s_t)$ is the residual consumption. Under quite reasonable assumptions the necessary and sufficient first order conditions for the consumer-worker-investor's problem are given by:

(15) $$n_t: \quad u_1(c(s_t), 1-n(s_t))\, w(s_t) = u_2(c(s_t), 1-n(s_t)), \text{ and}$$

(16) $$z_{t+1}(s_{t+j}): \quad u_1(c(s_t), 1-n(s_t))\, q_t(s_{t+j}) = \beta^j u_1(c(s_{t+j}), 1-n(s_{t+j}))\, dF^*(s_{t+j}; s_t)$$

where $dF^*(s_{t+j}; s_t)$ denotes the conditional probability of state s_{t+j} occurring in period t+j given that the period t state is s_t. Notice that under this construct consumers recursively purchase securities but do not directly decide the economy's level of investment. Neither do they own capital directly.

There is one firm which acts competitively. It issues state contingent claims of the type indicated and undertakes its investment and hiring decisions so as to maximize the value of these claims, the number of which exhausts the firm's dividends on a state by state basis. These dividends represent the firm's output less total wage payments and less output assigned to investment. Taking state claim prices as given, the firm solves the following recursive problem every period:

(17) $$v^f(k_0^f, \ddot{e}_0) = \max_{\{i_t^f\},\{n_t^f\}} \sum_{t=0}^{\infty} \int_{s_{t+j}} q_t(s_{t+j})\, z_t^{\,f}(s_{t+j})$$

$$\text{s.t.} \quad z_t^{\,f}(s_{t+j}) \equiv d^f(s_{t+j}) = f(k^f(s_{t+j}), n^f(s_{t+j}))\lambda(s_{t+j}) - w(s_{t+j})\, n^f(s_{t+j}) - i^f(s_{t+j})$$

$$k^f(s_{t+j+1}) = (1-\Omega)k^f(s_{t+j}) + i^f(s_{t+j})$$

where $d^f(s_{t+j})$ denotes the aggregate dividend paid by the firm in state s_{t+j}. We that the maximand in the above problem is well defined. Given that the economy is in state s_t , under reasonable assumptions the necessary and sufficient first order conditions for problem (17) are given by:

(18) $$n^f(s_t):\ q_t(s_t)[f_2(k^f(s_t), n^f(s_t))\,\lambda(s_t) - w(s_t)] = 0, \text{ and}$$

(19) $$i^f(s_t):\ -q_t(s_t) + \int_{s_{t+1}\in S_{t+1}} q_t(s_{t+1})\,[f_1(k^f(s_{t+1}), n^f(s_{t+1}))\,\lambda(s_{t+1}) + (1-\Omega)]\, dF^*(s_{t+1;}s_t) = 0.$$

We are now in a position to define equilibrium.

Definition: A decentralized equilibrium for this economy is a wage function $w(s_t)$ and a set of state-contingent price functions $\left\{q_t(s_{t+j})\right\}_{j=1,\, s_{t+j}\in S_{t+j}}^{\infty}$, such that (i) equations (15), (16), (18), and (19) are satisfied for all s_t and (ii) the following market clearing conditions are satisfied:

(20) $$n^f(s_t) = \int_0^1 n(s_t)d\gamma = n(s_t) = N(s_t);\ \ k^f(s_t) = K_t, \text{ and}$$

(21) $$z^f(s_t) = \int_0^1 z(s_t)d\tilde{a} = z(s_t) = Z(s_t) \text{ for all date-states}$$

$(s_t) \in S_t$, and for all times t; i.e., demand equals supply in the labor and securities markets. By Walras law, demand and supply must also be equated in the goods market; therefore

$$c(s_t) = \int c(s_t)d\gamma = \int [n(s_t)w(s_t) + z_t(s_t)d\gamma] = C(s_t)$$

for all t and for all $s_t \in S_t$. We may thus replace individual firm and agent quantities in equations (15), (16), (18), and (19) by the corresponding aggregate variable.

It remains to study the implications of this equilibrium concept for the dynamic behavior of aggregates. From (18), since $q_t(s_t) = 1$, we may conclude that

(22) $$w(s_t) = f_2(K(s_t), N(s_t))\lambda(s_t)$$

Combining this result with equation (15) gives

(23) $$u_1(C(s_t), 1\text{-}N(s_t))\, f_2(K(s_t), N(s_t))\lambda(s_t) = u_2(C(s_t), 1\text{-}N(s_t)),$$

for all states (s_t). Substituting the expressions for $q_t(s_{t+1})$, from equation (16) into equation (19) and observing that $q_t(s_t) = 1$, we obtain:

(24) $$1 = \beta\int \frac{u_1(C(s_{t+1}), 1-N(s_{t+1}))}{u_1(C(s_t), 1-N(s_t))}\left[f_1(K(s_{t+1}), N(s_{t+1}))\lambda(s_{t+1}) + (1-\Omega)\right] dF^*(s_{t+1}, s_t)$$

or, more conventionally, for any $s_t = (K_t, \lambda_t)$

(25) $$u_1(C(K_t, \lambda_t), 1-N(K_t, \lambda_t)) = \beta\int u_1(C(K_{t+1}, \lambda_{t+1}), 1-N(K_{t+1}, \lambda_{t+1}))$$
$$\left[f_1(K_{t+1}, N(K_{t+1}, \lambda_{t+1}))\lambda_{t+1} + (1-\Omega)\right] dF(\lambda_{t+1}; \lambda_t)$$

where $K_{t+1} = (1 - \Omega)K_t + I_t$. Equations (23) and (24) are, once again, the necessary and sufficient first-order-conditions for the stochastic growth paradigm (1). The set-up of this section thus constitutes an alternative to the PMB decentralization schemes, one that allows for a more active firm.

4. A Second Concept: Only the Equity Security is Traded

We retain the set-up of a continuum of consumer-worker-investors and one firm that behaves competitively. Under this second interpretation, there exists a legitimate stock market in which equity claims to the firm's net income stream are traded. Furthermore, ownership of this share is the only vehicle for savings by the consumer-worker-investors: no other assets are traded. We return to denoting the period t ex-dividend price of this equity security by q_t^e. The behavior of consumers is characterized by the solution to the following problem.

(26)
$$V(z_0, K_0, \ddot{e}_0) = \max_{\{z_{t+1},\, n_t\}} E\Big(\sum_{t=0}^{\infty} \hat{a}^t\, u(c_t, 1-n_t)\Big)$$

$$\text{s.t.}\quad c_t + q_t^e z_{t+1} \le (q_t^e + d_t) z_t + w_t n_t$$

where z_t represents the fraction of the single equity share held by the agent in period t, and d_t denotes the period t dividend to be defined shortly. Under standard assumptions the necessary and sufficient first order conditions are:

(27) n_t: $\quad u_1(c_t, 1 - n_t)\, w_t = u_2(c_t, l - n_t)$, and

(28) z_t: $\quad q_t^e u_1(c_t, 1-n_t) = \hat{a} \int u_1(c_{t+1}, 1-n_{t+1})\, (q_{t+1}^e + d_{t+1})\, dF(\ddot{e}_{t+1}; \ddot{e}_t)$.

The latter equation has the unique non-explosive solution:

(29)
$$q_t^e = E_t\left(\sum_{j=1}^{\infty} \beta^j \frac{u_1(c_{t+j}, 1-n_{t+j})}{u_1(c_t, 1-n_t)} \right) d_{t+j}$$

The representative firm begins period t with the stock of capital k_t^f carried over from the previous period, and one equity share outstanding $z_t^f = 1$. After observing the realization of the technology shock λ_t, it hires labor n_t^f taking the period equilibrium wage as given and produces and sells its output $f(k_t^f,\ n_t^f)\lambda_t$. The proceeds of the output sale are used to pay the wage bill, $w_t n_t^f$, to finance investments i_t^f**,** under the knowledge of the equation of motion on capital stock ($k_{t+1}^f\ =\ (1-\Omega)\, k_t^f + i_t^f$) and, residually, to pay dividends:

(30)
$$d_t\ =\ f(k_t^f,\ n_t^f)\lambda_t - w_t n_t^f - i_t^f\ .$$

In this setting of effectively complete markets, the firm's objective function is clear: maximize the pre-dividend stock market value of the firm, $d_t + q_t^e$, period by period. Substituting from (29), we can summarize the representative firm's decision problem as

(31)
$$V^f(k_0^f, \text{ë}_0) = \max_{\{(n_t);\{i_t)\}} (d_0 + q_0^e)$$

$$\text{s.t.}\qquad q_0^e + d_0\ =\ E\left(\sum_{t=0}^{\infty} \text{â}^j \frac{u_1(c_t, 1-n_t)}{u_1(c_0, 1-n_0)}\ d_t\right)$$

$$d_t\ =\ f(k_t^f,\ n_t^f)\,\text{ë}_t - w_t n_t^f - i_t^f$$

$$k_{t+1}^f\ =\ (1-\text{Ù})\, k_t^f + i_t^f\ .$$

Formulation (31) requires that shareholders convey to the firm a complete listing of their future intertemporal marginal rates of substitution. In the present complete markets setting and, a fortiori, in a homogenous agent environment, there would be perfect unanimity vis-a-vis the information to be provided. Alternatively, the shareholders could appoint one of their own members to manage the firm, knowing that his preference for future consumption is an exact representation of their own.

If we observe that problem (31) admits an equivalent sequential formulation this informational requirement can be further reduced. It may be recursively expressed as:

$$V^f(k_t^f, \lambda_t) = \max_{n_t^f,\, i_t^f} \left\{ \left[f(k_t^f, n_t^f)\lambda_t - w_t n_t^f - i_t^f \right] \right.$$

(32) $$\left. + \beta \frac{u_1(c_{t+1}, 1-n_{t+1})}{u_1(c_t, 1-n_t)} \int V^f(k_{t+1}^f, \lambda_{t+1})\, dF(\lambda_{t+1}; \lambda_t) \right\}$$

$$\text{s.t.} \qquad k_{t+1}^f = (1-\Omega)\, k_t^f + i_t^f .$$

The necessary and sufficient first-order conditions for problem (32) are

(33) $$n_t^f: \quad f_2(k_t^f,\, n_t^f)\,\lambda_t = w_t$$

(34) $$i_t^f: \quad -1 + E_t\left\{ \beta \frac{u_1(c_{t+1}, 1-n_{t+1})}{u_1(c_t, 1-n_t)} \left[f_1(k_{t+1}^f,\, n_{t+1}^f)\lambda_t + (1-\Omega) \right] \right\} = 0$$

Under this recursive formulation, the firm's investment decision only requires the knowledge of the typical shareholder's preference for current as opposed to next period's dividend. As before, shareholders would be unanimous and a single agent could be appointed to provide the required information.

Definition: Equilibrium in this economy is a wage function $w_t = w(K_t, \lambda_t)$ and a share price function $q^e(K_t, \lambda_t)$ such that (27), (28), (33) and (34) are satisfied, along with the usual market clearing conditions:

$$n_t^f = \int_0^1 n_t d\gamma = n_t = N_t,$$

(35) $$k_t = K_t, \text{ and}$$

$$z_t^f = \int_0^1 z_t d\gamma = z_t = 1 .$$

$$c_t = \int_0^1 c_t d\gamma = C_t$$

Notice that with these substitutions, equations (27) and (33) together give equation (23), while equations (34) and (25) are also identical. Once again we arrive at the necessary and sufficient conditions for the one good stochastic growth paradigm.

5. Delegated management

In this section, we go one step further and discuss the extent to which the stochastic growth model can be viewed as describing the time series properties of a decentralized economy in which firms' management is delegated to "firm managers" and incentive issues are present[3].

Our motivation is straightforward: firm owners (shareholders) rarely manage the firm they own. Typically management is delegated. This fact does not generate any new controversy, however, if it is assumed that managers implement the investment and hiring policies that are requested by (unanimous) shareholders; in that case, they behave like ordinary workers and receive a wage. But modern financial economics has emphasized that the world cannot be understood on these grounds and that incentive issues are pervasive. Our goal here is to bring macroeconomics one step closer to micro by introducing incentive issues at the general equilibrium level and discussing their macroeconomic implications.

Here we tackle this issue in the simplest context where managers are assumed to behave in their own best interest, given the remuneration scheme provided for them by shareholders. This is in contrast to a situation where they are requested to follow a certain strategy and monitored at a cost, possibly deviating from the prescribed policy depending on their own incentives and presumably penalized if they are caught pursuing their own agenda. Note that we

[3] Another extension in the same spirit is provided by Shorish and Spear (1996) who propose an agency theoretic extension of the Lucas (1978) asset pricing model.

do not detail the incentive problems that lead firm owners to propose a specific remuneration scheme that differs from a simple wage contract; rather, we are concerned to understand what schemes are compatible with the aggregate equilibrium allocations of the stochastic growth model.

Our description of the delegated management model is straightforward. There are two agent types: shareholder-workers and firm managers. The representative shareholder-worker is endowed with the standard problem, e.g. problem (26), that we repeat here for ease of discussion:

$$(26) \qquad V^s(z_0, k_0, \lambda_0) = \max_{\{z_{t+1}, n_t\}} E\left(\sum_{t=0}^{\infty} \beta^t u(c_t^s, 1 - n_t) \right)$$

$$\text{s.t.} \quad c_t^s + q_t^e z_{t+1} \le z_t (q_t^e + d_t) + w_t n_t$$

The only modification will be in the definition of dividends; the notation is otherwise unchanged.

Firm managers are assumed to maximize their lifetime utility of consumption. We do not make explicit their disutility for work or effort, which presumably gives rise to the incentive issue at the heart of our problem. Here we assume the manager's period consumption is limited to his remuneration without his being able to borrow or save. This extreme assumption helps clarify the issue at stake. It will be relaxed later. The manager solves:

$$(36) \qquad \begin{aligned} &V^m(k_0, \lambda_0) = \max_{\{n_t^f, i_t\}} E\left(\sum_{t=0}^{\infty} \beta^t v(c_t^m) \right) \\ &\text{s.t.} \\ &0 \le c_t^m \le g^m(CF_t) \\ &g^m(CF_t) + d_t = CF_t;\ d_t \ge 0 \\ &CF_t = f(k_t, n_t^f)\lambda_t - n_t^f w_t - i_t^f \\ &k_{t+1}^f = (1 - \delta)k_t^f + i_t^f;\ k_0 \text{ given.} \end{aligned}$$

In problem (36) the manager's compensation is presumed to be a function exclusively of the firm's real free cash flow CF_t. This latter quantity is simply its output (revenue) from which are subtracted its wage payments to workers and its investment. Since the free cash flow is then paid either to the manager or the shareholders as a dividend, our hypothesized compensation function can be roughly described as profit sharing.

Our first result is summarized in Theorem 5.1. All proofs are to be found in the Appendix.

Theorem 5.1: Suppose that u and v are increasing, concave and continuously differentiable, and that $g^m(CF_t)$ is continuously differentiable with $g_1^m(.) > 0$. Then the 'delegated management' economy has the same aggregate equilibrium investment function and labor services as the economy corresponding to problem (1) if and only if there is a constant $L > 0$ such that

$$Lv_1(g^m(CF_t))g_1^m(CF_t) = u_1(c_t, 1-n_t), \tag{37}$$

where c_t, n_t, denote the optimal consumption and labor service functions which solve problem (1).[4]

In substance, Theorem 5.1 asserts that if equation (37) is satisfied the time path of an economy with delegated management will be exactly identical to that of the standard neo-classical growth economy. In other words, the time series properties of the latter economy correspond as well to those of a decentralized economy with delegated management. This is an additional extension in the range of applicability of the neo-classical stochastic growth model, one that we believe opens up an interesting set of issues.

At this point, one may wonder if, under such conditions, the decentralized equilibrium indeed corresponds to a Pareto optimal allocation of resources, i.e., is condition (37) compatible with a first best equilibrium for the 'delegated management' economy? The answer is provided by Theorem 5.2, and it is highly restrictive.

Theorem 5.2 : Consider economies (26), (36) and (1), respectively, and suppose condition (37) is satisfied. Then the equilibrium of the delegated management economy is P.O. if and only if the manager's contract is of the form

$$g^m(CF_t) = \theta CF_t$$

for some constant $0 < \theta < 1$.

The conditions of Theorem 5.2 are demanding. The following corollary demonstrates that in the pure context of problems (1) and (26), (36), that is, in the absence of additional markets or income sources for the agents of our economy, these conditions typically preclude competitively determined labor income to go to the worker-shareholders (in contrast with the typical assumption made in Sections 2-4).

Corollary 5.2: Under the conditions of a CRS Cobb-Douglas technology and equally risk averse managers and shareholder workers each with CES utility, condition (37) is not compatible with a Pareto optimum if worker shareholders receive the competitively determined wage income.

The essence of Corollary 5.2 is to point out that it will be difficult to achieve complete risk sharing in a competitive world where one agent receives at a minimum a fraction $(1-\alpha)$ (.64

[4] Note that all CF_t, c_t, n_t are all functions of the state variables (K_t,λ_t). We suppress this identification for notational simplicity.

under standard parameterizations) of total income, while the other receives only a portion of the remainder after investment has been subtracted from it.

This result leaves us in an uncomfortable position. By implementing a remuneration policy $g^m(CF_t)$ satisfying condition (37), worker-shareholders insure an investment policy that is identical to the policy in force in the problem (1) economy, which we know led to the firm's value being maximized and Pareto optimal consumption and employment decisions in that context. But they now have to pay their firm managers, according to a scheme $g^m(CF_t)$ satisfying (37) and at an average level that is market determined. Finally, our last result asserts that "there is something left on the table", that the income distribution between the two agent types is suboptimal. Of course this is not something totally surprising in an economy with incentive issues. But these results would seem to suggest that in these circumstances other arrangements are likely to prevail between shareholders and managers, ones where the incentive scheme of managers will deviate from condition (37), *thus leading to a dynamic path for the economy that deviates from the one described by the neo-classical stochastic growth model*.

At this stage, however, it is worthwhile to question our hypothesis of no borrowing or saving for firm managers and to address the issue of what other financial instruments and income sources might be available.

Indeed, as we have noted (Section 3), the stochastic growth model is one of (implicitly) complete markets. Once we have more than one agent type, as in our delegated management economy, this implicit assumption demands a set-up where a complete set of security markets is effectively available. The reasoning underlying Theorem 5.1 appears , however, to be robust to introducing other sources of income for managers, including income from security trading. Under these circumstances, equation (37) would become

(38) $$Lv_1(c_t^m)g_1^m(CF_t) = u_1(c_t, 1-n_t)$$

while the Pareto optimal risk sharing condition would be

(39) $$Hv_1(c_t^m) = u_1(c_t, 1-n_t)$$, for some constant H

These latter conditions maintain the necessity of a remuneration scheme for firm managers that is linear in cash flows.

Now if we suppose that security markets are indeed complete, equation (39) may be expected to obtain "independently" as a consequence of security trading. But then the conclusion appears to be that imposing in addition the condition of a linear remuneration scheme for firms' managers will suffice to guarantee that equation (38) is satisfied. In that case we have operational necessary and sufficient conditions for an "equivalent" decentralization of the neo-classical stochastic growth model with delegated management. These considerations are summarized in our final Theorem:

Theorem 5.3: Suppose securities markets are complete. Then

a) the equilibrium of the "delegated management" economy is Pareto optimal, and
b) its aggregate time series properties are identical to those of the standard neoclassical stochastic growth model (problem (1))

if and only if the managers' contracts are linear in the firms' cash flows.

If contracts are not linear, managers will deviate from the first-best, value maximizing investment policy; the equilibrium will not be optimal. Although the fact that securities markets are complete implies that there will redistributive efficiency, allocative efficiency will not prevail.

Theorem 5.3 provides us the basis for opening up the discussion of the macroeconomic implications of deviations from linear remuneration schemes. Corollary 5.3 illustrates.

Corollary 5. 3. The time series properties of a delegated management economy when markets are complete and managers are remunerated with stock options will differ from the time series properties of the stochastic growth model.

Proof: Stock options or other price based compensation arrangements are not linear in CF_t. The conclusion follows from Theorem 5.3.

The implications of Corollary 5.3 are substantial. As executive compensation becomes more and more incentive based (viz., compensation in the form of stock options), the standard one good growth model and its numerous extensions will become a less relevant abstraction. Since the one good model reasonably approximates the behavior of many macroaggregates, these results also suggest that the statistical properties of aggregate time series may be altered as the economy becomes more incentive based.

The assumption of complete markets (retained because of the complete markets interpretation of problem (1)) is also at issue and may be challenged along the following lines. It is well understood that incentive based compensation has the intention of harmonizing shareholder and manager interests by correlating the manager's wealth position more closely with that of the shareholders. Under the assumption of market completeness where managers and shareholders insure one another against income shocks, some of the alignment is implicitly lost. With a non linear contract and an absence of complete markets for pooling risk, it is not clear how closely the consequent time series arising from problem (26), (36) will resemble those of (1).

6. Conclusion

In this paper we have sought to provide various perspectives on decentralizing the stochastic growth model, all of which allow investment decisions to be undertaken by firms. When managers are passive, Sections 3-4 provide two possible interpretations. When managers are no longer passive but are influenced by the structure of their compensation function, it turns out that the behavior of the one good growth model will be replicated only if the contract has a very specific form. Since these contracts are not normally observed, this observation suggests that either the one good growth model is not appropriate to an economy with incentive based compensation, or that the historical time series regularities may not be observed in the future.

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Appendix

Proof of Theorem 5.1:

Under our assumptions problem (36) has recursive representation

$$V^s(z_t;k_t,\lambda_t) = \max_{\{z_{t+1}s,n_t^s\}} \{u(z_t(q_t^e + d_t) + w_t n_t - q_t^e z_{t+1}, 1-n_t)$$

$$+ \hat{a}\int V^s(z_{t+1};k_{t+1}\,\ddot{e}_{t+1})dF(\ddot{e}_{t+1};\ddot{e}_t)\}$$

and necessary and sufficient first order conditions

(A1) $\quad n_t: \quad u_1(c_t^s,1-n_t)w_t = u_2(c_t^s,1-n_t)$

(A2) $\quad z_{t+1}: \quad u_1(c_t^s,1-n_t)\,q_t^e = \hat{a}\int u_1(c_{t+1}^s,1-n_{t+1})[q_{t+1}^e + d_{t+1}]dF(\ddot{e}_{t+1};\ddot{e}_t)$.

Under the same sufficient conditions, the manager's problem has recursive representation:

(A3) $\quad V^M(k_t,\lambda_t) = \max_{\{i_t,n_t^f\}}\{V(g^M(cF_t)) + \beta\int V^M(k_{t+1},\lambda_{t+1})dF(\lambda_{t+1},\lambda_t)\}$

The necessary and sufficient first order conditions to problem (A3) are

(A4) $\quad u_1(g^m(CF_t))g_1^m(CF_t)[f_2(k_t,n_t^f)\lambda_t - w_t] = 0$

(A5) $v_1\left(g^m(CF_t)\right)g_1^m(CF_t) = \int v_1\left(g^m(CF_{t+1})\right)g_1^m(CF_{t+1})\left[f_1\left(k_{t+1},n_{t+1}^f\right)\lambda_{t+1} + (1-\Omega)\right]dF(\lambda_{t+1};\lambda_t)$ where this latter representation was obtained using a standard application of the envelope theorem.

By assumption, $v_1(\ ) > 0$ and $g_1^m(\ ) > 0$ so that we may assert, from (A4), that

(A6) $$f_2\left(k_t, n_t^f\right)\lambda_t = w_t$$

Combining (A6) and (A1) and recognizing that in equilibrium $n_t = n_t^f$ we may also assert that

(A7) $$u_1\left(c_t^s, 1-n_t\right)f_2\left(k_t, n_t^f\right)\ddot{e}_t = u_2\left(c_t^s, 1-n_t\right) \text{ for every state } (k_t, \lambda_t).$$

By requirement (37), we can re-express equation (A5) as

(A8) $$\left(\frac{1}{L}\right)u_1\left(c_t^s, 1-n_t\right) = \hat{a}\int\left(\frac{1}{L}\right)u_1\left(c_{t+1}^s, 1-n_{t+1}\right)\left[f_1\left(k_{t+1}, n_t\right)\lambda_{t+1} + (1-\grave{U})\right]dF(\lambda_{t+1};\lambda_t)$$

where we have also substituted $n_t^f = n_t$. Canceling $\left(\frac{1}{L}\right)$ terms in equation (A8) gives us (in conjunction with (A7)) the necessary and sufficient first order conditions for problem (1).

$\Rightarrow$ Since economies (1) and (26)-(36) have the same equilibrium investment and labor service functions, the set of feasible equilibrium capital stock, shock pairs is the same. Let us approximate this set by a discretized counterpart, and denote it by $K \times \Lambda$.

With regard to the economy described by problems (26) and (36), for each $(k_i, \lambda_i) \in K \times \Lambda$, let us make the following identification:

$$A(k_i, \lambda_i) \equiv v_1\left(g^M(CF(k_i, \lambda_i))\right)g_1^M(CF(k_i, \lambda_i)),$$

for each $(k_i, \lambda_i) \in K \times \Lambda = \{(k_i, \lambda_i)\}_{i=1,2,\ldots,N}$.

Optimality condition (A5) can then be expressed as:

(A9) $$\left(A(k_1,\lambda_1),\ldots,A(k_N,\lambda_N)\right) = W\begin{pmatrix} A(k_1,\lambda_1) \\ A(k_2,\lambda_2) \\ \vdots \\ A(k_N,\lambda_N) \end{pmatrix}$$

where the entries of W_{ij} are of the form

$$w_{ij} = \beta\pi_{ij}\left[f_1\left(k_j, n\left(k_j,\lambda_j\right)\right)\lambda_j + (1-\Omega)\right]$$

and the probabilities ð$_{ij}$ are defined by

$$\pi_{ij} = \begin{cases} 0, \text{ if } k_j \neq (1-\Omega)k_i + i(k_i,\lambda_i) \\ dF\left(\lambda_j;\lambda_i\right), \text{ if } k_j = (1-\Omega)k_i + i\left(k_i,\lambda_j\right) \end{cases}$$

We note that by assumption all the $\{A(k_i,\lambda_i)\}$ terms are strictly positive.

In a like fashion, let us identify

$$B(k_i,\lambda_i) = u_1\left(c^s(k_i,\lambda_i), 1-n(k_i,\lambda_i)\right)$$

where the utility function arguments represent the optimal policy functions for problem (1). Since the investment and labor service functions for problems (26), (36) and problem (1) are identical we can express the optimality condition for investment for this latter problem as

(A10) $$\left(B(k_i,\lambda_i),\ldots,B(k_N,\lambda_N)\right) = W\begin{pmatrix} B(k_i,\lambda_i) \\ \vdots \\ B(k_N,\lambda_N) \end{pmatrix}$$

where W is the same matrix as above.

The system of equations (A9) and (A10) are homogenous and have strictly positive solutions $\{A(k_i,\lambda_i)\}$and $\{B(k_i,\lambda_i)\}$respectively. It follows (Theorem 3, Hoffman and Kunze (1965)) that there must exist a constant L>0 such that

$$\left(B(k_1,\lambda_1),\ldots,B(k_N,\lambda_N)\right) = L\left(A(k_1,\lambda_1),\ldots,A(k_N,\lambda_N)\right)$$

as required by (37). The same result holds under a continuum state up to a set of measure zero.☝

Proof of theorem 5.2:

A Pareto Optimal allocation for economy (26), (27) under the above assumptions is defined as the solution to

$$\max_{\{i_t, c_t^s, c_t^m, n_t\}} E\left(\sum_{t=0}^{\infty} \beta^t [u(c_t^s, 1-n_t) + \Psi V(c_t^m)]\right)$$

s.t. $c_t^s + c_t^m + i_t \le f(k_t, n_t)\lambda_t$

$k_{t+1} = (1\text{-}\Omega) k_t + i_t; k_0$ given,

for some constant ψ. The necessary and sufficient first order conditions are

(A11) $u_1(c_t^s, 1-n_t) = \Psi v_1(c_t^m)$, and

(A12) $u_1(c_t^s, 1-n_t) = \beta \int u_1(c_{t+1}^s, 1-n_t)[f_1(k_{t+1}, n_t)\lambda_{t+1} + (1-\Omega)] dF(\lambda_{t+1}; \lambda_1)$

in addition to the standard FOC on n_t.

If (37) and (A11) are to be jointly satisfied,

Then $g_1^m(CF_t)L \equiv \Psi$, or

$g_1^m(CF_t) \equiv \theta$, a constant. Hence

$g^m(CF_t) = a + \theta CF_t$, for constants a and θ. Since both $d_t \ge 0$ and $g^m(CF_t) \ge 0$, and $g^m(CF_t) + d_t \equiv CF_t$, if $CF_t = 0$, $g^m(CF_t) = 0$ and thus a=0. But with a=0, the preceding conditions impose 0<θ<1.

$\Leftarrow$ if the contract has the indicated form, then (37) reduces to

$$Lv_1(\theta CF_t)\theta = u_1((1-\theta)CF_t + w_t n_t, 1-n_t)$$

which is (A11) with Lθ=ψ.

Proof of Corollary 5.2:

Let us make the by-now-standard identifications that $y_t = f(k_t, n_t)\lambda_t = k_t^{\alpha} n_t^{1-\alpha}\lambda_t$, and that the shareholder's utility is CES and separable in consumption and leisure (as, e.g., in Hansen (1985)). Let $v(c_t^M)$ also be of the CES family and let the CRRA of both parties coincide.

Pareto Optimality in conjunction with (37) implies that for some $0 < \xi < 1$, $g^m(cF_t) = a + \xi CF_t$, and that under the requirement $d_t \geq 0$, a = 0. If wages are competitively determined, $n_t w_t = (1-\alpha)y_t$, and thus $CF_t = \alpha y_t - i_t$. Under Pareto optimality, there must exist a $\varphi > 0$ such that

(A13) $\varphi v_1(\xi(\alpha y_t - i_t)) = u_1((1-\alpha)y_t + (1-\xi)\alpha y_t - (1-\xi)i_t)$, which under CES reduces to

$$\varphi(\xi(\alpha y_t - i_t))^{-\gamma} = ((1-\alpha)y_t(1-\xi)\alpha y_t + (1-\xi)i_t)^{-\gamma},$$

or, $\theta(\xi(\alpha y_t - i_t)) = (1-\alpha)y_t + (1-\xi)\alpha y_t - (1-\xi)i_t$,

where $\theta = \varphi^{-1/\gamma}$. Combining like terms yields

$$\theta\xi\alpha y_t - \theta\xi i_t = [(1-\alpha) + (1-\xi)\alpha]y_t - (1-\xi)i_t.$$

Equating coefficients of like terms gives

$\theta\xi = (1-\xi)$, and

$$\theta\xi\alpha = (1-\alpha) + (1-\xi)\alpha$$

Substituting the expression for $\theta\xi$ into the second equation yields

$$(1-\xi)\alpha = (1-\alpha) + (1-\xi)\alpha.$$

This latter equation is only satisfied if $\alpha = 1$; i.e., there is no competitively determined wage income.

Proof of Theorem 5.3:

The manager and the shareholder-worker-investor trade state claims among themselves. We borrow the notation for state claims from Section 3; otherwise the notation of the current section is employed.

Under the assumption of complete markets, the problems confronting the shareholder-worker-investor and the manager become, respectively:

$$V^s(\{z_0^s(s_j)\}_{j=1}^{\infty}, z_0^s(s_0); k_0, \lambda_0) = \max_{\{z^s(s_{t+j})\},\{n_t\}} E(\sum_{t=0}^{\infty} \beta^t u(c_t^s, 1-n_t))$$

$$\text{s.t. } c^s(s_t) + \sum_{J=1}^{\infty}\sum_{s_{t+j}} q_t(s_{t+j}) z_{t+1}^s(s_{t+j}) + q_t^c z_{t+1} \le \sum_{J=1}^{\infty}\sum_{s_{t+j}} q_t(s_{t+j}) z_t^s(s_{t+j}) + z^s(s_t) + n_t w_t + q_t^e(z_t + d_t),$$

and

$$v^m(\{z_0^m(s_j)\}_{j=1}^{\infty}, z_0^M(s_0); k_0, \lambda_0) = \max_{\{i_t\},\{n_t^f\},\{z^M(s_{t+j})\}} E(\sum_{t=0}^{\infty} \beta^t v(c_t^M))$$

$$\text{s.t. } c^M(s_t) + \sum_{J=1}^{\infty}\sum_{s+j} q_t(s_{t+j}) z_{t+1}^M(s_{t+j}) \le \sum_{J=1}^{\infty}\sum_{s_{t+j}} q_t(s_{t+J}) z_t^m(s_{t+j}) + g^m(CF_t)$$

$d_t + g^m(CF_t) = CF_t = f(k_t, n_t^f)\lambda_t - w_t n_t - i_t$, and

$k_{t+1} = (1-\Omega)k_t + i_t$

Under our maintained assumptions the necessary and sufficient first order conditions are, respectively

(i) $z_t : u_1(c_t^s, 1-n_t) q_t^e = \beta \int u_1(c_{t+1}^s, 1-n_{t+1})[q_{t+1}^e + d_{t+1}^e] dF(\lambda_{t+1}; \lambda_t)$

(ii) $n_t : u_1(c_t^s, 1-n_t) w_t = u_2(c_t^s, 1-n_t^s)$

(iii) $z^{s}_{t+1}(s_{t+j}): u_1(c^s(s_t),1-n(s_t))q_t(s_{t+j}) = \beta^j u_1(c^s(s_{t+j}),1-n(s_{t+j}))dF^*(s_{t+j};s_t)$

and

(iv) $n^f_t : v_1(c^M_t)g_1(cF_t)[f_2(k_t,n^{-f}_t)\lambda_t - w_t] = 0$

(v) $i_t : v_1(c^m_t)g_1(CF_t) = \beta\int v_1(c^m_{t+1})g_1(CF_{t+1})\ [f_1(k_{t+1},n^f_{t+1})\lambda_{t+1} + (1-\Omega)]dF(\lambda_{t+1};\lambda_t)$

(vi) $z^m_{t+1}(s_{t+j}) : v_1(c^m_t)q_t(s_{t+j)}) = \beta^j v_1(c^m(s_{t+j}))dF^*(s_{t+j};s_t)$.

In equilibrium $n^s_t = n^f_t$. Since $g_1(CF_t) > 0$ and $v_1(c^m_t) > 0$

$$w_t = f_2(k_t,n^s_t)\lambda_t$$

Substituting for w_t in equation (ii) gives,

(A14) $u_1(c^s_t,1-n^s_t)f_2(k_t,n^s_t)\lambda_t = u_2(c^s_t,1-n^s_t)$.

Since for any state s_{t+1}, by (iii) and (vi)

$$q_t(s_{t+1}) = \beta\frac{u_1(c(s_{t+1}),1-n(s_{t+1}))}{u_1(c(s_t),1-n(s_t))} = \beta\frac{v_1(c^m(s_{t+j}))}{v_1(c^m(s_t))}$$

we may write equation (v) as

$$1 = \beta\int\frac{v_1(c^m(s_{t+1}))g_1(CF(s_{t+j})}{v_1(c^m(s_t))g_1(CF(s_t))}[f_1(k_{t+1},n_{t+1})\lambda_{t+1} + (1-\Omega)]dF(\lambda_{t+1};\lambda_t) \quad \text{or}$$

(A15) $1 = \beta\int\frac{u_1(c^s(s_{t+1}),1-n(s_{t+1}))}{u_1(c^s(s_t),1-n(s_t)}[f_1(k_{t+1},n_{t+1})\lambda_{t+1} + (1-\Omega)]dF(\lambda_{t+1};\lambda_t)$

since $g_1(CF)$ is constant.

By (A14) and (A15), the aggregate time series of the “delegated management’ economy will coincide with the time series of the economy summarized by Problem (1).

$\Leftarrow$ Since the aggregate time series of the delegated management economy coincide with those of the economy described by problem (1), by Theorem (1), it must be that for every state

$$Lv_1(c^M(s_t))g_1^M\left(CF(s_t)\right) = u_1(c^s(s_t), 1 - n(s_t))$$

for some constant L. Since the allocation is also Pareto optimal, it must simultaneously be the case that

$$v_1(c^M(s_t)) = \Psi u_1(c^s(s_t), 1 - n(s_t))$$

for some constant Ψ and all states s_t.

Thus $g_1^M(s_t) \equiv \frac{\Psi}{L}$, and the contract is linear.