# Should we base procurement rules on the competition of linear incentive contracts?*

François MARECHAL†

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**Abstract**

The study of optimal procurement contracts under informational asymmetries generally assumes that the cost disturbance affecting contractor's cost function is not observed by the principal. We assume here that this variable (which may represent environmental or geology conditions...) can be observed in the process of the contract. Thus, the principal is now able to make the payment contingent on the realization of this variable. In this context, the aim of this paper is to compare a linear incentive contract with a "modified" fixed-price contract, which allows the payment to the selected contractor to be independent upon his bid in the case of a high-cost value of exogenous uncertainty.

Keywords : Procurement Bidding, Linear Incentive Contracts, Fixed-Price Contract.

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†DEEP-HEC, University of Lausanne.

E-mail address: francois.marechal@hec.unil.ch

## 1 Introduction

In 1999, direct procurement spending on the part of governments around the world amounted to an estimated \$5.433 trillion (U.S.) or approximately 18% of total world economic output[1]. Public procurement particularly represents about 11% of the EU's GDP[2]. Given the important size of goods and services bought by governments, the design of contracts minimizing acquisition costs is a significant question which has been studied in the mechanism design literature[3] as a Principal/Agent problem. In such a situation, the government (principal) employs a firm (agent) to perform some task. However, contracts between principal and agent are quite difficult to design under asymmetries of information (adverse selection and moral hazard) and exogenous uncertainties.

Laffont and Tirole (1987) and McAfee and McMillan (1987a) characterize the optimal contract when several risk-neutral bidders compete for the award of a contract. They show that the optimal allocation can be implemented by offering the firms a menu of linear contracts (namely, the optimal contract is linear in realized costs but non-linear in bids). When bidders are risk averse, the design of the optimal contract is still an open question. However, McAfee and McMillan (1986) characterize the optimal contract that encompasses risk aversion, adverse selection and moral hazard, but with the a priori restriction to linear contract. The optimal linear contract is called a linear incentive contract and combines features of the cost-plus contract and the fixed-price contract. Indeed, the government's payment depends on both the firm's bid and the firm's actual costs, according to a pre-arranged ratio. Linear incentive contracts are planned by the US procurement regulation[4] and are notably used by the US Department of Defense for weapons acquisition[5].

A common feature of McAfee and McMillan (1986, 1987a) and Laffont and Tirole (1987) is that the *ex post* realized cost of the project is observed by the principal whereas the efficiency

[1] Source: Oxford Analytica Academic Database.

[2] Source: http://europa.eu.int/business/en/topics/publicproc/

[3] See Laffont and Tirole (1993) for a survey of this literature.

[4] See the *Federal Acquisition Regulation* (Subpart 16-4).

[5] For a detailed analysis of linear incentive contracts, see McAfee and McMillan (1988).

parameter of each firm and the winner's effort level are private information. Furthermore, in McAfee and McMillan (1986), the cost function of the firms involves a shock which represents exogenous uncertainties and which is not observed by the principal. Note that the cost function of Laffont and Tirole (1987) does not involve a shock. Nevertheless, they show that the optimal contract is robust to the introduction of a random disturbance in the cost function[6]. So, a common feature of optimal contracts is that the cost shock is not observed *ex post* by the principal. Thus, the form of the payment can not be contingent on this cost disturbance.

However, in procurement practice, the magnitude of exogenous uncertainties may be observed during the production of the project. We can think for instance of environmental or geology conditions.... Resulting from this observation, there may be a place for renegotiation. This argument may explain why optimal contracts are not widely used in practice. Indeed, most projects are awarded by means of fixed-price contracts[7]. The price is then arranged before the work is begun. Under a fixed-price contract, the firm is given the appropriate incentive to minimize its costs. However, it forces the firm to bear all the risks due to exogenous uncertainties. In practice, *ex post* adaptations of the initial payment by means of change orders allow a shift of risk from the agent to the principal, so that initial fixed-price contracts may involve a renegotiation. In Bajari and Tadelis (2001) *e.g.*, the design of the project may be incomplete and the probability of *ex post* change orders is endogenous.

In this paper, we depart from this analysis, considering complete contracts. Following McAfee and McMillan (1986), we consider that the procurement problem is one of *ex ante* asymmetric information with moral hazard. We also consider that the *ex post* realized cost of the project is affected by exogenous uncertainties represented by a random variable $\theta$. However, unlike McAfee and McMillan (1986), we assume that $\theta$ is observed after the selected contractor has chosen its cost-reducing effort. We also assume that $\theta$ can be contractible. Thus, the principal is able to make the payment to the selected firm contingent on the

[6] Obviously, this results holds when the principal and the potential contractors are risk-neutral (see Laffont and Tirole (1993)).

[7] Cost-plus contracts are also often used. The firm is then reimbursed for costs plus a stipulated fee.

realization of $\theta$.

In this framework, we first derive the optimal linear incentive ($LI$) contracts when the payment depends on the realization of the cost shock, *i.e.* the payment is "state-dependent". As we consider that $\theta$ can take on one of two values, we now have two different cost-share parameters for the two values of $\theta$. However, we show that the expected price under this "state-dependent" $LI$ contract is equal to the one under McAfee and McMillan's (1986) $LI$ contract.

Then, we design a new contract which specifies the following payment. In the case of a low-cost state, the payment corresponds to the one of a fixed-price contract. In the case of a high-cost state, the winner is reimbursed for the *ex post* realized cost plus the disutility of his effort. This kind of procedure can be called a "modified fixed-price" ($MFP$) contract. Although the effort level is not observable, the payment can be contingent on it since the moral hazard problem is solved. Indeed, we show that the selected agent chooses the first best effort level. If the principal was restricted to choose between an $LI$ contract and an $MFP$ contract, the goal of this paper is to know which contract minimizes the principal's expected payment to the selected agent.

Following McAfee and McMillan (1986), in the special case of risk-neutral bidders, the optimal $LI$ contract trades off stimulating competition in the initial bidding against giving the firm incentives to reduce its production costs. Under an $MFP$ contract, we show that the selected agent chooses the first best effort level. However, the bidding competition effect now depends upon the exogenous probability of realization of a high-cost state. Intuitively, an $MFP$ contract yields a lower expected price than an $LI$ contract if the gain from inducing the first best effort level is greater than the (potential) loss in stimulating competition in the initial bidding.

## 2 The optimal *LI* contract when the payment is "state dependent"

Following McAfee and McMillan (1986), we consider that $n$ bidders compete for the realization of a project. However, in contrast to McAfee and McMillan (1986), we restrict attention to risk-neutral bidders. Given that agent $i$ is selected, the actual *ex post* cost of the project is

$$C_i = c_i - e_i + \theta,$$

where $c_i$ is the efficiency parameter (including opportunity costs) of agent $i$, $e_i$ represents the cost-reduction due to agent $i$'s effort and $\theta$ denotes a random variable representing exogenous uncertainties. The principal neither observes $c_i$ nor $e_i$. However, it is common knowledge that $c_i$ is independently drawn from a distribution $F(.)$, which corresponding density $f(.)$ has support $[c^-, c^+]$. We assume that $F(.)$ has a monotone hazard rate ($F/f$ non-decreasing). For agent $i$, the choice of an effort $e_i$ yields a disutility (in monetary units) $h(e_i) = e_i^2/2d$ (with $d > 0$), which can not be incorporated in his bid. The function $h(.)$ is also common knowledge. Following McAfee and McMillan (1986), we assume that the *ex post* realized cost $C_i$ of the selected firm is observed. However, the main departure from McAfee and McMillan (1986) is to assume that $\theta$ is also observed after the selected contractor exerts its effort in reducing production costs. $\theta$ is assumed to be contractible and the payment to the winner can be contingent on its realization. We assume that $\theta$ can take on one of two values: $\theta_H$ (with probability $p$) and $\theta_L$ (with probability $(1-p)$). We also assume without loss of generality that

$$E(\theta) = p\theta_H + (1-p)\theta_L = 0, \text{ with } \theta_H > 0 > \theta_L.$$

Note that the main departure from McAfee and McMillan (1986) is to assume that $\theta$ is observed and contractible. Dunne and Loewenstein (1995) emphasize[8] that in McAfee and

[8] In contrast to our model, Dunne and Loewenstein (1995) analyse the optimal linear incentive contract when $\theta$ is privately observed by the selected agent. This agent then reports (or misreports) the value of $\theta$ and monitoring is assumed to be costly for the principal.

McMillan (1986), $\theta$ can be inferred by the principal since $C_i$ is observed, but the payment can not be contingent on the realization of $\theta$. In our model, if $\theta$ was not contractible, we would have a special case of that of McAfee and McMillan (1986) with a binary uncertainty, with risk-neutral bidders and with a quadratic disutility of effort function.

Define $C_{iH} = c_i - e_i + \theta_H$ and $C_{iL} = c_i - e_i + \theta_L$. Particularly, the sequence of events under an *LI* contract is as follows. The principal first commits to the payment rule

$$P_i = \begin{cases} \alpha_H C_{iH} + (1-\alpha_H)\, b_i = b_i + \alpha_H \,(C_{iH} - b_i) \text{ if } \theta = \theta_H, \text{ with } 0 \le \alpha_H < 1 \\ \alpha_L C_{iL} + (1-\alpha_L)\, b_i = b_i + \alpha_L \,(C_{iL} - b_i) \text{ if } \theta = \theta_L, \text{ with } 0 \le \alpha_L < 1 \end{cases} \tag{1}$$

where $b_i$ denotes the bid submitted by agent $i$.

Thus, the principal commits to reimburse a fraction $\alpha_H$ of the *ex post* realized cost if $\theta = \theta_H$ and a fraction $\alpha_L$ otherwise[9]. The contract is awarded by means of a first-price sealed-bid auction. Then, the winner chooses his cost-reduction effort. $\theta$ and $C_i$ are observed and the winner gets the payment $P_i$. The optimal *LI* contract is designed by the cost-share parameters $\alpha_H$ and $\alpha_L$ which minimize the principal's expected payment while taking into account bidder's responses. In contrast to McAfee and McMillan (1986), bidders are assumed to be risk-neutral, so the problem of risk-sharing between the principal and the selected contractor disappears. The optimal choices of $\alpha_H$ and $\alpha_L$ are only determined by a trade-off between stimulating bidding competition and giving the selected firm incentives to reduce its production costs. The larger the parameters $\alpha_H$ and $\alpha_L$ are, the lower is the government's expected payment because of the bidding competition effect, but the higher is the government's expected payment because of the moral hazard effect.

Under an *LI* contract, agent $i$'s expected profit (where expectation is taken over $\theta$) when he submits a bid $b_i$ is

$$\begin{aligned} E(\pi) &= p\,[b_i + \alpha_H\,(C_{iH} - b_i) - C_{iH} - h(e_i)] + (1-p)\,[b_i + \alpha_L\,(C_{iL} - b_i) - C_{iL} - h(e_i)] \\ &= [1 - p(\alpha_H - \alpha_L) - \alpha_L]\,(b_i - c_i + e_i) - h(e_i) + p\,(\alpha_H - \alpha_L)\,\theta_H. \end{aligned}$$

[9] In McAfee and McMillan (1986), the payment to the selected agent is $P_i = b_i + \alpha\,(C_i - b_i)$. The cost-share parameter $\alpha$ is designed to minimize the expected payment, while expectation is taken over $\theta$.

The selected agent chooses his effort level to maximize his expected profit. $e_i^{LI*}$ thus satisfies

$$e_i^{LI*} = h'^{-1}\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right] \;\Rightarrow\; e_i^{LI*} = d\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right].$$

Consider now the bidding strategy of agent $i$. Let $b(.)$ denotes the strictly increasing bidding strategy employed by all his opponents. Since the contract is awarded to the lowest bidder, $i$ wins with a bid $b_i$ if

$$b_i < b(c_j) \;\; \forall j \neq i,$$

*i.e.* with probability $\left[1 - F\left(b^{-1}(b_i)\right)\right]^{n-1}$. Agent $i$'s *interim* expected profit is then

$$\begin{aligned} E(\pi)_{interim} &= \left\{\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right](b_i - c_i + e_i) - h(e_i) + p(\alpha_H - \alpha_L)\theta_H\right\} \\ &\quad \left[1 - F\left(b^{-1}(b_i)\right)\right]^{n-1}. \end{aligned}$$

Following the methodology in McAfee and McMillan (1987b), we can show that the symmetric Nash equilibrium bidding strategy $b^{LI}(c_i)$ of agent $i$ is given by

$$b^{LI}(c_i) = c_i + \frac{\int_{c_i}^{c^+}(1 - F(s))^{n-1}ds}{(1 - F(c_i))^{n-1}} - e_i^{LI*} + \frac{h\left(e_i^{LI*}\right)}{\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right]} - \frac{p(\alpha_H - \alpha_L)\theta_H}{\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right]},$$

with $b^{LI}(c^+) = c^+ - e_i^{LI*} + \frac{h\left(e_i^{LI*}\right)}{[1-p(\alpha_H-\alpha_L)-\alpha_L]} - \frac{p(\alpha_H-\alpha_L)\theta_H}{[1-p(\alpha_H-\alpha_L)-\alpha_L]}$.

Given $b^{LI}(c_i)$, the payment rule (1) and the fact that the principal is risk neutral, the principal's expected payment is

$$\begin{aligned} P^{LI} &= \left\{p\left[b^{LI}(c_i) + \alpha_H\left(C_{iH} - b^{LI}(c_i)\right)\right] + (1-p)\left[b^{LI}(c_i) + \alpha_L\left(C_{iL} - b^{LI}(c_i)\right)\right]\right\} \quad (2) \\ &\quad n(1 - F(c_i))^{n-1} f(c_i)\, dc_i \\ &= \int_{c^-}^{c^+}\left\{\begin{array}{c} \left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right] b^{LI}(c_i) + \left[p(\alpha_H - \alpha_L) + \alpha_L\right](c_i - e_i^{LI*}) \\ +p(\alpha_H - \alpha_L)\theta_H \end{array}\right\} \\ &\quad n(1 - F(c_i))^{n-1} f(c_i)\, dc_i, \end{aligned}$$

where $n\left[1 - F(c_i)\right]^{n-1} f(c_i)$ is the probability density of the lowest expected cost of the $n$ bidders. Then we have[10]

$$\begin{aligned} P^{LI} &= c^- + n\left[1 - p(\alpha_H - \alpha_L) - \alpha_L\right]\int_{c^-}^{c^+}(1 - F(c_i))^{n-1} F(c_i)\, dc_i \quad (3) \\ &\quad + \frac{1}{2}d\left[(p(\alpha_H - \alpha_L) + \alpha_L)^2 - 1\right] + \int_{c^-}^{c^+}(1 - F(c_i))^n dc_i \end{aligned}$$

[10] See the appendix.

The optimal cost-share parameters, $\alpha_H^*$ and $\alpha_L^*$, which minimize the principal's expected payment thus satisfy[11]

$$\begin{cases} \frac{\partial P^{LI}}{\partial \alpha_H} = 0 \\ \frac{\partial P^{LI}}{\partial \alpha_L} = 0 \end{cases} \Rightarrow \begin{cases} \alpha_H^* = \frac{n\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)dc_i - d(1-p)\alpha_L^*}{1-p} \\ \alpha_L^* = \frac{n\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)dc_i - dp\alpha_H^*}{d(1-p)} \end{cases} \tag{4}$$

These two conditions are equivalent. So (4) becomes

$$\alpha_H^* = \frac{n\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)\,dc_i - d(1-p)\alpha_L^*}{1-p}.$$

We have an infinity of solutions for $(\alpha_L^*,\ \alpha_H^*)$. The only thing that matters is the relation between these two parameters. Replacing $\alpha_H^*$ by its value in (3) yields the expected payment under an *LI* contract

$$P^{LI} = c^- + \int_{c^-}^{c^+}(1-F(c_i))^n\,dc_i - \frac{\left[d - n\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)\,dc_i\right]^2}{2d}.$$

Note that this expected payment is the same as it would be under the state-independent contract of McAfee and McMillan (1986). Furthermore, note that if the principal chooses $\alpha_H^* = \alpha_L^*$, then we have

$$\alpha_H^* = \alpha_L^* = \frac{n\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)\,dc_i}{d},$$

and this value is equal to the cost-share parameter $\alpha^*$ that would be chosen under McAfee and McMillan's (1986) contract given our assumptions.

The fact that the expected payment under McAfee and McMillan's (1986) contract is not modified when the payment becomes "state-dependent" can be easily justified. Indeed, even if the contract involves two cost-share parameters, the form of the payment is still an *LI* contract whatever the realization of $\theta$ is. Therefore, the payment is only taken in expectation over $\theta$. The optimal bid and effort level of agent $i$ are adjusted given the "state-dependent" payment rule but the expected payment remains unchanged.

[11] We can verify that the second order conditions are satisfied, since

$$\begin{cases} \frac{\partial^2 P}{\partial \alpha_H^2} = dp^2 > 0 \\ \frac{\partial^2 P}{\partial \alpha_L^2} = d(1-p)^2 > 0 \end{cases}$$

# 3 The *MFP* contract

We now turn to the analysis of the *MFP* contract. The sequence of events under the *MFP* contract is modelled as follows. The principal commits to the payment rule[12]

$$P_i^{MFP} = \begin{cases} b_i^{MFP} \text{ if } \theta = \theta_L \\ C_{iH} + h\left(e_i^{MFP*}\right) \text{ if } \theta = \theta_H \end{cases} \quad (5)$$

The contract is awarded by means of a first-price sealed-bid auction. Each potential contractor submits a bid $b_i^{MFP}$. The lowest bidder is selected. Then he chooses his optimal level, $e_i^{MFP*}$, of cost-reduction effort. The realization of $\theta$ is then observed and the selected contractor is paid according to the payment rule (5). Indeed, if $\theta = \theta_L$, the payment is a fixed-price. Otherwise, if $\theta = \theta_H$, the principal reimburses the *ex-post* realized cost $C_{iH} = c_i - e_i^{MFP*} + \theta_H$ plus the disutility, $h\left(e_i^{MFP*}\right)$, of agent $i$'s effort. So, with probability $p$, it is common knowledge that the payment to the winner will not depend upon his bid. Our intuition is that this procedure is rather similar to many procurement practices, even if there is no renegotiation involved in our model[13]. Under an *MFP* contract, the winner's expected payment is

$$P_i^{MFP} = p\left[C_{iH} + h\left(e_i^{MFP*}\right)\right] + (1-p)\, b_i^{MFP}.$$

Given the payment rule of the *MFP* contract, agent $i$'s expected profit when he submits a bid $b_i^{MFP}$ and exerts an effort $e_i^{MFP}$ is

$$\begin{aligned} E\pi_i &= \left[C_{iH} - C_{iH} - h\left(e_i^{MFP}\right) + h\left(e_i^{MFP}\right)\right] p + \left[b_i^{MFP} - C_{iL} - h\left(e_i^{MFP}\right)\right](1-p) \\ &= (1-p)\left[b_i^{MFP} - C_{iL} - h\left(e_i^{MFP}\right)\right]. \end{aligned}$$

Agent $i$ chooses the effort level to maximize $E\pi_i$. $e_i^{MFP*}$ thus satisfies

$$h'\left(e_i^{MFP*}\right) = 1 \;\Rightarrow\; e_i^{MFP*} = d.$$

[12] In order to establish a comparison with the *LI* contract, this assumption of commitment is consistent with the assumption of commitment to the cost-share parameters $\alpha_H$ and $\alpha_L$.

[13] See Bajari and Tadelis (2001) for a model analysing the choice between fixed-price contracts and cost-plus contracts in a context of costly renegotiation.

The winner chooses the first best effort level, corresponding to the effort level under a fixed-price contract. However, unlike a fixed-price contract, the principal now bears all the risk linked to exogenous uncertainties. Indeed, given the payment rule, the selected agent knows that with probability $p$, he will not get any profit, but he knows ex-*interim* with certainty the total cost $C_{iL}$ he will have to bear with probability $(1-p)$. So, the selected agent is actually proposed a "modified" fixed-price contract. Indeed, the *ex-post* total cost is not affected by uncertainty and the payment is lower since multiplied by $(1-p)$. Intuitively, replacing a fixed-price contract by an *MFP* contract is valuable for the agent if the gain in shifting risks from the agent to the principal offsets the payment reduction.

Under an *MFP* contract, agent $i$'s *interim* expected profit when he submits a bid $b_i^{MFP}$ is

$$E\pi_{inteim} = (1-p)\left[b_i^{MFP} - C_{iL} - h\left(e_i^{MFP*}\right)\right]\left[1 - F\left(\left(b^{MFP}\right)^{-1}\left(b_i^{MFP}\right)\right)\right]^{n-1}. \quad (6)$$

Each bidder $i$ chooses $b_i^{MFP}$ to maximize (6). We can show that the symmetric Nash equilibrium bidding strategy is given by

$$b^{MFP}(c_i) = c_i + \frac{\int_{c_i}^{c^+}(1-F(s))^{n-1}ds}{(1-F(c_i))^{n-1}} - \left(e_i^{MFP*} - h\left(e_i^{MFP*}\right)\right) + \theta_L,$$

with $b^{MFP}(c^+) = c^+ - \left(e_i^{MFP*} - h\left(e_i^{MFP*}\right)\right) + \theta_L$. Given $e_i^{MFP*}$, we have

$$b^{MFP}(c_i) = c_i + \frac{\int_{c_i}^{c^+}(1-F(s))^{n-1}ds}{(1-F(c_i))^{n-1}} - \frac{d}{2} + \theta_L.$$

Given $b^{MFP}(c_i)$, the payment rule (5) and the fact that the principal is risk neutral, the principal's expected payment is

$$P^{MFP} = \int_{c^-}^{c^+}\left[p\left(C_{iH} + h\left(e_i^{MFP*}\right)\right) + (1-p)\,b^{MFP}(c_i)\right] n(1-F(c_i))^{n-1}f(c_i)dc_i,$$

and thus

$$P^{MFP} = c^- + \int_{c^-}^{c^+}(1-F(c_i))^n dc_i + n(1-p)\int_{c^-}^{c^+}(1-F(c_i))^{n-1}F(c_i)\,dc_i - \frac{d}{2}.$$

We can notice that $P^{MFP}$ is decreasing in $p$. This remark can be justified. When the payment reimburses the *ex post* total cost plus the disutility of effort, no rent is left to the selected contractor since the most efficient contractor is selected and chooses the first best effort level. Then, the payment is lower when $p$ approaches 1, but is not equal to 1 (if $p = 1$, the payment does not depend upon agent's bid, so the principal can not select the most efficient bidder).

Furthermore, we can justify the choice of the form of the payment when $\theta_H$ is observed. As soon as $\theta$ is observed, information becomes complete. Indeed, the effort level is at the first best and the principal can infer the cost $c_i$ of the selected contractor since he has submitted his bid. Therefore, when $\theta$ is observed, the principal knows all the components of $C_i$. So, *ex post,* the principal has to regulate a firm under complete information. The optimal regulation policy is then to reimburse the *ex post* realized cost plus the disutility of the firm's effort[14].

# 4 *LI* or *MFP* contracts ?

We can compare the expected price under an *LI* contract with the expected price under an *MFP* contract, while emphasizing the bidding competition effect and the moral hazard effect. First compare how both contracts are affected by the moral hazard effect. The difference between the increase in expected price of the *LI* contract and the *MFP* contract is then $\frac{d}{2}\alpha^{*2}$. A sufficient condition for the *MFP* contract to be chosen is then that the reduction in expected price, due to the bidding competition effect, must be stronger under an *MFP* contract than under an *LI* contract. This condition can be written as :

$$p > \alpha^*.$$

If $p < \alpha^*$, the *LI* contract dominates if

$$n(\alpha^* - p)\int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i > \frac{d}{2}\alpha^*.$$

Given the value of $\alpha^*$, this inequality becomes

$$p < \frac{n}{2d}\int_{c^-}^{c^+} F(c_i)\,[1 - F(c_i)]^{n-1}\, dc_i. \tag{7}$$

However, the *LI* contract is defined when $\alpha^* < 1$, i.e.

$$\frac{n}{d}\int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i < 1. \tag{8}$$

[14] See Laffont and Tirole (1993).

From (7) et (8), it is obvious that a necessary condition for the *LI* contract to be chosen is $p < 1/2$. The following proposition summarizes the comparison of contracts.

**Proposition 1** *: Si $p < \frac{1}{2}$, the principal has to choose an MFP contract if*

$$p > \frac{n}{2d} \int_{c^-}^{c^+} F(c_i) \left[1 - F(c_i)\right]^{n-1} dc_i,$$

*and an LI contract otherwise. If $p > \frac{1}{2}$, the MFP contract should be preferred.*

Finally, the choice of a contract depends on the number of bidders, the size of the moral hazard effect, the probability of a high-cost state $\theta_H$ and a term which reflects the bidding competition effects[15].

# 5 Conclusion

This article has proposed a comparison between expected prices under *LI* contracts and *MFP* contracts when the payment can be contingent on the realization of a random variable which represents exogenous uncertainties and which can only take on one of two values. We have shown that the expected price under McAfee and McMillan's (1986) *LI* contract is the same as under a "state-dependent" *LI* contract. We have also designed an *MFP* contract which allows the payment to the selected contractor to be independent upon his bid in the case of a high-cost value of exogenous uncertainty. Then, we have derived conditions under which the *MFP* contract may dominate the *LI* contract.

Note finally that our analysis is a first stab at comparing *LI* and *MFP* contracts. Indeed, we have adopted a positive approach to model the *MFP* contract. The design of the optimal contract under the informational structure considered is an open question, even in the case of risk-neutral bidders. We leave it for further research.

[15] More exactly, the expected rents stemming from the bidder's private cost information is equal to $n(1-\alpha^*) \int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i$ under an *LI* contract, and is equal to $n(1-p) \int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i$ under an *MFP* contract.

**Appendix**

We can compute

$$\int_{c^-}^{c^+} (c_i)\, n\, (1 - F(c_i))^{n-1} f(c_i)\, dc_i = c^- + \int_{c^-}^{c^+} (1 - F(c_i))^n\, dc_i$$

and

$$\begin{aligned} \int_{c^-}^{c^+} \left[b^{LI}(c_i)\right] n\, (1 - F(c_i))^{n-1} f(c_i)\, dc_i &= c^- + \int_{c^-}^{c^+} (1 - F(c_i))^n\, dc_i - e_i^{LI*} \\ &+ n \int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i \\ &+ \frac{h\left(e_i^{LI*}\right)}{[1 - p(\alpha_H - \alpha_L) - \alpha_L]} \end{aligned}$$

Replacing in (2), we get

$$\begin{aligned} P^{LI} &= c^- + \int_{c^-}^{c^+} (1 - F(c_i))^n\, dc_i + n\, [1 - p(\alpha_H - \alpha_L) - \alpha_L] \int_{c^-}^{c^+} (1 - F(c_i))^{n-1} F(c_i)\, dc_i \\ &\quad - e_i^{LI*} + h\left(e_i^{LI*}\right) \end{aligned}$$

Replacing $e_i^{LI*}$ by its value yields equation (3).

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