Public capital and private investment, a real option approach*

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Abstract

In this paper, public investment provision takes place in a stochastic environnement. The role of the government is to remove a part of the uncertainty faced by the firm. If the government simply maximizes the value of the firm, then the optimal tax is smaller under imperfect competition than it is under perfect competition since more public capital reduces the selling price. But if the government seeks to maximize the consumer surplus, tax and public capital provision are also a mean to correct the market and the optimal tax is then higher.

JEL: E22, E62,H41

Key words: irreversible investment, public capital, uncertainty.

1. Introduction

In the US, large public expenditures are currently undertaken which bear a large part of the macroeconomic risk while in the European Union the "stability pact" which contrains public expenditures becomes strongly criticized. But how does public investment affect the economic performance ? Such a question has yet received a large attention especially since it has provided a way to explain the productivity slowdown, Aschauer [1989] shows that an increase of 1% in the public-private capital ratio would increase by 0.39 % the total productivity in the economy. This has given rise to a number of empirical works

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as well as to debates about the productivity of public spending (see Gramlich [1994] and Shioji [2001]).

Barro [1990] proposes a theoretical approach (following Arrow and Kurz [1970]) in which public capital appears as a productive factor. He considers an endogenous growth model with public expenditures that enter the production function as flows of services and a balanced government budget for each period. Growth is then maximum when the tax rate equals the elasticity of public capital with respect to output. Many extensions have been proposed: for instance, Glomm and Ravikumar [1994] introduces congestion, Cashin [1995] considers the productivity of the stock of public capital and not of the flow of its services. Moreover, Burguet and Fernàndez-Ruiz [1998] relax the assumption on the government budget, allowing for borrowing and analyzing the possibility of poverty traps.

All this literature about public investment affecting private production remains in a deterministic framework. But what is the optimal tax rate if the productivity of the public good is stochastic as it would be the case in an uncertain environment? Turnovsky (1999) includes stochastic features into an endogenous growth model with productive public spending. However, he neglects one important characteristic of investment decision: the irreversible nature of capital expenditure. It is now largely admitted (see Dixit and Pindyck [1994]) that the assumption of a perfectly flexible private capital is no longer realistic in a stochastic world. Investment decisions are for sure affected by the joined facts that investment is irreversible and generates returns that are uncertain. In fact, a firm that may face bad news and which cannot easily sell its capital may prefer to postpone some investment projects, that is, to accumulate less capital for a given state of nature. This significantly alters the productivity of the private sector and one may wonder how the public sector is in turn affected: what happens for public investment, that is, for the optimal tax rate and the public capital provision?

In this paper, we extend usual models on irreversible investment under uncertainty by introducing the stock of public capital as an input for the private sector, which seems more realistic than considering the flow of public spending or the services provided by the government. Public investment takes place in a stochastic environment. Public capital then increases the productivity of private capital which is assumed to be fully irreversible. Implications for the firm can be generalized for the economy assuming a representative firm. This implies periods with no investment at the aggregate level which is at least relevant for developing countries. A more realistic modelling would consider heterogeneity among firms which would significantly complicate the model without probably affect the nature of the results we are interested in. In our model, the government has an intertemporal budget constraint, i.e. taxes are collected each period to fund the public debt. We provide a partial equilibrium analysis (as it is standard in models of irreversible investment under uncertainty); we first consider the case of perfect competition and then turn to issues about imperfect competition which allow for different objectives for the government: it may seek to maximize the value of the firm or rather the consumer surplus. Even under uncertainty, the optimal tax rate is then constant and does not depend on the size of uncertainty, it is exactly the same as the one that would prevail in a deterministic world. Note that Park and Philippopoulos (2002) shows that a constant tax rate avoids any intertemporal distortion. Nevertheless the optimal provision of public capital is negatively affected by uncertainty. We show that the government has an insurance role since it removes part of the uncertainty faced by the firm. Such a role for the government has already been suggested by Rodrik (1998): observing that the positive correlation between openness of economies and the government size of these economies is stronger when terms-of-trade risk is higher, he deduces that government spending may play a risk-reducing role. Our paper can therefore be considered as an attempt to show how the public sector uses public investment as a risk-reducing strategy.

Comparing cases under perfect competition and under imperfect competition, results depend on the objective of the government. If the government simply maximizes the value of the firm, the optimal tax is then smaller under imperfect competition than under perfect competition because having more public capital is not so good for the firm since it reduces the selling price. But if the government seeks to maximize the consumer surplus, tax and public capital provision are means to correct the market and the optimal tax is then higher than under perfect competition. Section two presents the model under perfect competition and section three extends it to imperfect competition.

2. Providing public capital in a stochastic world

2.1. The program of the firm

We consider public capital as a kind of public good that is provided by the government to the firms. Following the literature, an increase in the amount of public capital raises private productivity. It is a pure public good so there is no congestion issue. The production function has a Cobb-Douglas form[1]:

$$Y(t) = A(t)K(t)^{\alpha}Kg^{\beta} \tag{2.1}$$

with $\alpha + \beta < 1$. Kg is the amount of public good ; production is continuously perturbed by shocks since parameter $A(t)$ is stochastic and moves according to a geometric Brownian motion[2]:

$$\frac{dA(t)}{A(t)} = \mu dt + \sigma dz(t)$$

[1] Labor can be introduced in the production function, one could interpret the production function in per capita terms.

[2] Output price could also be modelled as a geometric Brownian motion. Such an assumption would alter neither the methodology nor the nature of the results.

with $dz = \varepsilon\sqrt{dt}$ where $\varepsilon\tilde{}N(0,1), E(\varepsilon_i\varepsilon_j) = 0\ \forall i, j\ with\ i \neq j.$

The private sector must deal with irreversibility in the installed private capital; once investment is implemented there is a sunk cost, which does not allow them to reduce the total stock. The problem of the firm is to maximize its value (the discounted sum of its cash flows) by choosing the optimal stock of private capital, given that there is uncertainty and irreversibility:

$$\left\{\begin{array}{l} \max_{I(t)} V(0) = E_0 \int_0^{+\infty} \left[(1-\tau)A(t)K(t)^{\alpha}(Kg)^{\beta} - kI(t)\right] e^{-rt}dt \\ \text{sc.} \left| \begin{array}{l} dA(t) = \mu A(t)dt + \sigma A(t)dz(t) \\ I(t) = dK(t) \geq 0 \\ K(t) = 0 \quad \forall t < 0 \\ Kg \text{ and } \tau \text{ given} \end{array}\right. \end{array}\right. \tag{2.2}$$

We assume that the government and the private sector pay the same constant price k for the capital. r is the discount rate of both government and private firms[3]. The government taxes the cash-flows at a rate τ to finance the public expenditures. The private sector takes taxes and public expenditure as given to solve the problem.

2.1.1. Deriving the desired stock of capital

The Bellman Equation of this problem is defined as follows (see Pindyck [1988]):

$$rv(t) = (1-\tau)\alpha A(t)K(t)^{\alpha-1}(Kg)^{\beta} + \frac{E\,(dv)}{dt} \tag{2.3}$$

with $v(t)$ being the marginal value of the firm. The solution is then:

$$v(t) = \underbrace{\frac{(1-\tau)A(t)\alpha K(t)^{\alpha-1}(Kg)^{\beta}}{r-\mu}}_{\text{Discounted value of future cash-flows}} + \underbrace{(1-\tau)Z_K(K(t))A(t)^{\lambda}}_{\text{Option to invest}} \tag{2.4}$$

where $Z_k < 0$ is the derivative of $Z(K)$ with respect to $K(t)$ and $\lambda = 0.5 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2r/\sigma^2} > 1$ thus, $\partial\lambda/\partial\sigma^2 < 0$. The value of the last unit of private capital is given by equation (2.4). It encompasses the discounted present value of future cash-flows given $A(t)$, less the option to invest this unit later since having one more marginal unit of capital implies to give up this option (and thus prevents to wait for better realizations of the stochastic variable). We will have to solve for Z_k and we will show it is actually negative. Note that the higher the uncertainty, the larger the value of the option the firm has to give up to invest in the marginal unit and thus, the smaller the

[3] In a different setting, Arrow and Lind (1970) show that uncertainty should not prevent from using the same discount rate for private firms and government.

value of the marginal unit. This is the main conclusion of models of irreversibility and uncertainty, and for plausible values of the model's parameters, this option value may be non-negligible, and may thus significantly affect the optimal stock of capital (again, see Pindyck [1988]).

The desired capital stock is obtained through "value matching" and "smooth pasting" conditions that are standard in the irreversible investment under uncertainty literature (see Dixit and Pindyck [1994]) :

$$(1-\tau)Z_K(K)A(t)^{\lambda} + \frac{(1-\tau)A(t)\alpha K(t)^{\alpha-1}(Kg)^{\beta}}{r-\mu} = k \tag{2.5}$$

$$(1-\tau)\lambda Z_K(K)A(t)^{\lambda-1} + \frac{(1-\tau)\alpha K(t)^{\alpha-1}(Kg)^{\beta}}{r-\mu} = 0 \tag{2.6}$$

These optimality conditions allow to derive the value of the desired stock of capital K^*as well as Z_k^*, for a given value of $A(t)$. Firms observe the value of the parameter $A(t)$ and then can choose the desired stock of capital as follows :

$$K^d(t) = \left[\frac{\lambda-1}{\lambda k}\frac{(1-\tau)A(t)\alpha Kg^{\beta}}{(r-\mu)}\right]^{\frac{1}{1-\alpha}} \tag{2.7}$$

$$Z_k(K(t)) = -\left(\frac{k}{(1-\tau)(\lambda-1)}\right)^{1-\lambda}\left(\frac{\alpha Kg^{\beta}}{(r-\mu)\lambda}\right)^{\lambda} K^d(t)^{\lambda(\alpha-1)} \tag{2.8}$$

$$\begin{array}{cc} \dfrac{\partial K^d(t)}{\partial \tau} < 0 & \dfrac{\partial K^d(t)}{\partial Kg} > 0 \\ \dfrac{\partial Z_k}{\partial \tau} < 0 & \dfrac{\partial Z_k}{\partial Kg} > 0 \end{array} \tag{2.9}$$

It is worth noting that taxes which negatively affect the marginal cash flow reduce the desired capital while the amount of public capital which increases the marginal productivity of capital leads to a higher desired stock of capital. Nevertheless, more taxes allow government to provide more public good, but the firm does not internalize the externality generated by taxes. The tax also reduces the value of Z_k, therefore it reduces the marginal option value; this contrasts with the effect of the public good which has a positive impact.

Another way to study the impact of the government intervention is to focus on the level of the stochastic variable required to invest given an installed stock of private capital, $K(t)$. Such a threshold may also be derived from the value matching and smooth pasting conditions:

$$A^*(t) = \frac{k}{(1-\tau)}\frac{(r-\mu)}{\alpha K(t)^{\alpha-1}Kg^{\beta}}\left(\frac{\lambda}{\lambda-1}\right) \tag{2.10}$$

Effects on this threshold of the tax rate and of the stock of public capital are consistent with those on the desired stock of capital that have already been examined:

$$\frac{\partial A^*(t)}{\partial \tau} > 0 \qquad \frac{\partial A^*(t)}{\partial Kg} < 0$$

2.1.2. Deriving the initial value of the firm

Given the desired capital stock derived before, we can express $V^d(t)$, the value of the firm when the installed stock of capital is the desired one. Since we know the expression for $\nu^d(t)$ (the marginal value of the firm when the installed capital is the desired one), $V^d(t)$ may be computed as follows:

$$\begin{aligned} V^d(t) &= \int_0^{+\infty} v^d(K)dK = \int_0^{K^d} \frac{(1-\tau)\alpha A(t)K(t)^{\alpha-1}(Kg)^{\beta}}{r-\mu} dK(t) \\ &+ \int_{K^d}^{+\infty} (1-\tau)Z_K(K(t))A(t)^{\lambda} dK(t) \end{aligned}$$

$$\Leftrightarrow \quad V^d(t) = \frac{(1-\tau)A(t)K^d(t)^{\alpha}Kg^{\beta}}{r-\mu} + \left(\frac{1}{\lambda(1-\alpha)+1}\right)\left(\frac{k}{(\lambda-1)}\right)^{1-\lambda}\left(\frac{\alpha A(t)(1-\tau)Kg^{\beta}}{(r-\mu)\lambda}\right)^{\lambda} K^d(t)^{\lambda(\alpha-1)+1} \tag{2.11}$$

assuming $\lambda(1-\alpha) > 1$ to ensure the convergence of the integral.

We assume that the firm has initially no capital: it only starts to invest at the time $t = 0$ when the government installs the capital Kg. Due to the specification of the cash flows, that are positive whatever the realization of the stochastic variable, at time $t = 0$, the firm will then jump for sure to its desired capital stock $K^d(0) > 0$. Thus, the initial value of the firm is such that given the amount of public capital and the realization of the stochastic variable at time $t = 0$, the installed capital stock $K(0)$ corresponds to the desired stock $K^d(0)$. Replacing $K^d(0)$ by its expression given by equation (2.7), the initial value of the firm is thus :

$$V(0) = V^d(0) = \left(\frac{A(0)}{r-\mu}\right)^{\frac{1}{1-\alpha}} \left(\frac{(\lambda-1)\alpha}{\lambda k}\right)^{\frac{\alpha}{1-\alpha}} (1-\tau)^{\frac{1}{1-\alpha}} Kg^{\frac{\beta}{1-\alpha}} \left[1 + \frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right] \tag{2.12}$$

Not surprisingly, this value is an increasing function of the stock of public capital and a decreasing function of the tax rate. The effect of uncertainty on this initial value is given by:

$$\frac{\partial V(0)}{\partial \sigma^2} = \underbrace{\frac{\partial V(0)}{\partial \lambda}}_{>0} \underbrace{\frac{\partial \lambda}{\partial \sigma^2}}_{<0} < 0$$

There exists two opposite effects of uncertainty on $V(0)$. On the one hand, more uncertainty induces the firms to install less capital ($K^d(0)$ is smaller) thus reducing the current cash-flow ; on the other hand, a larger uncertainty increases the option value part of $V(0)$ which relates to future cash flows. Clearly here, the first effect prevails and more uncertainty reduces the initial value of the firm.

2.2. The program of the government

Until now, public expenditures K_g and taxes τ are taken as given by the firm when deciding how much to invest. We must then consider the problem of the government: how should the level of taxes and the stock of public capital be determined ? Note that the model dealt here is a partial equilibrium analysis. Therefore the objective of the government will be to maximize the value of the firm subject to its intertemporal budget constraint.

2.2.1. The government budget constraint

At the beginning of the program, the government defines Kg, the optimal level of public capital to be provided once for all. Expenses are then kKg with k being the unit price of capital. This public debt is completely funded in future taxes on the instantaneous profits of the firms. Therefore, we are supposing that there is a "Ricardian equivalence" in the sense that the public debt is completely funded in future revenues[4]. Since the model is stochastic, future tax revenues are subject to uncertainty and the budget constraint is such that the expected present value of the revenues must be equal to the expenses in terms of public capital. Moreover, the expected present value of the revenue derives from the tax rate applied to the expected value of the future cash flows of the firm. This latter stream is given by $V(0)$, which is the value of the firm at period 0. The government budget is thus:

$$\tau V(0) = kKg \tag{2.13}$$

Note that this implies a precise time schedule in the realization of private and public investments : at time $t = 0$ the government observes the realization of the stochastic variable. It can then deduce the amount of capital the firm wants to install and the initial value of the firm, depending on the levels of tax and public capital. Using this information the government decides how much to tax and how much public goods to provide, which implies (given k) the initial amount of debt.

[4]However, the name of Ricardian Equivalence is more related with result that the public debt is not regarded by the agents as wealth. Here, the term is used in the sense of Walsh (1998) and Sargent (1982).

2.2.2. Deriving the optimal tax rate

The program of the government is to choose the levels of tax and of public capital, that maximize the value of the firm:

$$\begin{cases} Max_{\{\tau,Kg\}}V(0) = \left(\frac{A(0)}{r-\mu}\right)^{\frac{1}{1-\alpha}} \left(\frac{(\lambda-1)\alpha}{\lambda k}\right)^{\frac{\alpha}{1-\alpha}} (1-\tau)^{\frac{1}{1-\alpha}} Kg^{\frac{\beta}{1-\alpha}} \left[1+\frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right] \\ \text{sc. } \tau V(0) = kKg \end{cases}$$

Substituting for Kg using the budget constraint, the problem of the public sector can be restated as:

$$Max_{\tau}V(0) = \Psi(1-\tau)^{\frac{1}{1-\alpha-\beta}}\tau^{\frac{\beta}{1-\alpha-\beta}} \tag{2.14}$$

with $\Psi = \left(\frac{A(0)}{(r-\mu)k^{(\alpha+\beta)}}\right)^{\frac{1}{1-\alpha-\beta}} \left(\frac{(\lambda-1)\alpha}{\lambda}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left[1+\frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right]^{\frac{1-\alpha}{1-\alpha-\beta}}$

The first order condition of the problem is then (note that Ψ does not depend upon τ):

$$\frac{\partial V}{\partial \tau} = -\frac{\Psi}{1-\alpha-\beta}(1-\tau)^{\frac{1}{1-\alpha-\beta}-1}\tau^{\frac{\beta}{1-\alpha-\beta}} + \frac{\beta\Psi}{1-\alpha-\beta}(1-\tau)^{\frac{1}{1-\alpha-\beta}}\tau^{\frac{\beta}{1-\alpha-\beta}-1} = 0 \tag{2.15}$$

Moreover, it can be easily be checked that

$$\frac{\partial^2 V}{\partial \tau^2} < 0$$

The optimal tax rate τ_p^* is thus :

$$\tau_p^* = \frac{\beta}{1+\beta} \tag{2.16}$$

The model gives rise to an inversely U-shaped relationship between the level of tax and the value of the firm, as in Barro [1990] and in the subsequent extensions like Cashin [1995] and Bajo-Rubio [2000]. This arises because taxation plays an ambiguous role since it also allows for a provision of public capital and therefore represents a kind of externality. On the one hand, the tax has a direct negative impact through $(1-\tau)^{\frac{1}{1-\alpha-\beta}}$, since it reduces the after-tax value. On the other hand, a higher tax allows for more public capital and this has a positive impact on the value of the firm through the parameter $\tau^{\frac{\beta}{1-\alpha-\beta}}$. Since the model exhibits decreasing returns with respect to the stock of public capital, combining both effects of the tax will generate a concave relationship : the positive effect initially prevails while for taxes higher than τ_p^*, the reverse applies. Equation (2.16) can be interpreted, like in Barro [1990], as a natural efficiency condition for the public sector, where the government is equalizing the positive benefits of its actions with the negative distortions it generates. Here, the public sector is equalizing the marginal benefits of the public expenditures with the marginal cost due to taxation.

Since $\partial\tau_p^*/\partial\beta > 0$, the stronger the impact β of the public capital on the production of the firm, the higher the optimal tax. Intuition is very clear: the greater the impact of the public good on the production level, the higher the provision of public good should be, and the higher the tax needed to finance it.

2.2.3. The optimal provision of public capital

Using equations (2.13) , (2.16) and (2.14), the optimal level of public capital can be expressed as:

$$Kg_p^* = (1+\beta)^{\frac{\alpha-2}{1-\alpha-\beta}}\beta^{\frac{1-\alpha}{1-\alpha-\beta}}\left(\frac{A(0)}{(r-\mu)k}\right)^{\frac{1}{1-\alpha-\beta}}\left(\frac{(\lambda-1)\alpha}{\lambda}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left[1+\frac{\alpha}{\lambda\left[(1-\alpha)\lambda+1\right]}\right]^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{2.17}$$

2.3. Effect of uncertainty

2.3.1. Effect uncertainty on the optimal tax rate

The effect of uncertainty on the optimal tax rate is given by:

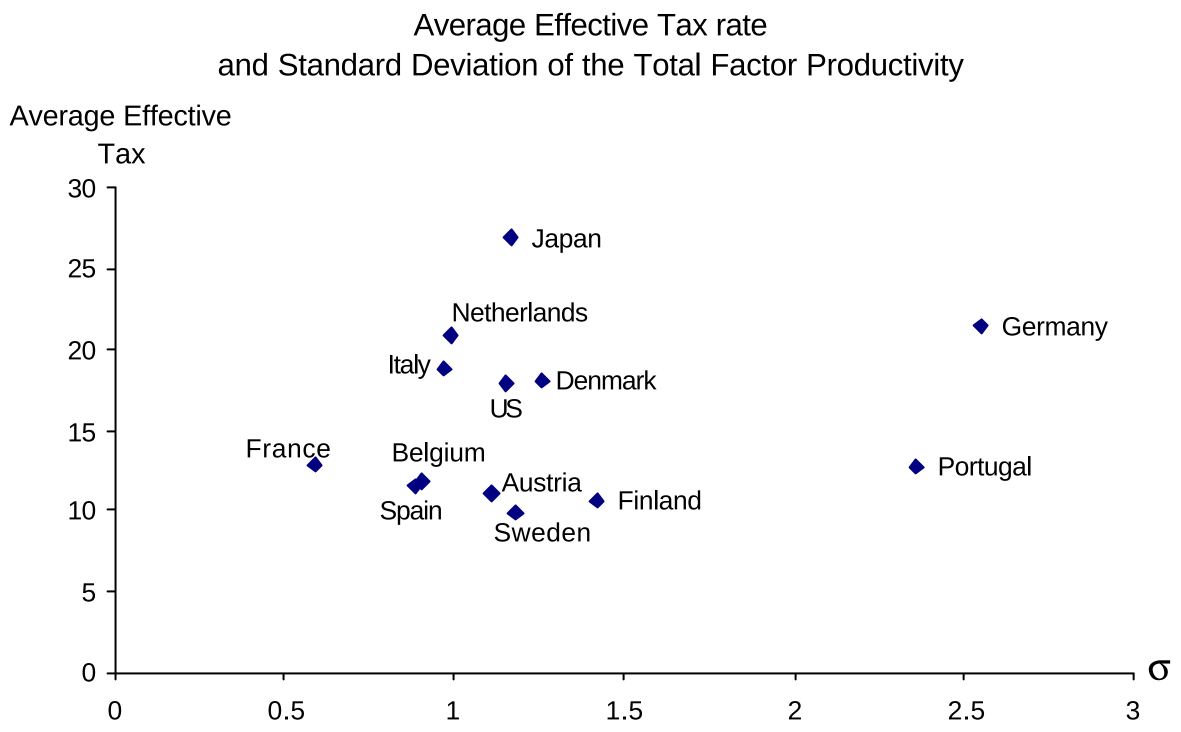
$$\frac{\partial\tau_p^*}{\partial\sigma^2} = 0 \tag{2.18}$$

Hence an increase in uncertainty has no effect on the tax rate, which is quite remarkable. This happens because the impact of the tax (either positive, through the amount of public good it generates, or negative since reducing the current profit) on the current cash flow is the same as that on the option value since the tax is levied on current cash flow as well as on future ones. Its optimal level has therefore nothing to do with the size of uncertainty. Comparing with the deterministic counterpart of this model, it is clear that the introduction of uncertainty does not change the optimal level of tax since the tax rate only depends on how productive is the public capital.

As an illustration of our results, we present some empirical data that do not reject our theoretical statements. Figure 1 shows that the average effective tax rate appears to be unrelated to the standard deviation errors of the total factor productivity[5]. We used the BACH - Banck for the Accounts of Companies Harmonised available at the European Commission to calculate the average effective tax rate for 13 countries (11 europeans countries + Japan and US), which is a ratio of the tax on profits over the value added in the manufacturing industry .

[5]Nidodème (2001) provides a good discussion about the methodologies on effective tax rates; we use here the methodology called "microbackward". For the standard deviation errors of TFP, the values come from the estimated standard deviation of errors of an autoregressive model.

It can be seen that indeed the slightly positive trend line is not significant ($t - test = 0.533$ and $R^2 = 0.0252$). Due to the very restrictive sample, a more careful empirical test should be conducted ; however it provides an indication of no relationship between the tax rate and the uncertainty parameter.



Source: OECD Economic Perspectives, june 2001
European Commission BACH - Bank for the Accounts of Companies Harmonised

2.3.2. Effect of uncertainty on the optimal provision of public capital

Contrary to the optimal tax rate, the corresponding optimal provision of public capital is affected by uncertainty. It arises from the fact that the amount of public good the government can provide, given any tax rate, directly depends on the future tax revenues which are determined by the future cash flows of the firm (see equation (2.13)).

$$\frac{\partial K g_p^*}{\partial \sigma^2} = \frac{\tau}{k} \underbrace{\frac{\partial V(0)}{\partial \sigma^2}}_{<0} + \frac{V(0)}{k} \underbrace{\frac{\partial \tau}{\partial \sigma^2}}_{0} < 0 \qquad (2.19)$$

This results from the two effects of uncertainty on the initial value: as we have seen below, the negative effect of uncertainty (which applies through the current cash flow) prevails.

2.3.3. Uncertainty and the role of the government

The government has an interesting role in this model since it bears the risk of stochastic tax revenues while offering as a counterpart a deterministic initial amount Kg to the private sector.

First, the government offers less public good in the stochastic world when uncertainty is larger (see equation (2.19)). This is due to the fact that uncertainty generates an option value which reduces the marginal productivity of private capital which in turn negatively affects the productivity of the public good. Second, it can be shown that any smaller government intervention would reduce the desired stock of private capital for any realization of the stochastic variable, or symmetrically, for a given installed stock of capital, the level of the stochastic variable required to invest would be higher. Therefore, it can be argued that the model generates some positive relationship between the public investment and the private investment. As the government decides the provision of public good once for all on the basis of the expected value of its revenue, the private sector benefits from a kind of insurance scheme: during bad realizations ($A(t) < A(0)e^{\mu t}$) of the stochastic process, the government is providing more public good than the expected revenues, but in good times ($A(t) > A(0)e^{\mu t}$) the reverse applies, this mechanism keeping the intertemporal budget balanced. The government has therefore an insurance role.

Note that in Barro [1990] public capital positively affects the economy since it is financed by a tax paid by n firms, so each firm only bears a small part of the cost while the public capital entirely benefits to any of them. Here, the public capital is financed by one representative firm. So the role of the government is not to provide a good which exhibits the special characteristics of a public good but rather to remove a part of uncertainty from the firm towards the government.

3. Market power and public capital provision in a stochastic world

Imperfect competition allows considering two different objectives for the government. It could be assumed that the government simply maximizes the value of the firm, but under imperfect competition, the government may rather seek to maximize consumer surplus. Both cases are studied in this section.

Assuming a monopolistic competition framework (see Dixit and Stiglitz [1977]), the firm is no longer price taker but takes into account the following demand function:

$$P(t) = bY(t)^{-\theta} \tag{3.1}$$

with $\theta < 1$ (the price elasticity is greater than unity). Moreover, the cash-flow becomes:

$$P(t)Y(t) = bA(t)^{1-\theta}K(t)^{\alpha(1-\theta)}Kg^{\beta(1-\theta)} \tag{3.2}$$

As before, the cash-flow is affected by uncertainty through the productivity parameter, but now it is a concave function of the uncertain variable (note that a power function of a Geometric Brownian Motion is again a Geometric Brownian Motion).

3.1. The program of the firm

The problem of firm may now be written:

$$\begin{array}{l} \max_{I_t} W(0) = E_0 \int_0^{+\infty} \left[(1-\tau)A'(t)K(t)^{\alpha'} (Kg)^{\beta'} - kI(t)\right] e^{-rt}dt \\ \text{sc.} \left| \begin{array}{l} dA(t) = \mu A(t)dt + \sigma A(t)dz(t) \\ I(t) = dK(t) \geq 0 \\ K(0) = 0 \\ Kg \text{ and } \tau \text{ given} \end{array} \right. \end{array} \tag{3.3}$$

where the $A'(t) = bA(t)^{1-\theta}$; $\alpha' = \alpha(1-\theta)$ and $\beta' = \beta(1-\theta)$

The Bellman equation is defined as:

$$rw(t) = (1-\tau)\alpha' A'(t)K(t)^{\alpha'-1}(Kg)^{\beta'} + \frac{E(dw)}{dt} \tag{3.4}$$

with $w(t)$ being the marginal value of the firm. Using the value matching and smooth pasting conditions leads to:

$$K_{mc}^{d*} = \left[\frac{(\lambda'-1)}{\lambda' k\delta}(1-\tau)A'(t)\alpha'(Kg)^{\beta'}\right]^{\frac{1}{1-\alpha'}} \tag{3.5}$$

with $\delta = r - \mu'$, $\lambda' = 0.5 - \mu'/\sigma'^2 + \sqrt{(\mu'/\sigma'^2 - 0.5)^2 + 2r/\sigma'^2}$ where $\mu' = (1-\theta)\mu + \frac{1}{2}(1-\theta)\theta\sigma^2$ and $\sigma'^2 = (1-\theta)^2\sigma^2$

After variables changes, the result looks like the previous one, except that now, uncertainty appears at two levels: as in the competitive case, there exists an irreversibility effect (through λ') which is now combined with a Jensen effect (through δ). Since the cash-flow is a concave function of the stochastic variable $A(t)$, both effects play in the same direction, and uncertainty unambiguously leads to a smaller desired capital stock. Moreover:

$$\begin{aligned} W(0) &= W^d(0) = \left(\frac{A'(0)}{\delta}\right)^{\frac{1}{1-\alpha'}} \left(\frac{(\lambda'-1)\alpha'}{\lambda' k}\right)^{\frac{\alpha'}{1-\alpha'}} (1-\tau)^{\frac{1}{1-\alpha'}} Kg^{\frac{\beta'}{1-\alpha'}} \\ &\quad . \left[1 + \frac{\alpha'}{\lambda'[(1-\alpha')\lambda' + 1]}\right] \end{aligned}$$

3.2. The program of the government

We consider two different objectives for the government: maximizing the value of the firm and maximizing the consumer's surplus.

3.2.1. Maximizing the value of the firm

It can be seen that the problem is close to the previous case of perfect competition

$$
\begin{aligned}
&Max_{\tau} W(0) = \Psi'(1-\tau)^{\frac{1}{1-\alpha'-\beta'}} \tau^{\frac{\beta'}{1-\alpha'-\beta'}} \\
&\text{with } \Psi' = \left(\frac{A'(0)}{\delta k^{(\alpha'+\beta')}}\right)^{\frac{1}{1-\alpha'-\beta'}} \left(\frac{(\lambda'-1)\alpha'}{\lambda'}\right)^{\frac{\alpha'}{1-\alpha'-\beta'}} \left[1+\frac{\alpha'}{\lambda'[(1-\alpha')\lambda'+1]}\right]^{\frac{1-\alpha'}{1-\alpha'-\beta'}}
\end{aligned}
\tag{3.6}
$$

The government must then follow the following optimal rule:

$$
\tau^*_{mcf} = \frac{\beta'}{\beta'+1} = \frac{\beta(1-\theta)}{\beta(1-\theta)+1} \tag{3.7}
$$

$$
\frac{\partial \tau^*_{mcf}}{\partial \theta} < 0 \tag{3.8}
$$

The higher the power of the firm (which is an increasing function of θ), the less effective the public capital (β' will drop), and thus the smaller the optimal tax rate. The economic interpretation is as follows. A higher market power means that the firm wants to produce less at a higher price, while public expenditures induce lower prices and higher quantities. Therefore, the value of the firm that has a higher market power is maximized by a smaller government intervention (smaller tax rate and less public capital). So finally, imperfect competition leads to a smaller optimal tax. Moreover,

$$
\lim_{\theta \to 0} \tau^*_{mcf} \to \tau^*_p
$$

When the power of the firms becomes negligible, the optimal tax rate under imperfect competition (τ^*_{mcf}) converges to the optimal tax rate under perfect competition (τ^*).

It is also possible to derive the stock of public capital:

$$
Kg_{mcf} = \left(\frac{(\beta')^{1-\alpha'}}{(1+\beta')^{2-\alpha'}}\right)^{\frac{1}{1-\alpha'-\beta'}} \left(\frac{A'(0)}{\delta k}\right)^{\frac{1}{1-\alpha'-\beta'}} \left(\frac{(\lambda'-1)\alpha'}{\lambda'}\right)^{\frac{\alpha'}{1-\alpha'-\beta'}} \tag{3.9}
$$

$$
. \left[1+\frac{\alpha'}{\lambda'[(1-\alpha')\lambda'+1]}\right]^{\frac{1-\alpha'}{1-\alpha'-\beta'}} \tag{3.10}
$$

3.2.2. Maximizing the consumer Surplus

In an imperfect competition set up, the government may seek to maximize the expected lifetime consumer surplus. It is a better measure for welfare than the sole value of the firms. But since we are in a stochastic world with irreversible investment, the exact expected lifetime consumer surplus cannot be computed. Nevertheless, it can be approximated using the expected *long term* consumer surplus, given the observed realization of the uncertain variable. The problem of the government becomes then:

$$
\begin{array}{c} Max_{\{\tau,Kg\}} S_{LT}(0) \\ \text{sc. } \tau W(0) = kKg \end{array}
$$

It can be shown (see the appendix) that the problem can be rewritten as:

$$
\underset{\tau}{Max} S_{LT}(0) = \phi(1-\tau)^{\frac{\beta'+\alpha'}{(1-\alpha'-\beta')}} \tau^{\frac{\beta'}{(1-\alpha'-\beta')}} \tag{3.11}
$$

where: $\phi = \left(\frac{\alpha'(\lambda'-1)}{\delta k\lambda'}\right)^{\frac{\alpha'}{1-\alpha'}} \left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right) \left(\frac{\theta}{1-\theta}\right) A'(0)^{\frac{1}{1-\alpha'}} \left(\frac{\Psi'}{k}\right)^{\frac{\beta'}{1-\alpha'}}$

The first order condition gives:

$$
\tau^*_{mcs} = \frac{\beta}{\alpha + 2\beta} \tag{3.12}
$$

$$
\frac{\partial \tau^*_{mcs}}{\partial \theta} = 0 \quad \frac{\partial \tau^*_{mcs}}{\partial \sigma^2} = 0 \tag{3.13}
$$

Observe that the tax rate depends neither on the uncertainty nor on the degree of market imperfection. As in the perfect competition case, the optimal tax rate is related to the elasticity of public capital; nevertheless, it now depends on the elasticity of private capital as well: the more efficient the private capital, the smaller the optimal tax rate. This comes from the fact that the government considers the consumer surplus as the objective; it thus implements a tax rate that leads to the maximum production each period, given its budget constraint (indeed we have: $\partial Y(t)/\partial \tau|_{\tau=\tau^*_{mcs}} = 0$). The government must therefore take into account the negative effect of the taxes on the current production and this is why the optimal value τ^*_{mcs} depends on the elasticity of private capital. Moreover, the optimal tax rate is higher than in the perfect competition model whatever the degree of imperfection.

The optimal amount of public capital is defined as:

$$
Kg_{mcs} = \left[\left(\frac{A'(0)}{\delta k}\right)\left(\frac{(\lambda'-1)\alpha'}{\lambda'}\right)^{\alpha'}\left[1 + \frac{\alpha'}{\lambda'\left[(1-\alpha')\lambda'+1\right]}\right]^{1-\alpha'}\left(\frac{(\alpha+\beta)\,\beta^{1-\alpha'}}{(\alpha+2\beta)^{2-\alpha'}}\right)\right]^{\frac{1}{1-\alpha'-\beta'}} \tag{3.14}
$$

3.3. Comparing the three cases

Looking at the optimal tax rates under perfect competition, under imperfect competition with the firm's value maximization, and under imperfect competition with the consumer surplus maximization, we get:

$$
\tau^*_{mcs} > \tau^*_p > \tau^*_{mcf} \tag{3.15}
$$

Government intervention generates a correction in the level of imperfection in the market: higher tax revenues imply higher public capital which induces higher production. Equation (3.15) therefore expresses the fact that if the government maximizes the value of the firm under imperfect competition, the optimal tax rate is lower than under perfect competition, whereas if the government maximizes the consumer surplus, the optimal tax rate is higher than under perfect competition, inducing a correction of imperfection in the market. Moreover:

$$Kg_{mcs} > Kg_{mcf} \tag{3.16}$$

The provision of public capital is higher when the government maximizes the consumer's surplus. In this latter case moreover, as far as the desired level of private capital is concerned, the negative effect of a higher taxation prevails on the positive effect due to a higher public capital. Indeed, it can easily been shown that maximizing the value of the firm to obtain the optimal tax and public capital (see equations (3.5) and (3.6)) reduces to maximizing the desired private capital ; therefore, the desired capital is of course greater than when maximizing the consumers surplus.

$$K_{mcs}^{d*} < K_{mcf}^{d*} \tag{3.17}$$

Finally, output is higher when the government maximizes the consumer's surplus rather than the value of the firm. This is not surprising since maximizing consumers surplus leads to maximizing production (see equations (6.1) and (6.11)). Therefore, under imperfect competition, the government can induce a higher production and correct the market imperfection through a higher taxation and a higher provision of public capital.

4. Conclusion

In this paper, we have studied the issue of the optimal provision of public capital under uncertainty. Whatever the competition in the market, the tax rate will not depend on the degree of uncertainty but only on technological parameters and market power. Nevertheless, the optimal stock of public capital will be negatively affected by uncertainty. The government has an insurance role since it collects taxes from future cash-flows that are stochastic and provides an initial amount of public capital.

Under imperfect competition, two cases have been studied, the first one keeps the value of the firm as the objective function of the government, while in the second case the government maximizes the consumer surplus. For the first case, the optimal tax rate depends on the technological parameters and on the market power of the firms. The higher the power of the firm, the less the tax rate, since more public good induces more production and a smaller selling price. But in the second case the optimal tax rate is higher than under perfect competition since it is a mean to correct the market imperfection.

A more realistic modelling with heterogenous firms should now be considered in order to avoid periods with no investment in the country ; nevertheless, there is no reason for the insurance role of the government to disappear in such a framework. More interesting would probably be to allow for successive public investments.

5. References

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6. Appendix: deriving the expected conditional consumer's surplus (following Bertola [1998])

The consumer surplus is defined as:

$$\begin{aligned} S(t) &= \int_0^{Q(t)} bq(t)^{-\theta}.dq - P\left(Q(t)\right)Q(t) = \left[\frac{b}{1-\theta}q(t)^{1-\theta}\right]_0^{Q(t)} - bQ(t)^{1-\theta} \\ &= \frac{b}{1-\theta}Q(t)^{1-\theta} - bQ(t)^{1-\theta} = \frac{b\theta}{1-\theta}Q(t)^{1-\theta} = \frac{b\theta}{1-\theta}\left[A(t)K(t)^{\alpha}Kg^{\beta}\right]^{1-\theta} \quad (6.1) \\ &= \frac{\theta}{1-\theta}A'(t)K(t)^{\alpha'}Kg^{\beta'} \quad (6.2) \end{aligned}$$

with $A'(t) = bA(t)^{1-\theta}; \alpha' = \alpha(1-\theta)$ and $\beta' = \beta(1-\theta)$.

From the optimality conditions for the firm we know that:

$$\alpha' A'(t)K(t)^{\alpha'-1}Kg^{\beta'} = \frac{\delta k\lambda'}{(1-\tau)(\lambda'-1)} \quad (6.3)$$

with: $\delta = r - \mu' + \frac{1}{2}\theta(1-\theta)\sigma^2$ and $\lambda' = 0.5 - \mu'/\sigma'^2 + \sqrt{(\mu'/\sigma'^2 - 0.5)^2 + 2r/\sigma'^2}$ where $\mu' = (1-\theta)\mu + \frac{1}{2}(1-\theta)\theta\sigma^2$ and $\sigma'^2 = (1-\theta)^2\sigma^2$ which gives the expression of the desired capital stock.

Let us define the marginal cash flow $X(t)$:

$$X(t) = \alpha' A'(t)K(t)^{\alpha'-1}Kg^{\beta'}$$

The marginal cash-flow thus follows a regulated Brownian motion. That is, when the firm is not investing, $X(t)$ follows a geometric Brownian motion with mean $M = (1-\theta)\mu - \theta(1-\theta)\frac{\sigma^2}{2}$ and variance $\Sigma = (1-\theta)\sigma$. When the firm is investing the marginal cash-flow equals then the right-hand side of equation (6.3) which is constant : $X(t) = c$ with $c = \delta k\lambda'/(\lambda'-1)$

Defining $\varepsilon(t) = \ln X(t)$, then

$$d\varepsilon(t) = \frac{dX(t)}{X(t)} - \frac{1}{2}\frac{d^2X(t)}{X(t)^2} \Rightarrow E\left[d\varepsilon(t)\right] = M - \frac{1}{2}\Sigma^2 > 0 \Leftrightarrow \mu > \frac{\sigma^2}{2} \quad (6.4)$$

which ensure the existence of a density function whose expression is:

$$w(\varepsilon(t)) = \rho e^{\rho(\varepsilon(t)-\ln c)} \quad (6.5)$$

with $\rho = \frac{2M}{\Sigma^2}$

Let us express $S(t)$ as a function of $X(t)$:

$$S(t) = \frac{\theta}{1-\theta} A'(t) K(t)^{\alpha'} K g^{\beta'} = z X(t)^{\frac{\alpha'}{\alpha'-1}} \tag{6.6}$$

$$\Rightarrow S(t) = z.e^{\varepsilon(t)\frac{\alpha'}{\alpha'-1}} \tag{6.7}$$

$$\Rightarrow \varepsilon(t) = \ln\left[\left(\frac{S(t)}{z}\right)^{\frac{\alpha'-1}{\alpha'}}\right] = g(S(t)) \tag{6.8}$$

where: $z = \frac{\theta}{1-\theta}\left[(\alpha')^{\alpha'} A'(t) K g^{\beta'}\right]^{\frac{1}{1-\alpha'}}$

A variable change allows then to deduce the density function of $S(t)$ when knowing that of $X(t)$ (see Bertola [1998] or Bentolila and Bertola [1990]):

$$f(S(t)) = -w\left[g(S(t))\right] \frac{\partial g(S(t))}{\partial S(t)} \tag{6.9}$$

$$= \left(\frac{\rho(1-\alpha')}{\alpha'}\right) z^{\frac{\rho(1-\alpha')}{\alpha'}} c^{-\rho} S(t)^{\frac{\rho(\alpha'-1)}{\alpha'}-1} \tag{6.10}$$

We then can find the expected long-term value of the consumer surplus:

$$E_{LT} S(0) = \int_{S(0)}^{\infty} f(S(t)) S(t) dS(t) = \int_{S(0)}^{\infty} \left(\frac{\rho(1-\alpha'))}{\alpha'}\right) z^{\frac{\rho(1-\alpha')}{\alpha'}} c^{-\rho} S(t)^{\frac{\rho(\alpha'-1)}{\alpha'}} dS(t)$$

which converges if $\alpha' < \rho(1-\alpha')$; then:

$$E_{LT} S(0) = \left[\left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right) c^{-\rho} z^{\frac{\rho(1-\alpha')}{\alpha'}} S(t)^{\frac{\rho(\alpha'-1)}{\alpha'}+1}\right]_{S(0)}^{\infty} \tag{6.11}$$

$$= \left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right) c^{-\rho} \left(\frac{S(0)}{z}\right)^{\frac{\rho(\alpha'-1)}{\alpha'}} S(0) \tag{6.12}$$

$$= \left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right) c^{-\rho} X^{\rho}(0) S(0) \tag{6.13}$$

$$= \left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right) S(0) \tag{6.14}$$

$$= \chi A'(0)^{\frac{1}{1-\alpha'}} K g^{\frac{\beta'}{1-\alpha'}} \tag{6.15}$$

with $\chi = \left(\frac{\alpha'(1-\tau)(\lambda'-1)}{\delta k \lambda'}\right)^{\frac{\alpha'}{1-\alpha'}} \left(\frac{\rho(1-\alpha')}{\rho(\alpha'-1)+\alpha'}\right)\left(\frac{\theta}{1-\theta}\right)$

The intertemporal constraint of the government is:

$$\tau W(0) = kKg \tag{6.16}$$

But we know that the value of the firm is given by:

$$W(0) = \Psi'(1-\tau)^{\frac{1}{1-\alpha'-\beta'}}\,\tau^{\frac{\beta'}{1-\alpha'-\beta'}} \tag{6.17}$$

combining both equations:

$$Kg = \frac{\Psi'(1-\tau)^{\frac{1}{1-\alpha'-\beta'}}\,\tau^{\frac{1-\alpha'}{1-\alpha'-\beta'}}}{k} \tag{6.18}$$

Substituting Kg for its expression in the expected long-term consumer surplus gives equation (3.11) in the text.