Optimal Debt Design and the Role of Bankruptcy*

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Abstract

This paper integrates the problem of designing corporate bankruptcy rules into a theory of optimal debt structure. We show that, in an incomplete-contracts framework with imperfect renegotiation, having multiple creditors increases a firm's debt capacity while increasing its incentives to default strategically. The optimal debt contract gives creditors claims that are jointly inconsistent in case of default. Bankruptcy rules, therefore, are a necessary part of the overall financing contract, to make claims consistent and to prevent a value reducing run for the assets of the firm. It is not optimal to treat creditors asymmetrically in default, but creditors may be protected by different security rights.

Keywords: Bankruptcy, Debt Structure, Contracts.
JEL Classification: G3, K2.

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1 Introduction

Bankruptcy law regulates conflicts between a debtor and her creditors, and among creditors themselves. The complexity of modern bankruptcy law stems primarily from the fact that most firms have more than one creditor. But this raises what seems like a paradox: if bankruptcy with multiple creditors is so complex, why would a firm contract with several creditors in the first place? Put differently: if conflicts of interest must be resolved ex post anyhow and these resolutions are costly, why create them ex ante?

This paper argues that the ex post conflicts and the rules mitigating them are designed so as to maximize the debtor's ability to commit to repay. In choosing how many creditors to borrow from and how to structure their claims, a firm takes into account the effects on its incentives to strategically default. As has been well documented by, for example, Asquith et al. (1994) in the case of workouts in financial distress, multiple creditors make such contract renegotiation more difficult. This ex-post inefficiency has a positive incentive effect, as it forces the firm to honor several claims instead of only one if it wants to avoid incurring the inefficiency.[1] But the higher repayment obligations also make strategic default more attractive, in which case inefficient renegotiation destroys value. In other words, the firm, on the one hand, wants to exploit the incentives of individual creditors to "run for the assets" when the firm could pay and, on the other hand, mitigate the effect of multiple liquidation when several creditors pursue their claims individually.

To provide an intuition for why two investors are better than one in our model, consider a firm negotiating with two investors for outside finance. Following Hart and Moore (1998), we describe the ex-post latitude of managers in repaying its financiers by assuming that the project generates some unverifiable cash flows in addition to the verifiable assets. In this setup, repayment is limited by how much asset value the financiers can credibly threaten to liquidate. Under perfect renegotiation (or with one single creditor), the firm's commitment ability is, in principle, given by the amount of assets available for foreclosure should the firm default. However, this constraint is relaxed if there are frictions in the negotiation process that force the firm to renegotiate individually with its creditors. The firm can then promise ex ante up to the full amount of available assets to each one of the investors. When the firm

[1] This is in the spirit of the literature on strategic capital structure design (Hart and Moore (1998), Bolton and Scharfstein (1996), Dewatripont and Tirole (1994), and Berglöf and von Thadden (1994)).

only defaults on one investor ex post, this investor has the right to foreclose on the firm's assets to collect her debt. As each creditor has this individual right, the firm must buy out each investor indivudually if it wants to protect its assets. If the firm defaults on both creditors, and one creditor calls the sheriff to enforce the payment, the other creditor can file for bankruptcy. In this case, the sum of the two claims will be larger than the available amount of verifiable assets and individual claims must be adjusted.

This is the essential role of bankruptcy in our model. Bankruptcy provides a means to pare down individual claims if the debtor is unable or unwilling to honor them and the creditors must confiscate what is available. We thus emphasize the view, shared by many scholars of law and economics, but rarely modeled in full, that "bankruptcy is a situation in which existing claims are inconsistent " (Hart, 1995). In our model, the need for bankruptcy rules arises endogenously because the inconsistency of the claims of creditors is not the result of chance or irrationality, but is the result of optimal contract design.

Our model brings out the distinction between debt collection and bankruptcy that is a fundamental feature of debt finance in practice and legal theory. Debt collection law is basically bilateral and defines the rights of a single creditor in a bilateral conflict with the debtor. In our model, each creditor has the right to collect his debt if the debtor defaults again him, and as every creditor wields this threat, the debtor is under strong pressure to pay out. Yet, if the debtor defaults against several creditors, these threats cannot be executed simultaneously, individual debt collection is no longer possible, and the debtor is in bankruptcy. In this sense, we model bankruptcy as "collective debt collection" (Jackson, 1986).

Our results lead to the prediction that corporate debt should be "undersecured", i.e., that the promised debt payments should be larger than the value of assets available to support creditor claims. This is consistent with the observation in many countries of low retrieval rates of creditors, in particular junior creditors, once firms are in bankruptcy (see, e.g., Weiss (1990)). In fact, the model predicts that in most parameter constellations, the debtor should retain some of her assets in bankruptcy. As a corollary, absolute priority - the notion that creditors must be satisfied fully in bankruptcy before owners are to retain something - is violated in the present model. Again, this is consistent with the empirical literature (see, e.g., Franks and Torous (1989)).

Unlike most other contributions, this paper is consistent with the observation that solvent firms sometimes actually enter into bankruptcy procedures. We study how the prospect of such a procedure affects the choice of capital structure ex ante. In particular, individual security rights may need to be allocated asymmetrically. The reason is that an optimal contract should make it as expensive as possible for the debtor to pay off individual creditors ex post at the expense of others in order to avoid repaying the debt in full.

By this design of individual claims, the parties can rule out an asymmetric treatment of creditors ex post. This corresponds to, and gives an efficiency justification for, the widespread use of "equal treatment" rules in bankruptcy legislation, such as the Trust Indenture Act in the U.S. In fact, the asymmetric treatment of creditors, particularly between local and out-of-state creditors, was one of the main driving forces behind the push for a federal bankruptcy law in the United States in the 19th century (Berglof and Rosenthal, 2003). If the debtor wants his repayment promise to be credible, he should not be able to play off creditors against each other. In case of strategic bankruptcy, both creditors should have the right to confiscate assets.

This brings us back to our initial puzzle why the contracting parties introduce the ex post conflict between creditors (the inconsistency of individual claims given the firm's asset base), together with the design of a procedure (bankruptcy) to deal with this conflict. While the independent liquidation threats of multiple claims put pressure on the debtor to pay out more than he would under fully coordinated claims, the threat is inefficient ex post if it must be realized. We show that giving each creditor the right to trigger bankruptcy improves strictly upon the no-bankruptcy contract, in which liquidation claims are not coordinated after default.

This simple model of bankruptcy captures some important elements of existing bankruptcy procedures. Bankruptcy is triggered when a creditor files to prevent his claims from being eroded through debt collection of other creditors. The procedure demands an "automatic stay" ensuring that liquidation claims are executed simultaneously, and distributes liquidation values according to a pre-specified rule. Hence, bankruptcy in our model minimizes the ex-post inefficiency resulting from the execution of the ex-ante threats of debt finance.

Interestingly, however, giving the creditors the right to trigger bankruptcy is not always sufficient to rule out runs for the assets. If the optimal debt level and aggregate security rights are both sufficiently small, then individual creditors may have an incentive not to trigger bankruptcy ex post, but rather run for the assets. This is because they stand to gain relatively little from bankruptcy (where they only get their security rights), but they recover their full debt claim if they are the first to foreclose. In such a constellation it is optimal to give the debtor the right to trigger bankruptcy. If the debtor does not gain from bankruptcy because it is ex ante optimal to fully liquidate under bankruptcy, then he will not trigger bankruptcy, but in this case there is no efficiency case for bankruptcy in the first place (a run for the assets achieves the same allocation as an "orderly" full liquidation). But if the debtor gains from bankruptcy (because he retains some of the assets in bankruptcy), he has a strict incentive to trigger bankruptcy if the creditors do not. In this sense, our model makes a case for debtor-friendly bankruptcy legislation.

Our paper touches on two large strands of the literature that up to now - unfortunately, as we argue - have rarely been brought together: the literature on corporate bankruptcy and on capital structure. An important part of the by now large literature on bankruptcy law focuses on optimal procedural and substantial rules, taking as given pre-existing debt contracts and the decision to enter bankruptcy (see Bebchuk (1988), Aghion, Hart and Moore (1992) or Cornelli and Felli (1998)). This work rightly points out that the choice of capital structure influences what happens and what should happen in bankruptcy. Yet, it is silent on the determinants of debt structure, which is problematic as the choice of bankruptcy procedures, or more generally debtor-creditor law, will impact on the firm's capital structure decision.[2] In fact, as our paper shows, the two issues of ex-post creditor interaction and ex-ante choice of debt structure are interrelated and should be studied in conjunction.[3] [4]

[2] An important exception is the work by Harris and Raviv (1995) who study the impact of different games played ex post on the ex-ante efficiency of the contract. Different from us, Harris and Raviv (1995) are only concerned with games between the debtor and one single investor.

[3] Relatedly, on a comparative international level Rajan and Zingales (1995), White (1996), and LaPorta et al. (1998) show important correlations between financing patterns and legal rules (without attempting and being able to say much about the causalities).

[4] Our model thus takes a different perspective from the work of Bizer and De Marzo (1992), who also identify an externality between multiple creditors. In their model, this externality is eliminated in an optimal contract with one single creditor that makes borrowing from additional sources unattractive for the firm. Their work is important in that it highlights the problem of potential externalities among multiple creditors and shows how to avoid them, but is of lesser interest for the study of bankruptcy, where multiple

Another interesting avenue of research has looked at the bankruptcy problem from an ex-ante perspective. Building on the early work of Bulow and Shoven (1978), contributions such as those by Bebchuk and Picker (1996), Berkovitch, Israel, and Zender (1998), or Schwartz (1998) analyse the impact of bankruptcy on debtors' incentives prior to bankruptcy. Cornelli and Felli (1997) have considered ex-ante incentives by creditors, and Berkovitch and Israel (1999) and Povel (1999) focus on the problem of information transmission between debtor and creditor, with interesting recommendations concerning whether a code should allow debtors or creditors, or both, to trigger bankruptcy. Their analysis, unlike ours, is concerned with asymmetric information among investors. These ex-ante analyses are not concerned with the key question of our paper, which is the role of multiple creditors in bankruptcy. In fact, all of these articles consider the conflict between a debtor and a single creditor.

There are only very few papers that have addressed multiple creditor problems from an ex-ante perspective. Of interest in our context are, in particular, the contributions by Bolton and Scharfstein (1996), Winton (1995), Khalid, Martimort, and Parigi (2001), and Hege and Mella-Barral (2001).[5] In Bolton and Scharfstein's (1996) theory of debt structure, multiple lending relationships can be optimal, but need not. In their model, multiple (two) creditors increase firm value on the one hand because of increased bargaining pressure in strategic default, but decrease firm value on the other hand because of less efficient continuation choices in liquidity default. The optimal number of creditors emerges as a trade-off between these two tendencies. Their analysis does not consider the problem of ex-post conflicts of creditors and their implications for bankruptcy. Furthermore, they are not concerned with the optimal allocation of individual security rights and their impact on default and bankruptcy. Finally, on a more technical note, their model does not feature strategic default in equilibrium. Therefore, our tradeoff between the costs and benefits of liquidation is different from theirs: in our model, high bankruptcy liquidation is good to reduce the incentives for strategic

creditors are the rule.

[5] An interesting contribution from a law-and-economics perspective is Kordana and Posner (1999), who study the complex voting features associated with the American Chapter 11. Like our paper, they discuss the tradeoff between reducing the cost of liquidation by lowering individual pre-bankruptcy entitlements and discouraging strategic default. However, they do not model the full ex-ante contracting problem.

default and bad because strategic default can occur in equilibrium after all.

Winton (1995) approaches the problem of multiple creditors from the perspective of costly state verification, thereby generalizing the work of Townsend (1979) and Gale and Hellwig (1985). His results provide a theoretical rationale for seniority and absolute priority, and predict an ordering of monitoring activities among investors. These monitoring activities are reactions to financial distress and can therefore be interpreted as gradual bankruptcy provisions. Different from our work, in Winton (1995) the debtor borrows from several creditors by assumption.[6] Khalid, Martimort and Parigi (2001) take Winton's (1995) work one step further by studying finance with multiple creditors under costly state verification as a multi-principal-agent problem. While they derive interesting insights about information flows and monitoring externalities between the debtor and the creditors, they have little to say about creditor conflicts, the allocation of security rights, and the design of bankruptcy rules.

Hege and Mella-Barral (2001) consider a dynamic model of debt renegotiation with multipe creditors. In their model, all debt claims are identical and equity can make opportunistic debt exchange offers to force concessions on coupon payments. Different from our focus in this paper, Hege and Mella-Barral (2001) study the timing and pricing consequences of those exchange offers. While their model is richer than ours in the dynamic aspect of renegotiation, it is not concerned with the design of individual claims and bankruptcy.[7]

Although written from a different perspective, the paper by Diamond and

[6] It is actually interesting to note that Winton (1995) himself plays down the link of his theory to bankruptcy. He rather stresses examples such as asset securitization or reinsurance, where the individual verification activities do not necessarily imply that a joint "bankruptcy" decision is negotiated and implemented.

[7] In a paper with a somewhat different focus, Bisin and Rampini (2000) are interested in the ex-ante incentive effects of bankruptcy in an environment in which a debtor can borrow from several lenders to smooth consumption. They show that a bankruptcy-like contract allows the main lender to relax the debtor's incentive compatibility constraint, because it is a means for the main lender to commit to confiscate returns in low-return states (which is not optimal for consumption smoothing reasons, but increases the borrower's effort incentives). Different from our model, in their model lending is for consumption smoothing and not production (so the focus is rather on consumer bankruptcy), and exclusive lending contracts are superior to contracts with multiple creditors, but cannot be enforced by assumption.

Rajan (2001) shares an important insight with our paper. They study how a relationship-specific lender can make her loans liquid, given that liquidity is valuable because of liquidity shocks to the lender and that relationship-specific loans can only be sold at a discount to non-specialized lenders. They argue that by financing themselves through dispersed demand deposits, a relationship lender can credibly commit to not appropriate future rents from relationship lending, which will allow her to reduce the discount to be paid ex ante. Hence, by locking herself into a bargaining relationship with multiple lenders, the relationship lender can improve her debt capacity.

The remainder of this paper is organized as follows. In Section 2 we describe the basic structure of the model. Section 3 studies optimal contracting under the assumption that bankruptcy is triggered automatically. Section 4 extends the model and provides a more detailed institutional analysis of the bankruptcy process, in particular the issues of triggering rights and seniority. Section 5 concludes with a discussion of the role of contracts versus the law and of the problem of multilateral renegotiation. Appendix A contains some technical arguments, and Appendix B extends the analysis to the technically more involved case of efficient ex-post liquidation.

2 The Model

A firm can invest I units of funds at date 0 and lives for two periods after that date. As in the "incomplete contracts" literature on debt contracts following Hart and Moore (1998), at date 1, the firm has assets in place worth A which generate a cash flow Y. Asset value A at date 1 is verifiable and deterministic, known to everybody in advance. Cash flow, Y, is observable, but not verifiable, and accrues to the firm's management. The assumption that only A is verifiable will play a crucial role in what follows; in particular, foreclosure on the firm's property by the sheriff can only reach A, not Y. If the firm is not liquidated at date 1, final firm value V is realized at date 2, where V is a continuous random variable with cumulative density function $F(V)$ and support $[\underline{V}, \overline{V}]$. We assume that F is differentiable on $(\underline{V}, \overline{V})$ with density f, and will extend the definition of F and f to all of $[0, \overline{V}]$ in the obvious way. In this paper we shall assume for simplicity that V is non-verifiable, i.e. that management cannot credibly promise to transfer value to creditors at date 2.[8] Hence, we focus on short-term debt and ignore issues such as debt maturity structure and debt rescheduling.[9]

At date 1, the firm receives a signal v about V. To simplify the presentation (no loss of generality here), we assume that the signal is perfect, i.e. that the firm knows V already at date 1. For our analysis it is irrelevant whether V is also observed by the investors (as long as it is not verifiable). Short term cash flow Y, realized at date 1, is given by

$$Y = \begin{cases} 0 & \text{with proba } 1-q \\ Y_H & \text{with proba } q. \end{cases}$$

At date 0, the firm is run by a risk-neutral owner/manager who wishes to raise I from external investors. Investors are risk-neutral and competitive. This implies that the firm has all the bargaining power at the financing stage, for simplicity we will assume that it has it as well at the refinancing stage. The firm is financed by $n \geq 1$ investors who each put up $I_i > 0, \sum I_i = I$, $i = 1, ..., n$. We focus from now on on the case of two creditors, the case of more than two creditors being a simple extension. Investors provide finance against the promise by management to repay $P_i, i = 1, 2$, at date 1. If an investor does not receive this payment at date 1, she has the right to foreclose on the firm's assets or force the firm into bankruptcy, according to rules which we describe below.

From a contract-theoretic perspective, it is clear that investors' debt collection rights must depend on the set of creditors who attempt to collect. Yet, in real-world contracting, not of all these contingencies are considered ex ante. In fact, debt contracts typically specify individual, non-interactive collection rights, which are in practice governed by debt collection law, and are less complete concerning collective debt collection. Debt contracts typically specify certain rights in those situations, through covenants, priority rules or collateral assignments, but leave much of the creditors' interactive collection rights to bankruptcy law and judges, as a way to implement multilateral debt collection. Our approach to modelling these decisions by the law and the courts is to ask what a contract would optimally stipulate if it included complete provisions for multilateral debt collection. Hence, we interpret bankruptcy as multilateral debt collection.

[8]Formally, we need to assume that the support of V is $\{0\} \cup [\underline{V}, \overline{V}]$, where 0 has point mass 0 (so that management can always claim at date 2 that it has nothing to pay out).

[9]See Gertner and Scharfstein (1991) and Berglöf and von Thadden (1994) for models that address some of these issues.

In the simple framework considered here, the multilateral framework reduces to two outside investors, and we denote by $D_i \leq A$ their individual collection rights, i.e. the amount investor i can foreclose if the other investor does not foreclose. We further denote by C_i the individual claims under multilateral debt collection (bankruptcy). Clearly, the total bankruptcy claims cannot exceed the asset value: $C_1 + C_2 \leq A$.

We model the issue of default, foreclosure, and bankruptcy as a dynamic game. At date 1, the firm is supposed to pay out P_i to the creditors. If it does not do so, it defaults and bargains over the repayment. A crucial assumption of the model is that there are frictions in multilateral bargaining. This means that the creditors as a group cannot get together with the firm to negotiate their way efficiently around bankruptcy.[10] If this were possible, there would be no difference between bankruptcy and "negotiations in the shadow of bankruptcy", in which case the theory of bankruptcy would be largely trivial. To be concrete, we assume that out-of-bankruptcy bargaining is bilateral, i.e. that creditors are too dispersed to negotiate collectively and with one voice with the debtor.

This bargaining either leads to payments, to (individual) foreclosure or to bankruptcy. To begin with, we analyse a simplified model by assuming that bankruptcy is triggered automatically if several (both) creditors attempt to collect their debt simultaneously. In Section 4, we generalise this model to include the decision to trigger bankruptcy. Formally, we describe this sequence of events by the following extensive form game between the firm and its creditors at date 1:

1. Nature determines Y and V.
2. The firm pays out $p_i \leq P_i$, $i = 1, 2$.
3. If $p_i = P_i$, $i = 1, 2$, the game is over and the firm receives V at date 2.
4. If $p_i < P_i$ for one or both creditors, these creditors simultaneously choose to accept (strategy a) or to (attempt to) foreclose (f).

If one creditor does not accept p_i, he collects his debt individually and receives assets worth D_i; if both creditors do not accept their p_i the firm goes bankrupt and each creditor gets C_i. If $L \leq A$ is the total amount of assets

[10] For empirical evidence on inefficiencies in workouts, see, for example, Gilson et al. (1990) or Asquith, Gertner, and Scharfstein (1994).

liquidated, the firm continues on the scale $(1 - L/A)$. This means that the firm and its owners obtain $(1 - L/A)V$ at date 2. This assumption amounts to assuming that long-term firm value is produced with constant returns to scale.[11]

Interest rates across periods are normalized to 0. In order to simplify the analysis, we impose some further restrictions on the parameters. First, we assume that management never wants to liquidate the firm voluntarily:

$$\underline{V} \geq A. \tag{1}$$

This assumption describes the more interesting case of the bankruptcy problem: although it is efficient ex post not to liquidate assets, the parties are forced to do so to motivate the debtor to repay. But the assumption may appear restrictive, and it indeed simplifies the analysis. In the appendix we discuss the general case $\underline{V} \geq 0$ and show that all our results hold more generally.

Second, we impose the following monotonicity condition on the distribution of V:

$$xf(x) \text{ is non-decreasing on } (\underline{V}, \overline{V}). \tag{2}$$

This assumption is satisfied by all distributions with non-decreasing density (including the uniform), all densities that decrease at most linearly, and non-monotone combinations of these (for example Λ- or V-shaped densities). The assumption is not very restrictive and allows us to simplify some arguments.[12] Finally, we assume that cash flows in the the good state are sufficiently high so as to avoid liquidity constraints in that state; specifically, we impose

$$Y_H \geq 2A \tag{3}$$

Clearly, this assumption is reasonable with a two-point distribution of cash flows (the high-cash-flow state is supposed to capture the case without liquidity constraints, the low-cash-flow state that with liquidity constraints).

[11] As for example in Hart and Moore (1998) and Harris and Raviv (1995).

[12] By allowing to extend some local arguments globally, it simplifies, for example, the φ-function in Section 3 and therefore allows a simple characterization of debt capacity. Without it, the necessary condition for existence would look more complicated. Nothing important would change if we dropped this assumption (except for some arguments who become longer).

3 Optimal Contracts

Our model is a simplified illustration of a structure in which creditors are separated and face coordination problems in bargaining with the firm in case of default. Debt renegotiation, therefore, is more difficult than in a hypothetical benchmark case of one creditor, a problem that we model by assuming that bargaining must be done individually and not collectively. Strictly speaking, this corresponds to a simple model of exchange offers.[13]

To simplify notation, we let $D = D_1 + D_2$ and $C = C_1 + C_2$ denote aggregate claims. Furthermore, we normalize notation by assuming that creditor 1 is the one with a (weakly) higher individual claim:

$$D_1 \geq D/2 \geq D_2. \tag{4}$$

Note that, once (4) is fixed, a further such normalization is not possible with respect to the collective claims C_i.

A contract is given by four numbers (D, C, D_1, C_1) with $0 \leq D \leq 2A$, $0 \leq C \leq A$, $D/2 \leq D_1 \leq D$, $0 \leq C_1 \leq C$. We will first analyze the interaction at date 1 for any given contract and then study the choice of contract at date 0, assuming that the contracting parties anticipate how the contract influences their behavior later on.

3.1 Date 1 interaction:

Consider first the case in which the firm has nothing to pay out, $Y = 0$. Therefore, $r_1 = r_2 = 0$, and the creditors' problem in stage 5 of the game played at date 1 is given by the following simple matrix game

	a	f
a	$0, 0$	$0, D_2$
f	$D_1, 0$	C_1, C_2

(5)

Clearly, (f, f) is always a Nash equilibrium of this game.[14]

Now consider the more interesting case $Y = Y_H$. How much management will pay out and to what extent it actually wants to prevent liquidation,

[13] As studied, for example, by Detriagache and Garella (1996).

[14] The equilibrium is unique if and only if $C_1 > 0$ and $C_2 > 0$ (which will always be the case here). In the sequel, we will ignore non-uniqueness caused by indifference, in order to avoid trivial non-existence problems.

depends on the contract (D, C, D_1, C_1) and on the equilibrium played by the creditors in the foreclosure game, which, in turn, is influenced by the firm's payout offer (p_1, p_2). The foreclosure game is given by the following modification of matrix (5):

$$\begin{array}{c|c|c|} & a & f \\ \hline a & p_1, p_2 & p_1, D_2 \\ \hline f & D_1, p_2 & C_1, C_2 \\ \hline \end{array} \tag{6}$$

In this game, the firm can always induce the outcome (f, f) by offering $p_1 = p_2 = 0$. Outcome (a, a) is an equilibrium (no liquidation takes place), if and only if $p_1 \geq D_1$ and $p_2 \geq D_2$. If we rule out trivial multiplicities by resolving indifferences in such a way that the ex-ante optimization problem has a solution (as is standard practice in agency theory), (a, a) is the unique equilibrium of (6) if and only if

$$p_i \geq D_i, i = 1, 2, \tag{7}$$

and

$$p_1 > C_1 \text{ or } p_2 > C_2. \tag{8}$$

We shall see that the latter condition will be slack and therefore ignore it now. Thus (7) implies that the firm can induce outcome (a, a) by setting $p_i = D_i$, $i = 1, 2$. Therefore, we will also call D_i creditor i's nominal claim or the face value of his debt.

By a similar reasoning, the firm can induce the asymmetric outcome (a, f) by offering $p_1 = C_1$ and $p_2 = 0$, and analogously (f, a). Here, the firm defaults and treats its creditors asymmetrically: it does not pay creditor 2 and has him collect his debt, and it pays creditor 1 just enough to prevent him from sending the firm into bankruptcy. Note that if $C_2 < D_2$ creditor 1 exerts a positive externality on creditor 2, because if creditor 1 refused the firm's reduced payout, the firm would go bankrupt and creditor 2 would obtain less.[15]

Going back one stage in the bargaining game, we can now ask which

[15] Such asymmetry of treatment between the two creditors is typically illegal in most jurisdictions. As we construct the interaction from first principles, we cannot rule out such behaviour. Rather, true to our approach, we shall show that optimality considerations will forbid such asymmetries.

of the four outcomes in matrix (6) the debtor wants to induce at date 1.[16] Clearly, this depends on the parameters C, D, C_1, D_1 of the initial contract.

The debtor's incentives can be understood graphically, if we fix total nominal claims D and total bankruptcy claims C, and vary C_1 and D_1. Because liquidation is costly to the debtor by (1), default is least interesting for high firm values V and most interesting for low V. Asymmetric default (involving some liquidation and some payout) is therefore to be expected for an intermediate range of V. Furthermore, the debtor's incentives to default asymmetrically against creditor 1 (have creditor 1 collect his claim D_1 and pay $C - C_1$ in cash to creditor 2), increase with increasing C_1, as the debtor gets away with a smaller cash payment. Hence, the intermediate range of V, in which asymmetric default against creditor 1 is ex-post optimal for the debtor, increases with C_1 and decreases with D_1. The converse argument holds for asymmetric default against creditor 2. Interestingly, if one works out the details (see the Appendix), it turns out that these intermediate ranges actually disappear if C_1 is neither too large nor too small compared to D_1. More precisely, it can be shown that asymmetric default against neither creditor is ex-post optimal if

$$\frac{D}{C}D_1 + D - \frac{D^2}{C} \leq C_1 \leq \frac{D}{C}D_1 - D + C \tag{9}$$

in which case the firm optimally induces

- outcome (a, a) if and only if $\frac{V}{A} \geq \frac{D}{C}$
- outcome (f, f) if and only if $\frac{V}{A} < \frac{D}{C}$.

For given aggregate values C and D, the region defined by condition (9) can be simply described graphically in the $D_1 - C_1$ - plane. Depending on the relative sizes of C and D (the three cases are $D < C$, $C < D < 2C$, or $D > 2C$), the two straight lines in (9) partition the relevant rectangle $[D/2, D] \times [0, C]$ in three different ways. Figures 1 and 2 show the two cases that will later turn out to be the relevant ones. The qualitative features in all three cases are the same.

The two straight lines in (9) define three possible regimes in $C_1 - D_1-$ space. In regime 1 (points in the rectangle above line (a) - this set is empty

[16] Of course, at date 0 management prefers the (a, a) outcome because liquidation is inefficient. But at date 1, its preferences are guided by the D_i, C_i, and no longer by overall efficiency considerations.

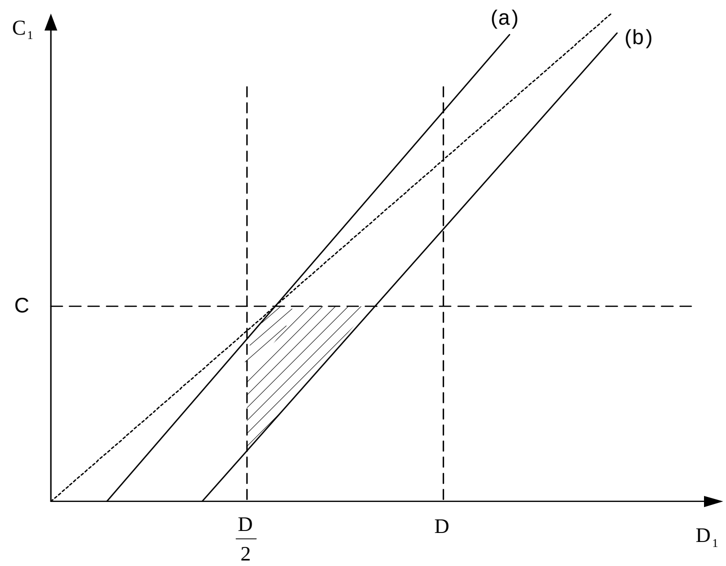

Figure 1: The three regimes if $C < D < 2C$

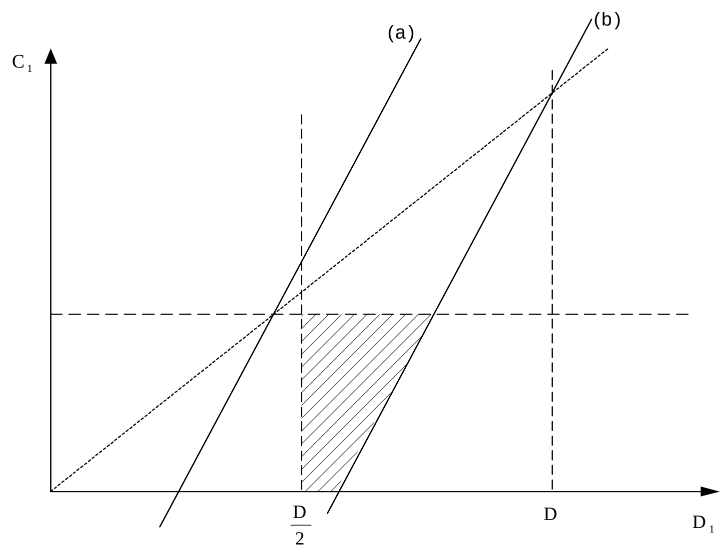

Figure 2: The three regimes if $D > 2C$

if $D > 2C$), creditor 1's bankruptcy claim is relatively high compared to his individual claim (and to that of creditor 2). In this regime it is never optimal to induce partial liquidation by creditor 2, because that would require paying off creditor 1's bankruptcy claim in cash and still have a sizeable foreclosure by creditor 2. Yet, all the other three outcomes are possible, depending on the size of V. If V is high, the firm repays completely, if V is low, the firm goes in strategic bankruptcy, and for intermediate values of V there is partial liquidation by creditor 1.

In regime 2, things are similar, with the roles of creditor 1 and 2 reversed. In regime 3 (points in the rectangle between the lines (a) and (b), defined by (9)), the individual claims of the two creditors are not too different from each other, and, if $C < D < 2C$, not too different from their bankruptcy claims. Therefore, it is never optimal to induce partial liquidation: the creditor experiences a sizeable liquidation by whoever he chooses to default upon and still needs to pay some cash to the other creditor to keep her from sending the firm into bankruptcy.

The choice among the two alternatives (a, a) and (f, f) can easily be understood by comparing the respective payoffs. With (a, a) the debtor's payoff is

$$Y_H - D + V, \tag{10}$$

while the payoff under (f, f) is

$$Y_H + (1 - \frac{C}{A})V. \tag{11}$$

Comparing (10) and (11) shows that the debtor prefers (a, a) over (f, f) iff

$$\frac{D}{C} \leq \frac{V}{A}. \tag{12}$$

Hence, in the case of intermediate individual claims, (9), the firm prefers to pay out if the continuation value V is higher than the threshold tA, where

$$t = \frac{D}{C}, \tag{13}$$

and prefers strategic bankruptcy otherwise.[17]

[17] If (9) does not hold, there is an interval around tA such that there is partial default for values of V in this interval.

Because the optimal payout in case (a, a) is D_1 and D_2, we can interpret these values as the face values of the individual debt claims. Differently put, in order for the initial contract not to be trivially renegotiated in case of full repayment, it should stipulate debt repayments of $P_i = D_i$. This is consistent with our earlier definition of D_1 and D_2 as individual liquidation rights, because in all jurisdictions we know of, an individual creditor has the right to sue the debtor for the full amount of her due debt, unless the firm seeks bankruptcy protection. Consistent with this interpretation, we can interpret C_1 and C_2 as collateral: these are asset values that each creditor must get under collective liquidation, regardless of how the overall debt is settled.[18]

3.2 Date 0 contracting:

We now turn to the contract design problem at date 0, which consists of choosing the optimal individual claims D_i (the face values) and bankruptcy claims C_i. A first important observation is that it is never optimal to induce default and partial liquidation.

Proposition 1 (Individual Debt Design): *Any optimal contract satisfies (9) and therefore does not entail asymmetric default ex-post.*

The proof of Proposition 1 is given in the appendix. To get an intuition for what condition (9) implies for the distribution of individual debt and bankruptcy claims, consider first the case $C < D < 2C$ (Figure 1), when aggregate bankruptcy claims are relatively high (more than 50 percent of total face value). In this case, it is optimal to make the creditors not too unequal (D_1 should be close to $D/2$) and to give the larger creditor a large share of the bankruptcy liquidation rights (C_1 should be large). This prevents the debtor from strategically defaulting against creditor 1 (because this would cost D_1 in asset value) and from strategically defaulting against creditor 2 (this would cost less in asset value, but C_1 in cash to prevent bankruptcy).

On the other hand, if $D > 2C$ (Figure 2), nominal debt claims are high and bankruptcy liquidation rights are relatively less important, so that it is

[18] Strictly speaking, the C_i are the creditors' bankruptcy receipts. Hence, in practice the C_i will be weakly greater than the value of the collateral. But as in many bankruptcies creditors get little more than their collateral, the collateral interpretation is a good approximation.

even possible to give either of the two creditors all these liquidation rights. In this case, what prevents partial default and liquidation are the high individual foreclosure rights.

Proposition 1 can be understood as follows. Suppose that the initial contract violates (9) by assigning a too low collateral value to the larger creditor 1 (and thus gives too much collateral to creditor 2): $C_1 < \frac{D}{C}D_1 + D - \frac{D^2}{C}$. In this case, we know from the discussion in the last subsection that there is an intermediate range of long-term firm values V such that the firm defaults asymmetrically against creditor 2. But decreasing D_1 (keeping overall debt D constant, i.e. bringing the two face values closer to each other), reduces this range, because the cost in terms of asset loss, $D - D_1$, increases. Hence, there is more (in terms of realizations of V) full repayment (D) and more bankruptcy liquidation (C). The former effect is unambiguously efficiency increasing, whereas the latter may be efficiency decreasing (if bankruptcy liquidation involves more liquidation than partial liquidation). However, the effect on total liquidation of more frequent full repayment D_1 always dominates the effect of more bankrutpcy liquidation $D_1 - C$. Hence, it is possible to reduce the relative importance of investor 1, such that total expected liquidation (which measures ex-ante efficiency) is decreased.

Proposition 1 is of interest for several reasons. First, it shows that optimal contracts rule out a too unequal division of nominal claims among the creditors. In particular, one creditor (1) can only have a significantly higher nominal claim than the other (creditor 2), if total debt is high and overall collateralization low ($D > 2C$), and if this claim is well collateralized (C_1 relatively high, to make partial default against creditor 2 costly). Second, optimal contracts exclude default with partial liquidation. In this sense, optimal contracts are symmetric: creditors are treated equally ex post. This feature is in line with the many provisions in different bankruptcy codes that prohibit the unequal treatment of creditors.[19] What is interesting is that the traditional justifications for such equal treatment rules are ethical: it is deemed unjust to treat similar creditors differently. Our model justifies such behavior on efficiency grounds: unequal treatment ex-post provides a way to pay off one creditor cheaply, which goes against the ex-ante goal of the contract: extract as much money from the creditor ex-post in order to avoid liquidation.[20]

[19] The *par conditio omnium creditorum* (equal treatment of all creditors) in Roman law is the mother of these clauses.

[20] The cash transfer of C_i made to prevent creditor i from triggering bankruptcy, also

The third point of interest of Proposition 1 is that it shows that optimal contracts only depend on the aggregate claims C and D. This is because an optimal contract uses the prisonner's-dilemma game between the creditors to maximizes the debtor's repayment incentives: either the debtor repays in full or she goes bankrupt; intermediate deals, which would involve the comparison of individual debt terms, are ruled out by contract design. Of course, the individual debt contracts (D_1, C_1) and (D_2, C_2) can be tailored freely and will be priced accordingly (as long as the limits of (9) for the individual claims are respected). But the firm's repayment incentives and debt capacity depend only on aggregate debt and collateral.

We now characterize these optimal D and C. As we have discussed, ex post the firm gets $(1-C/A)V$ in the bad cash flow state and either $Y_H - D + V$ (if $V \geq tA$) or $Y_H + (1 - \frac{C}{A})V$ (if $V < tA$) in the good cash flow state. If we denote the probability that the firm repays in the good state by θ,

$$\theta = \text{Prob } (V \geq \frac{D}{C}A),$$

the firm's expected surplus at date 0 is, therefore,

$$\begin{aligned} S_0 &= (1-q)(1-\frac{C}{A})EV + q\theta(Y_H - D + E[V \Big| V \geq \frac{D}{C}A]) \qquad (14) \\ &\quad + q(1-\theta)(Y_H + (1-\frac{C}{A})E[V \Big| V < \frac{D}{C}A]). \end{aligned}$$

The investors' participation constraints are

$$(1-q)C_i + q\theta D_i + q(1-\theta)C_i = I_i, i = 1, 2, \qquad (15)$$

where the I_i are, in fact, defined by (15), and $I_1 + I_2 = I$. Furthermore, the feasibility constraints

$$0 \leq D_1, D_2, C_1, C_2, C \leq A \qquad (16)$$

and the constraints (9) and $D_1 \geq D/2$ must hold.

Summing (15), one gets[21]

has the flavor of a fraudulent conveyance, ruled out in most bankruptcy codes. A payment to one creditor is a fraudulent conveyance if other creditors of equal or higher seniority are being paid less without being compensated in bankruptcy.

[21] It can easily be seen that the investors' participation constraint must be binding.

$$(1-q\theta)C + q\theta D = I. \tag{17}$$

Hence, the contract optimization problem at date 0 can be reduced to

$$\max_{D,C} \quad S_0 \tag{18}$$

$$\text{s.t.} \quad (1-q\theta)C + q\theta D = I \tag{19}$$

$$0 \le C \le A \tag{20}$$

$$0 \le D \le 2A. \tag{21}$$

In order to solve this program, we can first simplify S_0 to

$$S_0 = qY_H + EV - (1-q)\frac{EV}{A}C - q\theta D - q(1-\theta)\frac{C}{A}E[V \left| V < \frac{D}{C}A\right] . \tag{22}$$

(22) indicates that there are two sources of inefficiency in the contracting problem. First, there is liquidation in the bad cash flow state (occuring with probability $1-q$), and second there is liquidation in the good cash flow state following strategic default (occuring with probability $q(1-\theta)$). To minimize these inefficiencies, it is optimal to keep bankruptcy liquidation, C, small. On the other hand, this increases the debtor's incentives to default strategically, which reduces cash payout and is therefore inefficient as well. The optimal C trades off these two inefficiencies.

By substituting t for D as defined in (13), dropping additive terms, and multiplying by $-A/q$, problem (18)-(21) can be equivalently written as

$$\min_{t,C} \quad [(1-q)EV/q + \theta tA + (1-\theta)E[V\,|V < tA]]\,C \tag{23}$$

$$\text{s.t.} \quad (1+q\theta(t-1))C = I \tag{24}$$

$$0 \le C \le A \tag{25}$$

$$0 \le tC \le 2A \tag{26}$$

We can use the participation constraint (24) to express C as a function of t (remembering that $\theta = 1 - F(At)$):

$$C = \varphi(t) \equiv \frac{I}{1 + q(1-F(At))(t-1)} \tag{27}$$

Denoting the objective function (23) by

$$H(C,t) = \left[(1-q)EV/q + (1-F(At))At + \int_0^{At} V dF(V)\right] C \tag{28}$$

the problem can finally be written as

$$\begin{aligned} \min_t \quad & H(\varphi(t),t) & (29)\\ \text{s.t.} \quad & t \geq 0 & (30)\\ & \varphi(t) \leq A & (31)\\ & t\varphi(t) \leq 2A & (32) \end{aligned}$$

Here, (31) is the upper constraint on C (the right hand side of (20)) and (32) the upper constraint on D (the right hand side of (21)).

Under our auxiliary assumption (2) that $xf(x)$ is non-decreasing, it is relatively easy to solve this program. In fact, it is easy to show that under this assumption φ has exactly one minimum, t_0, and that this satisfies $t_0 \geq \underline{V}/A \geq 1$. If

$$f(\underline{V})(\underline{V} - A) > 1 \tag{33}$$

the minimum is $t_0 = \underline{V}/A$ and is at a kink. If $f(\underline{V})(\underline{V} - A) < 1$, we have $t_0 > \underline{V}/A$ and $\varphi'(t_0) = 0$.[22] See Figures 3 and 4 for graphical examples of φ.

For later reference we also note that

$$\frac{d}{dt}(t\varphi(t)) > 0. \tag{34}$$

Comparing the two constraints of the optimization problem, we have $t\varphi(t)/2 < \varphi(t) \Leftrightarrow t < 2$. Hence, constraint (32) on D will be redundant if $t < 2$, and constraint (31) on C will be redundant if $t > 2$. Constraint (30) is always redundant. Since the constraint set defined by (30) - (32) is compact and the objective function is continuous, a solution to the contracting problem exists if and only if constraints (31) and (32) are compatible. If $t_0 > 2$ the two constraints are compatible iff $\varphi(2) \leq A$ (see Figure 3). If $t_0 < 2$ they are compatible iff $\varphi(t_0) \leq A$ (see Figure 4). Hence, a solution exists iff

$$\varphi(\min(2, t_0)) \leq A. \tag{35}$$

[22] Remember that φ is not differentiable at $t = \underline{V}/A$. For $t < \underline{V}/A$, $\varphi' < 0$. For $t > \underline{V}/A$, φ' has at most one zero by assumption. If $\lim_{t \searrow \underline{V}/A} \varphi'(t) > 0$ (which is condition (33)), φ is increasing for $t > \underline{V}/A$, and therefore $t_0 = \underline{V}/A$. Otherwise we have $t_0 > \underline{V}/A$.

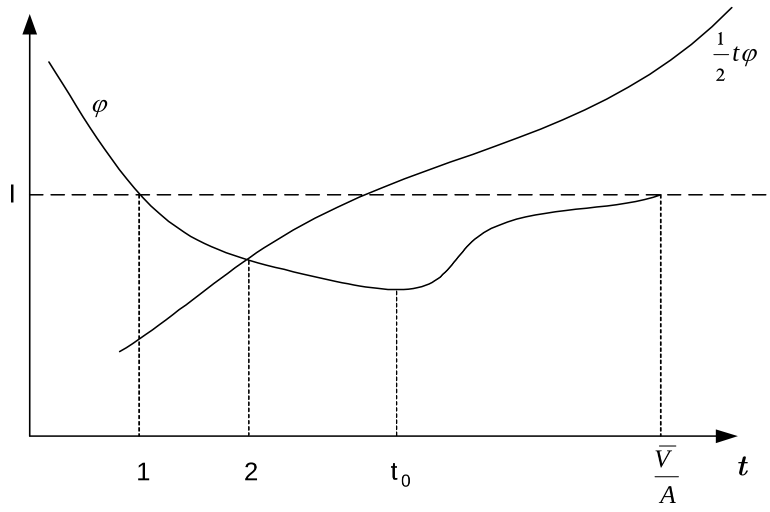

Figure 3: The constraints on C and D for $t_0 > 2$

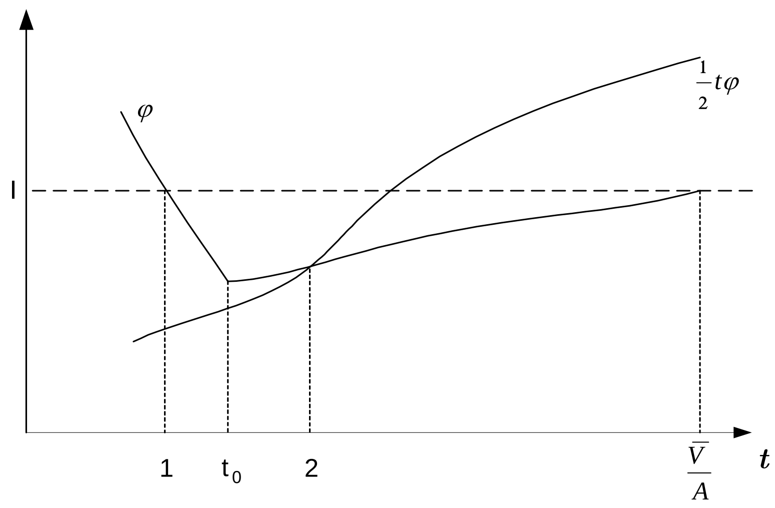

Figure 4: The constraints on C and D for $t_0 < 2$

Because φ is increasing for $t > t_0$, we have $\varphi(2) > \varphi(t_0)$ if $t_0 < 2$. Hence, $A \geq \varphi(2)$ is a sufficient condition for existence. Written out, the sufficient condition is

$$(1 + q - qF(2A))A \geq I. \tag{36}$$

Condition (36) is sufficient for existence but not necessary, as Figure 4 shows. Therefore, the left-hand side of (36) is a lower bound for the firm's debt capacity (the largest expected gross return the firm can credibly pledge to two creditors under any contract). An investment below this threshold can be financed, any amount above it may or may not be financed, depending on the distribution of long-term returns, F.

The economics of condition (36) can be understood as follows. Going back to the two-dimensional problem (23)-(26), the participation constraint (24) shows that the firm's exact debt capacity is given by

$$\begin{aligned} & \max_{t,C}(1 + q(1 - F(At))(t - 1))C && (37)\\ \leq\; & (1 + q(t - 1))C && (38)\\ =\; & (1 - q)C + qtC && (39)\\ \leq\; & (1 - q)A + q2A && (40)\\ =\; & (1 + q)A && (41) \end{aligned}$$

Hence, $(1 + q)A$ is an upper bound for the firm's debt capacity.[23] This is intuitive because $(1 + q)A = (1 - q)A + q2A$: the creditors can never get more than all assets in the bad state and each cash worth A in the good state. Condition (36) shows that this reasoning actually gives the exact debt capacity if $\underline{V} \geq 2A$, i.e. if long-term firm value is always very high. In this case, increasing C and t all the way up to their maximum values (A and $2A/A = 2$, respectively) maximises the investors' returns in (37). On the other hand, if $\underline{V} < 2A$, the debt capacity is lower than $(1 + q)A$ because the incentive for strategic default at low values of V provides a countervailing effect. In fact, in this case it can be impossible to raise an amount $I < (1 + q)A$ if I is sufficiently large, because the repayment obligations needed for the creditors to break even would be so high as to induce too much

[23] For the derivation, (38) uses the fact that at the maximum $t > 1$, because the factor in front of C in (37) is strictly smaller than 1 if $t < 1$, and equal to 1 and strictly increasing for $t = 1$. Inequality (40) uses (32), the upper bound on D.

strategic bankruptcy by the debtor.[24] However, (36) shows that the firm's debt capacity is always strictly greater than A.

We now turn to a fuller characterization of the optimal contract. It is easy to check that both partial derivatives of the objective function H are positive:

$$\begin{aligned} H_C(C,t) &= (1-q)EV/q + (1-F(At))At + \int_0^{At} V dF(V) > 0 \\ H_t(C,t) &= AC(1-F(At)) > 0 \end{aligned}$$

This is intuitive: increasing collective liquidation (C) and total payout obligations ($D = t\varphi(t)$, which is increasing in t by (34)), increases the firm's loss. Given that t_0 is the unique minimum of φ, this implies that $\frac{d}{dt}H(\varphi(t),t) > 0$ if $t > t_0$. Hence, the solution to the minimization problem (29) - (32) satisfies $t^* \leq t_0$. Furthermore, by explicit calculation one finds that for $t \leq \underline{V}/A$ (which implies $F(At) = 0$),

$$\begin{aligned} \frac{d}{dt}H(\varphi(t),t) &= \frac{I}{(1+q(t-1))^2}[A(1+q(t-1)) \\ & \quad -((1-q)EV + qAt)(1 - Af(At)(t-1))] \end{aligned} \tag{42}$$

Hence, for $t < \underline{V}/A$ (where $f = 0$),

$$\frac{d}{dt}H(\varphi(t),t) = -\frac{(1-q)I}{(1+q(t-1))^2}[EV - A] < 0$$

This implies that the solution to the problem satisfies $t^* \geq \underline{V}/A$, if this is compatible with the constraints. Which of the two constraints (31) or (32) can cause problems? Clearly, as long as $t < t_0$, constraint (31) (the upper bound on C) poses no problem because increasing t relaxes this constraint. However, because $\frac{d}{dt}(t\varphi(t)) > 0$, increasing t tightens constraint (32). Hence, if $\underline{V}/A > 2$ (constraint (31) is slack at $\underline{V}/A$) and if $\frac{V}{A}\varphi(\frac{V}{A}) > 2A$ (constraint (32) is violated at $\underline{V}/A$), it is necessary to decrease t below $\underline{V}/A$. This is quite plausible: If the long-term value of the firm is always very high, the firm is willing to increase the promised repayment above $2A$ in exchange for a further reduction of the liquidiation threat C. However, payments above $2A$ are not

[24] One can check that non-existence occurs, for example, if $\underline{V} < \frac{qA+I-A}{q}$, i.e. $I > A + q(\underline{V} - A)$, and F has sufficient mass to the left of $2A$.

incentive compatible ex post. Hence, the constraint on D, $t\varphi(t) \leq 2A$, will be binding for $t < \underline{V}/A$, although the firm would not default strategically even if D were yet higher.

Yet, whether or not the constraint on D binds, the above argument implies that at the optimum, $D > C$:

Proposition 2 (Undercollateralization): *The solution to the minimization problem (29) - (32) satisfies* $1 < t^* \leq t_0$. *Hence, under the optimal contract, aggregate debt is not fully collateralized:* $C^* < D^*$.

Proof: For the relationship between t, C, and D remember that $D = tC = t\varphi(t)$. We have already seen that $t^* \leq t_0$. For the lower bound assume first that $\underline{V} \geq 2A$. In this case, the existence condition (35) implies that $t = 2$ is feasible. By the argument following (42) above, decreasing t below 2 makes things strictly worse. Hence, $t^* \geq 2$.

If $\underline{V} < 2A$, constraint (32) is slack at $t = \underline{V}/A$. If $\varphi(\underline{V}/A) > A$, constraint (31) is violated at $t = \underline{V}/A$, which implies $t^* > \underline{V}/A \geq 1$.

If $\varphi(\underline{V}/A) \leq A$, $t = \underline{V}/A$ is feasible. By the argument following (42) above, it is not optimal to choose $t < \underline{V}/A$. This shows that $t^* > 1$ if $\underline{V} > A$. Whether or not it is optimal to choose $t > \underline{V}/A$ depends on the right-hand derivative of $H(\varphi(t), t)$ at $\underline{V}/A$. If this derivative is strictly negative, $t^* > \underline{V}/A$, otherwise $t^* = \underline{V}/A$. By (42), this right-hand derivative is strictly negative at $\underline{V}/A$ iff

$$(f(\underline{V})(\underline{V} - A)((1-q)EV + q\underline{V}) < (1-q)(EV - A)$$

which is always true if $\underline{V} = A$.■

The proposition states that debt should optimally be undercollateralized. Although this is what happens in practice, the result is not trivial. One can actually well conceive of overcollateralizing debt, for example for incentive or insurance reasons. Our analysis, however, emphasizes that collateral has the cost of destroying value ex post (because assets outside the firm are worth less than inside); therefore, too much collateral is not optimal.[25]

Proposition 2 only concerns aggregate values of debt and collateral. Whether it is optimal to fully or even overcollateralize individual claims will be analyzed in the next section.

[25] As shown in the appendix, for this argument to hold it is enough that the value of the assets inside the firm is higher than outside with a positive ex ante probability. Hence, the argument is completely general.

Proposition 2 has an immediate consequence for the optimal number of creditors in our model. If the firm contracted with only one creditor, this creditor would hold a liquidation claim of D regardless of the type of default. This implies that the creditor would be repaid D in both states: in the good state in cash (because the firm prefers paying out to liquidating the same amount), and in the bad state through liquidation (because the firm is cash constrained). Such a contract can be replicated in the two creditor case by setting $D = C$. The discussion in section 3.1 shows that in this case the firm never goes strategically bankrupt, just as with one creditor. Proposition 2 implies that such a contract is not optimal.

Proposition 3 (Optimal Number of Creditors): The firm strictly prefers to have two creditors rather than one.

Hence, the ability of the debtor to pledge his assets to each individual creditor strictly improves the terms of contracting. Because the debtor can credibly promise a higher cash payout than his bankruptcy liquidation value, the creditors can lower the amount of liquidation in bankruptcy. Although this strengthens the debtor's incentives for strategic bankruptcy, this effect does not outweigh the efficiency gains from reduced liquidation. Hence, the optimal contract differentiates between bankruptcy liquidation rights and individual debt collection rights.

A simple consequence of Propositions 2 and 3 is that the firm will be over-leveraged with respect to its asset base whenever there is finance for projects with $I > A$, i.e. for which the assets are insufficient ex post to cover the investment outlay. In this case, the face value of debt must exceed the firm's asset value: $D^* > A$. In particular, individual debt claims will be incompatible if the firm defaults on its debt.

Because the Coase Theorem does not apply in our model, strategic default and bankruptcy may occur in equilibrium. In fact, it is easy to see that if $\underline{V} < 2A$ and if the following condition holds, the debtor will default strategically for sufficiently low values of V:[26]

$$I > (1-q)A + q\underline{V} \tag{43}$$

The basic tradeoff faced by the debtor in this case can be illustrated in terms of the original two-dimensional contracting problem (23)-(26). On the one hand, minimising bankruptcy liquidation C is desirable. However, the participation constraint (24) shows that this comes at the cost of increasing t, which means that the face value of the debt ($D = tC = t\varphi(t)$) rises, which in turn favors strategic default. Yet, as long as it is possible to decrease C such that t does not exceed $\underline{V}/A$, strategic default is no issue and decreasing C is unambiguously advantageous. If the costs of strategic default are too high even at the margin, then $t^* = \underline{V}/A$; and there never is strategic default in equilibrium. Otherwise, we have $t^* > \underline{V}/A$, and the contract induces some strategic default. This will happen in particular (condition (43)), if investment needs I are relatively high. In this case, the parties must set the face value of debt sufficiently high, even if this means worsening the debtor's repayment incentives.

A final feature of the optimal contract worth investigating is what constraints are binding and under what conditions. To simplify the exposition, we again focus on the more realistic case $\underline{V} < 2A$ (otherwise, long-term firm value is very high almost surely - not a good assumption for the study of bankruptcy). Under this assumption, the constraint on D, (32), is slack at $t = \underline{V}/A$. It follows that if

$$I < q\underline{V} + (1-q)A$$

the firm is not fully liquidated in bankruptcy: $C^* < A$. On the other hand, if $I \geq q\underline{V} + (1-q)A$ and

$$f(\underline{V})(\underline{V} - A) > 1$$

[26] As seen in the proof of Proposition 3, if $\underline{V} < 2A$, constraint (32) is slack at $t = \underline{V}/A$. If $\varphi(\underline{V}/A) > A$, which is equivalent to (43), constraint (31) is violated at $t = \underline{V}/A$, which implies $t^* > \underline{V}/A \geq 1$. This implies the claim, because the firm defaults strategically iff $V < At$. I restrict attention to this sufficient condition, because the full characterization of strategic default (though not difficult) gets complicated because of the different regions of binding constraints, discussed before Proposition 2.

the firm is fully liquidated in bankruptcy: $C^* = A$.[27]

This observation is interesting because, technically speaking, an arrangement with $C^* < A$ violates absolute priority (see, e.g., Jackson, 1986). The point is that the constraint on C, (20) or (31), does not need to bind at the optimum. If this is the case, the debtor retains some of the assets after bankruptcy although the creditors are paid less than D. This is due to the fact that continuation in the present model is always efficient and that there are constant returns to scale. Therefore, the parties have a strong ex ante incentive not to punish the debtor too hard in case of bankruptcy.[28] On the other hand, if the initial investment I is sufficiently high and low realizations of V sufficiently likely ($f(\underline{V})$ sufficiently high), it is optimal to make maximum use of the liquidation threat and set $C = A$.

The basic point is fairly general: if the debtor has a comparative advantage using the assets, it is ex ante costly to separate her from them ex post, and, therefore, an optimal contract will aim at reducing this incidence as much as possible. This insight, simple as it is, is in sharp contrast with traditional legal reasoning that demands to satisfy creditors first in case of bankruptcy (absolute priority).

4 Bankruptcy versus Debt Collection

In the base model of the last section, we have assumed that a simultaneous attempt by creditors to collect their debt automatically triggers bankruptcy. In reality, of course, bankruptcy must be triggered by someone, and the base model is silent on this issue. In this section, we generalize the base model to a model in which bankruptcy is not an automatic consequence of simultaneous debt collection, but the result of a deliberate decision. This will at the same time shed light on the controversial question who should have the right to trigger bankruptcy. Finally, we will study the possibility of making individual claims senior.

[27] By the argument following (42), $t^* \geq \underline{V}/A$ if $\underline{V}/A < 2$. Hence, (31) is slack iff $\varphi(\underline{V}/A) < A$, which is equivalent to $I < q\underline{V} + (1-q)A$.

If $\varphi(\underline{V}/A) \geq A$, (31) binds a forteriori if $\frac{d}{dt}H(\varphi(t), t) \geq 0$ for all $t > \underline{V}/A$. As discussed earlier, this holds if $f(\underline{V})(\underline{V} - A) > 1$ (which is condition (33)). In this case $\varphi'(t) > 0$ for $t > \underline{V}/A$, hence $\frac{d}{dt}H(\varphi(t), t) = H_C + H_t\varphi' > 0$ for $t > \underline{V}/A$.

[28] As shown in the appendix, this argument continues to hold if liquidation ex post is efficient with a sufficiently small probability.

In order to define the more general model, we reconsider the assumption about simultaneous foreclosure in the debt collection games (5) and (6). While bankruptcy is a coordinated attempt to liquidate in default, simultaneous foreclosure is a priori uncoordinated. Hence, instead of the collective claims C_i exercised in the case of (f, f) in the game of Section 3, we will now assume that simultaneous foreclosure results in an uncoordinated run for the assets, in which the first to collect his debt liquidates D_i, and the second the rest, $A - D_i$.[29] Assuming that each creditor has the same probability of being first and that $D_1 + D_2 \geq A$, the payoff matrix becomes

	a	f
a	p_1, p_2	p_1, D_2
f	D_1, p_2	$\frac{1}{2}(A + D_1 - D_2), \frac{1}{2}(A + D_2 - D_1)$

(44)

where $p_1 = p_2 = 0$ in the case of $Y = 0$.

In this framework, which represents a "pre-bankruptcy", primitive state, ex-post interactions and ex-ante contracting will be as in Section 3, with the exception that $C = A$ (absolute priority) is fixed exogenously. This simplifies the problem of Section 3 considerably, but the resulting contract will typically not be optimal, because we have seen that the optimum typically has $C^* < A$. The reason is that the deadweight loss through complete liquidation more than outweighs the improved incentives for payout in the good state. Ex ante it is therefore optimal to reduce the threat of liquidation from A to C^*.

This can be done by introducing bankruptcy into the model; and more precisely, by giving each creditor or the firm the right to trigger bankruptcy when the firm defaults on (some of) its payments. Bankruptcy then means that individual debt collection is suspended, and creditors receive C_i instead of D_i.

The possibility of triggering bankruptcy changes nothing in the analysis of Section 3 if the firm has defaulted partially. In fact, the creditor who is defaulted upon, recuperates his full claim through individual debt collection, whereas the other creditor weakly prefers to accept the renegotiated payment from the firm. The firm prefers partial default to bankruptcy by revealed preferences.

If the firm has defaulted on both creditors and $Y = 0$ or $V < t^* A$, the analysis changes. In this case, creditor i when observing the attempt to

[29] This assumes that $D \geq A$, which is the interesting case to consider (otherwise there is no conflict ex post).

foreclose by creditor j will call bankruptcy if this makes him better off than waiting, i.e. if

$$C_i > A - D_j. \tag{45}$$

Since either of the two creditors may be last in line for debt collection, condition (45) must hold for $i = 1, 2$, if an (inefficient) run for the assets shall be avoided for sure. Adding up (45) for $i = 1, 2$ yields the joint condition

$$C + D > 2A, \tag{46}$$

which may be satisfied by the solution (D^*, C^*) found in the last section, but need not.

If D^* and C^* satisfy the joint condition, the next question is whether one can find values C_1 and D_1 such that $D_1 \geq D^*/2$ and (9) hold. If this is the case, the solution (D^*, C^*) can be implemented by giving each creditor the right to trigger bankruptcy. If this is not the case, or if $C^* + D^* < 2A$, this will not be possible, and at least one creditor will have an incentive to run for the assets even if he has the right to trigger bankruptcy.

In this case, however, another simple contractual remedy is available: allowing the firm to trigger bankruptcy. In fact, if $C^* < A$, the firm strictly prefers bankruptcy to uncoordinated liquidation. If on the other hand, $C^* = A$, the firm is indifferent, but then there is no efficiency case for bankruptcy in the first place.

We summarize our discussion by the following proposition, whose proof is completed in the appendix.

Proposition 4 (Implementation): *If $C^* + D^* > 2A$, optimal collective liquidation C^* can be implemented by giving each creditor the individual right to trigger bankruptcy following default. If $C^* + D^* < 2A$, the firm must have the right to trigger bankruptcy.*

It should be noted that the conditions imposed on individual claims in order to get the triggering incentives right in the case $C^* + D^* > 2A$ are non-negligeable. Figure 5 in the appendix (discussed in more detail in the appendix) provides an illustration of the additional restrictions that must be imposed on C_1 and D_1 (over and above (9)). If these restrictions are too strong for other reasons (if, for example, one creditor wants to be senior, see below), it is necessary to give the firm the right to trigger bankruptcy also in the case $C^* + D^* > 2A$.

Proposition 4 is interesting because it provides a new efficiency justification for debtor-friendly bankruptcy rules. There is a broad consensus that creditors should have the right to trigger bankruptcy. The consensus is less developed with respect to debtor rights. While most of the criticism of debtor-friendly rules concerns debtor-in-possession rules such as those of Chapter 11 in the U.S., it is also being argued that the individual right of a debtor to evade the discipline of debt collection is harmful. Proposition 4 shows that there is an efficiency reason for such a right: if the creditors do not have the ex-post incentives to select the efficient continuation decision, the debtor may have these incentives. Hence, any reform of possibly excessive managerial discretion rights under Chapter 11 should be careful not to throw out the child with the bathwater and scrap the debtor's right to trigger bankruptcy.[30]

A final question that is interesting to address in the framework of the present model concerns seniority. Usually a claim is said to be senior if it is satisfied before other claims. In our model, in the good state, both claims can be satisfied, in the bad state both are partially satisfied. To capture the main feature of seniority in our framework, we can ask how much a claim is paired down in bankruptcy. In particular, we can call the claim of creditor i senior if $C_i = D_i$ or $C_j = 0$: either creditor i receives his face value in liquidation or he gets all liquidation proceeds.

It is easy to see (e..g., Figures 1 and 2) that it is not optimal to set $C_i = D_i$ for any i. The reason is that this would make partial default too attractive, because it would be relatively cheap for the firm ex post to bribe creditor j not to liquidate (see the discussion in section 3.1). Hence, it is impossible to give a safe claim to a creditor. However, it may be possible to allocate all the liquidation rights to one creditor. If $C^* < D^* < 2C^*$ (Figure 1), only creditor 1 can be senior (C_1 cannot be set too small). Yet, if the optimal contract has $D^* > 2C^*$, one can give creditor 1 priority by setting $D^*/2 \leq D_1 \leq D^* - C^* + C^{*2}/D^*$ and $C_1 = C^*$. But one can equally well make creditor 2 senior (by setting $C_1 = 0$). If the reason for making this creditor senior is that he dislikes risky claims (this is outside the present model), the riskiness of his claim is minimized by setting $D_1 = D^*/2$ (see Figure 2). This arrangement thus has two debt claims of equal face value, one of which is highly secured, the other not at all.

We thus have shown that under the optimal contract, it is impossible

[30] On a related note, see Berkovitch, Israel and Zender (1998).

to collateralize one claim fully, i.e. to make the claim riskless. This is the case in practice, as 100 percent collateralized claims hardly exist. If aggregate liquidation rights are relatively small compared to nominal debt ($D^* > 2C^*$), each of the two creditors can be made senior in the sense that he receives all the bankruptcy proceeds. If aggregate liquidation rights are relatively high ($C^* < D^* < 2C^*$), only the larger creditor can be senior.

5 Conclusion

We have analyzed the design of bankruptcy rules and debt structure in an optimal-contracting perspective. If cash flow is not verifiable and only the collateral value of the firm is verifiable, then when a firm borrows from a single creditor and has all bargaining power, its debt capacity is limited to the value of its collateral. The reason is that the creditor can never expect to receive more than the collateral value in liquidation and in renegotiation. However, when a firm borrows from more than one creditor, it can increase its debt capacity by pledging its collateral value to more than one creditor and giving each the right to foreclose on its assets. If the debt structure of the firm is designed appropriately, this creates a commitment for the firm to pay out more in good states to prevent the exercise of individual foreclosure rights and thus raises the firm's debt capacity. Having multiple creditors thus helps to reduce the negative effects of the lack of commitment in contracting by distinguishing between individual foreclosure rights and joint liquidation rights achieved under bankruptcy.

Our theory provides a bridge between corporate finance and the legal theory of debtor-creditor law. The key distinction in debtor-creditor law in most jurisdictions is that between debt collection law and bankruptcy law. The former governs the interaction between the debtor and a single creditor, the latter the interaction between the debtor and several creditors.[31] Our analysis shows that this same distinction must be made in a contract-theoretic approach to debt. Individual foreclosure rights (corresponding to debt-collection law) are crucial to generate repayment incentives, but need to be complemented by collective liquidiation rights (corresponding to bankruptcy law) in order to maximize ex-post efficiency.

[31] The U.S. is a particularly telling example of this distinction. Here, the two bodies of law are even governed by different authorities: debt-collection law is state law, bankruptcy law is federal law.

Our results on debt structure and overleverage under multiple creditors depend on the fact that creditors have unilateral foreclosure rights that they can exercise in case of default, independently of what other creditors decide. These rights should be seen as an important element of investor protection. The renegotiation procedure modeled in this paper emphasises the effect of these rights since renegotiation is assumed to happen on an individual basis. The ensuing prisoner's dilemma situation forces the debtor to respect contractual claims as given by individual foreclosure rights whenever he wants to avoid default. The key assumption in this approach is that it is difficult and costly to bring the creditors together to renegotiate the debt contract collectively. Only bankruptcy brings all the contracting parties together at one table, but in bankruptcy the debtor is restricted in his bargaining, because bankruptcy is mostly concerned with the reconciliation of individual liquidation claims. This is the classical "vis attractiva" of bankruptcy.[32]

Yet, it is theoretically conceivable and possible in practice that the debtor can unite the group of creditors, or their representatives, and extract from them joint concessions under the threat of bankruptcy. If such workouts are frictionless, the theory presented in Sections 3 and 4 collapses into the one-creditor-case discussed in Proposition 3. More generally, any theory with multiple creditors and frictionless all-inclusive negotiations in the shadow of bankruptcy is likely to be of little interest. In practice, however, frictions in such negotiations can be substantial, and, in particular, increase with the number of creditors. One classical reason for these frictions is, of course, the hold-up problem of the individual creditor, which is precisely the reason for institutionalised bankruptcy rules as discussed in Section 4. Another reason is the legal uncertainty accompanying out-of-court debt renegotiations, if individual creditors have the possibility of contesting the new arrangement in court.

There is strong empirical evidence on the difficulty of out-of-court agreements. Among others, Gilson, John, and Lang (1990), and Asquith, Gertner, and Scharfstein (1994) find that out-of-court restructurings have a substantial risk of failure, and this risk is the higher, the larger the number of creditors, in particular public debtholders. Gilson (1997) argues that the relatively

[32] The notion of "vis attractiva" ("attracting force") is one of the classical principles of bankruptcy theory going back to Roman times. It formulates the historical experience that creditors typically fail to reach agreement when left to their own devices, and that only bankruptcy - when the debtor gives up the right to his estate - has the force to bring them together.

high transactions costs of private restructurings that he finds (compared to Chapter 11 cases) are the result of unanimity requirements and greater information asymmetries in out-of-court settlements.[33]

Further research is necessary to better characterize the effect of different renegotiation procedures, the role of courts in intervening in private contracts, and several other issues. In particular, we have considered one fixed bargaining game between debtor and creditors, and analyzed its implications for debt structure and bankruptcy. We believe that this bargaining game, a simple version of exchange offers, captures most of the important features and frictions of debt renegotiations, but it would be clearly useful to compare this structure to others. By doing this, the model may ultimately contribute to a comprehensive comparative analysis of the effects of various bankruptcy laws across countries and across time.

[33] For two (quite different) legal discussions of these issues, see Finkelstein (1993) and Schwartz (1998).

6 Appendix A. Proofs

For completeness, we first state the precise conditions for the different renegotiation outcomes at date 1, discussed verbally in Section 3.1.

Lemma 1: *The renegotiation game at date 1 has the following outcomes:*

1. *If* $C_1 > \frac{D}{C}D_1 - D + C$ *the firm optimally induces*
   - *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D-C+C_1}{D_1}$
   - *outcome* (f, a) *if and only if* $\frac{C-C_1}{C-D_1} \leq \frac{V}{A} < \frac{D-C+C_1}{D_1}$
   - *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{C-C_1}{C-D_1}$
2. *If* $C_1 < \frac{D}{C}D_1 + D - \frac{D^2}{C}$ *the firm optimally induces*
   - *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D-C_1}{D-D_1}$
   - *outcome* (a, f) *if and only if* $\frac{C_1}{C-D+D_1} \leq \frac{V}{A} < \frac{D-C_1}{D-D_1}$
   - *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{C_1}{C-D+D_1}$
3. *If* $\frac{D}{C}D_1 + D - \frac{D^2}{C} \leq C_1 \leq \frac{D}{C}D_1 - D + C$ *the firm optimally induces*
   - *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D}{C}$
   - *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{D}{C}$.

The proof of the proposition involves a somewhat lengthy comparison of alternatives and is available on request from the author. Note that these three regimes are exhaustive and exclusive.[34]

[34] This is because

$$\begin{aligned} & \frac{D}{C}D_1 - D + C \geq \frac{D}{C}D_1 + D - \frac{D^2}{C} \\ \Leftrightarrow \quad & (C - D)^2 \geq 0. \end{aligned}$$

Proof of Proposition 1: We will first show that it is not optimal to set

$$C_1 < \frac{D}{C}D_1 + D - \frac{D^2}{C} \tag{47}$$

(which is the condition in regime 2 of Lemma 1). Define the bounds derived in Lemma 1 as

$$\begin{aligned} V_L &= \frac{C_1}{C - D + D_1}A \\ V_H &= \frac{D - C_1}{D - D_1}A \end{aligned}$$

and recall that if the contract satisfies (47), ex post the firm optimally induces

- outcome (a, a) if and only if $V \geq V_H$
- outcome (a, f) if and only if $V_L \leq V < V_H$
- outcome (f, f) if and only if $V < V_L$.

We need to show that setting (D_1, C_1) in the region defined by (47) makes the parties ex ante worse off than setting (D_1, C_1) in the area defined by (9). To do this, we vary C_1, D_1 for $0 < C_1 < C$ and $\frac{C}{D}C_1 + D - C < D_1 < D$ such that the investors stay on their participation constraint.

The investors' (aggregate) participation constraint is

$$(1-q+qF(V_L))C+q(1-F(V_H))D+q(F(V_H)-F(V_L))(C_1+D-D_1) = I. \tag{48}$$

Hence, small changes dC_1 and dD_1 keep the investors indifferent iff

$$(F(V_H)-F(V_L))(dC_1-dD_1) = (D_1-C_1)f(V_H)dV_H+(D-C-D_1+C_1)f(V_L)dV_L \tag{49}$$

where $dV_i = \partial_{C_1}V_i dC_1 + \partial_{D_1}V_i dD_1$.

The total ex-ante deadweight loss under any choice (D_1, C_1) is

$$\begin{aligned} L &= q\left[\int_{\underline{V}}^{V_L}\left(\frac{C}{A}V - C\right)dF(V) + \int_{V_L}^{V_H}\left(\frac{D - D_1}{A}V - (D - D_1)\right)dF(V)\right] \\ &\quad +(1-q)(\frac{C}{A}EV - C) \end{aligned}$$

(remember that cash transfers constitute no loss). Hence, under the small change (dD_1, dC_1) we have, after some manipulations,

$$
\begin{aligned}
\frac{1}{q}dL &= (D-D_1)(\frac{V_H}{A}-1)f(V_H)dV_H + (C+D_1-D)(\frac{V_L}{A}-1)f(V_L)dV_L \\
&\quad + \left(F(V_H)-F(V_L)-\int_{V_L}^{V_H}\frac{V}{A}dF(V)\right)dD_1 \\
&= (D-D_1)(\frac{V_H}{A}-1)f(V_H)dV_H + (C+D_1-D)(\frac{V_L}{A}-1)f(V_L)dV_L \\
&\quad - \left((\frac{V_H}{A}-1)F(V_H)-(\frac{V_L}{A}-1)F(V_L)-\frac{1}{A}\int_{V_L}^{V_H}F(V)dV\right)dD_1
\end{aligned}
$$

where we have used partial integration for the last equality. Combining the last formula with (49) shows, after some straightforward manipulations, that increasing D_1 increases L if (47) holds. Hence, at the optimum (D_1, C_1) is either on or to the left of the line defined by (47) or on the line segment $C_1 = 0$, $D_1 > D - C$. The latter possibility can easily be discarded. Hence, a contract satisfying (47) is not optimal.

A similar argument shows that regime 1 of Lemma 1 is not possible under an optimal contract.■

Proof of Proposition 4: Assume first that

$$C + D > 2A. \tag{50}$$

Throughout the proof we assume that $D \geq A$ (the alternative is straightforward). As argued in the main text, if none of the two creditors shall prefer a run to bankruptcy, we must have

$$
\begin{aligned}
C_1 &> A - D + D_1 &(51)\\
C_1 &< C - A + D_1. &(52)
\end{aligned}
$$

The question is: Are these two conditions (plus the normalization $D_1 \geq D/2$) consistent with condition (9),

$$\frac{D}{C}D_1 + D - \frac{D^2}{C} \leq C_1 \leq \frac{D}{C}D_1 - D + C,$$

which we know from Proposition 1 an optimal contract must satisfy?

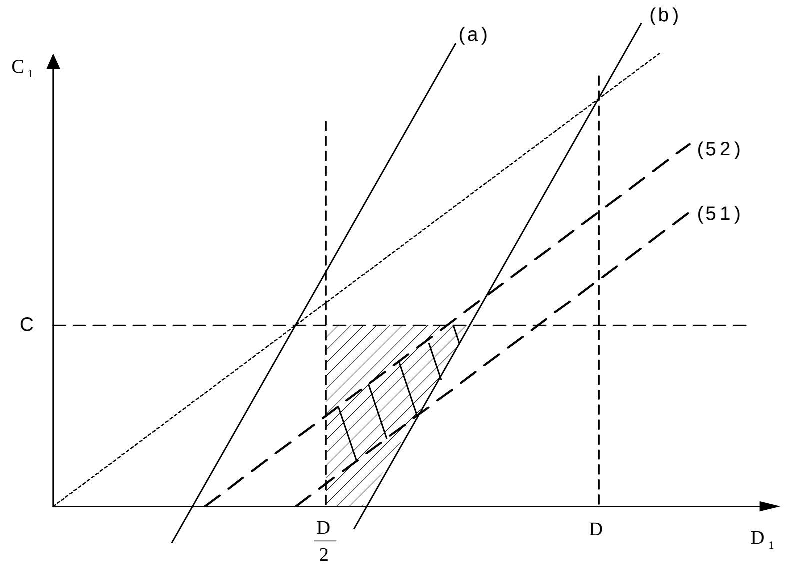

Figure 5: Compatibility of (51) and (52) with (9): $D > 2C$

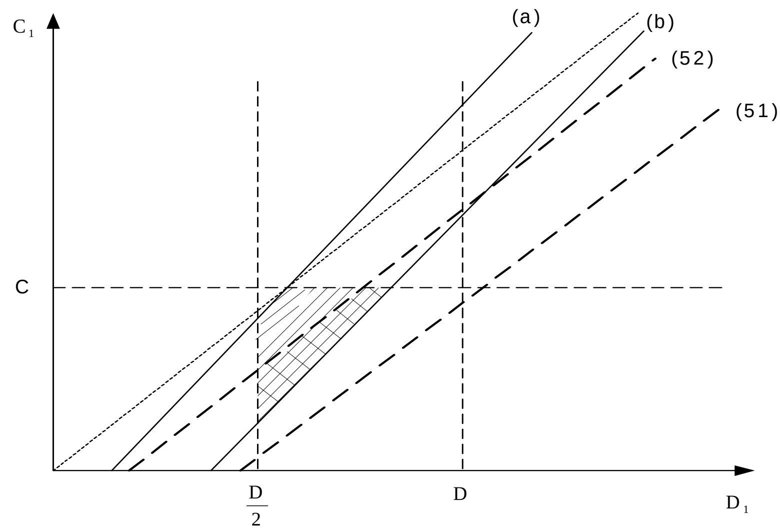

Figure 6: Compatibility of (51) and (52) with (9): $C < D < 2C$

Consider first the case $D > 2C$, for which Figure 5, which uses Figure 2, depicts condition (9) graphically. Condition (52) is compatible with (9) iff the straight line defined by (52) in $D_1 - C_1$ - space intersects the line $C_1 = \frac{D}{C}D_1 + D - \frac{D^2}{C}$ in the positive orthant (see Figure 5). The intersection is given by

$$\begin{aligned} C - A + D_1 &= \frac{D}{C}D_1 + D - \frac{D^2}{C} \\ \Leftrightarrow \quad D_1 &= D - C\frac{A-C}{D-C} \end{aligned}$$

Since $D - C\frac{A-C}{D-C} > D - C$, the intersection is indeed in the positive orthant.

Condition (51) is compatibel with (9) iff the intersection of the straight line defined by (51) with the line $D_1 = D/2$ lies below the line $C_1 = C$ (see Figure 5). The intersection is given by

$$C_1 = A - D + \frac{D}{2} = A - \frac{D}{2}$$

and we have $A - D/2 < C$ by (50).

The case $C < D \leq 2C$ is similar and omitted (see Figure 6 for an illustration). $D \leq C$ is impossible by Proposition 2.■

7 Appendix B. Inefficient continuation: the case $\underline{V} < A$

In this appendix, we briefly sketch how the analysis changes if the continuation of the firm can be inefficient ex post. Hence, we replace assumption (1) by the assumption $\underline{V} < A < \overline{V}$.

The main results do not change in this setting. However, some of the analysis becomes more complicated. Already Lemma 1 changes significantly, which reflects the different continuation incentives when $V < A$. In fact, the main insight for the case $V < A$ is that the debtor will always liquidate all his assets voluntarily ex post. This means that the debtor values an asset loss exactly like a cash loss. In particular, if $V < A$ the debtor prefers repayment (the cell (a, a)) over strategic bankruptcy (the cell (f, f)) iff $D < C$. This condition, and the analogous ones for asymmtric default, are different from the conditions in Lemma 1 and must therefore be added to the (already complicated) set of conditions in Lemma 1. However, a closer inspection shows that for the case of (9) (no partial liquidation) the additional restrictions mean very little. More precisely, if $D > C$, the default condition remains unchanged (i.e., the new conditions are slack); and for $D < C$, it becomes even simpler, becaus there never is strategic default.

Proposition 1 does not change at all when $\underline{V} < A$. The reason for why condition (9) is optimal ex ante is essentially the same as before: As seen in Proposition 1, it is strictly not optimal to induce partial liquidation when $V > A$ for sure, because it induces too much liquidation compared to what it achieves on the incentives front. However, if $V < A$ ex post, liquidation is efficient and the parties are therefore indifferent ex ante whether to induce partial liquidation or not. Hence, as long as there is some probability that $V > A$, the parties will strictly prefer to impose (9).

For the remainder of the analysis of Section 3.2, one additional argument is necessary when $\underline{V} < A$. As seen above, if $D < C$ the debtor will always repay D when solvent (regardless of V). Hence, there is no concern with strategic bankruptcy ex ante. This implies that it is optimal to increase D at the margin (which is costless in terms of incentives and liquidation loss) and lower C in exchange (which brings an efficiency gain if $V > A$ ex post). It follows that the optimal contract has $D \geq C$.

This now implies that the analysis remains essentially unchanged in the more general model. In fact, the debtor will default strategically if $V < A$ (because this costs him cash of C instead of cash of D - remember that he transforms his assets into cash, anyhow), and will default strategically if $A \leq V < tA$ (for the reasons discussed in Section 4.1). Taken together this means that the debtor will default strategically iff $V < tA$, exactly as before. The debtor's ex ante objective therefore is to maximize

$$(1-q)F(A)(A-C) + (1-q)\int_A^{\overline{V}} (1-\frac{C}{A})VdF(V) + qF(A)(Y+A-C)$$
$$+q\int_A^{At} Y + (1-\frac{C}{A})VdF(V) + q\int_{At}^{\overline{V}} Y - D + VdF(V)$$

which is a direct generalization of S_0 in (14). Hence, the deadweight loss to be minimized is

$$H(C,t) = \left[K + (1-F(At))At + \int_A^{At} VdF(V)\right] C \tag{53}$$

where

$$K = \frac{F(A)}{q}A + \frac{1-q}{q}\int_A^{\overline{V}} VdF(V)$$

The function H as defined in (53) is exactly the same as in (28), taking into account that $F(A) = 0$ in the case of Section 3.2 and that liquidation for $V < A$ is no deadweight loss. Similarly, the participation constraint does not change at all, and therefore, the φ - function linking C and t remains unchanged in the more general framework. Hence, the analysis can be conducted as before. Intuitively, what happens is that all the realizations $V < A$ lead to voluntary liquidation by the debtor, which gives him verifiable funds of A. This is the same as assuming that the distribution of V has $\underline{V} = A$ with a mass point at A.

Put differently, liquidation is efficient for $V < A$ and inefficient for $V > A$. In the former case, liquidation by the creditors simply is a transfer, with no ex-post efficiency consequences. Qualitatively, in the design of the initial contract, the efficiency consideration of the case $\underline{V} \geq A$ is therefore the only one that matters. Quantitatively, shifting weight in the distribution of V to the left of A will, of course, change things. For example, an inspection of the φ - function in (27) shows that this shift will shift φ uniformly upward. But this is only to be expected: if the firm's prospects become worse, its debt capacity goes down, and its bankruptcy loss will increase.

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