Subsidizing energy saving capital accumulation: a real option approach[1][2]

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Abstract

Some environmental policies, like tax credit, have tried to induce the acquisition of energy efficient units and the replacement of old energy inefficient vintages. However, they have faced the energy paradox that is a slow diffusion of new vintages. We develop a stochastic model of irreversible investment, in which firms also face embodied technological progress. We compare in a dynamic example a deterministic and a stochastic model with embodied technological progress. In the embodied case under uncertainty, the option to postpone replacement becomes very large, reducing drastically the effectiveness of a tax credit.

Key words : embodied technological progress, tax credit, option value
JEL classification : D81, E22,E62, H22

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1. Introduction

Global climate change has resulted in policy makers becoming more and more concerned about energy conversation investments. Indeed, public policies, like tax-incentives, have been developed, which aim at the adoption of energy saving machines and equipment. Nevertheless, these policies face the so-called "energy paradox": very attractive investment opportunities in energy efficient capital are ignored by investors, even if these opportunities have very high ex-ante rates of return. The diffusion of apparently cost-effective energy-efficient technologies is very low. Empirically, it has been shown by Walsh (1987,1989) that tax incentives decrease investment and by Dubin and Henson (1988) that the relationship between investment and tax incentives is statistically insignificant. Such a lack of effectiveness has substantially reduced policy makers' enthusiasm.

From a theoretical point of view, Hasset and Metcalf (1992) explains the discrepancy between tax incentives and investment using the combination of irreversibility of investment and uncertainty. They construct a model in which residential energy conservation investment is irreversible, and the price of energy as well as the cost of energy conservation capital are stochastic. In this framework, the authors study the decision of households to invest in one project. There is an option value to invest that leads to postponing the decision to invest. Households require a better environment to acquire new energy-efficient machines than they would if there were no irreversibility or no uncertainty. This is why a tax incentive may not be sufficient to trigger investment.

Nevertheless, such modelling only focuses on the household decision to invest, while energy conservation investment is also an issue for firms. Moreover, it only considers one project, which implies that it ignores the main grounds for investment. Investment is driven by two motives: capacity expansion and replacement. Pindyck (1988) proposes the first stochastic model of capacity expansion, in which investment is irreversible. His model deals with homogenous units, that is, all the machines are similar. Therefore, the firm has to reach full capacity before undertaking investment, which seems to be an unrealistic assumption. Due to the assumption of homogenous units, the replacement of machines is ignored in his model. However, replacement is an important feature of investment decisions, and technological progress is highly embodied in new machines, *i.e.* it is investment-specific. Greenwood et. al. (1997) argues that almost 60% of the US post-war growth can be accounted for by investment-specific technological progress. Therefore, when examining possible explanations for the energy paradox, irreversibility and uncertainty are certainly part of the story, but the fact that the energy-saving technological progress is largely investment-specific should be considered as well.

In this paper, we use a model developed in Pommeret and Cruz (2003) which exhibits these three characteristics (uncertainty, irreversibility and embodiment) to assess the efficiency of a tax credit. Investment is irreversible, the price of energy evolves stochastically, and new machines and equipment, due to embodied technological progress, are more efficient in terms of energy requirements. Capital and energy are assumed to be complementary. Furthermore, labour has no adjustment cost. We compare the effects of the tax credit we obtain in such a framework with those which result in a model of disembodied technological progress under uncertainty and those that emerge from the deterministic counterpart of the embodied case developed in Boucekkine and Pommeret (2001).

Our results show that uncertainty, irreversibility and the embodied technological progress assumption are crucial to explain the low degree of tax credit effectiveness. In a dynamic example, we compare the stochastic and deterministic cases of embodied/disembodied technological progress. In our example, firms reduce the optimal scrapping age with the tax credit; however, due to uncertainty, irreversibility and embodiment, the option to postpone the machines renders this impact almost negligible compared with those which exist in the deterministic case or in the disembodied case under uncertainty. Even if the returns on new equipment and machines are high, the acquisition is postponed because of uncertainty, irreversibility and embodiment. This leads to a slowdown in the replacement and the diffusion of more energy-efficient machines. Our model, therefore, can provide some theoretical explanations for the empirical observation about the ineffectiveness of tax credit, and the so-called energy paradox.

The deterministic model of embodied technological progress and the impact of a tax credit are presented in the next section. Section 3 describes the same model under uncertainty, and we also deduce the theoretical impact of tax credits. A dynamic example is performed in section 4 to compare the stochastic and deterministic cases in the disembodied and embodied models. Section 5 concludes.

2. Optimal capital stock under embodied technological progress

2.1. Presentation of the model

We consider that energy-saving technical progress is embodied in the new capital goods acquired by the firm. The firm's problem is:

$$\max \int_0^{\infty} \left[P(t)Q(t) - Pe(t)\, E(t) - w(t)L(t) - kI(t) \right] e^{-rt} dt \qquad (1)$$

subject to constraints taking embodiment into account :

$$P(t) = bQ(t)^{-\theta} \quad \text{with } \theta < 1 \qquad (2)$$

$$Q(t) = AK(t)^{\beta} L(t)^{1-\beta} \qquad (3)$$

$$K(t) = \int_{t-T(t)}^{t} I(z)dz \qquad (4)$$

$$Pe(t) = \overline{Pe}\, e^{\mu t} \text{ with } \mu < r \qquad (5)$$

$$E(t) = \int_{t-T(t)}^{t} I(z) e^{-\gamma z} dz \text{ with } \gamma < r \qquad (6)$$

$$w(t) = w \qquad (7)$$

$$I(t) = dK(t) \geq 0 \qquad (8)$$

$$K(t) \geq K_{eff} \qquad (9)$$

P(t) is the market price of the good produced by the firm, *Q(t)* is the production, the demand price elasticity is $(-1/\theta)$, *K(t)* is capital, *L(t)* is labour. *E(t)* stands for the energy use and *I(t)* is investment; *Pe(t)* is energy price; the wage rate *w* and the purchase cost of capital *k* are supposed to be constant for reasons of simplicity. *r* is the discount rate, μ is the energy price rate of growth, and $\gamma>0$ represents the rate of energy-saving technical progress. We assume that there is no physical depreciation. Moreover, we assume that $\mu<r$ and $\gamma<r$. If $\mu>r$, the firm would have an incentive to infinitely get into debt to buy an infinite amount of energy. $\gamma<r$ is a standard assumption in the exogenous growth literature since it permits a bounded objective function.

The Cobb-Douglas production function exhibits constant returns to scale but there are operating costs whose size depends on the energy requirement of the capital: for any capital use *K(t)* there is a corresponding given energy requirement $K(t)e^{-\gamma t}$ \$K(t). Such a complementarity is assumed in order to be consistent with the results of several studies showing that capital and energy are complements (see, for instance, Hudson and Jorgenson (1974), or Berndt and Wood (1975)).

T(t) denotes the age of the oldest machine still in use at *t* or scrapping age. Moreover, the capital variable is effective capital, since only active machines are taken into account in the definition of the capital stock. Note that only the new machines incorporate the latest technological advances, *i.e.* are more energy-saving than the machines acquired in the past. Such an assumption is consistent with Terborgh (1949) and Smith (1961), in which it is hypothesized that the operation cost of a machine is a decreasing function of its vintage[3]. However, the rate of technical progress γ enters linearly into their operation costs functions, whereas it is exponential in our model.

We assume that labour may be adjusted immediately and without any cost and this standard problem reduces to the following conditions for optimal inputs use:

$$L^*(t) = \left[\frac{A^{1-\theta}b(1-\beta)(1-\theta)}{w}\right]^{\frac{1}{1-(1-\beta)(1-\theta)}} K(t)^{\frac{\beta(1-\theta)}{1-(1-\beta)(1-\theta)}} \qquad (10)$$

with $B = \left(A^{(1-\theta)}b\right)^{\frac{1}{1-(1-\beta)(1-\theta)}}\left[1-(1-\beta)(1-\theta)\right]\left[(1-\beta)(1-\theta)\right]^{\frac{(1-\beta)(1-\theta)}{1-(1-\beta)(1-\theta)}} w^{\frac{(1-\beta)(1-\theta)}{(1-\beta)(1-\theta)-1}}$

The vintage structure does matter in capital accumulation decisions, investment and scrapping. Noting that *J(t)=T(t+J(t))* is the lifetime of a machine of vintage *t*, the problem may be transformed, following Malcomson (1975), into a more tractable one (see the Boucekkine and Pommeret, 2001) and the following first order conditions then result:

$$\int_t^{J(t)+t} \left\{B\alpha[K(\tau)]^{\alpha-1} - \overline{Pe}\ e^{\mu\tau-\gamma t}\right\} e^{-r(\tau-t)}d\tau = k(t) \qquad (11)$$

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[3] In contrast, Brems (1967) assumes a constant operation cost.

$$\alpha B[K(t)]^{\alpha-1} = \overline{Pe}\ e^{-\gamma[t-T(t)]+\mu t} \tag{12}$$

with $\alpha = [\beta(1-\theta)]/[1-(1-\beta)(1-\theta)]$. Note that $0<\alpha<1$.

Equation (11) gives the optimal investment rule according to which the firm should invest at time *t* until the discounted marginal productivity during the whole lifetime of the capital acquired in *t* exactly compensates for both its discounted operation cost and its marginal purchase cost in *t*. Equation (12) is the scrapping condition: it states that a machine should be scrapped as soon as its marginal productivity (which is the same for any machine whatever its age) no longer covers its operating cost (which rises with its age).

It is shown in Boucekkine and Pommeret (2001) that if $\gamma=\mu$, the Terborgh-Smith result $T^*(t)=J^*(t)=T$ is also reproduced in our case with *T* given by:

$$e^{-rT} = \frac{\gamma-r}{\gamma}\left[1 - e^{-\gamma T}\left(\frac{r}{r-\gamma} + \frac{rk}{\overline{Pe}}\right)\right] \tag{13}$$

Moreover, the optimal capital stock is:

$$K^{E*}(t) = \left[\frac{\alpha B}{\overline{Pe}} e^{-\gamma T}\right]^{\frac{1}{1-\alpha}} = K^{E*} \tag{14}$$

2.2. Tax credit and optimal stock of capital

The tax credit *s* is modelled as a cut in the investment acquisition cost. The optimal scrapping time then becomes:

$$e^{-rT} = \frac{\gamma-r}{\gamma}\left[1 - e^{-\gamma T}\left(\frac{r}{r-\gamma} + \frac{rk(1-s)}{\overline{Pe}}\right)\right]$$

and the optimal stock of capital can be deduced from equation (14). The behavior of the optimal scrapping time and of the optimal capital stock with respect to a tax credit are as follows :

$$\frac{\partial T}{\partial s} = \frac{1}{-\gamma(1-s)-\frac{\gamma}{r-\gamma}\frac{\overline{Pe}}{k}\left(1-e^{(\gamma-r)T}\right)} < 0 \tag{15}$$

$$\frac{\partial K^{E*}}{\partial s} = \underbrace{\frac{\partial K^{E*}(T,\overline{Pe})}{\partial T}}_{<0} \frac{\partial T}{\partial s} = \frac{K^{E*}}{\underbrace{(1-\alpha)(1-s)+(1-\alpha)\frac{\overline{Pe}}{rk}}_{A} \underbrace{+(1-\alpha)\frac{\overline{Pe}}{rk}\left[\frac{r\left(1-e^{(\gamma-r)T}\right)}{r-\gamma}-1\right]}_{B}} \tag{16}$$

First note that there is a negative relationship between K^{E*} and a given scrapping age *T*. The underlying mechanism is the following. The greater the age of the operated machines, the greater the operation cost associated with those machines, and thus the higher the marginal productivity required for all machines whatever their age due to the optimality condition (12). Since the production function exhibits decreasing returns with respect to capital, a higher marginal productivity can only be achieved by lowering the stock of capital.

The tax credit affects the optimal capital stock through the optimal scrapping age: the higher the tax credit, the more attractive it becomes to scrap old machines and replace them with new ones whose costs are subsidized. This negative relationship between the tax credit and the scrapping age is expressed in equation (15). Moreover, the total effect of the credit tax on the effective stock of capital can be broken down into two parts :

-A>0 results from the effect of the tax credit in the absence of embodied technological progress
-B<0 results from the effect of the tax credit due to the embodied technological progress.

On the one hand, embodiment makes it even more attractive to acquire new machines. On the other hand, it also encourages scrapping. This leads to an ambiguous effect on the effective stock of capital. In the simulations exercise proposed in the last section, the positive effect prevails.

3. Embodied technological progress under uncertainty

Now we consider the same programme with the exception that the energy price is stochastic. The problem the firm has to solve then becomes:

$$\max E\left[\int_0^\infty \left[P(t)Q(t) - Pe(t)\,E(t) - w(t)L(t) - k(t)I(t)\right]e^{-rt}dt\right] \qquad (17)$$

subject to constraints taking uncertainty into account:

$$P(t) = bQ(t)^{-\theta} \qquad \text{with } \theta < 1 \qquad (18)$$

$$Q(t) = AK_{eff}(t)^{\beta}L(t)^{1-\beta} \qquad (19)$$

$$K_{eff}(t) = \int_{\tau o}^{t} I(z)dz \qquad (20)$$

$$dPe(t) = \mu Pe(t)dt + \sigma Pe(t)dz(t) \qquad (21)$$

$$E(t) = \int_{\tau o}^{t} I(z)e^{-\gamma z}dz \text{ with } \gamma < r \qquad (22)$$

$$w(t) = \overline{w} \qquad (23)$$

$$I(t) = dK(t) \geq 0 \qquad (24)$$

$$K(t) \geq K_{eff} \qquad (25)$$

μ is the deterministic energy price trend which is disturbed by exogenous random shocks. *dz(t)* is the increment of a standard Wiener process (*E(dz)=0* and *V(dz)=dt*). σ is the size of uncertainty, that is, it indicates how strongly this price reacts to the shocks.

3.1. The value of a marginal unit of capital

Based on the Bellman equation which has to be satisfied, it is possible to derive the value of a unit of capital depending on the date of acquisition of this unit and on whether this unit is currently used or not (see Cruz and Pommeret (2003)). In the case of disembodied technological progress, the acquisition date of the unit is of no relevance. We must distinguish between the following values:

- $V(Pe(t),\tau,t)$ is the value of a unit of capital at time t acquired at time τ and currently used.

-$W(t,\tau)$ is the value at time t of a unit of capital acquired at time τ and not currently used (it is not currently providing any cash-flow).

-$O(t)$ is the value at time t of a unit that has not already been acquired.

3.2. Optimality conditions

In the case of disembodied technological progress (or no technological progress, see Pindyck (1988)), the firm must first decide whether to invest or not depending on the relative values of the desired capital stock (given the observed value of the uncertain variable) and of the stock already installed. In the case in which it is not optimal to invest, the firm must then decide whether to use all the unit capital it has installed or only a part of it. Since any unit of capital displays the same characteristics because technological progress benefits all units, the firm will decide to first reuse old units before investing in new ones. In a way, the decisions about using installed units and about investing in new ones are taken independently since there is no incentive for the firm to replace old units by new ones.

This is no longer the case if technological progress is embodied because capital units differ according to their installation date. The intuition is the following: since a new capital unit may be a lot more energy efficient than an old one, i.e. saves energy, it may be attractive for the firm to stop using an old unit and invest in a new one even if there is an additional acquisition cost for the new one. Therefore, the firm will simultaneously have to decide whether to invest or not and to determine the maximum age of a machine still in use. Indeed, these two decisions are now closely linked.

3.2.1. Utilization rule

The firm uses an old machine acquired at time τ until the realization of the energy is $Pe*^{E}(t)$ such that it becomes indifferent whether it is used or unused: the value of the oldest machine used must be the same whether it is used or not. Since the model is stochastic, the transition between these two values of the unit must be smooth for the firm to be functioning at the optimum. These two conditions are the usual value matching and smooth pasting conditions.

$$V(t,\tau) = W(t,\tau) \qquad \text{for } Pe(t) = Pe^{*E}(t) \qquad (26)$$

$$\frac{\partial V(t,\tau)}{\partial Pe(t)} = \frac{\partial W(t,\tau)}{\partial Pe(t)} \qquad \text{for } Pe(t) = Pe^{*E}(t) \qquad (27)$$

Taking into account the Bellman equations $V(t,\tau)$ and $W(t,\tau)$ have to satisfy, this leads to (see Cruz and Pommeret (2003)):

$$V(t,\tau) = \frac{\alpha B}{r}K_{eff}(t)^{\alpha-1} - \frac{Pe(t)}{r-\mu}e^{-\gamma\tau} + \underbrace{\frac{\beta_2}{\beta_1-\beta_2}\left[\alpha B K_{eff}(t)^{\alpha-1}\right]^{1-\beta_1} e^{-\gamma\beta_1\tau}\left(\frac{1}{r} - \frac{(\beta_2-1)}{\beta_2(r-\mu)}\right)}_{b_1(K_{eff}(t),\tau)} Pe(t)^{\beta_1}$$

(28)

where $\beta_1 = \frac{1}{2} - \frac{(\mu-\gamma)}{\sigma^2} + \sqrt{\left(\frac{(\mu-\gamma)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ with $\partial\beta_1 / \partial\sigma^2 < 0$ and

$\beta_2 = \frac{1}{2} - \frac{(\mu-\gamma)}{\sigma^2} - \sqrt{\left(\frac{(\mu-\gamma)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$ with $\partial\beta_2 / \partial\sigma^2 > 0$

The value of the option to stop using the unit $\left(b_1(K_{eff}(t),\tau)Pe(t)^{\beta_2}\right)$ negatively depends on its acquisition date τ: the earlier the unit has been installed, the less value it has to have the opportunity to stop using it.

3.2.2. Investment rule

The firm invests for an energy price realization such that, for a given effective stock of capital, it is indifferent between acquiring one more unit and doing nothing. Therefore, it invests until the value of a newly used unit exactly compensates for the constant cost *k* to acquire it and for the value of the option to invest in the future the firm has to give up (it corresponds to the value matching condition). In order for the firm to be at the optimum of this stochastic program, the standard smooth pasting condition has to be satisfied as well. For given effective capital stock and technology levels, these optimality conditions express the energy price level at which it is optimal for the firm to invest in a new unit. This expression may also be converted into that of the optimal effective stock of capital as a function of the observed energy price level and of the current level of technology.

$$V(t,\tau = t) = O(t) + k \qquad (29)$$

$$\frac{\partial V(t,t)}{\partial Pe(t)} = \frac{\partial O(t)}{\partial Pe(t)} \qquad (30)$$

When making investment decisions, firms do not care about how much capital they have, but about how much capital they decide to use. Due to the embodied technology, it may be interesting for the firm to acquire new units that are more energy saving even if all the old units are not used. Under embodied technological progress, the installed stock of capital (which may be in excess) is no longer the determinant for investment. What is more interesting is the effectively used stock of capital. Since the firm can determine the age of the oldest capital unit in use to adjust the used stock to its optimal level, the desired level of used capital always coincides with the effective level and the expression for the desired level of effectively used capital is valid whatever the realization of the uncertain variable. This makes it possible to derive the expression of the optimal acquisition date of the oldest machine as a function of the energy price.

3.3. Optimal effective capital stock

Given the observed level of the energy price and the current state of the energy-saving technology, it is optimal for the firm to have an effective capital stock equal to $K_{eff}*(t)$ which is given by the following implicit equation (see Cruz and Pommeret (2003) for the derivation):

$$\left[\alpha BK^{*}_{eff}(t)^{(\alpha-1)}\right]^{(1-\beta_1)}\left[\frac{\beta_2-1}{\beta_2(r-\mu)}-\frac{1}{r}\right]Pe(t)^{\beta_1}e^{-\beta_1\gamma t}+\frac{\alpha BK^{*}_{eff}(t)^{(\alpha-1)}}{r}=\frac{Pe(t)e^{-\gamma t}}{r-\mu}\left(\frac{\beta_2-1}{\beta_2}\right)+k \tag{31}$$

Note that, due to potential decreases in the optimal age of the oldest machine used, it is possible for the optimal effective capital stock to be decreasing even if the total installed stock of capital is irreversible (as we have already seen, under embodiment, the installed stock of capital does not really matter as far as the firm's decisions are concerned).

3.4. Optimal age of the oldest machine used

Since, in this model, temporarily not using a machine is not associated with any cost, there is no incentive for the firm to definitively scrap any machine. Thus we only derive an optimal age for the oldest machine used but not really an optimal scrapping age. This is a significant departure from what is obtained in a deterministic environment (see Boucekkine and Pommeret, 2001). Using equation (31) and taking into account the fact that the unit is only used if the cash-flow it provides is positive (see Cruz and Pommeret (2003) for the derivation), we can derive an implicit expression for the optimal acquisition date of the oldest machine as a function of the observed price:

$$\tau o^{*}(t)=\frac{-1}{\gamma}\left[\frac{rk}{Pe(t)}\ln+\frac{(\beta_2-1)r}{\beta_2(r-\mu)}e^{-\gamma t}-\left(\frac{(\beta_2-1)r}{\beta_2(r-\mu)}-1\right)e^{-\gamma\beta_1 t}e^{\gamma\tau o^{*}(t)(\beta_1-1)}\right] \tag{32}$$

Given the observed level of the energy price and the current state of the energy-saving technology, the firm desires to use only capital units that have been acquired at time $\tau o^*(t)$ or more recently. Firms dealing with uncertainty are more reluctant to renew the machines; replacement is postponed. It is also possible to see that a higher energy price reduces the age of the oldest machine; one could claim that the model can reproduce the "cleansing effect": firms would tend to use newer machines in periods of higher energy prices and eventually acquire new units. Moreover, for a given energy price, as time passes, new technology becomes available and we have seen that the optimal effective stock of capital increases. We can also use the optimal age to stop using new units. Defining $T^*=t-\tau o^*$, equation (32) can then be written as:

$$e^{-\gamma\beta_1 T^{*}(t)}=\frac{\beta_2(\mu-r)}{\beta_2\mu-r}\left[1-e^{-\gamma T^{*}(t)}\left(\frac{(\beta_2-1)r}{\beta_2(r-\mu)}+\frac{rk}{Pe(t)}e^{\gamma t}\right)\right] \tag{33}$$

3.5. Tax credit and optimal stock of capital

Introducing a tax credit s, the optimal effective stock of capital (see equation (31) becomes:

$$\left[\alpha BK^*_{eff}(t)^{(\alpha-1)}\right]^{(1-\beta_1)}\left[\frac{\beta_2-1}{\beta_2(r-\mu)}-\frac{1}{r}\right]Pe(t)^{\beta_1}e^{-\beta_1\gamma t}+\frac{\alpha BK^*_{eff}(t)^{(\alpha-1)}}{r} = \frac{Pe(t)e^{-\gamma t}}{r-\mu}\left(\frac{\beta_2-1}{\beta_2}\right)+k(1-s) \tag{34}$$

and the optimal age of the oldest machine will be:

$$e^{-\gamma\beta_1 T^*(t)} = \frac{\beta_2(\mu-r)}{\beta_2\mu-r}\left[1-e^{-\gamma T^*(t)}\left(\frac{(\beta_2-1)r}{\beta_2(r-\mu)}+\frac{rk(1-s)}{Pe(t)}e^{\gamma t}\right)\right]$$

The behaviour of the optimal age of the oldest machine and of the optimal effective capital stock with respect to a tax credit are as follows :

$$\frac{\partial T^*(t)}{\partial s} = \left[\underbrace{-\gamma(1-s)-\frac{\gamma}{r-\gamma}\frac{\overline{Pe}}{k}(1-e^{(\gamma-r)T})}_{D}+\underbrace{\frac{\gamma Pe(t)e^{-\gamma t}}{k}\left(\frac{1-e^{(\gamma-r)T}}{r-\gamma}+\frac{\beta_2-1}{\beta_2}\right)}_{E}\right.$$
$$\left.\underbrace{+(1-\alpha)\beta_1\left[\alpha BK^*_{eff}(t)^{\alpha-1}\right]^{1-\beta_1}\left[\left(\frac{\beta_2-1}{\beta_2}\frac{1}{(r-\mu)}\right)-\frac{1}{r}\right]\frac{(Pe(t)e^{-\gamma t})^{\beta_1}}{k}}_{C}\right]^{-1}$$

$$\frac{\partial K^*_{eff}(t)}{\partial s} = \left[\underbrace{(1-\alpha)(1-s)+\frac{(1-\alpha)Pe(t)e^{-\gamma t}}{rk}}_{A}+\underbrace{\frac{(1-\alpha)Pe(t)e^{-\gamma t}}{rk}\left[\left(\frac{r(\beta_2-1)}{\beta_2(r-\mu)}\right)-1\right]}_{B'}\right.$$
$$\left.\underbrace{-\frac{(1-\alpha)\beta_1\left[\alpha BK^*_{eff}(t)^{\alpha-1}\right]^{1-\beta_1}(Pe(t)e^{-\gamma t})^{\beta_1}}{k}\left[\left(\frac{\beta_2-1}{\beta_2(r-\mu)}\right)-\frac{1}{r}\right]}_{C}\right]^{-1}K^*_{eff}(t)$$

Now, in this stochastic case, the total effect on the effective stock of capital can be broken down into 3 parts:
-A>0 results from the tax credit if technological progress is disembodied.
-B'>0 results from the tax credit due to the embodied technological progress affecting the option to invest in the future. Embodied technological progress increases the option value to invest in the future, therefore discouraging even more investment and reducing the effect of the tax credit. In the simulations exercise it can be shown that this effect is very strong and strongly reduces the effect of the tax credit.
-C<0 results from the tax credit due to the option to reuse old units under embodied technological progress. Unambiguously, the tax credit increases the option to reuse old units (since the marginal cost is reduced by the tax credit), therefore favoring the effective stock of capital.

The impact of the tax credit on the optimal age can be also divided into 3 parts:
-D<0 gives the effect of the tax credit in a deterministic framework: lowering the price of new machines due to the tax credit creates an incentive to scrap older machines and therefore reduces the age of the oldest machine.
-E is the correction one should include because of the option to invest in new machines. A priori it has an ambiguous effect on the age of the oldest machine.
-C<0 results from the option to reuse machines. The tax credit reduces this option therefore increasing the age of the oldest machine.

In the dynamic example proposed in the next section, E>0, and the total effect of the tax credit is to lower the age of the oldest machine.

4. Dynamics of the effective stock of capital, age of the oldest machine and tax credit

In this dynamic example, simulations are driven over 100 periods. In order to get the dynamics of *Pe(t)*, a geometric Brownian motion is simulated using parameters γ=μ=0.02, σ^2=0.04 and *Pe(0)=10* as a starting value. Figure 1 shows the result for *Pe(t)*. The firm observes the energy price and derives how much effective capital to use. It then has to decide whether to use more or fewer old units and, at the same time, decide whether it should invest in new units or not.

Figure 2 shows the dynamics of the total stock of capital in the deterministic case. It exhibits the usual echoes effects. Since the optimal effective stock of capital is constant, the positive effect of the tax credit accumulates over time. Considering the total capital stock in a stochastic framework with disembodied technological progress, Figure 3 leads to a dynamics consistent with investment occurring infrequently and in bursts. Due to tax credits, the initial investment is higher. Since in this case technological progress reduces the energy requirements of all installed machines, the tax credit unambiguously results in higher initial investment and higher capital over the whole period. However, the positive effect of the credit tax is less striking when one takes into account the fact that technological progress is embodied in new machines. In fact, the effective total stock of capital is barely increased in the model of embodied technological progress under uncertainty (Figure 4); also note that firms leave their scrapping policy almost unchanged (Figures 6 and 7).

Figure 5 compares the percentage increase in the total stock of capital over time, depending on whether we consider a stochastic environment or a deterministic one and whether technological progress is disembodied or embodied. First note that under disembodiment, the tax credit is more efficient under uncertainty than in a deterministic framework. This is clearly due to the fact that we take capacity expansion into account since it contradicts the result of Hasset and Metcalf (1992) based on a single investment project. Second, introducing embodiment into the stochastic environment drastically reduces the effect of the tax credit under uncertainty while the reverse takes place in a deterministic environment. This means that taking embodiment into account is crucial when assessing the effectiveness of a tax credit.

Figure 1: Energy Price Dynamics over 100 periods

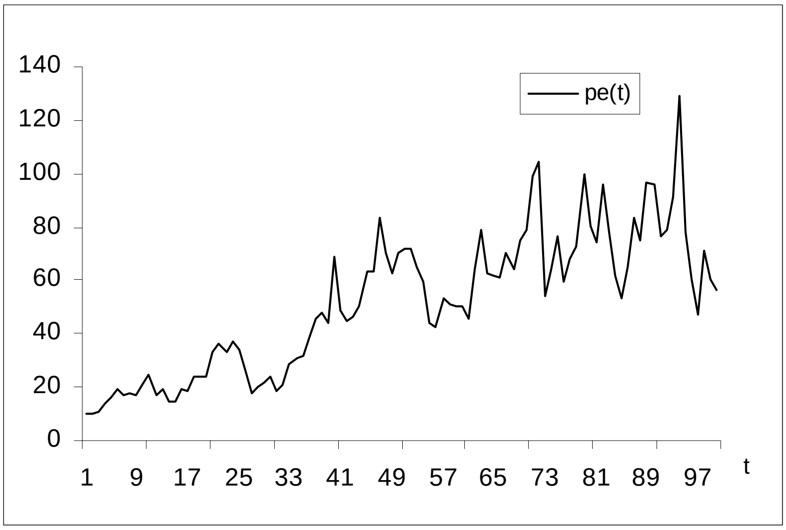
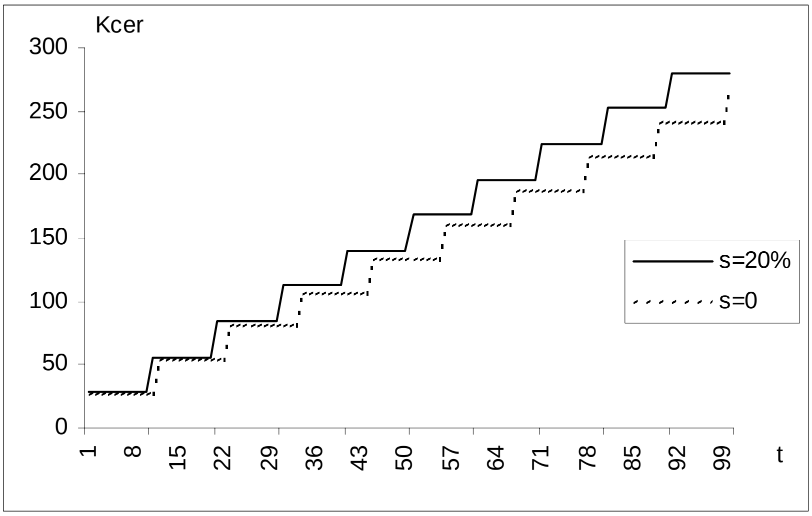
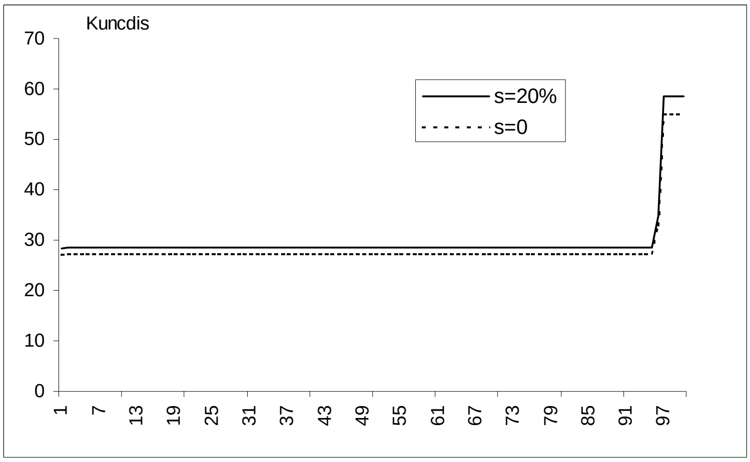


Figure 2: Total capital dynamics with and without tax credit; deterministic case under embodiment



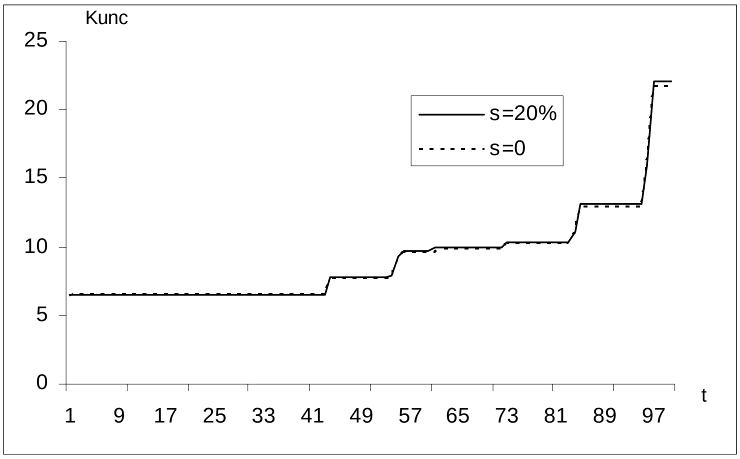
"Kcer" stands for the embodied capital stock under certainty

Figure 3: Total capital dynamics with and without tax credit; stochastic case under disembodiment



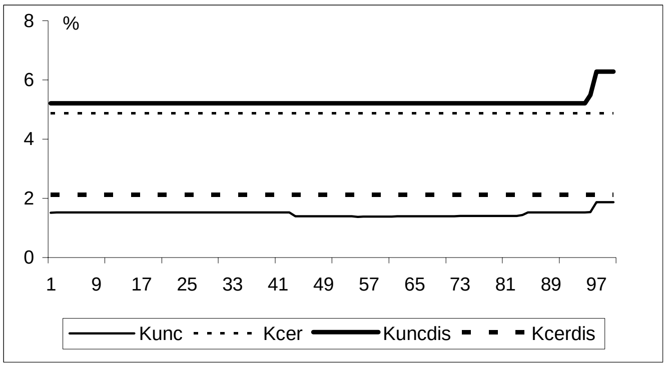
"Kuncdis" represents the disembodied capital stock under uncertainty

Figure 4: Total capital dynamics with and without tax credit; stochastic case under embodiment



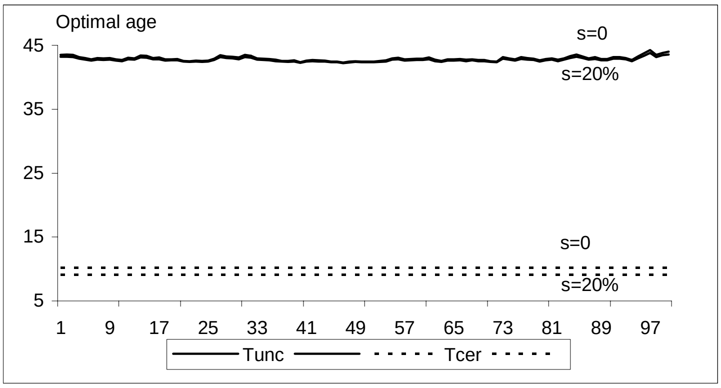
"Kunc" stands for the embodied capital stock under uncertainty

Figure 5: Percentage increase in capital due to tax credit



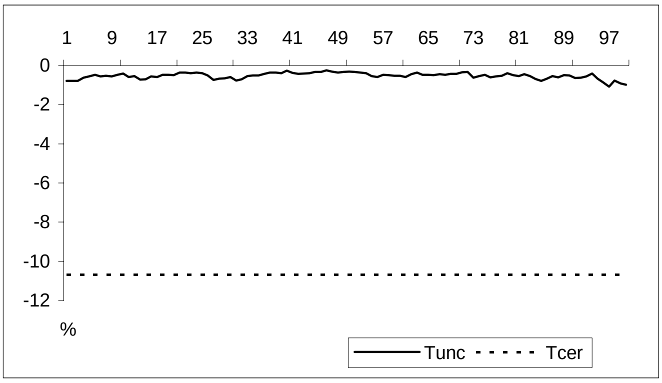
"Kunc" stands for the embodied capital stock under uncertainty
"Kcer" stands for the embodied capital stock under certainty
"Kuncdis" stands for the disembodied capital stock under uncertainty
"Kcerdis" stands for the disembodied capital stock under certainty

Figure 6: Optimal age of the oldest machine with and without tax credit



"Tunc" stands for the optimal age of the oldest machine under uncertainty
"Tcer" stands for the optimal age of the oldest machine under certainty

Figure 7: Percentage decrease in the optimal scrapping age due to tax credit



"Tunc" stands for the optimal age of the oldest machine under uncertainty
"Tcer" stands for the optimal age of the oldest machine under certainty

5. Conclusion

Policies focusing on the implementation of energy-efficient machines face the so-called energy paradox: efficient machines in terms of energy requirements, even if profitable for the firms, have a very low diffusion rate. Furthermore, some of these policies, such as tax credits, have been shown to be inefficient. In this paper, we propose a stochastic model that accounts for heterogeneous units and technological progress being investment-specific. We show that the assumption of embodied technological progress is very important when assessing the effectiveness of a tax credit. The existence of an option value (due to uncertainty and irreversibility) is not sufficient to explain the relative inefficiency of the tax credit in a capacity expansion framework. Indeed, in the disembodied case, the total impact on the capital stock is even higher than in the deterministic embodied case. Due to the combination of the option value and of embodiment, firms postpone replacement, and following a tax credit, they do not significantly increase their capital stock and reduce the scrapping age of the oldest machines. When devising energy conservation policies therefore, policy makers should not only take the option value into account as a feature of the investment decision. They should consider embodiment as well.

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