Contracts with Endogenous Information*

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Abstract

I derive a general formulation of a Bayes Nash revelation game in a linear environment with endogenous information, whose precision depends on a covert choice of effort. I show that in nonstochastic mechanisms the first order approach to reporting (the Mirrless approach) can be complemented by a first order approach to effort spent on information acquisition if and only if i) the agent's posterior is independent of the agent's effort, ii) an increase in the agent's effort increases the risk (in the sense of Rothschild and Stiglitz (1970)) in the ex ante distribution of the posterior mean, and iii) does so at a decreasing rate. Experiment structures with these characteristics arise when posteriors resulting from high (low) signals are higher (lower) for more informative signals (i.e., in monotone environments). The marginal value of information is ambiguous. Sufficient conditions for a negative or positive marginal value are derived. Contracts that encourage (discourage) information acquisition are more (less) sensitive to the agent's information than their counterparts for exogenous information structures. Distortions at the top arise if the agent may choose the precision of information directly.

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1 Introduction

1.1 Motivation

A vast literature on contracting and mechanism design has investigated the consequences of asymmetric information on the efficiency and distributive properties of allocations. In most of this literature the model's primitive is the agent's information structure. However, in some economic problems it is reasonable to assume that agents do only possess information because they expect to make use of it. Moreover, their effort to gather information is often unobservable to others. Consequently, in such situations an information acquisition technology rather than the information structure itself should be taken as the model's primitive, and contracts serve the double role of motivating the acquisition of information and ensuring its truthful revelation. This double role renders the implementation problem quite hard to solve. To gain insights despite this complexity the literature to date resorts to simple information acquisition technologies, with little exceptions all-or-nothing information acquisition. While insightful, this approach begs the question how general the insights are that emerge from it. This paper is a step towards a general theory of information acquisition in a branch of mechanism design theory. I characterize necessary and sufficient conditions on information acquisition technologies that make a first order approach to Bayes Nash implementation with endogenous information in linear environments workable. The features of optimal contracts that emerge from this approach will be shared by any model that relies on a first order characterization for the amount of information acquired. Hence, within the straightjacket of a first order approach, these predictions are robust.

1.2 Approach and Results

There are two key obstacles to characterizing optimal mechanisms with endogenous information in a reasonably rich informational environment, arising at the *ex post* and at the *ex ante* stage, respectively. *Ex post*, at the reporting stage, covert information acquisition introduces a problem of multi-dimensional screening: the agent knows the signal he received and its precision. However, in the linear environment, when attention is restricted to non-stochastic mechanisms, the agent's preference over contracts depends effectively only on the mean calculated from the posterior distribution. Since this is a one dimensional statistic the problem at the reporting stage is reduced to the well known one-dimensional screening problem. *Ex ante*, however, the problem remains multi-dimensional in the sense that the agent's *strategy* is multi-dimensional. Given some effort the principal wants to implement in some message game, the agent can deviate and exert a "wrong" effort level and lie about his type in a sequentially rational fashion. To deal with this dimensionality issue one needs to impose a stochastic experiment structure, where the agent's effort controls the frequency of receiving different posteriors, but the posteriors themselves are independent of effort. This allows to split the two-dimensional incentive constraint into two one-dimensional ones in the sense that truthful reporting is sequentially rational *for any effort level the agent chose* ex ante.

In this construction the agent's effort choice depends only on his effort's influence on the *ex ante* distribution of the conditional expectation. A key result of this paper is that his optimal choice of effort is characterized by a first order condition for any implementable contract if and only if an increase in his effort *increases the riskiness* of this distribution in the sense of Rothschild and Stiglitz (1970) at a decreasing rate. To obtain this kind of influence I assume that an increase in effort increases the likelihood of receiving more informative posteriors in the sense of a first order stochastic dominance shift with decreasing returns. A posterior is said to be more informative if posteriors arising from higher (lower) than average signals are higher (lower) for more informative signals. Intuitively, more informative signals are better correlated with the true state. Under natural conditions an increase in effort induces a single mean preserving spread, and all distributions for different effort levels cross at the prior mean and at the bounds of the support.

This identification of improvements in information and increases in risk has two main implications. First, the marginal value of information depends on quasi-attitudes[1] towards risk. Since implementable indirect utility profiles are convex in types (Rochet (1985)) the agent is a quasi-risk lover and information is always of value to him at the margin. In contrast, the shape of the principal's indirect utility function depends on the curvature of his direct utility function and the distribution of types. At the margin, more information can either be a blessing or a curse to him[2]. Sufficient conditions for both cases are provided. Second, it has immediate consequences for the optimal supply arrangement. The optimal production schedule differs from the fixed information counterpart everywhere but at the top, the prior mean, and at the bottom. Contracts that encourage (discourage) information acquisition are more sensitive to the agent's information than their fixed information counterparts. In the former case, when the agent's cost perception is surprisingly low he is rewarded by an extra increase in production that increases his informational

[1]Recall that quasi-attitudes toward risk refer to the shape of *indirect* utility functions which take account of the shape of contracts.

[2]This confirms results of Green and Stokey (1981), who do, however, not relate their results to risk.

rent at the margin, and punished if his cost perception is surprisingly high. Since information and risk are equivalent within the straightjacket of a first order approach to information acquisition every model that relies on a first order approach will have these stylized properties.

Finally, I test the robustness of these results with respect to the experiment structure. More precisely, I show that the stochastic experiment structure is almost necessary for a first order approach to work. If posteriors depend on the agent's choice of effort, one needs to analyze his deviation strategies in tandem, and imposing two separate incentive constraints does generally *not* do the trick. To make a first order approach workable in this context one needs to make sure that the agent's two stage problem of choosing some effort level and (mis-)reporting his type in a sequentially rational fashion forms a globally concave problem in effort as of the first stage. Without specific knowledge about the experiment structure this seems practically impossible to guarantee. However, whenever a first order approach works in the deterministic experiment model, the characterization of the optimal contract that emerges from it is essentially identical to the one of the stochastic experiment model. In this sense, there is essentially no loss of generality imposing a stochastic experiment structure. *Essentially*, because a deterministic experiment structure naturally generates a moving support in the distribution of the posterior mean. As a consequence, the bounds are very informative about effort and a *distortion at the top* emerges.

1.3 Literature

Information acquisition followed by reporting games has been studied by several authors before. As an example of a linear environment, I extend the Baron Myerson (1982) model allowing for endogenous information structures. Crémer et al (1998a) have pursued the same exercise before with a fixed-cost-all-or-nothing information acquisition technology, which induces the most extreme form of a mean preserving spread in the ex ante distribution of the posterior mean. Almost all their results survive in the more general model. More generally the present model demonstrates the robustness of high powered extreme reward schemes to foster information acquisition and of equalized reward schemes to discourage information acquisition.

Demski and Sappington (1987) were the first to discuss information acquisition in the principal agent problem; more recently Lewis and Sappington (1993), Crémer and Khalil (1992), Crémer, Khalil, and Rochet (1998b), Sobel (1993), and Kessler (1998) have studied procurement with information acquisition with differing timing assumptions on contracting and information acquisition. Dai and Lewis (2003) study a model of sequential screening with two possible precisions of information that obey the same ordering as those in the present paper. They show that experts with differentially precise information can be screened by the extent of decision authority embodied in contracts. Moreover, the ambiguity of the value of information to the principal is overcome with sequential screening.

The present paper shows that this literature has developed robust insights despite its use of tractable and simple information acquisition technologies and identifies these robust predictions. In particular, results are robust to the extent that they rely on the differential informativeness of extreme versus intermediate outcomes. To take a particular example, consider Szalay (2003), showing that a principal may want to restrict an agent's freedom of choice and force him to depart from prior optimal choices even if ex post objectives of principal and agent coincide. The reason is that the agent is the more likely to propose intermediate choices if he shirked on information acquisition. This result was obtained with a variant of an all or nothing information acquisition technology where the agent's effort corresponds to the probability of receiving perfectly informative versus completely uninformative signals. Since this technology induces an ordering of information structures as considered here, the present paper shows that these results are much more general than the specific technology used to derive them.

The information structures used in the present paper connect the contracting literature to a literature on the value of information in decision problems, a line of research that has been initiated by Blackwell (1951), and Karlin and Rubin (1956), and further pursued by Lehmann (1988), and most recently by Athey and Levin (2001). The combination of these two literatures delivers a powerful approach, that should prove useful to study further applications, because the predictions of the model are robust within a large class of information gathering technologies. One such application, left for future research, is the study of optimal auctions with endogenous information. To date, the literature on auctions with endogenous information has - to my knowledge - restricted attention to a class of mechanisms, e.g., first versus second price auctions, notably Tan (1992), Hausch and Li (1993), Stegemann (1996), and more recently Persico (2000).[3]

Information acquisition and mechanism design has also been studied using different implementation concepts. Bergemann and Välimäki (2002) discuss incentives for information acquisition in ex post efficient mechanisms. They show that incentives for information acquisition in a private value environment are related to supermodularity in the agents' payoff functions.[4] In contrast to

[3]Persico's result that the auction format with the higher risk sensitivity induces more information acquisition corresponds to the result that the marginal value of information for the agent is positive.

[4]They note that efficient mechanisms in the linear environment can be based on conditional expectations.

their paper, the present one treats a problem in Bayes Nash implementation with ex post participation constraints and shows that the agent may acquire too much or too little information although his payoff function is submodular in the state and the contracting variable. The reason for this discrepancy is that Bayes Nash implementation makes use of the endogenous distribution of types, whereas ex post implementation does not.

The present paper is related to the literature on multidimensional screening, notably McAfee and McMillan (1988)[5], although my model allows for a much simpler solution to the screening problem, because the dimensionality can be reduced to one by a transformation of the type space. Finally, the present paper is related to the literature on the validity of the first order approach (Rogerson (1985) and Jewitt (1988)). It is interesting to note that the first order approach goes through with much more ease when asymmetric information is added. This is because a main problem in problems of pure moral hazard is to ensure the monotonicity of contracts. There is no need to do this when there is adverse selection, because monotonicity of contracts is a necessary condition for implementability (Guesnerie and Laffont (1984)).

The paper is organized as follows. Sections 2 to 4 contain the main theory. In section 2 I spell out the main model. Section 3 contains the main result on the validity of the first order approach. Section 4 derives the statistical foundation of the second order stochastic dominance relation in the distribution of the conditional expectation in a monotone environment. Sections 5 and 6 contain the main implications of the theory. Section 5 derives some results on the value of information, section 6 discusses the form of optimal contracts. Section 7 derives an alternative formulation using nonstochastic experiments and shows that the first order approach is typically not valid in this framework but would deliver - if valid- essentially the same structural predictions except for distortions at the top. Section 8 concludes. Long proofs are in the appendix.

2 The Model

The model is a variant of the Baron and Myerson (1982) model where I allow for general, endogenous information structures. A principal contracts with an agent for the production of a good. The good is perfectly divisible so output can be produced in any quantity, q. q is perfectly observable and contractable. The agent receives a monetary transfer t from the principal and has costs of producing the quantity q equal to βq. Both parties are risk neutral with respect to transfers. The

[5] See also Armstrong and Rochet (1999) and Rochet and Stole (2003) for overviews of multidimensional screening problems.

principal derives gross surplus $V(q)$ from consumption, where $V(q)$ is defined on $[0,\infty)$ with[6] $V_q(q) > 0,\ V_{qq}(q) < 0,\ \lim_{q\to 0} V_q(q) = \infty,\ \lim_{q\to\infty} V_q(q) = 0$. Thus the principal's net utility is

$$V(q) - t$$

The agent's payoff from receiving the transfer t and producing the amount q is given as

$$t - \beta q$$

Ex ante the principal and the agent do not know the precise value of β, but share a common prior about it, which is supported on $\left[\underline{\beta}, \overline{\beta}\right]$. Once the principal has committed himself to the terms of the contract but before production takes place, the agent may acquire additional information about β. Information acquisition is modeled as a costly choice of effort e, that influences the frequency of performing certain experiments.

An *experiment* is the realization of two random variables, S and I, and a resulting posterior with cdf $H(\beta|\, s, i)$, which is *independent* of effort e. The variable S is called the signal, I is called the informativeness parameter (one may think of this as the precision of the signal). Typical realizations of these variables are $s \in [\underline{s}, \overline{s}]$ and $i \in [0,1]$, respectively. The marginal distributions of s and i are *independent* of each other and fully supported with densities $k(s)$ and $l(i,e)$, respectively, and cdfs $K(s)$ and $L(i,e)$, respectively[7]. Experiments can be ordered in the sense that high values of s indicate high costs: $\int_{\underline{\beta}}^{\overline{\beta}} \beta dH(\beta|\, s, i)$ is increasing in s. Below I will also introduce an ordering in the i dimension. For the time being this is not important, but the important properties are the embodied independence assumptions. The cost of effort is $g(e)$, a strictly convex function, that satisfies $g_e(e) > 0,\ g_{ee}(e) > 0,\ \lim_{e\to 0} g_e(e) = 0,\ \lim_{e\to\infty} g_e(e) = \infty$.

The game has the following time structure:

[6] Throughout the paper subscripts will denote derivatives of the function with respect to the respective argument.

[7] An intuitive example of this experiment structure -although discrete instead of continuous- would take the signal s as a red light on a junction, with signal realizations {red, orange, green} and the informativeness of the signal as {good,bad}. The informativeness refers to whether I expect to have priority if I receive a green realization. Assume that informativeness depends only on whether the junction is in Napels or in Zürich, say. I cannot conclude from the colour of the signal whether I am in Napels or in Zürich. Neither can I conclude from the fact that I am in Zürich that the signal should be red or green. Hence s and i are independent. Moreover, the frequency with which the signal changes coulour is (probably) the same at junctions in Napels and Zürich. However, if I do know that I am in Zürich this changes my posterior belief relative to the one I would have in Napels, whether I will receive priority on the junction when the signal is green.

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P offers a menu of contracts	A exerts effort e	$s$and i realized and observed by A	A accepts a contract or refuses to participate	A learns the true costs β produces and delivers

First, the principal offers a contract. Then the agent chooses an effort level, e, that determines the distribution of experiments. The experiment is realized and observed by the agent. Given this information he decides whether to sign the contract. If the agent does not sign a contract the game ends. If contracts are signed production and transfers take place. I assume that the agent's choice of effort is not observable to the principal and that the value of the signal as well as the informativeness is the agent's private knowledge.

3 Justifying a First Order Approach

As is customary, I will characterize optimal solutions to the contracting problem taking as given that the principal whishes to implement a given level of effort, and will say very little about the optimal choice of effort to implement[8].

I think of contracting in terms of mechanism design. A mechanism is a tuple $\{q(\cdot), t(\cdot)\}$ which specifies quantities of production and transfers to the agent as a function of a message m, the agent sends to the principal when he signs the contract. Invoking the *Revelation Principle* I can restrict attention to direct, incentive compatible mechanisms, $\{q(\cdot), t(\cdot)\}$ that depend only on a reported tuple of signal realization and experiment $(\hat{s}, \hat{\imath})$; effort, once sunk, is irrelevant at the communication stage, since posteriors are independent of effort. Hence, one can write the principal's problem as follows:

$$\max_{q(\cdot),\, t(\cdot)} \int_0^1 \int_{\underline{s}}^{\overline{s}} V(q\,(s,i)) - t\,(s,i)\, dK\,(s)\, dL\,(i,e) \tag{1}$$

s.t.

$$\forall s,i : \int_{\underline{\beta}}^{\overline{\beta}} t(s,i) - \beta q(s,i) dH\,(\beta\,|s,i\,) \geq \int_{\underline{\beta}}^{\overline{\beta}} t(\hat{s},\hat{\imath}) - \beta q(\hat{s},\hat{\imath}) dH\,(\beta\,|s,i\,)\ \forall \hat{s},\hat{\imath} \tag{2}$$

$$\int_{\underline{\beta}}^{\overline{\beta}} t(s,i) - \beta q(s,i) dH\,(\beta\,|s,i\,) \geq 0\, \forall s,i \tag{3}$$

[8] As is well known from the problem of pure moral hazard, the problem of determining the optimal choice of effort has almost no regularity structure.

$$e \in \arg\max_{e} \left\{ \int_0^1 \int_{\underline{s}}^{\overline{s}} \left(\int_{\underline{\beta}}^{\overline{\beta}} t(s,i) - \beta q(s,i) dH\left(\beta \left| s,i\right.\right) \right) dK\left(s\right) dL\left(i,e\right) - g(e) \right\} \tag{4}$$

(2) requires that the agent finds it optimal to report the true signal value and the true signal informativeness. (3) ensures that the agent finds it optimal to participate for all possible realizations of signal and informativenss. (4) imposes that the agent's choice of how much effort to acquire is optimal given the contract the principal offers. Observe that the agent's ex ante expected utility net of costs of information acquisition is always nonnegative. As (2) is independent of e, (2) and (4) together imply that joint deviations of misrepresenting the type and choosing an effort level different from the one the principal wants to implement do not pay, since truthtelling about s and i is sequentially optimal given *any* effort e.

The screening problem is multi-dimensional, and therefore potentially extremely complicated. However, similar to Biais et. al. (2000) in a different context we can observe that in a nonstochastic mechanism only the one dimensional statistic $\int_{\underline{\beta}}^{\overline{\beta}} \beta dH\left(\beta \left| s,i\right.\right)$ is available for screening purposes.[9] Since the agent's conditional expectation is the relevant contracting variable I directly place assumptions on this variable. Denote the function

$$\pi\left(s,i\right) = \int_{\underline{\beta}}^{\overline{\beta}} \beta dH\left(\beta \left| s,i\right.\right)$$

Suppose that $\pi\left(s,i\right) = \theta$ for some real number θ. Given that $\pi\left(s,i\right)$ is increasing in s, the function is invertible and the signal that generated a value of the conditional expectation equal to θ satisfies $s = \pi^{-1}\left(\theta,i\right)$. Ex ante, i.e., before s and i are realized, the value of the conditional expectation is a random variable itself, Θ say. The cdf of θ conditional on i is

$$F_i\left(\theta,i\right) = \begin{cases} 0 & for\, \theta < \pi\left(\underline{s},i\right) \\ K\left(\pi^{-1}\left(\theta,i\right)\right) & for\, \pi\left(\underline{s},i\right) \leq \theta \leq \pi\left(\overline{s},i\right) \\ 1 & \theta > \pi\left(\overline{s},i\right) \end{cases}$$

Let $F\left(\theta,e\right)$ denote the unconditional cdf of θ. We have

$$F\left(\theta,e\right) = \int_0^1 F_i\left(\theta,i\right) dL\left(i,e\right) \tag{5}$$

By our assumptions θ is independent of effort and its distribution is fully supported on an interval $\left[\underline{\theta},\overline{\theta}\right]$, independent of effort where $\underline{\theta} = \min_i \pi\left(\underline{s},i\right)$ and $\overline{\theta} = \max_i \pi\left(\overline{s},i\right)$. Using this change of variables one can state (1) s.t. (2), (3) and (4), equivalently as a message game with messages

[9] Bergemann and Välimäki (2002) have noted that this is also the relevant contracting variable in ex post efficient mechanisms in the linear environment, since efficient mechanisms are non-stochastic.

$\hat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ about "perceived costs". In this formulation, the principal's problem is

$$\max_{q(\theta),\, t(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q\left(\theta\right)\right) - t\left(\theta\right)\right) dF\left(\theta, e\right)$$

$$s.t.$$

$$t\left(\theta\right) - \theta q\left(\theta\right) \geq t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right) \ \forall \theta, \hat{\theta}$$

$$t\left(\theta\right) - \theta q\left(\theta\right) \geq 0 \forall \theta$$

$$e \in \arg\max_{e} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \left(t\left(\theta\right) - \theta q\left(\theta\right)\right) dF\left(\theta, e\right) - g\left(e\right) \right\}$$

In order to solve this problem one needs to be able to replace the final constraint by a first order condition.

Proposition 1 *The principal's problem* (1) *s.t.* (2), (3) *and* (4) *is equivalent to the following problem*

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q\left(\theta\right)\right) - \left(\theta + \frac{F\left(\theta, e\right)}{f\left(\theta, e\right)}\right) q\left(\theta\right) \right) f\left(\theta, e\right) d\theta \tag{6}$$

$$+\mu \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e\left(\theta, e\right) q\left(\theta\right) d\theta - g_e\left(e\right) \right)$$

$$s.t. q_{\theta}\left(\theta\right) \leq 0$$

for some Lagrange multiplier μ *if and only if*[10]

$$\int_{\underline{\theta}}^{y} F_e\left(\theta, e\right) d\theta \geq 0 \forall y; \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e\left(\theta, e\right) d\theta = 0 \right) \tag{7}$$

and

$$\int_{\underline{\theta}}^{y} F_{ee}\left(\theta, e\right) d\theta \leq 0 \forall y; \left(\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}\left(\theta, e\right) d\theta = 0 \right) \tag{8}$$

or equivalently, if and only if an increase in e induces a mean preserving spread in the sense of Rothschild and Stiglitz (1970), at a decreasing rate.

It is well known[11] that the set of implementable contracts satisfies $t(\theta) = \theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tau) d\tau$ and $q_{\theta}\left(\theta\right) \leq 0$. Substituting out transfers and integrating by parts one obtains the principal's objective function: the principal maximizes expected surplus net of the agent's virtual surplus (Myerson

[10] The observation in brackets is tautologically satisfied in the present context by the law of iterated expectations: $E\Theta$ must be independent of effort. By an integration by parts, this is found equivalent to $\int_{\underline{\theta}}^{\bar{\theta}} F_e\left(\theta, e\right) d\theta = 0$, from where $\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}\left(\theta, e\right) d\theta = 0$ follows as well.

[11] For convenience of the reader the derivation is reproduced in the appendix. A more detailed treatment is found in Fudenberg and Tirole (1991), chap 7.

(1981)). Proceeding likewise for the agent's expected utility one obtains the expression in the constraint to problem (6). After an integration by parts the agent's first order condition can then be expressed as

$$-\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau q_\theta(\theta)\, d\theta - g_e(e) \tag{9}$$

From (9) it is obvious that (7) renders the agent's marginal value of information positive, and that (8) renders it concave. The agent is in fact a quasi-risk lover because his indirect utility under any implementable contract is a convex function of θ (Rochet (1985)). Therefore he likes increases in risk in the distribution of types in the sense of Rothschild and Stiglitz (1970)[12]. Moreover, since the first order condition must be a valid description for *any* nonincreasing quantity schedule, conditions (7) and (8) are also necessary. The complete proof is in the appendix.

The upshot of proposition 1 is that one can complement the Mirrlees approach to reporting by a first order approach to information acquisition, which yields a fairly easily tractable problem. Before I proceed applying the approach to the specific context of procurement, I characterize sufficient conditions on the Bayesian updating process that induce second order stochastic dominance shifts in the distribution of Θ.[13]

4 Informativeness of Experiments in Monotone Environments

In this section I show that within the class of posteriors that are comparable with respect to likelihood ratios a necessary and sufficient condition for increasing risk in the distribution of θ is that the likelihood ratio of the distribution of β conditional on the experiment is increasing as one moves away from average signals. Signals higher than ES induce higher (lower) posteriors if i is larger, i.e., if they are more informative. Moreover, I will show that an increase in effort induces a *single* mean preserving spread relation in the ex ante distribution of the posterior expectation.

Recall that $H(\beta\,|s, i)$ is the posterior cdf of the state given the experiment, and that $L(i, e)$ is

[12] Dai and Lewis (2003), in a two experiment model, also report the positive marginal value of information result for the agent, if the two experiments can be compared with respect to their riskiness.

[13] In this reduced form representation it appears that the first order approach goes through with considerable more ease than in problems of pure moral hazard. However, some restrictiveness enters the problem through the requirement that an increase in effort must increase the riskiness of the distribution in the sense of Rothschild and Stiglitz (1970). Moreover, further restrictions will be introduces when backing up the reduced form with a Bayesian updating process that delivers second order stochastic dominance in the distribution of θ.

the cdf of the informativeness parameter. Moreover, S and I are independent with joint density $k(s)\,l(i,e)$. So far I have used the assumptions that a high signal indicates a high β, (10), and that higher effort shifts the distribution of the informativeness parameter i with a first order stochastic dominance impact with decreasing returns (11):

$$\forall s' < s'' : H(\beta\,|s', i) \leq H(\beta\,|s'', i)\,\forall i \tag{10}$$

$$L_e(i,e)\,, -L_{ee}(i,e) \leq 0 \forall i \tag{11}$$

where the inequalities are strict on a set of positive measure. Assume now in addition that posteriors satisfy the following likelihood ratio condition:

$$\text{For } \beta > \tilde{\beta} \text{ and } i > i' : \ \frac{h(\beta\,|s,i)}{h\left(\tilde{\beta}\,|s,i\right)} \gtreqqless \frac{h(\beta\,|s,i')}{h\left(\tilde{\beta}\,|s,i'\right)}\ \forall s \gtreqqless ES \tag{12}$$

If the agent receives a signal which is higher (lower) than ex ante expected, then it is relatively more likely that indeed the state is high (low) for a higher realization of i.[14] In this sense higher indexed signals are more informative than lower indexed signals. Following Milgrom (1981) I shall say that for given s two experiments i and i' are *equivalent* if for any β and $\tilde{\beta}$

$$h(\beta\,|s,i)\,h\left(\tilde{\beta}\,|s,i'\right) = h(\beta\,|s,i')\,h\left(\tilde{\beta}\,|s,i\right)$$

An experiment is called *comparable* if for any given s either (12) holds or if the experiments are equivalent.

Lemma 1 *Assume that experiments are comparable. Then*

$$\pi_i(s,i) \lesseqqgtr 0 \, for \, s \lesseqqgtr ES$$

for any prior distribution if and only if (12) *holds.*

Relative to the prior mean the agent revises his posterior expectation upwards if he receives a signal higher than ex ante expected, downwards if he receives a downward surprise. If and only if he receives the expected signal, $s = ES$, no revision takes place. The upward (downward) revision for surprisingly high (low) signals is the larger the higher is i. As a consequence the conditional expectation functions for different i all cross only once and do so at the prior mean $E\Theta$. Since an increase in effort makes more informative experiments more likely and more informative experiments correspond to stronger outward revisions of posterior means, this concept of informativeness is closely tied to the notion of increasing risk:

[14] As I show in the appendix, with $r(i\,|\beta,s,e)$ the posterior densitiy of informativeness given β, s, for some fixed e, the monotone likelihood ratio condition can equivalently be expressed as $\frac{r(i|\beta,s,e)}{r(i|\tilde{\beta},s,e)} > \frac{r(i'|\beta,s,e)}{r(i'|\tilde{\beta},s,e)}$.

Proposition 2 *Assuming* (10), (11), *and that experiments are comparable,* $F(\theta, e)$ *satisfies* (7) *and* (8) *for any prior distribution of signals and experiments if and only if* (12) *holds. Moreover, an increase in effort induces a single mean preserving spread*

$$\begin{aligned} F_e(\underline{\theta}, e) &= F_e(E\theta, e) = F_e(\overline{\theta}, e) = 0 \\ F_e(\theta, e) &> 0\, for\, \theta \in (\underline{\theta}, E\theta) \\ F_e(\theta, e) &< 0\, for\, \theta \in (E\theta, \overline{\theta}) \end{aligned}$$

Intuitively, an increase in effort makes extreme cost perceptions more likely since i) an increase in effort makes performing a more informative experiment more likely and ii) a more informative experiment results in a stronger revision of the conditional expectation. The distribution of Θ inherits the decreasing returns property of the shifting technology. All distributions of $F(\theta, e)$ for different levels of effort satisfy a triple crossing property, and cross at the prior mean $E\Theta$. It is in fact also true that the distribution of θ inherits the likelihood ratio property, (12), i.e., one has $\frac{\partial}{\partial\theta}\left(\frac{f_e(\theta,e)}{f(\theta,e)}\right) \gtreqless 0$ for $\theta \gtreqless E\Theta$. This follows directly from Milgrom's (1981) proposition 3.

A relatively large class of distributions satisfies condition (12)[15] As an example consider the signal structure studied in Ottaviani and Sorensen (2001). With a slight departure of our notation let the cost be $\beta = b + \Delta\beta$ for some $b > \frac{1}{2}$, take the marginal of $\Delta\beta$ and s as the uniform on $[-1, 1]$, and take a posterior $h(\Delta\beta | s, i) = \frac{1+is\Delta\beta}{2}$. It is easy to see that this posterior satisfies (12). One verifies that $\pi(s, i) = \frac{is}{3}$ and $\pi_{is}(s, i) = \frac{1}{3} > 0$.

In the remainder of this paper I apply the first order approach to study the specific problem of procurement. The first step is to sign the multiplier μ. The second is to characterize the structure of optimal contracts.

5 The Value of Information

In this section I establish two results. First, the value of information to the principal is positive, i.e., it is always better to implement a positive amount of effort than to implement zero effort. Second, the marginal value of information to the principal is ambigous and depends on his quasi-attitudes toward risk, i.e., on the shape of his indirect utility function.

[15] The concept has been used extensively in statistical decision theory (Karlin and Rubin (1956), Lehmann (1988), and Athey and Levin (2001)) because it allows to order a larger class of distributions with respect to their informativeness than Blackwell's (1951) criterion.

Consider first the absolute value of information to the principal, i.e., the difference in expected utility when he implements a positive amount of effort and zero effort. Implementing $e = 0$ requires that information has no value to the agent, neither for his decision what type to report conditional on participating, nor on his decision whether or not to participate. This means that production must be independent of the agent's announced type and that the transfer is so high that even type $\overline{\theta}$ breaks even ex post. It is clear that no contract can do *worse* than this. In particular a better contract is one where contract and effort level correspond to the Nash Equilibrium of a simultaneous move game (rather than the Stackelberg equilibrium). For the sake of the argument suppose that the family of distributions $\{F(\theta, e)\}_{e\geq 0}$ has increasing hazard rates[16] so that

$$\frac{\partial}{\partial \theta}\left(\theta + \frac{F(\theta, e)}{f(\theta, e)}\right) \geq 0 \forall e$$

Suppose that the agent exerts effort $\hat{e}$. Then the principal's best response in the simultaneous move game is to offer a contract with associated production schedule that satisfies the Baron Myerson (1982) condition -obtained by a point-wise maximization of (6) with $\mu \equiv 0$–

$$V_q\left(q^{BM}(\theta, \hat{e})\right) = \theta + \frac{F(\theta, \hat{e})}{f(\theta, \hat{e})} \tag{13}$$

Conversely the agent's best response to $q^{BM}(\theta, \hat{e})$ is the solution to

$$\left.\int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, q^{BM}(\theta, \hat{e})\, d\theta - g_e(e)\right|_{e'=e^*} = 0 \tag{14}$$

Contract offer and effort choice are in simultaneous equilibrium if $e' = \hat{e}$. In this equilibrium, the principal extracts some rent, and therefore he does better than under the pessimistic contract. Since the principal will be able to do *even better* if he takes the effect of the contract on the effort choice into account, information has a strictly positive value to him. This result is stated formally in the following proposition, whose prove is found in the appendix.

Proposition 3 *The value of information to the principal is positive.*

Consider now the marginal value of information, that is the change in the principal's utility resulting from a marginal increase in effort. One way to address this question is by a local approach, that is by a small increase in effort around $\hat{e}$ as defined by the system of equations (13) and (14). Would the principal benefit from a small increase in effort? An increase in the agent's effort

[16] This is not crucial, i.e., the argument goes trough exactly the same way if this condition is violated. However, as is well known (Bagnoli and Bergstrom (1989)) many of the well known and commonly used distributions have globally nondecreasing hazard rates.

increases the likelihood of more extreme cost perceptions. The principal benefits ex post if the agent's signal is better than expected but is harmed if the agent perceives his cost as being higher. Whether the principal likes to consume such a lottery depends on the shape of his indirect utility function. In turn the shape of the indirect utility function depends on the curvature of the direct utility function and on properties of the family of distributions $\{F(\theta,e)\}_{e\geq 0}$. Define

$$\rho(q) = \frac{-V''(q)}{V'(q)}$$

$\rho(q)$ is the Arrow-Pratt measure of absolute risk aversion with respect to production shocks in the function $V(q)$.

Proposition 4 *i) If $\rho'(q) \geq 0$ and $\frac{F(\theta,e)}{f(\theta,e)}$ is convex in θ for all e, then the marginal value of information to the principal is negative $(\mu < 0)$.*
ii) Suppose that $\rho'(q) \leq 0$ and $\frac{F(\theta,e)}{f(\theta,e)}$ is concave in θ for all e. Then there exists $z > 0$ such that $\underline{\theta} \leq z$ implies that the marginal value of information to the principal is positive $(\mu > 0)$.

Non decreasing absolute risk aversion in the direct utility function $V(q)$ plus a convex hazard rate are sufficient to render the principal's indirect utility function globally concave. Therefore he behaves as a *quasi-risk averter* and is harmed by a small increase in effort. However, if $V(q)$ has the more natural property of non increasing absolute risk aversion and the hazard rate is concave then the opposite may happen. However, this case is somewhat more subtle because it is impossible to render the principal's indirect utility function globally convex. Since both cases are possible I will characterize optimal contracts for both constellations.

6 The Structure of Contracts

Let $\{q^*(\theta), t^*(\theta)\}\,\forall\theta$ denote a contract that optimally implements a given amount of effort in a truthtelling equilibrium. I shall characterize such contracts, taking their existence for granted.[17] The main obstacle to this analysis is that the size of the multiplier μ is unknown. A global treatment necessicates the use of dynamic optimization and delivers little additional insights. Therefore it is useful to characterize the solution for effort levels that are easy to implement in the sense of the following lemma.

[17] Conditions for existence of solutions for exogenous type distributions can be found in Guesnerie and Laffont (1984). With a suitable adjustement for the endogeneity of information their results could be carried over. Finally, since the objective function is quasi-concave in q, if a solution exists, it is unique.

Lemma 2 *If*

$$E\left[\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta\leq E\theta\right]\geq\max\left\{\frac{F(E\theta,e)}{f(E\theta,e)},E\theta-\underline{\theta}\right\} \tag{15}$$

then $|\mu|\leq 1$.

$|\mu|$ measures the utility loss due to the need to give extra (less) incentives for information gathering when marginal costs of information gathering increase uniformly by a small amount. One way to place a bound on this loss is to find a simple contract that continues to implement a given level of effort when marginal cost of effort increase (decrease) by a small amount. Since at the margin the agent's effort makes extreme cost perceptions more likely, one such simple contract rewards (punishes) the agent with a small extra amount of production (a small reduction thereof) if cost perceptions are surprisingly low. The statistic given in the lemma measures how well one can infer from a surprisingly favorable message the effort level the agent chose, relative to the need to do so. The easier this inference the smaller the utility loss due to the need to provide incentives for information acquisition, and hence the smaller $|\mu|$.[18] The advantage of bounding the absolute value of the multiplier by 1 is that one can characterize the solution to the contracting problem without recourse to control techniques:

Proposition 5 *Suppose that* (15) *is satisfied, that* $\frac{\partial}{\partial\theta}\frac{F(\theta,e)}{f(\theta,e)}\geq 0$ *and that* $\left|\frac{\partial}{\partial\theta}\frac{F_e(\theta,e)}{f(\theta,e)}\right|\leq 1$. *Then the optimal quantity schedule is characterized by*

$$V_q(q^*(\theta))=\theta+\frac{F(\theta,e)}{f(\theta,e)}-\mu\frac{F_e(\theta,e)}{f(\theta,e)} \tag{16}$$

The production schedule coincides with the Baron Myerson schedule at the top, at the prior mean, and at the bottom. Otherwise, there is an additional distortion, whose direction depends on whether the principal wants to encourage or discourage information acquisition. In the former case production is increased for surprisingly low cost perceptions and decreased for surprisingly bad cost assessments. The sensitivity of the production scheme with respect to the agent's information is increased to provide extra incentives for information acquisition. In the latter case, the reverse happens and production is more equalized in order to dampen the agent's interest in additional information. The size of the additional distortion depends on how informative a given message is

[18] This result is reminiscent of Kim's (1995) result in a context of pure moral hazard, stating that one information system (in his notation) $\frac{g_e(\theta,e)}{g(\theta,e)}$ is better than another, $\frac{f_e(\theta,e)}{f(\theta,e)}$, if the distribution of $\frac{g_e(\theta,e)}{g(\theta,e)}$ is a mean preserving spread of the distribution of $\frac{f_e(\theta,e)}{f(\theta,e)}$. In the present context an effort level is easier to implement if the distribution of $\frac{F_e(\theta,e)}{f(\theta,e)}$ is more spread out.

about the agent's unobserved effort choice.[19] The bound on $\left|\frac{\partial}{\partial\theta}\frac{F_e(\theta,e)}{f(\theta,e)}\right|$ requires that the likelihood ratio $\frac{f_e(\theta,e)}{f(\theta,e)}$ be finite, which is not so much of a concern for distributions with finite support. However, one can replace the bound by any positive number z if one replaces at the same time the bound on $|\mu|$ by z^{-1}. Without this restriction all the qualitative results remain unchanged but bunching cannot be ruled out easily.

Within the straightjacket of the first order approach these conclusions seem very robust. One may however wonder which of these features depend on the assumptions underlying the justification of the first order approach itself. Since some order structure on experiments is necessary to get any structure, there is little use questioning the assumption of ordered likelihood ratios. However, one may wonder whether the assumption of stochastic versus deterministic experiment structures is crucial. It turns out that the property of no distortion at the top depends on this result, but the other features of the contract do not.

7 Deterministic Experiments and Distortions at the Top

This section derives two results. First, it is demonstrated that the stochastic experiment structure is almost necessary to find closed form solutions for the contracting problem. Second, when the solution to the contracting problem with deterministic experiments admits a solution, the exaggeration feature -rewarding exceptionally low cost assessments and punishing exeptionally high cost assessments - is shown to be robust, but the no distortion at the top feature is not.

7.1 (Non-) Validity of a First Order Approach

Suppose the agent chooses directly the posterior distribution, $H(\beta\,|s,e)$. Denote the posterior expectation of the agent by

$$\pi(s,e) = \int_{\underline{\beta}}^{\overline{\beta}} \beta dH(\beta\,|s,e)$$

The marginal of S is again taken to be independent of effort. Like in the previous model assume that high signals indicate high costs, $\pi_s(s,e) \geq 0 \forall (s,e)$, and assume that the posteriors have the likelihood property ordering (12) with i replaced by e. By lemma 1 this implies $\pi_e(s,e) \lesseqgtr 0 \forall e$

[19] The term $\frac{F_e(\theta,e)}{f(\theta,e)}$ has an interpretation in terms of hypothesis testing. Write $\frac{F_e(\theta,e)}{F(\theta,e)}/\frac{f(\theta,e)}{F(\theta,e)}$. $\frac{F_e(\theta,e)}{F(\theta,e)}$ is the derivative of the log-likelihood if the statistician observes only if the values in a sample are smaller than θ and wants to compute the optimal value of e. This measure is important in the contract because the production at θ changes the rent of all types who are at least as efficient as θ. Division by $\frac{f(\theta,e)}{F(\theta,e)}$ normalizes by the conditional density.

for $s \lesseqgtr ES$. To replace the decreasing returns assumption assume $\pi_{ee}(s,e) \geq 0 \forall (s,e)$. On top, consistent with lemma 1, but somewhat stronger, assume that

$$\pi_{se}(s,e) \geq 0 \forall (s,e)$$

In contrast to the previous model $\theta(e)$ depends on the agent's effort. Moreover, the support of the random variable $\Theta(e)$ depends on effort. I'll address this issue the following way: I restrict attention to message games where communication is only about the signal realization, but not about the effort level the agent chose, and derive conditions where this can be done without loss of generality. Thus, the principal solves the following problem:

$$\max_{q(s),\, t(s)} \int_{\underline{s}}^{\overline{s}} V(q(s)) - t(s)\, dK(s) \tag{17}$$

s.t.

$$\forall s\, t(s) - \pi(s,e)\, q(s) \geq t(\hat{s}) - \pi(s,e)\, q(\hat{s})\ \forall \hat{s} \tag{18}$$

$$\forall s\, t(s) - \pi(s,e)\, q(s) \geq 0 \tag{19}$$

$$e \in \arg\max_{e} \left\{ \int_{\underline{s}}^{\overline{s}} (t(s) - \pi(s,e)\, q(s))\, dK(s) - g(e) \right\} \tag{20}$$

For e that satisfies (20) and all s :

$$\int_{\underline{s}}^{\overline{s}} (t(s) - \pi(s,e)\, q(s))\, dK(s) - g(e) \geq \int_{\underline{s}}^{\overline{s}} (t(\hat{s}) - \pi(\hat{s},\hat{e})\, q(\hat{s}))\, dK(s) - g(\hat{e})\ \forall (\hat{s},\hat{e}) \tag{21}$$

The constraints (18) and (20) are the usual one-dimensional incentive constraints; given the agent picks the desired effort level, truthtelling about s must be optimal; given that he will tell the truth once information is acquired, he should be willing to acquire the right amount of information. I shall call problem (17), s.t. (18), (20), and (19) the "local problem". Strictly speaking the incentive constraints in this problem are redundant because they are also embodied in constraint (21), which ensures that joint deviations in the sense that the agent chooses an effort level different from the one the principal wishes to implement and lies optimally afterwards do not pay. However, the "global problem" which arises if constraint (21) is added to the "local problem", makes the principal's problem intractable. Therefore, I solve the local problem and provide sufficient conditions such that the solution to the local problem satisfies constraint (21).

Proposition 6 *The "local" problem* (17), *subject to* (18), (19), *and* (20) *is solved if and only if*

$q(s)$ is nonincreasing in s, $t(s) = \pi(s,e)\,q(s) + \int_s^{\overline{s}} \pi_\tau(\tau,e)\,q(\tau)\,d\tau$, and $q(s)$ solves the problem

$$\max_{q(s)} \int_{\underline{s}}^{\overline{s}} \left(V(q(s)) - \left(\pi(s,e) + \pi_s(s,e)\frac{K(s)}{k(s)} \right) q(s) \right) k(s)\,ds \qquad (22)$$

$$+\mu \left(-\int_{\underline{s}}^{\overline{s}} \pi_e(s,e)\,q(s)\,k(s)\,ds - g_e(e) \right)$$

for some Lagrange multiplier μ. If the solution to this reduced problem satisfies for all $(\hat{s},\hat{e})$

$$q_s(\hat{s}) \geq -\frac{\pi_{ee}(\hat{s},\hat{e})\,\pi_{\hat{s}}(\hat{s},\hat{e})}{(\pi_e(\hat{s},\hat{e}))^2}\,q(\hat{s}) \qquad (23)$$

then the solution to the reduced problem is also a solution to the global problem (i.e., satisfies also (21)).

The crucial difference is the justifiability of the first order approach: under what conditions will it not pay for the agent to deviate in the two dimensions in tandem? (23) is a sufficient condition that rules such deviations. It is *heavily restrictive* and requires strict convexity of the posterior expectation in effort. This difficulty is due to the fact that the problem of choosing a sequence of decisions, where each decision is taken sequentially rational, does not in general give rise to a concave problem as of the first stage, even if the payoff function at each subsequent stage is strictly concave in the choice variable at this stage. However, something like this is needed to ensure that the agent's choice of what effort to choose, followed by what message to choose about s has a unique solution. Except in special cases, i.e., with special information structures and special utility functions it seems that this condition cannot be checked.[20]

7.2 Distortions at the Top

Suppose the local approach were valid. How do optimal contracts that arise from this model compare to the ones of the stochastic experiments model? To avoid technicalities I shall again assume the effort level that is implemented is easy to implement (lemma 2) in the sense that $|\mu|$ is small. The following results follow directly from a pointwise maximization of (22) :

Proposition 7 *Suppose that $\pi_s(s,e)\frac{K(s)}{k(s)}$ is nondecreasing in s for all e and either the principal wants to encourage information acquisition or that $|\mu|\,\pi_{es}(s,e) \leq \pi_s(s,e)$. Then, the quantity schedule in an optimal contract satisfies*

$$V_q(q^*(s)) = \pi(s,e) + \pi_s(s,e)\frac{K(s)}{k(s)} + \mu\pi_e(s,e)$$

In particular, there is a distortion at the top.

[20] For an example where this approach *can* be used see Szalay (2003), section 5 "Extensions".

The sign of $\pi_e(s,e)$ is the reverse of the sign of $\pi_e^{-1}(\theta,e)$. Hence, by lemma 1, contracts that encourage information acquisition reward the agent for surprisingly favorable cost assessments and punish him otherwise. Interestingly however, under the same assumptions as in the stochastic experiment model, there is now a distortion at the top. The intuition for this result is easiest to see if one changes notation in accordance with the previous one. Noting that $F(\theta,e) = K\left(\pi^{-1}(\theta,e)\right)$ and $f(\theta,e) = \frac{k\left(\pi^{-1}(\theta,e)\right)}{\pi_s(\pi^{-1}(\theta,e),e)}$ one can express the optimal quantity schedule, wherever $\pi_e^{-1}(\theta,e) \neq 0$, equivalently as follows:

$$V_q(q^*(\theta)) = \theta + \frac{F(\theta,e)}{f(\theta,e)} + \mu \frac{\frac{\partial}{\partial e}F(\theta,e)}{f(\theta,e)} \frac{\pi_e\left(\pi^{-1}(\theta,e),e\right)}{\pi_e^{-1}(\theta,e)}$$

Since the distribution of the posterior mean has a moving support, extreme expectations are only possible if the agent exerts the right level of effort. Therefore the bounds $\underline{\theta}(e)$ and $\overline{\theta}(e)$ are very informative about the agent's effort and the agent is rewarded at $\underline{\theta}(e)$ and punished at $\overline{\theta}(e)$.

8 Conclusion

The main result of the paper is that the first order approach to information acquisition is justified if and only if the agent's information gathering increases risk in the ex ante distribution of the conditional expectation in the sense of Rothschild and Stiglitz (1970). Sufficient conditions on experiment structures are provided that generate such an ordering. The robust results that follow from the approach are that contracts that encourage information acquisition are more sensitive to the agent's information relative to their fixed information counterparts. The reverse is true when information acquisition must be discouraged. A tractable modeling of information acquisition must almost necessarily rely on a stochastic experiment structure, where the agent's effort controls the frequency of receiving different posteriors which are independent of the agent's effort. The approach gives rise to a reduced form which is relatively easy to handle. It can be used to address multi-agent mechanism design problems in the linear, private values environment, e.g., optimal auction design.

9 Appendix

Proof of proposition 1, preliminaries. Truthtelling: For convenience I summarize the known features of the contract. For a more extensive treatment, see Fudenberg and Tirole (1991). Let $u\left(\theta,\hat{\theta}\right) = t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right)$ and $u(\theta) = \max_{\hat{\theta}} t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right)$. In a truthtelling equilibrium $\hat{\theta} = \theta$. By the envelope theorem, $u_\theta(\theta) = -q(\theta)$. Moreover, the least efficient type $\overline{\theta}$, is indifferent between participating and not, $u\left(\overline{\theta}\right) = 0$. Hence $u(\theta) = -\int_\theta^{\overline{\theta}} u_\theta(\tau)\, d\tau = \int_\theta^{\overline{\theta}} q(\tau) d\tau$. The first order condition $t_{\hat{\theta}}\left(\hat{\theta}\right) - \theta q_{\hat{\theta}}\left(\hat{\theta}\right)\Big|_{\hat{\theta}=\theta} = 0$ holds almost everywhere. Hence $\left(t_{\hat{\theta}\hat{\theta}}\left(\hat{\theta}\right) - \theta q_{\hat{\theta}\hat{\theta}}\left(\hat{\theta}\right)\right) d\hat{\theta} - q_{\hat{\theta}}\left(\hat{\theta}\right) d\theta = 0$, a.e., so that $q_{\hat{\theta}}\left(\hat{\theta}\right) \leq 0$ is necessary for truthtelling to be locally optimal. Finally, monotonicity makes the local first order condition sufficient for a global optimum in truthtelling. Substituting $t(\theta) = \theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau$ into the objective one has

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau \right) \right) f(\theta, e)\, d\theta \tag{24}$$

Integration by parts delivers the representation in terms of expected surplus net of the agent's expected virtual surplus (Myerson (1981)), $\int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) q(\theta) \right) f(\theta, e)\, d\theta$.

Consider now the effort constraint. After substitution of $t(\theta) = \theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau$ one has

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} (t(\theta) - \theta q(\theta))\, dF(\theta, e) &= \int_{\underline{\theta}}^{\bar{\theta}} \int_\theta^{\overline{\theta}} q(\tau) d\tau dF(\theta, e) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} F(\theta, e)\, q(\theta)\, d\theta \end{aligned}$$

Differentiating, and integrating by parts, using the property of nonmoving supports, one has

$$\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, q(\theta)\, d\theta = -\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau q_\theta(\theta)\, d\theta \geq 0$$

where the inequality follows from the implementability condition $q_\theta(\theta) \leq 0$. Differentiating once more

$$\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}(\theta, e)\, q(\theta)\, d\theta = -\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} F_{ee}(\tau, e)\, d\tau q_\theta(\theta)\, d\theta \leq 0$$

since $\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}(\theta, e)\, d\theta = 0$ and $\int_{\underline{\theta}}^{\theta} F_{ee}(\tau, e)\, d\tau \leq 0 \forall \theta$ and $q_\theta(\theta) \leq 0 \forall \theta$. Hence, if (7) and (8) hold, then the agent faces a strictly concave problem in effort and the first order condition in conjunction with the implementability conditions $t(\theta) = \theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau$ and $q_\theta(\theta) \leq 0 \forall \theta$ is necessary and sufficient for

$$e \in \arg\max_e \left\{ \int_{\underline{\theta}}^{\bar{\theta}} (t(\theta) - \theta q(\theta))\, dF(\theta, e) - g(e) \right\}$$

To see the necessity part, suppose that (7) does not hold. For concreteness, suppose that $F_e(\theta, e) < 0$ on $(\underline{\theta}, \theta_1)$ and $F_e(\theta, e) \geq 0$ else such that $\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, d\theta = 0$. A contract that satisfies the implementability condition is $\tilde{q}(\theta) = \tilde{q} > 0$ for $\theta \in [\underline{\theta}, \theta_1)$ and $\tilde{q}(\theta) = 0$ else. But then

$$\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, \tilde{q}(\theta)\, d\theta < 0 \forall e$$

and the first order condition is neither necessary nor sufficient for the optimal choice of e. Likewise suppose that (7) does hold but that (8) does not hold and suppose that $F_{ee}(\theta, e) > 0$ on $(\underline{\theta}, \theta_1)$ and $F_{ee}(\theta, e) \leq 0$ else such that $\int_{\underline{\theta}}^{\overline{\theta}} F_{ee}(\theta, e)\, d\theta = 0$. In this case under the implementable contract $\tilde{q}(\theta)$, $\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, \tilde{q}(\theta)\, d\theta > 0 \forall e$ but

$$\int_{\underline{\theta}}^{\overline{\theta}} F_{ee}(\theta, e)\, \tilde{q}(\theta)\, d\theta > 0 \forall e$$

Consequently, the first order condition is neither necessary (the optimal choice may be $e = 0$) nor sufficient (the value of e that solves the first order condition may correspond to a minimum.) ■

Proof of Lemma 1. For $\beta > \tilde{\beta}$ and $i > i'$: $\frac{h(\beta|s,i)}{h(\tilde{\beta}|s,i)} \gtreqqless \frac{h(\beta|s,i')}{h(\tilde{\beta}|s,i')}$ $\forall s \gtreqqless ES$. Take $s > ES$ so that

$$h(\beta\,|s,i)\, h\left(\tilde{\beta}\,|s,i'\right) > h(\beta\,|s,i')\, h\left(\tilde{\beta}\,|s,i\right)$$

Let $p(\beta)$, and $P(\beta)$ denote the prior density and cdf, respectively, of β. Choose some β^* such that $0 < P(\beta^*) < 1$. Integrate this inequality over $\beta > \beta^*$

$$h\left(\tilde{\beta}\,|s,i'\right) \int_{\beta>\beta^*} h(\beta\,|s,i)\, dP(\beta) > h\left(\tilde{\beta}\,|s,i\right) \int_{\beta>\beta^*} h(\beta\,|s,i')\, dP(\beta)$$

which is equivalent to

$$h\left(\tilde{\beta}\,|s,i'\right)(1 - H(\beta^*\,|s,i)) > h\left(\tilde{\beta}\,|s,i\right)(1 - H(\beta^*\,|s,i'))$$

Integrating over $\tilde{\beta} \leq \beta^*$ one finds that

$$H(\beta^*\,|s,i')(1 - H(\beta^*\,|s,i)) > H(\beta^*\,|s,i)(1 - H(\beta^*\,|s,i'))$$

or equivalently

$$\frac{H(\beta^*\,|s,i')}{(1 - H(\beta^*\,|s,i'))} > \frac{H(\beta^*\,|s,i)}{(1 - H(\beta^*\,|s,i))}$$

which implies

$$H(\beta^*\,|s,i') > H(\beta^*\,|s,i)$$

Since this is true for any β^* such that $0 < P(\beta^*) < 1$, when integrated the inequality becomes

$$\int_{\underline{\beta}}^{\overline{\beta}} H(\beta^*\,|s,i')\, d\beta^* < \int_{\underline{\beta}}^{\overline{\beta}} H(\beta^*\,|s,i)\, d\beta^*$$

integrating by parts

$$\overline{\beta}-\int_{\underline{\beta}}^{\overline{\beta}}\beta^{*}h\left(\beta^{*}\left|s,i'\right.\right)d\beta^{*}<\overline{\beta}-\int_{\underline{\beta}}^{\overline{\beta}}\beta^{*}h\left(\beta^{*}\left|s,i\right.\right)d\beta^{*}$$

and hence

$$\int_{\underline{\beta}}^{\overline{\beta}}\beta^{*}h\left(\beta^{*}\left|s,i\right.\right)d\beta^{*}>\int_{\underline{\beta}}^{\overline{\beta}}\beta^{*}h\left(\beta^{*}\left|s,i'\right.\right)d\beta^{*}$$

Analogous arguments show that signal $s = ES$ is neutral, and signals $s < ES$ induce lower conditional expectations when the experiment is more informative. Hence

$$\pi_i(s,i) \lesseqqgtr 0 \, for \, s \lesseqqgtr ES$$

Since signals are comparable, it is easy to see that (12) is also necessary. ■

Proof of Proposition 2. Since $l(i,e)$ has full support for all e, the distribution of θ has a nonmoving support $F(\underline{\theta},e)=0\forall e$ and $F\left(\overline{\theta},e\right)=1\forall e$. Hence $F_e(\underline{\theta},e)=F_e\left(\overline{\theta},e\right)=0$. By the law of iterated expectations $E\tilde{\theta}=E\tilde{\beta}$ for all e. Since $\int_{\underline{\theta}}^{\overline{\theta}}\theta dF(\theta,e)=\overline{\theta}-\int_{\underline{\theta}}^{\overline{\theta}}F(\theta,e)\,d\theta$, this is equivalent to $\int_{\underline{\theta}}^{\overline{\theta}}F_e(\tau,e)\,d\tau=0\forall e$.

By an integration by parts

$$\begin{aligned}F(\theta,e) &= \int_0^1 F_i(\theta,i)\,dL(i,e)\\ &= F_i(\theta,i)\,L(i,e)|_0^1-\int_0^1 k\left(\pi^{-1}(\theta,i)\right)\pi_i^{-1}(\theta,i)\,L(i,e)\,di\end{aligned}$$

since $F_i(\theta,i)$ is locally constant for $\theta\notin[\pi(\underline{s},i),\pi(\overline{s},i)]$. Taking derivatives with respect to e, since $L(1,e)=1\forall e$, we have

$$F_e(\theta,e)=-\int_0^1 k\left(\pi^{-1}(\theta,i)\right)\pi_i^{-1}(\theta,i)\,L_e(i,e)\,di$$

$\pi_i^{-1}(\theta,i)=\frac{ds}{di}$ measures how the signal that induces a conditional expected value of β equal to some value θ changes with the experiment. The sign of $\pi_i^{-1}(\theta,i)$ is the reverse of the sign of $\pi_i(\theta,i)$. Hence, by lemma 1 we have

$$\pi_i(\theta,i)\gtreqqless 0\Leftrightarrow\theta\lesseqqgtr E\tilde{\theta}$$

and hence

$$\begin{aligned}F_e(\theta,e) &> 0\,for\,\theta\in(\underline{\theta},E\theta)\\ F_e(\theta,e) &< 0\,for\,\theta\in\left(E\theta,\overline{\theta}\right)\end{aligned}$$

It follows that $\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau \geq 0 \forall \theta$. Recalling that by the law of iterated expectations, $\int_{\underline{\theta}}^{\overline{\theta}} F_e(\tau, e)\, d\tau = 0 \forall e$, it follows that a higher value of effort changes the ex ante distribution of θ in the sense of second order stochastic dominance, i.e., makes it more risky in the sense of Rothschild and Stiglitz (1970).

Finally, since $F_e(\underline{\theta}, e) = F_e(\overline{\theta}, e) = F_e\left(E\tilde{\theta}, e\right) = 0 \forall e$ we have $F_{ee}(\underline{\theta}, e) = F_{ee}(\overline{\theta}, e) = F_{ee}\left(E\tilde{\theta}, e\right) = 0 \forall e$. By the same token, the law of iterated expectations holds for all e, so that

$$\frac{d}{de}\left(\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, d\theta\right) = \int_{\underline{\theta}}^{\overline{\theta}} F_{ee}(\theta, e)\, d\theta = 0$$

Since $L_{ee}(i, e)$ and $L_e(i, e)$ have opposing signs for all i, we have

$$\begin{aligned} F_{ee}(\theta, e) &< 0\, for\, \theta \in (\underline{\theta}, E\theta) \\ F_{ee}(\theta, e) &> 0\, for\, \theta \in \left(E\theta, \overline{\theta}\right) \end{aligned}$$

and hence

$$\begin{aligned} \int_{\underline{\theta}}^{\theta} F_{ee}(\tau, e)\, d\tau &\leq 0 \forall \theta \\ \int_{\underline{\theta}}^{\overline{\theta}} F_{ee}(\tau, e)\, d\tau &= 0 \end{aligned}$$

This proves sufficiency. For necessity note that the marginals of the experiment are arbitrary. If the condition

$$\pi_i(s, i) \lesseqqgtr 0\, for\, s \lesseqqgtr ES$$

fails there exist distributions $K(s)$ and $L(i, e)$ that put essentially all mass on the event where the condition fails. ■

Proof of Proposition 3. $e = 0$ is optimal for the agent if and only if $q(\theta) = q$ and $t - \overline{\theta} q \geq 0$. The best such contract from the principal's perspective solves

$$\max_{q,t} \int_{\underline{\theta}}^{\overline{\theta}} (V(q) - t)\, dF(\theta, e)$$

$$s.t.\ t - \overline{\theta} q \geq 0$$

The optimal contract in this class satisfies

$$V_q(q)|_{q=\hat{q}} = \overline{\theta}$$

and $\hat{t} = \overline{\theta}\hat{q}$. However, this contract is very costly to the principal. Suppose instead the principal offers a naive contract

$$q(\theta, \hat{e}) = V_q^{-1}\left(\theta + \frac{F(\theta, \hat{e})}{f(\theta, \hat{e})}\right)$$

where $\hat{e}$ satisfies

$$\int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, q(\theta, \hat{e})\, d\theta - g_e(e)\bigg|_{e=\hat{e}} = 0$$

This naive contract corresponds to the case where the principal neglects his influence on the agent's effort choice but offers a contract which elicits information truthfully. For simplicity in this argument we assume that $\theta + \frac{F(\theta,\hat{e})}{f(\theta,\hat{e})}$ is nondecreasing in θ, however this is not essential. Even with some bunching, the principal manages to get some share from the surplus. Since the principal extracts some rents, this contract dominates the contract $\{\hat{t}, \hat{q}\}$. ■

Proof of Proposition 4. It is easiest to work with condition (24) for the principal's expected indirect utility:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta q(\theta) + \int_{\theta}^{\bar{\theta}} q(\tau) d\tau \right) \right) f(\theta, e)\, d\theta$$

Let $\phi(q) = -\rho(q)^{-1}$. The change in expected indirect utility due to a small increase in effort is

$$\begin{aligned}
& \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta q(\theta) + \int_{\theta}^{\bar{\theta}} q(\tau) d\tau \right) \right) f_e(\theta, e)\, d\theta \\
= & -\int_{\underline{\theta}}^{\bar{\theta}} \left(V_q(q(\theta)) - \theta \right) q_\theta(\theta)\, F_e(\theta, e)\, d\theta \\
= & -\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{V_q(q(\theta)) - \theta}{V_{qq}(q(\theta))} \frac{\partial}{\partial\theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \right) F_e(\theta, e)\, d\theta \\
= & -\int_{\underline{\theta}}^{\bar{\theta}} \frac{V_q(q(\theta))}{V_{qq}(q(\theta))} \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial\theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) F_e(\theta, e)\, d\theta \\
= & \int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau \frac{\partial}{\partial\theta} \left(\phi(q(\theta)) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial\theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \right) \right) d\theta \\
= & \int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau\, \phi(q(\theta)) \left(\begin{array}{c} \phi'(q(\theta)) \frac{\frac{\partial}{\partial\theta}\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right)^2}{\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right)^2} \frac{F(\theta,e)}{f(\theta,e)} \\ + \frac{\frac{\partial}{\partial\theta}\left(\frac{F(\theta,e)}{f(\theta,e)}\right)\theta - \frac{F(\theta,e)}{f(\theta,e)}}{\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right)^2} \frac{\partial}{\partial\theta}\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) \\ + \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial^2}{\partial\theta^2}\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) \end{array} \right) \right) d\theta
\end{aligned}$$

where the first equality uses an integration by parts and non-moving supports, the second and third use condition (24), and the fourth uses another integration by parts, and the final equality follows from straightforward algebra. Observe that $\phi(q(\theta)) \leq 0$ and $\phi'(q) \lesseqgtr 0 \Leftrightarrow \rho'(q) \lesseqgtr 0$. Since $\frac{\partial}{\partial\theta}\left(\frac{F(\theta,e)}{f(\theta,e)}\right)\theta - \frac{F(\theta,e)}{f(\theta,e)}\Big|_{\theta=\underline{\theta}} = \underline{\theta} > 0$ and

$$\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial\theta}\left(\frac{F(\theta, e)}{f(\theta, e)} \right)\theta - \frac{F(\theta, e)}{f(\theta, e)} \right) = \frac{\partial^2}{\partial\theta^2}\left(\frac{F(\theta, e)}{f(\theta, e)} \right)\theta$$

the reader easily verifies that a convex hazard rate and $\rho'(q) \geq 0$ delivers a negative marginal value of information.

Consider now the reverse case. A concave hazard rate and $\rho'(q) \leq 0$ render the first and final line in brackets negative. The middle term is ambigous since it must be positive at $\underline{\theta}$. For a concave hazard rate the term $\frac{\partial}{\partial\theta}\left(\frac{F(\theta,e)}{f(\theta,e)}\right)\theta - \frac{F(\theta,e)}{f(\theta,e)}$ is decreasing in θ, as just shown. Decreasing $\underline{\theta}$ decreases the middle term uniformly. The conclusion follows. ■

Proof of Lemma 2. With a slight abuse of notation let $\hat{g}(e) = g(e) + ec$ denote the true cost of effort function. An increase in c increases the marginal cost of effort uniformly. Since we consider interior solutions with strictly positive information acquisition the solution continues to be interior when c is slightly increased from 0. Let

$$W(c) = \max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) q(\theta) \right) f(\theta,e)\, d\theta$$
$$+\mu \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta,e)\, q(\theta)\, d\theta - g_e(e) - c \right)$$

for given e. $-\mu = W_c(c)$ measures the change in the principal's utility due to a uniform increase in the agent's marginal cost of effort. Let $q(\theta,c)$ denote the quantity schedule that maximizes $W(c)$. Since the expression in brackets following the multiplier in the second line is identically equal to zero I shall drop it in the remainder of the proof. For $c > 0$ and small we have

$$\begin{aligned} W(c) \quad &\geq \quad \int_{\underline{\theta}}^{E\theta} \left(V(q(\theta,0)+\varepsilon) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right)(q(\theta,0)+\varepsilon) \right) f(\theta,e)\, d\theta \\ &\quad + \int_{E\theta}^{\bar{\theta}} \left(V(q(\theta,0)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) q(\theta,0) \right) f(\theta,e)\, d\theta \end{aligned}$$

where ε is defined by the relation

$$\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)(q(\theta,0)+\varepsilon)\, d\theta + \int_{E\theta}^{\bar{\theta}} F_e(\theta,e)\, q(\theta,0)\, d\theta - g_e(e) - c = 0$$

i.e., by

$$\varepsilon \int_{\underline{\theta}}^{E\theta} F_e(\theta,e)\, d\theta = c$$

Since $q(\theta,c)$ is nonincreasing, and there is no distortion at the top

$$\begin{aligned} \int_{\underline{\theta}}^{E\theta} V(q(\theta,0)+\varepsilon) f(\theta,e)\, d\theta \quad &\geq \quad \int_{\underline{\theta}}^{E\theta} \left(V(q(\theta,0)) f(\theta,e) + \varepsilon V_q(q(\underline{\theta},0))\right) f(\theta,e)\, d\theta \\ &= \quad \int_{\underline{\theta}}^{E\theta} \left(V(q(\theta,0)) f(\theta,e) + \varepsilon\underline{\theta}\right) f(\theta,e)\, d\theta \end{aligned}$$

Hence

$$\begin{aligned} W(c) \quad &\geq \quad W(0) + \varepsilon \int_{\underline{\theta}}^{E\theta} \left(\underline{\theta} - \theta - \frac{F(\theta,e)}{f(\theta,e)} \right) f(\theta,e)\, d\theta \\ &= \quad W(0) + c \frac{(\underline{\theta} - E\theta)}{\frac{\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)d\theta}{F(E\theta,e)}} \end{aligned}$$

Hence

$$\frac{W(c)-W(0)}{c}\geq\frac{(\underline{\theta}-E\theta)}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

Taking limits as c goes to zero

$$\lim_{c\to 0}\frac{W(c)-W(0)}{c}=-\mu\geq\frac{(\underline{\theta}-E\theta)}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

and hence

$$\mu\leq\frac{(E\theta-\underline{\theta})}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

For $c<0$ one needs to keep the agent from increasing his effort choice. This is achieved by decreasing production by ε if costs are lower than expected. In this case

$$\begin{aligned}W(c) &\geq \int_{\underline{\theta}}^{E\theta}\left(V(q(\theta,0)-\varepsilon)-\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}\right)(q(\theta,0)-\varepsilon)\right)f(\theta,e)\,d\theta \\ &\quad+\int_{E\theta}^{\bar{\theta}}\left(V(q(\theta,0))-\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}\right)q(\theta,0)\right)f(\theta,e)\,d\theta\end{aligned}$$

where ε is defined by the relation

$$\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)(q(\theta,0)-\varepsilon)\,d\theta+\int_{E\theta}^{\bar{\theta}}F_e(\theta,e)\,q(\theta,0)\,d\theta-g_e(e)-c=0$$

$$\varepsilon\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)\,d\theta=-c$$

Since

$$\begin{aligned}\int_{\underline{\theta}}^{E\theta}V(q(\theta,0)-\varepsilon)f(\theta,e)\,d\theta &\geq \int_{\underline{\theta}}^{E\theta}(V(q(\theta,0))f(\theta,e)-\varepsilon V_q(q(E\theta,0)))f(\theta,e)\,d\theta \\ &= \int_{\underline{\theta}}^{E\theta}\left(V(q(\theta,0))f(\theta,e)-\varepsilon\left(E\theta+\frac{F(E\theta,e)}{f(E\theta,e)}\right)\right)f(\theta,e)\,d\theta\end{aligned}$$

one can state

$$\begin{aligned}W(c) &\geq W(0)+\varepsilon\int_{\underline{\theta}}^{E\theta}\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}-E\theta-\frac{F(E\theta,e)}{f(E\theta,e)}\right)f(\theta,e)\,d\theta \\ &= W(0)+c\frac{\frac{F(E\theta,e)}{f(E\theta,e)}}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}\end{aligned}$$

Hence

$$(W(0)-W(c))\leq -c\frac{\frac{F(E\theta,e)}{f(E\theta,e)}}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

ande hence, since $c<0$

$$\lim_{-c\to 0}\frac{(W(0)-W(c))}{-c}=-\mu\leq\frac{\frac{F(E\theta,e)}{f(E\theta,e)}}{\frac{\int_{\underline{\theta}}^{E\theta}F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

and hence

$$\mu \geq -\frac{\frac{F(E\theta,e)}{f(E\theta,e)}}{\frac{\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

Therefore

$$-\frac{\frac{F(E\theta,e)}{f(E\theta,e)}}{\frac{\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)d\theta}{F(E\theta,e)}} \leq \mu \leq \frac{(E\theta - \underline{\theta})}{\frac{\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)d\theta}{F(E\theta,e)}}$$

Now

$$\frac{\int_{\underline{\theta}}^{E\theta} F_e(\theta,e)\, d\theta}{F(E\theta,e)} = \frac{\int_{\underline{\theta}}^{E\theta} \frac{F_e(\theta,e)}{f(\theta,e)} f(\theta,e)\, d\theta}{F(E\theta,e)} = E\left[\left.\frac{F_e(\theta,e)}{f(\theta,e)}\right| \theta \leq E\theta\right]$$

Hence if

$$E\left[\left.\frac{F_e(\theta,e)}{f(\theta,e)}\right| \theta \leq E\theta\right] \geq \max\left\{\frac{F(E\theta,e)}{f(E\theta,e)}, E\theta - \underline{\theta}\right\}$$

then

$$|\mu| \leq 1$$

■

Proof of Proposition 6. Consider first the local problem. Similar to proposition 1 one observes that almost everywhere $t_{\hat{s}}(\hat{s}) - \pi(s,e)\, q_{\hat{s}}(\hat{s})|_{\hat{s}=s} = 0$ in a truthtelling equilibrium. For fixed e let $u(s) = \max_{\hat{s}} t_{\hat{s}}(\hat{s}) - \pi(s,e)\, q_{\hat{s}}(\hat{s})$. It follows that $u_s(s) = -\pi(s,e)\, q(s)$. Since $u(\overline{s}) = 0$ one has $u(s) = -\int_s^{\overline{s}} u_\tau(\tau)\, d\tau = \int_s^{\overline{s}} \pi_\tau(\tau,e)\, q(\tau)\, d\tau$. Since $\pi_s(s,e) \geq 0$ only monotonic contracts are implementable. By the usual methods one shows that monotonicity of contracts plus truthtelling satisfying a first order condition is sufficient for a global optimum in truthtelling. This is left to the reader.

When will the cross deviation constraint be nonbinding? Suppose the principal wants to implement some level of effort e, and suppose he offers some contract $\{t(\hat{s}), q(\hat{s})\}\, \forall \hat{s}$. Suppose further that the agent spend any effort level $\tilde{e}$ which may or may not coincide with e. Where the contract is differentiable, a necessary condition for the agent's optimal report is

$$t_{\hat{s}}(\hat{s}) - \pi(s,\tilde{e})\, q_{\hat{s}}(\hat{s}) = 0$$

Let $\hat{s}^*$ solve this problem for given $(e, \tilde{e}, s)$ and let

$$\hat{s} = \phi(e, \tilde{e}, s)$$

denote the agent's best response function. Let

$$U(\tilde{e}) := \int_{\underline{s}}^{\overline{s}} t(\phi(e,\tilde{e},s)) - \pi(s,\tilde{e})\, q(\phi(e,\tilde{e},s))\, dK(s) - g(\tilde{e})$$

At the ex ante stage the agent will solve

$$\max_{\tilde{e}} U(\tilde{e})$$

Taking derivatives we find

$$\begin{aligned} U_{\tilde{e}}(\tilde{e}) &= \int_{\underline{s}}^{\overline{s}} \left(t_{\hat{s}}(\phi(e,\tilde{e},s)) - \pi(s,\tilde{e})\, q_{\hat{s}}(\phi(e,\tilde{e},s))\right) \phi_{\tilde{e}}(e,\tilde{e},s)\, dK(s) \\ &\quad - \int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}}(s,\tilde{e})\, q(\phi(e,\tilde{e},s))\, dK(s) - g_{\tilde{e}}(\tilde{e}) \end{aligned}$$

By the envelope theorem it is obvious that choosing $\tilde{e} = e$ and reporting $\hat{s} = s$ subsequently satisfies the first order necessary condition for an optimum.

By the same theorem, the first line is identically equal to zero for all $\tilde{e}$. Hence, differentiating once more we find

$$\begin{aligned} U_{\tilde{e}\tilde{e}}(\tilde{e}) &= -\int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}}(s,\tilde{e})\, q_{\hat{s}}(\phi(e,\tilde{e},s))\, \phi_{\tilde{e}}(e,\tilde{e},s)\, dK(s) \\ &\quad - \int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}\tilde{e}}(s,\tilde{e})\, q(\phi(e,\tilde{e},s))\, dK(s) - g_{\tilde{e}\tilde{e}}(\tilde{e}) \end{aligned}$$

By the implicit function theorem applied to $t_{\hat{s}}(\hat{s}) - \pi(s,\tilde{e})\, q_{\hat{s}}(\hat{s}) = 0$ we have $\left(t_{\hat{s}\hat{s}}(\hat{s}) - \pi(s,\tilde{e})\, q_{\hat{s}\hat{s}}(\hat{s})\right) d\hat{s} - \pi_{\tilde{e}}(s,\tilde{e})\, q_{\hat{s}}(\hat{s})\, d\tilde{e} = 0$ and hence

$$\phi_{\tilde{e}}(e,\tilde{e},s) = \frac{d\hat{s}}{d\tilde{e}} = \frac{\pi_{\tilde{e}}(s,\tilde{e})\, q_{\hat{s}}(\hat{s})}{t_{\hat{s}\hat{s}}(\hat{s}) - \pi(s,\tilde{e})\, q_{\hat{s}\hat{s}}(\hat{s})}$$

After substitution

$$\begin{aligned} U_{\tilde{e}\tilde{e}}(\tilde{e}) &= -\int_{\underline{s}}^{\overline{s}} \frac{\left(q_{\hat{s}}(\phi(e,\tilde{e},s))\, \pi_{\tilde{e}}(s,\tilde{e})\right)^2}{t_{\hat{s}\hat{s}}(\hat{s}) - \pi(s,\tilde{e})\, q_{\hat{s}\hat{s}}(\hat{s})} dK(s) \\ &\quad - \int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}\tilde{e}}(s,\tilde{e})\, q(\phi(e,\tilde{e},s))\, dK(s) - g_{\tilde{e}\tilde{e}}(\tilde{e}) \end{aligned}$$

Suppose the principal offers the "naive" contract

$$t(\hat{s}) = \pi(\hat{s},e)\, q(\hat{s}) + \int_{\hat{s}}^{\overline{s}} \pi_{\hat{s}}(\tau,e)\, q(\tau)\, d\tau$$

where $q(\hat{s})$ is the optimal quantity if the agent chose the right effort level.

As of the reporting stage the agent's utility is

$$U(\hat{s}) := \pi(\hat{s},e)\, q(\hat{s}) + \int_{\hat{s}}^{\overline{s}} \pi_{\hat{s}}(\tau,e)\, q(\tau)\, d\tau - \pi(s,\tilde{e})\, q(\hat{s})$$

One finds that

$$U_{\hat{s}}(\hat{s}) = \left(\pi(\hat{s},e) - \pi(s,\tilde{e})\right) q_{\hat{s}}(\hat{s})$$

and

$$U_{\hat{s}\hat{s}}(\hat{s}) = \pi_{\hat{s}}(\hat{s}, e)\, q_{\hat{s}}(\hat{s}) + (\pi(\hat{s}, e) - \pi(s, \tilde{e}))\, q_{\hat{s}\hat{s}}(\hat{s})$$

Substituting $U_{\hat{s}\hat{s}}(\hat{s})$ for $t_{\hat{s}\hat{s}}(\hat{s}) - \pi(s, e)\, q_{\hat{s}\hat{s}}(\hat{s})$ one obtains

$$\begin{aligned}
U_{\tilde{e}\tilde{e}}(\tilde{e}) &= -\int_{\underline{s}}^{\overline{s}} \frac{(q_{\hat{s}}(\hat{s})\, \pi_e(s, e))^2}{\pi_{\hat{s}}(\hat{s}, e)\, q_{\hat{s}}(\hat{s}) + (\pi(\hat{s}, e) - \pi(s, \tilde{e}))\, q_{\hat{s}\hat{s}}(\hat{s})} dK(s) \\
&\quad - \int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}\tilde{e}}(s, \tilde{e})\, q(\hat{s})\, dK(s) - g_{\tilde{e}\tilde{e}}(\tilde{e}) \\
&= -\int_{\underline{s}}^{\overline{s}} \frac{(q_{\hat{s}}(\hat{s})\, \pi_e(s, e))^2}{\pi_{\hat{s}}(\hat{s}, e)\, q_{\hat{s}}(\hat{s}) + (\pi(\hat{s}, e) - \pi(s, \tilde{e}))\, q_{\hat{s}}(\hat{s})\, \frac{q_{\hat{s}\hat{s}}(\hat{s})}{q_{\hat{s}}(\hat{s})}} dK(s) \\
&\quad - \int_{\underline{s}}^{\overline{s}} \pi_{\tilde{e}\tilde{e}}(s, \tilde{e})\, q(\hat{s})\, dK(s) - g_{\tilde{e}\tilde{e}}(\tilde{e}) \\
&= -\int_{\underline{s}}^{\overline{s}} \left(\frac{(q_{\hat{s}}(\hat{s})\, \pi_e(s, e))^2}{\pi_{\hat{s}}(\hat{s}, e)\, q_{\hat{s}}(\hat{s})} + \pi_{\tilde{e}\tilde{e}}(s, \tilde{e})\, q(\hat{s}) \right) dK(s) - g_{\tilde{e}\tilde{e}}(\tilde{e})
\end{aligned}$$

where the last equality makes use of the fact that $(\pi(\hat{s}, e) - \pi(s, \tilde{e}))\, q_{\hat{s}}(\hat{s}) = 0$ is the agent's first order condition for the optimal report $\hat{s}$.The sufficient condition is that the integrand is pointwise nonnegative. ■

An Alternative Proof of Lemma 1. Another way to see this is by noting the equivalence of the monotone likelihood of the posterior distribution given s, i with the monotone likelihood ratio in the posterior of i given β, s and e and then apply Milgrom's (1981) proof. In particular for $\beta > \tilde{\beta}$, $i > i'$, and $s > ES$ one can write

$$\frac{\frac{h(\beta|s,i)l(i,e)}{\int_0^1 h(\beta|s,i)l(i,e)di}}{\frac{h(\tilde{\beta}|s,i)l(i,e)}{\int_0^1 h(\tilde{\beta}|s,i)l(i,e)di}} > \frac{\frac{h(\beta|s,i')l(i',e)}{\int_0^1 h(\beta|s,i')l(i',e)di}}{\frac{h(\tilde{\beta}|s,i')l(i',e)}{\int_0^1 h(\tilde{\beta}|s,i')l(i',e)di}} \tag{25}$$

because $\frac{l(i,e)}{l(i,e)} = \frac{l(i',e)}{l(i',e)} = 1$, $\int_0^1 h(\beta|s,i)\, l(i,e)\, di = \int_0^1 h(\beta|s,i')\, l(i',e)\, di$, and $\int_0^1 h\left(\tilde{\beta}|s,i\right) l(i,e)\, di = \int_0^1 h\left(\tilde{\beta}|s,i'\right) l(i',e)\, di$. Since s is fixed one may write (25) equivalently as

$$\frac{\frac{h(\beta|s,i)k(s)l(i,e)}{\int_0^1 h(\beta|s,i)k(s)l(i,e)di}}{\frac{h(\tilde{\beta}|s,i)k(s)l(i,e)}{\int_0^1 h(\tilde{\beta}|s,i)k(s)l(i,e)di}} > \frac{\frac{h(\beta|s,i')k(s)l(i',e)}{\int_0^1 h(\beta|s,i')k(s)l(i',e)di}}{\frac{h(\tilde{\beta}|s,i')k(s)l(i',e)}{\int_0^1 h(\tilde{\beta}|s,i')k(s)l(i',e)di}} \tag{26}$$

Let $r(i|\beta, s, e)$ denote the posterior density of i conditional on β, s for some fixed e. By definition

$$\frac{h(\beta|s,i)\, k(s)\, l(i,e)}{\int_0^1 h(\beta|s,i)\, k(s)\, l(i,e)\, di} = r(i|\beta, s, e)$$

Likewise one derives similar terms for the remaining three expressions in (26). Hence (26) is in fact equivalent to

$$\frac{r(i|\beta, s, e)}{r\left(i\left|\tilde{\beta}, s, e\right.\right)} > \frac{r(i'|\beta, s, e)}{r\left(i'\left|\tilde{\beta}, s, e\right.\right)}$$

From here on one may essentially follow Milgrom's (1981) proof. With $p(\beta)$, and $P(\beta)$ the prior density and cdf, respectively, of β, choose some β^* such that $0 < P(\beta^*) < 1$. Integrate this inequality over $\beta > \beta^*$

$$\frac{\int_{\beta>\beta^*} r(i|\beta,s,e)\,dP(\beta)}{r\left(i\left|\tilde{\beta},s,e\right.\right)} > \frac{\int_{\beta>\beta^*} r(i'|\beta,s,e)\,dP(\beta)}{r\left(i'\left|\tilde{\beta},s,e\right.\right)}$$

or equivalently

$$\frac{r\left(i'\left|\tilde{\beta},s,e\right.\right)}{\int_{\beta>\beta^*} r(i'|\beta,s,e)\,dP(\beta)} > \frac{r\left(i\left|\tilde{\beta},s,e\right.\right)}{\int_{\beta>\beta^*} r(i|\beta,s,e)\,dP(\beta)}$$

integrating this expression over $\tilde{\beta} \leq \beta^*$ yields

$$\frac{\int_{\tilde{\beta}\leq\beta^*} r\left(i'\left|\tilde{\beta},s,e\right.\right)dP\left(\tilde{\beta}\right)}{\int_{\beta>\beta^*} r(i'|\beta,s,e)\,dP(\beta)} > \frac{\int_{\tilde{\beta}\leq\beta^*} r\left(i\left|\tilde{\beta},s,e\right.\right)dP\left(\tilde{\beta}\right)}{\int_{\beta>\beta^*} r(i|\beta,s,e)\,dP(\beta)}$$

By Bayes' Theorem we have

$$r(i|\beta,s,e)\,p(\beta)\,k(s) = h(\beta|\,s,i)\,l(i,e)\,k(s)$$

Similar expressions are found for the remaining three integrands. After substitution one has

$$\frac{l(i',e)\int_{\tilde{\beta}\leq\beta^*} h\left(\tilde{\beta}\right|s,i'\left.\right)d\tilde{\beta}}{l(i',e)\int_{\beta>\beta^*} h\left(\tilde{\beta}\right|s,i'\left.\right)d\tilde{\beta}} > \frac{l(i,e)\int_{\tilde{\beta}\leq\beta^*} h(\beta|\,s,i)\,d\beta}{l(i,e)\int_{\beta>\beta^*} h(\beta|\,s,i)\,d\beta}$$

and this is equivalent to

$$\frac{H(\beta^*|\,s,i')}{1-H(\beta^*|\,s,i')} > \frac{H(\beta^*|\,s,i)}{1-H(\beta^*|\,s,i)}$$

which implies that $H(\beta^*|\,s,i') > H(\beta^*|\,s,i)$, and hence the result. ■

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