May 2004

Fertility, Volatility, and Growth

by

Aude Pommeret and William T. Smith*

Empirically, growth rates are negatively correlated with birth rates; they are also correlated with production risk. We argue that these stylized facts are related, and arise jointly from the decision of how many children to have in a risky economic environment.

JEL Classifications: D80, J13, O41

Keywords: stochastic growth, fertility, volatility

*University of Lausanne and University of Memphis, respectively. Correspondence may be addressed to Bill Smith, Department of Economics, Fogelman College of Business and Economics, University of Memphis, TN 38152, USA. Phone: (901) 678-3675. email: wtsmith@memphis.edu.

Why do poor countries get trapped in a vicious circle of high birth rates and low growth rates? This paper will develop a unified theory of how production volatility affects the growth rate of an economy by altering both saving decisions and the decision to have children. The model may be particularly relevant in some developing countries, where production volatilities are much greater than in developed economies [Turnovsky and Chattopadhyay (2003)].

In developing such a theory, we are motivated by two "stylized facts" about long-term growth that have emerged from cross-sectional empirical studies. First, *countries with low birth rates tend to grow faster than countries with high birth rates*. For instance, Coale and Hoover (1958) and Blancet (1991) established a negative correlation between birth rates and growth rates.[1] Second, in a cross-section, *growth rates are correlated with the volatilities of GDP growth.* The sign of the correlation is disputed. On the one hand, Ramey and Ramey (1995) and Aizenman and Marion (1993, 1997) detect a negative correlation between mean growth rates and their volatility. On the other hand, Kormendi and Meguire (1985) and Grier and Tullock (1989) found a positive correlation between the standard deviation of output growth and mean growth rates. Despite Gavin and Hausmann (1995) – who found no significant correlation at all -- there seems to be a consensus that risk seems to matter for growth.

We contend that these two observations are inextricably linked to the decision of how many children to have in a risky economic environment. To develop this proposition, we integrate two previously distinct strands of growth theory, the literature on endogenous fertility, and the recent literature on stochastic growth.

There is ample empirical evidence that a variety of economic variables (per capita income, wages, education, and urbanization, for example) have significant effects on birth rates [Wahl (1985), Behrman (1990), Schultz (1989), Barro and Lee (1994)]. This has led to a

[1] Kelley (1988), Srinivasan (1988), and Simon (1989) have challenged this proposition. However, Yip and Zhang (1996) argue convincingly that the weak correlation between population growth and economic growth in cross-country studies occurs because these studies do not control for exogenous, country-specific factors.

theoretical literature that tries to explain the simultaneous determination of fertility and growth rates [Becker and Barro (1988), Barro and Becker (1989), Becker, Murphy, and Tamura (1990), Pavilos (1995, 2001), Galor and Weil (1996), Tamura (1996), Yip and Zhang (1996)]. In these models, people derive pleasure from having children, but raising children entails costs, in terms of output or foregone time. So far, this literature has not allowed for the possibility that uncertainty may affect fertility choices. Nevertheless, it is now admitted that uncertainty "matters" for growth even in the long run because it alters saving behavior and affects portfolio choices between alternative capital investments [Corsetti (1997), Turnovsky (1993, 2000a, 2000b, 2000c, 2000d, 2003), Grinols and Turnovsky (1993, 1994, 1998), Asea and Turnovsky (1998), Smith (1996, 1999), Turnovsky and Chattopadhyay (2003), Chaterjee, Giuliano, and Turnovsky (2001), Yoichi and Turnovsky (2002), Montfort and Pommeret (2001) and Kenc (2004)]. However, this literature ignores *fertility* decisions.

In this paper we link these literatures by incorporating fertility choice into a stochastic growth model. We construct a stochastic version of the model by Yip and Zhang (1996): Raising a child requires the diversion of time and effort away from the production of goods and services. This reduces growth, as in Yip and Zhang's (1996) nonstochastic model. However, it also reduces the variance of production, much as changes in employment alter it in models of stochastic growth with wage income [Turnovsky (2003), Kenc (2004)]. Since having more children reduces both the mean and the variance of production, these in turn alter saving decisions. The saving and fertility decisions interact simultaneously to determine the growth rate. We show that the effect of output volatility on both the birth rate and the growth rate depends crucially on preferences.

The outline of the paper is as follows. Section 1 develops the model. Section 2 analyzes the effect of risk on fertility and growth. Section 4 concludes.

1. Technology and Preferences

Consider an economy populated by a large number of identical households. Time is continuous. Each family is endowed with a fixed amount of time in each period – normalized to unity – which can be spent either in production or in child-rearing activities. Following Yip and Zhang (1996) let $\phi(n)$ be the amount of time required for child rearing if the birth rate is n, where $\phi' > 0$.[2] Without much loss in generality, we set $\phi(n) = n$ to simplify exposition.

The production function (in per capita terms) is

$$dy = A(1-n)^{1-\alpha} k[dt + \sigma dz]. \qquad (1)$$

where dz is the increment to a standard-normal Wiener process and time indices have been deleted for simplicity. Output is random, with a mean of $A(1-n)^{1-\alpha} k$ and a variance of $A^2(1-n)^{2(1-\alpha)}\sigma^2 k^2$. If $\alpha > 0$ and $\sigma^2 = 0$, this reduces to the non-stochastic production function in Yip and Zhang (1996). If $\sigma^2 > 0$ but $\alpha = 0$, then labor supply disappears, and we have the production function suggested by Eaton (1981), which is now standard in the stochastic growth literature. If $\sigma^2 = 0$ and $\alpha = 0$, we would recover the simple linear, nonstochastic (a-k) technology that has played such a major part in modern growth theory [Romer (1986), Manuelli and Jones (1990)].[3] Notice that an increase in the birth rate reduces the variance of output.

The per capita capital stock accumulates according to

$$dk = dy - nkdt - cdt. \qquad (2)$$

The family has an infinite planning horizon. We follow Becker and Barro (1988) and Yip and Zhang (1996) in assuming that it derives utility from both consumption c and the birth rate.[4]

---

[2] In general, ϕ may be either convex or concave. If it is convex, stability in a non-stochastic model requires an upper limit to the increasing returns. See Yip and Zhang (1996, p. 321).
[3] Yip and Zhang (1996) embellish the model with a Romer (1986) externality in order to generate an a-k technology. Since all endogenous growth models with balanced growth paths end up with a-k technology, it will suffice for the purposes of this paper to assume it at the start.
[4] See Becker and Barro (1988) and Barro and Sala-i-Martin (1999) for a discussion of these preferences and for a justification of both the continuous-time formulation and the assumption of infinite horizons. As noted

A time-separable specification of utility would confound preferences for substitution over time with risk aversion and with ordinal preferences between c and n. In order to disentangle these different aspects of preferences we employ a form of Generalized Isoelastic (GIE) utility:

$$(1-\gamma)U(t) = \left\{ \left[cn^{\theta} \right]^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\rho dt} \left[(1-\gamma) E_t U(t+dt) \right]^{\frac{1}{1-\gamma}\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \tag{3}$$

The aggregator $x = cn^{\theta}$ is the period felicity function; $\theta \geq 0$ governs the taste for having children. $\varepsilon > 0$ is the intertemporal elasticity of substitution for riskless paths of x; $\gamma > 0$ is the coefficient of relative risk aversion for timeless gambles of x. This nests several familiar cases. If $\theta = 0$, Equation (1) reduces to the GIE preferences, defined over consumption alone, proposed by Svensson (1989).[5] If in addition $\gamma = 1/\varepsilon$ we recover conventional time-separable, isoelastic preferences. If $\gamma = 1/\varepsilon$ and $\theta > 0$ we have time-separable preferences over c and n.[6] An important benchmark case is $\gamma = 1$ and $\varepsilon = 1$; this captures time-separable, logarithmic intertemporal and risk preferences.

The representative household maximizes lifetime utility (3); given the production function (1) and the resource constraint (2).

2. Equilibrium

The solution to this problem satisfies the following first-order conditions:[7]

$$c = n^{\theta(\varepsilon-1)} Bk \tag{5}$$

---

by Yip and Zhang (1996), the model can be reformulated as a continuous-time model with overlapping generations [Blanchard (1985), Weil (1989b), Buiter (1988)] without changing its conclusions.

[5] Discrete time GIE preferences were developed by Epstein and Zin (1989, 1991) and Weil (1989a, 1990). Continuous time versions were developed by Svensson (1989) and Duffie and Epstein (1992a). GIE preferences have been seen extensive use in asset pricing [Epstein (1988), Epstein and Zin (1991) and Duffie and Epstein (1992b), Smith (2001)], macroeconomics [Weil (1989a), Tallarini (2000), growth [Obstfeld (1994), Smith (1999), Chatterjee, Giuliano, and Turnovsky (2001), Guiliano and Turnovsky (2003)], the welfare cost of macroeconomic volatility [Epaulard and Pommeret (2003a), and resource extraction [Epaulard and Pommeret (2003b), Smith and Son (2004)].

[6] Our specification generalizes that of Yip and Zhang (1996) both in using recursive preferences and allowing the felicity function to be non-separable in c and n. Their felicity function is $\ln c + n^{1-\omega}/(1-\omega)$. Our preferences reduce to theirs if we set $\gamma = \varepsilon = \theta = 1$ in our model and $\omega = 1$ in theirs.

[7] The derivation, along with all other mathematical details, is relegated to an appendix, available on request.

$$\theta n^{\theta(\varepsilon-1)-1} B = (1-\alpha) A (1-n)^{-\alpha} + 1 - \gamma A^2 (1-\alpha)(1-n)^{1-2\alpha} \sigma^2 \tag{6}$$

where $$B\left(n,\sigma^2\right) = \varepsilon\rho + (1-\varepsilon)\underbrace{\left[A(1-n)^{1-\alpha} - n - \frac{1}{2}\gamma A^2 (1-n)^{2(1-\alpha)} \sigma^2 \right]}_{\text{certainty equivalent of the production rate of return}}. \tag{7}$$

Equation (5) is the consumption function. The marginal propensity to consume is a linear function of the certainty equivalent rate of return to capital. As shown by Weil (1990), the intertemporal elasticity of substitution governs the sign of the effect of risk σ^2 on consumption: an increase in risk (given a birth rate n) reduces the certainty equivalent rate of return, which in turn may increase or decrease consumption depending upon whether $\varepsilon < 1$ or $\varepsilon > 1$. If there are log preferences towards intertemporal substitution $(\varepsilon = 1)$, risk has no effect on consumption. Notice that the birth rate alters consumption by changing the certainty equivalent rate of return.

Equation (6) governs fertility choice. Having a child has a cost in terms of the lost production caused by working less, as in Yip and Zhang (1996). Now however, having a child also reduces the variance of production. The right hand side of Equation (6) is the net (or certainty equivalent) marginal cost of raising a child; it is the decrease in the certainty equivalent rate of return caused by increasing *n.* This is depicted with the positively sloped curve $MC\left(n,\sigma^2\right)$ in the Figure 1.[8] The left hand side of Equation (6) is the marginal benefit of having kids, given by the marginal utility of having children. Notice that the marginal utility of fertility is an increasing function of consumption. This means that having kids alters marginal utility both directly, as in Yip and Zhang (1996), and indirectly by changing consumption. The negatively sloped curve $MB\left(n,\sigma^2\right)$ in Figure 1 depicts the marginal benefit.[9]

---

[8] We assume that labor's share exceeds one half $(\alpha < 1/2)$ and that $1 > \gamma A \sigma^2 / 2$; these inequalities guarantee that the certainty equivalent rate of return is a decreasing, concave function of n.

[9] For convenience, we depict the case where MB is negative sloped. This is always true if $\varepsilon \le 1$. If $\varepsilon > 1$ it is possible for MB to be positively sloped. However, a sufficient condition for it be negatively sloped when $\varepsilon > 1$ is that $\theta(1-\theta)(\varepsilon-1) > 1$. Even if MB is positively sloped, the ensuing comparative statics still obtain as long as MB is not steeper than MC, a condition implied by the necessary second-order conditions.

The equilibrium birth rate n^* is determined where $MB(n^*, \sigma^2) = MC(n^*, \sigma^2)$.

3. Risk, Fertility, and Growth

How does risk affect the birth rate and growth?

To address this question it is useful to first consider the special case with logarithmic preferences for intertemporal substitution. In this case consumption is independent of the certainty equivalent rate of return [from Equations (5) and (7), $c = \rho k$], so the marginal benefit of having children is independent of risk [$\partial MB / \partial \sigma^2 = 0$]. For a given n an increase in σ^2 unambiguously lowers the certainty equivalent rate of return. This lowers the marginal cost of having children [$\partial MC / \partial \sigma^2 < 0$], shifting the MC curve to the right. Therefore the birth rate will increase. Intuitively, people self-insure against production risk by investing more time in the "riskless" investment of having children. Moreover, the expected growth rate of the economy is only affected by uncertainty on the production through the change in the birth rate: in this case, more uncertainty unambiguously reduces the growth rate.

Proposition 1. *In the case of a log utility function, an increase in risk unambiguously raises the birth rate, which in turn reduces the growth rate*

Now consider the general model with GIE preferences. An increase in risk still reduces the marginal cost of having children, exactly as in the log case. However, by changing the certainty equivalent rate of return it also changes consumption, which alters the marginal benefit of having children. If people like to substitute over time, so that $\varepsilon > 1$, then the increase in risk increases consumption, and with it the marginal benefit of having children [$\partial MB / \partial \sigma^2 > 0$]: the MB curve shifts up. This reinforces the effect of the decrease in MC so that the birth rate unambiguously increases. This is shown in Figure 2. However, if people don't like to substitute over time, so that $\varepsilon < 1$, the opposite occurs: consumption falls as risk increases, so MB shifts down [$\partial MB/\partial\sigma^2 < 0$]. In this case, the effect on MB offsets the effect on MC. In general, we have

$$\frac{\partial n^*}{\partial\sigma^2} = \frac{\partial MC/\partial\sigma^2 - \partial MB/\partial\sigma^2}{\partial MB/\partial n - \partial MC/\partial n}. \tag{8}$$

We have found numerical examples, when $\varepsilon < 1$, where the birth rate will decrease when risk increases.[10]

What about growth? Using Equations (2), (5), (6) and (7) the equilibrium growth rate is

$$\frac{dk}{k} = \left\{\varepsilon\left[A\left(1-n^*\right)^{1-\alpha} - n^* - \rho\right] + (1-\varepsilon)\gamma A^2\left(1-n^*\right)^{2(1-\alpha)}\frac{\sigma^2}{2}\right\}dt + A\left(1-n^*\right)^{1-\alpha}\sigma dz. \tag{9}$$

Define the expected growth rate of capital (the expression in braces) by μ_k^*. An increase in uncertainty affects the expected growth rate directly and indirectly: The direct effect is to alter the saving decision; the indirect effect is to increase the birth rate, which changes the risk and expected return of capital, feeding back to change the saving decision. The net effect of an increase in σ^2 on μ_k^*.is

$$\frac{\partial\mu_k^*}{\partial\sigma^2} = -\left\{\varepsilon\left[(1-\alpha)A\left(1-n^*\right)^{1-\alpha} + 1\right] + (1-\varepsilon)\gamma A^2\left(1-n^*\right)^{2(1-\alpha)}\frac{\sigma^2}{2}\right\}\frac{\partial n^*}{\partial\sigma^2} + (1-\varepsilon)\frac{\gamma}{2}A^2\left(1-n^*\right)^{2(1-\alpha)} \tag{10}$$

From our previous discussion we know that if $\varepsilon \geq 1$ then $\partial n^*/\partial\sigma^2 > 0$. Equation (10) then implies that if $\varepsilon \geq 1$ it will also be true that $\partial\mu_k^*/\partial\sigma^2 < 0$. However, if $\varepsilon < 1$ then $\partial\mu_k^*/\partial\sigma^2$ cannot be signed ; in particular, using plausible values for the parameters (see the appendix) we obtain that the expected growth rate may be increased by σ^2. Therefore

Proposition 2. *An increase in risk unambiguously raises the birth rate and reduces the growth rate only if* $\varepsilon \geq 1$. *If* $\varepsilon < 1$ *it is possible for an increase in risk to reduce the birth rate and raise the growth rate*

[10] To save space we leave these calibrations in the appendix.

5. Conclusion

The premise of this paper is that birth rates and growth rates are jointly affected by production uncertainty. Our analysis points toward -- and suggests a way of achieving -- the integration of the literatures on stochastic growth and on fertility. The crucial parameter governing the effect of risk on fertility and growth is the intertemporal elasticity of substitution ε. If $\varepsilon \geq 1$ our model predicts that countries with more volatile GDPs should have higher birth rates and lower growth rates than those with less volatile GDPs. If $\varepsilon < 1$ the converse holds: risky countries should have lower birth rates and higher growth rates than less risky countries. The conventional wisdom is that $\varepsilon < 1$ [Hall (1988), Epstein and Zin (1991), Attanasio and Weber (1989), Atkeson and Ogaki (1996), Campbell, Lo, and MacKinlay (1997), Giovannini and Weil (1993), Normandin and St. Amour (1998)], so the presumption would seem to be that risk dampens fecundity and stimulates growth. However, there is some evidence in favor of $\varepsilon > 1$ [Attanasio and Weber (1993), Bufman and Leiderman (1990), Koskievic (1999)]. Moreover, we should not forget that models used to evaluate the value of the intertemporal elasticity of substitution are precisely those that fail at reproducing the risk premium and the risk free rate. Deaton (1992) also warns that ". . . it is quite unsafe to make any inference about intertemporal substitution from representative agent models." Such evaluations may therefore not be taken as definitive. This suggests that the model proposed in this paper may provide an explanation for developing countries with risky technologies that are trapped in equilibria with high birth rates and low growth rates.

Figure 1

ε < 1 and γ > 1

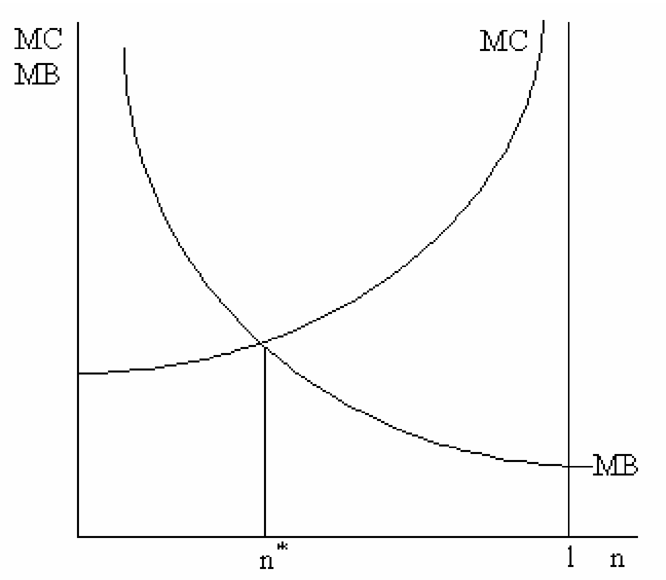
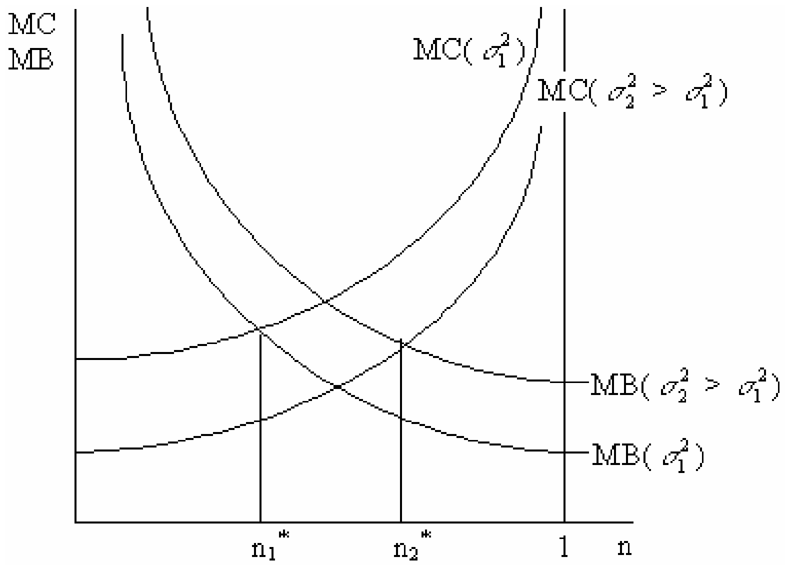


Figure 2

ε > 1 and γ > 1



References

Aizenman, A. and N. Marion, 1993, Policy Uncertainty, Persistence, and Growth, Review of International Economics 1, 145-163.

Aizenman, A. and N. Marion, 1997, Volatility and Investment: Interpreting Evidence from Developing Countries, Economica 66, 157-181.

Asea, P.K. and S.J. Turnovsky, 1998, Capital Taxation and Risk-Taking in a Small Open Economy, Journal of Public Economics 68, 55-90.

Attanasio, O. and G. Weber, 1989, Intertemporal Substitution, Risk Aversion, and the Euler Equation for Consumption, Economic Journal 99, 59-73.

Atkeson, A. and M. Ogaki, 1996, Wealth-Varying Elasticities of Substitution: Evidence from Panel and Aggregate Data, Journal of Monetary Economics 38, 507-534.

Attanasio, O. and G. Weber, 1993, Consumption Growth, the Interest Rate, and Aggregation, Review of Economic Studies 60, 411-445.

Barro, R. and G. Becker, 1989, Fertility Choice in a Model of Economic Growth, Econometrica 57, 481-501.

Barro, R. and J. Lee, 1994, Sources of Economic Growth. Carnegie-Rochester Series on Economic Policy.

Barro, R. and X. Sala-i-Martin, 1995, Economic Growth (McGraw-Hill: New York).

Becker, G. and R. Barro, 1988, Reformulation of the Economic Theory of Fertility, Quarterly Journal of Economics 103, 1-25.

Becker, G., K.M. Murphy, and A. Tamura, 1990, Human Capital, Fertility, and Economic Growth, Journal of Political Economy 98, S12-S37.

Behrman, J.R., 1990, Women's Schooling and Non-Market Productivity: A Survey and a Reappraisal. Working Paper, University of Pennsylvania.

Blancet, D., 1991, Estimating the Relationship Between Population Growth and Aggregate Economic Growth in Developing Countries: Methodological Problems, Discussion Paper, Institut National d'Etudes Demographiques: Paris.

Blanchard, O., 1985, Debt, Deficits, and Finite Horizons, Journal of Political Economy, 223-247.

Bufman, G. and G. Leiderman, 1990, Consumption and Asset Returns Under Non-Expected Utility, Economics Letters 34, 231-235.

Buiter, W. 1988, Debt, Population Growth, and Productivity Growth, Economic Journal, 279-293.

Campbell, J., A. Lo, and A.C. MacKinlay, 1997, The Econometrics of Financial Markets, (Princeton University Press: Princeton, NJ).

Chaterjee, S., P. Giuliano, and S. Turnovsky, 2001, Capital Income Taxes and Growth in a Stochastic Economy: A Numerical Analysis of the Role of Risk Aversion and Intertemporal Substitution, Working Paper, University of Georgia.

Corsetti, G., 1997, A Portfolio Approach to Endogenous Growth: Equilibrium and Optimal Policy, Journal of Economic Dynamics and Control 21, 1627-1644.

Coale, A. and E. Hoover, 1958, Population Growth and Economic Development in Low Income Countries: A Case Study of India's Prospects (Princeton University Press: Princeton, NJ).

Cooley, T. F., 1995, Frontiers of Business Cycle Research, Princeton, Princeton University Press.

Deaton, A. 1992, Understanding Consumption (Clarendon Press: Oxford).

Duffie, D. and L. Epstein, 1992a, Stochastic Differential Utility, Econometrica 60, 353-394.

Duffie, D. and L. Epstein, 1992b, Asset Pricing With Stochastic Differential Utility, Review of Financial Studies 5, 411-436.

Eaton, J., 1981, Fiscal Policy, Inflation, and the Accumulation of Risky Capital, Review of Economic Studies 48, 435-445.

Epaulard, A. and A. Pommeret, 2003a, Recursive Utility, Endogenous Growth, and the Welfare Cost of Volatility, Review of Economic Dynamics 6, 672-684.

Epaulard, A. and A. Pommeret, 2003b, Optimally Eating a Stochastic Cake, A Recursive Utility Approach, Resource and Energy Economics 25, 129-139.

Epstein, L., 1988, Risk Aversion and Asset Prices, Journal of Monetary Economics 22, 179-192.

Epstein, L. and S. Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, Econometrica 57, 937-969.

Epstein, L. and S. Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis, Journal of Political Economy 99, 263-268.

Gali, J., 1994, Government Size and Macroeconomic Stability, European Economic Review 38, 117-132.

Galor, O. and D. Weil, 1996, The Gender Gap, Fertility, and Growth. American Economic Review 86, 374-387.

Gavin, M. and R. Hausmann, 1995, Overcoming Volatility in Latin America. Inter-American Bank.

Gertler, M. and E.L Grinols, 1982, Monetary Randomness and Investment, Journal of Monetary Economics 10, 239-258.

Giovannini, A. and P. Jorion, 1993, Time Series Tests of Non-Expected Utility Asset Pricing, European Economic Review 37, 1083-1100.

Grier, K.B. and G. Tullock, 1989, An Empirical Analysis of Cross-National Economic Growth: 1951-1980, Journal of Monetary Economics 24, 48-69.

Grinols, E.L. and S.J. Turnovsky, 1993, Risk, the Financial Market, and Macroeconomic Performance, Journal of Economic Dynamics and Control 17, 1-36.

Grinols, E.L. and S.J. Turnovsky, 1994, Exchange Rate Determination and Asset Prices in a Small Open Economy, Journal of International Economics 36, 75-97.

Grinols, E.L. and S.J. Turnovsky, 1998, Risk, Optimal Government Finance and Monetary Policies in a Growing Economy, Economica 65, 401-427.

Guiliano, P. and S. Turnovsky, 2003, Intertemporal Substitution, Risk Aversion, and Economic Performance in a Stochastically Growing Economy, Journal of International Money and Finance 59, 267-295.

Hall, R., 1988, Intertemporal Substitution in Consumption, Journal of Political Economy 96, 339-357.

Kelley, A.C., 1988, Economic Consequences of Population Change in the Third World, Journal of Economic Literature 26, 1685-1728.

Kenc, T., 2004. Taxation, Risk-Taking and Growth: A Continuous-Time Stochastic General Equilibrium Analysis with Labor-Leisure Choice, Journal of Economic Dynamics and Control 28, 1511-1539.

Koskievic, J.M., 1999, An Intertemporal Model of Consumption/Leisure With Non-Expected Utility Preferences, Economics Letters 64, 285-289.

Jones, L.E. and R. E. Manuelli, 1990, A Convex Model of Equilibrium Growth: Theory and Policy Implications, Journal of Political Economy 98, 1008-1038.

Kormendi, R. and P. Meguire, 1985. Macroeconomic Determinants of Growth: Cross-Country Evidence, Journal of Monetary Economics 16, 141-163.

Montfort, P. and A. Pommeret, 2001, Fiscal Harmonization Under Uncertainty, Working Paper, IRES.

Normandin, M. and P. St. Amour, 1998, Substitution, Risk Aversion, Taste Shocks and Equity Premia, Journal of Applied Econometrics 13, 265-281.

Obstfeld, M., 1994, Risk-Taking, Global Diversification, and Growth, American Economic Review 84, 1310-1329.

Pavilos, T., 1995, Endogenous Fertility, Multiple Growth Paths, and Economic Convergence, Journal of Economic Dynamics and Control 19, 1489-1510.

Pavilos, T., 2001, Social Norms, Fertility, and Economic Development, Journal of Economic Dynamics and Control 25, 1919-1934.

Ramey, G. and V. Ramey, 1995, Cross-Country Evidence on the Link Between Volatility and Growth, American Economic Review 85, 1138-1151.

Romer, P.M., 1986, Increasing Returns and Long Run Growth, Journal of Political Economy 94, 1002-1037.

Schultz, T.P., 1989, Returns to Women's Education, PHRWD Background Paper 89/001, The World Bank, Population, Health, and Nutrition Department, Washington D.C.

Simon, J., 1989, On Aggregate Empirical Studies Relating Population Variable to Economic Development, Population Development Review 15, 323-332.

Smith, W., 1996a, Taxes, Uncertainty, and Long-Term Growth, European Economic Review 40, 1647-1664.

Smith, W., 1996b, Feasibility and Transversality Conditions for Models of Portfolio Choice With Non-Expected Utility in Continuous-Time, Economics Letters 30, 123-131.

Smith, W., 1998, Risk, the Spirit of Capitalism, and Long-Term Growth: the Implications of a Preference for Capital, Journal of Macroeconomics 21, 241-163.

Smith, W. 2001, How Does the Spirit of Capitalism Affect Asset Prices," Review of Financial Studies 14, 1215-1232.

Smith, W., and Y. Son, 2004, Can the Desire to Conserve Our Natural Resources Be Self-Defeating?, forthcoming, Journal of Environmental Economics and Management.

Svensson, L., 1989, Portfolio Choice With Non-Expected Utility in Continuous-Time, Economics Letters 30, 313-347.

Srinivasan, T.N., 1988, Population Growth and Economic Development, Journal of Policy Modeling 10, 7-28.

Tallarini, T.D., 2000, Risk-Sensitive Real Business Cycles, Journal of Monetary Economics 45, 27-42.

Tamura, R., 1996, From Decay to Growth: A Demographic transition to Economic Growth, Journal of Economic Dynamics and Control 20, 1237-1261.

Turnovsky, S.J., 1993, The Impact of Terms of Trade of Trade Shocks in a Small Open Economy: A Stochastic Analysis, Journal of International Money and Finance 12, 278-297.

Turnovsky, S.J., 2000a, Methods of Macroeconomic Dynamics (M.I.T. Press, Cambridge).

Turnovsky, S.J., 2000b, International Macroeconomic Dynamics (M.I.T. Press, Cambridge).

Turnovsky, S.J., 2000c, Fiscal Policy, Elastic Labor Supply and Endogenous Growth, Journal of Monetary Economics 45: 185-210.

Turnovsky, S.J. 2003, Government Policy in a Stochastic Growth Model with Elastic Labor Supply, Journal of Public Economic Theory 2: 389-433.

Turnovsky, S.J., and P. Chattopadhyay, 2003, Volatility and Growth in Developing Economies: Some Numerical Results and Empirical Evidence, Journal of International Economics 59, 267-295.

Wahl, J.B., 1985, Fertility in America: Historical Patterns and Wealth Effects on the Quantity and Quality of Children. Ph.D. dissertation, University of Chicago.

Weil, P., 1989a, The Equity Premium and the Risk-free Rate Puzzle, Journal of Monetary Economics 24, 401-421.

Weil, P., 1989b, Overlapping Families of Infinitely Lived Agents, Journal of Public Economics, 183-198.

Weil, P., 1990, Nonexpected Utility in Macroeconomics, Quarterly Journal of Economics 105, 29-42.

Yip, C.K. and J. Zhang, 1996, Population Growth and Economic Growth: A Reconsideration, Economics Letters 52, 319-324.

Yoichi, G. and S. J. Turnovsky, 2002, Alternative Government Financing and Stochastic Endogenous Growth, Journal of Economic Dynamics and Control 26, 681-706.

Appendices to "Fertility, Volatility, and Growth"

by

Aude Pommeret and William T. Smith

I. *Derivation of the consumption and fertility policies*

The program may be written:

$$(1-\gamma)J\left[k(t)\right]=\max_{c(t),n(t)}\left\{\left[c(t)n(t)^{\theta}\right]^{\frac{\varepsilon-1}{\varepsilon}}dt+e^{-\rho dt}\left[(1-\gamma)E_t J\left[k(t+dt)\right]\right]^{\frac{1}{1-\gamma}\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \quad \text{(A.1)}$$

subject to

$$dk = \left(A(1-n)^{1-\alpha}k - nk - c\right)dt + A(1-n)^{1-\alpha}k\sigma dz \quad \text{(A.2)}$$

where $J\left[k(t)\right]$ is the value function. We conjecture that it is of the form:

$$J\left[k(t)\right]=B^{\frac{1-\gamma}{1-\varepsilon}}\frac{k(t)^{1-\gamma}}{1-\gamma} \quad \text{(A.3)}$$

where B is a constant to be determined. The Bellman equation then becomes:

$$(1-\gamma)B^{\frac{1-\gamma}{1-\varepsilon}}\frac{k(t)^{1-\gamma}}{1-\gamma}=\max_{C,n}\left\{\left[cn^{\theta}\right]^{\frac{\varepsilon-1}{\varepsilon}}dt+e^{-\rho dt}B^{\frac{-1}{\varepsilon}}\left[E_t k(t+dt)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \quad \text{(A.4)}$$

Applying It's lemma we have:

$$\frac{E_t\left[k(t+dt)^{1-\gamma}\right]}{k(t)}=k(t)^{1-\gamma}+(1-\gamma)k(t)^{-\gamma}E_t(dk)-\frac{1}{2}\gamma(1-\gamma)k(t)^{-1-\gamma}E_t(dk^2)$$
$$=k(t)^{1-\gamma}\left[1+(1-\gamma)\left(A(1-n)^{1-\alpha}-n-c/k\right)dt-\gamma(1-\gamma)A^2(1-n)^{2(1-\alpha)}(\sigma^2/2)dt\right] \quad \text{(A.5)}$$

Therefore:

$$(1-\gamma)B^{\frac{1-\gamma}{1-\varepsilon}}\frac{k(t)^{1-\gamma}}{1-\gamma}=\max_{c,n}\left\{\left[cn^{\theta}\right]^{\frac{\varepsilon-1}{\varepsilon}}dt\right.$$
$$\left.+B^{\frac{-1}{\varepsilon}}k(t)^{\frac{\varepsilon-1}{\varepsilon}}\left[1+\frac{(\varepsilon-1)}{\varepsilon}\left(A(1-n)^{1-\alpha}-n-c/k\right)dt-\gamma\frac{(\varepsilon-1)}{\varepsilon}A^2(1-n)^{2(1-\alpha)}(\sigma^2/2)dt-\rho dt\right]\right\}^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \quad \text{(A.6)}$$

The first order conditions lead to:

$$c=n^{\theta(\varepsilon-1)}Bk \quad \text{(A.7)}$$

$$\theta n^{\theta(\varepsilon-1)-1}B = (1-\alpha)A(1-n)^{-\alpha} + 1 - \gamma A^2(1-\alpha)(1-n)^{1-2\alpha}\sigma^2 \tag{A.8}$$

Replacing consumption by its expression in the Bellman equation leads to the expression in Equation (7) of the text for *B*.

I. Equilibrium and Comparative Statics

A. Preliminary Results: the Certainty Equivalent Rate of Return and the Birth Rate

Recall from Equation (7) of the text that the certainty equivalent rate of return is

$$R(n,\sigma^2) = A(1-n)^{1-\alpha} - n - \gamma A^2(1-n)^{2(1-\alpha)}\frac{\sigma^2}{2}. \tag{a.1}$$

Notice that

$$R(0,\sigma^2) = A - \gamma A^2\frac{\sigma^2}{2}. \tag{a.2}$$

For any reasonable calibration of the model $R(0,\sigma^2) > 0.$ Henceforth we will assume this condition is satisfied.

Notice also that $R(1,\sigma^2) = -1.$

Next calculate the first and second derivatives of $R(n,\sigma^2)$ with respect to n:

$$R_n = -(1-\alpha)A(1-n)^{-\alpha} - 1 + (1-\alpha)\gamma A^2(1-n)^{1-2\alpha}\sigma^2, \tag{a.3}$$

$$R_{nn} = -\alpha(1-\alpha)A(1-n)^{-\alpha-1} - (1-\alpha)(1-2\alpha)\gamma A^2(1-n)^{-2\alpha}\sigma^2. \tag{a.4}$$

Assuming that $1 > 2\alpha$, $R_{nn} < 0$ globally. It remains to sign R_n.

First notice that

$$R_n(0,\sigma^2) = -1 - (1-\alpha)(A - \gamma A^2\sigma^2). \tag{a.5}$$

Assuming that $1 - \gamma A\sigma^2 > 0$ -- which also seems reasonable – it follows that $R_n(0,\sigma^2) < 0$.

Second, notice that – given $1 > 2\alpha$ -- $R_n(1,\sigma^2) \to -\infty.$

Since $R_{nn} < 0$, $R_n(n,\sigma^2) < 0$ for $0 \le n \le 1$.

Lemma 1. *On the unit interval the certainty equivalent rate of return is a monotonically decreasing and concave function on* n*: For* $0 \le n \le 1,\ R_n\left(n,\sigma^2\right) < 0$ *and* $R_{nn}\left(n,\sigma^2\right) < 0.$

B. *Existence*

Use Equation (a.1) to write Equation (7) in the text as

$$\theta n^{\theta(\varepsilon-1)-1}\left[\varepsilon\rho + (1-\varepsilon)R\left(n,\sigma^2\right)\right] = -R_n\left(n,\sigma^2\right). \qquad \text{(a.7)}$$

The right hand side of this equation is the net (or certainty-equivalent) marginal cost of having children, $MC\left(n,\sigma^2\right) = -R_n\left(n,\sigma^2\right)$. From Lemma 1 we can immediately deduce that MC is positive and increasing with n. It tends to ∞ as n tends to one.

The left hand side of Equation (a.7) is the marginal benefit of having children, $MB\left(n,\sigma^2\right) = \theta n^{\theta(\varepsilon-1)-1}\left[\varepsilon\rho + (1-\varepsilon)R\left(n,\sigma^2\right)\right]$. The feasibility condition -- $\varepsilon\rho + (1-\varepsilon)R\left(n,\sigma^2\right) > 0$ --ensures the $MB\left(n,\sigma^2\right) > 0.$

The slope of $MB\left(n,\sigma^2\right)$ is

$$MB'(n) = \theta n^{\theta(\varepsilon-1)-1}\left\{\frac{\left[\theta(\varepsilon-1)-1\right]\left[\varepsilon\rho + (1-\varepsilon)R(n)\right]}{n} + (1-\varepsilon)R'(n)\right\}. \qquad \text{(a.8)}$$

Use Equation (a.7) to write this as

$$MB_n\left(n,\sigma^2\right) = \theta n^{\theta(\varepsilon-1)-2}\left[\varepsilon\rho + (1-\varepsilon)R(n)\right]\left[\theta(\varepsilon-1) - 1 + \theta(\varepsilon-1)n^{\theta(\varepsilon-1)}\right] \qquad \text{(a.9)}$$

On inspection, we have

Lemma 2. *A sufficient condition for* $MB\left(n,\sigma^2\right)$ *to be negatively sloped is that* $\varepsilon \le 1.$

What if $\varepsilon > 1$? Consider the expression $\theta(\varepsilon-1) - 1 + \theta(\varepsilon-1)n^{\theta(\varepsilon-1)}$ in Equation (a.9). This will be negative if and only if

$$n^{\theta(\varepsilon-1)} < \frac{1-\theta(\varepsilon-1)}{\theta(\varepsilon-1)}. \qquad \text{(a.10)}$$

The left hand side of this inequality in monotonically increasing and concave in n; it equals zero when $n = 0$ and one when $n = 1$. Therefore a sufficient condition for $MB\left(n,\sigma^2\right)$ to be globally negatively sloped is

$$\varepsilon < \frac{1}{2\theta} + 1 \qquad \text{(a.11)}$$

We will assume this condition holds as well. This leads to

Proposition*. If inequality (a.11) holds, then there exists a unique equilibrium birth rate between zero and unity.*

We note in passing that it would suffice for the purposes of comparative statics, to impose the weaker "stability" condition that, if the MB is positively sloped, it not be steeper than the MC curve:

$$MB_n\left(n,\sigma^2\right) < MC_n\left(n,\sigma^2\right). \qquad \text{(a.12)}$$

This is ensured by the necessary 2nd order condition.

C. *Comparative Statics of Risk*

The comparative static effects of risk on the birth rate follow directly from differentiation of Equation (a.7):

$$\frac{\partial n}{\partial \sigma^2} = \frac{MC_{\sigma^2} - MB_{\sigma^2}}{MB_n - MC_n}. \qquad \text{(a.13)}$$

Now, the effect of risk on marginal costs in unambiguously negative:

$$MC_{\sigma^2} = \frac{\partial MC\left(n,\sigma^2\right)}{\partial \sigma^2} = -\frac{\partial R_n\left(n,\sigma^2\right)}{\partial \sigma^2} = -(1-\alpha)\gamma A^2\left(1-n\right)^{(1-2\alpha)} < 0. \qquad \text{(a.14)}$$

However, risk will increase or decrease the marginal benefit, depending upon the magnitude of ε:

$$MB_{\sigma^2} = \frac{\partial MB\left(n,\sigma^2\right)}{\partial \sigma^2} = \left(1-\varepsilon\right)\theta n^{\theta(\varepsilon-1)-1}\frac{\partial R\left(n,\sigma^2\right)}{\partial \sigma^2} = \left(1-\varepsilon\right)\theta n^{\theta(\varepsilon-1)-1}\frac{\gamma}{2}A^2\left(1-n\right)^{2(1-\alpha)}. \qquad \text{(a.15)}$$

If $\varepsilon > 1$, an increase in risk raises MB and therefore increases *n*. In the knife-edge case of log utility $\left(\varepsilon = 1\right)$ it will have no affect on MB and again, an increase in risk raises *n*.

For the case $\varepsilon < 1$ define

$$f(n) = MC_{\sigma^2} - MB_{\sigma^2} = (1-\varepsilon)\theta n^{\theta(\varepsilon-1)-1}\frac{\gamma}{2}A^2(1-n)^{2(1-\alpha)} - (1-\alpha)\gamma A^2(1-n)^{(1-2\alpha)} \qquad \text{(a.16)}$$

Note that $f(n)$ has the same sign as

$$g(n) = (1-\varepsilon)\theta n^{\theta(\varepsilon-1)-1}\frac{1}{2}(1-n) - (1-\alpha). \qquad \text{(a.17)}$$

Now $g(0) \to \infty$ since $\theta\left(\varepsilon-1\right)-1 < 0$ and $g(1) = -(1-\alpha)$. Finally

$$g'(n) = (1-\varepsilon)\theta\left[\theta(\varepsilon-1)-1\right]n^{\theta(\varepsilon-1)-2}\frac{1}{2}(1-n) - (1-\varepsilon)\theta n^{\theta(\varepsilon-1)-1}\frac{1}{2} < 0 \qquad \text{(a.18)}$$

This means that $g(n)$ is first positive and then negative. It follows that uncertainty reduces n for small initial levels of the birth rate and raises it for large initial levels.

D. Calibrated Examples

We want to appraise whether for an intertemporal elasticity of substitution less than unity, it is still possible that the stronger the uncertainty the larger the birth rate and the smaller the growth rate, even if there exists an effect through savings. We first calibrate a benchmark economy, using the following parameters: The production elasticity of labor is 60%, so $\alpha = 0.4$; the standard deviation of the production shock is set at 6% (close to the mean for the set of OECD countries in Gali (1994)). For the preferences parameters, we will consider a range of plausible values except for the rate of time preference, which is set at is set to 4%. The scale parameter *A* is set in order to get reasonable magnitudes for the birth rate and the economy growth.

Calibration parameters-benchmark

θ	**ε**	**γ**	**A**	**α**	**ρ**	**σ**
1.5	0.9	5	0.25	0.4	0.04	0.06

Starting values for the preferences parameters are set at plausible values. In particular, since the model can be reinterpreted as n being leisure, θ is set as the elasticity on leisure in utility which is 1.5 in the real business cycles literature (Cooley (1995)). Other values for these preferences parameters are investigated as well.

The results are as follows:

		n	μ_k	$\sigma^2{}_k$
θ =1.5 A=0.25	**ε =0.9; γ =5; σ =0.06**	9.30%	7.10%	0.00020011
	ε =0.91; γ =5; σ =0.06	7.32%	10.94%	0.00020539
	ε =0.9; **γ =30**; σ =0.06	9.29%	7.14%	0.00020014
	ε =0.9; γ =5; **σ =0.2**	9.10%	7.77%	0.01393393
θ =2 A=0.25	ε =0.9; γ =5; **σ =0.06**	12.29%	3.70%	0.00019224
	ε =0.9; γ =5; **σ =0.5**	12.02%	4.44%	0.01339842
θ =5 A=0.75	**ε** =0.9; **γ** =5; **σ =0.06**	33.92%	14.23%	0.00123176
	ε =0.9; **γ** =5; **σ =0.5**	32.09%	20.08%	0.08837987

For each value of θ, an increase in uncertainty always leads to a downwards shift on the MB curve. This in turn generates a smaller birth rate. Since the intertemporal elasticity of substitution is less than one, the mean growth rate is positively affected by uncertainty through the birthrate. In these numerical examples, this positive effect prevails over the negative one applying through consumption. These results contradict the ones obtained analytically for $\varepsilon \geq 1$.