Systematic monetary policy and persistence.

Luca Bindelli* Université de Lausanne

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Abstract

Woodford (1999 and 2003) has raised the theoretical possibility that in a standard, forward looking sticky price model, an independent channel of inertia might arise as a result of policy behavior. We analyze this assertion empirically, and estimate a standard model, in which the monetary authority is assumed to commit to an optimal rule. We contribute to the existing literature by identifying the purely policy induced persistence present in the model. We also analyze the role of the structural parameters reflecting policy preferences and price flexibility in altering the policy induced, as well as the overall persistence properties of the model. We find that such a model is able to replicate most of the data's moments. In constrast to previous empirical literature, lagged terms in both modelled Phillips and IS curves are found to be either insignificant or very small. Commitment policy alone can explain a substantial part of output persistence. While the pricing mechanism at the heart of this model helps transfer output persistence into inflation persistence, commitment policy manages to undo this link by undershooting the inflation target following a positive 'cost push' shock so that inflation persistence is slightly reduced compared to the discretionary policy case.

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*Contact address: Université de Lausanne, HEC, BFSH1, 1015 Lausanne (Switzerland). E-mail: luca.bindelli@unil.ch. I wish to thank Philippe Bacchetta, Andreas Fischer, Charles Goodhart, Jean Imbs, Eric Jondeau, as well as participants at the 9th Spring Meeting of Young Economists (Warsaw, April 2004), 21st Symposium on Banking and Monetary Economics (Nice, June 2004), 9th Annual Conference of SPiE (Lisbon, October 2004), 29th Simposio de Analisis Economico (Pamplona, December 2004) and SSES Annual Meeting (Zurich, March 2005) for comments and discussions. The usual disclaimer applies.

1 Introduction

In two important contributions, Chari, Kehoe and Mc Grattan (2000) and Huang and Liu (2002) conclude that the sticky price mechanism at the heart of recent New Keynesian macromodels is hardly able to generate any persistence beyond the exogenously assumed period of price stickiness. For reasonable parameter values used to calibrate their model, they demonstrate that monetary shocks have a limited effect on output in both impact and duration. Importantly, the Phillips curve underlying these models implies that the persistence properties of inflation stem mainly from that of marginal cost. Since output is one of marginal cost's main component, observed persistence in inflation could then hardly be explained by such models. This result has led researchers to look at mechanisms which could emphasize additional channels through which more persistent responses to monetary shocks might arise. In addressing this issue, some studies have emphasized labour market frictions. The idea backing these studies is to reduce the sensitivity of real marginal costs to output fluctuations.[1]

The general equilibrium literature cited above explores the impact of random policy shocks i.e. the unsystematic component of central bank's actions and its role in affecting the dynamic properties of inflation and output. However, as McCallum (1999a) convincingly argues, random monetary shocks account for a very small fraction of total policy instrument variability (with respect to the variability induced by systematic policy behavior). Indeed, it is likely that monetary policy behavior is hardly affected by an unsystematic component. Even more importantly, the way given shocks affect the economy is in general closely linked to the systematic behavior of the monetary authorities. In accordance with this view, a recent strand of literature has integrated endogenous policy making in this class of models. For example, the central bank optimizes its response to shocks on a day to day basis or chooses to behave according to an instrument rule, whereby it evaluates the current and possible future state of the economy and chooses to act accordingly (usually by choosing the short term interest rate as its instrument).[2] Such studies have also generally been confronted with the difficulties inherent to purely forward looking models in explaining the persistence in the data; they have consequently either added lagged terms in the equations reflecting private agent's decision rules, or assumed an inertial central bank objective to start with (i.e. an interest rate smoothing objective). The so-called backward looking behavior in private agent's decision rules is still a matter of considerable debate and this essay tries to shed some light on this issue. In an early theoretical analysis, Woodford (1999) has shown that the central bank might

[1] For example, Huang and Liu (2002) show that wage staggering is better at producing persistence. Christiano *et al.* (2001) confirm this result and show furthermore that the contribution of price staggering to overall persistence is very poor, so that assuming wage staggering performs nearly as well as when both staggering mechanisms are assumed.

[2] Typical instrument rules have mainly focused on the so called 'Taylor rule'.

be willing to optimally induce some inertia. If the central bank is precommited to a rule, its decision procedure will affect present and future private sector expectations. However, given the current state of the economy, an optimal plan will typically not allow the central bank's current decisions to depend exclusively on future events. Rather, as Woodford (2003) clearly puts it:

"Optimal policy must take account of the advantages of the anticipation of the policy at earlier dates; and for this reason it must generally be history dependent rather than purely forward looking. Past conditions should be taken into account in choosing the current policy setting, because it is desirable that people be able, at the earlier time, to count on the fact that the central bank will subsequently do so."

Accordingly, it is optimal for the monetary authority to respond to both contemporaneous and lagged shocks affecting the economy. This behavior will in turn introduce an additional and independent channel of inertia. In light of all this, it is quite surprising how little work has sought to analyze empirically this type of optimal policy rule in a standard New Keynesian model. We argue that a commitment to a policy rule might be a good approximation of central bank behavior when the period of analysis reflects a stable and credible policy regime. Since our benchmark economy is purely forward looking, the only assumed inherent source of persistence resulting from private sector behavior arises from serially correlated shocks to the economy. More precisely, we assume serially correlated preference and money demand shocks on the demand side and serially correlated 'cost push' shocks on the supply side. Such a choice is motivated by our willingness to focus on the role of monetary policy in generating persistence, without specifying any definite form of persistence that would initially arise from private sector behavior or decisions.[3] Importantly, as shown by Steinsson (2003) and Ireland (2004), the presence of supply shocks can be derived from the model's microfoundations.

Our main goal is to identify and isolate the role of monetary policy in affecting the dynamics of endogenous variables. In a relevant time period, we estimate the standard general equilibrium model outlined above, in which a central bank chooses to commit, and see whether it can match the broad characteristics of US data. As seen above, there is still a lack of consensus on the importance of backward looking behavior on the agent's decision behavior. Similarly, and because persistence might mistakenly be attributed to serially correlated shocks alone, another goal of the paper is to examine the quantitative importance of lagged terms on the structural relationships of the model. Finally, we estimate the importance of policy commitment in affecting persistence.

The standard model is shown to be a reasonably good description of the 1987-1999 period when the central bank is assumed to commit to a rule. On the whole, the persistence properties of the output and the interest rate are well reproduced, but the model generates slightly lower inertia in inflation than observed. Modelled output and inflation volatilities are close to their empirical equivalent. Interest rate

[3] Except for the assumed pricing mechanism.

volatility is however somewhat lower than in the data. Controlling for lagged terms in our model structural equations reveals that the backward looking behavior is not important if the central bank follows an optimal rule. Moreover, we find that an optimal commitment policy has a important role in generating large and persistent effects on output. However, this type of policy implies that in reaction to a supply shock, the central bank is willing to let output stay below its target level for a long period of time. Hence, it will generate a faster fall in inflation followed by a slight (but long lasting) undershooting of the inflation target: thus, inflation persistence is slightly lower than in the discretionary policy case. Finally, adding a smoothing parameter to the policy rule followed by the central bank is not necessary to reproduce the observed interest rate volatility and persistence.

In the next section, we briefly describe our framework and discuss how optimal commitment policy might induce inertia. We explore more in detail the persistence generating mechanism in section 3. We present our empirical methodology and the results in section 4. Section 5 concludes.

2 A Standard Model

In order to preserve the generality of our argument, we specify a framework that is now fairly standard in the literature. Following Walsh (2003), we consider a standard small scale New Keynesian economy in which prices are staggered *à la* Calvo, and the production function is linear in labour.[4] The price setting behavior is derived as the product of optimization by monopolistically competitive firms subject to constraints on the frequency of the price adjustment. Moreover, Calvo's partial adjustment rule stipulates that each period firms are allowed to adjusts their price with a fixed probability $1-\omega$. The central equations of our model economy are as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t + \varepsilon_{s,t}, \tag{1}$$

$$y_t = E_t y_{t+1} - 1/\sigma(i_{2t} - E_t \pi_{t+1}) + \varepsilon_{d,t}, \tag{2}$$

$$i_{2t} = i_t - \varepsilon_{i,t}, \tag{3}$$

$$mc_t = (\sigma + \eta) y_t, \tag{4}$$

$$\varepsilon_{s,t} = \rho_s \varepsilon_{s,t-1} + \xi_{s,t},\ \ \varepsilon_{d,t} = \rho_d \varepsilon_{d,t-1} + \xi_{d,t},\ \ \varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \xi_{i,t}. \tag{5}$$

[4] The main difference with Walsh (2003) is that in our setup, we add a shock to interest rate to the aggregate demand equation. In contrast, Woodford (2003) uses an interest rate shock, but has no preference shock as defined below.

$\beta < 1$ is a subjective discount rate and $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} > 0$ is interpretable as a price flexibility parameter. For instance, as the probability of adjusting price increases, so does κ. $\sigma > 0$ measures the inverse of the intertemporal elasticity of substitution of consumption while $\eta > 0$ measures that of labor supply. π_t is defined as the inflation rate, y_t the output gap, defined as the distance between the actual and natural level of output i.e. the output that would prevail if prices were fully flexible; mc_t is the deviation of real marginal cost from its steady state level and i_t is the nominal short term interest rate expressed as a deviation from its steady state value.[5] Equation (1) is the Phillips Curve resulting from Calvo's pricing rule; it describes the price adjustment mechanism and is akin to a traditional AS supply curve except for its forward looking component. Equation (2) represents the forward looking IS curve and can be derived from the traditional Euler condition along with the assumption that no investment is present in our closed economy model, so that consumption equals output. Equation (3) describes the relation between the central bank control variable i_t, and i_{2t}, defined as the nominal rate relevant to the economy. The latter can deviate from the interest rate set by the monetary authority up to a stochastic component, $\varepsilon_{i,t}$.[6] This type of behavior is widely believed to be consistent with real policy practice. For instance, in recent years, the US monetary policy has been conducted by controlling the Fed fund rate.[7] Under the assumption that labor market is competitive and output is linear in labor, a linear relation between real marginal cost and the output gap, as described by (4), emerges.[8] $\varepsilon_{d,t}$ is a shock to current demand and may be interpreted as a preference shock.[9] Steinsson (2003) and Ireland (2004) provide some theoretical foundations to the presence of a shock term in the Phillips curve. Both present models were the elasticity of demand for each intermediate good is assumed to be time-varying. In a monopolisic setup, and under the assumption that prices are fully flexible, price will be set equal to a markup above nominal marginal cost.[10] Since the markup is function of demand elasticity, the representative firm faces time varying monopoly power.[11] We can thus interpret $\varepsilon_{s,t}$ as a disturbance to the firm's desired markup. Throughout the paper, we refer to $\varepsilon_{s,t}$ as a 'cost push' shock, as defined in Galí *et al.* (1999). Finally, $\xi_{j,t}$, where $j = [i, d, s]$, are uncorrelated innovations,

[5]This steady state value is sometimes refered to as being the natural rate of interest, which is the rate compatible with a natural level of output (hence a zero output gap). Throughout our analysis, this rate is assumed to be constant.

[6]It can be interpreted as a shock to money demand or as a shock to the natural rate of interest.

[7]On the characterization of the Federal Reserve policy, see for example Goodfriend (1991) or Erceg and Levin (2003).

[8]We avoid the modelisation of the labor market which is somewhat difficult and controversial in order to preserve simplicity of exposition and focus on our primary interest.

[9]Ireland (2004) shows formally how such shock can be derived from a New Keynesian model microfoundations.

[10]In addition, we only consider representative firms, so that all firms will set the same price when they can adjust. The real marginal cost is expressed as the inverse of the markup.

[11]Whenever the probability of adjusting prices is lower than one, the actual markup will differ from the desired one.

normally distributed with mean zero and standard deviation given by $\sigma_j(\xi)$.

2.1 Monetary Policy

The problem of monetary policy is formulated as an optimal response to shocks hitting the economy. Assume furthermore that the monetary authority behaves optimally according to a *targeting* rule. Suppose for instance, that the central bank seeks to minimize the following loss function:

$$W_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i L_{t+i}, \tag{6}$$

and the loss at each period is given by,

$$L_{t+i} = \left[\chi(\pi_{t+i} - \pi^*)^2 + \lambda(y_{t+i} - y^*)^2 \right], \ for \ i \geq 0.$$

λ represents the weight the central bank places on output gap stabilization and χ the weight placed on inflation stabilization. π^* and y^* represent the target variables. We will assume that the target level for inflation is normalized to zero, while that of output is given by the natural level. The target for the output gap is thus also zero. For a standard sticky price economy such as ours, Woodford (2003) shows formally how the deviations of the expected discounted utility of the representative agent around the steady state level of utility are approximated by the above loss function.[12]

In what follows, two different assumptions about monetary policy behavior will be discussed: discretionary and commitment policy. The fundamental difference between these approaches lies in the central bank's ability to make a credible promise about its future actions. It will turn out, as presented more formally in the next section, that this distinction is crucial for the equilibrium dynamics of the model economy and its persistence properties. In addition, such distinction will enable us to isolate the overall effect commitment policy has on inertia.

2.2 Analytical Solutions

We present below the solutions under both discretionary and commitment policy. As in Walsh (2003), we use the Minimal State Variable (MSV) solution approach introduced by Mc Callum (1999b). Since our model economy is very similar to Walsh (2003) and Woodford (2003), derivations of the first order conditions and equilibrium dynamics are left to the appendix.[13]

[12] With a positive inflation target (6) can be simply viewed as the representation of the central bank's preferences. These may then not be compatible with welfare maximizing objectives.

[13] More precisely, our equilibrium path for inflation and output are identical, while that of the interest rate slightly differs from these authors.

2.2.1 Discretion

In order to better understand why a central bank might be willing to introduce some inertia in its response to shocks, it is useful to first present the situation in which the monetary authority acts on a discretionary basis. Because the decisions of the central bank are not binding for the future, and thus cannot affect agent's future expectations, discretionary policymaking requires that optimization takes place period by period taking initial conditions as given.[14] Accordingly, using the law of iterated expectations, the central bank optimization problem can be written as:

$$\min_{\{i_t,\pi_t,y_t\}} E_t \left\{ \begin{array}{c} \frac{1}{2}(\chi\pi_t^2+\lambda y_t^2) \\ +\gamma_t(y_t-y_{t+1}+1/\sigma(i_t-\pi_{t+1})-1/\sigma\varepsilon_{i,t}-\varepsilon_{d,t}) \\ +\varphi_t(\pi_t-\beta\pi_{t+1}-\psi y_t-\varepsilon_{s,t}) \end{array} \right\}, \tag{7}$$

where γ_t and φ_t are the Lagragians associated to the constraints, and $\psi=\kappa(\sigma+\eta)$. As already said, the central bank's control variable is the short term interest rate i_t. This control over i_t enables the monetary authority to set whatever short term rate it chooses, given optimal paths for π_t, in order to satisfy equation (2) above and achieve the desired value of y_t. As a consequence, the IS curve imposes no constraints on the central bank's behavior.[15] After combining the relevant first order conditions, we obtain:

$$\pi_t=-\frac{\lambda}{\chi\psi}y_t. \tag{8}$$

This condition describes the optimal targeting rule under discretionary policy. The central bank, by adjusting its instrument, ensures that that the target criterion (8) is satisfied at any time. This rule corresponds to the well known 'lean against the wind' policy, whereby a central bank contracts demand whenever inflation is above target. Suppose the central bank becomes more concerned about output volatility through an increase in λ, then for a given deviation of inflation from its target (here, zero), equation (8) tells us that it will let output react with a smaller magnitude, therefore decreasing output volatility.[16] The same will happen if prices become more sticky (ψ decreases). In this case, the central bank acknowledges that prices will adjust more slowly, so that inflation will return back to its target more slowly. For a given initial deviation of inflation from its target value, this means that a more persistent deviation from the target value will increase overall inflation volatility. Since the central bank reacts on a period by period basis, for inflation to return to its targeted value, output will need as well to deviate for a longer period of time from its target value. In addition, the central bank policy preferences (trade-off) does not change, hence it will accept an increase in inflation volatility only if the overall

[14] We are ignoring the possibility of reputational equilibria.

[15] This would not be true in case there are restrictions or costs attached to the variation of interest rates.

[16] The same reasoning applies to a decrease in χ. What matters here is the *relative* preference shift of the monetary authority.

output volatility decreases, i.e. output will deviate to a lower extent from its mean target value of zero in each period as reflected in (8). Replacing this policy rule in the structural equations yields the equilibrium output and inflation :

$$y_t = e\ \varepsilon_{s,t},\ \text{and}\ \ \pi_t = f\ \varepsilon_{s,t}.$$

where $e = -\frac{\chi\psi}{\lambda[1-\beta\rho_s]+\chi\psi^2} < 0$, $f = \frac{\lambda}{\lambda[1-\beta\rho_s]+\chi\psi^2} > 0$. The equilibrium short term interest rate is

$$i_t = g\ \varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t},$$

where $g = \rho_s f - (1-\rho_s)\,\sigma e$. Note from this equation that the equilibrium nominal interest rate varies in order to exactly compensate any demand shock (that is, either $\varepsilon_{i,t}$ or $\varepsilon_{d,t}$). Given that the monetary authority enjoys full credibility, and given that it entirely compensates any demand shock, the only reason for inflation and output to deviate from the targeted values is to face a 'cost push' shock which will generate a trade-off in the policy objective and thus allow for a temporary deviation of both inflation and output from targeted values. Equilibrium output and inflation dynamics are thus solely depending on 'cost push' shocks and the only source of persistence pertains exclusively to the shock process itself.[17] Accordingly, the pricing mechanism itself does not play any role in generating persistence. Reacting on a period by period basis, the central bank cannot manipulate private sector's expectations. This explains why neither structural rigidity parameters nor policy objectives parameters do play any role in affecting inflation persistence in this case.

2.2.2 Commitment

In this case, the central bank makes a stand as to its current and future behavior and sticks to it. The minimization problem is given by

$$\min_{\{i_{t+j},\pi_{t+j},y_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \begin{array}{c} \frac{1}{2}(\chi\pi_{t+j}^2 + \lambda y_{t+j}^2) \\ +\gamma_{t+j}(y_{t+j} - y_{t+1+j} + 1/\sigma(i_{t+j} - \pi_{t+1+j}) - 1/\sigma\varepsilon_{i,t+j} - \varepsilon_{d,t+j}) \\ +\varphi_{t+j}(\pi_{t+j} - \beta\pi_{t+1+j} - \psi y_{t+j} - \varepsilon_{s,t+j}) \end{array} \right\}. \tag{9}$$

and the monetary authority reacts according to the following rule:

$$\pi_t = -\frac{\lambda}{\chi\psi}(y_t - y_{t-1}). \tag{10}$$

This equation reflects the fact that the central bank wishes to implement a rule that links inflation to *variations* in the output gap.[18] This type of policy response

[17] The impulse response functions following a supply shock are simply given by $IRF^d_{y,\varepsilon_s}(n) = e\rho_s^n$, $IRF^d_{\pi,\varepsilon_s}(n) = f\rho_s^n$, and $IRF^d_{i,\varepsilon_s}(n) = g\rho_s^n$,where $IRF_{w,x}(n) = \frac{\partial w_{t+n}}{\partial x_t}$.

[18] The initial period ($t = 0$) targetting rule implies an identical behavior to the one obtained under discretionary policy.

reflects what Woodford (2003) calls *history dependence* of policy. By not making any commitment about future actions, a discretionary policymaker cannot, by definition, affect private sector expectations. Hence, the optimal response under discretion is to reduce y_t and then let future output gap values revert back to trend as inflation falls back to target. On the other side, making a commitment about current and future policy allows the central bank to directly manipulate private sector expectations. As apparent from equation (10), whenever actual inflation is above target value, the output gap is expected to decline.[19] Since inflation today depends on future output gap values, this has the immediate effect of dampening inflation.[20] Also, the central bank may then take advantage of this future dependence and manage to reduce the initial impact on inflation. But for the central bank to be able to bring exactly this kind of dynamic responses, it needs to make sure that the private sector will indeed understand but also believe in this policy. The central bank can credibly commit, hence manipulate expectations, when it calls on past economics conditions, which in turn depends on past policy actions. It is in this sense that monetary policy introduces *history dependence*. Accordingly, the monetary authority reacts not only to current 'cost push' shocks, but also to past shocks. Furthermore, past output gaps now affect current output gap and inflation.[21] The impact of variations of λ and ψ on policy rule (10) is very similar to that discussed in the discretionary case. The only difference lies in the fact that following a deviation of inflation from its target, it is the difference in output gap that will adjust to bring inflation back to target instead of the output gap level. Since such variations will alter the relationship between inflation and past output gap, both λ and ψ are critical to the determination of the policy induced persistence in the model. Equilibrium output and inflation are given by:

$$y_t = a y_{t-1} + b\varepsilon_{s,t}, \text{ and } \pi_t = c y_{t-1} + d\varepsilon_{s,t},$$

where $a = \frac{\left[\lambda+\lambda\beta+\chi\psi^2-\left((\lambda+\lambda\beta+\chi\psi^2)^2-4\beta\lambda^2\right)^{-1/2}\right]}{2\lambda\beta} > 0$, $b = -\frac{\chi\psi}{\lambda[1+\beta(1-a-\rho_s)]+\chi\psi^2} < 0$, $c = \frac{\lambda}{\chi\psi}(1-a) > 0$, and $d = \frac{\lambda}{\lambda[1+\beta(1-a-\rho_s)]+\chi\psi^2} > 0$. The equilibrium interest rate is now given by:

$$i_t = \Omega y_{t-1} + \Gamma\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t},$$

[19] Similarly, an inflation rate below target value will require the central bank to let the output gap grow through time.

[20] We will see that this means commitment achieves a better outcome in terms of (6).

[21] In Clarida, Galí and Gertler (1999), the authors find a rule under commitment policy that is very similar to equation (8), except that the coefficient on output is bigger. In their paper, Clarida *et al.* (1999) restrict the class of rules under analysis to the general form that arises under discretion and look for the optimum within this class of rules. Put differently, they constrain the policymaker to react only to contemporaneous supply shocks. They find that, conditional on the class of rules considered, that reaction to inflationary pressures is bigger under the commitment case. The difference is that they do not consider a global (unconstrained) optimum.

where $\Omega = a[\sigma(a-1)+c]$ and $\Gamma = [\sigma b(a+\rho_s-1)+(cb+\rho_s d)]$. As long as $a > 0$, inertia in the interest rate, inflation and the output gap is now present. Thus, in the case of commitment, the monetary authority introduces a separate and independent channel through which inertia might arise. This is true even if there is no inherent source of persistence in the model i.e. if $\rho_s = 0$. Because the central bank lets past output affect current inflation through (10), price stickiness as well as policy preferences have a role to play in determining the persistence properties of inflation, output, and the interest rate. Note that the parameter a refers only to the fact that the central bank commitment implies dependence upon past economic conditions as reflected by the presence of lagged output gap (i.e. a is zero under discretionary policy).

We have seen in this section that a unique stable MSV equilibrium exists under both optimal discretionary and commitment policy. Our assumptions presuppose that the monetary authority is able to enforce either (8) or (10) in order to reach its desired objective without explicitly specifying its implementation through an interest rate setting rule. This issue will however not be considered in our presentation since it is extensively discussed in Woodford (2003).

3 Inertia and the role of policy commitment

While the dynamics of the New Keynesian model have been already widely studied, we do not have, to our knowledge, studies focusing on the structural characterization of inertia due to policy alone (i.e. inertia as a function of the model's structural parameters). The presence of commitment and the related concept of *history dependence* affect the inertial properties of the macro variables. We have seen that such properties will not remain unaltered if, for example, the monetary authority becomes more averse to inflation volatility or if prices become more flexible. Thus, it might be interesting to know how structural parameters determine inertia in the presence of commitment. We propose first to characterize the endogenous variable's paths following a supply shock (the only shock involving a trade-off in the policy objective function). We address then the issue of structural inertia analysis by looking at the model's autocorrelation functions of inflation, output and the interest rate.

3.1 Impulse responses

In this section, we compute the theoretical impulse responses under commitment to a 'cost push' shock resulting from the solution above.[22] We obtain for the output gap:

$$
\begin{aligned}
IRF^{c}_{y,\varepsilon_s}(n) &= b, \qquad for\ n = 0, \\
&= b\left(a^n + \sum_{j=1}^{n} a^{n-j}\rho_s^j\right), \qquad for\ n \geq 1.
\end{aligned}
$$

The inertial properties of the output gap process are entirely determined by the evolution of λ, χ, ψ, and ρ_s.[23] The impulse response simply tells us that the higher a, the higher the inertia in the output gap process. Furthermore, there is no need for 'built in' inertia within the shock process to generate gradually decaying impulse responses. Thus, commitment generates an amplified ($b > e$, since $a > 0$) and more persistent output response with respect to the case of discretionary policy. In the next section, we will discuss in more detail how deep structural parameters alter a. The impulse response for inflation is:

$$
\begin{aligned}
IRF^{c}_{\pi,\varepsilon_s}(n) &= d, \qquad for\ n = 0, \\
&= cb\left(a^{n-1} + \sum_{j=1}^{n-1} a^{(n-1)-j}\rho_s^j\right) + d\rho_s^n, \\
&= c\ IRF^{c}_{y,\varepsilon_s}(n-1) + d\rho_s^n, \quad for\ n \geq 1.
\end{aligned}
$$

Finally, the impulse response of the interest rate is given by:

$$
\begin{aligned}
IRF^{c}_{i,\varepsilon_s}(n) &= \Gamma, \qquad for\ n = 0, \\
&= \Omega b\left(a^{n-1} + \sum_{j=1}^{n-1} a^{(n-1)-j}\rho_s^j\right) + \Gamma\rho_s^n, \\
&= \Omega\ IRF^{c}_{y,\varepsilon_s}(n-1) + \Gamma\rho_s^n, \quad for\ n \geq 1.
\end{aligned}
$$

The inertial properties of inflation and interest rate following a supply shock depend on the persistence parameter of the supply shock as well as policy preferences and price flexibility through the parameter a. Since we are interested in isolating the monetary policy effect on the variables dynamics, we present below the impulse response to a supply shock that would arise if supply shocks to the economy were i.i.d.:

[22] Because of the compensation by policy of any demand shock, output and inflation only react to supply shocks. The compensation operates through interest rate movements. This can be seen from the impulse response functions of interest rate to interest rate and preference shocks that are given by $IRF^{c}_{i,\varepsilon_i}(n) = -\rho_i^n$ and $IRF^{c}_{i,\varepsilon_s}(n) = \sigma\rho_d^n$ respectively. Note that both are indentical under discretionary or commitment policy regime.

[23] Recall that the first three structural parameters determine a. Note that although the same parameters also enter in b, the latter only has an initial level effect on the impulse response function.

$$
\begin{aligned}
IRF^c_{0,y,\varepsilon_s}(n) &= b_0, \qquad for\ n=0, \\
&= b_0 a^n, \qquad for\ n \geq 1.
\end{aligned}
$$

$$
\begin{aligned}
IRF^c_{0,\pi,\varepsilon_s}(n) &= d_0, \qquad for\ n=0, \\
&= c b_0 a^{n-1}, \\
&= c\ IRF^c_{0,y,\varepsilon_s}(n-1), \quad for\ n \geq 1.
\end{aligned}
$$

$$
\begin{aligned}
IRF^c_{0,i,\varepsilon_s}(n) &= \Gamma, \qquad for\ n=0, \\
&= \Omega b_0 a^{n-1}, \\
&= \Omega\ IRF^c_{0,y,\varepsilon_s}(n-1), \quad for\ n \geq 1.
\end{aligned}
$$

Where subscripts $_0$ denote parameter values accounting for $\rho_s \rightarrow 0$. Through the presence of policy commitment, inflation and interest rate impulse responses following a supply shock are now entirely determined, up to a scaling factor, by the output gap impulse response from the previous period (for $n \geq 1$). Under discretionary and commitment cases, we have plotted in Figure 1 the impulse response functions of output, inflation and the interest rate following a supply shock. Parameter values are set to: $\psi = 0.05$, $\lambda = 0.2$, $\rho_s = 0.3$, $\sigma = 2$, and $\beta = 0.99$. Finally, we set $\chi = 1$ so that λ can readily be interpreted as the relative weight a central bank places on output stabilization. Similarly, in order to visualize the impact of the sole monetary policy commitment effect, the figures present the impulse responses under the assumption of i.i.d supply shocks (labelled 'MP induced'). Inflation's impulse response display an initial positive impact that gradually declines and turns into a long lasting deflationary episode (undershooting of inflation target) before going back to its initial value. Also, the supply shock generates an immediate fall in output as expected from policy rule (10). If the central bank precommits to a given policy rule, it will keep output below its potential level for several periods after the shock. This is achieved by an immediate raise in the interest rate that progressively decrease and turns into a long lasting but small decrease in the interest rate. Private agents understand this and believe that the central bank will indeed do so, and thus will incorporate this fact into their current expectations about inflation. Future expected inflation is lower and a slight deflation is induced.[24] Note that if the supply shock is not serially correlated,

[24] On the other hand, inducing a small undershooting of the target inflation rate in the case of discretionary policy is not credible for private agents because after the initial shock, the agents understand that the central bank only reacts to contemporaneous output gap movements and consequently is not willing to induce 'undershooting' in the future. That it why this type of policy is considered as suboptimal with respect to commitment. In the literature, this relative inefficiency has been termed the *stabilization bias* (see Woodford (2003) for example).

so that all persistence in the model stem from policy commitment alone, the interest rate does not increase initially. Knowing that the shock will die out next period, an initial negative impact of the supply shock on the output gap is sufficient to generate expectations of a deflation during the following period, so that the central bank might simply dampen the deflationary effect by slightly (and initially) lowering the interest rate. The possibility that a positive serially correlated supply shock might keep future inflation above its targeted value implies a positive and stronger interest rate response.

In order to clarify the way interest rates affect endogenous variables, note that despite the fact that the central bank uses the current short term interest rate as its instrument, output and inflation also respond to variations in the future short term rates rates. Accordingly, we first rearrange and iterate equation (2) forward to obtain:

$$y_t = E_t y_\infty - 1/\sigma \sum_{j=0}^{\infty} E_t(i_{t+j} - \pi_{t+1+j}) + \sum_{j=0}^{\infty}[\rho_d^j \varepsilon_{d,t} - 1/\sigma \rho_i^j \varepsilon_{i,t}], \qquad (11)$$

where $E_t y_\infty$ can be interpreted as the long run output gap under the policy regime in question. Assuming that this long run output gap equals its targeted value of zero, we can see that aggregate demand, not only depends on current short term rates, but on expected future rates as well. As seen earlier, because of policy commitment, a channel through which private sector expectations are affected by current policy actions is introduced. Clearly then, the way in which commitment policy should alter inflation and output dynamics runs mostly through the private sector's expectations about future interest rates. Such exact relationship may sound somewhat unrealistic, but one should not forget that our setup allows for deviations from the central bank controlled interest rate, as represented by shocks on the monetary control variable. Besides, the fact that the private sector effectively anticipates future movements in the monetary instrument is both intuitive and realistic.[25] As shown in (11), small but persistent change in current short term rate can influence future interest rates. If agents believe that moderate adjustments of short rates could have a significant and credible impact on future ones, achieving a given stabilization goal without inducing too much volatility in the short term rates would seem possible. Hence, compared to a discretionary policy case, the central bank's impact on interest rates will lessen after a given shock, because private agents understand and believe that it can take advantage of its possibility to affect future outcomes. Accordingly, the central bank manages to smooth the interest rate movements so that stabilization of inflation and output achieves better results in terms of equation (6).

[25] As an alternative interpretation, one may think also of an implicit term structure that links short to long term rates through future expected short term rates values.

3.2 Persistence

In the appendix, we derive the autocorrelation functions for $y_t = ay_{t-1} + b\varepsilon_{s,t}$, $\pi_t = cy_{t-1} + d\varepsilon_{s,t}$, and $i_t = \Omega y_{t-1} + \Gamma\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}$ respectively. In this section, we focus on the persistence of the variables represented by the first order autocorrelations. These are given by:

$$\rho_y^c(1) = \frac{\rho_s + a}{1 + a\rho_s}.$$

$$\rho_\pi^c(1) = \left\{b^2c^2\left[\rho_s + a\right] + d^2\rho_s\left(1 - a^2\right)\left(1 - a\rho_s\right) + bcd\left[\left(\rho_s^2 + 1\right)\left(1 - a^2\right)\right]\right\} \left[c^2b^2\left(1 + a\rho_s\right) + d^2\left(1 - a^2\right)\left(1 - a\rho_s\right) + 2bcd\rho_s\left(1 - a^2\right)\right]^{-1}$$

$$\rho_i^c(1) = \left\{\begin{array}{c} \{b^2\Omega^2\left[\rho_s + a\right] + \Gamma^2\rho_s\left(1 - a^2\right)\left(1 - a\rho_s\right) \\ +b\Omega\Gamma\left[\left(\rho_s^2 + 1\right)\left(1 - a^2\right)\right]\}\sigma_{\varepsilon_s}(0) \\ +\left(1 - a^2\right)\left(1 - a\rho_s\right)\left(\rho_i\sigma_{\varepsilon_i}(0) + \sigma^2\rho_d\sigma_{\varepsilon_d}(0)\right) \end{array}\right\} \left\{\begin{array}{c} \left[\Omega^2b^2\left(1 + a\rho_s\right) + \Gamma^2\left(1 - a^2\right)\left(1 - a\rho_s\right) + 2b\Omega\Gamma\rho_s\left(1 - a^2\right)\right]\sigma_{\varepsilon_s}(0) \\ +\left(1 - a^2\right)\left(1 - a\rho_s\right)\left(\sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0)\right) \end{array}\right\}^{-1}$$

Note that output and inflation persistence are independent of the shocks' volatility, while that of interest rate depends on $\sigma_{\varepsilon_s}(0)$, $\sigma_{\varepsilon_i}(0)$ and $\sigma_{\varepsilon_d}(0)$. This should come as no surprise. As a matter of fact, since we know that demand shocks are entirely compensated by variations in the interest rate under the optimal policy, output and inflation are left unaffected.

What is the amount of inertia induced by commitment policy? In the general case where shocks are assumed to be serially correlated, it is relatively easy to identify the source of persistence stemming from policy commitment alone. If policy is discretionary, the equilibrium outcome only depends on shocks the only source of inertia present in the model. Comparing the persistence properties of this equilibrium with those obtained under commitment (for the same structural parameter values), allows us to identify the contribution of policy to overall persistence.[26] In the limiting case where all shocks are i.i.d.(i.e. $\rho_j \to 0$, for $j = s, d, i$), we can interpret the autocorrelation functions for both y_t and π_t as arising exclusively from monetary policy behavior. Accordingly, equations (11) and (12) can be simplified :

$$\lim_{\rho_j \to 0} \rho_y^c(1) = a \tag{12}$$

[26] From equilibrium solutions above, we have in the discretionary case: $\rho_j^d(1) = \rho_s$ where $j = [\pi, y]$, and $\rho_i^d(1) = \left[g^2\rho_s\sigma_{\varepsilon_s}(0) + \rho_i\sigma_{\varepsilon_i}(0) + \sigma^2\rho_d\sigma_{\varepsilon_d}(0)\right]\left[g^2\sigma_{\varepsilon_s}(0) + \sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0)\right]^{-1}$.

$$\lim_{\rho_j \to 0} \rho_\pi^c(1) = \frac{b_0^2 c^2 a + b_0 c d_0 (1 - a^2)}{c^2 b_0^2 + d_0^2 (1 - a^2)}, \quad (13)$$

$$\lim_{\rho_j \to 0} \rho_i^c(1) = \frac{\left[b^2 \Omega^2 a + b \Omega \Gamma (1 - a^2)\right] \sigma_{\varepsilon_s}(0)}{\left[\Omega^2 b^2 + \Gamma^2 (1 - a^2)\right] \sigma_{\varepsilon_s}(0) + (1 - a^2)\left(\sigma_{\varepsilon_i}(0) + \sigma^2 \sigma_{\varepsilon_d}(0)\right)} \quad (14)$$

where Γ_0 , $b_0 < 0$ and $d_0 > 0$ denote respectively Γ, b and d with $\rho_j \to 0$. We move next to the ability of several structural parameters to alter inertia in the model.

3.3 The role of policy preferences and price flexibility when commitment is the only source of persistence

We have displayed in Figures 2 to 9, reduced form parameters a, b_0, c, d_0, Ω and Γ_0 and theoretical output gap, inflation and interest rate persistence (still defined as the first lag correlation) under the assumption that shocks are i.i.d..[27] We let λ and ψ vary within $[0.01; 0.6]$ and $[0.01; 0.2]$ respectively. As it can be seen from Figure 2, the autoregressive parameter a, which also represents output gap persistence, is increasing in λ and decreasing in ψ. Recall from equation (10), that as λ approaches zero, there is no role left for output stabilization and consequently no role either for responding to past shock. That is why, on impact, a small but positive λ takes us away from a 'corner solution' and increases output persistence drastically. By definition, with i.i.d. shocks, the parameters b_0 and d_0 only govern the initial level impact after a supply shock. Figures 3 and 5 show that b_0 is decreasing (in absolute value) and d_0 increases whenever the central bank increases its aversion toward output variability or prices become more sticky. While this result is fairly intuitive in the case of variations in λ, variations in ψ imply somewhat subtler dynamic responses in equilibrium. Both results reflect partly our discussion about the implementation of the targeting rule.[28] The slight difference results from the distinction between a comparative analysis with respect to a given shock and a given deviation of inflation from its target value. Consider for instance a decrease in ψ. As prices become more sticky, inflation should fall back on its target value more slowly. We have seen that this implies that the central bank will let output remain below its target value for a longer period of time. Knowing this, what should firms do in response to a given supply shock? In our setup, the firm is forward looking and sets its price taking into account all future expected movements in the output gap.[29] Since firms expect that the output gap will remain below its trend for a longer period of time, the firms that

[27] We keep the same values as before for the remaining structural parameters, i.e. $\sigma = 2$, $\chi = 1$ and $\beta = 0.99$.

[28] We also discuss this below when plotting impulse response functions for an altenative λ and ψ.

[29] This can easily be seen by iterating equation (1) forward. Remember that in our setup the marginal cost is a linear function of the output gap.

are able to adjust their price in the initial period will respond by increasing their price more strongly to compensate for future decreases in inflation due to negative output gaps. This is why inflation increases more on impact if prices are more rigid. Hence d_0 increases. The central bank is still willing to tolerate such increase in inflation by mitigating it with a smaller initial shock on the output gap (b_0 decreases in absolute value). As it is apparent from Figure 4, for a given level of price rigidity, the contribution of past output gap movements to inflation variability, as measured by c, is always positive in λ. Price rigidity generally adds to output driven inflation variability, but not always though. As one approaches full price rigidity, the central bank is ready to let output stay below the target level for a very long period of time after a shock, because firms have a reduced probability of adjusting their prices. This effect raises output driven inflation variability. We term this effect 'duration effect'. On the other side, a lower shock on impact occurs on the output gap process because policy preferences did not change, and this relatively decreases output driven inflation variability. We call this 'magnitude effect'. This shock is also smaller the higher the preference for output stabilization because the central bank implicitly accepts more inflation volatility. That is why c decreases faster in ψ the higher λ. Accordingly, output driven inflation variability increases up to the point where the 'duration effect' is outweighed by the 'magnitude effect' as implied by the presence of monetary policy commitment. Figure 6 shows that the contribution of past output gap movements to interest rate variability, measured by Ω, can be either positive or negative depending on λ and ψ. When preference for output stabilization is sufficiently large, the interest rate reacts positively to past output movements. This reflects a stronger stabilization role of interest rates on output gap variations. On the contrary, as preferences shift toward inflation stability, the interest rate may correlate negatively with past output movements. The central bank may become so concerned about inflation volatility that it accepts additional output gap volatility by letting the output gap to further deviate from its target. As already shown by Figure 1 above, Γ_0 is generally slightly negative (see also Figure 7). However, if the monetary authority is strongly concerned about inflation volatility (λ is low), then it might react positively to a supply shock. Figure 8 shows that the first lag theoretical inflation autocorrelation is always negative but increasing in both λ and ψ. The central bank thus induces sufficiently strong negative correlation between output and inflation to undo the positive impact of output serial correlation on inflation persistence. In other words, despite the fact that the output process contributes positively to inflation inertia through c, the policy response implying a target 'undershooting' might completely eliminate the previous positive correlation, possibly reporting much of observed inflation persistence on the supply shock correlation alone.

Finally, we wish to quantify the inertia in the interest rate that is due to commitment policy alone. Since supply and demand shocks volatilities determine the level of interest rate inertia, we need to specify their values. We set $\sigma_{\varepsilon_i}(0) = \sigma_{\varepsilon_d}(0) = 0$ to start with so that interest rate inertia is reduced to:

$$\lim_{\rho_j \to 0} \rho_i^c(1) = \frac{b^2\Omega^2 a + b\Omega\Gamma\left(1-a^2\right)}{\Omega^2 b^2 + \Gamma^2\left(1-a^2\right)}$$

In the absence of demand shocks, the interest rate inertia would only depend upon structural parameters and is independent of supply shock volatility. The resulting interest rate inertia is illustrated by Figure 9. Interest rate persistence is, as for inflation and output, increasing in λ and decreasing in ψ. However, the presence of demand shocks is likely to decrease sharply commitment induced interest rate inertia. We have also plotted interest inertia, as defined by (14), in function of $\sigma_{\varepsilon_i}(0)$ and $\sigma_{\varepsilon_d}(0)$ and let them vary within $[0.01; 1]$ (figure not presented here). To this end we have set $\sigma_{\varepsilon_s}(0) = 0.1$, $\lambda = 0.2$ and $\psi = 0.05$. Results indicate that commitment induced inertia remains positive but falls to negligible level (arbitrarily close to zero), attributing much of the interest rate inertia as shock driven. We know that the central bank entirely compensates any demand shock irrespective of any commitment to a intertemporal policy rule. Hence, only the demand shock's dynamics determines the response of the interest rate. If demand shock volatilities increase relative to that of supply shocks, interest rate inertia is largely dominated by the three shocks persistence. If shocks persistence is brought to zero, we should expect interest rate inertia to decrease, eventually reaching the zero bound.

To better understand the mere effect of commitment on equilibrium dynamics through ψ and λ when the economy faces a supply shock, we have plotted in Figures 10 and 11 the impulse responses of output and inflation for different values of ψ and λ (using the same parameter values set in Figure 1). By expressing λ relative to $\chi = 1$, we can identify λ and ψ as the two central parameters governing the dynamics of our endogenous variables.

First, consider in Figure 10 the case where the central bank precommits and prices become relatively inflexible ($\psi = 0.01$). The higher price rigidity requires output to deviate from its target for a longer period of time in order to generate disinflation. Because a reduced proportion of firms manage to adjust their price in each period, the central bank knows it will take more time to affect prices and hence inflation. Since the policy preference parameter did not change, we know from our discussion about Figures 3 and 5 that a bigger shock on impact affect inflation while a smaller one affects output. In other words, the largest part of a given supply shock is absorbed by inflation. The central bank will also increase interest rates in a more pronounced way, both because the initial impact on inflation is bigger and because prices are more sticky. Output persistence increases and, through the Calvo pricing mechanism, so does inflation persistence. However, the stronger initial impact of the supply shock on inflation is matched by a higher response in interest rate meaning that initial inflation shoot-up is rapidly turning into a deflation as in Figure 1. This fall will certainly decrease inflation persistence. However, as we have seen in Figure 8, this decrease is not sufficient to decrease overall inflation persistence due to commitment (for the same chosen parameter values). Interest rate inertia increases as well (Figure 9).

Next, in Figure 11, we have pictured an increase in the parameter λ. In this case, the monetary authority not only reduces its response to the initial shock on impact but responds more strongly to variations in the output gap. Policy responses are thus milder, in the sense that interest rate variations are dampened. This makes the output gap more persistent. In turn, due to the Calvo pricing mechanism, price level changes are reduced in response to smaller expected movements in output, implying relatively more persistence in inflation, as confirmed again by Figure 8.

4 Results

4.1 Econometric Methodology

The assessment of the model performance and the estimation of the structural parameters rely on the approach suggested by Söderlind (1999). Our model, and more generally, any linear rational expectation model under optimal policy, can be integrated into a setup evolving according to:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0} \end{bmatrix}, \qquad (15)$$

where x_{1t} is a vector of n_1 predetermined variables (x_{10} is given), x_{2t} is a vector of n_2 non predetermined variables (forward looking), u_t a vector of k policy instruments and ξ_{t+1} an n_1 vector of innovations to x_{1t}. Our policy objective can be expressed as

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t), \qquad (16)$$

where $x_t = [x_{1t}\ x_{2t}]'$, and the matrix Q (symmetric) is a function of the structural parameters. The solution to the above problem can be cast into a state space model which can be directly estimated via maximum likelihood. The procedure is the following: A first guess of the parameter vector is used in the solution algorithm. This gives rise to a system of linear difference equations that can then be framed into a state space model in which a Kalman Filter is then used to build up a likelihood function. We then iterate over the entire parameter space to find the parameter vector maximizing the likelihood.[30] We use data found on the FRED database of the Federal Reserve Bank of St. Louis. It consists of quarterly data for the period from 1987Q4 to 1999Q4.[31] The inflation rate is measured as the annualized quarterly change in the GDP deflator and the output gap is the percentage deviation of real GDP from

[30] Given that data for three series are used, at least three shocks need to be specified in order to avoid a singular covariance matrix when calculating the likelihood. The application to our benchmark model economy is briefly discussed in the appendix.

[31] We choose this sample because it excludes the disinflationary period in the early 1980's and is characterized by a stable monetary policy regime.

potential GDP as calculated by the Congressional Budget Office. Finally, the interest rate is the annualized 3 month T-bill rate, which is calculated as the average of daily rates (we use the deviation from the unconditional mean in our estimations). All series are seasonally adjusted except for the 3 Month T-bill series (see Figure 12).

4.2 Model Estimation

Table 1 summarizes our estimation results under optimal commitment policy for the system of equations as given by (1)-(6). Note that in all our estimations, we have constrained $\chi = 1$, so that λ is the weight the central bank places on output stability relative to inflation. All our parameter estimates are found to be significant. Our estimate for β is slightly lower than conventional values but still well within theoretical bounds. As is well known, the literature does not reach firm conclusions as to the value of $1/\sigma$. However, it is generally calibrated between zero and one depending on the studies whereas $1/\eta$ is generally calibrated around 3. In our setup, σ is estimated as being close to two and η near 0.5. Thus, our estimates can also be considered as compatible with standard calibrated values. χ is estimated close to one, which is a common assumed value in the literature. A value of 0.182 for λ suggest that the Federal Reserve policy has responded to output gaps movements in a moderate way throughout our period of analysis. This estimate is within usual values found in the literature, which generally range between 0.1 and 0.25. Concerning the autoregressive parameters of the shocks, we find relatively persistent supply shocks (root of the AR process is 0.697). Interest rate shocks are moderately persistent (0.522), while preference shocks are very persistent (0.946). The value of ω, which reflects the probability of leaving the price unchanged in pricing equation (1), implies that the estimated average time between price changes is roughly six quarters. Concerning the structural parameters of the Phillips curve, $\psi = \kappa(\sigma + \eta)$ is estimated to be approximately equal to 0.085. This figure stands in contrast with some previous partial equilibrium results on the Phillips curve for the US when output gap is used as the main driving variable of inflation.[32] This suggests that a linear relationship between the labor share and the output gap may well be a reasonable assumption. Turning now to the reduced form solution parameter estimates, we have:

$$y_t = 0.828\ y_{t-1} - 0.887\ \varepsilon_{s,t} \tag{17}$$

$$\pi_t = 0.369\ y_{t-1} + 1.899\ \varepsilon_{s,t} \tag{18}$$

$$i_t = 0.025\ y_{t-1} + 0.08\ \varepsilon_{s,t} + 1.969\ \varepsilon_{d,t} - \ \varepsilon_{i,t}. \tag{19}$$

[32]For instance, Galí and Gertler (1999) find a negative parameter estimate for the contemporaneous output gap.

The autoregressive root for the output gap is quite large at roughly 0.828. This is suggestive that the presence of commitment policy (through a) may indeed strongly affects the inertial properties of output. For the equilibrium inflation rate, we see that the coefficient on lagged output is estimated at 0.369, which implies that, due to the presence of a commitment policy, a non negligible component of output variability may be transferred to inflation. In the same time, the inflation rate reacts strongly to shock variations (coefficient is 1.899), this reflects the strong response of monetary policy to inflation variability following 'cost push' shocks. The equilibrium interest rate is only modestly affected by the lagged output gap and 'cost push' shocks, more so by both demand shocks (recall that these are exactly compensated by the central bank). This indicates that the central bank only needs to make modest changes in the interest rate to generate substantial movements in future output.

In Figure 13 are displayed the estimated impulse responses for inflation, the output gap and the interest rate following a supply shock (using Table 1 parameter values). The central bank increases slightly the interest rate for the first 3 to 4 quarters, and drives the output gap further down as it helps reducing the initial impact on inflation. As the central bank slowly decreases the interest rate, inflation expectations are lower and private agents expect a slight but long lasting undershooting of the inflation target since the output gap only smoothly returns to its initial level.[33] Using a smoothing algorithm, we also have estimated the supply shocks that have hit the economy during that period. The supply shock series is presented in Figure 14. The positive 'cost push' shock period from the second quarter of 1990 to the end 1991 is probably due to the burst of the war in Kuwait. In contrast, the end of the period is characterized by negative 'cost push' shocks. We believe that throughout this very same period, increasing industry competitiveness, mostly in technology intensive sectors, may have indeed produced downward shocks to the desired markups.

4.3 The Importance of 'Built in' Inertia

Before addressing the issue of the quantitative contribution of monetary policy to the overall persistence, we wish to discuss an important issue, still matter of considerable debate. As a matter of fact, additional 'built-in' inertia such as backward looking pricing behavior, is not included in our benchmark setup.[34] Consequently, the only

[33] It is certainly defensible to contend that inducing a slight but long lasting deflation in response to a supply shock is relatively unlikely to happen in the real world, even though our estimated magnitude of deflation is small. After a supply shock, inflation peaks at roughly -0.2 after 10 quarters. Note however that the average inflation rate for the period 1987-1999 is roughly two percent. If we consider that this number is sufficiently close to the true target value over the period and the model still is a good approximation in such 'neighborhoods', then the validity of the argument might be attenuated. This would simply reduce inflation to 1.8 instead (i.e. a disinflation would occur).

[34] The presence of a lagged inflation term on the right hand side of (1) is not necessarily reflective of backward looking price setting. A standard rational forward looking "Taylor type" pricing contract

sources of inertia arise from correlated disturbances to the economy. However, because other sources of inertia, stemming from private sector behavior, are not present in our benchmark forward looking setup, we may mistakenly attribute variable dynamics to correlated shocks whereas those are driven by other frictions not present in our model economy. For example, several authors have argued that a lagged inflation term should enter the forward looking Phillips curve which is unable to capture the fact that inflation is highly persistent (see Fuhrer and Moore (1995)). A similar justification has been pointed out for the presence of a lagged output term in the standard IS curve. The contribution of such inertial behavior can be evaluated by specifying an aggregate supply and demand containing lagged terms. To this purpose, we replace (1) and (2) by:

$$\pi_t = \beta \left[\alpha_\pi \pi_{t-1} + (1-\alpha_\pi) E_t \pi_{t+1}\right] + \kappa mc_t + \varepsilon_{s,t},$$

$$y_t = \alpha_y y_{t-1} + (1-\alpha_y) E_t y_{t+1} - 1/\sigma (i_{2t} - E_t \pi_{t+1}) + \varepsilon_{d,t}.$$

The resulting model economy has been estimated under the assumption of an optimal commitment policy. The results are presented in Table 2. Strikingly, virtually no backward looking pricing behavior is detected. α_π is very small and not significantly different from zero. α_y is found significant and slightly higher at 0.047, but still very small. As a consequence, other parameter estimates are left almost unaffected by the presence of lagged terms in the Phillips and IS curves, suggesting that forward looking behavior is predominant on both demand and supply sides. In a similar setup, Ireland (2004) models the central bank behavior as a modified Taylor rule. His results for the post 1980 period also show that lagged terms parameter estimates are not significant.[35] In order to check for the stability of the results displayed in Table 2, we present in Table 3 and 4 the estimations results constraining in turn $\alpha_\pi = 0$ and $\alpha_y = 0$. If we force $\alpha_\pi = 0$, we find negligible changes in the parameter estimates. Only supply shock innovation volatility slightly increases. α_y estimates remain virtually identical, and the overall fit is even slightly improved (as evident from the LogLikelihood measure). In Table 4, we see that forcing the lagged output term to zero slightly decreases the LogLikelihood of the model. The lagged inflation parameter estimate is now higher, but still not significant at a 5% confidence level.

Our evidence thus suggest that frictions such as backward looking pricing behavior should not alter significantly the model variables, in particular their persistence properties. Hence, we will rely on our benchmark, forward looking model economy as a valid representation of the data. All in all, the inertia present in our economy can be consistently attributed either to monetary policy or to correlated shocks in the economy alone.

model yields precisely a Phillips curve in which lagged inflation terms appear.

[35] On the other hand, using optimal discretionary policy, Söderström *et al.* (2003) argue that backward looking behavior is needed in the Phillips curve, but that the forward looking IS curve is a good approximation of actual behavior.

4.4 Shocks vs. Policy Induced Persistence

In trying to assess further the model performance at replicating volatility and persistence features of the data, we have computed some unconditional moments for inflation, the output gap and the short term interest rate resulting from the estimates obtained in Table 1.[36] The results are presented in Table 5. The estimated standard deviation for inflation is very close to that displayed by the data. The first autocorrelation is relatively close to its empirical counterpart and within a two standard errors band. Output persistence and volatility are well accounted for as they lie generally within one standard error above the estimated moments of the data. The unconditional moments for the short term interest rate are well reproduced. The estimated volatility is however slightly lower. Accordingly, and contrary to what has generally been argued, adding a term for a change in the interest rate in the central bank's objective function is not necessary to produce the low volatility found in the data when the central bank is committed to a rule. Overall, the model performs relatively well, but has difficulties at replicating the third lag autocorrelation in inflation. In the same Table 5, we display moments estimated that are obtained when forcing the supply shock persistence to alternative values, keeping other parameters from our initial model estimation unchanged. We do this experiment for an intermediate value of supply shock persistence ($\rho_s = 0.3$) and for a limiting economy where no other source of persistence other than policy commitment induced is present ($\rho_s = 0.001$). As persistence in the supply shock is decreased, inflation inertia falls turning into a negative correlation as the model tends toward zero 'built in' persistence. This means that policy commitment actually manages to induces a slight *negative* inflation correlation. Consequently, the only source of positive serial correlation in inflation is supply shock driven.[37] Such a result was already anticipated when discussing Figure 8. The volatility in inflation also decreases the smaller ρ_s is. This is not surprising since lower persistence means that firms will adjust initial prices to a smaller extent because they anticipate that the shock will not propagate for long. This implies also that the output gap will display lower volatility as the shock becomes less persistent. Output persistence decreases slightly as the supply shock becomes less persistent, but a substantial amount remains attributable to commitment policy alone. Concerning the interest rate, we observe only marginal changes (at a 4 digit level generally) in persistence and volatility as ρ_s decreases. Remember that in this case the interest rate dynamics are also affected by the two demand shocks the central bank is compensating for. Following a supply shock, we already know that the interest rate reacts very little. This happens to be true for any degree of persistence in the supply shock so that most of the interest rate dynamics are due to the central bank compensating

[36] Note that all moments below and in successive tables could also be directly calculated from our reduced form solution. To this purpose, we need to express the supply shock volatility in terms of its innovation volatility. Since $\varepsilon_t^s = (1-\rho^s L)\ \xi_t^s = \sum_{i=0}^{\infty} (\rho^s)^i\, \xi_{t-i}^s$, we have $\sigma_s(\varepsilon) = \frac{\sigma_s(\xi)}{1-(\rho^s)^2}$.

[37] Interestingly, Ireland (2004) finds that, for the post 1980 period, the most important contributor to movements in inflation is indeed the 'cost push' shock.

for the demand shocks. Table 7 shows particularly well the interest rate persistence and volatility for alternative values of persistence in the demand shock processes keeping ρ_s to its estimated value (0.697). As it can be seen from those estimations, once demand shock persistence is given lower values, most of the interest rate inertia disappears. The volatility also drastically decreases as much of the dynamics are cut off by the decrease in demand shock driven variability.[38]

Recall from our discussion in previous sections that if the central bank behaves in a discretionary manner, it acts on a day to day basis, responding to contemporaneous shocks only. The equilibrium outcome then implies no role for policy or for price rigidity mechanism in altering the persistence properties of the model variables. Thus, only exogenous shocks' inertia is transmitted to output, interest rate and inflation. Comparing the moments obtained under such hypothetical outcome helps define the commitment policy contribution to persistence when 'built in' persistence is still present through the serial correlation of shocks. Accordingly, the estimated parameter vector presented in Table 1 is used to solve for the state space system resulting from optimization under discretionary policy. The resulting unconditional moments are displayed in Table 6. In this case, inflation persistence almost matches that of the data. Model inflation is too volatile. On the contrary, output volatility and persistence are too low. The same comment applies for the interest rate's inertial properties. As expected, the interest rate volatility is too high. Thus, for the estimated structural parameter values under commitment, the main differences the discretionary and the commitment policy are: higher inflation variability and persistence, lower output gap volatility and persistence, and finally, higher interest rate variability and lower persistence. A model estimated under discretion when supply shock inertia decreases keeping other parameters constant is also provided. Since output and inflation dynamics are entirely dictated by the supply shock, the persistence properties entirely reflect those of the shock. Inflation volatility decreases as firms post lower prices following a supply shock. Output volatility also decreases as output fall following the shock is expected to be smaller and less persistent. In the discretionary case, interest rate inertia increases and volatility decreases as the supply shock becomes less persistent. The central bank needs to react more strongly to supply shocks. As a matter of fact, it cannot exploit future expected movements in the interest rates as under commitment; therefore less persistent shocks imply smaller deviation of inflation and output from their target values and thus require less interest rate movements, hence volatility decreases. Simultaneously, other shock persistence do not vary and represent thus a bigger proportion of interest rate induced dynamics. Since overall volatility decreases, the first order autocorrelation increases slightly.

[38] One might be tempted to argue that the interest rate dynamics are too heavily depending on serial correlation in demand shock. However, we think our specification is reasonable in this context since we have checked for the presence of lagged endogenous dynamics and found poor or little empirical relevance. In addition, our high preference shock persistence estimate is realistic. Indeed, changes in tastes can intuitively be considered as long lasting.

5 Conclusion

Implementing a commitment policy is welfare enhancing from a theoretical viewpoint. However, we believe that commitment is not only a theoretically desirable feature, but is also compatible with real world assumed policy goal and behavior by central banks. Indeed, we have seen that a simple forward looking general equilibrium model can be a reasonably good description of reality when the policy followed by the monetary authority is a rule. Second, the presence of lagged terms in both Phillips and IS curve is not empirically relevant, so that model inertia remain unaltered when these variables are controlled for. Third, we have seen that a substantial part of output persistence can be generated through a policy concerned with variations in output, but that commitment policy responses also involve two distinct effects on inflation inertia: i) A slight undershooting, implying relatively less inflation persistence, because the central bank manages to reduce inflation faster; ii) A positive effect, which reflects the gradual return of inflation to its target from *below*. Since the former slightly dominates, much of inflation inertia is reported to serially correlated 'cost push' shocks alone; iii) Interest rate inertia is mainly demand shock driven. One might be tempted to argue that the interest rate dynamics are too heavily depending on serial correlation in both demand shocks. However, we think that our specification is reasonable in this context since we have controlled for the presence of lagged endogenous dynamics and found poor or little empirical support for their presence. In addition, and in our view, a high preference shock persistence estimate is realistic. Indeed, changes in tastes can intuitively be considered as long lasting.

Finally, we think a key point in our setup concerns the informational assumptions made. For example, the central bank targets clear and fixed inflation and output goals. While this is a good first approximation, it is likely that policy targets are not always clearly pinned down by the private sector. One reason is simply because most central banks do not target precise values. In that respect, the monetary authorities' assumed credibility and the fact that agents have potentially access to all the available information in the economy, including the central bank's behavior is probably only a good starting point. In a recent study, Erceg and Levin (2003) explore a model in which the private sector faces a signal extraction problem about the central bank's targeted inflation value.[39] Discussing the 'Volcker' disinflation period, these authors show that the 'transparency' of the monetary policy regime also affects considerably inflation persistence properties. Alternatively, the central bank might have difficulties in identifying whether a shock pertains to demand factors, supply, or a mix of both. We should probably expect the latter situation to predominate in the real world. Another point concerns the potential existing lags between a change in the interest rates and its impact on output and inflation. It would be probably worth studying how such lags would alter optimal policy, and consequently, the persistence properties

[39] Similarly, Ehrmann and Smets (2003) explore in a small model for the euro area, the implications of incomplete information about potential output for the conduct of monetary policy.

of real variables. Finally, if one is willing to accept that supply shock autocorrelation is an acceptable feature of New Keynesian modelling, the forward looking behavior that characterizes these models could well then be a good starting point for policy evaluation analysis. In that case, further research should probably take into account investment decisions both on firms' and consumers' sides.[40]

[40] Note however that Sveen and Weinke (2004) integrate capital (with adjustment cost) in a New Keynesian model and find very little difference in inflation dynamics: capital accumulation has two counteracting effects on marginal costs. On one side, an increase in aggregate demand increases production and marginal cost. On the other, an increase in investment increases economic productive capacity, thereby decreasing the marginal cost.

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6 Appendix

6.1 Solutions

6.1.1 Discretion

The first order conditions for i_t, π_t and y_t are respectively given by:

$$\begin{aligned} 1/\sigma E_t \gamma_t &= 0, \\ E_t \left(\chi \pi_t + \varphi_t \right) &= 0, \\ E_t \left(\lambda y_t - \psi \varphi_t \right) &= 0. \end{aligned}$$

Notice that the Lagrangian associated with the aggregate demand curve is always equal to zero. As argued in the text, this simply states that equation (2) is irrelevant in determining the equilibrium outcome for π_t and y_t. Combining the last two optimal conditions yields:

$$\pi_t = -\frac{\lambda}{\chi \psi} y_t.$$

The central bank can thus implement the behavior dictated by the first order conditions in a precise manner. Replacing this expression in the Phillips curve, we have:

$$-\frac{\lambda}{\chi \psi} y_t = \beta E_t \left(-\frac{\lambda}{\chi \psi} y_{t+1} \right) + \psi y_t + \varepsilon_{s,t},$$

$$(1 + \frac{\chi \psi^2}{\lambda}) y_t = \beta E_t y_{t+1} - \frac{\chi \psi}{\lambda} \varepsilon_{s,t}.$$

To solve this equation, guess a solution of the form: $y_t = e \ \varepsilon_{s,t}$. Since the shocks is assumed AR(1), we have that $E_t y_{t+1} = \rho_s \ e \ \varepsilon_{s,t}$. We then have:

$$e = -\frac{\chi \psi}{\lambda \left[1 - \beta \rho_s \right] + \chi \psi^2}.$$

Thus, equilibrium output and inflation are given by:

$$y_t = e \ \varepsilon_{s,t},$$

and

$$\pi_t = f \ \varepsilon_{s,t}.$$

where $f = \frac{\lambda}{\lambda[1-\beta\rho_s]+\chi\psi^2}$. The short term interest rate is now

$$\begin{aligned}
i_t &= \sigma(E_t y_{t+1} - y_t) + E_t\pi_{t+1} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}, \\
&= \sigma(-\frac{\rho_s\chi\psi}{\lambda\left[1-\beta\rho_s\right]+\chi\psi^2}\varepsilon_{s,t} + \frac{\chi\psi}{\lambda\left[1-\beta\rho_s\right]+\chi\psi^2}\varepsilon_{s,t}) + \frac{\rho_s\lambda}{\lambda\left[1-\beta\rho_s\right]+\chi\psi^2}\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}, \\
&= \frac{\rho_s\lambda + (1-\rho_s)\chi\psi\sigma}{\lambda\left[1-\beta\rho_s\right]+\chi\psi^2}\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}.
\end{aligned}$$

6.1.2 Commitment

In this case the first order conditions are given by:

$$\begin{aligned}
1/\sigma E_t\gamma_{t+j} &= 0, \qquad j \geq 0, \\
E_t\left(\chi\pi_{t+j} + \varphi_{t+j} - \varphi_{t-1+j}\right) &= 0, \qquad j \geq 1, \\
E_t\left(\lambda y_{t+j} - \psi\varphi_{t+j}\right) &= 0, \qquad j \geq 0.
\end{aligned}$$

The first order condition for the start up inflation value (when $j = 0$) is $\chi\pi_t + \varphi_t = 0$. This introduces some inconsistency in the commitment approach. Note in particular that at time t, the central bank would set $\pi_t = -\frac{1}{\chi}\varphi_t$ and promises then to set $\pi_{t+j} = -\frac{1}{\chi}(\varphi_{t+j} - \varphi_{t+j-1})$ in the future. But when the central bank reaches period $t + j$, it will prefer to set $\pi_{t+j} = -\frac{1}{\chi}\varphi_{t+j}$, as its optimization plan would suggest. The *timeless perspective* approach, as defined in Woodford (2003), circumvents this problem by assuming that the optimal commitment policy has been chosen in the past and that current values of inflation and output gap satisfy the second first order condition above. This means that we abstract from the initial condition above.[41] To rationalize this choice, we think of commitment as a policy regime that is effective, and understood by rational agents as such, for a sufficiently long period of time before initial conditions are set. We perceive this behavior as economically relevant for the issue at stake.[42] By combining the last two equation, we obtain for time t:

$$\pi_t = -\frac{\lambda}{\chi\psi}(y_t - y_{t-1}). \tag{20}$$

Note that this equation holds also in realized values because when period t comes, the central bank can observe by assumption both π_t and y_t. Using equation (20) back in the Phillips Curve, we obtain an expectational linear difference equation for the output gap of the form:

$$-\frac{\lambda}{\chi\psi}(y_t - y_{t-1}) = \beta E_t\left[-\frac{\lambda}{\chi\psi}(y_{t+1} - y_t)\right] + \psi y_t + \varepsilon_{s,t},$$

[41] Hence, throughought the text, we will loosely speak of commitment as a *timeless perspective* policy

[42] Mc Callum and Nelson (2000) argue for instance that commitment is convincing when one is concerned with macroeconomic performance "...within and across [policy] regimes,....".

rearranging, we finally get:

$$(1+\beta+\frac{\chi\psi^2}{\lambda})y_t = \beta E_t y_{t+1} + y_{t-1} - \frac{\chi\psi}{\lambda}\varepsilon_{s,t}. \tag{21}$$

Now conjecture a solution of the form $y_t = ay_{t-1} + b\varepsilon_{s,t}$. Then, we have $E_t y_{t+1} = a^2 y_{t-1} + (a+\rho)b\varepsilon_{s,t}$. Plug this back in (21), the equation becomes:

$$(1+\beta+\frac{\chi\psi^2}{\lambda})(ay_{t-1}+b\varepsilon_{s,t}) = \beta\left[a^2 y_{t-1} + (a+\rho_s)b\varepsilon_{s,t}\right] + y_{t-1} - \frac{\chi\psi}{\lambda}\varepsilon_{s,t}.$$

By rearranging, one finally gets,

$$\left[\beta a^2 - (1+\beta+\frac{\chi\psi^2}{\lambda})a + 1\right] y_{t-1} + \left[\beta(a+\rho_s)b - \frac{\chi\psi}{\lambda} - b(1+\beta+\frac{\chi\psi^2}{\lambda})\right]\varepsilon_{s,t} = 0.$$

The solution for b to the above equation is easily obtained as one gets,

$$b = -\frac{\chi\psi}{\lambda\left[1+\beta\left(1-a-\rho_s\right)\right]+\chi\psi^2}.$$

Similarly, a unique solution for a is obtained by solving the equation $\beta\lambda a^2 - (\lambda + \lambda\beta + \chi\psi^2)a + \lambda = 0$, and imposing $a < 1$ for stability, which gives:

$$a = \frac{\lambda + \lambda\beta + \chi\psi^2 - \sqrt{\left(\lambda+\lambda\beta+\chi\psi^2\right)^2 - 4\beta\lambda^2}}{2\lambda\beta}.$$

By using the above solutions in our conjectured output gap dynamic equation and plugging it back into (20), we obtain:

$$\pi_t = cy_{t-1} + d\varepsilon_{s,t}, \tag{22}$$

where $c = \frac{\lambda}{\chi\psi}(1-a)$, and $d = \frac{\lambda}{\lambda[1+\beta(1-a-\rho_s)]+\chi\psi^2}$. Finally,

$$\begin{aligned}
i_t &= \sigma(E_t y_{t+1} - y_t) + E_t\pi_{t+1} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}, \\
&= \sigma\left[a^2 y_{t-1} + (a+\rho_s)b\varepsilon_{s,t} - ay_{t-1} - b\varepsilon_{s,t}\right] + acy_{t-1} + (cb+\rho_s d)\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}, \\
&= a\left[\sigma(a-1)+c\right]y_{t-1} + \left[\sigma b\left(a+\rho_s-1\right) + (cb+\rho_s d)\right]\varepsilon_{s,t} - \varepsilon_{i,t} + \sigma\varepsilon_{d,t}.
\end{aligned}$$

6.2 Autocorrelations

In what follows, we assume covariance stationarity of all variables. $\sigma_y(k)$ denotes $cov(y_t, y_{t+k})$ and $\sigma_{y\varepsilon_s}(k)$ denotes $cov(y_t, \varepsilon_{s,t+k})$.

$$
\begin{aligned}
\sigma_y(0) &= a^2\ var(y_t) + b^2 var(\varepsilon_{s,t}) + 2ab\ cov(y_{t-1}, \varepsilon_{s,t}) \\
&= a^2\sigma_y(0) + b^2\sigma_{\varepsilon_s}(0) + 2ab\rho_s\ cov(y_{t-1}, \varepsilon_{s,t-1}), \\
\sigma_y(0) &= \frac{1}{1-a^2}\left[b^2\sigma_{\varepsilon_s}(0) + 2ab\rho_s\sigma_{y\varepsilon_s}(0)\right], \\
&= \frac{b^2}{1-a^2}\left(1 + \frac{2a\rho_s}{1-a\rho_s}\right)\sigma_{\varepsilon_s}(0) = \frac{b^2}{1-a^2}\left(\frac{1+a\rho_s}{1-a\rho_s}\right)\sigma_{\varepsilon_s}(0).
\end{aligned}
$$

The last equality is obtained by noting that $\sigma_{y\varepsilon_s}(0) = cov(y_t, \varepsilon_{s,t}) = cov(ay_{t-1} + b\varepsilon_{s,t}, \varepsilon_{s,t}) = b\sigma_{\varepsilon_s}(0) + cov(ay_{t-1}, \varepsilon_{s,t}) = b\sigma_{\varepsilon_s}(0) + cov(ay_{t-1}, \rho_s\varepsilon_{s,t-1} + \xi_{s,t}) = b\sigma_{\varepsilon_s}(0) + a\rho_s\sigma_{y\varepsilon_s}(0)$. From our stationarity assumption, we have then $\sigma_{y\varepsilon_s}(0) = \frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s}$. For the covariance we have,

$$
\begin{aligned}
\sigma_y(k) &= a^2\sigma_y(k) + b^2\sigma_{\varepsilon_s}(k) + ab\left[cov(y_{t-1}, \varepsilon_{s,t+k}) + cov(y_{t+k-1}, \varepsilon_{s,t})\right] \\
&= a^2\sigma_y(k) + b^2\rho_s^k\sigma_{\varepsilon_s}(0) \\
&\quad + ab\left[\rho_s^{k+1}cov(y_{t-1}, \varepsilon_{s,t-1}) + cov(a^{k-1}y_t + b(\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j})\varepsilon_{s,t}, \varepsilon_{s,t})\right],
\end{aligned}
$$

thus,

$$
\begin{aligned}
\sigma_y(k) &= \frac{1}{1-a^2}\left[\left(b^2\rho_s^k + ab^2\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)\sigma_{\varepsilon_s}(0) + ab(\rho_s^{k+1} + a^{k-1})\sigma_{y\varepsilon_s}(0)\right], \\
&= \frac{1}{1-a^2}\left[\left(b^2\rho_s^k + ab^2\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)\sigma_{\varepsilon_s}(0) + ab(\rho_s^{k+1} + a^{k-1})\frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s}\right], \\
&= \frac{b^2}{1-a^2}\left[\rho_s^k + a\sum_{j=1}^{k-1}\rho^j a^{k-1-j} + \frac{a(\rho_s^{k+1} + a^{k-1})}{1-a\rho_s}\right]\sigma_{\varepsilon_s}(0).
\end{aligned}
$$

We can write the autocorrelogram for the output gap (in the commitment policy case) as,

$$
\rho_y^c(k) = \frac{\sigma_y(k)}{\sigma_y(0)} = \frac{(1-a\rho)\left(\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right) + a(\rho_s^{k+1} + a^{k-1})}{1+a\rho_s}.
$$

Using the same method for inflation, we obtain:

$$
\begin{aligned}
\sigma_\pi(0) &= c^2\sigma_y(0) + d^2\sigma_{\varepsilon_s}(0) + 2cd\rho_s\sigma_{y\varepsilon_s}(0), \\
&= \frac{c^2b^2}{1-a^2}\frac{1+a\rho_s}{1-a\rho_s}\sigma_{\varepsilon_s}(0) + d^2\sigma_{\varepsilon_s}(0) + 2cd\rho_s\frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s}, \\
&= \frac{c^2b^2(1+a\rho_s) + d^2(1-a^2)(1-a\rho_s) + 2bcd\rho_s(1-a^2)}{(1-a^2)(1-a\rho_s)}\sigma_{\varepsilon_s}(0).
\end{aligned}
$$

$$\begin{aligned}
\sigma_\pi(k) &= c^2\sigma_y(k) + d^2\rho_s^k\sigma_{\varepsilon_s}(0) + cd\left[(\rho_s^{k+1} + a^{k-1})\sigma_{y\varepsilon_s}(0) + b\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\sigma_{\varepsilon_s}(0)\right], \\
&= \frac{b^2c^2}{1-a^2}\left[\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j} + \frac{a(\rho_s^{k+1}+a^{k-1})}{1-a\rho_s}\right]\sigma_{\varepsilon_s}(0) \\
&\quad + d^2\rho_s^k\sigma_{\varepsilon_s}(0) + cd\left[(\rho_s^{k+1}+a^{k-1})\frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s} + b\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\sigma_{\varepsilon_s}(0)\right], \\
&= \left[\begin{array}{c}
\frac{b^2c^2\left[\left(\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)(1-a\rho_s) + a(\rho_s^{k+1}+a^{k-1})\right]}{(1-a^2)(1-a\rho_s)} \\
+\frac{d^2\rho_s^k\left(1-a^2\right)(1-a\rho_s) + bcd\left[(\rho_s^{k+1}+a^{k-1})\left(1-a^2\right) + \sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\left(1-a^2\right)(1-a\rho_s)\right]}{(1-a^2)(1-a\rho_s)}
\end{array}\right]\sigma_{\varepsilon_s}(0),
\end{aligned}$$

$$\begin{aligned}
\rho_\pi^c(k) &= \frac{1}{c^2b^2\left(1+a\rho_s\right) + d^2\left(1-a^2\right)\left(1-a\rho_s\right) + 2bcd\rho_s\left(1-a^2\right)} \\
&\quad \left\{\begin{array}{c}
b^2c^2\left[\left(\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)\left(1-a\rho_s\right) + a(\rho_s^{k+1}+a^{k-1})\right] \\
+d^2\rho_s^k\left(1-a^2\right)\left(1-a\rho_s\right) \\
+bcd\left[(\rho_s^{k+1}+a^{k-1})\left(1-a^2\right) + \sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\left(1-a^2\right)\left(1-a\rho_s\right)\right]
\end{array}\right\}
\end{aligned}$$

In the case of the short term interest rate, we have:

$$\begin{aligned}
\sigma_i(0) &= \Omega^2\sigma_y(0) + \Gamma^2\sigma_{\varepsilon_s}(0) + 2\Omega\Gamma\rho_s\sigma_{y\varepsilon_s}(0) + \sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0), \\
&= \frac{\Omega^2b^2}{1-a^2}\frac{1+a\rho_s}{1-a\rho_s}\sigma_{\varepsilon_s}(0) + \Gamma^2\sigma_{\varepsilon_s}(0) + 2\Omega\Gamma\rho_s\frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s} + \sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0), \\
&= \frac{\Omega^2b^2\left(1+a\rho_s\right) + \Gamma^2\left(1-a^2\right)\left(1-a\rho_s\right) + 2b\Omega\Gamma\rho_s\left(1-a^2\right)}{\left(1-a^2\right)\left(1-a\rho_s\right)}\sigma_{\varepsilon_s}(0) + \sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0).
\end{aligned}$$

$$\begin{aligned}
\sigma_i(k) &= \Omega^2\sigma_y(k) + \Gamma^2\rho_s^k\sigma_{\varepsilon_s}(0) + \Omega\Gamma\left[(\rho_s^{k+1}+a^{k-1})\sigma_{y\varepsilon_s}(0) + b\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\sigma_{\varepsilon_s}(0)\right] \\
&\quad + \rho_i^k\sigma_{\varepsilon_i}(0) + \sigma^2\rho_d^k\sigma_{\varepsilon_d}(0), \\
&= \frac{\Omega^2b^2}{1-a^2}\left[\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j} + \frac{a(\rho_s^{k+1}+a^{k-1})}{1-a\rho_s}\right]\sigma_{\varepsilon_s}(0) \\
&\quad + \Gamma^2\rho_s^k\sigma_{\varepsilon_s}(0) + \Omega\Gamma\left[(\rho_s^{k+1}+a^{k-1})\frac{b\sigma_{\varepsilon_s}(0)}{1-a\rho_s} + b\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\sigma_{\varepsilon_s}(0)\right] \\
&\quad + \rho_i^k\sigma_{\varepsilon_i}(0) + \sigma^2\rho_d^k\sigma_{\varepsilon_d}(0),
\end{aligned}$$

$$= \left[\begin{array}{c} \frac{\Omega^2 b^2 \left[\left(\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)(1-a\rho_s)+a(\rho_s^{k+1}+a^{k-1})\right]}{(1-a^2)(1-a\rho_s)} \\ +\frac{\Gamma^2\rho_s^k\left(1-a^2\right)(1-a\rho_s)+b\Omega\Gamma\left[(\rho_s^{k+1}+a^{k-1})\left(1-a^2\right)+\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\left(1-a^2\right)(1-a\rho_s)\right]}{(1-a^2)(1-a\rho_s)} \end{array} \right] \sigma_{\varepsilon_s}(0)$$
$$+\rho_i^k \sigma_{\varepsilon_i}(0) + \sigma^2 \rho_d^k \sigma_{\varepsilon_d}(0),$$

$$\rho_i^c(k) = \left\{ \begin{array}{c} \left[\Omega^2 b^2 (1+a\rho_s) + \Gamma^2(1-a^2)(1-a\rho_s) + 2b\Omega\Gamma\rho_s(1-a^2)\right]\sigma_{\varepsilon_s}(0) \\ +(1-a^2)(1-a\rho_s)\left(\sigma_{\varepsilon_i}(0) + \sigma^2\sigma_{\varepsilon_d}(0)\right) \end{array} \right\}^{-1}$$
$$\left\{ \begin{array}{c} \{b^2\Omega^2\left[\left(\rho_s^k + a\sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}\right)(1-a\rho_s) + a(\rho_s^{k+1}+a^{k-1})\right] \\ +\Gamma^2\rho_s^k(1-a^2)(1-a\rho_s) \\ +b\Omega\Gamma\left[(\rho_s^{k+1}+a^{k-1})(1-a^2) + \sum_{j=1}^{k-1}\rho_s^j a^{k-1-j}(1-a^2)(1-a\rho_s)\right]\}\sigma_{\varepsilon_s}(0) \\ +(1-a^2)(1-a\rho_s)\left(\rho_i^k\sigma_{\varepsilon_i}(0) + \sigma^2\rho_d^k\sigma_{\varepsilon_d}(0)\right) \end{array} \right\}$$

6.3 The Model in State Space Form

First write:

$$\begin{aligned} x_{1t} &= [\pi_{t-1}\ \ \pi_{t-2}\ \ \pi_{t-3}\ \pi_{t-4}\ \ y_{t-1}\ \ y_{t-2}\ \ y_{t-3}\ \ y_{t-4}\ \ i_{t-1}\ i_{t-2}\ \ i_{t-3}\ \ i_{t-4}\ \varepsilon_{i,t}\ \ \varepsilon_{s,t}\ \ \varepsilon_{d,t}]', \\ x_{2t} &= [\pi_t\ \ y_t]', \\ \xi_{t+1} &= \left[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \xi_t^i\ \ \xi_t^s\ \ \xi_t^d\right]', \\ u_t &= i_{t.} \end{aligned}$$

The model can then be written as

$$x_{t+1} = Ax_t + Bu_t + \epsilon_{t,}$$

where

$$
A=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\rho^i&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\rho^s&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\rho^y&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{\beta}&0&\frac{1}{\beta}&-\frac{\kappa}{\beta}(\sigma+\eta)\\
0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{\sigma}&\frac{1}{\beta\sigma}&-1&-\frac{1}{\beta\sigma}&1+\frac{\kappa}{\beta\sigma}(\sigma+\eta)
\end{bmatrix},
$$

and

$$B=[\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1/\sigma\ ]',$$

are the matrices containing the structural parameters of the model. We denote Σ_ϵ the covariance matrix of ϵ_t which is diagonal, and $x_{t+1} = [x_{1t+1}\ E_t x_{2t+1}]'$, $\epsilon_t = [\xi_t\ \mathbf{0}_{n2\times 1}]'$.[43]

6.3.1 Solving under optimal policy

To solve the central bank optimization problem, first recall that our central bank loss function is given by

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t x_t' Q x_t,$$

where Q is a symmetric matrix. Since we assume the central bank is able to commit to a constant policy rule, its optimization problem can be stated as the following Lagrangian

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t' Q x_t + 2\phi_{t+1}' [A x_t + B u_t + \epsilon_t - x_{t+1}] \right\}.$$

[43] The additional lags in y, π and i appearing in x_{1t} are needed to calculated the unconditional moments below.

The solution for the commitment case can be expressed in a reduced form as:[44]

$$\begin{bmatrix} x_{1t+1} \\ \phi_{2t+1} \end{bmatrix} = M \begin{bmatrix} x_{1t} \\ \phi_{2t} \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0} \end{bmatrix}, \tag{23}$$

with x_{10} given and $\phi_{20} = \mathbf{0}$, and

$$\begin{bmatrix} x_{2t} \\ u_t \\ \phi_{1t} \end{bmatrix} = C \begin{bmatrix} x_{1t} \\ \phi_{2t} \end{bmatrix}. \tag{24}$$

This state space representation is estimated by transposing it into a Kalman Filter framework used to build up the likelihood function of the data. Write the state space model transition equation as

$$X_{s,t+1} = M\ X_{s,t} + \epsilon_t. \tag{25}$$

The measurement equation is,

$$X_{m,t} = C_1\ X_{s,t}, \tag{26}$$

where $X_{s,t} = [x_{1t} \quad \phi_{2t}]'$ and $X_{m,t} = [x_{2t} \quad u_t]'$.[45] Note that ϕ_{1t} has disappeared from the measurement equation since it does not add any useful information to the dynamics of the forward looking endogenous variables.[46] At each guess of the parameter vector, the solution algorithm gives an optimal policy rule and a system of linear difference equations for the model variables i.e. our state space model. To obtain unconditional moments of the state and endogenous variables simply write:

$$\Sigma_{X_s} = M\ \Sigma_{X_s} M' + \Sigma_\epsilon,$$

and the solution is:

$$vec(\Sigma_{X_s}) = (I - M \otimes M)^{-1} vec(\Sigma_\epsilon).$$

where Σ_{X_s} is the variance covariance matrix of the states. Finally, we have $\Sigma_{X_m} = C_1\ \Sigma_{X_s}\ C_1'$. A similar solution procedure can be applied under discretionary optimal policy.[47]

[44] See Söderlind (1999) for details.

[45] In our estimated system, we add in the transition and measurement equation two vectors of constants c and d respectively. Also a vector of shocks η_t is added to the measurement equation (with covariance Σ_η).

[46] We thus eliminate the rows in C that are unnecessary. This yields C_1.

[47] See Söderlind (1999) or Söderström *et al.* (2003) for an exposition.

Table 1: Commitment policy

Parameter	Estimate	St.err
β	0.978	0.006
σ	1.969	0.005
ω	0.836	0.009
η	0.414	0.001
ρ_d	0.946	0.028
ρ_s	0.697	0.024
ρ_i	0.522	0.001
$\sigma_d(\xi)$	0.196	0.022
$\sigma_s(\xi)$	0.421	0.021
$\sigma_i(\xi)$	0.086	0.008
λ	0.182	0.010
LogL	-176.44	

Table 2: Commitment policy

Parameter	Estimate	St.err
β	0.973	0.006
σ	1.9701	0.004
ω	0.828	0.010
η	0.414	0.001
ρ_d	0.955	0.029
ρ_s	0.681	0.023
ρ_i	0.522	0.001
$\sigma_d(\xi)$	0.196	0.020
$\sigma_s(\xi)$	0.340	0.020
$\sigma_i(\xi)$	0.083	0.007
λ	0.185	0.010
α_π	0.009	0.013
α_y	0.047	0.006
LogL	-175.93	

Table 3: Commitment policy with $\alpha_\pi = 0$

Parameter	Estimate	St.err
β	0.964	0.007
σ	1.968	0.004
ω	0.822	0.010
η	0.415	0.001
ρ_d	0.971	0.027
ρ_s	0.671	0.025
ρ_i	0.524	0.002
$\sigma_d(\xi)$	0.199	0.019
$\sigma_s(\xi)$	0.388	0.022
$\sigma_i(\xi)$	0.059	0.014
λ	0.196	0.012
α_π	0	
α_y	0.048	0.005
LogL	-174.39	

Table 4: Commitment policy with $\alpha_y = 0$

Parameter	Estimate	St.err
β	0.980	0.006
σ	1.968	0.004
ω	0.838	0.008
η	0.414	0.001
ρ_d	0.945	0.028
ρ_s	0.689	0.022
ρ_i	0.521	0.001
$\sigma_d(\xi)$	0.196	0.019
$\sigma_s(\xi)$	0.410	0.020
$\sigma_i(\xi)$	0.088	0.006
λ	0.182	0.009
α_π	0.020	0.011
α_y	0	
LogL	-176.54	

Table 5: Unconditional moments: Commitment[48]

Inflation	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.55	0.69	0.08	0.99	1.02	0.15
$\boldsymbol{\rho}_s = 0.3$	0.19			0.49		
$\boldsymbol{\rho}_s = 0.001$	-0.09			0.36		

Output	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.97	0.91	0.08	1.79	1.65	0.16
$\boldsymbol{\rho}_s = 0.3$	0.90			0.52		
$\boldsymbol{\rho}_s = 0.001$	0.83			0.29		

Int. Rate	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.94	0.96	0.07	1.20	1.50	0.19
$\boldsymbol{\rho}_s = 0.3$	0.94			1.20		
$\boldsymbol{\rho}_s = 0.001$	0.94			1.20		

[48] Tables 5,6, and 7 display both first order autocorrelation and standard deviation. The standard errors for the data series are calculated using bootstrapping techniques (200 sample replications).

Table 6: Unconditional moments: Discretion

Inflation	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.70	0.69	0.08	1.64	1.02	0.15
$\boldsymbol{\rho}_s = 0.3$	0.30			0.59		
$\boldsymbol{\rho}_s = 0.001$	0.001			0.41		

Output	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.70	0.91	0.08	0.76	1.65	0.16
$\boldsymbol{\rho}_s = 0.3$	0.30			0.28		
$\boldsymbol{\rho}_s = 0.001$	0.001			0.19		

Int. Rate	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_s = 0.697$	0.79	0.96	0.07	2.00	1.50	0.19
$\boldsymbol{\rho}_s = 0.3$	0.83			1.32		
$\boldsymbol{\rho}_s = 0.001$	0.86			1.26		

Table 7: Unconditional moments: Commitment with $\boldsymbol{\rho}_s = 0.697$

Int. Rate	Persistence	$\rho(1)$		Volatility	$\boldsymbol{\sigma}$	
	Estimated	Data	s.e.(Data)	Estimated	Data	s.e.(Data)
$\boldsymbol{\rho}_d = \boldsymbol{\rho}_i = 0.3$	0.30	0.96	0.07	0.42	1.50	0.19
$\boldsymbol{\rho}_d = \boldsymbol{\rho}_i = 0.001$	0.01			0.40		

Figure 1: Impulse Responses with $\psi = 0.05$ and $\lambda = 0.2$

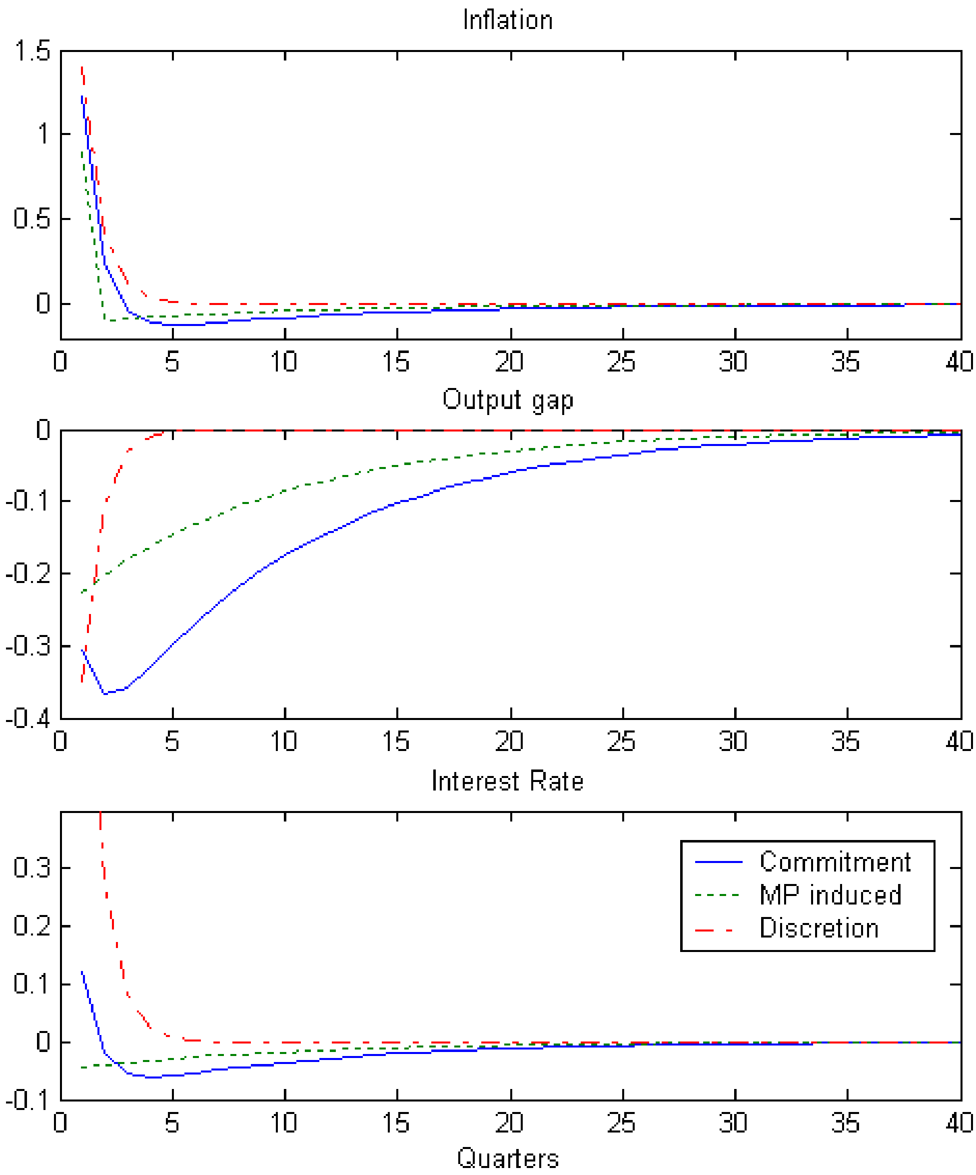

Figure 2: Parameter a (Output persistence)

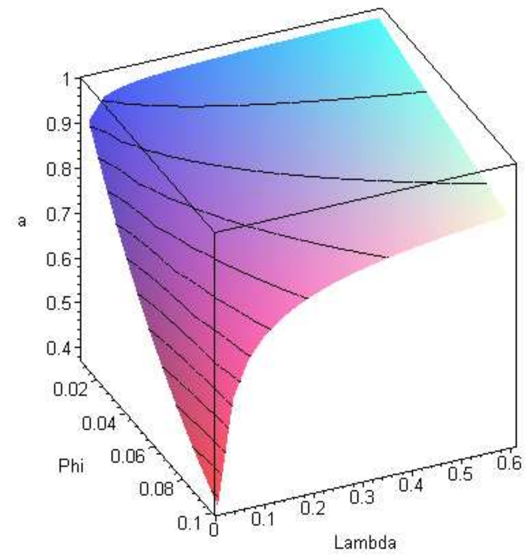

Figure 3: Parameter b_0

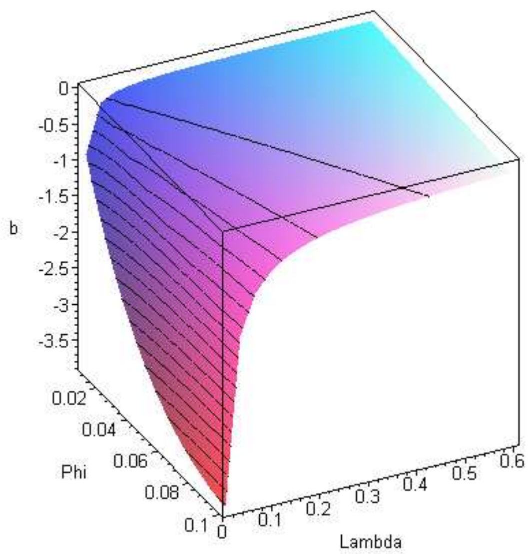

Figure 4: Parameter c

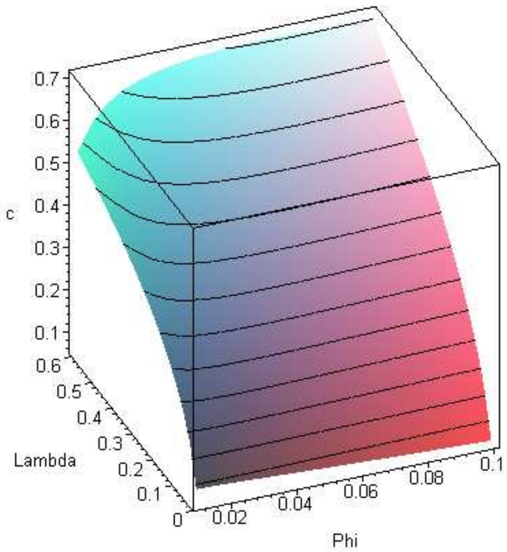

Figure 5: Parameter d_0

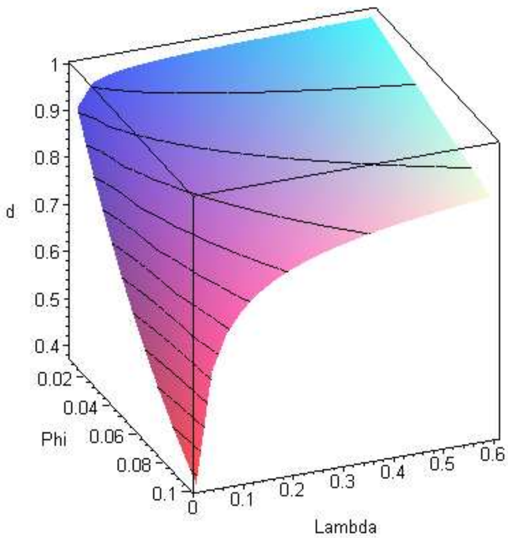

Figure 6: Parameter Ω

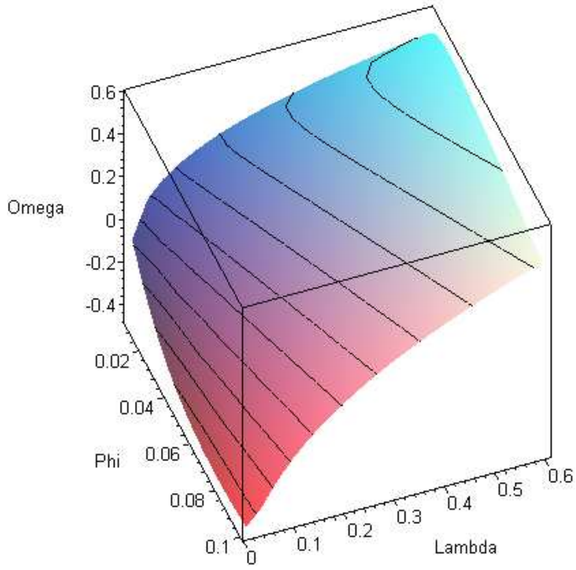

Figure 7: Parameter Γ_0

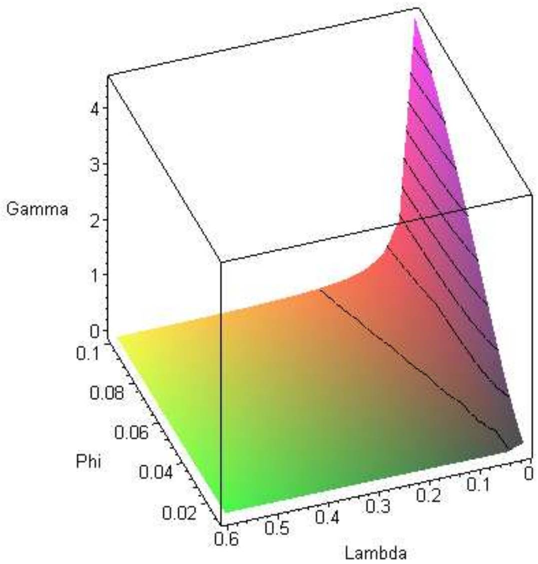

Figure 8: Inflation Persistence

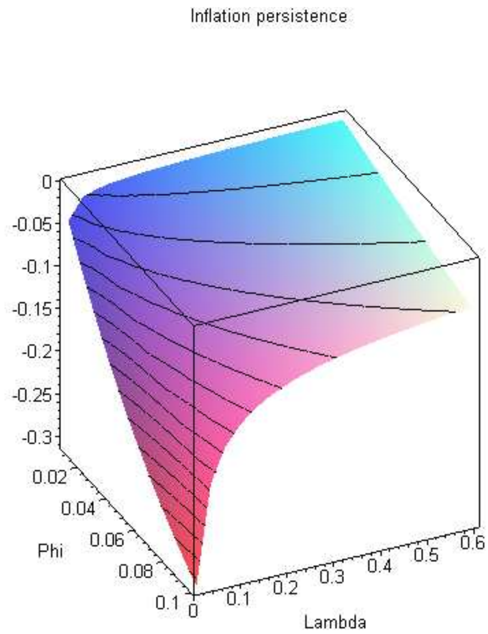

Figure 9: Interest Rate Persistence

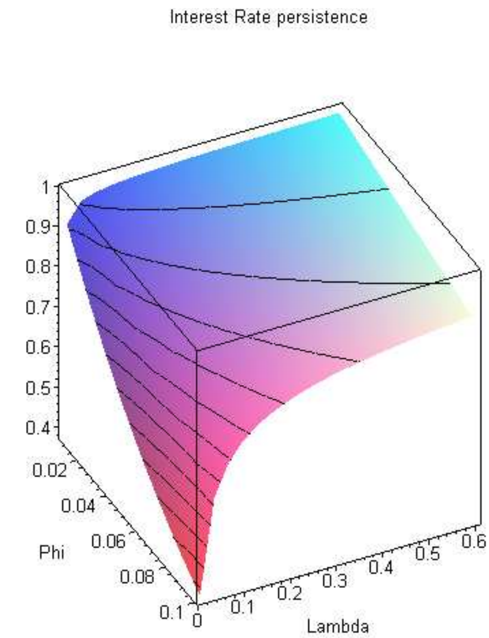

Figure 10: Impulse responses with $\psi = 0.02$ and $\lambda = 0.2$

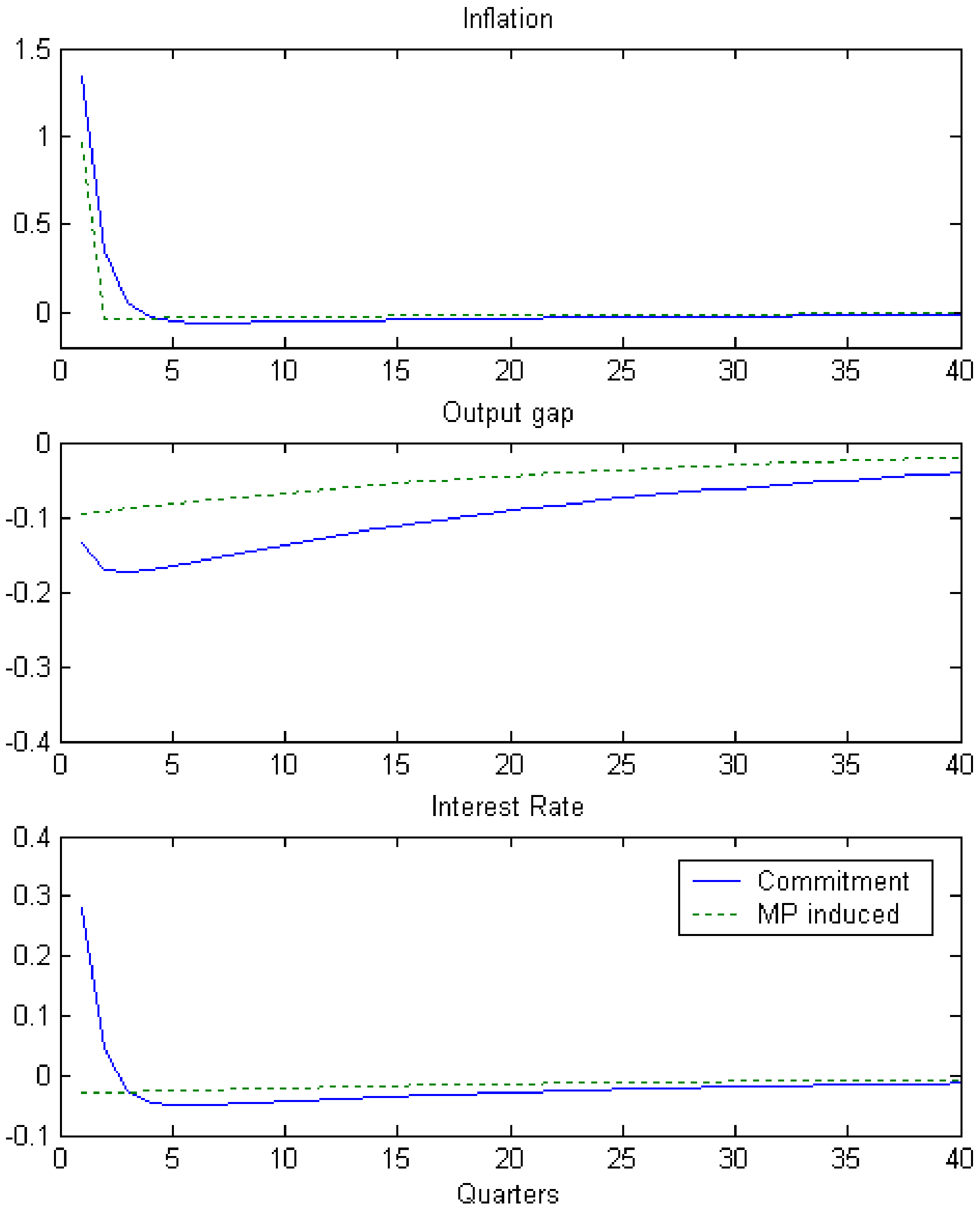

Figure 11: Impulse responses with $\psi = 0.05$ and $\lambda = 0.6$

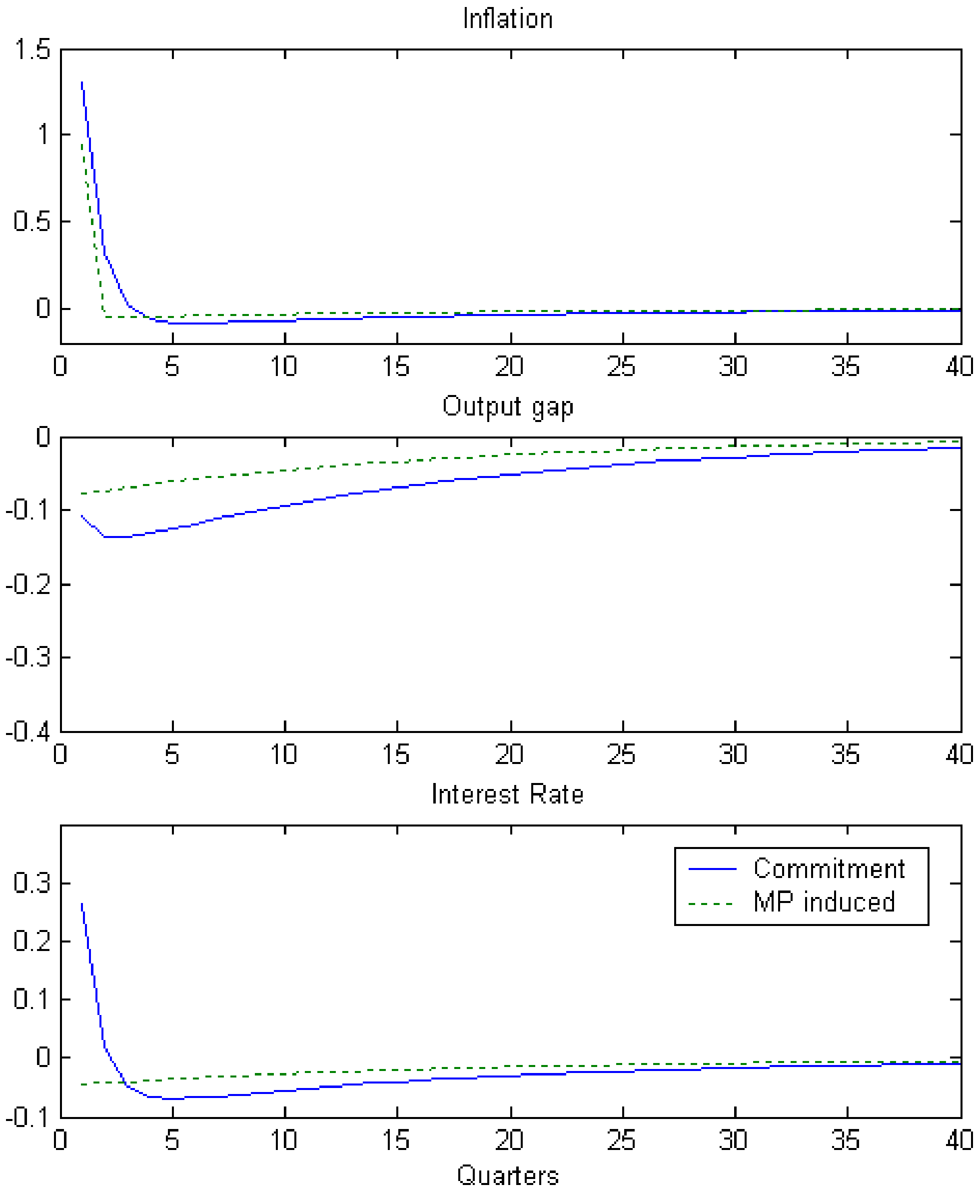

Figure 12: Data series 1987Q4-1999Q4

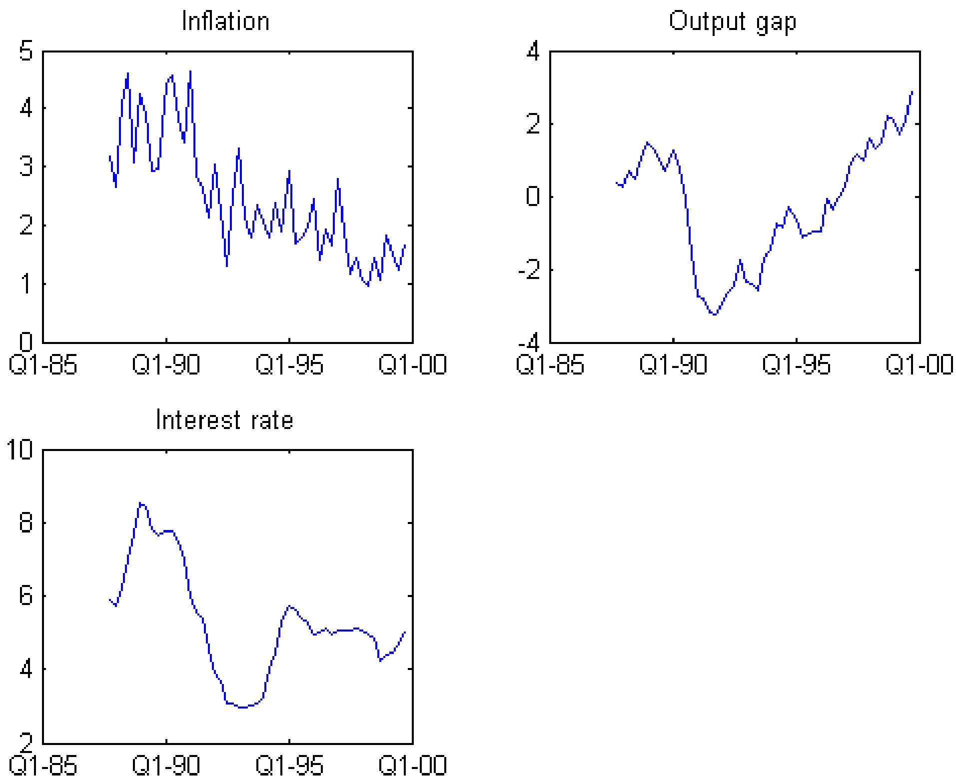

Figure 13: Impulse Responses to a Supply Shock: Commitment Policy

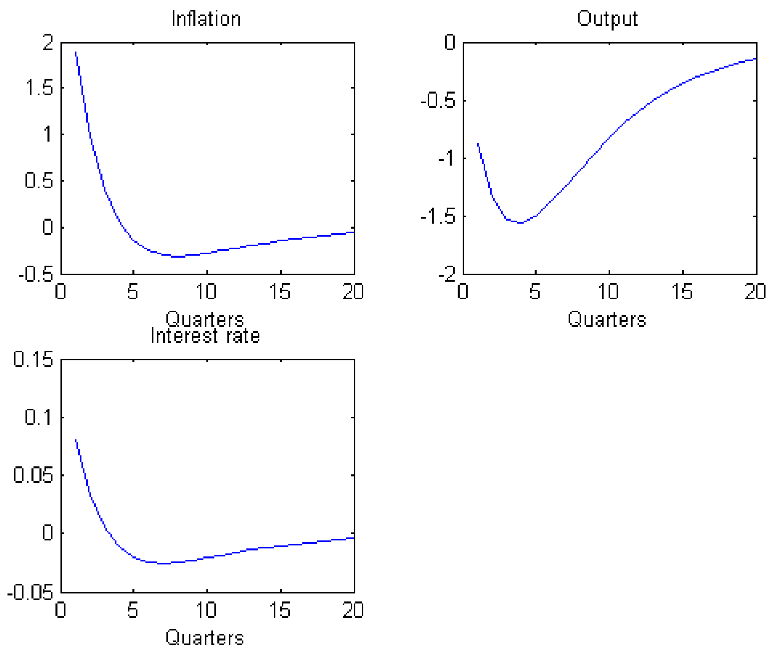

Figure 14: Smoothed Estimate of Supply Shock