Distribution Risk and Equity Returns*

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Abstract

In this paper we entertain the hypothesis that observed variations in income shares are the result of changes in the balance of power between workers and capital owners in labor relations. We show that this view implies that income share variations represent a risk factor of first-order importance for the owners of capital and, consequently, are a crucial determinant of the return to equity. When both risks are calibrated to observations, this *distribution risk* dominates in importance the usual *systematic risk* for the pricing of assets. We also show that distribution risks may originate in non-traded idiosyncratic income shocks.

JEL classification: E3; G1

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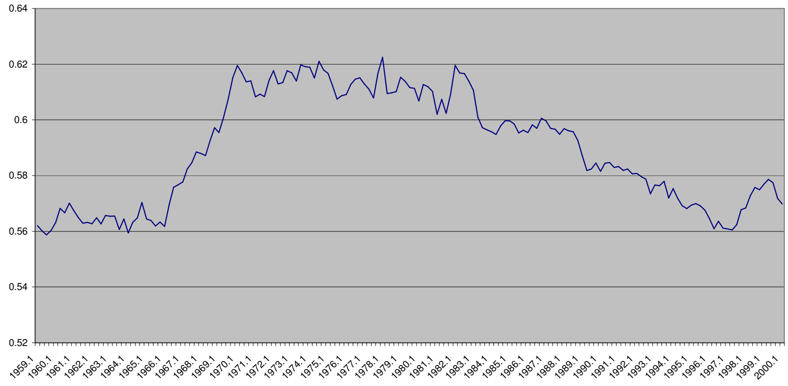
1 Introduction

In all developed economies, capital and labor income shares display persistent variations over time. For the US (see Figure 1), the variability of the (log) wage share over the postwar period is approximately 3.45% on an annual basis with a first-order autocorrelation larger than .97. Such variations are incompatible with the hypothesis of Walrasian labor markets, at least under standard assumptions on the aggregate technology.[1]

Figure 1: U.S. Labor Share 1959-2000



Source: National Income and Product Accounts (NIPA) published by Bureau of Economic Analysis (BEA); US labor share is defined as : Compensation of employees (deflated by consumption expenditures index) / Real GDP.

In this paper we entertain the hypothesis that the observed variations in income shares are the result of infrequent changes in the balance of power between workers and capital owners in labor relations. The overall context is one of non-competitive labor markets where political (partially via taxes on labor and capital) and social forces (notably via the action of trade unions) influence the sharing of value added between capital and labor. We show that this view implies that income share variations represent a risk factor of first-order importance for the owners of capital and, consequently, are a crucial determinant of the return to equity. We demonstrate that, when both risks are calibrated to observations (including the statistical properties of income shares), what may be called *distribution risk* dominates in importance the usual *systematic risk* for the pricing of financial assets.

We make our point in the standard, separable utility infinite-horizon, production economy paradigm characteristic of many studies of macroeconomic and financial equilibrium. Our economy is one with two classes of agents - shareholders and workers - where

[1] In this sense these variations falsify most existing business cycle models. We pursue a purely technological explanation for income share variations in Section 6.

the allocation of resources conforms to the maximization of a social welfare function under aggregate feasibility constraints. There are, however, two distinguishing features to this economy. First, the welfare weights in the social welfare function are time-varying and this risk is uninsurable. Second, there is limited financial market participation: workers do not trade financial assets. Relative to a standard business cycle model, our economy features an additional source of uncertainty, resulting in variations in the shares of income going to capital and labor. Yet, the distribution of income risk in our economy is period-by-period Pareto optimal. Our specific modeling of labor relations is borrowed from Danthine and Donaldson (2002). The equivalent central planning formulation may, however, be interpreted as summarizing a broader class of labor market arrangements.

We study the extent to which these considerations allow the model to replicate the basic financial stylized facts. These include not only the mean equity and risk-free returns (and thus the equity premium; cf. Mehra and Prescott (1985)), but also their respective standard deviations and correlations with aggregate consumption growth. Everywhere the standard Lucas (1978) asset pricing methodology is employed. As an added model discipline, we also examine the extent to which the aggregate variations attendant to the uncertainty in factor shares are consistent with the observed properties of the business cycle.

The outline of the paper is as follows. Section 2 proposes a simple model of risk sharing which gives rise to variation in factor shares. Section 3 reports the outcome of numerically solving the model and displays the impact of distribution risk in an economy with no other source of shocks. In Section 4 we add aggregate uncertainty. We show that the financial properties of the model are, to a large extent, determined by the characteristics of the income share shock while the macroeconomic properties follow from the properties assumed for the aggregate technology shock. Section 5 discusses the relative importance of various parameters. Section 6 contrasts our economy with one where factor share variations are purely technology driven. Section 7 offers an alternative motivation - non-traded idiosyncratic income shocks - for the variable factor share mechanism. While the number of studies seeking to explain the financial stylized facts is already very large (see Kocherlakota (1996) and Mehra and Prescott (2003) for excellent surveys), few have focused purely on distribution considerations alone. We review the related theoretical and empirical literature in Section 8. Section 9 concludes the paper.

2 The Model Economy

Imagine an economy with two agents, a worker and a rentier shareholder/bondholder. The worker's utility is denoted by $v(W)$, where W stands for his consumption level, the shareholder's utility is $u(C)$, with C being his consumption level. The worker inelastically supplies one unit of labor. The capital stock is K, investment I, the per-period rate of capital depreciation Ω, and the production function $f(\cdot)\lambda_t$ with λ_t a stochastic shock to the technology. The aggregate constraints of this economy are standard:

$$\begin{aligned} &C_t + W_t + I_t \leq f(K_t, 1)\lambda_t \\ &K_{t+1} = (1 - \Omega)K_t + I_t, K_0 \text{ given.} \end{aligned}$$

Now assume there is a benevolent central planner maximizing a weighted sum of the two agent's utilities given these aggregate constraints with the distinguishing feature that the welfare weight μ is a random variable whose stochastic process is known but exogenously given to the central planner.

$$\max_{\{C_t, I_t, W_t\}} E_0 \left\{ \sum_{t=0}^{\infty} [\tilde{\mu}_t v(W_t)) + u(C_t)] \right\} \tag{1}$$

This is the economy we will investigate. In effect we interpret the observed variations in factor income shares as the result of a bargaining process between capitalists and workers. We will show that such an economy not only rationalizes the evolution of income shares but also possesses many features of the real economy. Adding the hypothesis that one class of agents only - shareholders - price the outstanding financial assets, we will show that the implied distribution risk is of first-order significance for the properties of the equity market.

In the section that follows, we propose a specific decentralized interpretation for such an economy, borrowed from Danthine and Donaldson (2002).

2.1 Workers

We postulate a continuum of workers distributed on $[0, 1]$ who each supplies one unit of labor inelastically and consumes his wage. This extreme assumption forces any worker income smoothing activities to remain in the context of their employment relationship with the firm. Workers are viewed, somewhat paternalistically, as permanent members of the firm with which they have a lifetime employment relationship. The essence of the employment contract is that workers supply one unit of labor in exchange for a wage that is substantially less variable than their marginal productivity. Note that, by the nature of the contract to be analyzed, a worker's income risk cannot be decreased by his participation in the financial markets.

Accordingly, workers solve

$$\max_{\{c_t^w, n_t^w\}} E_0 \sum_{t=0}^{\infty} \beta^t v\left(c_t^w\right) \text{ s.t} \tag{2}$$
$$c_t^w \leq w_t n_t^w$$
$$n_t^w \leq 1$$

In the above problem $v(\cdot)$ denotes a representative worker's period utility function, β his subjective discount factor and c_t^w his period t consumption; w_t is his period t wage and n_t^w his period t labor supply. Conforming to notational custom, in this and all other problem formulations, E_t stands for the period t expectations operator under rational expectations.[2]

The solution to this problem is particularly simple:

$$\begin{aligned} c_t^w &= w_t \\ n_t^w &= 1; \end{aligned}$$

That is, workers consume their wages and work their full time endowment.

2.2 Shareholders

A continuum of shareholders indexed to the unit interval is also assumed. Shareholders are rentiers; they consume their dividend and interest income payments and trade securities in the financial markets. They own all the securities "traded" in the economy and do not supply labor services. Their problem is:

$$\max_{\{c_t, z_t, b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{3}$$
s.t.
$$c_t + q_t^e z_{t+1} + q_t^b b_{t+1} \leq \left(q_t^e + d_t\right) z_t + b_t$$

[2] We assume all relevant information is public knowledge. For ease of exposition we specify the process on the relevant state variables, $dF(\cdot)$, when we spell out the problem of the firm. The expectations operators at the individual worker and shareholder levels apply to stochastic processes - on wages, dividends and consumption - that are direct transformations of $dF(\cdot)$.

In this decision problem $u(\cdot)$ denotes the period utility function of the representative shareholder and c_t his period t consumption. For simplicity, the shareholder's subjective discount factor β coincides with that of the worker. The shareholder's decision variables, z_t and b_t, denote, respectively, his period t stockholdings and bondholdings. The corresponding period t prices of these securities are q_t^e and q_t^b, the former with an associated period t dividend payment d_t. The bonds considered are one period discount bonds (paying one unit of consumption with certainty after one period).

Under standard concavity and differentiability assumptions, the necessary and sufficient first order equations for Problem (3) are

$$u_1(c_t)\,q_t^e = \beta E_t\left\{u_1(c_{t+1})\left[q_{t+1}^e + d_{t+1}\right]\right\} \tag{4}$$

$$u_1(c_t)\,q_t^b = \beta E_t\left\{u_1(c_{t+1})\right\} \tag{5}$$

Equation (4) has the unique non-explosive solution.

$$q_t^e = E_t \sum_{j=1}^{\infty} \beta^j \frac{u_1(c_{t+j})}{u_1(c_t)} d_{t+j}. \tag{6}$$

2.3 The firm

For notational simplicity, there is one firm that behaves competitively and lives forever. The capital structure of this firm is composed of one perfectly divisible share and b one period default free bonds.[3]

With homogenous firm owners, the firm's objective clearly is to maximize its pre-dividend stock market value, $d_t + q_t^e$, on a period by period basis. The key decision variable is the level of investment i_t, given that a long-term labor contract defines the relation between the firm and its workers. The latter stipulates that in exchange for delivering one unit of labor per period for their lifetime, workers receive a wage income which corresponds to an optimal risk sharing arrangement with firm owners. The firm's decision problem may thus be represented as:

[3]With our focus on multiperiod labor market arrangements and the presence of debt in the firm's capital structure, it would be inconsistent to admit a one period firm as commonly the case in this literature.

$$\max_{\{i_t,n_t\}} d_t + q_t^e \equiv d_t + E_t \left\{ \beta \frac{u_1(c_{t+1})}{u_1(c_t)} \left(q_{t+1}^e + d_{t+1} \right) \right\} \tag{7}$$

s.t.

$$d_t = f(k_t, n_t)\tilde{\lambda}_t - n_t w_t - i_t - b + b q_t^b \tag{8}$$

$$k_{t+1} = (1 - \Omega) k_t + i_t; \quad k_0 \text{ given} \tag{9}$$

$$n_t = 1$$

$$\tilde{\mu}_t v_1(w_t) = u_1(c_t) \tag{10}$$

$$dF(\lambda_{t+1}, \mu_{t+1}; \lambda_t, \mu_t) \text{ and initial values } \lambda_0, \mu_0 \text{ given.}$$

In equation (8), $f(\cdot)$ is the firm's (increasing, concave, and differentiable) production technology, λ_t is an aggregate shock to productivity, n_t denotes the period t level of hours (employment) engaged by the firm, k_t is period t capital stock (with depreciation rate Ω) and i_t its period t investment. Dividends, d_t, are output less the aggregate wage bill $(n_t w_t)$, the net interest payment $(b - bq_t^b)$ and the level of investment.

Equation (9) is the standard equation of motion on the firm's capital stock. Constraint (10) summarizes the terms of the contract. It is one designed to effect optimal risk sharing between workers and firm owners on a period by period basis, that is, once μ_t has been determined. As the only non-standard element in this arrangement we postulate that a new value μ_t is drawn at the end of each period (that is, at the end of $t-1$ for μ_t). This new value of μ applies for the next period and is taken as a given by both parties. We mean this to represent the hypothesis that the relative weight of the two classes of agents is determined in a process that, to a large extent, escapes the economic sphere. We thus take the view that the low frequency movements in the wage share are the outcome of interactions taking place at the social and political levels and that we do not attempt to model. When at the negotiating table, economic agents make sure that income risk is efficiently allocated taking that reality as a given.

We capture the uncertainties inherent in this process by postulating that $\tilde{\mu}_t$ follows an exogenously specified stochastic process. The joint conditional density of $(\tilde{\lambda}_t, \tilde{\mu}_t)$ is given by $dF(\lambda_{t+1}, \mu_{t+1}; \lambda_t, \mu_t)$. The parameter μ_t determines not only the average consumption shares going to the agents in this economy but also their relative variability. The calibration of the stochastic process governing the value of μ_t will be guided by the desire to replicate the variation in income shares observed in the US economy.

The necessary and sufficient first order conditions for the firm's problem are :

$$\begin{aligned} u_1(c_t) &= \beta E_t \{u_1(c_{t+1})[f_1(k_{t+1}, n_{t+1})\lambda_{t+1} + (1-\Omega)]\} \qquad (11) \\ \mu_t v_1(w_t) &= u_1(c_t) \\ n_t &= 1. \end{aligned}$$

2.4 Equilibrium

Let capital letters denote equilibrium quantities; market clearing then requires that the following conditions be satisfied. In all cases the variable θ indexes the continua of economic participants.

$$\begin{aligned} n_t &= \int_0^1 n_t^w d\theta = \int_0^1 1 d\theta \equiv 1 \qquad (12) \\ \int_0^1 c_t^w d\theta &= \int_0^1 w_t d\theta \equiv W_t \\ \int_0^1 c_t d\theta &\equiv C_t \\ z_t &= \int_0^1 z_t d\theta = 1 \\ b &= \int_0^1 b d\theta = B \\ k_t &\equiv K_t \text{ and } i_t \equiv I_t. \end{aligned}$$

In equilibrium, aggregate variables are related according to

$$Y_t = f(K_t, 1)\lambda_t = C_t + W_t + I_t. \qquad (13)$$

Total consumption $C_t + W_t$ will be later labeled TC_t. In this economy workers are explicitly (though not implicitly) passive and the shareholder's activities have the effect only of pricing financial assets after receiving their dividend and bond income. Equilibrium is thus characterized by imposing the relevant market clearing conditions on the optimality conditions of the firm. This yields

$$u_1(C(s_t)) = \beta \int u_1(C(s_{t+1}))[f(K_{t+1}, 1)\lambda_{t+1} + (1-\Omega)] dF(\cdot) \qquad (14)$$

$$\mu_t v_1(W(s_t)) = u_1(C(s_t)), \qquad (15)$$

where we explicitly recognize the dependence of all the equilibrium quantities on the economy's state variables $s_t \equiv (K_t, \lambda_t, \mu_t)$.

More formally we define equilibrium as follows :

Definition : Equilibrium for the economy defined in equations (2) - (7) is a triple of functions $C(s_t), W(s_t)$, and $I(s_t)$ which jointly satisfy equations (13), (14) and (15).

It may be observed in particular that equations (14) and (15) are the necessary and sufficient first order conditions for the functional equation

$$\begin{aligned} J(s_t) &= \max_{C(s_t),I(s_t)} \{\mu_t v\left[f(K_t,1)\lambda_t - C(s_t) - I(s_t)\right] \\ &\quad + u(C(s_t)) + \beta \int J\left[(1-\Omega)K_t + I(s_t), \lambda_{t+1}, \mu_{t+1}\right] dF(\cdot)\}, \end{aligned} \tag{16}$$

which, in turn, is the functional equation for the central planning formulation of our model:

$$J(s_0) = \max_{\{C(s_t),I(s_t),W(s_t)\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\mu_t v\left(W(s_t)\right) + u(C(s_t))\right] \right\} \tag{17}$$

$$\begin{aligned} \text{s.t.} \quad & C(s_t) + I(s_t) + W(s_t) \leq f(K_t,1)\lambda_t \\ & K_{t+1} = (1-\Omega)K_t + I_t, \quad K_0 \text{ given.} \\ & \lambda_0, \mu_0, dF(\cdot) \text{ specified a priori.} \end{aligned}$$

Representation (17), which is nothing else that problem (1), confirms that all mutually advantageous exchanges of risks between workers and shareholders are effected, but for the impossibility of their contracting ex ante against future idiosyncratic income variation as their respective bargaining powers wax and wane.

In this economy the standard Lucas (1978) asset pricing methodology is employed to value financial assets; in particular, given the shareholders equilibrium consumption function, $C(s_t)$, the equity and risk free debt security prices are computed as the solution to the following equations, respectively:

$$q^e(s_t) = \beta \int \frac{u_1(C(s_{t+1}))}{u_1(C(s_t))}[q^e(s_{t+1}) + d(s_{t+1})]dF(\cdot) \tag{18}$$

and

$$q^b(s_t) = \beta \int \frac{u_1(C(s_{t+1}))}{u_1(C(s_t))} dF(\cdot), \tag{19}$$

where $d(s_t)$, the aggregate dividend, satisfies

$$d(s_t) = f(K_t,1)\lambda_t - W(s_t) - I(s_t) - B + q^b(s_t)B. \tag{20}$$

Lastly, the period-by-period returns to the ownership of these securities are constructed according to,

$$1 + r^e_{t,t+1} = \frac{q^e(s_{t+1}) + d(s_{t+1})}{q^e(s_t)}, \text{ and} \tag{21}$$

$$1 + r^b_{t,t+1} = \frac{1}{q^b(s_t)} \tag{22}$$

2.5 Numerical Procedures and Calibration

Using standard discrete space methodologies, we solve expression (16) by value function iteration (see Christiano (1988)) to obtain accurate approximations to the economy's optimal policy functions $C(s_t)$ and $I(s_t)$. Once the values of capital stock which identify the economy's long run stationary distribution and the equilibrium consumption and dividend functions have been identified, solving (18) and (19) to obtain the equilibrium stock and bond prices amounts to solving a system of linear equations. All financial and business cycle statistics are computed on the basis of artificially constructed return series of 400,001 periods in length.

In all cases the shareholder's period utility function is hypothesized to be logarithmic, $u(C) = \ln(C)$, while the period utility function of the representative worker is postulated as $v(W) = \frac{W^{1-\gamma}}{1-\gamma}$, with γ assuming a variety of values. The production function common to all simulation runs is the customary $f(K, 1) = LK_t^{\alpha}$, with L a scale parameter. The parameter α is typically calibrated to reproduce the observed share of capital in total value added. Estimations on this number vary: the most commonly used value is .36. Cooley and Prescott (1995) justify a higher value of .4 by the inclusion of a measure of imputed income for government capital. Gollin (2002) argues that this number is likely to be too high because of improperly accounting, as capital income, of the labor income of the self-employed. The various adjustments he proposes to correct for this leads him to estimating the 1992 U.S. capital income share in a range [.23 -.34] (instead of the .4 obtained for that particular year under a naive calculation). In our economy, income shares are jointly determined by the value of the parameter α, the average value of μ and the risk aversion parameter γ. Our basic scenario will be one where the capital share is .3 ($\alpha = .3$) and where $\{\mu_t\}$ is such that the average wage share matches its competitive counterpart. We then study the impact of alternative values of μ which represent circumstances where workers are effectively paying a premium for income insurance provided by firm owners and thus receive a lower average share of value added.

Lastly, we make two distinct assumptions for the shock process on the share parameter. We first explore the implications of an economy with no technology shock ($\lambda_t \equiv 1$) where μ_t is assumed to follow a two state Markov chain; that is,

$$dF(\mu_{t+1};\, \mu_t) = \begin{array}{c} \\ \mu_1 \\ \mu_2 \end{array} \begin{array}{c} \begin{array}{cc} \mu_1 & \mu_2 \end{array} \\ \left[\begin{array}{cc} \pi & 1-\pi \\ 1-\pi & \pi \end{array} \right] \end{array}. \tag{23}$$

In this case, μ_1, μ_2 and π are chosen to result in a process on factor shares which reasonably corresponds to the data.

We then complete the model by allowing for aggregate uncertainty as well. A joint stochastic process on the risk sharing parameter μ and the multiplicative technology shock parameter λ is assumed. Together they follow a four state Markov chain with transition matrix:

$$dF(\lambda_{t+1},\ \mu_{t+1};\ \mu_t\ \lambda_t) : \begin{array}{c} \\ (\lambda_1,\mu_1) \\ (\lambda_1,\mu_2) \\ (\lambda_2,\mu_1) \\ (\lambda_2,\mu_2) \end{array} \begin{array}{c} \begin{array}{cccc} (\lambda_1,\mu_1) & (\lambda_1,\mu_2) & (\lambda_2,\mu_1) & (\lambda_2,\mu_2) \end{array} \\ \left[\begin{array}{cccc} \psi & \pi & \sigma & H \\ \pi+\Delta & \psi-\Delta & H & \sigma \\ \sigma & H & \psi-\Delta & \pi+\Delta \\ H & \sigma & \pi & \psi \end{array} \right] \end{array} \tag{24}$$

This choice of matrix admits a high degree of flexibility. Parameters $\lambda_1, \mu_1,\ \lambda_2, \mu_2$, ψ and σ principally determine the behavior of the technology shock and their values are chosen so that the economy's equilibrium output series, when subjected to the Hodrick-Prescott (HP) detrending procedures, matches the standard deviation of output for the U.S. economy as well as its first order autocorrelation. The remaining parameters are selected to replicate the time series properties of the wage share.

The targeted income share data can be described as follows. Let Y_t denote output in period t. For the period 1947.1 – 1998.1, the behavior of the wage share through time is well approximated by a first order autoregressive process with corr $[\ln\left(\frac{W}{Y}\right)_t, \ln\left(\frac{W}{Y}\right)_{t-1}] =$ .974, SD($\ln\left(\frac{W}{Y}\right)_t$) = 3.45%, and corr $[\ln\left(\frac{W}{Y}\right)_t, Y_t^{HP}] = -.053$. The average value of the wage share in this data series is .57. As explained before, this value needs to be adjusted for mis-measurement problems. We use a benchmark value of .7. All statistics apply to U.S. data.

3 An Economy with Distribution Risk only

We first study the properties of our economy in the absence of technology shocks, i.e., with distribution shocks exclusively. The corresponding transition matrix is (23). Table

(1) below presents a sampling of results for one possible (μ_1, μ_2) pair; analogous results for the U.S. economy and for a classic pure real business cycle (RBC) study (Hansen (1985)) are provided for comparison purposes.

The intention of this exercise is purely suggestive. The RBC literature tells us that in the absence of aggregate shocks it would be surprising to match the macro data adequately, and Table 1 confirms this message. Yet given this very artificial context, the financial results displayed in Table 1 are surprisingly representative. While still falling short of the analogous U.S. figure by a substantial margin, the 2.32% premium is extremely high relative to what is typically obtained for this class of models (see Kocherlakota (1998)).[4] This improvement comes from both the equity and risk free returns with the former increasing and the latter decreasing by roughly the same amount relative to the mean 4% value typically observed for both securities in this class of stationary models with $\beta = .99$. With respect to the various return standard deviations, the match is also quite good: the risky return and the premium are only about 7% more volatile than what is observed in the data, and the risk free rate only 40% less so. Dividend volatility and the correlation of equity return with consumption growth are also close to what is observed. All in all, the present exercise suggests that distribution risk may well be a first-order importance determinant of financial returns. This message, however, needs to be confirmed in a more realistic economy.

4 Adding Aggregate Uncertainty

We now add aggregate uncertainty to distribution risk. The corresponding probability transition matrix is described by (24). The results of this exercise are presented in Table 2, where Case 1 admits variation in both the share parameter μ and the technology shock λ (aggregate uncertainty) while Case 2 admits aggregate uncertainty alone.

With plausible aggregate shocks the model economy may now be evaluated in the light of business cycle data. Case 2 reproduces the results routinely obtained in the business cycle literature. Case 1 demonstrates that distributional risk does not disturb the performance of the macroeconomy. The following differences are worth noticing, however: distribution risk adds some extra volatility in aggregate consumption and investment. It should be viewed positively as consumption is excessively smooth in the standard model. For this benchmark calibration there is excessive wage volatility suggesting that there

[4]If we shift the basis of comparison to a sample of non-U.S. countries, this particular result looks even better: Goetzmann and Jorion (1998) report that for a sample of non-U.S. countries the premium averages 3.5% when adjusted to factor out the effects of war-related market interruptions.

Table 1
The Benchmark Case: Pure Distribution Risk

Panel A: Financial and Wage Share Statistics

	U.S. data[iii]			Hansen's (1985) Model			Risk Sharing Economy[i]		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
r_e	6.98	18.2	.23	4.11	.52	.36	5.32	17.66	.35
r_f	.80	5.67	.12	4.10	.37	.06	3.00	3.42	-.42
r_p	6.18	19.1		.01	.27		2.32	17.68	
d_{t+1}/d_t		11.98			9.36			13.44	
W/Y	57	3.45		64	0		.70	4.13	
		(d)			(d)			(d)	
corr($\ln W/Y, Y^{HP}$)		-.053			0			-.0073	
corr($\ln(W/Y)_t, (\ln W/Y)_{t-1}$)		.97			0			.92	
corr($r_{e,}, C_{t+1}/C_t$)		-			-			.69	
corr($r_{e,}, TC_{t+1}/TC_t$)[ii]		.06			.78			-.54	
	Panel B: Aggregates								
	(e)	(f)		(e)	(f)		(e)	(f)	
output	1.76	-		1.79	-		.07	-	
total consumption	1.29	.85		.54	.88		.75	.176	
shareholder consumption	-	-		-	-		9.78	.025	
investment	8.60	.92		5.78	.99		2.72	-.055	
wages				.54	.88		1.90	.063	
capital stock	.63	.04		1.35	.98		.24	1.00	

(i) $\mu_1 = 73$, $\mu_2 = 228$, $\alpha = .30$, $\beta = .99$, $\Omega = .025$, $\gamma = 3$, corr $(\mu_t, \mu_{t-1}) = .98$, $\pi = .98$, $B \equiv 0, L = 1.25$

(ii) In the case of Hansen (1985), this is the figure for representative agent consumption.

(iii) Data sources: Mehra and Prescott (1985), Mankiw and Zeldes (1991), Mehra (1998), Campbell (1999), Kocherlakota (1996), Bansal (2004).

(a) expected values in percent, annual frequency

(b) standard deviation in percent, annual frequency

(c) correlation with growth rate of output, annual frequency

(d) indicated correlations: wage share on a quarterly frequency; others annualized

(e) standard deviation in percent

(f) correlation with output

Table 2

Adding Aggregate Shocks

Panel A: Financial and Wage Share Statistics

	Case 1			Case 2		
	(a)	(b)	(c)	(a)	(b)	(c)
r_e	5.83	22.11	.48	4.21	4.78	.68
r_f	2.71	4.40	.09	4.04	1.68	.13
r_p	3.12	21.88		.17	4.25	
d_{t+1}/d_t		16.57			3.90	
W/Y	70	5.69		71	2.17	
		(d)			(d)	
$\text{corr}(\ln W/Y, Y^{HP})$		-.07			-.04	
$\text{corr}(\ln(W/Y)_t, (\ln W/Y)_{t-1})$		.92			.89	
$\text{corr}(r_{e,}, C_{t+1}/C_t)$		.69			.79	
$\text{corr}(r_{e,}, TC_{t+1}/TC_t)^{(ii)}$		.06			.79	
	Panel B: Macro-aggregates					
	(e)	(f)		(e)	(f)	
output	1.77	-		1.77	-	
total consumption	1.16	.62		.87	.95	
shareholder consumption	11.35	.62		2.75	.95	
investment	6.77	.85		5.41	.98	
wages	1.97	-.003		.69	.95	
capital stock	.59	.06		.48	.06	

Columns (a) - (f); same interpretation as Table 1

Both cases: $\alpha = .30, \beta = .99, \Omega = .025, \gamma = 3$

Case 1: $\lambda_1 = 1.056, \lambda_2 = .944, \mu_1 = 73, \mu_2 = 228, \psi = .98376, \pi = .00133, \sigma = .00533, H = .00967, \Delta = .02, L = 1.25$ yielding $\rho_{\lambda_t \mu_t} = -.61$, $\rho_{\lambda_t \lambda_{t+1}} = \rho_{\mu_t \mu_{t+1}} = .97$.

Case 2: $\mu_1 = \mu_2 = 150.5 = \frac{73+228}{2}$

is insufficient income insurance. More interestingly, wages are acyclical in Case 1 while they are strongly pro-cyclical in the more standard Case 2. The cyclicity of wages has been the subject of much controversy since Dunlop (1938) and Tarshis (1938) provided evidence suggesting that wages are nearly acyclical. Some have seen this observation as a source of falsification of the standard real business cycle model. Our interpretation of the labor market provides a potential resolution of this puzzle.

On the financial front, the performance of the complete model is extremely good delivering an equity premium of 3.12%.[5] By contrast, Case 2 confirms that the standard real business cycle model is entirely prone to the equity premium puzzle, delivering a very low premium of .17%.[6]

As in the pure distribution risk economy, the volatility of equity returns and of the premium is somewhat in excess of what is observed while the volatility of the risk free return is slightly too low. It is clear that these volatilities are, to a large extent, attributable to the share variations.

It may be argued that this model is falsified by the excessive dividend volatility relative to what is observed. Before accepting this conclusion one should, however, consider that the 11.98% figure reported in Table 1 for the US economy corresponds to the distributed dividends series which is a smoothed out series. In our artificial economy, there are no retained earnings and no dividend smoothing. Our dividend series is closer to a free-cash-flows series which is, inevitably, more variable.

The wage share statistics are of special interest since they bear on the specifics of our model. As to the volatility of the wage share itself, it may come as a surprise that, even in the absence of variation in μ, substantial share variation (W/Y) is observed (Table 2, Case 2, SD(W/Y) = 2.17%). In a more standard representative agent model with our production technology and competitively determined factor share, the SD(W/Y) $\equiv 0$, irrespective of the degree of output uncertainty. The result in Case 2 obtains because the risk sharing contract ($\mu v_1(W_t) = u_1(C_t)$) does not imply a linear relationship between the increase in output net of investment and the resulting marginal increases in worker and shareholder consumption. This is attributable to the differences in the two agents' degrees of relative risk aversion. One sees from Table 2 that, at -.07 the Corr ($\ln W/Y, Y^{HP}$) is close to the observed -.04 while the autocorrelation of $\ln W/Y$ is a little

[5] Note that this is an unlevered equity return.

[6] In a purely competitive model with only technology shocks - e.g. Hansen (1985) - we would anticipate even a much smaller value. The .17% is attributable to the large technology shocks (relative to Hansen (1985)) necessary to get the output variation to a level that corresponds to the data. That such larger shocks are required is due to the high average level of risk aversion in the economy since the fraction of income to the more risk averse workers ($\gamma = 3$) is 70%.

on the low side (at .92 vs. .97 for the observations). The most significant difference on this score between Cases 1 and 2 is the dramatic improvement in the correlation between equity return and aggregate consumption: it is too high in the standard RBC model (at .79) and almost exactly correct in our benchmark case.

All in all, the economy with both distribution risk and aggregate shocks performs extremely well over an unusually wide range of financial and macroeconomic statistics. Specifically, it does contribute a large equity premium for a production model that otherwise behave remarkably close to the US economy. It also appears that the financial characteristics of the model are largely determined by the income sharing mechanism, while the business cycle properties are, to a large extent, determined by the technological uncertainty. For this specification at least, the incorporation of either source of uncertainty in the same model reinforces the attractive features of the other.

The intuition for our results is relatively straightforward. With a negative correlation between μ and λ, justified by the observed counter-cyclicity of the wage share (corr(μ_t, λ_t)=-.63 in the benchmark case), low productivity shocks coincide roughly half the time with a high μ_t realization, that is, with situations where the bargaining power of capitalists is low. In these circumstances, the normally low payment to capital owners is further reduced by the above average income share going to labor, and vice versa in periods of high productivity shocks. These events have the consequence of making capital fundamentally riskier. As a result, stocks are less attractive to investors, bonds more attractive, and the premium rises.

It is of interest to observe that our results so far have been obtained in the absence of a reinforcing financial leverage effect: in all simulations $B = 0$, so that the bond is being priced and its return determined in zero net supply. Table 3 reports the results obtained for the benchmark case modified to include various (modest) levels of financial leverage. Only the financial results are reported, as Modigliani-Miller obtains and the real variables are unchanged relative to the benchmark. Note that the properties of the risk free return are unchanged as well: while financial leverage modifies the properties of dividends, it does not alter the consumption of shareholders whose total income (interest + dividends) is unaltered. The equity premium increases from 3.12% to 3.64% for $B = 3$. The most significant impact of financial leverage is on the standard deviation of dividend growth and of equity returns which both increase to very high levels for the higher values of B. It is clear that the import of financial leverage pales alongside the role played by operating leverage. Note for instance that Case 1 of Table 5 produces an equity premium of 7.78% as opposed to the premium of 3.64% obtained with financial leverage for an almost identical level of dividend volatility. We do not explore financial leverage ratio above $B = 3$ (or a debt-equity ratio of 11%) because, beyond this debt level, dividends are occasionally negative and they become excessively volatile.

Table 3

Adding Financial Leverage

	Various B					
	B=1 ($\frac{D}{E} = 3.5\%$)		B=2 ($\frac{D}{E} = 7\%$)		B=3 ($\frac{D}{E} = 11\%$)	
	(a)	(b)	(a)	(b)	(a)	(b)
r^e	5.99	23.46	6.16	25.09	6.36	26.86
r^f	2.71	4.40	2.71	4.40	2.71	4.40
r^p	3.27	23.25	3.45	24.82	3.64	26.69
$\frac{d_{t+1}}{d_t}$		18.86		22.31		27.54

Note: (a) average values; (b) standard deviations

A final insight is forthcoming if one writes down the representative agent model directly analogous to the risk sharing construct as presented in problem (17):

$$\max_{\{C(s_t),I(s_t)\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(C(s_t))\right] \right\} \tag{25}$$

$$\begin{aligned} \text{s.t.} \quad & C(s_t) + I(s_t) \le f(K_t, 1)\lambda_t \\ & K_{t+1} = (1-\Omega)K_t + I_t, \quad K_0 \text{ given.} \\ & \lambda_0, \mu_0, dF(\cdot) \text{ specified a priori.} \end{aligned}$$

Problems (17) and (25) (similarly parameterized; i.e., for both models $\alpha = .30, \beta = .99, \gamma^{\text{rep.agent}} = \gamma^{\text{shareholder}} = 1$ and the same stochastic process on $\{\lambda_t\}$) have identical steady state capital stocks, the same aggregate consumption, etc. It thus follows that

$$\begin{aligned} C_t &= f(K_t, 1)\lambda_t - W_t - I_t \\ &\sim C_t^{\text{rep.agent}} - W_t. \end{aligned} \tag{26}$$

Equation (26) effectively means that the wage bill serves a role similar to a postulated external habit. Relative to the representative agent construct, the wage bill is the postulated habit, and shareholder consumption in Problem (17) is nothing more than the representative agent consumption in excess of the "habit".

There are a number of dimensions along which the risk sharing "wage bill-habit" replicates the complex non-linear habit evolution process found in the seminal paper by

Campbell and Cochrane (1999). One of them is the cyclical pattern in risk aversion obtained in both models. The effective underlying mechanism is very different, however. In Campbell and Cochrane (1999), an external moving habit is postulated to express the desire of investors to maintain their relative societal consumption position ("keeping up with the Jones", see Abel (1990)). For the risk sharing model, this feature need not be an hypothesized appendage, but rather arises entirely endogenously. For both models, the asset pricer's consumption and his habit co-vary positively with output, although in the risk sharing cases this also requires either (i) low risk aversion on the part of workers (cf. Table 4, cases with $\gamma = 1$, and $\gamma = 2$), or (ii) non negative correlation between μ and λ (Table 3, left most case). In addition the "wage bill-habit" is less variable than the analogous representative agent consumption series.

In the following section the sensitivity of our results to parameter changes is examined as a first step to gaining a better understanding of why they are observed in the first place.

5 Comparative Dynamics and Welfare Assessment.

The message of the two previous sections has been that distributional considerations – the relative shares of income going to capital and labor – are highly influential in determining the financial characteristics of equilibrium in a way that is in general harmony with business cycle phenomena. In this section we explore how these results are influenced by the various model parameters, in particular by γ, μ, E_μ, and Corr (λ_t, μ_t), Corr (μ_t, μ_{t+1}).

5.1 Changing in $\rho_{\lambda\mu}$

Intuitively one expects that an important determinant of our results is the correlation between the two shocks. In particular, this correlation significantly determines the inherent riskiness, for the capital owners, of the labor market arrangements characteristic of this economy. The negative correlation adopted in our benchmark (meaning that capitalists' weight in the social welfare function is low precisely when the economy is less productive) is motivated by the observation that the wage share is countercyclical, and indeed this countercyclicity is reflected in the results of Table 2. Table 4 confirms our intuition. The first case considered is one where $\rho_{\lambda\mu} = 0$ and indeed in that case the premium decreases to (a still respectable) 2.52%. This lower premium is explained by a decrease in the volatility of shareholder's consumption and dividend growth. The wage share remains countercyclical (a result explained by the mechanism discussed in the previous sub-section for the case where there are no distribution shocks) but the correlation falls in absolute value. And the correlation between equity returns and aggregate consumption becomes counterfactually negative. Conversely a $\rho_{\lambda\mu}$ more negative than in the benchmark case produces an increase in shareholders' consumption volatility, in dividend volatility and higher premia. It is worth noticing that the volatility of aggregate consumption and investment (but not GDP) are affected as well. Although the correlation between aggregate consumption and equity returns is excessive, it has the attractive consequence of producing a remarkable 3.89% equity premium.

5.2 Changes in γ and in E_μ.

For our choice of functional forms the income sharing condition, equation (10) reduces to

$$\mu^{-1}C_t = W_t^{\gamma}, \quad \gamma > 0.$$

For a fixed μ, a smaller value of γ implies a larger value of W_t, provided $W_t > 1$, which is the case for all scenarios considered in this paper. This implies that as workers become less risk averse, the share of income going to workers increases. As a consequence, shareholder consumption growth volatility increases while simultaneously being restricted to a region of greater utility function curvature: the operating leverage effect is thus stronger. This reduces the demand for risky stocks, increases the demand for risk free assets, and increases the premium.

This intuition is explored in Table 5 where we report the financial and aggregate statistics for scenarios of varying γ, centered on our benchmark of $\gamma = 3$. Confirming our intuition, the fraction of income going to workers is greatest when they are less risk averse. In fact, the mean consumption of the firm owners in the $\gamma = 1$ case (.065) is less than one seventh its value when $\gamma = 4$ (.4725). As a result, shareholders' consumption growth (dividend growth) is much more volatile, thus increasing the premium to a level of 5.74% in the $\gamma = 2$ case and even an excessive 7.78% in the $\gamma = 1$ case. Another direct consequence of this observation is the increased volatility of the return on equity and the premium, which now lies at the upper bounds of acceptability. We observe that $\text{corr}((W/Y)_t, Y_t^{HP})$ and $\text{corr}(\ln(W/Y)_t, \ln(W/Y)_{t-1})$ are largely unaffected by the changes in γ, while correlation between equity returns and aggregate consumption turns more positive, as desired to match the observations when γ decreases.

On the macroeconomic side, there are no major effects of varying the parameter γ. The most significant impact is on the properties of wages. As workers' risk aversion decrease, wages become more highly pro-cyclical and their volatility falls. The latter may be seen as somewhat of a curiosum: as the workers become *less* risk averse, their

Table 4

Comparative Dynamics: Changes in $\rho_{\lambda\mu}$

Panel A: Financial and Wage Share Statistics

	$\rho_{\lambda\mu} = 0$			$\rho_{\lambda\mu} = -.96$			$\rho_{\lambda\mu} = -1$		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
r_e	5.43	18.42	.21	6.20	24.93	.68	6.24	25.26	.71
r_f	2.92	3.74	.03	2.31	3.98	.14	2.26	3.93	.15
r_p	2.51	18.34		3.89	24.85		3.98	25.20	
d_{t+1}/d_t		14.10			18.46			18.72	
W/Y	70	4.73		70	6.33		70	6.41	
		(d)			(d)			(d)	
$\text{corr}(\ln W/Y, Y^{HP})$		-.04			-.11			-.11	
$\text{corr}(\ln(W/Y)_t, (\ln W/Y)_{t-1})$		.92			.91			.91	
$\text{corr}(r_{e,}, C_{t+1}/C_t)$		.70			.68			.68	
$\text{corr}(r_{e,}, TC_{t+1}/TC_t)$		-.06			.36			.44	

Panel B: Aggregates

	$\rho_{\lambda\mu} = 0$		$\rho_{\lambda\mu} = -.96$		$\rho_{\lambda\mu} = -1$	
	(e)	(f)	(e)	(f)	(e)	(f)
output	1.78	-	1.78	-	1.78	-
total consumption	1.15	.75	.57	.26	.43	.16
shareholder consumption	10.17	.26	12.41	.96	12.56	.99
investment	5.96	.87	8.37	.96	12.56	.99
wages	2.02	.33	1.51	-.71	1.44	-.93
capital stock	.53	.07	.72	.097	.74	.1

Columns (a) - (f), same as Tables (1), (2); all parameters conforms to those underlying Case 1 of Table 2 except:

- $\rho_{\lambda_t \mu_t} = 0$: $\psi = .97237, \pi = .01262, \sigma = .01487, H = .00013, \Delta = .03$
- $\rho_{\lambda_t \mu_t} = -.96$: $\psi = .98499, \pi = .00001, \sigma = .00051, H = .01449, \Delta = .02$
- $\rho_{\lambda_t \mu_t} = -1$: $\psi = .985, \pi = 0, \sigma = 0, H = .015, \Delta = 0$

Table 5

Comparative Dynamics: Changes in Workers' Risk Aversion γ

Panel A: Financial and Wage Share Statistics

	$\gamma=1$			$\gamma=2$			$\gamma=4$		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
r_e	8.32	38.59	.34	7.24	31.85	.40	4.97	14.77	.54
r_f	.54	8.58	.08	1.50	6.80	.08	3.42	2.90	.10
r_p	7.78	38.77		5.74	31.79		1.54	14.50	
d_{t+1}/d_t		27.75			23.24			11.34	
W/Y	77	2.29		74	3.79		65	6.67	
		(d)			(d)			(d)	
corr$(\ln W/Y, Y^{HP})$		-.06			-.06			-.06	
corr$(\ln(W/Y)_t, (\ln W/Y)_{t-1})$		.86			.90			.93	
corr$(r_{e,}, C_{t+1}/C_t)$		.63			.65			.72	
corr$(r_{e,}, TC_{t+1}/TC_t)$		.29			.16			.10	
				Panel B: Aggregates					
	(e)	(f)		(e)	(f)		(e)	(f)	
output	1.78	-		1.77	-		1.77	-	
total consumption	.83	.92		.94	.79		1.21	.60	
shareholder consumption	17.71	.47		15.29	.54		8.03	.72	
investment	5.78	.97		6.19	.92		6.78	.84	
wages	.81	.72		1.29	.26		2.24	-.12	
capital stock	.51	.09		.54	.07		.60	.05	

Columns (a) - (f), same as Tables (1), (2); all parameters except γ conform to those underlying Case 1 of Table 2.

consumption volatility *decreases*. Both results are easy to explain, however. When γ decreases, shareholders' bargaining power diminishes and their share of income falls. Workers' consumption (W) then makes up an increasing fraction of aggregate consumption and the properties of the two series (W and TC) naturally converge.

Note that these results further refute the assertion that a reasonable replication of the financial data requires the assumption of an excessive level of risk aversion. In our model construct, what appears to be key is the relative income shares and their respective volatilities.

5.3 Other comparative dynamic tests

In this section we briefly report on other tests for which we do not provide full tabular results.

As just argued, from the perspective of financial return characteristics, the main impact of the parameter γ is to help determine the relative income shares. This is even more obviously the case for the average μ and variation in this quantity can be expected to have the same effect (with an increase in μ going in the same direction as a reduction in γ). This intuition is confirmed in simulations. As for the case of changes in the risk aversion parameter, the size of the wage share is of overwhelming importance. With an increase in E_μ (workers become more successful as bargainers on average), the share of income to labor rises. As a further benefit to the negotiation success, the volatility of their wages also diminishes(in percentage terms). On the shareholders' side the reversed consequences together reinforce one another to push up the premium: not only does the fundamental income uncertainty more fully reside with the shareholder but also the income share reduction forces the risk to be borne by shareholders on the more concave portion of their period utility surface: relative to a representative agent economy there is a larger habit. As a result, shareholders behave in a more risk averse fashion, and attempt to reduce their holdings of the risky security and increase their holdings of the risk free one. This increases the premium, though at the price of excessive return volatility. For the benchmark Case 1 of Table 2, $Ec^s = .30$ corresponding to $E\mu = 150$; when $E_\mu = 75$ (same $\frac{\mu_1}{\mu_2}$ ratio), $Ec^s = .45$; if $E_\mu = 250, Ec^s = .21$. In the first case the premium is 2.05%; in the second it is 4.12%.

We also analyzed the effects of the time series characteristics of the share process on financial equilibrium, and, in particular its autocorrelation. The results of this exercise do not entirely conform to intuition. One would expect that the more persistent the variations in μ, the harder it would be for shareholders to bear the distribution risk and the higher the premium they would require for holding the risky asset. This is not systematically the case and the passage from $\rho_{\mu\mu} = .97$ to $\rho_{\mu\mu} = .5$ in fact increases the premium from 3.12% to 4.91% (The reference is once again Case 1 of Table 2 where $\rho_{\mu\mu} = .97$). The above change (to $\rho_{\mu\mu} = .5$) has, however, far reaching counterfactual consequences at the macro level. These are both difficult to interpret and lead to a categorical rejection of this parametrization. First, when the income sharing parameter becomes less persistent, its impact on macroeconomic volatility becomes much smaller and for technology shocks of the same size as in the benchmark, output becomes excessively smooth. Both shareholders' and total consumption relative volatility become excessive. More significatively, the autocorrelation of the (log-) wage share becomes much too low (.68) and the return to equity becomes strongly negatively correlated with consumption growth, two observations that disqualify this calibration.

5.4 Welfare Considerations

So far we have imposed on shareholders and workers a specific arrangement by which workers do not participate in financial markets but obtain income insurance from their employers. Besides having significant descriptive power, this form of arrangement has a long standing tradition in the labor market literature (under the heading "implicit contracts", see, e.g., Rosen, 1994). While such an arrangement can be viewed as resulting from the same non-economic forces that also lead to variations in the respective bargaining power of the two parties, one could equally well argue that, apart for these variations in μ, the underlying arrangement has been willingly agreed upon and is socially optimal, for example, because of the presence of significant fixed costs to financial market participation. If the latter interpretation is adopted, one would then want to argue that shareholders should be no worse off under the chosen arrangement than under the Walrasian alternative. This would imply that they should receive an appropriate premium as a compensation for the income insurance they provide.

In that spirit it is natural to ask what is the average value of μ (i.e. the average income share, maintaining the same relative μ_1-μ_2 variation) for which firm owners are as well off in our economy as in the corresponding Walrasian equilibrium. The latter is defined as the solution to the functional equation:

$$\begin{aligned} J(K_t) &= \max_{I_t}\{u(f(K_t,1)\lambda_t - I_t - W_t) + \beta \int J(K_{t+1},\lambda_{t+1})dF(\lambda_{t+1},\lambda_t)\} \quad (27) \\ K_{t+1} &= (1-\Omega)K_t + I_t;\ I_t \geq 0, \end{aligned}$$

with $\{W_t\}$ given. Walrasian equilibrium, in particular, is captured not only by regarding the wage sequence as exogenous from the firm owner perspective, but also by imposing on the first order condition to problem (27), the requirement that

$$W_t = (1 - \alpha)\, f(K_t, 1)\lambda_t. \tag{28}$$

Not surprisingly, incentive compatibility requires that the portion of output extracted by workers be somewhat smaller than what underlies Tables (2)-(5). Under problem (27)-(28), firm owners experience a smaller income uncertainty and they receive 30% of total output. In order to agree to an income sharing scheme with variable shares, their average share of income must increase. Our computations show that shareholders are as well off under the defined labor arrangement scheme (including variable income shares, i.e., distribution risk) if their share of income is increased by a little over 1% of GDP, i.e., is equal to 31% instead of 30%. This is obtained for $(\mu_1,\ \mu_2) = (61,\ 190)$. With this higher share of income to firm owners, and under our benchmark calibration for the other parameters, the equity premium is 2.80%. The entire set of statistics is reported in Table 6.

5.5 Explaining the Market Value to National Income ratio

In a recent article, Mehra (1998) has reminded us that the (1985) Mehra-Prescott asset pricing model is also unable to match the observed volatility of the q^e/NI (Value of market equity as captured by the S&P500 to national income ratio) for the U.S. economy. For the sample period (1929-1993) the value of this ratio is in the range [.45, 1.90]; for the postwar period (1946-1993) the corresponding range is [.48, 1.33]. It is striking that the standard Mehra-Prescott (1985) model, with logarithmic utility for the representative agent, generates a range [.54,.69] for the same ratio. We observe (Table 7) that the distribution risk model is able to generate vastly more variation in this important ratio. The benchmark case produces a multiple between the minimum and the maximum values of this ratio, at 3.4, that stands in between the value observed in the postwar period, 2.77, and the one obtained for the entire Mehra sample, 4.22. It is also worth noting that our model would suggest an alternative explanation to that of McGrattan and Prescott (2005) for the steep rise in the q^e/NI ratio observed since the 1960s in the U.S.. Whereas those authors attribute the phenomenon to less oppressive capital taxation, our model suggests that the observed reduction in the labor share could be responsible as well. The (modest) reduction in the labor share from .73 for the case $\mu_1 = 121; \mu_2 = 380$ to .70 for our benchmark, for example, produces an increase of 41% in the mean value of the ratio q^e/NI. This interpretation of the facts receives strong support from Boldrin and Peralta-Alva (2005).

Table 6

Incentive compatible case

Panel A: Financial and Wage Share Statistics

	$\mu_1 = 61, \mu_2 = 190$		
	(a)	(b)	(c)
r_e	5.65	20.70	.49
r_f	2.85	4.12	.08
r_p	2.80	20.51	
d_{t+1}/d_t		15.58	
W/Y	69	5.99	
		(d)	
$\text{corr}(\ln W/Y, Y^{HP})$		-.06	
$\text{corr}(\ln(W/Y)_t, (\ln W/Y)_{t-1})$		.92	
$\text{corr}(r_{e,}, C_{t+1}/C_t)$		.69	
$\text{corr}(r_{e,}, TC_{t+1}/TC_t)^{(ii)}$		.05	
	Panel B: Aggregates		
	(e)	(f)	
output	1.77	-	
total consumption	1.18	.60	
shareholder consumption	10.77	.64	
investment	6.83	.84	
wages	2.10	-.04	
capital stock	.60	.06	

(a)-(f) as in Table 1

Parameters: $\alpha = .30, \beta = .99, \Omega = .025, \gamma = 3, \text{corr}(\mu, \mu_{-1}) = .98$; other parameters as in Table 2

Table 7

Properties of the q^e/Y ratio

	$\mu_1 = 36; \mu_2 = 114$	Benchmark	$\mu_1 = 121; \mu_2 = 380$
$E(W/Y)$	.66	.70	.73
$SD(q^e/Y)$	1.01	.87	.69
$\max(q^e/Y)$	14.85	3.43	2.53
$\min(q^e/Y)$	1.71	1.00	.66
$E(q^e/Y)$	3.11	2.07	1.46

All parameters other than μ_1, μ_2 are the same across all 3 cases; Benchmark is Case 1 of Table 2

6 Technology Driven Variations in Factor Shares

In this section we briefly entertain the hypothesis that the observed variations in factor shares might be the result of time variations in the parameters of the aggregate production function. Let us assume a representative agent with utility of consumption $u(C_t) = \ln C_t$ and an aggregate technology given by

$$f(K_t, 1) = LK_t^{\alpha_t},$$

where α_t varies stochastically through time according to a known Markovian process. For consistent comparisons, we have assumed that $n_t \equiv 1$ and that all investment is financed out of the capital share of income:

$$d(K_t, \alpha_t) = \alpha_t f(K_t, 1) - I(K_t, \alpha_t). \tag{29}$$

Such a model formulation can be summarized by the recursive functional equation.

$$\begin{aligned} \hat{J}(K_t, \alpha_t) \quad = \quad & \max_{I(K_t,\alpha_t)} \{u(C(K_t, \alpha_t)) + \beta \textstyle\int \hat{J}(K_{t+1}, \alpha_{t+1}) dH(\alpha_{t+1}, \alpha_t)\} \text{ s.t.} \qquad (30) \\ & C(K_t, \alpha_t) + I(K_t, \alpha_t) \leq LK_t^{\alpha_t}, \\ & K_{t+1} = I(K_t, \alpha_t) + (1 - \Omega)K_t; K_0 \text{ given}, \\ & dH(\alpha_{t+1}; \alpha_t) \text{ specified, Markovian.} \end{aligned}$$

(31)

This formulation differs from the earlier one in several essential ways. First, the consumption of the representative agent equals the sum total of his wage and dividend income, a fact that suggests it will be nearly impossible – while maintaining plausible output and consumption variations – to achieve substantial MRS variability. Second, for this formulation, the share of income to capital cannot be specified independently of the average level of capital stock: a lower $E\alpha$ (to match a 70% worker share) is coincident with lower equilibrium capital stock, output, etc. All variants of the risk sharing model displayed the same steady state level of capital stock, since α was unchanging.

These remarks together suggest that it will be very difficult to achieve the same results as in the earlier formulation where factor shares (over a limited range) could be expressed substantially independently of the capital stock process, an assertion that is borne out in the results of Table 8. In this case the process on α_t follows a two state Markov process. It was possible to parameterize this process so as to replicate simultaneously the detrended output variability and the properties of the labor share as per Case 1, Table 2. This was accomplished by using the same transition matrix as in the cases underlying Tables 2-4, while assigning the role of μ_t to α_t. That is, we chose to assume no other shocks and set $\lambda_t \equiv 1$, thus isolating the consequences of a variable α_t. This also implies that the process governing the evolution of $\tilde{\alpha}_t$ coincides exactly with the process governing the evolution of $\tilde{\mu}_t$ in the comparison cases. One may thus choose to compare the results in Table 8 with those of Table 1, where there are no aggregate shocks, or with those of Tables 2 and following, where macro shocks are present and macroeconomic volatility is comparable. For full consistency, $u(C_t) = \ln C_t$ was also maintained.

Table 8

Technology induced variations in income shares

	(a)	(b)	(c)
r_e	4.111	1.12	.33
r_f	4.111	1.04	.19
r_p	.00	.24	
d_{t+1}/d_t		11.09	
W_t/Y_t	69.	1.82	
		(d)	
$\text{corr}(\ln(W/Y)_t, Y_t^{HP})$		-.03	
$\text{corr}(\ln(W/Y)_t, \ln(W/Y)_{t-1})$		.99	
$\text{corr}(r_{e,}, C_{t+1}/C_t)$		.97	
	(e)		(f)
Output	1.78		–
Consumption	.45		.20
Investment	8.65		.97
Wages	1.06		.99
Capital stock	.73		.14

(a) - (f) as in prior Tables.
$\alpha_1 = .322, \alpha_2 = .285, \beta = .99, \Omega = .025, \psi = .97237, \pi = .0162, \sigma = .01487, H = .00013, \Delta = .002$

As is clearly seen, there is no measurable premium, and the return standard deviations are quite low relative to previous cases. The business cycle properties of the model, including the properties of the wage share, are, however, quite acceptable (except for the very high correlation of wages with output and the low relative volatility of consumption).

These results underline the fact that factor share variations are not an additional source of risk in a representative agent model: what the agent does not get in the form of wages, he receives in the form of dividend payments. By contrast, income share variations are (highly) relevant in economies with two separate agent classes as is the case in the other models proposed herein.

7 An Alternative Interpretation of the Sharing Mechanism

The variable sharing parameter $\tilde{\mu}$ can also be rationalized in a manner which differs significantly from the motivation heretofore proposed (where variation in $\tilde{\mu}$ was postulated to reflect changes in the relative bargaining strengths of workers and firm owners). In particular, we will demonstrate that variable sharing will also arise endogenously in purely competitive contexts where workers receive an idiosyncratic income shock against which they are unable to insure (restricted participation).

Let us denote this income shock by ξ_t when received in period t, and let us suppose that ξ_t and λ_t are both highly autocorrelated and cross correlated. To make clear the intuition underlying the development to follow we first review the story underlying the analogous complete markets case.

Suppose, as we assume, that workers are more risk averse than shareholders. They will thus wish to sell contingent commodities (or state claims) which pay off in their high future income states, and purchase claims that pay off in their low future income states. Since, with a competitive labor market and Cobb-Douglas technology, low aggregate income states for workers are also typically low income states for shareholders, it is very likely that the equilibrium price of claims paying in low income states will be higher than their high income state counterparts. On balance the claims that workers sell will have less aggregate value than the claims they purchase. On net they thus willingly transfer positive amounts of current consumption to the shareholders in the context of their trading activities in exchange for a more stable future consumption stream. It is in this sense that the shareholders are "compensated" for providing income insurance to workers in a competitive equilibrium context. Under competitive labor markets and a reasonable model calibration, workers' wage income will be relatively smooth, especially as compared with firm owner income. This fact suggests that workers will mostly seek to insure against variation in their idiosyncratic income component. In any event, under market completeness, for all states (k, ξ, λ), $v_1(\bar{C}(k,\ \xi,\ \lambda)) = \hat{\theta}\, u_1\,(C(k,\ \xi,\ \lambda))$ for some constant $\hat{\theta}$ after all trades have been executed.

Suppose now that workers (for reasons of implicit moral hazard, etc.) cannot trade claims that distinguish among $(\xi_t,\, \lambda_t)$ realizations. The only mechanism open to workers for future consumption stabilization is to save via the acquisition of positive quantities of (k_t, t) indexed claims from shareholders, at a net cost to themselves. These claims pay one unit of the consumption good if the particular (k_t, t) state is observed irrespective of what (ξ_t, λ_t) is simultaneously realized. In a sense they represent (k_t, t) conditional discount bonds. Consider two workers with identical preferences and initial wealth. Such trading restrictions mean that the period $\hat{t}$ wealth of a worker who has experienced a favorable sequence of high idiosyncratic income stocks ξ_t in periods $t < \hat{t}$, and the period $\hat{t}$ wealth of an otherwise identical worker who has experienced a sequence of low ξ_t shocks, $t < \hat{t}$, will be the same. The former will have enjoyed greater consumption and higher welfare than the latter but this does not translate into the potential for higher wealth accumulation. If that were to be the case, it would imply that the high idiosyncratic income worker could sell contingent commodities that pay in high future ξ_t incomes states and use the proceeds to buy more (k_t, t) based claims – something prohibited by the market structure.

By accumulating (k_t, t) based contingent commodities, workers can partially insure against low wage income states (ceteris paribus, low capital states are low wage states) but not specifically against low ξ_t or low λ_t states. By selling capital based contingent commodities to workers, shareholders insure workers but only partially so, as these securities ignore (ξ_t, λ_t) differences. In equilibrium the ratio $u_1(C(k_t, \xi_t, \lambda_t))/v_1(\bar{C}(k_t, \xi_t, \lambda_t)) = \theta(\xi_t, \lambda_t)$ will thus vary, though generally to a lesser extent than in the absence of any claims trading whatsoever. It is this quantity $\theta(\xi_t,\ \lambda_t)$ that we identify with $\tilde{\mu}_t$.

The formalism behind these ideas is presented below, where first the generalized planning problem and, second, the details of its competitive decentralization are discussed. Appendix 1 provides a more detailed development, including the recursive counterpart.[7]

$$\max_{\{C_t, \bar{C}_t, I_t\}} E\left\{\sum_{t=0}^{\infty} \beta^t \left[u(C_t) + \gamma(\xi_t, \lambda_t) v(\bar{C}_t)\right]\right\} \tag{32}$$

subject to:

$$\begin{aligned} C_t + \bar{C}_t + I_t &\leq f(K_t)\lambda_t + \xi_t \\ K_{t+1} &= (1-\Omega) K_t + I_t \\ K_0 &= \bar{K}, (\xi_0, \lambda_0) = (\bar{\xi}, \bar{\lambda}) \text{ given.} \end{aligned}$$

In formulation (32), $\gamma(\xi_t,\ \lambda_t)$ is the variable sharing parameter which is postulated to be a 1 to 1 function of the economy's two sources of uncertainty, the worker's endowment shock and the firm's technology shock, and $\bar{C}_t$ denotes period t worker consumption;

[7] We present the non-recursive valuation equilibrium representation as the underlying mechanism is more transparent in this case.

otherwise the notation is the same as previously. The planner's decision variables may be written as functions of the economy's capital stock K_t and the history, up to and including the present, of the economy's exogenous shocks $\{(\xi_t,\ \lambda_t)\}_{t=0}^{T} \equiv z^t$; we also identify $(z^{t-1},\ (\xi_t,\ \lambda_t)) = z^t$. Hence, $C_t = C(K_t,\ z^t)$, $\bar{C}_t = \bar{C}(K_t,\ z^t)$ and $I_t = (K_t,\ z^t)$.

In decentralizing planning problem (32), shareholders are presumed to have access to a complete set of contingent commodities markets. They therefore solve the following problem:

$$\max_{\{C_t, I_t, n_t\}} E\left\{\sum_{t=0}^{\infty} \beta^t u(C, (K_t, z^t))\right\} \tag{33}$$

subject to

$$\sum_{t=0}^{\infty} \sum_{(k_t,\, z^t)} P(K_t,\ z^t)\ \left[C(K_t,\ z^t) + I\,(K_t,\ z^t) + W\,(K_t,\ z^t)\ n(K_t,\ z^t)\ -\ \lambda_t f(K_t)\right]\ \leq\ 0$$

$$K_{t+1}(z^t) = (1-\Omega)K_t(z^{t-1}) + I\left(K_t, z^t\right), n\left(K_t, z^t\right) \leq 1. \tag{34}$$

$K_0, (\xi_0, \lambda_0)$ given.

Since the owner has access to complete markets he is confronted with one overall budget constraint (34). Let θ denote the multplier on this constraint. The relevant differential optimality condition is thus

$$\beta^t \pi(z^t) u_1(C(K_t, z^t)) = \theta\, P(K_t, z^t), \tag{35}$$

where $\pi(z^t)$ denotes the probability of the indicated sequence of events, and $P(K_t, z^t)$ is the period t present value (relative to $t = 0$) price of consumption (output).

As noted, the worker is presumed to be highly constrained in his trading activities. Neither can he trade contingent commodities indexed by ξ_t, described earlier, nor commodities indexed by λ_t. Workers may thus transfer wealth across different realizations of (K_t, t), but not across realizations $(K_t, t, \hat{\xi}_t, \hat{\lambda}_t)$ and $(K_t, t, \xi_t', \lambda_t')$. As a result, in each period, workers face multiple budget constraints, each one indexed by a possible realization of (ξ_t, λ_t). These constraints allow the worker to trade contingent commodities only across future (ξ_t, λ_t) states which coincide with the particular (ξ, λ) experienced today.

In every period t, workers thus solve

$$\max_{\{n_{t+j}^w, \bar{C}_{t+j}\}} E\left\{\sum_{j=1}^{\infty} \beta^{t+j} v(\bar{C}(K_{t+j}, z^{t+j}))\right\} \tag{36}$$

subject to:

$$\sum_{j=1}^{\infty} \sum_{k_{t+j}} P(K_{t+j},\ z^{t+j})\ \left[\left(\bar{C}(K_{t+j},\, z^{t+j}) - \xi_t\right) - W\,(K_{t+j},\, z^{t+j})\; n\,(K_{t+j},\, z^{t+j})\right] \;=\; 0 \tag{37}$$

$$\forall\ (\xi_t,\ \lambda_t)\ \in\ \Xi\ \times\ \Gamma$$

$$n^w(K_{t+j},\, z^{t+j})\ \leq\ 1$$

Making specific the remarks above, there is one constraint of type (37) for every possible $(\xi_t,\ \lambda_t)$ that is feasible going forward.

Let $\theta(\xi_t, \lambda_t)$ denote the multiplier identified with the constraint indexed by (ξ_t, λ_t). The relevant differential optimality condition from the workers' problem is:

$$\beta^t\, \pi(z^t)\, v'(\bar{C}(K_t,\, z^t)) = \theta\,(\xi_t,\, \lambda_t,)\, P\,(K_t,\, z^t). \tag{38}$$

In equilibrium,

$$v'(\bar{C}\,(K_t,\, z^t)) = \frac{\theta\,(\xi_t,\, \lambda_t)}{\hat{\theta}}\; u'\,(C\,(K_t,\, z^t)). \tag{39}$$

Define $\tilde{\mu}_t = \gamma\,(\xi_t,\, \lambda_t) = \frac{\hat{\theta}}{\theta\,(\xi_t,\, \lambda_t)}$ and the identification is complete (constraints are binding and multipliers are strictly positive). Because of an absence of market access workers cannot insure against all eventualities. Across different future states the marginal benefit to relaxing the relevant budget constraints will no longer be in fixed ratio but will change with the (ξ_t, λ_t) state, a fact mirrored by the variable $\gamma\,(\xi_t,\, \lambda_t)$ ratio.

Under this identification, roughly speaking $\tilde{\mu}_t \equiv \gamma(\tilde{\xi}_t,\, \tilde{\lambda}_t)$ will be highly persistent when $\tilde{\xi}_t$ and $\tilde{\lambda}_t$ are. Furthermore, ceteris paribus, we may identify a high level of ξ_t with a high value of μ_t and vice versa. More generally, whenever one agent is constrained in his options to trade contingent commodities (or state claims), then there will be incomplete insurance and this fact can be captured by a variable $\tilde{\mu}$ parameter as per our initial model identification.

The above equivalence allows us to connect our modeling effort to an important strand in the literature associated most closely with Constantinides and Duffie (1996). As emphasized in Constantinides and Duffie (1996), a reasonable replication of the financial stylized facts requires (i) uninsurable income shocks which are (ii) highly persistent. By the identifications in this section, the assumed properties on the $\tilde{\mu}_t$ process may be interpreted as fully satisfying these conditions.

8 Related Theoretical and Empirical Literature

The central advantage of production economies for the understanding of the pattern of financial returns is the added discipline they present to the exercise. Since the actions of the same economic agents give rise to both macroeconomic and financial phenomena, it is a minimum expression of consistency that the same model be expected to replicate the financial *and* macroeconomic stylized facts, at least along a limited set of dimensions. This has been our perspective. In this section we discuss other models with significant labor market features and their implications for financial return data.

8.1 The Theoretical Literature

Matching financial data in a production setting requires that the capital owner display a strong desire to smooth his consumption intertemporally (provoked by, e.g., a habit formation feature) while simultaneously acting in a context that makes it difficult to reallocate labor or capital to that same end. These latter restrictions essentially substitute for some form of market incompleteness: in either case agents are prevented from smoothing their consumption across states and dates. In most models it is the degree of restrictiveness in the labor market that ultimately holds sway vis-à-vis financial characteristics. There are four models, in particular, that we review. More detailed model descriptions may be found in Appendix 2; principal comparative output data is provided as available. In all cases notation is harmonized to be consistent with that adopted in this paper.

The first paper to emphasize the influence of labor market phenomena on equilibrium financial returns was Danthine et al. (1992). It proposed a model with shareholders, primary and secondary workers. These latter groups hold no securities (limited participation). The primary workers are assumed to have a permanent, full employment association with the firm. Their compensation is governed by a risk sharing arrangement identical to the one proposed in this paper. At the other extreme, the secondary workers' employment prospects are governed by a pure Walrasian mechanism, one that otherwise would lead to substantial income variation. In order to moderate this wage income variability, primary worker wages are postulated to be subject to a wage floor augmented by unemployment compensation (the wage floor is above the market clearing wage in some states) financed by a tax on corporate profits.[8] As a result of these later arrangements, all workers in the model experience income volatility less than what would occur under

[8]Thus the model generates unemployment among the secondary workers as both worker types supply labor inelastically.

a full Walrasian scenario. Whether directly – via wage insurance – or indirectly – via the unemployment tax – the net effect of worker income stabilization is to shift income risk onto the shareholders. The principle model results are presented in Table 9 below.

Table 9

Model Results: Danthine et al. (1992)[(i)]

Financial and Aggregate Statistics

	(a)[(ii)]	(b)
r^e	4.56	.84
r^f	3.98	.80
r^P	.58	.06
$d_t^{(iii)}$		5.36
		(d)
corr $(r_e, c_{t+1}/c_t)$		.06
	(e)	(f)
output	1.76	.69
total consumption	.34	.98
shareholder consumption	5.36	.99
investment	6.08	.99
wages[(iv)]	.22	.10
capital stock	.54	.03

(i) The reported statistics are drawn from Tables (3) and (4) in Danthine et al. (1992).
(ii) (a), (b), (d) – (f) as in Table 1.
(iii) The reported volatility is for the dividend annualized, not its growth rate.
(iv) Wages are equivalent to total worker compensation.

While the model is able to replicate the stylized business cycle facts very well and produces a premium substantially in excess of what is obtainable under a Hansen (1985) construct, the premium still falls significantly short of what is observed. Security return volatilities are also much too low. In effect, variable equilibrium labor supply in the secondary sector in conjunction with shareholder control over investment together provide too much opportunity for shareholder consumption smoothing. Indeed, shareholder consumption volatility is about half the level of the benchmark case of this paper (Table 2, case (1)); otherwise, the macro series are very similar. In a sense the current model is a simplified version of Danthine et al. (1992) where all workers are subject to the primary worker income determination mechanism, augmented with an extra source of risk affecting the mechanism of income sharing itself. This second source of uncertainty is fundamental to its superior results along the financial dimensions.[9]

Boldrin and Horvath (1995) propose a contracting mechanism that is similar to Danthine et al. (1992). In equilibrium, it also has the consequence that employees supply resources to firm owners in high income states and receive payments from them in low output ones.[10] In their set up, profits and hours both display high levels of variability in line with their respective empirical counterparts. As they do not present data on the pattern of financial returns characteristic of their model, it is difficult to directly compare their results with the other literature. By the nature of their model formulation, however, it is likely that their results would be similar to those in Danthine et al. (1992). Subsequent to Danthine et al. (1992), the literature approached the same set of issues more from the perspective of modifying shareholder preferences in order that they act in a more risk averse fashion and less from the "operating leverage" perspective of worker income insurance.

Jermann (1998) postulates a representative agent style model with habit formation (leading to a high MRS volatility) in conjunction with capital adjustment costs which make it difficult to smooth consumption via investment variation. The inability of the agent to smooth is strengthened by a fixed labor supply assumption. With these features his model is able to explain the business cycle stylized facts in conjunction with the mean premium quite well, but at a cost of excessive risk free rate volatility. See Table 10 below.

Boldrin et al. (2001) demonstrate, however, that the high premium in Jermann (1998) is lost if a Hansen (1985) style labor-leisure choice mechanism is introduced even while retaining the same adjustment cost specification. Thus modified, Jermann's (1998) model also has the unattractive feature that hours and output are negatively correlated. In this modified model there are two opportunities for the representative agent (and therefore the representative shareholder) to smooth his consumption stream – by adjusting his hours and investment (though at a cost) – and, taken together, these are very effective consumption smoothing devices. As a result the premium declines to .30%. The results in Jermann (1998), while important as a source in intuition, are thus not extremely robust.

Boldrin et al. (2001) also review a number of possible model features, and ultimately explore one with two sectors – one producing consumption and the other capital goods

[9] It is probably true, however, that the introduction of variable labor into the present model construct would reduce the premium.

[10] Much of the added generality in Boldrin and Horvath (1995) comes from their contracting mechanism which admits fixing wage and employment schedules many periods in advance. Due to computational problems, however, they only compute statistics for one period ahead contracts.

Table 10

Model Results: Jermann (1998)$^{(i)}$

Financial and Aggregate Statistics

	(a)$^{(ii)}$	(b)
r^e	7.00	19.86
r^f	.82	11.64
r^P	6.18	
d_{t+1}/d_t		8.44
	(e)	
output	1.76	
consumption	.86	
investment	4.64	

(i) The reported statistics are drawn from Tables (1) and (2) in Jermann (1998).
(ii) (a), (b), (e) as in Table 1.

– where the allocation of capital and labor to each sector must be chosen one period in advance of knowing the respective technology shocks. This has the consequence of reducing the ability of shifts in either factor of production to be used to smooth consumption significantly. It is the restrictions to labor market flows (between sectors) posterior to shock realizations, in particular, that they view as most crucial to their results. In conjunction with standard habit formation preferences these authors can explain the mean equity premium although investment volatility is a bit too low and the risk free rate again displays excessive volatility, so much so that its standard deviation substantially exceeds that of the return on equity. See Table 11 below. We note that the excessive risk free rate volatility of Jermann (1998) and Boldrin (2001) is not a general consequence of the distributional risk perspective.

Danthine and Donaldson (2002) revisits the original question posed in Danthine et al. (1992): to what extend can operating leverage cum income share variation simultaneously explain the business cycle and financial market stylized facts? It is an exploration that is accomplished in a slightly more abstract setting than in Danthine et al. (1992) whereby the latter's elaborate labor market set up (temporary and permanent classes of workers, etc.) is summarized by a "net" risk sharing mechanism nearly identical to the one considered here. With several additional features, such as costs of adjusting the capital stock, they also achieve an excellent and broad based fit to the data. See Table 12.

Generally speaking this case replicates the one presented in Table 2 except that returns seem to conform to the data slightly less satisfactorily. This is attributable to

Table 11

Model Results: Boldrin et al. (2001)$^{(i)}$

Financial and Aggregate Statistics

	(a)$^{(ii)}$	(b)
r^e	7.83	18.4
r^f	1.20	24.6
r^P	6.63	
	(e)	(f)
output	1.97	
total consumption	1.36	.76
investment	4.71	.96
hours	1.58	.78

(i) This data is drawn from Tables (1) and (2) in Boldrin et al. (2001).
(ii) (a), (b), (e), (f) as in Table 1.

Table 12

Model Results: Danthine and Donaldson (2002)$^{(i)}$

Financial and Aggregate Statistics

	(a)$^{(ii)}$	(b)	(c)
r^e	5.92	22.20	.26
r^f	2.46	4.05	.02
r^P	3.46	22.34	
d_{t+1}/d_t		16.72	
		(d)	
W/Y	.69	4.83	-.022
	(e)	(f)	
output	1.77		
total consumption	1.45	.96	
shareholder consumption	11.94	.38	
investment	3.05	.93	
capital stock	.27	-.005	

(i) The reported statistics are drawn from Table (4) in Danthine and Donaldson, (2002).
(ii) (a), (b), (c), (e), (f) as in Table (1).

the slightly lower W/Y ratio which results in a more modest operating leverage effect.

By contrast, the present paper may be viewed as decomposing the general results in Danthine and Donaldson (2002) into the distributional and aggregate shock related components. Not only is a richer range of phenomena explored, but alternative mechanisms for share variation are analyzed as well. We find that the attractive financial characteristics of the model may be principally attributed to the distributional variations whereas the business cycle properties have their origins in the standard technology shock set up. As such the two sources of shocks considered appear complementary.

Our final theoretical comments concern Guvenen (2005). He assumes a perspective which may be viewed as providing an alternative macro interpretation for the variable risk sharing feature of the present model. Rather than assuming workers and shareholders interacting in an uncertain bargaining context, Guvenen (2005) presumes that the population is divided into two groups with unequal financial market access.[11] Shareholders participate in both stock and bond markets while non-shareholders trade only bonds. Both groups supply labor inelastically to the firm and non-stockholders are modeled as being more risk averse.[12] With bond trading being their only mechanism for consumption smoothing, non-stockholders bid up bond prices, resulting in a low risk free rate. In equilibrium, stockholders end up insuring non-stockholders by increasing their debt holding exactly when a low productivity realization reduces both agents' income and vice versa. As a result, bond market events act to create a high level of volatility of shareholder consumption, volatility against which they can insure only via management of the capital stock. Although the effective extent of income insurance provided by shareholders to non-shareholders is not as great as in the present model, the fundamental idea is the same. Guvenen (2005) also goes on to show that the consumption of non-shareholders serves a role similar to that of a slow moving habit in his equilibrium asset pricing equation, a feature also present in our distributional risk sharing formulation.

We note that these results seem to be more favorable vis-à-vis "distribution risk" along the dimension of the return volatilities, but less so with regard to the business cycle stylized facts. In particular, investment is insufficiently volatile on an absolute and relative basis.

The substance of these theoretical contributions, broadly speaking, is as follows:

[11]The stockholder/non-stockholder apportionment is exogenous in Guvenen (2005).

[12]We suspect that if either agent (especially shareholders) were confronted by a labor-leisure choice (allowing the shareholders another dimension of consumption smoothing), the ability of Guvenen's (2004) model to replicate the financial stylized facts would be substantially compromised. The same may be said of the present paper. A mechanism such as in Dow (1995), where labor decisions must be taken in advance, is a possible mechanism for preserving labor supply inflexibility.

Table 13

Model Results: Guvenen (2005)$^{(i)}$

Financial and Aggregate Statistics

	(a)$^{(ii)}$	(b)	(c)
r^e	5.30	14.1	
r^f	1.98	5.73	
r^P	3.32	14.7	.30
	(e)		
output	2.4		
total consumption	2.3		
shareholder consumption	4.6		
non-shareholder consumption	1.1		
investment	2.7		
capital stock			

(i) These statistics are principally from Tables 2 and 11 of Guvenen (2003), which is the antecedent of Guvenen (2005). The latter version is considerably abbreviated, however, and lacks macro statistics. It is for this reason that we make the indicated choice.
(ii) (a), (b), (e) as in Table 1; (c) – correlation with output.

(1) Labor market arrangements have substantial impact on the volatility of profits and shareholder income. (2) In contexts where shareholders have limited ability to hedge this added income risk, its consequences for the equilibrium pricing of financial claims are profound and generally go in the direction of enhancing the models abilities to simultaneously replicate the stylized facts of the business cycle and financial markets. (3) Since the magnitude of the equity premium responds directly to low frequency income shocks, it is convenient – in the sense of allowing for a superior replication of financial data within a simple context – to have alternative sources of income variation beyond that arising from business cycle co-movements. Our risk sharing mechanism is one such source. (4) A reasonable representation of the financial stylized facts requires income shocks that cannot be insured (smoothed). This may take the form of technological restrictions, as in many of the papers detailed in the present section, or various forms of market incompleteness. Our distribution risk perspective entails aspects of both these perspectives.

8.2 Empirical Explorations

The focus of research in empirical finance is to explain the cross section of security returns, where the notion of cross section is in reference to sets of specifically constructed portfolios rather than individual issues.[13] Curiously, there are, to date, few studies that include labor market explanatory variables of any sort in the first stages of the Fama and MacBeth (1972) style regressions which constitute the fundamental technique employed in these exercises. There are two exceptions to this general rule: Jagannathan and Wang (1996) and Santos and Veronesi (2004). Jagannathan and Wang (1996) include the growth rate in per capita labor income as an explanatory variable, a fact that allows their model to outperform the standard CAPM. Such a variable is completely consistent with the model presented in this paper: an above-verage value of $\bar{\mu}_t$ in a particular period is consistent with a simultaneous high growth rate in per worker labor income, and vice versa.

Santos and Veronesi (2004) focus rather on the predictability of stock returns. Their labor market variable is the economy-wide labor income to total consumption ratio, a quantity that is perfectly positively correlated with μ_t in our model. What is of particular interest to us is the intuition provided by Santos and Veronesi (2004), which, by construction, applies to the "distributional risk" construct. They argue that the share of income due to wages, as with all other principal sectors of the economy is a stationary process. The significance of this fact for asset pricing is twofold: (1) If the share of income to labor is high and likely to remain so, investors' MRS variability will be relatively insensitive to events in the stock market and thus the market risk premium is likely to be small; and conversely. We note, however, that this ignores the operating leverage effect: a higher share of income to labor suggests a higher fundamental riskiness in the equity cash flow, something, per se, likely to increase the premium. For most cases presented in this paper this latter effects dominates the Santos and Veronesi (2004) intuition. (2) If the share of income from wages is above average it is likely to decline with the consequence that future dividend growth is likely to exceed consumption growth leading to high asset prices and returns. The time variation in the asset risk premium suggested by these comments, however, is fully a feature of the distributional risk model.

Regressing stock returns on lagged values of this variable leads to statistically significant coefficients and adjusted R^2 that exceed what would be obtained using, e.g., the lagged dividend-price ratio as the explanatory variable. We suspect that a similarly good fit could be obtained using data generated by our model. Including this ratio as

[13]The famous twenty-five Fama and French (1993) portfolios are the principal case in point.

an explanatory variable also allows the model to outperform the standard CAPM in explaining the returns to the twenty-five Fama and French (1993) portfolios. Given the results obtained here, none of this is surprising.

9 Concluding Comments

The principal implication of this paper is that asset return patterns have less to do with business cycle systematic risk factors and more to do with distributional effects. What appears to be key is that the insurance mechanism demands payments from the firm (shareholders) to workers precisely at times when shareholder cash flow is already low. We showed that this mechanism acts as a standard habit formation mechanism but one that arises endogenously via shareholder-worker interactions.

Not only is the model able to replicate simultaneously an astonishingly wide class of financial and business cycle stylized facts, but it is also able to do so with very low CRRAs for both shareholders and workers, a fact that reinforces the significance of the insurance mechanism per se. Both the equity premium and security return volatilities, in particular, match the data well. Of additional interest is the equity/output ratio, q^e/Y; not only is this ratio of an average level typical of the economy but its variation is also characteristic of what is observed. Financial leverage effects are also magnified to a degree unmatched in more standard representative agent paradigms.

The latest trends, for the U.S. at least, point in the direction of a falling labor share. Simultaneously, competitive pressures under the force of globalization appear to render traditional internal-to-the-firm insurance mechanisms, which are at the heart of the present paper, harder and harder to maintain (The plight of American automobile manufacturers is a dramatic case in point). On those counts, our model leads us to anticipate a decrease in equity premia. While our viewpoint is entirely specific, this prediction confirms those made recently by several researchers (Jagannathan, McGrattan and Scherbina, 2000, McGrattan and Prescott, 2000, Fama and French, 2002).

Our literature review together with the present contribution illustrates two distinct perspectives on the simultaneous replication of the business cycle and financial stylized facts. The first, as embodied in Jermann (1998) and Boldrin et al. (2001), focuses on representative agent economies in which the agent has simultaneously a strong desire to consume (habit formation) in conjunction with severe technological constraints to doing so (cost of adjusting capital and fixed labor supply in the case of Jermann (1998) and restrictions on shifting capital and labor intertemporally and across sectors in the case of Boldrin et al. (2001)). These features are required because the bulk of the representative agents' consumption, in either case, comes from wage income which, if the model is correctly calibrated to the business cycle stylized facts, is relatively smooth. Capital income must therefore be highly variable and of a pattern that tends to destabilize consumption. The technological constraints make the smoothing of capital income especially difficult. These features, per se, are nevertheless insufficient for an adequate replication of the financial stylized facts: the asset holding class must also display very high marginal risk aversion, something that habit formation preferences provides.

In heterogeneous agent models with distributional risk, however, neither high levels of effective risk aversion for the shareholding class nor complex technological restrictions are required. From the perspective of asset pricing, the provision of income insurance by the shareholding class to the working class is sufficient (it must be added that a somehow-restricted labor supply may be required as well). We note that income insurance is fundamentally a constraint on prices (in particular, wage) rather than quantities.

Our equivalence in Section 7 goes a long way to harmonize these perspectives by observing that partial income insurance is essentially equivalent to particular types of trading restrictions. Trading restrictions prevent outputs from being transferred across various dates and states, something that technological constraints equivalently accomplish. In addition, Section 4 demonstrates that labor income, relative to shareholder consumption, behaves much the same as a slow moving habit. In this sense distributional risk serves a dual role.

References

[1] Abel, A., "Asset Prices under Habit Formation and Catching Up with the Jones", *American Economic Review*, Papers and Proceedings, 80 (1990), 38-42.

[2] Bansal, R., "Long Run Risk and Risk Compensation in Equity markets," working paper, Duke University Fuqua School of Business, 2004

[3] Boldrin, M., Christiano L., and J. Fisher, "Habit Persistence, Asset Returns, and the Business Cycle", *American Economic Review*, 91 (2001), 149-166.

[4] Boldrin, M., and M. Horvath, "Labor Contracts and the Business Cycle", *Journal of Political Economy*, 103 (1995), 972-1004.

[5] Boldrin Michele and Adrian Peralta-Alva, "What did (really) happened to the US stock market: Accounting for the last 50 year", mimeo, U. of Miami, 2005

[6] Campbell, J.Y. and J. Cochrane, “By Force of Habit: A Comsumption-based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107 (1999), 205-251.

[7] Constantinides, G. and D. Duffie, “Asset Pricing with Heterogeneous Consumers”, *Journal of Political Economy*, 104 (1996), 219-240.

[8] Cooley, Thomas F. and Edward C. Prescott, “Economic Growth and Business Cycles,” Chapter 1 in *Frontiers of Business Cycle Research*, T.F. Cooley, ed., Princeton U. Press (1995).

[9] Danthine, J. P., and J. B. Donaldson, “Labor Relations and Asset Returns,” *Review of Economic Studies*, 69 (2002), 41-64.

[10] Danthine, J. P., Donaldson, J. B., and R. Mehra, “The Equity Premium and the allocation of Income Risk”, *Journal of Economic Dynamics and Control*, 16 (1992), 509-532.

[11] Dow, James P., “Real Business Cycles and labor Markets with Imperfectly Flexible Wages,” *European Economic Review*, 39 (1995), 1683-1696

[12] Dunlop, J., “The Movement of Real and Money Wage Rates”, Economic Journal, 48 (1938), 413-434.

[13] Fama E., and K. French, “Common Risk Factors in the Returns to Stocks and Bonds”, *Journal of Financial Economics*, 33 (1993), 3-56.

[14] Fama E., and K. French, “The Equity Premium”, *Journal of Finance*, xx (2002)

[15] Fama, E., and J. D. Mac Beth, “Risk, Return and Equilibrium: Empirical Tests”, *Journal of Political Economy*, 81 (1972), 607-636.

[16] Goetzman, W. and P. Jorion, “A Century of Global Stock Markets,” NBER Working Paper 5901 (1997).

[17] Gollin, Douglas, “Getting Income Shares Right,” *Journal of Political Economy*, 110 (2002), 458-474.

[18] Guvenen, Fatih, “A Parsimonious Macroeconomic Model for Asset Pricing. Habit Formation or Cross-Sectional Heterogeneity,” Rochester Center for Economic Research Working Paper No. 499 (2004).

[19] Hansen, G., "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16 (1985), 309-327.

[20] "The Declining Equity Premium", *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 2000

[21] Jermann, Urban J., "Asset Pricing in Production Economies", *Journal of Monetary Economics*, 41 (1998), 257-275.

[22] Kocherlakota, N. "The Equity Premium: It's Still a Puzzle, "*Journal Economic Literature*, 34 (1996), 42-71.

[23] Lucas, R. E., Jr., "Asset Prices in an Exchange Economy," *Econometrica*, 46 (1978), 1429-1444.

[24] McGrattan, Ellen R. and Edward C. Prescott, "Is the Stock Market Overvalued?," *Federal Reserve Bank of Minneapolis Quarterly Review* (2000).

[25] McGrattan, Ellen R. and Edward C. Prescott, "Taxes, Regulations, and the Value of U.S. and U.K. Corporations," Federal Reserve Bank of Minneapolis Staff Report (2005).

[26] Mehra, Rajnish,"On the Volatility of Stock prices: an exercise on Quantitative Theory," *International Journal of Systems Science*, 29 (1998), 1203-1211.

[27] Rosen, S., *Implicit Contract Theory*, Elgar (1994).

[28] Santos, T. and P. Veronesi, "Labor Income and Predictable Stock Returns", Columbia University, mimeo (2004)

[29] Tarshis, L. "Changes in Real and Money Wage rates", *Economic Journal*, 49 (1938), 150-54.

[30] Weil, Ph., "Comment", *NBER Macroeconomics Annual*, 7 (1992), 169-181.

APPENDIX 1: Background to Section 7

A. Planner's problem

We first detail the planner's problem, provide its equivalent recursive representation and then, in B, decentralize it.

1. Let $z = (\xi, \lambda)$ denote a shock realization and $z^T = ((\xi, \lambda)_t)_{t=1}^T$ a history of shock realizations. We also use the notation $z^T = (z^{T-1}, (\xi, \lambda))$.

Let $v(c, \bar{c}; (\xi, \lambda)) = u(c) + \gamma(\xi, \lambda)\bar{u}(\bar{c})$, $\gamma(\xi, \lambda) > 0$, be the instantaneous objective function of the planner. To simplify notation we set $n(z^T) = 1$ for all z^T.

Consider the following planner problem:

$$\max E \sum_t v(c(z^{t-1}, \xi, \lambda)), \bar{c}(z^{t-1}, (\xi, \lambda)); (\xi, \lambda) \tag{40}$$

subject to:

$$\begin{aligned} K_0 &= \bar{K},\ (\xi_0, \lambda_0) = (\bar{\xi}, \bar{\lambda}) \\ \xi_t + \lambda_t F(K(z^{t-1})) &= c(z^{t-1}, (\xi, \lambda)) + \bar{c}(z^{t-1}, (\xi, \lambda)) + I(z^{t-1}, (\xi, \lambda)) \\ K(z^t) &= (1 - \Omega)K(z^{t-1}) + I(z^{t-1}, (\xi, \lambda)) \end{aligned}$$

2. The latter has the following recursive representation:

$$\max_{(c,\bar{c},I)} V(K, (\xi, \lambda)) = v(c(z^{t-1}, (\xi, \lambda)), \bar{c}(z^{t-1}, (\xi, \lambda)); (\xi, \lambda)) + E_{(\xi,\lambda)}\beta V(K', (\xi', \lambda')) \tag{41}$$

subject to:

$$\begin{aligned} \xi + \lambda F(K) &= c(K,\ \xi, \lambda) + \bar{c}(K,\ \xi, \lambda)) + I(K,\ \xi, \lambda) \\ K' &= (1 - \Omega)K + I(\xi, \lambda)), \end{aligned}$$

where $V(K, (\xi, \lambda))$ denotes the value function associated with (40).

3. Remark: In the extensive formulations, probabilities are naturally defined. If (ξ, λ) are independent first order Markov processes, $\pi(z^T) = \Pi_{t=1}^T \pi(\xi_t|\xi_{t-1})\pi(\lambda_t|\lambda_{t-1})$. In the recursive formulations, they are just $\pi(\xi', \lambda'|\xi, \lambda)$.

4. Under either representation, the differential optimality conditions are:

$$\beta^T \pi(z^T) u'(c(z^{T-1}, (\xi, \lambda)) = p(z^{T-1}, (\lambda, \xi)) \tag{42}$$

$$\beta^T \pi(z^T)\gamma(\xi, \lambda)\bar{u}'(\bar{c}(z^{T-1}, (\xi, \lambda))) = p(z^{T-1}, (\lambda, \xi)) \tag{43}$$

$$E_{z^{T+1}|z^T} p(z^T, (\xi, \lambda))(\lambda F'(K(z^T)) + (1 - \Omega))) = p(z^T). \tag{44}$$

5. We make three observations concerning the recursive representation:

a) If, in the planner objective functions, the weights γ are a non trivial function of z^T, $T = 1, 2, ...$, then a recursive representation may not exist.

b) Total resources are entirely determined by K and the realizations (ξ, λ). Thus, given the form of the objective function, the planner's optimal consumption and investment choices satisfy the following equations

$$(I, c, \bar{c})(z^{T-1}, (\xi, \lambda)) = (I, c, \bar{c})(K, (\xi, \lambda)), (\text{for} K = K(z^{T-1})).$$

As the optimality conditions make clear, the multipliers satisfy the equations

$$p(z^{T-1}, (\lambda, \xi)) = p_T(K, (\lambda, \xi)),$$

(for $K = K(z^{T-1})$), with $p(\cdot)/\beta^T$ time invariant. As for the capital stock,

$$K(z^T) = K(K(z^{T-1}), (\xi, \lambda)).$$

We denote by ζ an element of the state space Z defined as follows:

$$Z = \{\zeta = (K, \lambda, \xi) : K = K(z^{T-1}), \text{ for some history } z^T\}$$

c) Consider the ratio of equation (42)/equation (43). All terms are strictly positive. The choice of the planner's welfare function thus delivers the following sharing rule:

$$\bar{u}'(\bar{c}(z^T))\gamma(\xi, \lambda) = u'(c(z^T)).$$

B. Competitive Markets

1. We next respond to the natural question: Can the planner allocations be decentralized as competitive equilibria? Under what form of competitive markets? The last question arises since workers and owners have different optimality conditions. They cannot face, therefore, the same constraint set. If they did, $\gamma(\lambda, \xi) = 1$. There are two possible decentralized formulations.

2. NON RECURSIVE FORMULATION

We use the planner multipliers $p(\cdot)$ to define the supporting prices. [Alternatively we could have used $p(\cdot)/\gamma(\cdot)$. The two choices support the same allocations, but they require different forms of budget constraints.] The state space is defined by the planner's optimal solution, that is $Z = \{\zeta = (K, \xi, \lambda) : K = K(z^{T-1})$, for some history $z^T\}$

(a) The owners face a set of complete markets and therefore face a unique budget constraint. The owners solve the following problem:

$$\max E_\zeta \sum \beta^t u(c(\cdot)) \tag{45}$$

subject to

$$\sum_t \sum_{\zeta \in Z} p_t(\zeta)[c_t(\zeta) + I_t(\zeta) + w_t(\zeta)n(\zeta) - \lambda f(K_{t-1}(\zeta))] \leq 0$$

$$K_t(\zeta) = (1 - \Omega)K_{t-1}(\zeta') + I_t(\zeta)$$

$$n_t(\zeta) \leq 1$$

where ζ' is the realized state in period $t - 1$.

(b) Observe that the first order conditions of the owners take the same form as the first order conditions of the planner. Probabilities $\pi_t(\xi, \lambda)$ are defined by $\pi(z^t)$, where $K(z^{t-1}) = K$ (in the planner solution).

(c) The worker faces several (rather than one) budget constraints. This is a result of the weights $\gamma(\cdot)$ used in the welfare function. Thus, the worker maximizes the utility function $E_\zeta \sum \beta^t \bar{u}(\bar{c}(\cdot))$ subject to a sequence of budget constraints. The sequence of constraints will depend on the particular specification of the weights $\gamma(\xi, \lambda)$.

If $\gamma(\lambda, \xi)$ is 1-to-1 in λ and ξ, the sequence is:

$$(A) \sum_t \sum_K p(K, \lambda, \xi)(\bar{c}(K, \xi, \lambda) - \xi) - w(K, \xi, \lambda) = 0, \text{ and } \bar{n}(\zeta) \leq 1, \ (\lambda, \xi) \in \Lambda \times \Xi;$$

if $\gamma(\lambda, \xi) = g(\xi)$ for some g 1-to-1 in ξ, then:

$$(B) \sum_{(t,\lambda,K)} p(K, \lambda, \xi)(\bar{c}(K, \xi, \lambda) - \xi) - w(K, \xi, \lambda) = 0, \text{ and } \bar{n}(\zeta) \leq 1, \xi \in \Xi;$$

if $\gamma(\lambda, \xi) = g(\lambda)$, for g 1-to-1 in λ, then

$$(C) \sum_{(t,\xi,K)} p(K, \lambda, \xi)(\bar{c}(K, \xi, \lambda) - \xi) - w(K, \xi, \lambda) = 0, \text{ and } \bar{n}(\zeta) \leq 1, \ \lambda \in \Lambda.$$

Once again, workers' first order conditions are:

$$\beta^T \pi(K, \lambda, \xi)\bar{u}'(\bar{c}(\cdot)) = \gamma(\lambda, \xi)p(K, \lambda, \xi)$$

where, since by definition of state space, $K = K(z^{T-1})$, for some z^{T-1}, and thus $\pi(K, \lambda, \xi) = \pi(z^T)$.

The workers face sequences of budget constraints. They are therefore able to freely allocate wealth within the time-events contingent commodities that appear in the sum, but not across states defining the constraints. Thus for instance, if the constraint is (A), the worker can transfer wealth across different realizations (K, t) for each given (ξ, λ), but not across (K, t, ξ', λ') and (K, t, ξ^*, λ^*).

(d) Remarks:

(1) Here we have simplified the problem quite a bit. The planner problem delivers a family of optimal solutions parameterized by $\gamma(\cdot)$. Decentralizing these optimal solutions will typically entail a transfer positive or negative above the wage income. Thus a more detailed analysis would entail proving that for some $\gamma(\cdot)$, the transfer is zero. Obviously, there exists a γ constant such that this is the case (this is the standard Negishi argument). It can be shown that this is also true for non trivial choices of maps $\gamma(\xi, \lambda)$, $\gamma(\xi)$ and $\gamma(\lambda)$.

(2) It is evident that the market arrangements as formalized by (A) provides the sharing rule

$$u'(c(z^T))\gamma(\xi, \lambda) = \bar{u}'(\bar{c}(z^T))$$

by (B)

$$u'(c(z^T))g(\xi) = \bar{u}'(\bar{c}(z^T))$$

and by (C)

$$u'(c(z^T))g(\lambda) = \bar{u}'(\bar{c}(z^T))$$

3. RECURSIVE FORMULATION

The owner has access to a set of complete Arrow securities. Arrow securities are indexed by elements of the state space ζ; they cost $q(\zeta)$ and they pay off one dollar if ζ realizes and zero otherwise. Thus, they maximize:

$$U(K, \xi, \lambda) = \max_{(c,I,b)} \; u(c) + \beta E_{(\xi,\lambda)} U(K', \xi', \lambda') \tag{46}$$

subject to:

$$p(\zeta)[c(\zeta) + I(\zeta) + w(\zeta)n(\zeta) - \lambda f(K)] + \sum_{\zeta' \in Z} b(\zeta') \leq b(\zeta)$$

$$K' = (1 - \Omega)K + I(K, \xi, \lambda)$$

and

$$n(\zeta) \leq 1, \quad \zeta \in Z.$$

Bear in mind that with reference to the planner problem as well as the extensive general equilibrium formulation, $p(\zeta) = \frac{p_t(\cdot)}{\beta^t}$, while $q(\zeta') = 1$.

For instance, if (A) is the right formulation (that is if $\gamma(\cdot)$ is 1-to-1 in (λ', ξ'), the workers maximize:

$$\bar{U}(K, \xi, \lambda) = \max_{(c,n,b)} u(\bar{c}(K, \xi, \lambda)) + \beta E_{(\xi,\lambda)} \bar{U}(K', \xi', \lambda') \tag{47}$$

subject to

$$p(K, \lambda, \xi)[(\bar{c}(K, \lambda, \xi) - \xi) - w(K, \lambda, \xi)] + \sum_{K'} q(K', \lambda, \xi) b(K', \lambda, \xi)$$
$$= b(K, \xi, \lambda), (\lambda, \xi) \in \Lambda \times \Xi.$$

and $\quad \bar{n}(\zeta) \leq 1, \ \zeta \in Z.$

Thus, the worker at the state realization (K, λ^*, ξ^*) can only buy Arrow securities assets indexed by (K', λ^*, ξ^*). In problem (47), $\bar{U} \cdot)$ denotes the worker's value function.

Remark: If we give up recursive representations, we can take general weights of the form $\gamma(z^T)$ delivering the sharing rule $\bar{u}'(\bar{c}(z^T))\gamma(z^T) = u'(c(z^T))$. The programming problem of the owner stays the same, modulo that prices are now functions of histories. The budget constraints of the worker can be expressed in a fairly general way as follows.

Let $\Gamma = \{\gamma : \gamma = \gamma(z^T)$ for some $z^T\}$ and let $Z^T(\gamma) = \{z^T : \gamma(z^T) = \gamma$, for all $T\}$;

then

$$\sum_t \sum_{z^T \in Z(\gamma)} p(z^T)[\bar{c}(z^T) - \xi) - w(z^T)] = 0, \text{ and } \bar{n}(z^T) \leq 1, \gamma \in \Gamma.$$

APPENDIX 2: Model Descriptions

Unless otherwise stated, when the model distinguishes between workers and firm owners, $u^s() = u(c) = \ell n(c)$ denotes the firm owner utility function, and $v(c) = \frac{c^{1-\gamma}}{1-\gamma}$ the utility function of the workers.

A. 2. 1. The Danthine and Donaldson (1992) Model

Danthine et al. (1992) can be viewed as exploring a special case of the distributional risk perspective. It presumes the shareholder-worker distinction, but in the context of a more elaborate representation of the labor market. Primary workers, those with a permanent association with the firm (no layoffs) are governed by a complete risk sharing arrangement stronger than the one considered in this paper: their labor supply, n_t^P, is fixed at one unit per period with wage payment governed by the relationship:

$$v_1^P(w^P(k,\,\lambda)) = \hat{\theta}\,u_1^s(c^s(k,\,\lambda)) \tag{48}$$

where $v^P(\;)$ is the period utility function of a permanent worker, and $\hat{\theta}$ is a constant.

Secondary workers receive partial income insurance in the form of an equilibrium wage floor $w^f(k,\,\lambda)$ financed by a lump-sum tax on shareholder incomes. Their wage income is then of the form

$$w^{\text{sec}}(k,\,\lambda) = \max\left\{w^f(k,\,\lambda),\; w(k,\,\lambda)\right\} \tag{49}$$

where $w(k,\,\lambda)$ is the secondary worker Walrasian wage which satisfies

$$f_3(k_t, n_t^P, n(k_t, \lambda_t)\lambda_t = w(k_t, \lambda_t) \tag{50}$$

In equation (50), $n_t = n(k_t, \lambda_t)$ denotes the Walrasian level of employment and

$$f(k_t,\,n_t^P,\;n_t)\lambda_t = k_t^\alpha (n_t^P)^{(1-\alpha)\nu}(n_t)^{(1-\alpha)(1-\nu)}\lambda_t$$

the period production technology.

In some states the wage floor may exceed the Walrasian wage leading to the unemployment of secondary workers. The fraction $(1 - n(k,\,\lambda))$ of unemployed secondary workers receives the aforementioned income transfer, $t\,(k,\,\lambda)$. The wage floor $w^f = w^f\,(k,\,\lambda)$ and the transfer $t = t\,(k,\,\lambda)$ jointly solve

$$\max_{\{w^f,\,t\}}\;\; \lambda\,u^s\,(c\,(k,\,\lambda)) + v^P\left(w^P(k,\,\lambda)\right) + n(k,\,\lambda)\,v(w^f) + (1 - n(k,\,\lambda)\,v(t(k,\,\lambda))$$

subject to:

$$w_f \geq t(k,\,\lambda)$$

$$1 \geq n\,(k,\,\lambda)$$

with the parameter λ chosen to yield an appropriate share of income to capital while $w^P(k,\,\lambda), n(k,\lambda)$ are determined, respectively, by equations (48) and (49).

Lastly, the firm solves

$$\max E\left(\sum_{t=0}^{\infty}\beta^t u^s(c_t)\right)$$

s.t.

$$\begin{aligned} c_t + i_t &\leq \pi(k_t, \lambda_t) \\ k_{t+1} &= (1-\Omega)k_t + i_t, k_0\text{given}, \end{aligned}$$

where

$$\pi(k_t, \lambda_t) = \max_{n(k_t,\lambda_t)} \left\{f(k_t, n_t^P, n(k_t,\lambda_t))\lambda_t - w^P(k_t,\lambda_t)n_t^P - n(k_t,\lambda_t)w^{\text{sec}}(k_t,\lambda_t) - t(k_t,\lambda_t)(1-n(k_t,\lambda_t))\right\} \tag{51}$$

Under this formulation the level of income insurance is stronger for primary workers and weaker for secondary workers than in the distributional risk formulation. Calibrated to yield an unemployment rate of around 5%, the financial and business cycle characteristics of this model are given in Table 9. For the case reported there, $\alpha = .36$, $\nu = 1/2$, $\gamma = 7$, $u(c) = logc$, $\beta = .99$, $\lambda_1 = 1.025$, $\lambda_2 = .975$, $\pi_{ii} = .975$, $\Omega = .025$.

A. 2. 2. The Jermann (1998) Model

Although Jermann (1998) builds up from a decentralized version of his model, fundamentally it is a representative agent construct where the quantity-financial equilibrium arises from decision rules that solve:

$$\max_{\{c_t, I_t\}} E\left(\sum_{t=1}^{\infty}\beta^t u(C_t, C_{t-1})\right)$$

s.t.

$$\begin{aligned} & C_t + I_t \leq f(K_t, A_t N_t) \\ & K_{t+1} = (1-\Omega)K_t + \phi(I_t/K_t)K_t, \\ \text{where} \quad & u(C_t, C_{t-1}) = \frac{(C_t - b\,C_{t-1})^{1-\tau}}{1-\tau} \\ & f(K_t, A_t N_t) = K_t^{\alpha}(N_t A_t)^{1-\alpha} \\ & \phi(I_t/K_t) = \frac{a_1}{1-1/\xi}\left(\frac{I_t}{K_t}\right)^{1-1/\xi} + a_2 \end{aligned}$$

where, for the case reported in Table 11, $\xi = .23, b = .82, \tau = 5, \beta = .99, \Omega = .025, \alpha = .64$ and $A_{t+1} = \rho A_t + \varepsilon_t$ with ρ=.99. The parameters a_1, a_2 are arbitrarily fixed to match the state state of the comparison (no costs of adjustment, etc.) standard RBC model.

A. 2. 3. The Boldrin et al. (2001) Model

Boldrin et al. (2001) take a perspective which eschews variations in factor shares in favor of a representative agent model where the agent has a strong desire to smooth his consumption coupled with profound technological restrictions to doing so. The former feature is induced by a habit formation preference structure

$$u\left(C_t, C_{t-1}\right) = \log(C_t - bC_{t-1}) - N_t$$

where, in their best case scenario, $b = .73$. The former feature is derived from their assumption of a two sector (firm) economy, one producing consumption goods according to

$$C_t \leq K_{c,t}^{\alpha}(\lambda_t N_{c,t})^{1-\alpha},$$

and the other producing capital goods as per

$$\begin{aligned} K_{c,t+1} + K_{i,t+1} &\leq K_{i,t}^{\alpha}(\lambda_t N_{i,t})^{1-\alpha} + (1-\Omega)(K_{c,t} + K_{i,t}) \\ N_{c,t} + N_{i,t} &\leq N_t. \end{aligned}$$

From the asset pricing perspective the key aspect is that $K_{c,t+1}, K_{i,t+1}, N_{c,t+1}, N_{i,t+1}$ must all be chosen at time t before the technology shock $\lambda_{t+1}$is realized. The reallocation of capital or labor between sectors ex post to the λ_{t+1} realization is prohibited. In this model context, the equity return on the market portfolio is a weighted average of the returns to shares in each of the two firms, where the weights are the respective shares of the aggregate capital stock invested in the two firms.

A. 2. 4 The Danthine and Donaldson (2002) Model

The central planning version of the Danthine and Donaldson (2002) model is as follows:

$$\max_{\{C_t, I_t, W_t\}} E\left(\sum_{t=0}^{\infty} \beta^t \left[\tilde{\mu}_t v(W_t) + u(C_t)\right]\right)$$

$$
\begin{aligned}
& C_t + W_t + I_t \leq f(K_t, 1)\tilde{\lambda}_t - g(I_t, K_t) \\
& K_{t+1} = (1-\Omega)K_t + I_t; K_0 \text{ given,} \\
\text{where} \quad & f(K_t, N_t)\tilde{\lambda}_t = Lk_t^{\alpha} N_t^{1-\alpha}\tilde{\lambda}_t, \\
& v(W_t) = \frac{W_t^{1-\gamma}}{1-\gamma}, u(C_t) = \ell n C_t, \text{ and} \\
& g(I_t, K_t) = (\phi/2)\,(1/K_t)\,(I_t - \Omega K_t)^2.
\end{aligned}
$$

For the case of Table 12, $\alpha = .36, L = 1.25, \beta = .99, \Omega = .025, \lambda_1 = 1.056, \lambda_2 = .944, \tilde{\mu}_1 = \ 339, \mu_2 = 1062, \psi = .97239, \pi = H = \Delta = 0, \sigma = .03, \gamma = 4$.

A. 2. 5. The Guvenen Model

Guvenen (2005) presents a decentralized set up. Since the financial markets are incomplete, there is no central planning counterpart.

Firms. The single firm chooses its investment plan and hires labor so as to maximize its value to its shareholders:

$$
V_t^F = \max_{\{I_{t+j}, n_{t+j}^f\}} E_t \left\{ \sum_{j=1}^{\infty} \beta^j \left(\frac{\Lambda_{t+j}}{\Lambda_t} \right) \left[\lambda_{t+j}(K_{t+j})^{\alpha} \left(n_{t+j}^f \right)^{1-\alpha} - W_{t+j} n_{t+j}^f - I_{t+j} \right] \right\}
$$

$$
\text{s.t. } K_{t+j} = (1-\Omega)K_{t+j-1} + \Phi\left(\frac{I_{t+j}}{K_{t+j}} \right) K_{t+j}
$$

where $\Phi()$ is as in Jermann (1998) (and identically parameterized) and $\frac{\Lambda_{t+j}}{\Lambda_t} = \frac{u_1^s(c_{t+j}^s)}{u_1^s(c_t^s)}$; $\left(c_{t+j}^s\right)$ is the representative shareholder's consumption in equilibrium.

Shareholders and Non Shareholders: Guvenen (2005) selects (K_t, λ_t, B_t)

as his state variables vector where B_t denotes the aggregate (one period discount) bond holdings across the non-shareholder class. The recursive representation of consumer-worker-investor i's problem is:

$$
V^i\left(b_t^i, s_t^i, K_t, B_t, \lambda_t\right) = \max_{\left\{c_t^i, b_{t+1}^i, s_{t+1}^i\right\}} \left\{ u^i(c_t^i) + \beta E_t V^i\left(b_{t+1}^i, s_{t+1}^i, K_{t+1}, B_{t+1}, \lambda_{t+1}\right) \right\}
$$

s.t.

$$
\begin{aligned}
& c_t^i + q^b\left(K_t, B_t, \lambda_t\right) b_{t+1}^i + q^e\left(K_t, B_t, \lambda_t\right) s_{t+1}^i \\
& \qquad \leq b_t^i + s_t^i\left(q^e\left(K_t, B_t, \lambda_t\right) + d\left(K_t, B_t, \lambda_t\right)\right) + w\left(K_t, B_t, \lambda_t\right) \quad (52)
\end{aligned}
$$

and given the equation of motion on the state variables.

For the shareholder class, $i = s, b_t^s > 0, s_t^s > 0$. For the non shareholder class, $i \ = \ ns, s_t^{ns} = 0$, and $b_t^{ns} = B_t$.

In equilibrium

$$\begin{aligned}\theta b_t^s + (1-\theta) b_t^{ns} &= 0 \\ \theta s_t^s &= 1 \\ n_t^f = \theta \cdot 1 + (1-\theta)1 &= 1\end{aligned}$$

where θ denotes the fraction of the population who are shareholders.

For the case reported in the text, $\theta = .2, \beta = .99, \gamma_s = 2$, $\gamma_{ns} = 10$, $\rho = .95$, $\sigma_e = .02$, $\alpha = .3$, $\Omega = .02$, $\xi = .23$ (this is a cost of adjusting capital parameter; see Jermann (1998)).