A Theory of Corporate Social Responsibility in Oligopolistic Markets*

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Abstract

This paper provides a theory of corporate social responsibility in imperfectly competitive markets. We consider a two-stage game where consumers have a preference from buying goods from firms that do CSR and where firms first decide simultaneously the amount per unit sold to give to social causes and then choose quantities. We find that firms will do CSR when products are complements but might not do it when products are substitutes. We characterize how contributions to social causes depend on costs of production and on the degree of product differentiation. Finally, we show that CSR increases quantities, prices and profits.

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1 Introduction

Lately corporate social responsibility has been in the spot light. Indexes measure the performance of socially responsible firms, and some specific funds – Ethical Funds – are being created for firms with the best social responsibility standards. The European Commission has published a document where it tries to incorporate corporate social responsibility (CSR from now on) concerns in European Union's policies.[1]

Another observable trend is social rating, a process by which firms are given a rate based on social and environmental criteria used in management.[2] The idea behind social rating is that investors have the power to change firms' behavior through this non financial rate. Disrespect for social and environmental issues represents a risk for the firm's image though financially it may not have a direct impact.

Marketing studies show that 70% of European consumers consider important a firm's commitment to CSR when buying a product or service and, moreover, 1 in 5 consumers would be willing to pay more for products that are socially and environmentally responsible. A more recent study, in 2003, reveals that more than eight in ten British consumers consider important that a firm shows a high degree of social responsibility, when making their purchasing decisions.[3] Also, half of consumers thinks that companies do not listen and respond to their social and environmental concerns.[4]

Many firms recognize that their social actions can bring them benefits. Some of the most important benefits of CSR seem to be: enhanced brand image and reputation, increased ability to attract and retain employees, and potentially reduced regulatory oversight.[5] There is a fast growing literature in economics and management on the benefits and costs of CSR.

This paper analyzes the impact of CSR on strategic interactions between firms using an oligopolistic framework. Firms play a two stage game where in the first stage they can commit to donate a given amount of money per each unit sold to social causes and in the second stage, given their first-stage commitments, compete in quantities. Consumers prefer to buy goods from social responsible firms. We call this model the CSR commitment game.

In this framework there are two direct effects of CSR on firms' profits: CSR raises revenues by increasing consumer demand but it reduces profits by the amount given to social causes. These two effects occur regardless of the behavior of the competitors. CSR also has a strategic effect on firms' profits. The strategic effect which arises from the influence of a firm's choice of CSR in the

[1] "Corporate Social Responsibility: a Business Contribution to Sustainable Development."

[2] An example is the ARESE, "Agence de Rating Social et Environnemental sur les Entreprises," one of the first social and environmental rating agencies.

[3] See www.mori.com/polls/2003/mori-csr.shtml.

[4] Mohr and Webb (2005) study empirically the impact of CSR on price and consumer responses. Their results indicate that CSR has a positive effect on consumers' evaluation of a firm and on their purchase intent. They also find that a low price does not appear to compensate for a low level of CSR.

[5] See www.csrnetwork.com.

rivals' choices. We find that the strategic effect is always positive in our model. This implies that a firm always wants to overinvest in CSR regardless of CSR making the firm tough or soft.

We apply Fudenberg and Tirole's (1984) taxonomy of business strategies to our model of CSR. We find that when goods are substitutes, CSR makes firms tough and so firms should follow a "top dog" strategy: overinvest in CSR to be *more* aggressive, and hence inducing the rival to be less aggressive. When goods are complements, CSR makes firms soft and so firms should follow a "fat cat" strategy: overinvest in CSR to be *less* aggressive, and hence inducing the rival also be less aggressive.

We provide conditions for an equilibrium of the CSR commitment game to exist where contributions to social causes are strictly positive. A necessary condition is that consumers' maximum marginal valuation of CSR compensates firms' marginal cost of CSR. This is also a sufficient condition if goods are complements. If goods are substitutes, there will only be an equilibrium with strictly positive contributions to social causes if the degree of product substitutability is not excessively high. Thus, we find that the conditions for CSR to exist are less restrictive when goods are complements than when they are substitutes.

Next, we study how costs of production and the degree of product differentiation influence contributions to social causes. We distinguish between the impact of these two variables on contributions per unit sold and on total contributions (the product of contributions per unit sold and output). We find that contributions to social causes *per unit sold* are increasing with the cost of production. Thus, a high cost firm will make higher contributions to social causes *per unit sold* than a low cost firm. We also find that if goods are complements, then an increase in the degree of product differentiation increases contributions to social causes *per unit sold*.

We show that the impact of cost of production on *total* contributions to social causes is ambiguous. On one hand, an increase in the cost of production raises contributions per unit sold which has a positive impact on total contributions. On the other hand, an increase in the cost of production reduces the number of units sold which has a negative impact on total contributions. The impact of the degree of product differentiation on *total* contributions to social causes is also ambiguous. If goods are complements an increase in the degree of product differentiation raises contributions per unit sold which has a positive impact on total contributions. However, an increase in the degree of product differentiation reduces the number of units sold which has a negative impact on total contributions.

Finally, we study the impact of CSR in market outcomes. To do that we compare the equilibrium outcomes of our CSR commitment game with the equilibrium outcomes of a game where firms are not able to commit to CSR. We find that CSR increases equilibrium quantities, prices, and profits.

The papers that are most closely related to ours are Bagnoli and Watts (2003), Polishchuk and Firsov (2005), Besley and Ghatak (2006), and Manasakis, Mitrokostas and Petrakis (2007).[6] Our paper makes three contributions to the CSR literature. First, we show that Fudenberg and Tirole's (1984) taxonomy of business strategies can be applied to study CSR. Second, we prove that firms will choose to engage in CSR when products are complements but might not do it if there is a high degree of product substitutability. Third, we describe how the amount of CSR depends on firms' costs and on the degree of product differentiation.

2 The CSR Commitment Game

In this section we set up the model, we describe the two-stage game played by firms, and we show how Fudenberg and Tirole's (1984) taxonomy of business strategies can be applied to CSR.

2.1 Consumers' Preferences

The representative consumers has preferences given by:

$$U(q_1, q_2; s_1, s_2) = \sum_{i=1,2} \left(\alpha q_i - \frac{1}{2}\beta q_i^2 + \theta(s_i q_i) - \frac{1}{2}\mu(s_i q_i)^2 \right) + \gamma q_1 q_2 \qquad (1)$$

where q_i is the units of good $i = 1, 2$, and $s_i q_i$ is the monetary contribution to social causes from buying good i. The parameters α, β, θ and μ are assumed to be strictly positive. The parameter γ measures the complementarity between goods and can be positive (goods are complements), negative (goods are substitutes) or zero (goods are independent). The parameter γ also influences the degree of product differentiation. The closer γ is to zero the higher the degree of product differentiation. Under this set-up, consumers not only care about the consumption of goods 1 and 2 but also about how much they will contribute to social causes via consumption of each good. For utility to be concave we need that:

$$\triangle = \frac{\partial U^2}{\partial q_1^2}\frac{\partial U^2}{\partial q_2^2} - \left(\frac{\partial U^2}{\partial q_1 \partial q_2}\right)^2 = \beta^2 + \beta\mu(s_1^2 + s_2^2) + \mu^2 s_1^2 s_2^2 - \gamma^2 > 0 \text{ for } i \neq j \quad (2)$$

So, a sufficient condition is that $|\gamma| < \beta$. The representative consumer solves the problem:

$$\max_{q_1, q_2} U(q_1, q_2; s_1, s_2) - \sum_{i=1,2} p_i q_i.$$

The solution to this problem gives us the inverse demand for each good:

$$p_i(q_1, q_2; s_i) = \alpha + \theta s_i - (\beta + \mu s_i^2) q_i + \gamma q_j, \; i \neq j = 1, 2. \qquad (3)$$

We see from (3) that an increase in contributions to social causes per unit sold by firm i raises the intercept (the term θs_i) and the slope (the term μs_i^2) of the

[6] We discuss these papers in Section 6.

inverse demand of good i. We assume that the parameter μ is sufficiently small such that CSR increases the inverse demand of good i for all output levels.[7]

2.2 Firms' Choices

We consider an oligopolistic market with two firms, 1 and 2. Firms have constant marginal costs of production given by $0 \leq c_i < \alpha$, $i = 1, 2$. In the first stage of the game each firm can commit simultaneously to give a s_i, with $i = 1, 2$, dollars per unit sold to social causes. In the second stage firms observe the commitments made in the first stage and decide simultaneously the quantities that each one will set in order to maximize profits. This game is solved by backward induction.

The second stage problem of firm i is:

$$\underset{q_i}{Max} \pi_i = \left[p_i(q_i, q_j, s_i) - (c_i + s_i) \right] q_i \tag{4}$$

where $p_i(q_i, q_j, s_i)$ is given by (3). The first-order condition is

$$\frac{\partial \pi_i}{\partial q_i} = -2\left(\beta + \mu s_i^2\right) q_i + \alpha + \theta s_i + \gamma q_j - (c_i + s_i) = 0.$$

From here we obtain firm i's best reply to quantities produced by firm j for given contribution of firm i to social causes

$$q_i\left(s_i, q_j\right) = \frac{\alpha - c_i + (\theta - 1)\, s_i}{2\left(\beta + \mu s_i^2\right)} + \frac{\gamma}{2\left(\beta + \mu s_i^2\right)} q_j, \ i = 1, 2 \neq j. \tag{5}$$

Quantities are strategic complements for $\gamma > 0$ and strategic substitutes for $\gamma < 0$. The equilibrium quantities produced by each firm as a function of contributions to social causes are given by the intersection of the best reply functions. The solution is

$$q_i\left(s_1, s_2\right) = \frac{2(\beta + \mu s_j^2)\left(\alpha - c_i + (\theta - 1)\, s_i\right) + \gamma\left(\alpha - c_j + (\theta - 1)\, s_j\right)}{4\left(\beta + \mu s_i^2\right)\left(\beta + \mu s_j^2\right) - \gamma^2}. \tag{6}$$

Doing the second step of the backward induction process, we have that in the first stage of the game, firms decide the per unit monetary value donated to social causes. They will do it in such a way that total payoffs are maximized:

$$\underset{s_i \geq 0}{Max} \pi_i(s_1, s_2) = \left[p_i(q_i(s_i, s_j), q_j(s_j, s_i), s_i) - s_i - c_i \right] q_i(s_i, s_j). \tag{7}$$

[7] Following Shy (1995), we can use the ratio $\delta \equiv \left(\frac{\partial q_i / \partial p_j}{\partial q_i / \partial p_i}\right)^2$ as a measure of product differentiation. In our case we have $\delta = \frac{\gamma^2}{\left(\beta + \mu s_i^2\right)^2}$. Note that $\delta \in [0, 1)$ with $\delta \rightarrow 0$ meaning high differentiated products and $\delta \rightarrow \frac{\beta^2}{\left(\beta + \mu s_i^2\right)^2}$ meaning low differentiation (i.e., homogeneous products). So, in our model CSR increases product differentiation.

Note that we are maximizing profits over a compact set since s_i must be smaller than $p_i - c_i$ and firms do not wish to set $p_i = \infty$ since that would yield them no sales and so zero profits. Meaning this that $s_i \in [0, \bar{s}]$ with $\bar{s}$ finite. This together with the continuity of the profit function assures the existence of a solution to the problem.

2.3 Taxonomy of Business Strategies

We will now show how Fudenberg and Tirole's (1984) taxonomy of business strategies can be applied to our model of CSR. Contributions can be seen as an investment in the broad sense of the word. Moreover they can influence rival's actions and, therefore, contributions can be considered strategic variables.

From (7) we obtain the effect of s_i on π_i which is given by:

$$\frac{\partial \pi_i(s_1, s_2)}{\partial s_i} = \left(\frac{\partial p_i}{\partial s_i} - 1\right) q_i + \left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial s_i}\right) q_i, \tag{8}$$

since the impact of s_i on π_i via q_i is of second-order. The first term in (8) is the direct effect and the second term the strategic effect. The direct effect can be interpreted as a cost minimizing effect, since it only considers the costs and benefits firm i incurs with its own choice of contributions per unit sold, regardless the impact of that choice on firm j's actions. The strategic effect results from the influence of firm i's choice of contributions per unit sold on firm j's quantity choice.

Let us interpret the direct effect first: $\left(\frac{\partial p_i}{\partial s_i} - 1\right) q_i$. The direct effect of contributions on profits has two components. On the one hand contributions increase demand but on the other hand they increase costs. The sign of $\frac{\partial p_i}{\partial s_i}$ measures the impact of an increase in s_i on the price that can be charged for good i. Since consumers like s_i this derivative is positive, that is, the firm can charge a higher price for the bundle (q_i, s_i'') than for the bundle (q_i, s_i'), for $s_i'' > s_i'$. The term -1 measures the impact of an increase in s_i on costs. This term is equal to -1 because a one dollar increase in s_i raises the cost of each unit sold by one dollar.

Let us now interpret the strategic effect: $\left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial s_i}\right) q_i$. The sign of $\partial p_i/\partial q_j$ is determined by the relation between goods, while the sign of $\partial q_j/\partial s_i$ can be related to the slope of second period's best response function of firm j and with the effect of s_i in firm j's price. To see why note that the sign of the strategic effect can be written as

$$sign\left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial s_i}\right) = sign\left(\frac{\partial p_j}{\partial q_i}\frac{\partial q_j}{\partial q_i}\frac{\partial q_i}{\partial s_i}\right) = sign\left(\frac{\partial p_j}{\partial q_i}\frac{\partial q_i}{\partial s_i}\right) sign\left(\frac{\partial BR_j}{\partial q_i}\right), \tag{9}$$

where BR_j stands for firm j's best response and the term $\frac{\partial p_j}{\partial q_i}\frac{\partial q_i}{\partial s_i} = \frac{\partial p_j}{\partial s_i}$ illustrates if firm i's contributions make the firm soft or tough. The $sign\left(\frac{\partial p_j}{\partial q_i}\frac{\partial q_i}{\partial s_i}\right)$ is given by the sign of $\frac{\partial p_j}{\partial q_i}$ which comes from the relation between goods in the utility function. We thus have that $\frac{\partial p_j}{\partial q_i} = \frac{\partial p_i}{\partial q_j} = \gamma$. Therefore, if goods are substitutes, $\gamma < 0$, then $\frac{\partial p_j}{\partial q_i}\frac{\partial q_i}{\partial s_i} < 0$, meaning that contributions to social causes make firm i tough. On the other hand, if goods are complements, $\gamma > 0$, then $\frac{\partial p_j}{\partial q_i}\frac{\partial q_i}{\partial s_i} > 0$, meaning that contributions to social causes make firm i soft.

In our model we have that

$$\frac{\partial \pi_i(s_1, s_2)}{\partial s_i} = (\theta - 2\mu s_i q_i - 1)\, q_i + \left(\gamma \frac{\partial q_j}{\partial s_i}\right) q_i,$$

where the first term is the direct effect and the second term the strategic effect. The sign of the strategic effect is given by

$$sign\left(\gamma \frac{\partial q_j}{\partial s_i}\right) = sign\left(\gamma \frac{\partial q_j}{\partial q_i}\frac{\partial q_i}{\partial s_i}\right) = \frac{\gamma^2}{2(\beta + \mu s_j^2)} sign\left(\frac{\partial q_i}{\partial s_i}\right). \tag{10}$$

We see from (10) that the sign of the strategic effect is positive under our assumption that CSR is demand expanding. So, considering only the strategic effect, firm i should overinvest in CSR regardless of CSR making the firm tough or soft.[8]

Fudenberg and Tirole's taxonomy of business strategies can now be applied to our model. When goods are substitutes, CSR makes firms tough and best responses are negatively sloped. In this case firms should follow a "top dog" strategy: overinvest in CSR to be *more* aggressive, and hence inducing the rival to be less aggressive. When goods are complements, CSR makes firms soft and best responses are positively sloped. In this case firms should follow a "fat cat" strategy: overinvest in CSR to be *less* aggressive, and hence inducing the rival also be less aggressive.

3 Existence of Equilibrium

Our first result provides conditions under which there exist a subgame perfect Nash equilibrium of the CSR commitment game where contributions to social causes are strictly positive.

Proposition 1 *The CSR commitment game has an equilibrium in which firms contribute to social causes if $\theta > 1$ and*

$$\gamma \in \left(\left\{\max\left(-\beta, -\frac{2\sqrt{2\mu\beta(\alpha - c_i)(\alpha - c_j)}}{(\theta - 1)}\right)\right\}, \beta\right).$$

By engaging in corporate social responsibility firms make their products more valuable to consumers. This demand expanding effect of CSR increases sales and raises revenues. However, firms also incur a cost by doing CSR, the total

[8] Here "overinvest" means that firms should choose a higher level of contributions to social causes per unit sold than the level they would choose if they did not take into account the strategic effect of CSR.

monetary contributions donated to social causes. Proposition 1 tells us that a necessary condition for firms to engage in CSR is that consumers' maximum marginal willingness to pay for a firm's social behavior, θ, must be higher than the marginal cost of increasing CSR to the firm, which is 1 dollar. Thus, only when consumers place a sufficiently high value on CSR will firms do practice it.

Proposition 1 also tells us that if goods are complements, $\gamma \in (0, \beta)$, then the assumption that consumers' maximal marginal willingness to pay for the firm's social behavior has to be higher than the marginal cost of CSR is a necessary and sufficient condition for firms to practice CSR. However, that is no longer the case if goods are substitutes, $\gamma \in (-\beta, 0)$. When the degree of product substitutability is high, that is, $\gamma \in \left(-\beta, -\frac{2}{(\theta-1)}\sqrt{2\mu\beta(\alpha - c_i)(\alpha - c_j)}\right)$, firms prefer not to engage in CSR. The intuition behind this result is as follows.

If goods are complements an increase in firm i's contributions to social causes will increase firm i's quantity sold but also its rival's quantity. Similarly, the more a rival donates, the higher will be a firm's quantity sold. So, in this case, the only requirement for the existence of CSR equilibrium is that CSR marginal cost is outweighed by the maximum marginal willingness to pay for CSR by consumers. In contrast, if goods are substitutes an increase in firm i's contributions will lower firm j's sales. The effect is larger the larger is the level of product substitutability. Thus, for the existence of an equilibrium with strictly positive contributions when goods are substitutes, there has to be a balance between θ and γ. For example, if consumers' marginal willingness to pay for CSR, θ, is not sufficiently high, then it is sufficient that substitutability is not to high itself for an equilibrium to exist. However, if consumers' marginal willingness to pay for CSR is sufficiently high, then a CSR equilibrium exists for any level of product substitutability.

4 Equilibrium Contributions to Social Causes

In this section we study how equilibrium contributions to social causes change with cost of production and the degree of product differentiation.

4.1 The Symmetric CSR Commitment Game

To simplify the analysis we start by assuming that firms have the same marginal cost of output ($c_1 = c_2 = c$). In this case the CSR commitment game becomes symmetric, and there exists a symmetric subgame perfect Nash equilibrium where $s_1 = s_2 = s$. Our next result characterizes equilibrium contributions per unit sold in the symmetric equilibrium.

Lemma 1: *In the symmetric CSR commitment game, equilibrium contributions to social causes per unit sold are the solution to*

$$s\left(\frac{\alpha - c}{\theta - 1}\right) = \frac{2\left(2\beta - \gamma\right)\left(\beta + \mu s^2\right)^2 - \gamma^2 \mu s^2}{\mu\left(4(\beta + \mu s^2)^2 + \gamma^2\right)}. \tag{11}$$

We use (11) to find out how equilibrium contributions to social causes per unit sold vary with the cost of production and the degree of product differentiation. Our next result summarizes the findings.

Proposition 2 *If* $\frac{\alpha-c}{\theta-1} > \frac{2\beta-\gamma}{\sqrt{5\mu\beta}}$, *then equilibrium contributions to social causes per unit sold are (i) increasing in* c, *and (ii) decreasing in* γ, *for* $\gamma > 0$.

Proposition 2 tells us that if consumers' maximum marginal willingness to pay for social behavior by the part of firms is not excessively high, then firms' contributions to social causes per unit sold are increasing with the cost of production. The intuition behind this result is that the higher the cost of production, the more it pays for firms to use CSR to expand the demand of their products. Proposition 2 also tells us that if consumers' maximum marginal willingness to pay for social behavior by the part of firms is not excessively high and goods are complements, then an increase in the degree of product differentiation (a decrease in γ) raises contributions to social causes per unit sold. In other words, everything else constant, if goods are complements, one should expect higher contributions to social causes per unit sold in markets where product differentiation is high than in markets where product differentiation is low.[9]

Let us now consider the impact of cost of production and the degree of product differentiation on total contributions to social causes. Total contributions of each firm are given by

$$S = sq(s), \tag{12}$$

where

$$q(s) = \frac{(\alpha - c) + (\theta - 1)\, s}{2\left(\beta + \mu s^2\right) - \gamma}. \tag{13}$$

Differentiating (12) with respect to $\tau \in \{c, \gamma\}$ we obtain

$$\frac{\partial S}{\partial \tau} = \left(q(s) + s\frac{\partial q}{\partial s}\right)\frac{\partial s}{\partial \tau} + s\frac{\partial q}{\partial \tau}. \tag{14}$$

From (13) we have that $\partial q/\partial c < 0$, $\partial q/\partial \gamma > 0$, for $\gamma > 0$, and

$$\frac{\partial q}{\partial s} = \frac{(\theta - 1)\left(2\left(\beta + \mu s^2\right) - \gamma\right) - 4\mu s\left((\alpha - c) + (\theta - 1)\, s\right)}{\left(2\left(\beta + \mu s^2\right) - \gamma\right)^2}.$$

After doing some algebra one can show that

$$q(s) + s\frac{\partial q}{\partial s} > 0.$$

These results together with Proposition 2 tell us that the impact of the cost of production on total equilibrium contributions to social causes is ambiguous. On the one hand, an increase in the cost of production raises per unit contributions

[9] When goods are substitutes the impact of γ on equilibrium contributions per unit sold is ambiguous.

which has a positive impact on total contributions. On the other hand, an increase in the cost of production reduces the number of units sold which has a negative impact on total contributions.

The impact of the degree of product differentiation on total equilibrium contributions is also ambiguous. If goods are complements an increase in the degree of product differentiation raises per unit contributions to social causes which has a positive impact on total contributions. However, an increase in the degree of product differentiation decreases the number of units sold which has a negative impact on total contributions.

4.2 The Asymmetric CSR Commitment Game

Do more efficient firms contribute more to social causes than less efficient firms? To answer this question we need to go back to the general version of the CSR commitment game where firms can have different costs of production. If we do that we find that equilibrium contributions per unit sold are the solution to

$$\left[2(\beta + \mu s_2^2)\left(\alpha - c_1 + \theta s_1 - s_1\right) + \gamma\left(\alpha - c_2 + \theta s_2 - s_2\right)\right] \frac{\mu s_1}{\phi(\theta - 1)} = \frac{2\phi - \gamma^2}{2\phi + \gamma^2}, \tag{15}$$

and

$$\left[2(\beta + \mu s_1^2)\left(\alpha - c_2 + \theta s_2 - s_2\right) + \gamma\left(\alpha - c_1 + \theta s_1 - s_1\right)\right] \frac{\mu s_2}{\phi(\theta - 1)} = \frac{2\phi - \gamma^2}{2\phi + \gamma^2}, \tag{16}$$

where $\phi = 2(\beta + \mu s_1^2)(\beta + \mu s_2^2)$. Diving (15) by (16) we have that

$$\frac{s_1}{s_2} = \frac{2(\beta + \mu s_1^2)\left(\alpha - c_2 + \theta s_2 - s_2\right) + \gamma\left(\alpha - c_1 + \theta s_1 - s_1\right)}{2(\beta + \mu s_2^2)\left(\alpha - c_1 + \theta s_1 - s_1\right) + \gamma\left(\alpha - c_2 + \theta s_2 - s_2\right)}$$

or

$$\frac{s_1}{s_2} = \frac{2(\beta + \mu s_1^2) + \gamma\left(\frac{\alpha - c_1 + \theta s_1 - s_1}{\alpha - c_2 + \theta s_2 - s_2}\right)}{2(\beta + \mu s_2^2)\left(\frac{\alpha - c_1 + \theta s_1 - s_1}{\alpha - c_2 + \theta s_2 - s_2}\right) + \gamma}. \tag{17}$$

The assumption that $|\gamma| < \beta$ implies that the impact of cost asymmetries on the denominator in the right-hand-side of (17) will be more important than the impact on the numerator. If $c_1 = c_2 = c$ we have a symmetric game and $s_1 = s_2$. Now, suppose that $c_1 < c < c_2$. In this case the denominator of (17) increases and so the right-hand-side of (17) decreases by comparison with the symmetric game. So, if $c_1 < c < c_2$, (17) will hold as an equality if the left-hand-side decreases by comparison with the symmetric game. This can only happen if $s_1 < s_2$. Thus, our model predicts that a low cost firm makes lower contributions to social causes *per unit sold* than a high cost firm.

Does the level of total contributions of each firm also depend on its cost? It turns out that the answer to this question is no. In our model firms' *total contributions* to social causes will be equal even if their costs are different. This result also follows from (15) and (16). From (6) we can rewrite (15) and (16) as

$$s_1 q_1(s_1, s_2) = \frac{\phi(\theta - 1)}{\mu(2\phi + \gamma^2)},$$

and

$$s_2 q_2(s_1, s_2) = \frac{\phi(\theta - 1)}{\mu(2\phi + \gamma^2)}.$$

Thus,

$$S_1 = S_2 = \frac{\phi(\theta - 1)}{\mu(2\phi + \gamma^2)},$$

where S_1 and S_2 denote the total contributions to social causes of firms 1 and 2, respectively. This happens because the higher sales volume of the low cost firm exactly compensates its smaller level of contributions per unit sold.

5 CSR and Market Outcomes

In this section we show how CSR influences market outcomes. We focus on the impact of CSR on equilibrium output, prices and profits. To do that we compare the predictions of our CSR commitment game to those of a game where firms cannot commit to CSR. As before, we focus on the symmetric game so $q_1(s) = q_2(s) = q(s)$, $p_1(s) = p_2(s) = p(s)$, and $\pi_1(s) = \pi_1(s) = \pi(s)$, where s is the equilibrium level of contributions per unit sold.

The equilibrium quantity produced by each firm in the symmetric equilibrium of the CSR commitment game is given by (13), the equilibrium price is

$$p\left(s\right) = \frac{\left(\alpha + c + s\left(1 + \theta\right)\right)\left(\beta + \mu s^2\right) - \gamma\left(c + s\right)}{2\left(\beta + \mu s^2\right) - \gamma},$$

and equilibrium profits are

$$\pi(s) = \frac{\left(\beta + \mu s^2\right)\left[\alpha - c + \left(\theta - 1\right)s\right]^2}{\left[2(\beta + \mu s^2) - \gamma\right]^2}.$$

When firms cannot commit to CSR they produce $q = (\alpha - c)/(2\beta - \gamma)$, they set price equal to $p = \left(\alpha\beta + (\beta - \gamma)c\right) / \left(2\beta - \gamma\right)$, and attain profits of $\pi = \beta\left(\alpha - c\right)^2 / \left(2\beta - \gamma\right)^2$. We now introduce a result that will be helpful for characterizing the impact of CSR on market outcomes.

Lemma 2 *In the symmetric CSR commitment game, equilibrium contributions to social causes per unit sold are bounded above by* $\bar{s} = \frac{\theta - 1}{\alpha - c}\frac{2\beta - \gamma}{2\mu}$.

Lemma 2 says that, in a symmetric equilibrium of the CSR commitment game, contributions per unit sold are bounded above by $\bar{s}$. This upper bound is quite intuitive since the marginal utility consumers derive from contributions to social causes is decreasing, meaning that from a certain level onwards marginal willingness to pay for CSR will fade out and the cost of CSR will overweight its benefit. We are now ready to state our last result.

Proposition 3 *In the equilibrium of the symmetric CSR commitment game: (i) $q(s) > q$, (ii) $p(s) > p$, and (iii) $\pi(s) > \pi$.*

This result shows that quantities, prices and profits will all be higher in the CSR commitment game than in a game where firms cannot commit to CSR. In our model consumers' perceive that products of firms engaging in CSR are of higher "quality." Firms know about this and use CSR to expand consumer demand. So, quantities produced when firms can commit to CSR are higher than if there can be no commitment to CSR. On the other hand, CSR means that firms will have higher unitary costs. This means that firms have to compensate the additional costs with a higher price. Thus, prices will be higher when firms can commit to CSR than when there is no possibility of commitment. Proposition 3 also shows that firms' profits increase with CSR since the demand expansion and the higher equilibrium prices compensate the cost of CSR.

6 Discussion

In this section we discuss the main assumption of the model. We compare our model to quality and advertising models. We also review the literature on CSR.

6.1 Assumptions of the Model

The assumption that firms can commit to give a dollar amount per unit sold is one possible way to model CSR among many. We could also have assumed that firms commit to give a percentage of sales or profits to social causes. The algebra would be slightly different but the main results of the paper would still hold. We chose to model CSR this way because we believe it is more reasonable to assume that firms are able to commit to give to social causes a dollar amount per unit sold or a percentage of their sales rather than a percentage of profits. Sales are easily verifiable while profits can be manipulated.

The assumption that an increase in CSR by firms increases consumer demand for both goods drives the impact of CSR on market outcomes. As we have seen, our model predicts that CSR raises quantities, prices and profits. Since we are using a partial equilibrium model this means that CSR creates a substitution effect away from the outside good. An alternative hypothesis would be to assume that CSR just shifts demand from one firm to the other. In this case consumers' social preferences would lead firms to a prisoner's dilemma and CSR would lower firms' profits.

6.2 Quality and Advertising Models

In a sense we introduce an additional unitary cost, so total cost of CSR for firms will depend on the sold quantity and total costs as a whole will also depend on sold quantities. From this point of view we can see CSR as a quality improvement in firms' goods, since it is as each unit of good is more expensive to produce with CSR than without it. The difference between our model from an advertising model is the fact that CSR is modeled as a variable cost rather than a fixed our a sunk cost as usually advertisement models do.[10] Even in models where advertisement costs are not fixed per se, they are invariant with quantities sold.[11] A formulation of the type $q = f(p, s)$ where s is interpreted as advertisement is not appropriate because in general consumers may not enjoy advertisement per se.[12] Application of advertisement models to marketing also different from our setup, since they are usually based on sales dynamics over time.[13]

6.3 CSR Literature

The literature on CSR can be divided into four main sets of papers: (i) CSR driven by managers and stakeholders, (ii) CSR driven by consumers, (iii) CSR driven by the labor force, and (iv) CSR driven by tax incentives. We will focus our review of the literature on the first three sets of papers.

6.3.1 CSR Driven by Managers and Stakeholders

Navarro (1988) examines the motives behind CSR in the context of profit maximization and managerial discretion. He argues that the profit motive may be nested within the managerial discretion motive and also predicts the reversal of the positive relation of changes in tax rates and tax deductible preferred expenditures. An empirical test of the theoretical model indicates that profit maximization is an important motive driving contributions. He also finds that CSR has a negative relation with debt versus equity in the firm's capital structure and a positive relation with increases in dividends.

Campbell, Gulas and Gruca (1999) investigate why some corporations give to social causes and others don't. They find a strong relationship between the personal attitudes of the charitable decision maker and the firm's giving behavior. They also find that firms with an history of charity giving justify it with altruistic motives while firms not giving to charity use business reasons as a justification.

Baron (2001) models strategic CSR as firms' response to private politics where lobbyists put pressure on firms to adopt more stringent environmental standards. He assumes that the initial environmental quality is at the first-best level so that lobbying leads to redistribution from firms to consumers, creating a distortion.

McWilliams and Siegel (2001) use a supply and demand theory of the firm to address the question of how much should a firm spend in CSR. They find

[10] For example, outdoors, media time, etc.

[11] See, for example, the advertisement models referred to in Tirole's "The Theory of Industrial Organization", where a fixed demand is considered and advertisement is seen as a form of non price competition and advertisement costs are "per consumer".

[12] As in the Dorfman-Steiner advertisement model.

[13] Known models are the Nerlove-Arrow, Vidale-Wolfe and Lanchester models, where we have a problem of optimal advertisement schedule over time, in order to achieve the optimal steady state level of goodwill or some modified variables including sales and market share.

that using cost benefit analysis it's possible to determine a level of CSR that maximizes profits and satisfies stakeholder demand for CSR. A set of hypothesis regarding the factors that can influence CSR are also developed allowing predictions of how CSR provision varies across industries, products and firms. Large and diversified firms were found to be more active in CSR. Results also support the neutrality of CSR in financial performance, that is, firms engaging in CSR and firms not engaging in CSR will have the same profit rates.[14]

Cespa and Cestone (2007) develop a model where CSR is related with managerial entrenchment. In their model stakeholders can affect CEO replacement and CEOs can make manager specific commitments in order to adopt a stakeholder friendly behavior. The likelihood of CEO replacement is reduced if the CEO aligns his preferences with those of stakeholders' through manager specific investment in CSR. Cespa and Cestone show that the congruence between managers' and stakeholders' objectives increases as the CEO increases investment in CSR expertise and that the CEO will be more willing to invest the more effective CSR is. The model also predicts that firms' incentives to engage in CSR are stronger when corporate control is more contestable.

6.3.2 CSR Driven by Consumers

Bagnoli and Watts (2003) present a model where firms compete for socially responsible consumers by linking the provision of a public good to sales of their private goods. In this context strategic CSR arises from competitive advantage seeking where the private provision of a public good becomes a by-product of product-market competition between firms. So, the provision of the public good will depend on consumers's willingness to pay as well as the structure of market competition. They analyse how the level of public good provided is affected by social responsible consumers, the competitive environment and how the type of public good provided can be influenced by strategic CSR.

Polishchuk and Firsov (2005) study CSR in a monopoly and in a perfectly competitive market. They model CSR as cause-related marketing (CRM).[15] They assume that consumers derive utility from consumption of a basic good and from their contributions to charity. Demands for the basic good and charity are mutually independent. A firm that engages in CRM incurs a fixed cost. They find that CRM results in a price for charity above the marginal cost and thus an additional source of profit for the firm. They also find that if a firm has a monopoly on the good market and introduces CRM, the outcome is equivalent to two separate monopolies on the markets for basic good and charity. However, if CRM is practiced on a competitive market, it makes all sides involved better-off.

Besley and Ghatak (2007) explore the feasibility and desirability of CSR. Considering two types of utility maximizing consumers, caring and neutral ones,

[14] Siegel and Vitaliano (2007) find that there is little evidence that large firms are more likely to be social responsible than smaller firms. However, they find that firms selling durable experience goods or credence services are much more likely to be socially responsible. Their results support the idea that CSR is a strategic variable for profit maximization.

[15] Cause Related Marketing is a business strategy where firms bundle their products and brands with contributions to designated charities.

and free entry of profit maximizing firms in the market, they show that CSR is consistent with profit maximization in competitive markets and that in equilibrium firms sell ethical and neutral brands and consumers self select according to their valuation of the public good. They assume that firms move first and offer a pair of price and amount of public good provided and consumers the decide to which firm to purchase.

Manasakis, Mitrokostas and Petrakis (2007) consider an oligopoly setting where consumers have social preferences and firms' owners decide if they want to hire socially responsible managers or self-interested managers. By hiring a socially responsible manager a firm can commit to CSR. They find that in equilibrium owners of firms will hire social responsible managers.

6.3.3 CSR Driven by the Labor Force

Greening and Turban (2000) find that applicants will be more attracted, and would prefer to work in firms with a good CSR reputation. Brekke and Nyborg (2008) model CSR as a way to improve firms' ability to hire highly motivated employees. They consider a competitive market for both firms and labour. Workers' utility decreases with effort but increases in the consumption of private goods, environmental quality and self-image as a socially responsible individual. Workers maximize utility choosing in which firm to work and their level of effort. Each firm emits pollution and can eliminate voluntarily its damage incurring in a fixed cost. This voluntary action is seen as CSR by workers. They find that profit maximizing firms will prefer to hire workers with higher concerns about social causes and will be able to do it at a lower wage. Moreover, the authors show that the average effort in socially responsible firms is higher than in non socially responsible firms.

7 Conclusion

This paper studies corporate social responsibility in imperfectly competitive markets where consumers value firms' contributions to social causes. By engaging in CSR firms can expand the demand they face and increase prices and sales.

The paper characterizes the direct and strategic effects of CSR on firms' profits. We show the strategic effect of CSR is always positive, meaning that firms will want to overinvest in CSR no matter if CSR makes them tough or soft. The paper also shows that firms will always engage in CSR when products are complements but might not do it when products are substitutes. The model is used to characterize how equilibrium contributions to social causes depend on firms' costs and on the degree of product differentiation. Finally, we show that CSR increases equilibrium quantities, prices and profits.

8 References

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9 Appendix

Proof of Proposition 1: Evaluating the derivative of profits with respect to firm's own contributions at $s_i = 0$ we obtain:

$$\left.\frac{\partial \pi_i(s_i, s_j)}{\partial s_i}\right|_{s_i=0} = \left(\gamma^2 \frac{\theta - 1}{4\beta(\beta + \mu s_j^2) - \gamma^2} + \theta - 1\right) q_i\left(s_i = 0, s_j\right).$$

Thus, if both terms are positive so will be $\left.\frac{\partial \pi_i(s_i,s_j)}{\partial s_i}\right|_{s_i=0}$. For the first term to be positive we must have $\theta > 1$. For the second term to be positive, $q_i\left(s_i = 0, s_j\right) > 0$, we must have that:

$$\frac{2\left(\beta + \mu s_j^2\right)(\alpha - c_i) + \gamma\left(\alpha - c_j - (1-\theta)\, s_j\right)}{4\beta\left(\beta + \mu s_j^2\right) - \gamma^2} > 0,$$

or

$$(2\beta + \gamma)(\alpha - c_i) + 2\left(\alpha - c_j\right)\mu s_j^2 + \gamma\left(\theta - 1\right) s_j > 0.$$

Since $\alpha > \max(c_1, c_2)$ and $|\gamma| < \beta$, the inequality always holds for $\gamma \in [0, \beta)$. We will now show that the inequality also holds for

$$\gamma \in \left(\max\left(-\beta, -\frac{2}{(\theta-1)}\sqrt{2\mu\beta(\alpha - c_i)(\alpha - c_j)}\right), 0\right). \tag{18}$$

To do that note that $|\gamma| < \beta$ implies $(2\beta + \gamma) > \beta$. So, if we can show that (18) implies

$$\beta(\alpha - c_i) + 2(\alpha - c_j)\mu s_j^2 + \gamma(\theta - 1)s_j > 0 \tag{19}$$

we are done. The LHS of (19) is convex in s_j and attains a global minimum at $s_j^{\min} = \frac{-\gamma(\theta-1)}{4(\alpha-c_j)\mu}$. Evaluating the LHS of (19) at $s_j^{\min}$ we obtain

$$2(\alpha - c_j)\mu\left(\frac{-\gamma(\theta - 1)}{4(\alpha - c_j)\mu}\right)^2 - \gamma^2\frac{(\theta-1)^2}{4(\alpha - c_j)\mu} + (\alpha - c_i)\beta,$$

which can be simplified to

$$-\frac{\gamma^2(\theta-1)^2}{8(\alpha - c_j)\mu} + (\alpha - c_i)\beta.$$

So, for γ such that

$$-\frac{\gamma^2(\theta-1)^2}{8(\alpha - c_j)\mu} + (\alpha - c_i)\beta > 0$$

or

$$-\frac{2}{(\theta-1)}\sqrt{2\mu\beta(\alpha - c_i)(\alpha - c_j)} < \gamma.$$

Q.E.D.

Proof of Lemma 1: Consider the first-order condition of profits:

$$\frac{\partial \pi_i(.)}{\partial s_i} = \left(p_i + \frac{\partial p_i}{\partial q_i} q_i - s_i - c_i\right) \frac{\partial q_i}{\partial s_i} + \left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial s_i} + \frac{\partial p_i}{\partial s_i} - 1\right) q_i = 0$$

From the envelope theorem we have that the first term is of second-order and so the condition simplifies to

$$\frac{\partial p_i(q_i, q_j, s_i)}{\partial s_i} - 1 + \frac{\partial p_i(q_i, q_j, s_i)}{\partial q_j}\frac{\partial q_j(s_i, s_j)}{\partial s_i} = 0$$

or

$$\theta - 2\mu s_i q_i(s_i, s_j) - 1 + \gamma \frac{\partial q_j(s_j, s_i)}{\partial s_i} = 0. \tag{20}$$

The derivative $\frac{\partial q_j(s_j,s_i)}{\partial s_i}$ is given by:

$$\frac{\partial q_j}{\partial s_i} = \frac{4\mu s_i(\alpha + \theta s_j - c_j - s_j) + \gamma(\theta - 1)}{4\left(\beta + \mu s_i^2\right)\left(\beta + \mu s_j^2\right) - \gamma^2} - \\ \frac{\left(2\left(\beta + \mu s_i^2\right)(\alpha + \theta s_j - c_j - s_j) + \gamma(\alpha + \theta s_i - c_i - s_i)\right)\left(\beta + \mu s_j^2\right)}{\left(4\left(\beta + \mu s_i^2\right)\left(\beta + \mu s_j^2\right) - \gamma^2\right)^2} 8\mu s_i.$$

Assuming that $c_1 = c_2 = c$, we have that $\frac{\partial q_j(s_j,s_i)}{\partial s_i}$ simplifies to:

$$\frac{\partial q_j}{\partial s_i} = \gamma \frac{(\theta - 1)\left(2\beta - \gamma - 2\mu s^2\right) - 4\mu s(\alpha - c)}{\left(2\mu s^2 + 2\beta - \gamma\right)^2\left(2\mu s^2 + 2\beta + \gamma\right)}.$$

This together with (20) yields:

$$\theta - 2\mu s q_i(s) - 1 + \gamma^2 \frac{(\theta - 1)\left(2\beta - \gamma - 2\mu s^2\right) + 4\mu s(c - \alpha)}{\left(2\mu s^2 + 2\beta - \gamma\right)^2\left(2\mu s^2 + 2\beta + \gamma\right)} = 0$$

Making use of (6) and rearranging terms we have that

$$s\left(\frac{\alpha - c}{\theta - 1}\right) = \frac{2(2\beta - \gamma)(\beta + \mu s^2)^2 - \gamma^2 \mu s^2}{\mu\left(4(\beta + \mu s^2)^2 + \gamma^2\right)}.$$

Q.E.D.

Proof of Proposition 2: Rewriting (11) we obtain

$$F = \frac{\mu(\alpha - c)\left(4(\beta + \mu s^2)^2 + \gamma^2\right) s}{\theta - 1} - 2(2\beta - \gamma)(\beta + \mu s^2)^2 + \gamma^2 \mu s^2 = 0. \tag{21}$$

From (21) we have that

$$\frac{\partial F}{\partial s} = \mu\left(\frac{\alpha - c}{\theta - 1} + 2s\right)\gamma^2 + 4\mu(\beta + \mu s^2)\left[\frac{\alpha - c}{\theta - 1}(\beta + 5\mu s^2) - 2(2\beta - \gamma)s\right].$$

A sufficient condition for $\partial F/\partial s$ to be strictly positive is that the term inside square brackets is nonnegative. The term inside square brackets is convex in s and attains a minimum at $s = \frac{\theta-1}{\alpha-c}\frac{2\beta-\gamma}{5\mu}$. So, if we can show that

$$\frac{\alpha-c}{\theta-1}\left[\beta+5\mu\left(\frac{\theta-1}{\alpha-c}\frac{2\beta-\gamma}{5\mu}\right)^2\right]-2\left(2\beta-\gamma\right)\frac{\theta-1}{\alpha-c}\frac{2\beta-\gamma}{5\mu}>0$$

we are done. The LHS of the above inequality reduces to

$$\frac{\alpha-c}{\theta-1}\beta-\frac{\theta-1}{\alpha-c}\frac{(2\beta-\gamma)^2}{5\mu}$$

which is positive provided that $\frac{\alpha-c}{\theta-1} > \frac{2\beta-\gamma}{\sqrt{5\mu\beta}}$. Thus,

$$\frac{\partial s}{\partial c}=-\frac{\partial F/\partial c}{\partial F/\partial s}=-\frac{1}{\partial F/\partial s}\frac{-\mu s\left(4(\beta+\mu s^2)^2+\gamma^2\right)}{\theta-1}>0.$$

and

$$\frac{\partial s}{\partial \gamma}=-\frac{\partial F/\partial \gamma}{\partial F/\partial s}=-\frac{2}{\partial F/\partial s}\left(\mu s\gamma\frac{\alpha-c}{\theta-1}+(\beta+\mu s^2)^2+\gamma\mu s^2\right).$$

The derivative $\partial s/\partial\gamma$ is negative for $\gamma > 0$. *Q.E.D.*

Proof of Lemma 2: Let

$$s=\frac{\theta-1}{\alpha-c}f(s)$$

where

$$f(s)=\frac{2\left(2\beta-\gamma\right)(\beta+\mu s^2)^2-\gamma^2\mu s^2}{\mu\left(4(\beta+\mu s^2)^2+\gamma^2\right)}.$$

We can rewrite $f\left(s\right)$ as

$$f(s)=\frac{2\left(2\beta-\gamma\right)-\frac{\gamma^2\mu s^2}{(\beta+\mu s^2)^2}}{4\mu+\frac{\gamma^2\mu}{(\beta+\mu s^2)^2}}$$

and taking limits as s goes to infinity, we can see that

$$f_\infty\equiv\lim_{s\to\infty}\frac{2\left(2\beta-\gamma\right)-\frac{\gamma^2\mu s^2}{(\beta+\mu s^2)^2}}{4\mu+\frac{\gamma^2\mu}{(\beta+\mu s^2)^2}}=\frac{2\beta-\gamma}{2\mu}$$

We can also show that $f\left(s\right) < f_\infty$ $\forall s > 0$. To see this we show that there is at most one extreme such that $s > 0$ and that this extreme is a global minimum below f_∞. Start with $f\left(0\right) = \frac{2(2\beta-\gamma)\beta^2}{\mu(4\beta^2+\gamma^2)}$. We prove that $f_\infty > f(0)$. We have that

$$f_\infty > f(0) \Leftrightarrow \frac{2\beta-\gamma}{2\mu} > \frac{2\left(2\beta-\gamma\right)\beta^2}{\mu\left(4\beta^2+\gamma^2\right)}.$$

or

$$\mu\left(4\beta^2+\gamma^2\right)(2\beta-\gamma) > 4\mu(2\beta-\gamma)\beta^2 \Leftrightarrow 4\beta^2+\gamma^2 > 4\beta^2 \Leftrightarrow \gamma^2 > 0,$$

which is always true. For the remaining values of $s > 0$ we can use first and second derivatives to study the behavior of f. Taking the first derivative of $f(s)$ and evaluating it at zero, we find three possible extrema:
i) $s_0 = 0$ for any γ
ii) $s_1 = \sqrt{\frac{-(2\beta-\gamma)-\gamma\sqrt{2}}{2\mu}}$, for $\gamma_1 \leq -2\beta\left(\sqrt{2}+1\right)$
iii) $s_2 = \sqrt{\frac{-(2\beta-\gamma)+\gamma\sqrt{2}}{2\mu}}$, for $\gamma_2 \geq 2\beta\left(\sqrt{2}-1\right) > 0$
But we still have another condition that needs to be verified: $|\gamma| < \beta$. Take γ_1 in equality. Then, since γ_1 is always negative it must be that

$$-\beta < \gamma_1 \Longleftrightarrow -\beta < -2\beta\left(\sqrt{2}+1\right) \Leftrightarrow \frac{1}{2} > \left(\sqrt{2}+1\right),$$

which is false. So $-\beta < \gamma_1$ does not hold, meaning that s_1 does not exist in the relevant range of γ in our model. Consider now γ_2 and take it as an equality. Hence we must have that $\gamma_2 < \beta$ since γ_2 is always positive. We have that

$$\gamma_2 < \beta \Leftrightarrow 2\beta\left(\sqrt{2}-1\right) < \beta \Leftrightarrow 0.828\,43 < 1,$$

which is always true. Then, in the range of γ relevant for our model s_2 exists. We are left with two possible extrema, s_0 and s_2.
For $\gamma \in (-\beta, 0)$, goods are substitutes, and we have that $f(s)$ is increasing in s since $\left.\frac{\partial f(s)}{\partial s}\right|_{s_0=0} = 0$ and $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_0=0} > 0$:

$$\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_0=0} = \frac{2\gamma^2\left(4\beta(\beta-\gamma)-\gamma^2\right)}{\left(4\beta^2+\gamma^2\right)^2}$$

the sign of this expression is given by the sign of the term $4\beta(\beta-\gamma)-\gamma^2$. This term is concave in γ and its roots are $-2\beta\left(\sqrt{2}+1\right)$ and $2\beta\left(\sqrt{2}-1\right)$ meaning that for $\gamma \in (-\beta, 0)$ the second derivative of $f(s)$ with respect to s is positive. Therefore $f(s) < f_\infty$, $\forall s \geq 0$.
For $\gamma = 0$ we have $f(s)$ is increasing in s since $\left.\frac{\partial f(s)}{\partial s}\right|_{s_0=0} = 0$ and $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_0=0} > 0$, meaning that $f(s) < f_\infty$, $\forall s \geq 0$.
For $\gamma \in (0, \beta)$ goods are complements and we can have two cases:
a) If $\gamma \in \left(0, 2\beta\left(\sqrt{2}-1\right)\right)$, then $f(s)$ is increasing in s because $\left.\frac{\partial f(s)}{\partial s}\right|_{s_0=0} = 0$ and $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_0=0} > 0$. So, $f(s) < f_\infty$, $\forall s \geq 0$.
b) If $\gamma \in \left(2\beta\left(\sqrt{2}-1\right), \beta\right)$, then $f(s)$ is decreasing in s for $s \in [0, s_2]$ and increasing in s for $s \in (s_2, \infty)$. This because we have two extremes, $\left.\frac{\partial f(s)}{\partial s}\right|_{s_0=0} =$

0 and $\left.\frac{\partial f(s)}{\partial s}\right|_{s_2} = 0$. Also $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_0=0} < 0$ implies that $s = 0$ is a local maximum and $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_2} > 0$ meaning that s_2 is a global minimum:

$$\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_2} = \frac{\left(6\gamma - 4\beta - 4\sqrt{2}\beta + 4\sqrt{2}\gamma\right)}{\left(10 + 7\sqrt{2}\right)\gamma}.$$

Thus, the sign of $\left.\frac{\partial^2 f(s)}{\partial s^2}\right|_{s_2}$ depends on the sign of $6\gamma - 4\beta - 4\sqrt{2}\beta + 4\sqrt{2}\gamma$ since the denominator is always positive because $\gamma > 2\beta\left(\sqrt{2} - 1\right)$. We can show that the numerator is always positive for $\gamma > 2\beta\left(\sqrt{2} - 1\right)$:

$$6\gamma - 4\beta - 4\sqrt{2}\beta + 4\sqrt{2}\gamma > 0 \Leftrightarrow \gamma\left(6 + 4\sqrt{2}\right) - 4\beta\left(1 + \sqrt{2}\right) > 0$$

or

$$\gamma > \frac{4\beta\left(1 + \sqrt{2}\right)}{\left(6 + 4\sqrt{2}\right)} \Leftrightarrow \gamma > \frac{2\beta\left(1 + \sqrt{2}\right)}{\left(3 + 2\sqrt{2}\right)} = 2\beta\left(\sqrt{2} - 1\right) \Leftrightarrow \gamma > 2\beta\left(\sqrt{2} - 1\right)$$

We have shown that $f(s) < f_\infty$, hence $s < \bar{s} = \frac{\theta - 1}{\alpha - c}\frac{2\beta - \gamma}{2\mu}$. *Q.E.D.*

Proof of Proposition 3: (i) Comparing $q(s)$ with q:

$$q(s) \geq q \Leftrightarrow \frac{(\alpha - c) + (\theta - 1)s}{2(\beta + \mu s^2) - \gamma} \geq \frac{(\alpha - c)}{2\beta - \gamma}$$

or

$$(2\beta - \gamma)(\alpha - c) + (2\beta - \gamma)(\theta - 1)s \geq \left(2\left(\beta + \mu s^2\right) - \gamma\right)(\alpha - c),$$

or

$$(2\beta - \gamma)(\alpha - c) + (2\beta - \gamma)(\theta - 1)s \geq (2\beta - \gamma)(\alpha - c) + 2\mu s^2(\alpha - c),$$

or

$$(2\beta - \gamma)(\theta - 1) \geq 2\mu s(\alpha - c),$$

which is equivalent to

$$s \leq \frac{\theta - 1}{\alpha - c}\frac{2\beta - \gamma}{2\mu} = \bar{s}$$

which holds by Lemma 2.
(ii) Comparing $p(s)$ with p:

$$p(s) \geq p \Leftrightarrow \frac{(\alpha + c + s(1 + \theta))\left(\beta + \mu s^2\right) - \gamma(c + s)}{2(\beta + \mu s^2) - \gamma} > \frac{(\alpha + c)\beta - \gamma c}{2\beta - \gamma},$$

or

$$s^2\mu(\theta + 1)(2\beta - \gamma) - s\mu\gamma(\alpha - c) + (2\beta - \gamma)[\beta(1 + \theta) - \gamma] > 0 \qquad (22)$$

If $\gamma \in (-\beta, 0]$ then $s^2\mu(\theta+1)(2\beta-\gamma) - s\mu\gamma(\alpha-c) + (2\beta-\gamma)(\beta(1+\theta)-\gamma) > 0$ and so $p(s) > p$. Consider now that $\gamma \in (0, \beta)$. In this case the term $-s\mu\gamma(\alpha-c) + (2\beta-\gamma)[\beta(1+\theta)-\gamma]$ is decreasing in s, so if this term is positive in the maximum possible value for s we can assure positiveness of (22). Evaluating $-s\mu\gamma(\alpha-c) + (2\beta-\gamma)[\beta(1+\theta)-\gamma]$ at $s = \bar{s}$ we obtain

$$-\frac{\theta-1}{\alpha-c}\frac{2\beta-\gamma}{2\mu}\mu\gamma(\alpha-c) + (2\beta-\gamma)[\beta(1+\theta)-\gamma]$$
$$= (2\beta-\gamma)\left(\frac{\theta+1}{2}\right)(2\beta-\gamma) > 0.$$

Therefore we can conclude that prices under CSR will increase.
(iii) Comparing $\pi(s)$ with π:

$$\pi(s) > \pi \Leftrightarrow \frac{\left(\beta+\mu s^2\right)[\alpha-c+(\theta-1)s]^2}{[2(\beta+\mu s^2)-\gamma]^2} > \frac{\beta(\alpha-c)^2}{(2\beta-\gamma)^2},$$

or

$$\left(\beta+\mu s^2\right)\left(\frac{[(\alpha-c)+(\theta-1)s]^2}{[2(\beta+\mu s^2)-\gamma]^2} - \frac{(\alpha-c)^2}{(2\beta-\gamma)^2}\right) + \frac{\mu s^2(\alpha-c)^2}{(2\beta-\gamma)^2} > 0.$$

A sufficient condition for $\pi(s) > \pi$ is that

$$\frac{[(\alpha-c)+(\theta-1)s]^2}{[2(\beta+\mu s^2)-\gamma]^2} - \frac{(\alpha-c)^2}{(2\beta-\gamma)^2} > 0$$

or

$$\frac{(\alpha-c)+(\theta-1)s}{2(\beta+\mu s^2)-\gamma} > \frac{(\alpha-c)}{(2\beta-\gamma)},$$

or

$$(2\beta-\gamma)(\theta-1) > 2\mu s(\alpha-c),$$

which is equivalent to

$$s < \frac{\theta-1}{\alpha-c}\frac{2\beta-\gamma}{2\mu} = \bar{s}$$

which holds by Lemma 2. *Q.E.D.*