Competition and Resource Sensitivity in Marriage and Roommate Markets*

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Abstract

We consider one-to-one matching markets in which agents can either be matched as pairs or remain single. In these so-called roommate markets agents are consumers and resources at the same time. We investigate two new properties that capture the effect newcomers have on incumbent agents. Competition sensitivity focuses on newcomers as additional consumers and requires that some incumbents will suffer if competition is caused by newcomers. Resource sensitivity focuses on newcomers as additional resources and requires that this is beneficial for some incumbents. For solvable roommate markets, we provide the first characterizations of the core using either competition or resource sensitivity. On the domain of all roommate markets, we obtain two associated impossibility results.

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1 Introduction

We consider one-to-one matching markets in which agents can either be matched as pairs or remain single. These markets are known as roommate markets and they include, as special cases, the well-known marriage markets (Gale and Shapley, 1962; Roth and Sotomayor, 1990). As simple as the roommate model may be, it is of conceptual importance as it lies in the intersection of network and coalition formation models[1] (for surveys and current research of network and coalition formation see Demange and Wooders, 2004; Jackson, 2008).

Loosely speaking, in these discrete markets the commodities to be traded are the agents themselves. Thus, agents are consumers and resources at the same time. We investigate two new properties that capture the effect newcomers have on incumbent agents: competition and resource sensitivity. *Competition sensitivity* focuses on newcomers as additional consumers and requires that some incumbents will suffer if competition is caused because newcomers initiate new trades. *Resource sensitivity* focuses on newcomers as additional resources and requires that some incumbents will benefit if there are new trades, i.e., the extra resources are consumed. The corresponding weak population sensitivity properties only consider situations when newcomers join one by one.

For marriage markets, both properties are closely related to population monotonicity, a solidarity property that requires that additional agents affect the incumbents in a similar way (either all incumbents are weakly better off or all incumbents are weakly worse off). Because of the polarization of interests that occurs in marriage markets, two specific versions of population monotonicity exist: own-side and other-side population monotonicity (Toda, 2006, indroduced the first of these specifications).[2] We show that in marriage markets, essentially own-side population monotonicity implies weak competition sensitivity (Lemma 1) and other-side population monotonicity implies weak resource sensitivity (Lemma 2). Our main results are two characterizations of the core by weak unanimity[3], Maskin monotonicity[4], and either (weak) competition or (weak) resource sensitivity for marriage markets and solvable roommate markets (Theorem 1) and two associated impossibility results on the general domain (Theorem 2).

Theorem 1 presents the first characterizations of the core for solvable roommate markets. One of Toda's (2006, Theorem 3.1) results can be interpreted as a corollary (Corollary 1) of our results. More importantly, Theorem 1 demonstrates that it is not really a solidarity property (population monotonicity) that is at work in Toda's (2006, Theorem 3.1) characterization of the core for marriage markets, but that it is the competition sensitivity aspect that is captured as well. Our results also imply a new characterization of the core for marriage markets (Corollary 3): a solution φ satisfies individual rationality, weak unanimity, Maskin monotonicity, and other-side population monotonicity if and only if it equals the core.[5]

[1] In a "roommate network" situation each agent is allowed or able to form only one link and in a "roommate coalition" situation only coalitions of size one or two can be formed.

[2] *Own-side population monotonicity*: if additional men (women) enter the market, then all incumbent men (women) are weakly worse off.

Other-side population monotonicity: if additional men (women) enter the market, then all incumbent women (men) are weakly better off.

[3] *Weak unanimity*: if a complete unanimously best matching exists, then it is chosen.

[4] *Maskin monotonicity*: if a matching is chosen in one market, then it is also chosen in a market that results from a Maskin monotonic transformation (which essentially means that the matching improved in the ranking of all agents).

[5] Can and Klaus (2010) consider the population sensitivity properties introduced here together with consistency. They show that some of Toda's "consistency results" do not extend to the domain of solvable roommate markets. Hence, the analysis of consistency together with the population sensitivity properties introduced here is not a simple

2 Roommate Markets

2.1 The Model

Gale and Shapley (1962, Example 3) introduced the very simple and appealing roommate markets as follows: "An even number of boys wish to divide up into pairs of roommates." A very common extension of this problem is to allow also for odd numbers of agents and to consider the formation of pairs and singletons (rooms can be occupied either by one or by two agents). In addition, we will extend the problem to variable sets of agents, e.g., because the allocation of dormitory rooms at a university occurs every year for different sets of students.

Let $\mathbb{N}$ be the set of potential agents[6] and $\mathcal{N}$ be the set of all non-empty finite subsets of $\mathbb{N}$, i.e., $\mathcal{N} = \{N \subseteq \mathbb{N} \mid \infty > |N| > 0\}$. For $N \in \mathcal{N}$, $L(N)$ denotes the set of all linear orders over N.[7] For $i \in N$, we interpret $R_i \in L(N)$ as agent i's preferences over sharing a room with any of the agents in $N \setminus \{i\}$ and having a room for himself; e.g., $R_i : j, k, i, l$ means that i would first like to share a room with j, then with k, and then i would prefer to stay alone rather than sharing the room with l. If $j \, P_i \, i$ then agent i finds agent j *acceptable* and if $i \, P_i \, j$ then agent i finds agent j *unacceptable*. $\mathcal{R}^N = \prod_N L(N)$ denotes the set of all preference profiles of agents in N (over agents in N). A *roommate market* consists of a set of agents $N \in \mathcal{N}$ and their preferences $R \in \mathcal{R}^N$ and is denoted by (N, R). A *marriage market* (Gale and Shapley, 1962) is a roommate market (N, R) such that N is the union of two disjoint sets M and W and each agent in M (respectively W) prefers being single to being matched with any other agent in M (respectively W).

A *matching* μ for roommate market (N, R) is a function $\mu : N \to N$ of order two, i.e., for all $i \in N$, $\mu(\mu(i)) = i$. Thus, at any matching μ, the set of agents is partitioned into pairs of agents who share a room and singletons (agents who do not share a room). Agent $\mu(i)$ is agent i's *match* (if $\mu(i) = i$ then i is matched to himself or *single*). For notational convenience, we often denote matchings as ordered lists, e.g., for $N = \{1, 2, 3, 4, 5\}$ and matching μ such that $\mu(1) = 3$, $\mu(2) = 2$ and $\mu(4) = 5$ we write $\mu = (3, 2, 1, 5, 4)$. For $S \subseteq N$, we denote by $\mu(S)$ the set of agents that are matched to agents in S, i.e., $\mu(S) = \{i \in N \mid \mu^{-1}(i) \in S\}$. We denote the set of matchings for roommate market (N, R) by $\mathcal{M}(N, R)$ (even though this set does not depend on preferences R). If it is clear which roommate market (N, R) we refer to, matchings are assumed to be elements of $\mathcal{M}(N, R)$. Since agents only care about their own matches, we use the same notation for preferences over agents and matchings: for all agents $i \in N$ and matchings μ, μ', $\mu \, R_i \, \mu'$ if and only if $\mu(i) \, R_i \, \mu'(i)$.

In the sequel, we consider three domains of roommate problems: the domain of all roommate markets, the domain of so-called solvable roommate markets (Definition 5), and the domain of marriage markets. To avoid notational complexity when introducing solutions and their properties, we use the generic domain of roommate markets $\mathfrak{D}$.

A *solution* φ on $\mathfrak{D}$ is a correspondence that associates with each roommate market $(N, R) \in \mathfrak{D}$ a nonempty subset of matchings, i.e., for all $(N, R) \in \mathfrak{D}$, $\varphi(N, R) \subseteq \mathcal{M}(N, R)$ and $\varphi(N, R) \neq \emptyset$.

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addition or extension of this work.

[6]Most results remain valid for a finite set of potential agents. We will explain throughout the article, which results depend on the set of potential agents to be infinite.

[7]A linear order over N is a binary relation $\bar{R}$ that satisfies *antisymmetry* (for all $i, j \in N$, if $i \, \bar{R} \, j$ and $j \, \bar{R} \, i$, then $i = j$), *transitivity* (for all $i, j, k \in N$, if $i \, \bar{R} \, j$ and $j \, \bar{R} \, k$, then $i \, \bar{R} \, k$), and *comparability* (for all $i, j \in N$, $i \, \bar{R} \, j$ or $j \, \bar{R} \, i$). By $\bar{P}$ we denote the asymmetric part of $\bar{R}$. Hence, given $i, j \in N$, $i \, \bar{P} \, j$ means that i is strictly preferred to j; $i \, \bar{R} \, j$ means that $i \, \bar{P} \, j$ or $i = j$ and that i is weakly preferred to j.

2.2 Basic Properties and the Core

We first introduce a voluntary participation condition based on the idea that no agent can be forced to share a room.

Definition 1. ***Individual Rationality***
A matching μ is *individually rational* for roommate market (N, R) if for all $i \in N$, $\mu(i)\ R_i\ i$. $IR(N, R)$ denotes the set of all these matchings. A solution φ on $\mathfrak{D}$ is *individually rational* if it only assigns individually rational matchings, i.e., for all $(N, R) \in \mathfrak{D}$, $\varphi(N, R) \subseteq IR(N, R)$.

Remark 1. ***Individual Rationality and Marriage Markets***
An individually rational matching for a marriage market respects the partition of agents into two types and never matches two men or two women. Hence, we embed marriage markets into our roommate market framework by an assumption on preferences (same gender agents are unacceptable) and individual rationality to ensure that no two agents of the same gender are matched. We refer to a marriage market for which matching agents of the same gender is not feasible as a *classical marriage market*.

Next, we introduce the well-known condition of Pareto optimality and the weaker conditions of unanimity and weak unanimity.

Definition 2. ***Pareto Optimality***
A matching μ is *Pareto optimal* for roommate market (N, R) if there is no other matching $\mu' \in \mathcal{M}(N, R)$ such that for all $i \in N$, $\mu'\ R_i\ \mu$ and for some $j \in N$, $\mu'\ P_j\ \mu$. $PO(N, R)$ denotes the set of all these matchings. A solution φ on $\mathfrak{D}$ is *Pareto optimal* if it only assigns Pareto optimal matchings, i.e., for all $(N, R) \in \mathfrak{D}$, $\varphi(N, R) \subseteq PO(N, R)$.

Definition 3. ***(Weak) Unanimity***
A matching μ is the *unanimously best matching* for (N, R) if it is such that for all $i, j \in N$, $\mu(i)\, R_i\, j$. A solution φ on $\mathfrak{D}$ is *unanimous* if it assigns the unanimously best matching whenever it exists, i.e., for all roommate markets $(N, R) \in \mathfrak{D}$ with a unanimously best matching μ, $\varphi(N, R) = \{\mu\}$. A solution φ on $\mathfrak{D}$ is *weakly unanimous* if it assigns the unanimously best matching whenever it exists and is complete (no agent is single).

Note that Pareto optimality implies unanimity and that unanimity implies weak unanimity.

The next property requires that two agents who are "mutually best agents" are always matched with each others.

Definition 4. ***Mutually Best***
Let (N, R) be a roommate market and $i, j \in N$ [possibly $i = j$] such that for all $k \in N$, $i\ R_j\ k$ and $j\ R_i\ k$. Then, i and j are *mutually best agents* for (N, R). A solution φ on $\mathfrak{D}$ is *mutually best* if it only assigns matchings at which all mutually best agents are matched, i.e., for all roommate markets $(N, R) \in \mathfrak{D}$, for all mutually best agents i and j, and for all $\mu \in \varphi(N, R)$, $\mu(i) = j$.

Note that our notion of mutually best is slightly stronger than that used in Toda (2006, because he consider mutually best man-woman pairs, he does not allow for a single mutually best agent $i = j$). Furthermore, mutually best implies unanimity and Pareto optimality and mutually best are logically unrelated.

Examples of solutions are immediately implied by two of the properties: define IR (respectively PO) as correspondences that assign to each roommate market the set of individually rational (respectively Pareto optimal) matchings.

Next, we define stability for roommate markets. A matching μ for roommate market (N, R) is blocked by a pair $\{i, j\} \subseteq N$ [possibly $i = j$] if $j \, P_i \, \mu(i)$ and $i \, P_j \, \mu(j)$. If $\{i, j\}$ blocks μ, then $\{i, j\}$ is called a *blocking pair* for μ. Note that a matching is individually rational if there is no blocking pair $\{i, j\}$ with $i = j$.

Definition 5. ***Stability and Solvability***
A matching μ is *stable* for roommate market (N, R) if there is no blocking pair for μ. $S(N, R)$ denotes the set of all these matchings. A roommate market is *solvable* if stable matchings exist, i.e., (N, R) is solvable if and only if $S(N, R) \neq \emptyset$. Furthermore, on the domain of solvable roommate markets, a solution φ is *stable* if it only assigns stable matchings, i.e., for all (N, R) such that $S(N, R) \neq \emptyset$, $\varphi(N, R) \subseteq S(N, R)$.

Another well-known concept for matching problems is the core.

Definition 6. ***Core***
A matching is in the (strict or strong) core if no coalition of agents can improve their welfare by rematching among themselves. For roommate market (N, R), $core(N, R) = \{\mu \in \mathcal{M}(N, R) \mid$ there exists no $S \subseteq N$ and no $\mu' \in \mathcal{M}(N, R)$ such that $\mu'(S) = S$, for all $i \in S$, $\mu'(i) \, R_i \, \mu(i)$, and for some $j \in S$, $\mu'(j) \, P_j \, \mu(j)\}$.

Similarly as in other matching models (e.g., marriage markets and college admissions markets), the core equals the set of stable matchings, i.e., for all (N, R), $core(N, R) = S(N, R)$. Hence, the core is a solution on the domain of solvable roommate markets, but not on the domain of all roommate markets. Gale and Shapley (1962) showed that all marriage markets are solvable and gave an example of an unsolvable roommate market (Gale and Shapley, 1962, Example 3).

Finally, we introduce Maskin monotonicity (Maskin, 1999): if a matching is chosen in one roommate market, then it is also chosen in a roommate market that results from a Maskin monotonic transformation, which essentially means that the matching (weakly) improved in the preference ranking of all agents.

Let (N, R) be a roommate market. Then, for any agent $i \in N$ and matching $\mu \in \mathcal{M}(N, R)$, the *lower contour set of R_i at μ* is $L_i(R_i, \mu) := \{\mu' \in \mathcal{M}(N, R) \mid \mu \, R_i \, \mu'\}$. For preference profiles $R, R' \in \mathcal{R}^N$ and matching $\mu \in \mathcal{M}(N, R)$, R' is a *Maskin monotonic transformation of R at μ* if for all $i \in N$, $L_i(R_i, \mu) \subseteq L_i(R'_i, \mu)$.

Definition 7. ***Maskin Monotonicity***
A solution φ on $\mathfrak{D}$ is *Maskin monotonic* if for all roommate markets $(N, R), (N, R') \in \mathfrak{D}$, and all $\mu \in \varphi(N, R)$ such that R' is a Maskin monotonic transformation of R at μ, $\mu \in \varphi(N, R')$.

Maskin monotonicity is one of the key concepts in implementation theory. However, here we focus on Maskin monotonicity as a desirable property in itself.

Proposition 1. *On the domains of marriage markets and of solvable roommate markets, the core satisfies individual rationality, Pareto optimality, unanimity, mutually best, stability, and Maskin monotonicity.*

Proof. It is easily checked that the core satisfies individual rationality, Pareto optimality, unanimity, mutually best, and stability. Sönmez's (1996) Proposition 1 applies to the domains of marriage markets and of solvable roommate markets and it therefore shows that the core is Maskin monotonic. □

2.3 Variable Population Properties

The next properties we consider concern population changes. More specifically, consider the change of a roommate market (N, R) when a finite set of agents or *newcomers* $\hat{N} \subseteq \mathbb{N} \backslash N$ shows up. Then, the new set of agents is $N' = N \cup \hat{N}$ and (N', R'), $R' \in \mathcal{R}^{N'}$, is an *extension of* (N, R) if agents in N extend their preferences to include the newcomers in $\hat{N}$, i.e.,

(i) for all $i \in N'$, $R'_i \in L(N')$ and

(ii) for all $j, k, l \in N$, $j \; R_l \; k$ if and only if $j \; R'_l \; k$.

Note that $R \in \mathcal{R}^N$ is the *restriction of* $R' \in \mathcal{R}^{N'}$ *to* N. We also denote the restriction of R' to N by R'_N.

Adding a set of newcomers $\hat{N}$ might be a positive or a negative change for any of the incumbents in N because it might mean

a negative change with more competition or

a positive change with more resources.

Before we capture both effects in two new properties called competition and resource sensitivity, we make a short excursion to the definition of population monotonicity for marriage markets. This property goes back to Thomson (1983), who also presents a survey of population monotonicity in various economic models (Thomson, 1995).

Population Monotonicity: When a change in the population is exogenous, it would be unfair if the agents who were not responsible for this change were treated unequally. Population monotonicity represents this idea of solidarity. However, for marriage markets this would mean that if a newcomer enters (e.g., a man) men and women are all affected in the same way (all weakly better off or all weakly worse off).

This might not be a natural condition for marriage markets because of a certain polarization imbedded in the market: a man might be considered good news for women (more choice), but bad news for men (more competition). Therefore, for marriage markets we can formulate two population monotonicity conditions that take the polarization aspect into account. The first one was introduced by Toda (2006) and we will refer to it as own-side population monotonicity: a solution φ is own-side population monotonic if for any marriage market $(M \cup W, R)$, if additional men [women] enter the market such that the new marriage market equals $(M' \cup W, R')$ [$(M \cup W', R')$], then – because of the possible negative effect of the extra competition – all men in M [women in W] weakly prefer $\varphi(M \cup W, R)$ to $\varphi(M' \cup W, R')$ [$\varphi(M \cup W', R')$].

We formalize own-side population monotonicity for marriage markets by restricting population changes to either a set of men or a set of women. Consistent with Toda's (2006) choice of extending preferences over matchings to sets of matchings, we apply the pessimistic view of comparing sets of matchings throughout this article.[8]

[8] Agents are pessimistic and always assume that the worst matching will be realized, i.e., given two sets of matchings A and B, an agent will compare the worst matching in A to the worst matching in B. Thus, if agent i weakly prefers

Definition 8. ***Own-Side Population Monotonicity for Marriage Markets***
On the domain of marriage markets, a solution φ is *own-side population monotonic* if the following holds. Let (N,R) be a marriage market and assume that (N',R'), $N'=N\cup\hat{N}$, is an extension of (N,R) and all newcomers $\hat{N}$ are men [women]. Then, for all $\mu\in\varphi(N,R)$ there exists $\mu'\in\varphi(N',R')$ such that $(*)$ for all men $m\in N$, $\mu(m)\ R'_m\ \mu'(m)$ [for all women $w\in N$, $\mu(w)\ R'_w\ \mu'(w)$].[9]

By the strictness of preferences, $(*)$ means that if all newcomers are men [women], then every man [woman] who is matched differently is strictly worse off, i.e., for all men $m\in N$, either $\mu(m)=\mu'(m)$ or $\mu(m)\ P'_m\ \mu'(m)$ [for all women $w\in N$, either $\mu(w)=\mu'(w)$ or $\mu(w)\ P'_w\ \mu'(w)$]. Hence, if a man m [woman w] has a new match at μ', then he [she] is worse off. Without specifying whether the set of newcomers consists of men or women, own-side population monotonicity implies that if $m,w\in N$ are newly matched at μ', then at least one of them is worse off (if newcomers are men, then man m is worse off and if newcomers are women, then woman w is worse off). This latter requirement that if two incumbents are newly matched at μ', then one of them suffers from the increased competition by the newcomers and is worse off, can be formulated as a new property, namely competition sensitivity. This property requires that the solution is sensitive to competition, which is a different requirement than the solidarity aspect that own-side population monotonicity reflects.

Definition 9. ***(Weak) Competition Sensitivity***
A solution φ on $\mathfrak{D}$ is *competition sensitive* if the following holds. Let $(N,R)\in\mathfrak{D}$ be a roommate market and assume that $(N',R')\in\mathfrak{D}$, $N'=N\cup\hat{N}$, is an extension of (N,R). Then, for all $\mu\in\varphi(N,R)$ there exists $\mu'\in\varphi(N',R')$ such that for all $i,j\in N$ [possibly $i=j$] that are *newly matched at* μ', at least one is worse off, i.e., if $i,j\in N$, $\mu(i)\neq j$, and $\mu'(i)=j$, then $\mu(i)\ P'_i\ \mu'(i)$ or $\mu(j)\ P'_j\ \mu'(j)$.[10] A solution φ on $\mathfrak{D}$ is *weakly competition sensitive* if we require competition sensitivity only when adding one newcomer at a time, i.e., $\hat{N}=\{n\}$. Note that the competition sensitivity defined in Klaus (2008, Definition 9) equals the weak competition sensitivity here.

On the domain of marriage markets, weak competition sensitivity is essentially a weaker property than own-side population monotonicity (individual rationality is added to ensure that no two agents of the same gender are matched, see Remark 1). The only reason why full competition sensitivity is not implied is that own-side population monotonicity only addresses the addition of same-gender newcomers. In the following lemma, we can slightly strengthen weak competition sensitivity by allowing for sets of same-gender newcomers (the proof is for this somewhat more general version of weak competition sensitivity).

Lemma 1. ***Own-Side Population Monotonicity $\Rightarrow$ Weak Competition Sensitivity***
On the domain of marriage markets, individual rationality and own-side population monotonicity imply weak competition sensitivity.

Proof. Let φ be a solution on the domain of marriage markets that is individually rational and own-side population monotonic. Let (N,R) be a marriage market and assume that marriage market

A to B, then for all $\mu\in A$ there exists $\mu'\in B$ such that $\mu\, R_i\, \mu'$. As already noted by Toda (2006), using an optimistic set comparison, i.e., comparing the best matchings, will not give the same results and using a standard set comparison that compares best to best and worst to worst matchings (see Barberà et al., 2004) will not change the results.

[9] Equivalently (Toda, 2006): $(*)$ for all men $m\in N$, $\mu(m)\ R_m\ \mu'(m)$ [for all women $w\in N$, $\mu(w)\ R_w\ \mu'(w)$].

[10] Equivalently, if agents in $\hat{N}$ are leaving: for all $i,j\in N$ [possibly $i=j$] that *are not matched at* μ *anymore*, at least one is better off, i.e., if $i,j\in N$, $\mu'(i)=j$, and $\mu(i)\neq j$, then $\mu(i)\ P'_i\ \mu'(i)$ or $\mu(j)\ P'_j\ \mu'(j)$.

(N', R'), $N' = N \cup \hat{N}$, is an extension of (N, R) such that all newcomers in $\hat{N}$ are men. By own-side population monotonicity, for all $\mu \in \varphi(N, R)$ there exists $\mu' \in \varphi(N', R')$ such that for all men $m \in N$, either $\mu(m) = \mu'(m)$ or $\mu(m)\ P'_m\ \mu'(m)$. Let $i, j \in N$, $\mu(i) \neq j$, and $\mu'(i) = j$. If $i \neq j$, then the pair $\{i, j\}$ consists of one man and one woman. Without loss of generality assume that i is the man and j the woman. Thus, by own-side population monotonicity, $\mu(i)\ P'_i\ \mu'(i)$. If $i = j$, then, by individual rationality, $\mu(i)\ P'_i\ \mu'(i)$.[11] Hence, φ is weakly competition sensitive. □

In the following example we demonstrate that for marriage markets, competition sensitivity does not imply own-side population monotonicity.

Example 1. Solution $\bar{\varphi}$ uses two important stable matchings that always exist for marriage markets: the man- and the woman-optimal stable matching (obtainable by applying Gale and Shapley's, 1962, deferred acceptance algorithm). We define $\bar{\varphi}$ as follows. For all marriage markets (N, R), if there are more men than women, then $\bar{\varphi}$ assigns the man-optimal stable matching and otherwise $\bar{\varphi}$ assigns the woman-optimal stable matching. Solution $\bar{\varphi}$ is individually rational and competition sensitive (Proposition 2), but it violates own-side population-monotonicity (see Appendix A). ◇

Next we introduce other-side population monotonicity for marriage markets: a solution φ is other-side population monotonic if for any marriage market $(M \cup W, R)$, if additional men [women] enter the market such that the new marriage market equals $(M' \cup W, R')$ $[(M \cup W', R')]$, then – because of the possible positive effect of the extra matching opportunities or resources – all women in W [men in M] weakly prefer $\varphi(M' \cup W, R')$ $[\varphi(M \cup W', R')]$ to $\varphi(M \cup W, R)$.

Definition 10. ***Other-Side Population Monotonicity for Marriage Markets***
On the domain of marriage markets, a solution φ is *other-side population monotonic* if the following holds. Let (N, R) be a marriage market and assume that (N', R'), $N' = N \cup \hat{N}$, is an extension of (N, R) and all newcomers $\hat{N}$ are men [women]. Then, for all $\mu' \in \varphi(N', R')$ there exists $\mu \in \varphi(N, R)$ such that $(**)$ for all women $w \in N$, $\mu'(w)\ R'_w\ \mu(w)$ [for all men $m \in N$, $\mu'(m)\ R'_m\ \mu(m)$].

By the strictness of preferences, $(**)$ means that if newcomers are men [women], then every woman [man] who is matched differently is strictly better off, i.e., for all women $w \in N$, either $\mu'(w) = \mu(w)$ or $\mu'(w)\ P'_w\ \mu(w)$ [for all men $m \in N$, either $\mu'(m) = \mu(m)$ or $\mu'(m)\ P'_m\ \mu(m)$]. Hence, if a woman w [man m] is unmatched from her match at μ, then she [he] is better off. Without specifying whether the set of newcomers consists of men or women, other-side population monotonicity implies that if $m, w \in N$ are not matched anymore at μ', then at least one of them is better off (if newcomers are men, then woman w is better off and if newcomers are women, then man m is better off). This latter requirement that if two incumbents were unmatched at μ', then one of them benefits from the increase of resources by the newcomers and is better off, can be formulated as a new property, namely resource sensitivity. This property requires that the solution is sensitive to an increase in resources, which is a different requirement than the solidarity aspect that other-side population monotonicity reflects.

Definition 11. ***(Weak) Resource Sensitivity***
A solution φ on $\mathfrak{D}$ is *resource sensitive* if the following holds. Let $(N, R) \in \mathfrak{D}$ be a roommate market and assume that $(N', R') \in \mathfrak{D}$, $N' = N \cup \hat{N}$, is an extension of (N, R). Then, for all $\mu' \in \varphi(N', R')$

[11] If i is a man, this latter implication would also be implied by own-side population monotonicity. However, if i is a woman, this concluding argument cannot be made solely by using own-side population monotonicity.

there exists $\mu \in \varphi(N,R)$ such that for all $i,j \in N$ [possibly $i=j$] that *are not matched at* μ' *anymore*, at least one is better off, i.e., if $i,j \in N$, $\mu(i)=j$, and $\mu'(i) \neq j$, then $\mu'(i)\, P'_i\, \mu(i)$ or $\mu'(j)\, P'_j\, \mu(j)$.[12] A solution φ on $\mathfrak{D}$ is *weakly resource sensitive* if we require resource sensitivity only when adding one newcomer at a time, i.e., $\hat{N}=\{n\}$. Note that the resource sensitivity defined in Klaus (2008, Definition 9) equals the weak resource sensitivity here.

On the domain of marriage markets, weak resource sensitivity is essentially a weaker property than other-side population monotonicity (individual rationality is added to ensure that no two agents of the same gender are matched, see Remark 1). The only reason why full resource sensitivity is not implied is that other-side population monotonicity only addresses the addition of same-gender newcomers. In the following lemma, we can slightly strengthen weak resource sensitivity by allowing for sets of same-gender newcomers (the proof is for this somewhat more general version of weak resource sensitivity).

Lemma 2. ***Other-Side Population Monotonicity*** $\Rightarrow$ ***Weak Resource Sensitivity***
On the domain of marriage markets, individual rationality and other-side population monotonicity imply weak resource sensitivity.

Proof. Let φ be a solution on the domain of marriage markets that is individually rational and other-side population monotonic. Let (N,R) be a marriage market and assume that marriage market (N',R'), $N'=N\cup\hat{N}$, is an extension of (N,R) such that all newcomers in $\hat{N}$ are men. By other-side population monotonicity, for all $\mu' \in \varphi(N',R')$ there exists $\mu \in \varphi(N,R)$ such that for all women $w \in N$, either $\mu'(w)=\mu(w)$ or $\mu'(w)\, P'_w\, \mu(w)$. Let $i,j \in N$, $\mu(i)=j$, and $\mu'(i)\neq j$. If $i \neq j$, then the pair $\{i,j\}$ consists of one man and one woman. Without loss of generality assume that i is the man and j the woman. Thus, by other-side population monotonicity, $\mu'(j)\, P'_j\, \mu(j)$. If $i=j$, then by individual rationality, $\mu'(i)\, P'_i\, \mu(i)$.[13] Hence, φ is weakly resource sensitive. □

Solution $\bar{\varphi}$ (Example 1) also demonstrates that for marriage markets, resource sensitivity does not imply other-side population monotonicity: solution $\bar{\varphi}$ is individually rational and resource sensitive (see Proposition 2), but it violates other-side population monotonicity (see Appendix A).

Proposition 2. *On the domains of marriage markets and of solvable roommate markets, any stable solution satisfies competition and resource sensitivity. In particular, the core satisfies competition and resource sensitivity.*

Proof. Let φ be a stable solution on the domain of solvable roommate markets [marriage markets]. Let (N,R) be a solvable roommate market [marriage market] and assume that (N',R'), $N'=N\cup\hat{N}$, is an extension of (N,R) that is solvable [a marriage market].

Competition Sensitivity: Assume that φ is not competition sensitive, i.e., there exist $\mu \in \varphi(N,R)$, $\mu' \in \varphi(N',R')$, and $i,j \in N$ [possibly $i=j$] such that $\mu'(i)=j$, $\mu(i)\neq j$, $\mu'(i)\, P'_i\, \mu(i)$, and $\mu'(j)\, P'_j\, \mu(j)$. Thus, $j\, P_i\, \mu(i)$ and $i\, P_j\, \mu(j)$. Hence, $\{i,j\}$ is a blocking pair for μ; contradicting $\mu \in \varphi(N,R) \subseteq S(N,R)$.

Resource Sensitivity: Assume that φ is not resource sensitive, i.e., there exist $\mu \in \varphi(N,R)$, $\mu' \in \varphi(N',R')$, and $i,j \in N$ [possibly $i=j$] such that $\mu(i)=j$, $\mu'(i)\neq j$, $\mu(i)\, P'_i\, \mu'(i)$, and $\mu(j)\, P'_j\, \mu'(j)$.

[12] Equivalently, if agents in $\hat{N}$ are leaving: for all $i,j \in N$ [possibly $i=j$] that are *newly matched at* μ at least one is worse off, i.e., if $i,j \in N$, $\mu'(i)\neq j$, and $\mu(i)=j$, then $\mu'(i)\, P'_i\, \mu(i)$ or $\mu'(j)\, P'_j\, \mu(j)$.

[13] If i is a woman, this latter implication would also be implied by other-side population monotonicity. However, if i is a man, this concluding argument cannot be made solely by using other-side population monotonicity.

Thus, $j\,P'_i\,\mu'(i)$, and $i\,P'_j\,\mu'(j)$. Hence, $\{i,j\}$ is a blocking pair for μ'; contradicting $\mu' \in \varphi(N',R') \subseteq S(N',R')$. □

3 Results

3.1 Relations between Properties

Lemma 3.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

weak unanimity and competition sensitivity imply mutually best.

Proof. Let φ be a solution on any of the domains in Lemma 3 that satisfies weak unanimity and competition sensitivity. Let (N,R) be a (marriage, solvable) roommate market such that agents $i,j \in N$ [possibly $i=j$] are mutually best. To prove that φ satisfies mutually best, we show that for all $\mu \in \varphi(N,R)$, $\mu(i)=j$.

Without loss of generality assume that $N \setminus \{i,j\} = \{1,\ldots,l\}$. Let $\hat{N} = \{k_1,\ldots,k_l\}$ be a set of newcomers and assume that (N',R'), $N' = N \cup \hat{N}$, is an extension of (N,R) such that agents 1 and k_1 are mutually best, agents 2 and k_2 are mutually best, ..., and agents l and k_l are mutually best [if (N,R) is a marriage market, then the gender of newcomers and preferences can be chosen such that (N',R') is also a marriage market]. For the solvable roommate market (N',R') there exists the complete and unanimously best matching ν' that matches agent i with agent j, agent 1 with agent k_1, agent 2 with agent k_2, ..., and agent l with agent k_l. Hence, by weak unanimity, $\varphi(N',R') = \{\nu'\}$. By competition sensitivity, for all $\mu \in \varphi(N,R)$, if agents i and j are newly matched at ν', then at least one is worse off. Since at ν' agents i and j are mutually best matched, for all $\mu \in \varphi(N,R)$, $\mu(i)=j$. □

On the domain of marriage markets, Toda (2006, Lemma 3.1) proves that weak unanimity and own-side population monotonicity imply mutually best. The proof of our Lemma 3 follows similar arguments as Toda's (2006, Lemma 3.1) proof for the corresponding marriage market result.

In Appendix B we prove a stronger version of Lemma 3 – Lemma 3' – using weak competition sensitivity instead of competition sensitivity. The proof of Lemma 3' (b) is more elaborate because when adding newcomers one by one, the resulting roommate markets have to always be solvable.

Finally, with Example 3 in Appendix B we illustrate why Lemmas 3 and 3' might not hold if the set of potential agents is finite.

Lemma 4.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

weak unanimity and resource sensitivity imply mutually best.

Proof. Let φ be a solution on any of the domains in Lemma 4 that satisfies unanimity and resource sensitivity. Let (N,R) be a (marriage, solvable) roommate market such that agents $i,j \in N$ [possibly $i=j$] are mutually best. To prove that φ satisfies mutually best, we show that for all $\mu \in \varphi(N,R)$, $\mu(i)=j$.

Let $\bar{N}=\{i,j\}$ and consider the reduced preferences $\bar{R}=R_{\bar{N}}$, i.e., $i\,\bar{R}_j\,j$ and $j\,\bar{R}_i\,i$ completely describe $\bar{R}$. There exists a unanimously best complete matching $\bar{\nu}$ for (marriage, solvable) roommate market $(\bar{N},\bar{R})$: $\bar{\nu}$ matches agent i with agent j. Hence, by weak unanimity, $\varphi(\bar{N},\bar{R})=\{\bar{\nu}\}$ and $\bar{\nu}(i)=j$.

Without loss of generality assume that $N=\bar{N}\cup\{1,\ldots,l\}$. Consider the extension (N,R) of $(\bar{N},\bar{R})$ that is obtained by adding newcomers $\hat{N}=\{1,\ldots,l\}$. By resource sensitivity, for all $\mu \in \varphi(N,R)$, if agents i and j are not matched at μ anymore, then at least one is better off. Since at $\bar{\nu}$ agents i and j are already mutually best matched, for all $\mu\in\varphi(N,R)$, $\mu(i)=j$. □

Lemma 4 establishes a corresponding result to Lemma 3 by using resource sensitivity instead of competition sensitivity. In Appendix B we prove a stronger version of Lemma 4 (a) and (c) – Lemma 4' (a) and (c) – using weak resource sensitivity instead of resource sensitivity. In Lemma 4' (b) we weaken resource sensitivity, but add Maskin monotonicity. The proof of Lemma 4' is more elaborate because newcomers have to be added one by one.

In Corollary 2 (see Section 3.3) we establish a result that complements Toda's (2006) Lemma 3.1: on the domain of marriage markets, individual rationality, weak unanimity, and other-side population monotonicity imply mutually best. We show in Lemma 8 (Section 3.3) that indeed in our roommate market framework we use individual rationality only to ensure that same gender agents are not matched (Remark 1).

Finally, we prove in Appendix B that Lemma 4 (b) cannot be strengthened by only using weak resource sensitivity instead of resource sensitivity (without adding Maskin monotonicity); we define a solution on the domain of solvable roommate markets that satisfies weak unanimity and weak resource sensitivity, but neither mutually best nor Maskin monotonicity (see Example 4 in Appendix B).

Lemma 5.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

mutually best and Maskin monotonicity imply individual rationality.

Proof. Let φ be a solution on any of the domains in Lemma 5 that satisfies mutually best and Maskin monotonicity, but not individual rationality. Thus, there exists a (marriage, solvable) roommate market (N,R), a matching $\mu\in\varphi(N,R)$, and an agent $i\in N$ such that $i\,P_i\,\mu(i)$.

We define $\tilde{R}\in\mathcal{R}^N$ by moving i on top of agent i's preferences and, for any $j\neq i$, by moving $\mu(j)$ on top of agent j's preferences [if (N,R) is a marriage market, then $(N,\tilde{R})$ is also a marriage market]. Note that $(N,\tilde{R})$ is solvable[14] and that $\tilde{R}$ is a Maskin monotonic transformation of R at μ. Hence, by Maskin monotonicity, $\mu\in\varphi(N,\tilde{R})$. Let $\tilde{\mu}$ be the matching obtained from μ by unmatching agents i and $\mu(i)$. By mutually best, $\varphi(N,\tilde{R})=\{\tilde{\mu}\}$. Since $\tilde{\mu}\neq\mu$ this is a contradiction to $\mu\in\varphi(N,\tilde{R})$. □

[14] Roommate market $(N,\tilde{R})$ has a unique core allocation that matches all agents in $N\setminus\{i,\mu(i)\}$ according to μ – agents i and $\mu(i)$ are single.

Toda (2006, Lemma 3.2) presents a different result to obtain individual rationality for marriage markets: own-side population monotonicity and mutually best imply individual rationality. Note that we prove in Lemma 8 (Section 3.3) that for the standard marriage market model where agents of the same gender are never matched, other-side population monotonicity (without requiring mutually best) implies individual rationality. We show that mutually best and either competition or resource sensitivity imply individual rationality in Appendix B (Lemmas 9 and 10).

3.2 Two Characterizations of the Core and two Impossibilities

Lemma 6.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

if a solution φ is mutually best and Maskin monotonic, then it is a subsolution of the core, i.e., on the domains of marriage markets and of solvable roommate markets, for all (N, R), $\varphi(N, R) \subseteq core(N, R)$.

(c) *On the domain of all roommate markets,*

no solution φ is mutually best and Maskin monotonic.

Proof. Let φ be a solution on any of the domains in Lemma 6 that satisfies mutually best and Maskin monotonicity. By Lemma 5, φ satisfies individual rationality.

To prove (a) and (b), let (N, R) be a solvable roommate market [marriage market] such that $\varphi(N, R) \nsubseteq core(N, R)$. To prove (c), let (N, R) be an unsolvable roommate market. In both cases there exists a matching $\mu \in \varphi(N, R)$ with a blocking pair $\{i, j\}$ for μ. By individual rationality, $i \neq j$.

We define $\tilde{R} \in \mathcal{R}^N$ by moving j on top of agent i's preferences, by moving i on top of agent j's preferences and, for any $k \in N \setminus \{i, j\}$, by moving $\mu(k)$ on top of agent k's preferences. Note that $(N, \tilde{R})$ is solvable[15] and that $\tilde{R}$ is a Maskin monotonic transformation of R at μ. Hence, by Maskin monotonicity, $\mu \in \varphi(N, \tilde{R})$. By mutually best, for all $\tilde{\mu} \in \varphi(N, \tilde{R})$, $\tilde{\mu}(i) = j$. Since $\mu(i) \neq j$ this is a contradiction to $\mu \in \varphi(N, \tilde{R})$.

For (a) and (b) this proves that $\varphi(N, R) \subseteq core(N, R)$ and for (c) this proves that mutually best and Maskin monotonicity are not compatible on the general domain of roommate markets. □

Lemma 7. *On the domains of marriage markets and of solvable roommate markets, there exists no Maskin monotonic strict subsolution of the core.*

Proof. Sönmez's (1996) Theorem 1 applies to the domains of marriage markets and of solvable roommate markets and it therefore shows that if a rule φ is Pareto optimal, individually rational, and Maskin monotonic, then it is a supersolution of the core, i.e., for all marriage markets and all solvable roommate markets (N, R), $\varphi(N, R) \supseteq core(N, R)$.

Thus, since any subsolution of the core satisfies Pareto optimality and individual rationality, there exists no Maskin monotonic strict subsolution of the core on the domains of marriage markets and of solvable roommate markets. □

[15] Roommate market $(N, \tilde{R})$ has a unique core allocation that matches agent i with agent j and all agents in $N \setminus \{i, j, \mu(i), \mu(j)\}$ according to μ – agent(s) $\mu(i)$ and $\mu(j)$ are either single or, if mutually acceptable, matched with each other.

Theorem 1. ***Two Characterizations of the Core***
On the domains of marriage markets and of solvable roommate markets, a solution φ satisfies

(1) *weak competition sensitivity,*

(2) *weak resource sensitivity,*

weak unanimity, and Maskin monotonicity if and only if it equals the core.

Proof. Let φ be a solution on any of the domains in Theorem 1. By Propositions 1 and 2, the core satisfies all properties listed in the theorem. Let φ satisfy weak unanimity, Maskin monotonicity,

(1) and weak competition sensitivity. Then, by Lemma 3' (a) and (b), φ satisfies mutually best.

(2) and weak resource sensitivity. Then, by Lemma 4' (a) and (b), φ satisfies mutually best.

Thus, φ satisfies Maskin monotonicity and mutually best. Hence, by Lemma 6 (a) and (b), φ is a subsolution of the core. Then, by Lemma 7, $\varphi = \mathit{core}$. □

Theorem 2. ***Two Impossibility Results***
On the domain of all roommate markets, no solution φ satisfies

(1) *weak competition sensitivity,*

(2) *weak resource sensitivity,*

weak unanimity, and Maskin monotonicity.

Proof. Let φ be a solution on the domain of all roommate markets. Let φ satisfy weak unanimity, Maskin monotonicity,

(1) and weak competition sensitivity. Then, by Lemma 3' (c), φ satisfies mutually best.

(2) and weak resource sensitivity. Then, by Lemma 4' (c), φ satisfies mutually best.

Thus, φ satisfies Maskin monotonicity and mutually best; contradicting Lemma 6 (c). □

With Example 3 in Appendix B we illustrate why Theorems 1 (1) and 2 (1) might not hold if the set of potential agents is finite.

We next show the independence of properties in Theorems 1 and 2.

The solution φ^s on the domains in Theorems 1 and 2 that always assigns the matching at which all agents are single satisfies Maskin monotonicity, (weak) competition and (weak) resource sensitivity, but not weak unanimity.

On the domains of marriage markets and of solvable roommate markets, any strict subsolution of the core satisfies (weak) unanimity, (weak) competition and (weak) resource sensitivity (Proposition 2), but not Maskin monotonicity (Lemma 7). For the domain of all roommate markets, solution φ^{mbs} that always assigns the matching where all mutually best agents are matched and everybody else is single satisfies (weak) unanimity, (weak) competition and (weak) resource sensitivity, but not Maskin monotonicity.

The Pareto solution PO on the domains in Theorems 1 and 2 satisfies (weak) unanimity and Maskin monotonicity, but – as the following two examples demonstrate – neither weak competition nor weak resource sensitivity.

Example 2. ***The Pareto Solution is neither Competition nor Resource Sensitive***
Competition Sensitivity: Consider the solvable roommate markets (N, R) and (N', R') such that

$N = \{1, 2\}$	$N' = \{1, 2, 3\}$
$R_1 : 1, 2$	$R'_1 : 1, 2, 3$
$R_2 : 1, 2$	$R'_2 : 3, 1, 2$
	$R'_3 : 2, 3, 1$
$core(N, R) = \{\bar{\mu}\}$	$core(N', R') = \{\mu'\}$
$\bar{\mu} = (1, 2)$	$\mu' = (1, 3, 2)$

Note that (N, R) and (N', R') are marriage markets such that agents 1 and 3 have the same gender. Let $\mu = (2, 1)$. Then, $PO(N, R) = \{\mu, \bar{\mu}\}$ and $PO(N', R') = \{\mu'\}$. Thus, for $\mu \in PO(N, R)$ there exists $\mu' \in PO(N', R')$ such that agent 1 is newly self-matched at μ' and better off. Hence, PO violates competition sensitivity.

Resource Sensitivity: Consider the marriage or solvable roommate markets (N, R) and (N', R') such that

$N = \{1\}$	$N' = \{1, 2\}$
$R_1 : 1$	$R'_1 : 1, 2$
	$R'_2 : 1, 2$
$core(N, R) = \{\mu\}$	$core(N', R') = \{\bar{\mu}\}$
$\mu = (1)$	$\bar{\mu} = (1, 2)$

Let $\mu' = (2, 1)$. Then, $PO(N, R) = \{\mu\}$ and $PO(N', R') = \{\mu', \bar{\mu}\}$. Thus, for $\mu' \in PO(N', R')$ there exists $\mu \in PO(N, R)$ such that agent 1 was self-matched at μ and is worse off at μ'. Hence, PO violates resource sensitivity. ◇

Finally, we briefly discuss the relation between both sensitivity conditions. Proposition 2 implies that (weak) competition and (weak) resource sensitivity are equivalent under stability. The following two solutions demonstrate that (weak) competition and (weak) resource sensitivity are logically independent. Recall that φ^{mbs} is the solution that always assigns the matching where all mutually best agents are matched and everybody else is single.

The following solution φ^{CS} on the domain of solvable roommate markets satisfies competition sensitivity, but not resource sensitivity. For all solvable roommate markets (N, R),

$$\varphi^{\text{CS}}(N, R) = \begin{cases} S(N, R) & \text{if } 1 \notin N, \\ \varphi^{mbs}(N, R) & \text{if } 1 \in N. \end{cases}$$

The following solution φ^{RS} on the domain of solvable roommate markets satisfies resource sensitivity, but not competition sensitivity. For all solvable roommate markets (N, R),

$$\varphi^{\text{RS}}(N, R) = \begin{cases} \varphi^{mbs}(N, R) & \text{if } 1 \notin N, \\ S(N, R) & \text{if } 1 \in N. \end{cases}$$

3.3 Marriage Markets and Population Monotonicity

In this section, we present some marriage market results involving one of the population monotonicity properties.

Corollary 1. *On the domain of marriage markets, a solution satisfies individual rationality, weak unanimity, own-side population monotonicity, and Maskin monotonicity if and only if it equals the core.*

Proof. On the domain of marriage markets, let φ satisfy individual rationality, weak unanimity, own-side population monotonicity, and Maskin monotonicity. By Lemma 1, φ satisfies weak competition sensitivity. By Theorem 1 (1), $\varphi = core$. □

For the classical domain of marriage markets for which matching agents of the same gender is not feasible, Toda (2006, Theorem 3.1) established Corollary 1 without individual rationality.

Example 3 in Appendix B can also be used to illustrate why Corollary 1 and Toda's (2006) Theorem 3.1 might not hold if the set of potential agents is finite.

The next result establishes a result that complements Toda's (2006, Lemma 3.1) by using other-side population monotonicity instead of own-side population monotonicity.

Corollary 2. *On the domain of marriage markets, individual rationality, weak unanimity, and other-side population monotonicity imply mutually best.*

Proof. Let φ be a solution on the domain of all marriage markets that satisfies individual rationality, weak unanimity, and other-side population monotonicity. By Lemma 2, φ satisfies weak resource sensitivity. By Lemma 4' (a), φ satisfies mutually best. □

Our results also imply a new characterization of the core for marriage markets.

Corollary 3. *On the domain of marriage markets, a solution satisfies individual rationality, weak unanimity, other-side population monotonicity, and Maskin monotonicity if and only if it equals the core.*

Proof. On the domain of marriage markets, let φ satisfy individual rationality, weak unanimity, other-side population monotonicity, and Maskin monotonicity. By Lemma 2, φ satisfies weak resource sensitivity. By Theorem 1 (2), $\varphi = core$. □

With the next lemma, we show that for the classical domain of marriage markets for which matching agents of the same gender is not feasible, we can drop individual rationality from Corollaries 2 and 3.

Lemma 8. *On the classical domain of marriage markets, other-side population monotonicity implies individual rationality.*

Proof. Let φ be a solution on the domain of classical marriage markets that satisfies other-side population monotonicity. Let (N, R) be a marriage market with $N = M \cup W$ and $\mu \in \varphi(N, R)$. Consider marriage market (W, R_W). Since at (W, R_W) only female agents are present, the only possible matching that can be assigned is matching μ^s where all women in W are single, i.e., $\varphi(W, R_W) = \{\mu^s\}$. By other-side population monotonicity, for $\mu \in \varphi(N, R)$, there exists $\mu' \in \varphi(W, R_W)$ such that for all $i \in W$, $\mu(i) R_i \mu'(i)$. Since $\varphi(W, R_W) = \{\mu^s\}$, for all $i \in W$, $\mu(i) R_i \mu^s(i) = i$, which proves individual rationality for women. Individual rationality for men is obtained by considering (M, R_M) instead of (W, R_W). □

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Appendix

A Solution $\bar{\varphi}$ is not Population Monotonic

Example 1. Solution $\bar{\varphi}$ defined on the domain of marriage markets
We denote the single-valued solution that assigns to any marriage market its man-optimal [woman-optimal] stable matching by φ^M [φ^W]. Then, for all marriage markets (N, R),

$$\bar{\varphi}(N,R) = \begin{cases} \varphi^M(N,R) & \text{if } |M| > |W|, \\ \varphi^W(N,R) & \text{otherwise.} \end{cases}$$

In order to guarantee that $\bar{\varphi} \neq \varphi^W$ [φ^M], we assume that there exist women $w_1, w_2 \in \mathbb{N}$ and men $m_1, m_2 \in \mathbb{N}$. $\diamond$

Proposition 3. *On the domain of marriage markets, solution $\bar{\varphi}$ is individually rational, competition and resource sensitive, but neither own-side nor other-side population monotonic.*

Proof. Since $\bar{\varphi}$ is a stable solution on the domain of marriage markets, it satisfies individual rationality and both sensitivity conditions (Proposition 2). The following examples demonstrate that $\bar{\varphi}$ is neither own-side nor other-side population monotonic. Assume that agents 1, 2, and 5 are men and agents 3 and 4 are women and consider roommate markets (N, R) and (N', R') such that

$N = \{1,2,3,4\}$	$N' = \{1,2,3,4,5\}$
$R_1 : 3,4,1,2$	$R'_1 : 3,4,1,2,5$
$R_2 : 4,3,2,1$	$R'_2 : 4,3,2,1,5$
$R_3 : 2,1,3,4$	$R'_3 : 2,1,3,4,5$
$R_4 : 1,2,4,3$	$R'_4 : 1,2,4,3,5$
	$R'_5 : 5,\ldots$
$\bar{\varphi}(N,R) = \{\bar{\mu}\}$	$\bar{\varphi}(N',R') = \{\mu'\}$
$\bar{\mu} = (4,3,2,1)$	$\mu' = (3,4,1,2,5)$

At marriage market (N, R) there are two men and two women, the woman-optimal stable matching is chosen at $\bar{\varphi}(N, R)$, and man 1 is matched to woman 4 – his second choice. At marriage market (N', R'), man 5 causes a switch to the man-optimal stable matching at $\bar{\varphi}(N', R')$ and man 1 is now matched to woman 3 – his first choice. This is a violation of own-side population monotonicity. On the other hand, at marriage market (N, R) woman 3 is matched to man 2 – her first choice. At marriage market (N', R'), woman 3 is matched to man 1 – her second choice. This is a violation of other-side population monotonicity. □

B Relations between Properties

First, we establish a stronger version of Lemma 3 by using weak competition sensitivity instead of competition sensitivity.

Lemma 3'.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

weak unanimity and weak competition sensitivity imply mutually best.

Proof. Let φ be a solution on any of the domains in Lemma 3' that satisfies weak unanimity and weak competition sensitivity, but not mutually best. Thus, there exists a (marriage, solvable) roommate market (N, R) and a matching $\mu \in \varphi(N, R)$ such that for two agents i and j [possibly $i = j$] that are mutually best, $\mu(i) \neq j$.

First, if roommate market (N, R) is solvable and not a marriage market, we add newcomers in order to guarantee solvability in later steps. Thus, if (N, R) is solvable then first go to Step 1 and otherwise go to Step 2 immediately.

Step 1: *Guaranteeing Solvability*
Assume that roommate market (N, R) is solvable and let $\mu' \in S(N, R)$.

Let k_1, k_2 be such that $k_1 \neq k_2$ and $\mu'(k_1) = k_2$. Consider the extension (N^1, R^1) of (N, R) that is obtained by adding a newcomer k_1' such that agent k_1 immediately prefers k_1' after k_2, agent k_1' finds only agent k_1 acceptable, and k_1' is unacceptable for all other agents $k \in N \setminus \{k_1\}$, i.e., $N^1 = N \cup \{k_1'\}$ and R^1 is such that $[k_2\ P^1_{k_1}\ k_1'$ and for no $k \in N$, $k_2\ P^1_{k_1}\ k\ P^1_{k_1}\ k_1')]$ and for all $k \in N \setminus \{k_1\}$, $k_1\ P^1_{k_1'}\ k_1'\ P^1_{k_1'}\ k$ and $k\ P^1_k\ k_1'$. Note that (N^1, R^1) is solvable.[16] By weak competition sensitivity, for $\mu \in \varphi(N, R)$, there exists $\mu^1 \in \varphi(N^1, R^1)$ such that if agents i and j are newly matched at μ^1, then at least one is worse off. Hence, there exists $\mu^1 \in \varphi(N^1, R^1)$ such that $\mu^1(i) \neq j$ (agents i and j are still mutually best at (N^1, R^1)).

We consider the extension (N^2, R^2) of (N^1, R^1) that is obtained by adding a newcomer k_2' such that agent k_2 immediately prefers k_2' after k_1, agent k_2' finds only agent k_2 acceptable, and k_2' is unacceptable for all other agents $k \in N^1 \setminus \{k_2\}$. Similarly as before it follows that (N^2, R^2) is solvable and there exists $\mu^2 \in \varphi(N^2, R^2)$ such that $\mu^2(i) \neq j$ (agents i and j are still mutually best at (N^2, R^2)).

Note that we add newcomers as described above for all k_1, k_2 such that $k_1 \neq k_2$ and $\mu'(k_1) = k_2$. This results in a solvable roommate market that for notational convenience we also denote by (N, R). For this matching market (N, R) there exists a corresponding stable matching μ' and a matching $\mu \in \varphi(N, R)$ such that for the two mutually best agents i and j, $\mu(i) \neq j$. The difference between this roommate market (N, R) and the original market is that now a newcomer who is added in the sequel will not cause instability because an agent k who is unmatched by the newcomer from his original stable partner at μ' can now match in a stable way with the "corresponding" added agent k' instead of creating a "roommate cycle".

Step 2: Without loss of generality assume that $N \setminus \{i, j\} = \{1, 2, \ldots, l\}$. First, consider the extension (N^1, R^1) of (N, R) that is obtained by adding a newcomer k_1 such that agents 1 and k_1 are mutually best and k_1 is unacceptable for all other agents in $\{2, \ldots, l\}$ [if (N, R) is a marriage market, then the gender of newcomer k_1 and preferences can be chosen such that (N^1, R^1) is also a marriage market]. By weak competition sensitivity, for $\mu \in \varphi(N, R)$, there exists $\mu^1 \in \varphi(N^1, R^1)$ such that if agents i and j are newly matched at μ^1, then at least one is worse off. Hence, there exists $\mu^1 \in \varphi(N^1, R^1)$ such that $\mu^1(i) \neq j$. Pairs $\{i, j\}$ and $\{1, k_1\}$ consist of mutually best agents.

We continue adding newcomers $k_2, \ldots, k_l$ in a similar fashion and end up with a roommate market (N^l, R^l) such that there exists $\mu^l \in \varphi(N^l, R^l)$ with $\mu^l(i) \neq j$. At (N^l, R^l) we can partition N^l in pairs $\{i, j\}, \{1, k_1\}, \ldots, \{l, k_l\}$ of mutually best agents. For the solvable roommate market (N^l, R^l) there exists the complete and unanimously best matching ν that matches agent i with agent j, agent 1 with agent k_1, agent 2 with agent k_2, …, and agent l with agent k_l. Hence, by weak unanimity, $\varphi(N^l, R^l) = \{\nu\}$, contradicting $\mu^l \in \varphi(N^l, R^l)$. □

[16] Roommate market (N^1, R^1) has at least the stable matching where all agents in N are matched according to μ' and agent k_1' is single.

The following three-agent example demonstrates why Lemmas 3 and 3' might not hold if the set of potential agents is finite.

Example 3. Assume that the set of potential agents is $\{1,2,3\}$ and denote by μ^s the matching where all agents are single.Then, for all (marriage, solvable) roommate markets (N, R),

$$\tilde{\varphi}(N,R) = \begin{cases} core(N,R) & \text{if } |N| < 3, \\ core(N,R) \cup \{\mu^s\} & \text{otherwise.} \end{cases}$$

It is easy to check that $\tilde{\varphi}$ satisfies weak unanimity, competition sensitivity, Maskin monotonicity, but not mutually best. $\diamond$

Next, we prove that Lemma 4 (a) and (c) can be strengthened by using weak resource sensitivity instead of resource sensitivity. In order to weaken resource sensitivity in Lemma 4 (b), we add Maskin monotonicity.

Lemma 4'.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets, Maskin monotonicity,*

(c) *On the domain of all roommate markets,*

weak unanimity and weak resource sensitivity imply mutually best.

Proof. We first give a direct proof for (a) and (c) and conclude with an indirect proof for (b).

Proof of (a) and (c): Let φ be a solution on the domain of all roommate markets [marriage markets] that satisfies weak unanimity and weak resource sensitivity. Let (N, R) be a roommate market [marriage market] such that agents $i, j \in N$ [possibly $i = j$] are mutually best. To prove that φ satisfies mutually best, we show that for all $\mu \in \varphi(N,R)$, $\mu(i) = j$.

Let $\bar{N} = \{i, j\}$ and consider the reduced preferences $\bar{R} = R_{\bar{N}}$, i.e., $i \bar{R}_j j$ and $j \bar{R}_i i$ completely describe $\bar{R}$. There exists a unanimously best complete matching $\bar{\nu}$ for marriage market $(\bar{N}, \bar{R})$: $\bar{\nu}$ matches agent i with agent j. Hence, by weak unanimity, $\varphi(\bar{N}, \bar{R}) = \{\bar{\nu}\}$ and $\bar{\nu}(i) = j$. In the sequel we will not use the single-valuedness of $\varphi(\bar{N}, \bar{R})$ but that for all $\mu' \in \varphi(\bar{N}, \bar{R})$, $\mu'(i) = j$.

Without loss of generality assume that $N = \bar{N} \cup \{1, \ldots, l\}$. First, consider the extension (N^1, R^1) of $(\bar{N}, \bar{R})$ that is obtained by adding newcomer 1 such that $N^1 = \bar{N} \cup \{1\}$ and $R^1 = R_{N^1}$ [if (N, R) is a marriage market, then (N^1, R^1) is also a marriage market].[17] By weak resource sensitivity, for all $\mu^1 \in \varphi(N^1, R^1)$, there exists $\mu' \in \varphi(\bar{N}, \bar{R})$ such that if agents i and j are not matched at μ^1 anymore, then at least one is better off. Then, since for all $\mu' \in \varphi(\bar{N}, \bar{R})$ agents i and j are already mutually best matched, for all $\mu^1 \in \varphi(N^1, R^1)$, $\mu^1(i) = j$.

We continue adding newcomers $2, \ldots, l$ in a similar fashion and end up with the original roommate market (N, R). By weak resource-sensitivity, for all $\mu \in \varphi(N, R)$, $\mu(i) = j$.

Proof of (b): Let φ be a solution on the domain of solvable roommate markets that satisfies Maskin monotonicity, weak unanimity, and weak competition sensitivity, but not mutually best. Thus, there exists a solvable roommate market (N, R) and a matching $\mu \in \varphi(N, R)$ such that for two agents i and j [possibly $i = j$] that are mutually best, $\mu(i) \neq j$.

We define $\tilde{R} \in \mathcal{R}^N$ by moving $\mu(i)$ just below agent j in agent i's preferences, by moving $\mu(j)$ just below agent i in agent j's preferences and, for all $k \in N \setminus \{i, j\}$, by moving $\mu(k)$ on top of agent k's preferences.

[17] Note that for solvable roommate markets this step might be problematic because even though (N, R) and $(\bar{N}, \bar{R})$ are solvable, (N^1, R^1) might not be solvable.

Note that $(N, \tilde{R})$ is solvable[18] and that $\tilde{R}$ is a Maskin monotonic transformation of R at μ. Hence, by Maskin monotonicity, $\mu \in \varphi(N, \tilde{R})$. Agents i and j that are still mutually best and $\mu(i) \neq j$.

Let $\bar{N} = N \setminus \{\mu(i), \mu(j)\}$ and consider the reduced preferences $\bar{R} = \tilde{R}_{\bar{N}}$. There exists a unanimously best complete matching $\bar{\nu}$ for the solvable roommate market $(\bar{N}, \bar{R})$: $\bar{\nu}$ matches agent i with agent j, and all agents in $\bar{N} \setminus \{i, j\}$ according to μ. Hence, by weak unanimity, $\varphi(\bar{N}, \bar{R}) = \{\bar{\nu}\}$ and $\bar{\nu}(i) = j$. In the sequel we will not use the single-valuedness of $\varphi(\bar{N}, \bar{R})$ but that for all $\mu' \in \varphi(\bar{N}, \bar{R})$, $\mu'(i) = j$.

If $\mu(i) \neq i$, then consider the extension (N^1, R^1) of $(\bar{N}, \bar{R})$ that is obtained by adding newcomer $\mu(i)$ such that $N^1 = \bar{N} \cup \{\mu(i)\}$ and $R^1 = R_{N^1}$.[19] By weak resource sensitivity, for all $\mu^1 \in \varphi(N^1, R^1)$, there exists $\mu' \in \varphi(\bar{N}, \bar{R})$ such that if agents i and j are not matched at μ^1 anymore, then at least one is better off. Then, since for all $\mu' \in \varphi(\bar{N}, \bar{R})$ agents i and j are already mutually best matched, for all $\mu^1 \in \varphi(N^1, R^1)$, $\mu^1(i) = j$.

If $\mu(j) \neq j$, then we add newcomer $\mu(j)$ in a similar fashion. So we end up with the original roommate market $(N, \tilde{R})$. By weak resource-sensitivity, for all $\mu^2 \in \varphi(N, \tilde{R})$, $\mu^2(i) = j$, contradicting $\mu(i) \neq j$. □

The following example, which is due to an anonymous referee (thanks), shows that we cannot strengthen Lemma 4 (b) by using weak resource sensitivity instead of resource sensitivity (and without adding Maskin monotonicity).

Example 4. ***Solution $\hat{\varphi}$ defined on the domain of solvable roommate markets***
We define solution $\hat{\varphi}$ using the following roommate market and matchings. Let $(\hat{N}, \hat{R})$ be an element of roommate market subdomain $\hat{\mathfrak{D}}$ if $\hat{N} = \{1, 2, 3, 4, 5, 6\}$ and

$\hat{N} = \{1, 2, 3, 4, 5, 6\}$
$\hat{R}_1 : 2, 3, 4, 1, \ldots$
$\hat{R}_2 : 3, 4, 1, 2, \ldots$
$\hat{R}_3 : 4, 1, 2, 3, \ldots$
$\hat{R}_4 : 1, 2, 3, 4, \ldots$
$\hat{R}_5 : 6, 5, \ldots$
$\hat{R}_6 : 5, 6, \ldots$

The unique stable matching $\hat{\mu} = (3, 4, 1, 2, 6, 5)$ for a roommate market $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ matches agents 1 and 3, agents 2 and 4, and the mutually best agents 5 and 6. Let $\hat{\mu}' = (3, 4, 1, 2, 5, 6)$ be the matching obtained from $\hat{\mu}$ by unmatching agents 5 and 6.

We now define $\hat{\varphi}$ as follows. For all solvable roommate markets (N, R),

$$\hat{\varphi}(N, R) = \begin{cases} \{\hat{\mu}, \hat{\mu}'\} & \text{if } (N, R) = (\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}, \\ core(N, R) & \text{otherwise.} \end{cases} \qquad \diamond$$

Proposition 4. *On the domain of solvable roommate markets, solution $\hat{\varphi}$ is weakly unanimous and weakly resource sensitive, but neither Maskin monotonic nor mutually best.*

[18]Roommate market $(N, \tilde{R})$ has a unique core allocation that matches agent i with agent j and all agents in $N \setminus \{i, j, \mu(i), \mu(j)\}$ according to μ – agent(s) $\mu(i)$ and $\mu(j)$ are either single or, if mutually acceptable, matched with each other.

[19]Roommate market (N^1, R^1) has a unique core allocation that matches agent i with agent j, all agents in $N^1 \setminus \{i, j, \mu(i)\}$ according to μ, and agent $\mu(i)$ is single.

Proof. It is easy to see that $\hat{\varphi}$ is weakly unanimous (since for $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ no complete unanimously best matching exists), and neither mutually best (since for $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ and at $\hat{\mu}'$ agents 5 and 6 are not mutually best matched) nor Maskin monotonic (since for $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$, $\hat{\mu}'$ is not chosen by $\hat{\varphi}$ anymore if agents 1, 2, 3, and 4 like their matches at $\hat{\mu}'$ best).

Note that the only solvable roommate markets for which $\hat{\varphi}$ does not assign the core are the ones in $\hat{\mathfrak{D}}$. Hence, by Proposition 2, as long as no roommate market in $\hat{\mathfrak{D}}$ is concerned, $\hat{\varphi}$ satisfies weak resource sensitivity. Thus, to complete the proof that $\hat{\varphi}$ satisfies weak resource sensitivity, we only have to consider (a) the addition of a newcomer to a roommate market $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ or (b) the addition of a newcomer that results in a roommate market $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$.

Case (a): Consider an extension (N', R') of $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ such that $N' = \hat{N} \cup \{n\}$. Then, by weak resource sensitivity, we have to show that for all $\mu' \in \hat{\varphi}(N', R')$, there exists $\mu \in \hat{\varphi}(\hat{N}, \hat{R})$ such that for all $i, j \in \hat{N}$ [possibly $i = j$] that are not matched at μ', at least one is better off. This follows from $\hat{\varphi}(N', R') = core(N', R')$, $\hat{\varphi}(\hat{N}, \hat{R}) \supseteq core(\hat{N}, \hat{R})$, and Proposition 2.

Case (b): Consider an extension $(\hat{N}, \hat{R}) \in \hat{\mathfrak{D}}$ of (N, R) such that $\hat{N} = N \cup \{n\}$. By the solvability of roommate market (N, R), $n \in \{5, 6\}$. Therefore, either $\hat{\varphi}(N, R) = \{(3, 4, 1, 2, 5)\}$ or $\hat{\varphi}(N, R) = \{(3, 4, 1, 2, 6)\}$. In both cases, the only agents that could be matched differently at $\hat{\varphi}(N, R)$ and $\hat{\varphi}(\hat{N}, \hat{R})$ are agent 5 or 6 and if they are, then they are better off. □

Lemma 9.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

mutually best and competition sensitivity imply individual rationality.

Proof. Let φ be a solution on any of the domains in Lemma 9 that satisfies mutually best and competition sensitivity. Let (N, R) be a (marriage, solvable) roommate market and $i \in N$. To prove that φ satisfies individual rationality, we show that for all $\mu \in \varphi(N, R)$, $\mu(i)\ R_i\ i$.

Without loss of generality assume that $N \setminus \{i\} = \{1, \ldots, l\}$. Let $\hat{N} = \{k_1, \ldots, k_l\}$ be a set of newcomers and assume that (N', R'), $N' = N \cup \hat{N}$, is an extension of (N, R) such that agents 1 and k_1 are mutually best, agents 2 and k_2 are mutually best, …, and agents l and k_l are mutually best [if (N, R) is a marriage market, then the gender of newcomers and preferences can be chosen such that (N', R') is also a marriage market]. For the (marriage, solvable) roommate market (N', R') mutually best implies that $\varphi(N', R') = \{\nu'\}$ where ν' is the matching that matches agent 1 with agent k_1, agent 2 with agent k_2, …, agent l with agent k_l, and leaves agent i single.

By competition sensitivity, for all $\mu \in \varphi(N, R)$, if agent i is newly matched at ν', then (s)he is worse off. Since $\nu'(i) = i$, for all $\mu \in \varphi(N, R)$, $\mu(i)\ R_i\ \nu(i)' = i$. □

Similarly to Lemma 3 and its stronger version Lemma 3' (Appendix B), one could strengthen Lemma 9 by using weak competition sensitivity instead of competition sensitivity.

An example similar to Example 3 (Appendix B) can be constructed to show why Lemma 9 might not hold if the set of potential agents is finite.

Lemma 10.

(a) *On the domain of marriage markets,*

(b) *On the domain of solvable roommate markets,*

(c) *On the domain of all roommate markets,*

mutually best and resource sensitivity imply individual rationality.

Proof. Let φ be a solution on any of the domains in Lemma 10 that satisfies mutually best and resource sensitivity. Let (N, R) be a (marriage, solvable) roommate market and $i \in N$. To prove that φ satisfies individual rationality, we show that for all $\mu \in \varphi(N, R)$, $\mu(i)\, R_i\, i$.

Let $\bar{N} = \{i\}$ and consider the reduced preferences $\bar{R} = R_{\bar{N}}$. Since there is only one agent in $\bar{N}$, $\varphi(\bar{N}, \bar{R}) = \{\bar{\nu}\}$ and $\bar{\nu}(i) = i$. Without loss of generality assume that $N = \bar{N} \cup \{1, \ldots, l\}$. Consider the extension (N, R) of $(\bar{N}, \bar{R})$ that is obtained by adding newcomers $\hat{N} = \{1, \ldots, l\}$. By resource sensitivity, for all $\mu \in \varphi(N, R)$, $\bar{\nu}$ is such that if i matched with (her)himself, then (s)he is better off. Since at $\bar{\nu}$ agents i is single, for all $\mu \in \varphi(N, R)$, $\mu(i)\, R_i\, i$. □

Similarly to Lemma 4 (a) and (c) and its stronger version Lemma 4' (a) and (c) (Appendix B), one could strengthen Lemma 10 (a) and (c) by using weak resource sensitivity instead of resource sensitivity.

Similarly to Lemma 4 (b) and its variant Lemma 4' (b), once could obtain a variant of Lemma 10 (b) by replacing resource sensitivity with weak resource sensitivity and Maskin monotonicity.

Finally, an example similar to Example 4 (Appendix B) can be constructed to show that when using weak resource sensitivity Maskin monotonicity cannot be dropped.