Labor Market Signaling with Overconfident Workers*

Luís Santos-Pinto†

Faculty of Business and Economics, University of Lausanne

This version: June 2, 2010

Abstract

I extend Spence's (1974) labor market signaling model by assuming some workers are overconfident and some underconfident. Overconfident (underconfident) workers underestimate (overestimate) their marginal cost of acquiring education. Firms cannot observe workers' productive abilities and cannot observe workers' beliefs. However, firms know the fraction of overconfident, underconfident, and high-ability workers in the economy. I find that the presence of overconfident and/or underconfident workers in the labor market compresses wages. I show that workers' biased beliefs reduce welfare when workers are sufficiently different in terms of productivity and cost of education. Finally, I show that if the fraction of overconfident workers is relatively low and workers are sufficiently similar in terms of productivity and cost of education, then biased beliefs improve welfare.

JEL Codes: D03; D82; J24; J31.
Keywords: Signaling; Labor Market; Behavioral Biases; Wages; Education.

*I thank Joel Sobel and Bettina Klaus for helpful comments and suggestions.
†Faculty of Business and Economics, University of Lausanne, Internef 53, CH-1015, Lausanne, Switzerland. Ph: 41-216923658. Fax: 41-216923435. E-mail address: LuisPedro.SantosPinto@unil.ch.

1 Introduction

This paper explores the implications of biased self-evaluations in the classic model of labor market signaling by Spence (1973). Firms are perfectly competitive and cannot observe workers' productive abilities, which may be either high or low. Some workers know their marginal cost of acquiring education but others do not. Overconfident workers believe that their marginal cost of acquiring education is low when, in fact, it is high. Underconfident workers believe that their marginal cost of acquiring education is high when, in fact, it is low. Firms cannot observe workers' beliefs but know the fraction of high-ability, overconfident and underconfident workers in the labor market.

The main finding of the paper is that wage compression can arise because of workers' biased self-evaluations. Wage compression is a key feature of labor markets and has important consequences for labor market performance. For example, Acemoglu and Pischke (1999) demonstrate that wage compression may encourage employers to offer and pay for general training. Lindquist (2005) shows that, when labour markets are competitive, even low degrees of wage compression lead to large welfare losses from costly unemployment among low-skilled workers.

The economics literature has proposed several explanations for wage compression, ranging from labor market institutions, to incentives not to sabotage colleagues competing in a tournament, to fairness considerations in wage-setting decisions by firms.[1]

I show that wage compression arises quite naturally in a competitive labor market where workers have biased beliefs about their marginal cost of acquiring education, firms do not know workers' skills and beliefs, and education is a signal of workers' productive abilities. The intuition behind this result is straightforward.

In a signaling equilibrium with biased workers, overconfident and unbiased high-ability workers choose a high education level and underconfident and unbiased low-ability workers choose a low education level. The optimal response of firms to the fact that overconfidence raises the proportion of low-ability workers in the high education group whereas underconfidence raises the proportion of high-ability workers in the low education group is to compress wages.

[1] I review the relevant literature in Section 4.

I also find that biased beliefs reduce welfare if workers are sufficiently different in terms of productivity and cost of education. When all workers are unbiased, Spence's (1974) model shows that if the two groups of workers are sufficiently different, then one can have a signaling equilibrium where investments in education are the efficient ones and the outcome is as if there was perfect information in the market place. Introducing a distortion–workers' biased beliefs–reduces welfare.

Finally, I show that if workers are sufficiently similar in terms of productivity and cost of education, then biased beliefs can improve welfare. When all workers are unbiased, Spence's (1974) model shows that if the two groups of workers are sufficiently similar in terms of productivity and cost of education, then there exist separating equilibria with overinvestment in education by the more productive group. In this case private information about productive ability reduces welfare. However, it is possible to improve market efficiency with an optimal tax-subsidy schedule that consists of a rising tax on education combined with a lump-sum transfer to everyone so that net tax revenues are zero. Spence (2002) shows that the optimal tax-subsidy schedule implies that high-ability workers will pay a net tax and choose their optimal level of education whereas low-ability workers receive a net subsidy.

The reason why workers' biased beliefs raise welfare are similar to those why a tax-subsidy schedule raises welfare. Overconfidence is like a "tax" on the education of unbiased high-ability workers because it lowers their wage and this brings their education level closer to the optimal. Underconfidence is like a "subsidy" for unbiased low-ability workers because it raises their wage for a given education level. This result is consistent with the theory of the second best. According to this theory, introducing a new distortion—workers' biased beliefs—in an environment where another distortion is already present—private information about skill—, may increase welfare.

Of course, welfare does not always rise when workers have biased beliefs and are sufficiently similar in terms of productivity and cost of education. If the fraction of overconfident workers is too high, unbiased high-ability workers are "overtaxed" and end up doing worse than high-ability workers. In this case there is a transfer of utility from unbiased high-ability to unbiased low-ability workers.

One implication of the welfare results is that policies that try to improve workers' self-evaluations will reduce welfare when the fraction of overconfident workers is low and workers are sufficiently similar in terms of productivity and cost of education. In contrast, if either (i) workers are sufficiently different or (ii) a significant fraction of workers is overconfident and workers are sufficiently similar, then improving self-evaluations improves welfare.

The assumption that some workers are overconfident and some underconfident is supported by robust empirical evidence on patterns of over- and underestimation in self-evaluation of skills. Overconfidence is a staple finding in psychology and has been shown to be present in individuals' self-assessments of performance in their jobs. According to Myers (1996), a textbook in social psychology, "(...) on nearly any dimension that is both subjective and socially desirable, most people see themselves as better than average."[2]

Kruger and Dunning (1999) find that it is the poorest performers who hold the least accurate evaluations of their skills and performances, grossly overestimating how well their performances stack up against those of their peers. They observe that students performing in the bottom 25% among their peers on tests of grammar, logical reasoning, and humor tend to think that they are performing above the 60% percentile. They also find that top performers consistently underestimate how superior their performances are relative to their peers. In Kruger and Dunning (1999) studies, the top 25% tended to think that their skills lay in the 70-75% percentile, although their performances fell roughly in the 87% percentile.[3]

This paper is an additional contribution to the growing literature on the impact of behavioral biases on markets and organizations. DellaVigna and Malmendier (2004), Glenn Ellison (2005), and Gabaix and Laibson (2006) study market interactions between sophisticated firms and biased consumers. They find that in competitive markets, biased consumers may be indirectly exploited by sophisticated consumers.

Sandroni and Squintani (2007) investigate the policy implications of overconfidence in insurance markets. They find that if a significant fraction of agents are overconfident and insurance firms cannot directly observe agents' beliefs, then compulsory insurance fails to make all agents better off because

[2] Baker et al. (1988) cite a survey of General Electric Company employees according to which 81 percent of a sample of white-collar clerical and technical workers rated their own performance as falling within the top 20 percent of their peers in similar jobs. Myers (1996) cites a study according to which, in Australia, 86 percent of people rate their job performance as above average.

[3] These patterns have been replicated among undergraduates completing a classroom exam (Dunning et al. 2003), medical students assessing their interviewing skills (Hodges et al. 2001), clerks evaluating their performance (Edwards et al. 2003), and medical laboratory technicians evaluating their on-the-job expertise (Haun et al. 2000).

it is detrimental to low-risk agents. Thus, behavioral biases may weaken asymmetric information rationales for government intervention in insurance markets because they may turn policies beneficial to all agents into wealth transfers between agents.

The paper closest related to mine is Fang and Moscarini (2005). They use a principal-agent model to study the implication of worker overconfidence on the firm's optimal wage-setting policies. Wage contracts provide incentives and affect workers' confidence in their own skills, by revealing private information of the firm about workers' skills. They find, using numerical examples, that overconfidence is a necessary condition for a firm to choose a non-differentiation wage policy (the most extreme form of wage compression). This happens because, when ability and effort are complements, a non-differentiation wage policy preserves worker overconfidence which in turn induces higher effort, offsetting the moral hazard inefficiency.

The paper is organized as follows. Section 2 sets-up the model. Sections 3 describes the equilibria. Section 4 shows that workers' biased beliefs compress wages. Section 5 discusses the impact of biased beliefs on welfare. Section 6 concludes. Proofs of all results are in the Appendix.

2 The Model

For each worker there are two possible productive abilities, low, θ_L, and high, θ_H, with $0 < \theta_L < \theta_H$. Nature determines a worker's productive ability and beliefs about marginal cost of acquiring education. The worker chooses a level of education, $e \geq 0$, based on her beliefs. Two firms, 1 and 2, observe the worker's education and then simultaneously make wage offers w_1 and w_2, with $w_i \geq 0$, $i = 1, 2$. The worker accepts the highest of the two wage offers, flipping a coin in case of a tie.

The payoff of a firm that employs a worker with ability θ and education e is $\pi(w, e, \theta) = y(e, \theta) - w$, where $y(e, \theta)$ is the worker's output. The payoff of a firm that does not employ a worker is zero. High-ability workers are more productive: $y_\theta(e, \theta) > 0$. Education does not reduce productivity, that is, $y_e(e, \theta) \geq 0$ where $y_e(e, \theta)$ is the marginal productivity of education for a worker of ability θ at education e. The marginal productivity of education is non-increasing with education: $y_{ee}(e, \theta) \leq 0$. The marginal productivity of education is non-decreasing with ability: $y_{e\theta}(e, \theta) \geq 0$.

There are four types of workers in the labor market. Unbiased high-ability workers have marginal cost of acquiring education $c_e(e, \theta_H)$ and know it. Unbiased low-ability workers have marginal cost of acquiring education $c_e(e, \theta_L)$ and know it. Overconfident workers believe their marginal cost of acquiring education is $c_e(e, \theta_H)$ when, in fact, it is $c_e(e, \theta_L)$. Underconfident workers believe their marginal cost of acquiring education is $c_e(e, \theta_L)$ when, in fact, it is $c_e(e, \theta_H)$. Let $\lambda \in (0, 1)$ be the fraction of high-ability workers, $\nu \in [0, \lambda]$ be the fraction of underconfident workers, and $\kappa \in [0, 1 - \lambda]$ be the fraction of overconfident workers. Firms cannot observe a worker's productive ability or beliefs, but know λ, κ and ν.

The utility of an employed worker is $u(w, e, \theta) = w - c(e, \theta)$, where w is the wage offer made by a firm and $c(e, \theta)$ is the cost to a worker with ability θ to obtaining education e. The utility of an unemployed worker is zero. The cost of no education is zero: $c(0, \theta) = 0$. The cost of education increases with education: $c_e(e, \theta) > 0$, where $c_e(e, \theta)$ is the marginal cost of education for a worker of ability θ at education e. The cost of education decreases with ability: $c_\theta(e, \theta) < 0$. The marginal cost of education increases with education: $c_{ee}(e, \theta) > 0$. The marginal cost of education decreases with ability: $c_{e\theta}(e, \theta) < 0$.[4]

3 Equilibria

In a separating equilibrium, education choices are determined by workers' beliefs about their marginal cost of acquiring education: underconfident and unbiased low-ability workers choose a low education level, e^{LU}, whereas overconfident and unbiased high-ability workers choose a high education level, e^{HO}, with $e^{HO} \in [\hat{e}^B, \bar{e}^B]$, and $e^{HO} > e^{LU}$. Firms cannot distinguish between underconfident and unbiased low-ability workers because, at the time wage offers are made, both types of workers have the same education level: e^{LU}. Similarly, firms cannot distinguish between overconfident and unbiased high-ability workers because both types of workers display the same education level e^{HO}. However, firms know λ, κ and ν.

Among all workers who choose an education level e^{LU} firms know that fraction $\alpha = \frac{\nu}{1-\lambda-\kappa+\nu}$ has high ability and fraction $1 - \alpha = \frac{1-\lambda-\kappa}{1-\lambda-\kappa+\nu}$ low-ability. Among all workers who choose an education level e^{HO} firms know

[4] This assumption is critical because it implies that workers who think they have a high marginal cost of aquiring education find signaling more costly than those who think they have a low marginal cost of acquring education.

that fraction $\beta = \frac{\kappa}{\lambda+\kappa-\nu}$ has low ability and fraction $1-\beta = \frac{\lambda-\nu}{\lambda+\kappa-\nu}$ high-ability. Thus, in a separating equilibrium, the firms' posterior belief that a worker has high ability after observing education level e are

$$\mu(\theta_H|e) = \begin{cases} \alpha, & \text{for } e < e^{HO} \\ 1-\beta, & \text{for } e \geq e^{HO} \end{cases}.$$

The firms' strategy is then

$$w(e) = \begin{cases} (1-\alpha)y(e,\theta_L) + \alpha y(e,\theta_H), & \text{for } e < e^{HO} \\ \beta y(e,\theta_L) + (1-\beta)y(e,\theta_H), & \text{for } e \geq e^{HO} \end{cases}. \tag{1}$$

The firms' strategy is derived from the assumption that firms make zero profits in equilibrium and that firms know λ, κ and ν. Competition between firms implies that the wage offered to each group of workers (those who choose e^{LU} and those who choose e^{HO}) must be a weighted average of the productivities of each type of worker in the group.

In a separating equilibrium underconfident and unbiased low-ability workers do not envy overconfident and unbiased high-ability workers, that is

$$\begin{aligned} &(1-\alpha)y(e^{LU},\theta_L) + \alpha y(e^{LU},\theta_H) - c(e^{LU},\theta_L) \\ &\geq \beta y(e^{HO},\theta_L) + (1-\beta)y(e^{HO},\theta_H) - c(e^{HO},\theta_L), \end{aligned} \tag{2}$$

and overconfident and unbiased high-ability workers do not envy underconfident and unbiased low-ability workers, that is

$$\begin{aligned} \beta y(e^{HO},\theta_L) + (1-\beta)y(e^{HO},\theta_H) - c(e^{HO},\theta_H) \geq \\ (1-\alpha)y(e^{LU},\theta_L) + \alpha y(e^{LU},\theta_H) - c(e^{LU},\theta_H). \end{aligned} \tag{3}$$

A necessary condition for separating equilibria to exist is that the wage paid to overconfident and unbiased low-ability workers is higher than that paid to underconfident and unbiased low-ability workers. Since in a separating equilibrium $e^{LU} < e^{HO}$ we see from (1) that if $\alpha + \beta \leq 1$, then the wage paid to overconfident and unbiased low-ability workers is less than that paid to underconfident and unbiased low-ability workers. Using the definitions of α and β, the inequality is equivalent to

$$(1-\lambda)\nu + \lambda\kappa < (1-\lambda)\lambda. \tag{4}$$

Condition (4) says that if the fractions of overconfident and underconfident workers are sufficiently small, education can serve as a signal of productive ability.[5] When the fraction of biased workers is too high, condition (4) is violated, separating equilibria may no longer exist.[6] I assume from now on that condition (4) is satisfied.

Let $e^*(\sigma,\alpha)$ solve $max_e\,[(1-\alpha)y(e,\theta_L)+\alpha y(e,\theta_H)-c(e,\theta_L)]$ and $e^*(\sigma,\beta)$ solve $max_e\,[\beta y(e,\theta_L)+(1-\beta)y(e,\theta_H)-c(e,\theta_H)]$, where $\sigma=(\theta_L,\theta_H)$. Let $w^*(\sigma,\alpha)=(1-\alpha)y(e^*(\sigma,\alpha),\theta_L)+\alpha y(e^*(\sigma,\alpha),\theta_H)$ and, in addition, let $w^*(\sigma,\beta)=\beta y(e^*(\sigma,\beta),\theta_L)+(1-\beta)y(e^*(\sigma,\beta),\theta_H)$. Finally, let $u^*(\sigma,\alpha)=w^*(\sigma,\alpha)-c(e^*(\sigma,\alpha),\theta_L))$ and $u^*(\sigma,\beta)=w^*(\sigma,\beta)-c(e^*(\sigma,\beta),\theta_H))$.

There are two qualitatively different kinds of separating equilibria. In one case it is too expensive for underconfident and unbiased low-ability workers to acquire education $e^*(\sigma,\beta)$, even if doing so would make firms believe that they are overconfident or unbiased high-ability workers and so cause them to pay the wage $w^*(\sigma,\beta)$, that is, $w^*(\sigma,\alpha)-c(e^*(\sigma,\alpha),\theta_L)>w^*(\sigma,\beta)-c(e^*(\sigma,\beta),\theta_L)$. In the other case, underconfident and unbiased low-ability workers prefer the wage $w^*(\sigma,\beta)$ and the education level $e^*(\sigma,\beta)$ of overconfident and unbiased high ability workers, that is,

$$w^*(\sigma,\alpha)-c(e^*(\sigma,\alpha),\theta_L)<w^*(\sigma,\beta)-c(e^*(\sigma,\beta),\theta_L). \tag{5}$$

When inequality (5) is satisfied overconfident and unbiased high-ability workers must choose an education level greater than $e^*(\sigma,\beta)$ to distinguish themselves from underconfident and unbiased low-ability workers, that is, $e^{HO}\in[\hat{e}^{HO},\bar{e}^{HO}]$, with $\hat{e}^{HO}>e^*(\sigma,\beta)$.[7]

Proposition 1: *In a separating equilibrium: (i) the education level of underconfident and unbiased low-ability workers is at least the first-best education*

[5] When all workers are unbiased, Spence's (1974) model shows that education can serve as a signal of productive ability if the marginal cost of education is decreasing with ability.

[6] There are always pooling equilibria where all types of workers choose the same education level e. The firms' posterior belief about a worker's productive ability after observing e must be the prior belief, $\mu(\theta_H|e)=\lambda$, which in turn implies that the equilibrium wage is $w=\lambda y(e,\theta_H)+(1-\lambda)y(e,\theta_L)$.

[7] Inequality (5) is satisfied if workers are sufficiently similar in terms of productivity and cost of acquiring education and the fraction of biased workers is not too high. To see this consider the case $y(e,\theta)=\theta e$ and $c(e,\theta)=e^2/2\theta$. We have $e^*(\sigma,\alpha)=\theta_L(\theta_L+\alpha\rho)$, $e^*(\sigma,\beta)=\theta_H(\theta_H-\beta\rho)$, where $\rho=\theta_H-\theta_L$. In this case inequality (5) becomes $\frac{\theta_L}{2}(\theta_L+\alpha\rho)^2<\frac{\theta_H}{2}(\theta_H-\beta\rho)^2\left(1-\frac{\rho}{\theta_L}\right)$. A necessary condition for this inequality to be satisfied is that $\rho<\theta_L\left[1-(\theta_L/\theta_H)^3\right]$.

level of low-ability workers–$e^{LU} = e^*(\sigma, \alpha) \geq e^*(\theta_L)$*–, (ii) the wage paid to underconfident and unbiased low-ability workers is greater than the first-best wage of low ability workers–*$w(e^{LU}) = w^*(\sigma, \alpha) > w^*(\theta_L)$*–, and (iii) the utility of unbiased low-ability workers is greater than the first-best utility of low ability workers–*$u(w(e^{LU}), e^{LU}, \theta_L) = u^*(\sigma, \alpha) > u^*(\theta_L)$.

The intuition behind Proposition 1 is as follows. Underconfident workers think (mistakenly) they have a high marginal cost of acquiring education and, like unbiased low-ability workers, choose a low education level. Firms observe this low education level but since they are unable to distinguish each type of worker, they pay a wage that is equal to the average product of underconfident and unbiased low-ability workers. This implies that, for a given education level, the marginal productivity of education is higher for underconfident and unbiased low-ability workers than for low-ability workers. Since the marginal cost of education is the same, the education level of underconfident and unbiased low-ability workers is at least the first-best education level of low-ability workers. This in turn implies that underconfident and unbiased low-ability workers are paid a higher wage than the first-best wage of low-ability workers. Finally, the utility of unbiased low-ability workers is higher than the first-best utility of low-ability workers because the favorable impact of underconfidence on the wage is of first-order whereas the unfavorable impact of underconfidence on the cost of education is of second-order.

Proposition 2: *If workers are sufficiently different in terms of productivity and cost of acquiring education or the fraction of biased workers is sufficiently high–inequality (5) is violated–, then: (i) the education level of overconfident and unbiased high-ability workers is at most the first-best education level of high-ability workers–*$e^{HO} = e^*(\sigma, \beta) \leq e^*(\theta_H)$*–, (ii) the wage paid to overconfident and unbiased high-ability workers is less than the first-best wage of high-ability workers–*$w(e^{HO}) = w^*(\sigma, \beta) < w^*(\theta_H)$*–, and (iii) the utility of unbiased high-ability workers is smaller than the first-best utility of high-ability workers–*$u(w(e^{HO}), e^{HO}, \theta_H) = u^*(\sigma, \beta) < u^*(\theta_H)$.

When workers are sufficiently different in terms of productivity and cost of education or the fraction of biased workers is sufficiently high, there is a unique separating equilibrium. Overconfident workers think (mistakenly) they have a low marginal cost of acquiring education and, like unbiased high-ability workers, choose a high education level. Firms observe this high education level but since they are unable to distinguish each type of worker, they pay a wage that is equal to the average product of overconfident and unbiased high-ability workers. This implies that, for a given education level, the marginal productivity of education is lower for overconfident and unbiased high-ability workers than for high-ability workers. Since the marginal cost of education is the same, the education level of overconfident and unbiased high-ability workers is at most the first-best education level of high-ability workers. This in turn implies that overconfident and unbiased high-ability workers are paid a lower wage than the first-best wage of high-ability workers. Finally, the utility of unbiased high-ability workers is lower than the first-best utility of high-ability workers because overconfidence shifts the education level away from the optimal one and reduces the wage.

Proposition 3 shows that if inequality (5) is satisfied, then the education and wage of overconfident and unbiased high-ability workers in the lowest (highest) separating equilibrium with biased workers is smaller than the education and wage, respectively, of high-ability workers in the lowest (highest) separating equilibrium with rational workers. Thus, the set of equilibria education-wage levels of overconfident and unbiased high-ability workers is "lower" than the set of education-wage levels of high-ability workers.

Proposition 3: *If workers are sufficiently similar in terms of productivity and cost of acquiring education and the fraction of biased workers is sufficiently small–inequality (5) is satisfied when $\alpha > 0$ and $\beta > 0$–, then: (i) the education level of overconfident and unbiased high-ability workers belongs to $[\hat{e}^{HO}, \bar{e}^{HO}]$, with $\hat{e}^{HO} < \hat{e}^{H}$ and $\bar{e}^{HO} < \bar{e}^{H}$, and (ii) the wage paid to overconfident and unbiased high-ability workers belongs to $[\hat{w}^{HO}, \bar{w}^{HO}]$, with $\hat{w}^{HO} < \hat{w}^{H}$ and $\bar{w}^{HO} < \bar{w}^{H}$.*

The intuition behind the result is as follows. In the lowest separating equilibrium with biased workers the incentive compatibility condition of underconfident and unbiased low-ability workers binds. We know from Proposition 1 part (iii) that the utility of an unbiased low-ability worker is higher than the first-best utility of a low-ability worker. Thus, for the incentive compatibility condition of underconfident and unbiased low-ability workers to bind, their education level must be lower than that of high-ability workers. The wage of overconfident and unbiased high-ability workers is smaller than the wage of high-ability workers because their education and productive ability are smaller than the education and productive ability, respectively, of high-ability workers.

4 Wage Compression

A key feature of imperfect labor markets is the presence of wage compression across skills (see Garibaldi, 2006, pp. 21). In the simplest static model of a competitive labor market, the relation between productivity and wages is straightforward: wages equal marginal product. Wage compression refers to a tendency of wages to be equalized across the skill distribution. For example, Campbell and Kamlani (1997) conducted a survey of 184 US firms and found that pay differentials represented about one half of the productivity differential between any two workers identical in all respects but productivity. Frank (1984a) examined wages and productivities of sales workers and university professors, and found that the more productive workers were paid less than their marginal product, while the least productive were paid more than their marginal product.

Economic theory offers two main explanations for wage compression. The first one identifies exogenous labor market frictions and institutions like mobility costs, trade-unions, efficiency wages, wage floors, or any institution which contributes to raise the reservation wage (e.g., generous unemployment benefits), as sources of wage compression. Freeman (1982) shows that unionized firms appear to have less wage dispersion than non–unionized ones.

The second type of explanations identify endogenous causes for wage compression. Frank (1984b) shows that if workers value status, then those who put a highest value on prestige will be willing to work for a wage that is lower than their marginal product in return for having lower-level workers around who in return are paid more than their marginal product. In contrast, Lazear (1989) and Milgrom and Roberts (1990) argue that wage inequalities may give rise to rent-seeking behavior within firms when workers change their behavior with the aim of ensuring wage increases. Wage compression reduces uncooperative behavior and may be efficient. Akerlof and Yellen (1990) posit that large wage differentials between groups may be perceive as unfair and lead to reduced effort.

In this paper I find that wage compression can arise due to workers' biased beliefs. I also find that workers' biases compress education levels, that is, the gradient education-ability is less steep with biased workers than with rational workers. Corollary 1 summarizes the results.

Corollary 1:
(i) If workers are sufficiently different in terms of productivity and cost of education–inequality (5) is violated when $\alpha = \beta = 0-$, then the equilib-

rium wage (education) spread with biased workers is smaller than the equilibrium wage (education) spread with rational workers: $\triangle w^*(\alpha,\beta) < \triangle w^*$ *(*$\triangle e^*(\alpha,\beta) \leq \triangle e^*$*);*

(ii) If workers are sufficiently similar in terms of productivity and cost of education and the fraction of biased workers is not too high–inequality (5) is satisfied when $\alpha > 0$ *and* $\beta > 0$*–, then the wage (education) spread in the lowest and in the highest separating equilibrium with biased workers is smaller than the wage (education) spread in the lowest and highest separating equilibrium with rational workers:* $\triangle \hat{w}^B < \triangle \hat{w}^R$ *and* $\triangle \bar{w}^B < \triangle \bar{w}^R$ *(*$\triangle \hat{e}^B < \triangle \hat{e}^R$ *and* $\triangle \bar{e}^B < \triangle \bar{e}^R$*).*

Corollary 1 follows from Propositions 1, 2 and 3. It shows that wage compression arises quite naturally in a setting where workers have biased beliefs, firms do not know workers' skills and beliefs, and workers use education as a signal of their productive ability. The intuition behind this result is as follows.

When workers are sufficiently different in terms of productivity and cost of education there is a unique separating equilibrium. The equilibrium education and wage spreads, in the separating equilibrium with biased workers, are $\triangle e^*(\alpha,\beta) = e^*(\sigma,\beta) - e^*(\sigma,\alpha)$ and $\triangle w^*(\alpha,\beta) = w^*(\sigma,\beta) - w^*(\sigma,\alpha)$, respectively. The same spreads in the separating equilibrium with rational workers are $\triangle e^* = e^*(\theta_H) - e^*(\theta_L)$ and $\triangle w^* = w^*(\theta_H) - w^*(\theta_L)$, respectively.

The wage paid to a worker in the low-education group is equal to the average product of that group. The presence of underconfidence in the low-education group implies that the average product of a worker in that group is greater than the product of a low-ability worker at any given education level. Since the marginal cost of education is the same, underconfident and unbiased low-ability workers will choose a higher education level and will be paid a higher wage than low-ability workers, that is, $e^*(\sigma,\alpha) \geq e^*(\theta_L)$ and $w^*(\sigma,\alpha) > w^*(\theta_L)$.

The wage offered to a worker in the high-education group is equal to the average product of that group. The presence of overconfidence in the high-education group implies that the average product of a worker in that group is lower than the product of a high-ability worker at any given education level. Since the marginal cost of education is the same, overconfident and unbiased high-ability workers will choose a lower education level and will be paid a lower wage than high-ability workers, that is, $e^*(\sigma,\beta) \leq e^*(\theta_H)$ and $w^*(\sigma,\beta) < w^*(\theta_H)$.

When workers are sufficiently similar in terms of productivity and cost of education and the fraction of biased workers is not too high, there is a continuum of separating equilibria. The equilibrium education and wage spreads, in the lowest separating equilibrium with biased workers, are $\triangle\hat{e}^B = \hat{e}^{HO} - e^*(\sigma, \alpha)$ and $\triangle\hat{w}^B = w(\hat{e}^{HO}) - w^*(\sigma, \alpha)$, respectively. The same spreads in the lowest separating equilibrium with rational workers are $\triangle\hat{e}^R = \hat{e}^H - e^*(\theta_L)$ and $\triangle\hat{w}^R = w(\hat{e}^H) - w^*(\theta_L)$, respectively.

In the lowest separating equilibrium, underconfident and unbiased low-ability workers are indifferent between getting their education-wage contract and the education-wage contract of overconfident and unbiased high-ability workers. We know from Proposition 1 part (iii) that the utility of unbiased-low ability workers is higher than the utility of low-ability workers. Thus, the only way to make sure that the incentive compatibility condition of underconfident and unbiased low-ability workers is binding is for overconfident and unbiased low-ability workers to choose a smaller education level and be paid a lower wage than high-ability workers, that is, $\hat{e}^{HO} < \hat{e}^H$ and $w(\hat{e}^{HO}) < w(\hat{e}^H)$.[8]

Corollary 1 shows that biased beliefs compress wages in the sense that the wage spread with biased workers is smaller than the wage spread with rational workers. This is a weak from of wage compression. Can biased beliefs explain a stronger form of wage compression? To show that they can, I specialize the model by assuming $y(e, \theta) = \theta$ and $c(e, \theta) = e^\gamma/\theta$, with $\gamma > 2$.[9].

Proposition 4: *In the specialized model, underconfident and unbiased low-ability workers receive wage $\theta_L + \alpha\rho$ and get zero education and overconfident and unbiased high-ability workers receive wage $\theta_H - \beta\rho$ and get education e^{HO}, where $[\rho\theta_L[1 - (\alpha + \beta)]]^{\frac{1}{\gamma}} \leq e^{HO} \leq [\rho\theta_H[1 - (\alpha + \beta)]]^{\frac{1}{\gamma}}$, and the wage spread is $\triangle w^B = \rho\,[1 - (\alpha + \beta)] < \rho = \triangle w^R = \triangle w^*$, with $\rho = \theta_H - \theta_L$.*

This result shows that if education has no impact on productivity, then workers' biased beliefs imply wage compression relative to productive abilities, that is, the wage spread is less than the productive ability spread.

[8]In the highest separating equilibrium the intuition is similar with the difference that it is the incentive compatibility constraint of overconfident and unbiased high-ability workers that binds.

[9]Under this specification education has no impact on productivity. A first order prediction of Spence's (1973) labor market signaling model is that a signal (such as education) commands a positive price in equilibrium, even when acquiring that signal has no impact on productivity.

5 Welfare Impact of Biased Beliefs

In this section I characterize the impact that workers' biased beliefs have on welfare. To do that I compare welfare levels with biased and rational workers. In both cases firms make zero profits so welfare is equal to the weighted average of the utilities of each group of workers.

To evaluate the utility of a biased worker I take the perspective of an outside observer who knows the worker's actual marginal cost of acquiring education.[10] Hence, welfare with biased workers is

$$\begin{aligned} W^B &= (\lambda - \nu)u(w(e^{HO}), e^{HO}, \theta_H) + \nu u(w(e^{LU}), e^{LU}, \theta_H) \\ &\quad + \kappa u(w(e^{HO}), e^{HO}, \theta_L) + (1 - \lambda - \kappa)u(w(e^{LU}), e^{LU}, \theta_L). \end{aligned} \tag{6}$$

My first welfare result shows that workers' biased beliefs reduce welfare when workers are sufficiently different in terms of productivity and cost of acquiring education.

Proposition 5: *If workers are sufficiently different in terms of productivity and cost of acquiring education–inequality (5) is violated when $\alpha = \beta = 0$–, then biased beliefs reduce welfare.*

The intuition behind this result is straightforward. Spence's (1974) model shows that if workers are unbiased and sufficiently different in terms of productivity and cost of education, then there is a unique separating equilibrium which is fully efficient. Thus, introducing a distortion in such a setting–workers' biased beliefs–reduces welfare.

Let us now consider the case where workers are sufficiently similar in terms of productivity and cost of education, that is, inequality (5) is satisfied when $\alpha = \beta = 0$. One of the main results of Spence's (1974) model is that in this case private information about ability reduces welfare. This happens because high-ability workers must overinvest in education (by comparison with the complete information education level) to distinguish themselves from low-ability workers.[11]

[10] This is the "hardest" test. An alternative would be to measure the utility of biased workers according to their perceived utility function.

[11] If inequality (5) is violated when $\alpha = \beta = 0$, that is, $w^*(\theta_L) - c(e^*(\theta_L), \theta_L) > w^*(\theta_H) - c(e^*(\theta_H, \theta_L)$, there is a continuum of separating equilibria where high-ability workers overinvest in education to distinguish themselves from low-ability workers. In this case $e^H \in [\hat{e}^H, \bar{e}^H]$ with $\hat{e}^H > e^*(\theta_H)$. These various separating equilibria can be Pareto ranked. In all of them a high-ability worker's utility is $y(e^H, \theta_H) - c(e^H, \theta_H)$, a low-ability

My second welfare shows that if workers are sufficiently similar in terms of productivity and cost of education and all biased workers are overconfident, then welfare is lower with biased workers than with rational ones.

Proposition 6: *If no worker is underconfident, workers are sufficiently similar in terms of productivity and cost of acquiring education, the fraction of overconfident workers is not too high–inequality (5) is satisfied when $\alpha = 0$ and $\beta > 0$–, then welfare in the lowest separating equilibrium with biased workers is smaller than welfare in the lowest separating equilibrium with rational workers.*

This result shows that overconfidence reduces welfare when workers are sufficiently similar in terms of productivity and cost of education and no worker is underconfident. This happens because the existence of overconfident workers and the absence of underconfident ones imply that the utility of unbiased high-ability workers is smaller than the utility of high-ability workers and the utility of unbiased low-ability workers is equal to that of low-ability workers.

Can biased beliefs improve welfare when workers are sufficiently similar in terms of productivity and cost of acquiring education? We know that the existence of underconfident workers implies that unbiased low-ability workers do better than low-ability workers. We also know that the education level of an unbiased high-ability worker is smaller than the education level of a high-ability worker. Thus, unbiased high-ability workers might do better than high-ability workers since they do not need to overinvest in education as much as high-ability workers do.

Proposition 7: *If workers are sufficiently similar in terms of productivity and cost of acquiring education, the fraction of biased workers is not too high–inequality (5) is satisfied when $\alpha > 0$ and $\beta > 0$–, and*

$$\hat{e}^H - \hat{e}^{HO} > \frac{\beta \left[y(\hat{e}^H, \theta_H) - y(\hat{e}^H, \theta_L) \right]}{-u_e(w(\hat{e}^H), \hat{e}^H, \theta_H) + \beta \left[y_e(\hat{e}^H, \theta_H) - y_e(\hat{e}^H, \theta_L) \right]}, \tag{7}$$

then the utility of an unbiased high-ability worker in the lowest separating equilibrium with biased workers is higher than the utility of a high-ability worker in the lowest separating equilibrium with rational workers.

worker's utility is $u^*(\theta_L)$, and firms earn zero profits. However, a high-ability worker does strictly better in equilibria where she gets a lower level of education (and a lower wage) since this brings her utility closer to the complete information utility $u^*(\theta_H)$. Thus, the separating equilibrium in which the high-ability worker gets education level $\hat{e}^H$ Pareto dominates all others.

The existence of biased workers has two effects on the utility of unbiased high-ability workers. On the one hand, it reduces the wage for a fixed education level since now unbiased high-ability workers are pooled with low-ability overconfident workers. On the other hand, it reduces the education level for a fixed wage. This moves the education level of unbiased high-ability workers close to the first-best (i.e., it reduces overinvestment in education). If the fraction of overconfident workers is not too high and the fraction of underconfident workers is not too low–inequality (7) is satisfied–, the reduction in the wage is less than the reduction in the cost of education. Therefore, the utility of unbiased high-ability workers is higher than the utility of high-ability workers.

To fully characterize the impact of biased beliefs on welfare when workers are sufficiently similar in terms of productivity and cost of education we also need to take into account the utility of biased workers. Generally, we cannot determine how the ex-post utilities of underconfident and overconfident workers compare to the utilities of high- and low-ability workers, respectively.

To answer this question I return to the specialized model where education has no impact on productivity. My last result provides bounds on the fractions of overconfident and underconfident workers under which biased beliefs improve welfare in the specialized model.

Proposition 8: *If*

$$\kappa < (1-\lambda)\frac{1-\frac{\rho}{\theta_H}}{1-(1-\lambda)\frac{\rho}{\theta_H}}, \tag{8}$$

and

$$\lambda(1-\kappa)\frac{\rho}{\theta_H} \leq \nu < \lambda\left(1-\frac{\kappa}{1-\lambda}\right), \tag{9}$$

then welfare in the most efficient separating equilibrium with biased workers is higher than welfare in the most efficient separating equilibrium with rational workers, that is, $\hat{W}^R < \hat{W}^B$.

Condition (8) provides an upper bound for the fraction of overconfident workers and condition (9) provides lower and upper bounds for the fraction of underconfident workers. When these conditions are satisfied, welfare in the most efficient separating equilibrium with biased workers is higher than welfare in the most efficient separating equilibrium with rational workers.

The existence of underconfident workers leads to a first-order increase in the wage of unbiased low-ability workers but only a second-order increase in their cost of education. Thus, unbiased low-ability workers do better than low-ability workers. The existence of biased workers lowers the wage and the cost of education of unbiased high-ability workers. If the fraction of overconfident workers is sufficiently low and the fraction of underconfident workers is sufficiently high, the fall in cost of education of unbiased high-ability workers is higher than the fall in the wage. Thus, unbiased high-ability workers do better than high-ability workers.

6 Conclusion

This paper extends Spence's (1974) model of labor market signaling by assuming that a fraction of workers in the labor market do not accurately evaluate their marginal cost of acquiring education. More precisely, some workers are overconfident and some underconfident. In addition, I assume that firms know the fractions of overconfident, underconfident, and high-ability workers.

I find that worker overconfidence and/or underconfidence compress wages and education levels. Wage compression arises because firms find it to be the optimal response to the fact that, in a signaling equilibrium with biased workers, overconfidence raises the proportion of low-ability workers in the high education group whereas underconfidence raises the proportion of high-ability workers in the low education group.

I also find that workers' biased beliefs always reduce welfare when workers are sufficiently different in terms of productivity and cost of education. In contrast, biased beliefs can improve welfare when workers are sufficiently similar in terms of productivity and cost of education and the fraction of overconfident workers is relatively small.

7 References

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8 Appendix

Proof of Proposition 1:
(i) The education level of underconfident and unbiased low-ability workers, e^{LU}, is the solution to $\max_e (1-\alpha)y(e,\theta_L)+\alpha y(e,\theta_H)-c(e,\theta_L)$. Thus, e^{LU} is implicitly defined by the first-order condition: $(1-\alpha)y_e(e,\theta_L)+\alpha y_e(e,\theta_H)-c_e(e,\theta_L)=0$. The second-order condition is satisfied since $y_{ee}\leq 0$, $c_{ee}>0$, and $\alpha>0$ imply $(1-\alpha)y_{ee}(e,\theta_L)+\alpha y_{ee}(e,\theta_L)-c_{ee}(e,\theta_L)<0$. From the implicit definition of e^{LU} and $y_{e\theta}\geq 0$ we have

$$\frac{\partial e^{LU}}{\partial \alpha} = -\frac{y_e(e,\theta_H)-y_e(e,\theta_L)}{(1-\alpha)y_{ee}(e,\theta_L)+\alpha y_{ee}(e,\theta_L)-c_{ee}(e,\theta_L)} \geq 0. \tag{10}$$

It follows from (10) that $e^{LU}\geq e^*(\theta_L)=\arg\max_e y(e,\theta_L)-c(e,\theta_L)$.
(ii) The wage of underconfident and unbiased low-ability workers is $w(e^{LU})=(1-\alpha)y(e^{LU},\theta_L)+\alpha y(e^{LU},\theta_H)$. The first-best wage of low-ability workers is $w^*(\theta_L)=y(e^*(\theta_L),\theta_L)$. From (i) we know that $e^{LU}\geq e^*(\theta_L)$ which implies implies $y(e^{LU},\theta_L)\geq y(e^*(\theta_L),\theta_L)$. This together with $\alpha>0$ and the definitions of $w(e^{LU})$ and $w^*(\theta_L)$ imply $w(e^{LU})>w^*(\theta_L)$.
(iii) The utility of education level e^{LU} for an unbiased low-ability worker is $u(w(e^{LU}),e^{LU},\theta_L)=(1-\alpha)y(e^{LU},\theta_L)+\alpha y(e^{LU},\theta_L)-c(e^{LU},\theta_L)$. We have $\frac{\partial u}{\partial \alpha}=\frac{\partial u}{\partial e}\frac{de^{LU}}{d\alpha}+\frac{\partial u}{\partial \alpha}$. The first term is zero from the Envelope Theorem and the sign of the second term is positive since $y_\theta>0$. Hence, $u(w(e^{LU}),e^{LU},\theta_L)>u^*(\theta_L)=y(e^*(\theta_L),\theta_L)-c(e^*(\theta_L),\theta_L)$. *Q.E.D.*

Proof of Proposition 2:
(i) If inequality (5) is violated, the education level of overconfident and unbiased high-ability workers, e^{HO}, is the solution to $\max_e \beta y(e,\theta_L)+(1-\beta)y(e,\theta_H)-c(e,\theta_H)$. Thus, e^{HO} is implicitly defined by the first-order condition $\beta y_e(e,\theta_L)+(1-\beta)y_e(e,\theta_H)-c_e(e,\theta_H)=0$. The second-order condition is satisfied since $y_{ee}\leq 0$, $c_{ee}>0$, and $\beta>0$ imply $\beta y_{ee}(e,\theta_L)+(1-\beta)y_{ee}(e,\theta_H)-c_{ee}(e,\theta_H)<0$. From the implicit definition of e^{HO} and $y_{e\theta}\geq 0$ we have

$$\frac{\partial e^{HO}}{\partial \alpha} = -\frac{-[y_e(e,\theta_H)-y_e(e,\theta_L)]}{\beta y_{ee}(e,\theta_L)+(1-\beta)y_{ee}(e,\theta_H)-c_{ee}(e,\theta_H)} \leq 0. \tag{11}$$

It follows from (11) that $e^{HO}\leq e^*(\theta_H)=\arg\max_e y(e,\theta_L)-c(e,\theta_L)$.
(ii) The wage of overconfident and unbiased high-ability workers is $w(e^{HO})=\beta y(e^{HO},\theta_L)+(1-\beta)y(e^{HO},\theta_H)$. The first-best wage of high-ability workers is

$w^*(\theta_H) = y(e^*(\theta_H), \theta_H)$. From (i) we know that $e^{HO} \leq e^*(\theta_H)$ which implies $y(e^{HO}, \theta_H) \leq y(e^*(\theta_H), \theta_H)$. This together with $\beta > 0$ and the definitions of $w(e^{HO})$ and $w^*(\theta_H)$ imply $w(e^{HO}) < w^*(\theta_H)$.
(iii) The utility of education level e^{HO} for an unbiased high-ability worker is $u(w(e^{HO}), e^{HO}, \theta_H) = \beta y(e^{HO}, \theta_L) + (1-\beta) y(e^{HO}, \theta_H) - c(e^{HO}, \theta_H)$. We have $\frac{\partial u}{\partial \beta} = \frac{\partial u}{\partial e}\frac{de^{HO}}{d\beta} + \frac{\partial u}{\partial \beta}$. The first term is zero from the Envelope Theorem and the sign of the second term is negative since $y_\theta > 0$. Hence, $u(w(e^{HO}), e^{HO}, \theta_H) < u^*(\theta_H) = y(e^*(\theta_H), \theta_H) - c(e^*(\theta_H), \theta_H)$. *Q.E.D.*

Proof of Proposition 3:
(i) In the lowest separating equilibrium with biased workers inequality (2) is binding. Denote the lowest e^{HO} that satisfies inequality (2) by $\hat{e}^{HO}$. In a separating equilibrium with rational workers (i.e., $\alpha = \beta = 0$), inequality (2) becomes

$$y(e^*(\theta_L), \theta_L) - c(e^*(\theta_L), \theta_L) \geq y(e^H, \theta_H) - c(e^H, \theta_L). \tag{12}$$

In the lowest separating equilibrium with rational workers inequality (12) is binding. Denote the lowest e^H that satisfies inequality (12) by $\hat{e}^H$. Can it be that $\hat{e}^{HO} \geq \hat{e}^H$? No, since in that case inequality (2) would not bind because

$$\begin{aligned}
&(1-\alpha) y(e^{LU}, \theta_L) + \alpha y(e^{LU}, \theta_H) - c(e^{LU}, \theta_L) > y(e^*(\theta_L), \theta_L) - c(e^*(\theta_L), \theta_L) \\
&= y(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_H) - [c(\hat{e}^H, \theta_L) - c(\hat{e}^H, \theta_H)] \geq y(\hat{e}^{HO}, \theta_H) \\
&- c(\hat{e}^{HO}, \theta_H) - \beta \left[y(\hat{e}^{HO}, \theta_H) - y(\hat{e}^{HO}, \theta_L) \right] - [c(\hat{e}^{HO}, \theta_L) - c(\hat{e}^{HO}, \theta_H)],
\end{aligned}$$

where the first inequality follows from Proposition 1, the equality follows from the definition of $\hat{e}^H$, and the last inequality follows from: (1) $\hat{e}^{HO} \geq \hat{e}^H > e^*(\theta_H)$ implies $y(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_H) \geq y(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_H)$, (2) $\hat{e}^{HO} \geq \hat{e}^H$ and $y_{e\theta}(e, \theta) \geq 0$ imply $y(\hat{e}^{HO}, \theta_H) - y(\hat{e}^{HO}, \theta_L) \geq y(\hat{e}^H, \theta_H) - y(\hat{e}^H, \theta_L)$, and (3) $\hat{e}^{HO} \geq \hat{e}^H$, $c_\theta(e, \theta) < 0$, and $c_{e\theta}(e, \theta) < 0$ imply $c(\hat{e}^{HO}, \theta_L) - c(\hat{e}^{HO}, \theta_H) \geq c(\hat{e}^H, \theta_L) - c(\hat{e}^H, \theta_H)$. Hence, it must be that $\hat{e}^{HO} < \hat{e}^H$.
In the highest separating equilibrium with biased workers inequality (3) is binding. Denote the highest e^{HO} that satisfies inequality (3) by $\bar{e}^{HO}$. In a separating equilibrium with rational workers (i.e., $\alpha = \beta = 0$) inequality (3) reduces to

$$y(e^H, \theta_H) - c(e^H, \theta_H) \geq y(e^*(\theta_L), \theta_L) - c(e^*(\theta_L), \theta_H). \tag{13}$$

In the highest separating equilibrium with rational workers inequality (13) is binding. Denote the highest e^H that satisfies inequality (13) by $\bar{e}^H$. Can it be that $\bar{e}^{HO} \geq \bar{e}^{H}$? No, since in that case inequality (3) is violated because

$$\begin{aligned} &y(e^{LU},\theta_L) - c(e^{LU},\theta_L) + \alpha\left[y(e^{LU},\theta_H) - y(e^{LU},\theta_L)\right] \\ &+\left[c(e^{LU},\theta_L) - c(e^{LU},\theta_H)\right] > y(e^*(\theta_L),\theta_L) - c(e^*(\theta_L),\theta_L) \\ &+\left[c(e^*(\theta_L),\theta_L) - c(e^*(\theta_L),\theta_H)\right] = y(\bar{e}^H,\theta_H) - c(\bar{e}^H,\theta_H) \\ &> y(\bar{e}^{HO},\theta_H) - c(\bar{e}^{HO},\theta_H) - \beta\left[y(\bar{e}^{HO},\theta_H) - y(\bar{e}^{HO},\theta_L)\right], \end{aligned}$$

where the first inequality follows from $c(e^{LU},\theta_L) - c(e^{LU},\theta_H) \geq c(e^*(\theta_L),\theta_L) - c(e^*(\theta_L),\theta_H)$ and Proposition 1, the equality follows from the definition of $\bar{e}^H$, and the last inequality follows from: (1) $\bar{e}^{HO} \geq \bar{e}^H > e^*(\theta_H)$ implies $y(\bar{e}^H,\theta_H) - c(\bar{e}^H,\theta_H) \geq y(\bar{e}^{HO},\theta_H) - c(\bar{e}^{HO},\theta_H)$, and (2) $y_\theta(e,\theta) > 0$ implies $y(\bar{e}^{HO},\theta_H) - y(\bar{e}^{HO},\theta_L) > 0$. Hence, it must be that $\bar{e}^{HO} < \bar{e}^H$.
(ii) If $\hat{e}^{HO} < \hat{e}^H$, then $w(\hat{e}^{HO}) = \beta y(\hat{e}^{HO},\theta_L) + (1-\beta)\, y(\hat{e}^{HO},\theta_H) < y(\hat{e}^H,\theta_H) = w(\hat{e}^H)$. If $\bar{e}^{HO} < \bar{e}^H$, then $w(\bar{e}^{HO}) = \beta y(\bar{e}^{HO},\theta_L) + (1-\beta) y(\bar{e}^{HO},\theta_H) < y(\bar{e}^H,\theta_H) = w(\bar{e}^H)$. *Q.E.D.*

Proof of Proposition 4: In the specialized model, the firms' strategy becomes

$$w(e) = \begin{cases} \theta_L + \alpha\rho, & \text{for } e < e^{HO} \\ \theta_H - \beta\rho, & \text{for } e \geq e^{HO} \end{cases}, \tag{14}$$

where $\rho = \theta_H - \theta_L$. Underconfident and unbiased low-ability workers choose zero education since education is not productive. So, $e^{LU} = e^*(\theta_L) = 0$. In equilibrium, underconfident and unbiased low-ability workers do not envy overconfident and unbiased high-ability workers, that is, $\theta_L + \alpha\rho \geq \theta_H - \beta\rho - (e^{HO})^\gamma/\theta_L$. The education level of overconfident and unbiased high-ability workers in the lowest separating equilibrium, $\hat{e}^B$, satisfies the incentive compatibility condition of underconfident and unbiased low-ability workers as an equality. Thus, $\hat{e}^{HO} = [\rho\theta_L[1-(\alpha+\beta)]]^{\frac{1}{\gamma}}$. In equilibrium, overconfident and unbiased high-ability workers do not envy underconfident and unbiased low-ability workers, that is, $\theta_H - \beta\rho - (e^{HO})^\gamma/\theta_H \geq \theta_L + \alpha\rho$. The education level of overconfident and unbiased high-ability workers in the highest separating equilibrium, $\bar{e}^{HO}$, satisfies the incentive compatibility condition of overconfident and unbiased high-ability workers as an equality. Thus, $\bar{e}^{HO} = [\rho\theta_H[1-(\alpha+\beta)]]^{\frac{1}{\gamma}}$. The equilibrium wage spread with biased workers is $\triangle w^B = (\theta_H - \beta\rho) - (\theta_L + \alpha\rho) = \rho\left[1-(\alpha+\beta)\right]$. The equilibrium wage spread with rational workers (i.e., $\alpha = \beta = \kappa = \nu = 0$) is $\triangle w^* = \triangle w^R = \theta_H - \theta_L = \rho$. Hence $\triangle w^B < \triangle w^R$. *Q.E.D.*

Proof of Proposition 5: If inequality (5) is satisfied when $\alpha = \beta = 0$, then there is a unique separating equilibrium with rational workers where low-ability workers get education $e^*(\theta_L)$ and attain utility $u^*(\theta_L)$ and high-ability workers get education $e^*(\theta_H)$ and attain utility $u^*(\theta_H)$. This separating equilibrium maximizes welfare. Hence, if inequality (5) is satisfied when $\alpha = \beta = 0$, welfare is higher with rational than with biased workers. *Q.E.D.*

Proof of Proposition 6: We have that

$$\begin{aligned}\hat{W}^B - \hat{W}^R = \lambda &\left[u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) - u(w(\hat{e}^H), \hat{e}^H, \theta_H)\right] \\ &+ (1-\lambda)\left[u(w(e^{LU}), e^{LU}, \theta_L) - u^*(\theta_L)\right] \\ &- \nu[u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) - u(w(e^{LU}), e^{LU}, \theta_H)]. \qquad (15)\end{aligned}$$

If $\nu = \alpha = 0$ we have $u(w(e^{LU}), e^{LU}, \theta_L) = u^*(\theta_L)$. Hence, (15) reduces to $\hat{W}^B - \hat{W}^R = \lambda\left[u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) - u(w(\hat{e}^H), \hat{e}^H, \theta_H)\right]$. In the lowest equilibrium with overconfident workers, the incentive compatibility condition of unbiased low-ability workers is binding: $u^*(\theta_L) = \beta y(\hat{e}^{HO}, \theta_L) + (1-\beta)y(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_L)$. In the most efficient equilibrium with rational workers, the incentive compatibility condition of low-ability workers is binding: $u^*(\theta_L) = y(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_L)$. Hence, we have $\beta y(\hat{e}^{HO}, \theta_L) + (1-\beta)y(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_L) = y(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_L)$. This is equivalent to

$$\begin{aligned}u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) = u&(w(\hat{e}^H), \hat{e}^H, \theta_H) \\ &+ \left[c(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_L)\right] - \left[c(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_L)\right].\end{aligned}$$

The assumption $c_{e\theta} < 0$ and $\hat{e}^H > \hat{e}^{HO}$ imply $c(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_L) < c(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_L)$. Thus, $u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) < u(w(\hat{e}^H), \hat{e}^H, \theta_H)$ which implies $\hat{W}^B < \hat{W}^R$ *Q.E.D.*

Proof of Proposition 7: The utility of an unbiased high-ability worker is $u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) = y(\hat{e}^{HO}, \theta_H) - c(\hat{e}^{HO}, \theta_H) - \beta\left[y(\hat{e}^{HO}, \theta_H) - y(\hat{e}^{HO}, \theta_L)\right]$. The utility of a low-ability worker is $u(w(\hat{e}^H), \hat{e}^H, \theta_H) = y(\hat{e}^H, \theta_H) - c(\hat{e}^H, \theta_H)$. Taking a second-order Taylor series expansion of $u(w(\hat{e}^{HO}), e^{HO}, \theta_H)$ around $\hat{e}^H$ we obtain

$$\begin{aligned}u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) &\approx u(w(\hat{e}^H), \hat{e}^H, \theta_H) - \beta\left[y(\hat{e}^H, \theta_H) - y(\hat{e}^H, \theta_L)\right] \\ &+ \left\{u_e(w(\hat{e}^H), \hat{e}^H, \theta_H) - \beta\left[y_e(\hat{e}^H, \theta_H) - y_e(\hat{e}^H, \theta_L)\right]\right\}(\hat{e}^{HO} - \hat{e}^H) \\ +\frac{1}{2}&\left\{u_{ee}(w(\hat{e}^H), \hat{e}^H, \theta_H) - \beta\left[y_{ee}(\hat{e}^H, \theta_H) - y_{ee}(\hat{e}^H, \theta_L)\right]\right\}(\hat{e}^{HO} - \hat{e}^H)^2. \quad (16)\end{aligned}$$

The second term on the right-hand side of (16) is negative since $\beta > 0$ and output increases with ability. The third term is positive since (i) $\hat{e}^H > e^*(\theta_H)$ implies $u_e(w(\hat{e}^H), \hat{e}^H, \theta_H) < 0$, (ii) the marginal productivity of education is weakly increasing with ability, and (iii) $\hat{e}^{HO} < \hat{e}^H$. The impact of the fourth term is of second-order since $(\hat{e}^{HO} - \hat{e}^H)^2$ is small. Hence, $u(w(\hat{e}^{HO}), \hat{e}^{HO}, \theta_H) > u(w(\hat{e}^H), \hat{e}^H, \theta_H)$ if (7) is satisfied. *Q.E.D.*

Proof of Proposition 8: Welfare in the most efficient separating equilibrium with biased workers is

$$\hat{W}^B = (\lambda - \nu)\left[\rho + \frac{\theta_L^2}{\theta_H} + \alpha\frac{\rho\theta_L}{\theta_H} - \beta\frac{\rho^2}{\theta_H}\right] + (1 - \lambda + \nu)(\theta_L + \alpha\rho). \tag{17}$$

Welfare in the most efficient separating equilibrium with rational workers (i.e., $\alpha = \beta = \kappa = \nu = 0$) is

$$\hat{W}^R = \lambda\left[\rho + \frac{\theta_L^2}{\theta_H}\right] + (1 - \lambda)\theta_L. \tag{18}$$

It follows from (17), (18) that welfare with biased workers is higher than welfare with rational workers if $\alpha > [\nu + (\alpha + \beta)(\lambda - \nu)]\frac{\rho}{\theta_H}$. Substituting α and β and simplifying terms we obtain

$$\nu > \left[\lambda(1 - \kappa) - \frac{(\lambda - \nu)^2}{\lambda + \kappa - \nu}\right]\frac{\rho}{\theta_H}. \tag{19}$$

Since $\lambda = \arg\min_{v\in[0,\lambda]} \frac{(\lambda-\nu)^2}{\lambda+\kappa-\nu}$ we have that $\nu > \lambda(1 - \kappa)\frac{\rho}{\theta_H}$ implies (19). A necessary condition for a separating equilibrium to exist is that the wage of underconfident and unbiased low-ability workers is less than the wage of overconfident and unbiased high-ability workers, that is, (4) must be satisfied. Solving (4) with respect to ν we obtain

$$\nu \leq \lambda\left(1 - \frac{\kappa}{1 - \lambda}\right). \tag{20}$$

For (19) and (20) to provide a lower and an upper bound for ν, respectively, it must be the case that

$$\lambda(1 - \kappa)\frac{\rho}{\theta_H} < \lambda\left(1 - \frac{\kappa}{1 - \lambda}\right).$$

Solving this inequality with respect to κ we obtain (8). Thus, (8) and (9) imply $\hat{W}^R < \hat{W}^B$. *Q.E.D.*