Entrepreneurial Optimism and the Market for New Issues

Luís Santos-Pinto and Michele Dell'Era*

Faculty of Business and Economics, University of Lausanne

Department of Economics, Bocconi University

This version: October 2, 2015

Abstract

This paper analyzes the impact of entrepreneurial optimism on the market for new issues. We find that the existence of optimists generates a new reason for entrepreneurs to own equity in their firms. We show that optimism is a natural explanation for why some new issues are underpriced and others overpriced. We also show that the impact of optimism on entrepreneurs' equity holdings depends on the number of optimists, absolute risk aversion, and cash flow variance. Optimism makes entrepreneurs worse off. In contrast, optimism can make outside investors better off when entrepreneurs signal firm value by retaining shares and underpricing.

JEL Codes: D82; G11; G14; G32.

Keywords: Optimism; Signaling; Market for New Issues; Underpricing; Overpricing.

*We are thankful to Andrea Galeotti, Amit Goyal, Mark Grinblatt, Chuan Hwang, Andreas Kohler, David Martimort, Dirk Niepelt, Pascal St-Amour, Katrin Tinn, seminar participants at University of Lausanne, Nova School of Business & Economics, University of Bern, Young Swiss Economists Meeting 2012, Swiss Society of Economics and Statistics Annual Meeting 2012, Royal Economic Society Annual Conference 2012, 27th Annual Congress of the European Economic Association 2012, 67th European Meetings of the Econometric Society 2013, World Finance Conference 2014, Research in Behavioral Finance Conference 2014, 42nd EARIE Conference 2015, and two anonymous referees for many insightful comments and suggestions.

Corresponding Author. Luís Santos-Pinto. Faculty of Business and Economics, University of Lausanne, Internef 535, CH-1015, Lausanne, Switzerland. Ph: 41-216923658. E-mail address: LuisPedro.SantosPinto@unil.ch.

1 Introduction

Leland and Pyle (1977) show that when entrepreneurs have private information about the mean return of their projects, the amount of their own funds invested in the project will be interpreted as a signal of its mean return. In equilibrium, the higher the mean return of the project, the greater the amount of equity that will be retained by the entrepreneur, and the higher will be the equity market valuation of the firm. However, signaling is costly because entrepreneurs are risk-averse and those with high expected value projects do not obtain full-insurance. Thus, signaling reduces the welfare losses caused by asymmetric information in equity markets but at a cost (second-best solution).[1]

Entrepreneurs' accurate beliefs about the value of their projects are the cornerstone of the signaling mechanism. Yet, scholarly work shows that entrepreneurs are typically overconfident about their skills and optimistic about the chances that their projects will be successful–e.g. Cooper et al. (1988), Wu and Knott (2006), and Landier and Thesmar (2009).

Optimistic individuals are more likely to become entrepreneurs according to Gentry and Hubbard (2000), Hurst and Lusardi (2004), Puri and Robinson (2007), and Cassar and Friedman (2009). There is also considerable evidence that entrepreneurs are more optimistic than other individuals. For example, Busenitz and Barney (1997) and Lowe and Ziedonis (2006) find that entrepreneurs are more optimistic than managers. Arabsheibani et al. (2000) find that self-employed are more optimistic than employees.

Entrepreneurs are also considered to be optimistic because they are not deterred by the evidence of unfavorable returns to entrepreneurship. Dunne et al. (1988) show that most businesses fail within a few years. Hamilton (2000) finds that after 10 years in business, median entrepreneurial earnings are 35 percent less than those on a paid job of the same duration. Moskovitz and Vissing-Jorgensen (2002) find that the returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio (private equity puzzle).

In this paper we study the impact of entrepreneurial optimism on the market for new

[1] When outside investors are risk-neutral, entrepreneurs are risk-averse, and the mean return of entrepreneurs' projects is known by both sides of the market, entrepreneurs are fully insured and welfare is maximized (first-best solution).

issues. To do that we extend Grinblatt and Hwang (1989) by including optimists and show how optimism affects the pricing of new issues, retained shares, and welfare.

Leland and Pyle (1977) propose the first model of equity market signaling. In this paper the only parameter unknown to outside investors is the mean return of a project and entrepreneurs signal firm value by retaining shares. Grinblatt and Hwang (1989) study the problem of entrepreneurs trying to signal mean and variance simultaneously. Two signals are needed to communicate these two pieces of information: retained shares and the degree of underpricing.[2] In Leland and Pyle (1977) as well as in Grinblatt and Hwang (1989) entrepreneurs are risk averse and have enough wealth to finance their projects entirely. As the need for external funds is assumed away, these papers focus on the role of the equity market in providing entrepreneurs an opportunity to diversify idiosyncratic risk.

We model the behavior of an entrepreneur who owns the rights to an investment project that requires a date 0 capital outlay of k. A project i yields a random cash flow of $\tilde{x}_i$ in date 1 and an independent random cash flow of $\mu_i + \tilde{y}_i$ in date 2. There exist two types of projects, i.e., $i = 1, 2$. The low expected value project has mean μ_1 and cash flow variance σ_1^2 and the high expected value project has mean μ_2 and cash flow variance σ_2^2, with $0 < k < \mu_1 < \mu_2 < \infty$ and $0 < \sigma_i^2 < \infty$, for $i = 1, 2$. There exist three types of entrepreneurs. A realist with a low expected value project knows his project has mean μ_1 and cash flow variance σ_1^2. A realist with a high expected value project knows his project has mean μ_2 and cash flow variance σ_2^2. An optimist believes to have a project with mean μ_2 and cash flow variance σ_2^2, when, in fact, he has a project with mean μ_1 and cash flow variance σ_1^2.

Entrepreneurs are risk averse and, to achieve a more diversified portfolio, market the projects to the investing public. There are two signals that can be employed, each observed by market participants in date 0. The first is the fraction of the new issue retained by the entrepreneur, denoted by α. The second is the amount by which the new issue is underpriced, denoted by D. The mean and the variance of a project's cash flows are unknown to outside investors in date 0 but they are revealed in date 1 with probability $r \in (0, 1]$. Outside investors are risk neutral, know about the existence of optimists but do

[2] Welch (1989), Allen and Faulhaber (1989), and Chemmanur (1993) are other prominent signaling models which can explain underpricing.

not know whether a particular entrepreneur is optimist or not. Outside investors observe retained shares α, underpricing per share D, and the offering price of the new issue P, and use this information to decide whether to buy equity or not.

Section 3 describes the impact of optimism on the market for new issues when outside investors are able to directly observe entrepreneurs' beliefs. We show that the existence of optimists generates a new reason for entrepreneurs to own equity in their firms. The intuition behind this result is as follows. When outside investors are able to directly observe entrepreneurs' beliefs and there are no optimists, all entrepreneurs choose to hold zero equity in their own firms to avoid facing any idiosyncratic risk. Let us now assume that there exists a fraction $\theta > 0$ of optimists among entrepreneurs who believe to have a high expected value project and that outside investors know about this. If that is the case, then outside investors are only willing to pay a price of $(1-\theta)\mu_2 + \theta\mu_1 = \mu_2 - \theta\Delta\mu$, with $\Delta\mu = \mu_2 - \mu_1$, for the equity of an entrepreneur who believes (either realistically or because he is an optimist) he has a high expected value project. Faced with an equity price of $\mu_2 - \theta\Delta\mu$ such an entrepreneur prefers not to fully insure because he thinks (either realistically or because he is an optimist) that the project is underpriced by outside investors by $\theta\Delta\mu$. Thus, regardless of risk aversion, the existence of optimists implies that entrepreneurs who believe to have a high expected value project retain shares and face idiosyncratic risk.

Throughout the rest of the paper we assume outside investors cannot directly observe entrepreneurs' beliefs and therefore information is asymmetric.

Section 4 studies the impact of optimism on the market for new issues when only the mean of a project's cash flows is private information of the entrepreneur. To perform this analysis we assume the two types of projects have the same variance, i.e., $\sigma_1^2 = \sigma_2^2 = \sigma^2$, and σ^2 is known to outside investors in date 0. In addition, we assume a project's mean is unknown to outside investors in date 0 but becomes known in date 1 with certainty, i.e., $r = 1$. This special case illustrates the model's relation to Leland and Pyle (1977).

In an efficient separating equilibrium, realists with low expected value projects do not retain shares whereas realists with high expected value projects and optimists retain α shares. The optimal response of outside investors to the fact that optimism raises the proportion of low expected value projects in the group of entrepreneurs who retain shares is to lower the stock price offered to that group. Hence, the existence of optimists makes it less profitable for realists with high expected value projects to sell equity because it reduces stock prices.

Note that entrepreneurs who retain shares do not, on average, misprice the $(1-\alpha)$ shares sold to outside investors since they receive a price of $\mu_2 - \theta\Delta\mu$ for their equity. However, a realist with a high expected value project *underprices* the $(1-\alpha)$ shares sold to outside investors by $\theta\Delta\mu$ whereas an optimist *overprices* them by $(1-\theta)\Delta\mu$. Hence, the existence of optimists is a natural explanation for why some new issues are underpriced while others are overpriced.[3]

Optimism also affects entrepreneurs' equity holdings α. When the fraction of optimists among entrepreneurs who signal is not too large, the more optimists there are, the *less* shares are retained. In contrast, when the fraction of optimists among entrepreneurs who signal is large enough and absolute risk aversion is either constant or increasing in wealth, the more optimists there are, the *more* shares are retained.

Next we turn to the welfare implications of optimism. Optimism leaves unchanged the utility of a realist with a low expected value project. It makes a realist with a high expected value project worse off. It either leaves unchanged or lowers the expected utility of an optimist if one takes the perspective of an outside observer who knows the actual type of a project. Lastly, optimism has no effect on the expected payoff of outside investors.

Section 5 describes the impact of optimism on the market for new issues when both the mean and the variance of the project's cash flows are private information of the entrepreneur. To keep the model close to Grinblatt and Hwang (1989) we assume cash flows are normally distributed and entrepreneurs have constant absolute risk aversion. In addition, we assume a project's type is revealed in date 1 with probability $r \in (0,1)$. This last assumption plays a critical role. First, it implies that there exists a primary (date 0) and a secondary (date 1) market for assets. Second, if r is 0 or 1 underpricing cannot be a signal.

[3] The underpricing of initial public offerings (IPOs) is a well-documented fact of empirical equity market research. While most IPOs are underpriced some are overpriced–see Krigman et al. (1999) and Leite (2004). A famous example of overpricing is the IPO of Facebook in 2012. According to Allen and Morris (2001) IPOs "(...) have received a great deal of attention in the academic literature. The reason perhaps is the extent to which underpricing and overpricing represent a violation of market efficiency. It is interesting to note that while game-theoretic techniques have provided many explanations of underpricing they have not been utilized to explain overpricing. Instead the explanations presented have relied on relaxing the assumption of rational behavior by investors."

In an efficient separating equilibrium, realists with low expected value projects retain no shares and do not underprice. Realists with high expected value projects and optimists signal by retaining shares in date 0 and the use of underpricing as an additional signal in date 0 depends on how large the variance of the high expected value project is.

When the variance of the high expected value project is not too large, realists with high expected value projects and optimists do not, on average, misprice the $(1-\alpha)$ shares sold to outside investors in date 0. However, a realist with a high expected value project underprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $\theta\Delta\mu$ and, if the project's type is not revealed, the α shares sold in date 1 by $\theta\Delta\mu$. An optimist overprices the $(1-\alpha)$ shares sold in date 0 by $(1-\theta)\Delta\mu$ and, if the project's type is not revealed, the α shares sold in date 1 by $(1-\theta)\Delta\mu$. Hence, optimism leads to underpricing and overpricing in the primary as well as in the secondary market for assets (but, on average, equity prices in dates 0 and 1 are in line with fundamentals).

When the variance of the high expected value project is large enough, realists with high expected value projects and optimists underprice, on average, the $(1-\alpha)$ shares sold to outside investors in date 0 by D. In this case, a realist with a high expected value project underprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $\theta\Delta\mu + D$ and, if the project's type is not revealed, the α shares sold in date 1 by $\theta\Delta\mu$. An optimist misprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $(1-\theta)\Delta\mu - D$ and, if the project's type is not revealed, overprices the α shares sold in date 1 by $(1-\theta)\Delta\mu$. In addition, we show that an increase in the fraction of optimists lowers retained shares but has an ambiguous effect on the average degree of underpricing per share.

We also show that optimism can make outside investors better off when realists with high expected value projects and optimists retain α shares in date 0 and, on average, underprice the $(1-\alpha)$ shares sold to outside investors in date 0. In this case optimism has two effects on outside investors' welfare. First, the greater the number of optimists, the more outside investors gain from financing realists with high expected value projects because of the increase in the volume of stocks that are underpriced. Second, the greater the number of optimists, the higher the number of projects where outside investors make losses due to overpricing. When the former effect dominates the latter the existence of optimists makes outside investors better off.

The above results were derived under the assumption that realists with high expected value projects and optimists sell the remaining α shares in the secondary market in date 1. This assumption is valid as long as the fraction of optimists among entrepreneurs who signal is not too high. When the fraction of optimists among entrepreneurs who signal is high enough, realists with high expected value projects and optimists prefer to retain the remaining α shares until the project's value is realized in date 2. We discuss this possibility at the end of Section 5.

Our paper contributes to the equity market signaling literature. In Leland and Pyle (1977) there is a unique stage in the equity raising process, entrepreneurs are only privately informed about the project's mean, and are perfectly informed about the project's type. Gale and Stiglitz (1989) extend Leland and Pyle (1977) by assuming two stages in the equity raising process, i.e., a primary and a secondary market for assets. Grinblatt and Hwang (1989) generalize Leland and Pyle (1977) by assuming that both the mean and the variance of a project's returns are unknown to outside investors in date 0 and that there is a primary and secondary market for assets. We extend Grinblatt and Hwang (1989) by including optimistic entrepreneurs.

Besides Gale and Stiglitz (1989) and Grinblatt and Hwang (1989), there have been several other extensions to the Leland and Pyle's (1977) model, e.g. by allowing for the possibility of non-linear contracts, by seeking "robust" contracts, and by focusing on particular aspects of initial public offerings processes. For example, Bajaj et al. (1998) endogeneize the scale of investment choice by entrepreneurs. In Tinn (2010) entrepreneurs can signal with investments and not with retained equity by assumption. She shows that when entrepreneurs have superior information about the value of their firms, their decision to invest in the newest technology becomes a positive signal to the market. This increases the expected market value of firms and encourages entrepreneurs to invest in such technology.

In Angeletos et al. (2010) entrepreneurs play a signaling game with financial traders. Entrepreneurs make investment decisions based on their expectations of the price at which they may sell their capital. Financial traders look at the entrepreneurs' activity as a signal of the profitability of the new investment opportunity. This interaction creates a speculative incentive for the entrepreneur to invest more than what warranted from his expectation of the fundamentals. As all entrepreneurs do the same, this will trigger asset prices to inflate since financial traders perceive this exuberance in part as a signal of good fundamentals. The anticipation of inflated prices can feed back to further exuberance in real economic activity, and so on.

Our paper also contributes to the behavioral corporate finance literature that assumes entrepreneurs or managers suffer from behavioral biases and explores the implications of these biases for decisions and market outcomes. For example, entrepreneurs or managers are often assumed to be too optimistic when assessing the productivity of their investment, the value of assets in place, or the prospects attached to mergers and acquisitions.

In DeMeza and Southey (1996) risk neutral entrepreneurs must choose the right mix of self-finance and debt-finance from risk neutral banks to develop their projects. Banks and realists know a project's true probability of success but optimists overestimate it. When all entrepreneurs are realists information is symmetric and the market is efficient. Hence, entrepreneurial optimism is a distortion in an environment otherwise free of distortions and so it lowers welfare.

In Manove and Padilla (1999) risk neutral banks use collateral requirements and interest rates to screen risk neutral entrepreneurs with good projects from those with bad ones. Optimistic entrepreneurs are willing to fully collateralize their loans and so collateral cannot be used to separate them from the realists. Collateral serves to protect the banks against the errors of optimistic entrepreneurs, but competition between banks reduces interest rates, which further encourages optimists. As a consequence banks lend too much, and thus entrepreneurial optimism reduces welfare.

Roll (1986) argues that optimism can lead to value destroying mergers and acquisitions. Malmendier and Tate (2008) show empirically that overconfident CEOs overestimate their ability to generate returns and, as a result, overpay for target companies and undertake value-destroying mergers. Heaton (2002) shows that optimistic managers overinvest when they have abundant internal funds whereas they cut investment when they need external financing since they view it as extremely costly. Malmendier and Tate (2005 and 2011) find empirical support for Heaton's (2002) predictions. Gervais et al. (2011) study capital budgeting and executive compensation in a setting with a risk averse manager with private information and a risk neutral shareholder. They find that some degree of manager's overconfidence creates value for the manager and the firm since it commits the manager to follow an optimal investment policy and exert effort.[4]

Finally, our paper contributes to the corporate finance literature that tries to explain portfolio underdiversification (see Polkovnichenko, 2005). Section 3 shows that when information is symmetric, realists with high expected value projects and optimists prefer to retain some shares in their projects and face idiosyncratic risk because they consider that their projects are underpriced by outside investors. Sections 4 and 5 show that when information is asymmetric, realists with high expected value and optimists do not hold a fully diversified portfolio, to signal their beliefs to outside investors. In all cases the model predicts that optimists will make losses from these disproportionately large holdings whereas realists with high expected value projects will make gains. This potentially testable implication could be compared to those of alternative explanations for underdiversification. For example, in Van Niewerburgh and Veldkamp (2010) investors gain from underdiversification because they optimally specialize and hold more shares on firms they are better informed about.

The rest of the paper is organized as follows. Section 2 outlines the model. Sections 3, 4 and 5 report the findings. Section 6 concludes the paper. The proofs are in the Appendix.

2 Set-Up

This section extends Grinblatt and Hwang (1989) by including optimists.

Consider a three date world in which each entrepreneur has a risky project i that requires a date 0 fixed investment of k. Project i yields a random cash flow of $\tilde{x}_i$ in date 1 and an independent random cash flow of $\mu_i + \tilde{y}_i$ in date 2; $\tilde{x}_i$ and $\tilde{y}_i$ have mean of zero and variance σ_i^2. We assume there exist only two types of projects, i.e., $i = 1, 2$. The low expected value project has mean μ_1 and cash flow variance σ_1^2 and the high expected value project has mean μ_2 and cash flow variance σ_2^2, with $0 < k < \mu_1 < \mu_2 < \infty$ and $0 < \sigma_i^2 < \infty$, for $i = 1, 2$.[5]

Entrepreneurs derive utility $u(w)$ from final wealth w and are risk averse, i.e., u is

[4] See also Shleifer and Vishny (2003), Landier and Thesmar (2009), and Malmendier and Tate (2005, 2011).

[5] In Grinblatt and Hwang (1989) there is a continnum of project types and the lower bound on μ, if it exists, is assumed to be less than or equal to k.

strictly increasing and concave.[6] At date 0 the entrepreneur decides whether to undertake the project. The entrepreneur initially pays the capital cost k out of personal wealth w_0 and then seeks equity financing to reduce the exposure to risk.

There are three types of entrepreneurs. A realist with a low expected value project knows his project has mean μ_1 and cash flow variance σ_1^2. A realist with a high expected value project knows his project has mean μ_2 and cash flow variance σ_2^2. An optimist believes to have a project with mean μ_2 and cash flow variance σ_2^2, when, in fact, he has a project with mean μ_1 and cash flow variance σ_1^2. Hence, while realists always know the true type of their project, optimists are always wrong (and unaware that they are wrong). Note that when $\sigma_1^2 > \sigma_2^2$ an optimist is overconfident in the sense that he underestimates the variance of the project's cash flows.[7] In contrast, when $\sigma_1^2 < \sigma_2^2$ an optimist is underconfident in the sense that he overestimates the variance of the project's cash flows.[8]

There are n projects in the economy, each associated to one entrepreneur, where n is a large number. There is a fraction $\pi = \Pr(\mu = \mu_2) \in (0, 1)$ of high expected value projects and a fraction $1 - \pi$ of low expected value projects. The high expected value projects are only held by realists. Hence, fraction π of entrepreneurs are realists with high expected value projects. The low expected value projects are held by optimists and realists. Fraction $\kappa < 1-\pi$ of entrepreneurs are optimists and fraction $1-\pi-\kappa$ are realists with low expected value projects. Note that the shares of realists and optimists are specified such that the unconditional expectation of the type of a typical project is always the same.

Entrepreneurs can employ two signals, each observed by outside investors in date 0. The fraction of the project retained by the entrepreneur, denoted by α, and the degree of underpricing of the new issue, denoted by D. In date 0 an entrepreneur decides the

[6] Grinblatt and Hwang (1989) assume mean-variance utility, i.e., $u(w) = E(w) - \rho V(w)/2$. This is equivalent to assuming normally distributed cash flows and exponential utility.

[7] The behavioral corporate finance literature distinguishes between optimism and overconfidence. Optimism is usually defined as an overestimation of the probability of good outcomes and an underestimation of the probability of bad outcomes, while overconfidence relates to underestimation of the risk or variance of future events. See, for example, DeLong et al. (1991), Goel and Thakor (2000), and Heaton (2002).

[8] To justify the behavior of an optimist when $\sigma_1^2 < \sigma_2^2$ one can assume that the date 2 expected utility of the high expected value project is greater than that of the low expected value project, i.e., $E[u(w_0 - k + \mu_2 + \tilde{x}_2 + \tilde{y}_2)] > E[u(w_0 - k + \mu_1 + \tilde{x}_1 + \tilde{y}_1)]$. For example, when a project's returns are normally distributed, entrepreneurs have constant absolute risk aversion of ρ, and $\sigma_1^2 < \sigma_2^2$, the assumption is equivalent to $\rho < \Delta\mu/(\sigma_2^2 - \sigma_1^2)$, i.e., there is an upper bound for ρ. Note that when $\sigma_1^2 \geq \sigma_2^2$, the assumption is satisfied for all ρ.

fraction of the project he wants to retain α and the degree of underpricing D. In date 1, after the realization of the date 1 cash flow becomes public information, an entrepreneur who retained fraction α of his project decides whether to sell or not sell the retained fraction α. The discount factor between periods is set to 1 for simplicity.[9]

Outside investors are risk neutral and cannot directly observe entrepreneurs' beliefs. They know that there is a fraction π of high expected value projects, only held by realists, and a fraction $1-\pi$ of low expected value projects held by optimists (fraction κ) and by realists (fraction $1-\pi-\kappa$). They also know the distribution of returns of the two types of projects. In the absence of signaling, outside investors do not know a project's type in date 0. They learn a project's type between dates 0 and 1 with probability $r \in (0,1]$. Otherwise, it remains unknown until date 2. In date 0, outside investors observe the fraction of the project retained by the entrepreneur α, underpricing per share D, and the offering price of the new issue P, and use this information to decide whether to buy equity or not.

After observing α, D, and P outside investors expect the value of the firm to be $\mu(\alpha, D, P)$ and value their portion of the project at $(1-\alpha)\mu(\alpha, D, P)$. Their actual cash payment to the entrepreneur in date 0 is $(1-\alpha)P$ where $P = \mu(\alpha, D, P) - D$. Perfect competition in the equity market implies that the optimal strategy of outside investors in the primary market is to buy equity if and only if $\mu(\alpha, D, P) - P \geq 0$. In the secondary market the stock price of a project is set equal to $\mu(\alpha, D, P)$ if the project's type is not perfectly revealed, and equal to its intrinsic expected value if it is.

In an efficient separating equilibrium, realists with low expected value projects retain no shares and do not underprice (thus bearing zero signaling cost). Realists with high expected value projects and optimists signal by retaining shares in date 0 and the use of underpricing as an additional signal in date 0 depends on large the variance of the high expected value project is. Among all entrepreneurs who signal, outside investors know that

[9] In Grinblatt and Hwang (1989) an entrepreneur can choose any budget feasible combination of three investments: a risk-free asset, the market portfolio, and equity shares in his own firm. The cash flows of the project are assumed to be uncorrelated with the returns of the market portfolio in periods 1 and 2. This assumption implies that the choice of the fraction of the market portfolio held by the entrepreneur is independent of the choice of fractional shareholdings. We keep this assumption and therefore do not model the choice of the fraction of the market portfolio held by the entrepreneur. Risk-free borrowing has no effects on the equilibrium values of retained shares and underpricing and so we also do not model the choice of the risk-free asset.

fraction $\theta = \frac{\kappa}{\pi+\kappa}$ has a low expected value project and fraction $1-\theta = \frac{\pi}{\pi+\kappa}$ has a high expected value project. Hence, outside investors' posterior belief that a project has a high expected value after having observed α, D and P is

$$\Pr((\mu_2, \sigma_2^2)|\alpha, D, P) = \begin{cases} 1-\theta, \text{ if } \alpha \geq \hat{\alpha},\ D \geq \hat{D}, \text{ and } P \geq \hat{P} \\ 0, \text{ otherwise} \end{cases}, \tag{1}$$

where $\hat{\alpha}$, $\hat{D}$, and $\hat{P}$ denote the least cost separating retained shares, underpricing per share, and offering price of stocks in date 0, respectively. It follows from (1) that, after having observed α, D and P, outside investors expect the value of the firm to be

$$\mu(\alpha, D, P) = \begin{cases} \mu_2 - \theta\Delta\mu, \text{ if } \alpha \geq \hat{\alpha},\ D \geq \hat{D}, \text{ and } P \geq \hat{P} \\ \mu_1, \text{ otherwise} \end{cases}.$$

The offering price of stocks in date 0 is

$$P = \mu(\alpha, D, P) - D = \begin{cases} \mu_2 - \theta\Delta\mu - D, \text{ if } \alpha \geq \hat{\alpha},\ D \geq \hat{D}, \text{ and } P \geq \hat{P} \\ \mu_1, \text{ otherwise} \end{cases}. \tag{2}$$

Since outside investors cannot distinguish optimists from realists with high expected value projects, they will never accept to pay more than $\mu_2 - \theta\Delta\mu$ to an entrepreneur who signals so it must be that $D \geq 0$. Since the offering price of an entrepreneur who signals cannot be negative it must also be that $D \leq \mu_2 - \theta\Delta\mu$. Note that nothing prevents the offering price of an entrepreneur who signals, $\mu_2 - \theta\Delta\mu - D$, to be less than the offering price of an entrepreneur who does not signal, μ_1.

Under these assumptions, the date 1 wealth of an entrepreneur who has a project with mean μ_i, cash flow variance σ_i^2, and who sells the remaining shares α at date 1 is given by

$$\tilde{w}_1(\mu_i, \sigma_i^2) = w_0 - k + (1-\alpha)[\mu(\alpha, D, P) - D] + \alpha(\tilde{\mu}_i + \tilde{x}_i), \tag{3}$$

where $\tilde{\mu}_i$ is equal to μ_i with probability r and to $\mu(\alpha, D, P)$ with probability $1-r$. In the latter case, outside investors use $\mu(\alpha, D, P)$ to evaluate the expected value of the project's date 2 cash flows. The expected value and the variance of $\tilde{\mu}_i$ are given by

$$E(\tilde{\mu}_i) = r\mu_i + (1-r)\mu(\alpha, D, P) \tag{4}$$

and

$$V(\tilde{\mu}_i) = r(1-r)[\mu_i - \mu(\alpha, D, P)]^2. \tag{5}$$

From equations (3), (4), and (5) we obtain the expected value and the variance of date 1 wealth,

$$E[\tilde{w}_1(\mu_i, \sigma_i^2)] = w_0 - k + (1-\alpha)[\mu(\alpha, D, P) - D] + \alpha[r\mu_i + (1-r)\mu(\alpha, D, P)] \tag{6}$$

and

$$V[\tilde{w}_1(\mu_i, \sigma_i^2)] = \alpha^2 r(1-r)[\mu_i - \mu(\alpha, D, P)]^2 + \alpha^2\sigma_i^2. \tag{7}$$

The objective of an entrepreneur who perceives to have a project with mean μ_i and cash flow variance σ_i^2 is to maximize his date 1 perceived expected utility $E[u(\tilde{w}_1(\mu_i, \sigma_i^2))]$.

In an efficient separating equilibrium, realists with low expected value projects do not envy entrepreneurs who signal:

$$u(w_0 - k + \mu_1) \geq E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu - D) + \alpha(\tilde{\mu}_1 + \tilde{x}_1))]. \tag{8}$$

Furthermore, entrepreneurs who signal (realists with high expected value projects and optimists) do not envy realists with low expected value projects:

$$E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu - D) + \alpha(\tilde{\mu}_2 + \tilde{x}_2))] \geq u(w_0 - k + \mu_1). \tag{9}$$

The assumption $0 < k < \mu_1$ implies that a realist with a low expected value project prefers undertaking the project and obtaining $u(w_0 - k + \mu_1)$ to not undertaking the project and obtaining $u(w_0)$. This implies that as long as (9) is satisfied a realist with a high expected value project (and an optimist) prefers to undertake the project. Thus, both types of projects will be undertaken in an efficient separating equilibrium.

3 Outside Investors know Entrepreneurs' Beliefs

This section describes the impact of optimism on the market for new issues when outside investors are able to directly observe entrepreneurs' beliefs. To perform this analysis we assume that the expected value and variance of a project's cash flows is unknown to outside investors in date 0 but becomes known in date 1 with certainty, i.e., $r = 1$.

Let us start by assuming that all entrepreneurs have correct beliefs about their projects and outside investors are able to directly observe entrepreneurs' beliefs in date 0. Since outside investors are risk neutral they are willing to pay equity price μ_1 to entrepreneurs who believe to have a low expected value project and equity price μ_2 to those who believe to have a high expected value project. Since entrepreneurs are risk averse, those who believe to have a low expected value project sell the project to outside investors at equity price μ_1 and those who believe to have a high expected value project sell the project at equity price μ_2. In this case there is full coverage, the first-best is attained, and welfare is maximized.

Suppose now that some entrepreneurs are optimists, outside investors are able to directly observe entrepreneurs' beliefs, know about the existence of optimists but do not know whether a particular entrepreneur is optimist or not. Like before, outside investors are willing to pay equity price μ_1 to entrepreneurs who believe (correctly) to have a low expected value project. Hence, these entrepreneurs sell their projects to outside investors at equity price μ_1 and get full coverage. In contrast, outside investors are only willing to pay equity price $\mu_2 - \theta\Delta\mu$ to entrepreneurs who believe (some correctly and some incorrectly) to have a high expected value project. This happens because outside investors know that fraction $1-\theta$ of these projects has high expected value and fraction θ has low expected value. Faced with an equity price of $\mu_2 - \theta\Delta\mu$, entrepreneurs who believe to have a high expected value project prefer not to fully insure because they think (either realistically or because they are optimists) that their projects are underpriced by outside investors.

Formally, an entrepreneur who believes to have a high expected value project prefers to retain α shares in the project and face risk rather than being fully insured at equity price $\mu_2 - \theta\Delta\mu$ when

$$u(w_0 - k + \mu_2 - \theta\Delta\mu) < E[u(\tilde{w}_1(\mu_2, \sigma_2^2))],$$

where

$$\begin{aligned}\tilde{w}_1(\mu_2, \sigma_2^2) &= w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_2 + \tilde{x}_2) \\ &= w_0 - k + \mu_2 - \theta\Delta\mu + \alpha\theta\Delta\mu + \alpha\tilde{x}_2.\end{aligned}$$

Letting $\bar{w} = w_0 - k + \mu_2 - \theta\Delta\mu$, the expected utility of $\tilde{w}_1(\mu_2, \sigma_2^2)$ may be approximated as follows

$$\begin{aligned}E[u(\tilde{w}_1(\mu_2, \sigma_2^2))] &\approx u(\bar{w} + \alpha\theta\Delta\mu) + \frac{1}{2}u''(\bar{w} + \alpha\theta\Delta\mu)\alpha^2\sigma_2^2 \\ &= u(\bar{w} + \alpha\theta\Delta\mu) + O\left(\alpha^2; \sigma_2\right) \\ &\approx u(\bar{w}) + u'(\bar{w})\alpha\theta\Delta\mu + \frac{1}{2}u''(\bar{w})(\alpha\theta\Delta\mu)^2 + O\left(\alpha^2; \sigma_2\right) \\ &= u(\bar{w}) + u'(\bar{w})\alpha\theta\Delta\mu + O(\alpha^2; \sigma_2, \theta, \Delta\mu).\end{aligned}$$

Note that because the last term in the expected utility is quadratic in α, as long as $\theta > 0$, there will be some (possibly very small) $\alpha > 0$ such that the expected utility of retaining α shares in the project exceeds the utility of selling the whole project to outside investors and getting the certain amount $\bar{w}$. Thus, regardless of risk aversion, the existence of optimists implies that entrepreneurs who believe to have a high expected value project (either realistically or because they are optimists) will retain some shares with a positive risk premium.

We now turn to the impact of optimism on welfare. Welfare is the sum of the expected utilities of each group of entrepreneurs since investors break even. Welfare in the absence of optimists is given by

$$W = n(1-\pi)\, u(w_0 - k + \mu_1) + n\pi u(w_0 - k + \mu_2),$$

since entrepreneurs with low expected value projects are fully insured at price μ_1 and entrepreneurs with high expected value projects are fully insured at price μ_2. To evaluate the expected utility of an optimist, we take the perspective of an outside observer who knows the actual project's value. Therefore, welfare in the presence of optimists is given by

$$\begin{aligned} W &= n(1-\pi-\kappa)\,u(w_0-k+\mu_1) \\ &\quad + n\kappa E[u(w_0-k+(1-\alpha)(\mu_2-\theta\Delta\mu)+\alpha(\mu_1+\tilde{x}_1))] \\ &\quad + n\pi E[u(w_0-k+(1-\alpha)(\mu_2-\theta\Delta\mu)+\alpha(\mu_2+\tilde{x}_2))]. \end{aligned} \tag{10}$$

The first term in (10) represents the utility of a realist with a low expected value project. This entrepreneur is fully insured at price μ_1. The second term represents the expected utility of an optimist from the perspective of an outside observer who knows that the project has expected value μ_1 and variance σ_1^2. The third term represents the expected utility of a realist with a high expected value project.

When outside investors are able to directly observe entrepreneurs' beliefs, the existence of optimists lowers welfare since the first-best is no longer achieved. Optimism does not affect the utility of a realist with a low expected value project since he is fully covered. It makes a realist with a high expected value project worse off because he receives a lower equity price for the $(1-\alpha)$ shares that he sells to outside investors and because he is exposed to risk on the α shares that he retains. Finally, the existence of optimists has an ambiguous impact on the expected utility of an optimist. On the one hand, an optimist receives a higher equity price for the $(1-\alpha)$ shares sold to outside investors. On the other hand, an optimist is exposed to risk on the α shares that he retains.

To illustrate these results we assume project's date 1 cash flows are normally distributed, i.e., $\tilde{x}_i \sim N(0,\sigma_i^2)$, and entrepreneurs have constant absolute risk aversion, i.e., $u(w) = -\exp(-\rho w)$, where $\rho > 0$ is the coefficient of absolute risk aversion. If $u(\tilde{w}(\mu_i,\sigma_i^2)) = -\exp\{-\rho[a+\alpha(\mu_i+\tilde{x}_i)]\}$ and $\tilde{x}_i \sim N(0,\sigma_i^2)$, then $E[u(\tilde{w}(\mu_i,\sigma_i^2))] = -\exp\left\{-\rho\left(a+\alpha\mu_i-\frac{\rho}{2}\alpha^2\sigma_i^2\right)\right\}$. The optimal retained shares for a realist with a high expected value project (and for an optimist) are the solution to

$$\begin{aligned} &\max_{\alpha\in[0,1]} -\exp\left\{-\rho\left[w_0-k+(1-\alpha)(\mu_2-\theta\Delta\mu)+\alpha\mu_2-\frac{\rho}{2}\alpha^2\sigma_2^2\right]\right\} \\ &\text{s.t.}\ \ -\exp\left\{-\rho\left[w_0-k+(1-\alpha)(\mu_2-\theta\Delta\mu)+\alpha\mu_2-\frac{\rho}{2}\alpha^2\sigma_2^2\right]\right\} \geq -\exp\left\{-\rho w_0\right\}, \end{aligned} \tag{11}$$

where the constraint guarantees that a realist with high expected value project (and an optimist) prefers undertaking the project retaining α shares to not undertaking it. This problem can be simplified to

$$\begin{aligned}&\min_{\alpha\in[0,1]}\left[(1-\alpha)\theta\Delta\mu+\frac{\rho}{2}\alpha^2\sigma_2^2\right]\\&\text{s.t. } w_0-k+\mu_2-\left[(1-\alpha)\theta\Delta\mu+\frac{\rho}{2}\alpha^2\sigma_2^2\right]\geq w_0.\end{aligned}$$

The solution to this problem is given by

$$\theta\Delta\mu=\rho\alpha\sigma_2^2, \tag{12}$$

i.e., the optimal retained shares equate the marginal benefit of retaining one more share–selling one less share to outside investors at a discount $\theta\Delta\mu$–to the marginal cost of retaining one more share–the disutility $\rho\alpha\sigma_2^2$ from the increase in risk exposure. Solving (12) with respect to α we obtain

$$\alpha^*=\frac{\theta\Delta\mu}{\rho\sigma_2^2}. \tag{13}$$

Hence, as long as there exist some optimists, realists with high expected value projects (and optimists) retain some shares in the project.[10] It follows from (13) that the more optimists there are, the more shares are retained. This happens because the marginal benefit of selling one less share to outside investors at a discount $\theta\Delta\mu$ is higher when there are more optimists.

The expected utility of a realist with a high expected value project under α^* is

$$-\exp\left\{-\rho\left[w_0-k+\mu_2-\theta\Delta\mu+\frac{1}{2}\theta^2\frac{(\Delta\mu)^2}{\rho\sigma_2^2}\right]\right\}. \tag{14}$$

We see from (14) that the more optimists there are, the lower is the expected utility of a realist with a high expected value project.[11] From the perspective of an outside observer,

[10] Note that α^* is less than 1 as long as $\theta<\rho\sigma_2^2/\Delta\mu$, i.e., as long as the fraction of optimists is sufficiently low. When $\theta\geq\rho\sigma_2^2/\Delta\mu$, a realist with a high expected value project (and an optimist) prefers undertaking the project without selling equity, i.e., $\alpha^*=1$.

[11] It follows from (14) that $\alpha=0$ does not solve (11). When $\alpha=0$ the utility of a realist with a high expected value project (and the perceived utility of an optimist) is $-\exp\{-\rho\left[w_0-k+\mu_2-\theta\Delta\mu\right]\}$ which is less than (14). This, in turn, implies that a realist with high expected value project (and an optimist) undertakes the project since $w_0-k+\mu_2-\theta\Delta\mu>w_0-k+\mu_1>w_0$.

the expected utility of an optimist is equal to

$$-\exp\left\{-\rho\left\{w_0-k+\mu_1+\Delta\mu\left[1-\left(1+\frac{\Delta\mu}{\rho\sigma_2^2}\right)\theta+\frac{\Delta\mu}{\rho\sigma_2^2}\left(1-\frac{\sigma_1^2}{2\sigma_2^2}\right)\theta^2\right]\right\}\right\}. \tag{15}$$

An optimist is better off (worse off) than a realist with a low expected value project when the term inside square brackets in (15) is positive (negative). The term inside square brackets is positive (negative) when θ is lower (higher) than:

$$\hat{\theta}=\frac{1+\frac{\Delta\mu}{\rho\sigma_2^2}-\sqrt{\left(1+\frac{\Delta\mu}{\rho\sigma_2^2}\right)^2-4\frac{\Delta\mu}{\rho\sigma_2^2}\left(1-\frac{\sigma_1^2}{2\sigma_2^2}\right)}}{2\frac{\Delta\mu}{\rho\sigma_2^2}\left(1-\frac{\sigma_1^2}{2\sigma_2^2}\right)}.$$

An optimist is better off when there are few optimists because he receives a high equity price for the $(1-\alpha^*)$ shares sold to outside investors–when there are few optimists $\mu_2-\theta\Delta\mu$ is close to μ_2–and because he is not very exposed to risk–when there are few optimists α^* is low. An optimist is worse off when there are many optimists because he receives a low equity price for the $(1-\alpha^*)$ shares sold to outside investors–when there are many optimists $\mu_2-\theta\Delta\mu$ is close to μ_1–and because he is very exposed to risk–when there are many optimists α^* is high.

4 Unknown Mean and Known Variance

This section studies the impact of optimism on the market for new issues when only the mean of the project's cash flows is private information of the entrepreneur.

To perform this analysis we assume that the two types of projects have the same variance, i.e., $\sigma_1^2=\sigma_2^2=\sigma^2$, and outside investors know σ^2 in date 0. In addition, we assume that the expected value of the project's cash flows is unknown to outside investors in date 0 but becomes known in date 1 with certainty, i.e., $r=1$.[12] This special case illustrates the model's relation to Leland and Pyle (1977).

In date 0 an entrepreneur decides the fraction of the project he wants to retain α.

[12] We assume $\sigma_1^2=\sigma_2^2$, otherwise outside investors would use their knowledge of the project's variance to find out the project's mean in date 0 and therefore there would be no asymmetric information. In Section 5 we consider the general model where $r\in(0,1)$ and $\sigma_1^2\neq\sigma_2^2$.

Outside investors cannot directly observe entrepreneurs' beliefs, know about the existence of optimists but do not know whether a particular entrepreneur is optimist or not. Outside investors observe retained shares α and the offering price of the new issue P and use this information to decide whether to buy equity or not. After observing α and P outside investors expect the value of the firm to be $\mu(\alpha, P)$.

In an efficient separating equilibrium, realists with low expected value projects retain no shares and realists with high expected value projects and optimists signal by retaining α shares in date 0. Hence, outside investors' posterior belief that a project has a high expected value after having observed α and P is

$$\Pr((\mu_2, \sigma^2)|\alpha, P) = \begin{cases} 1-\theta, \text{ if } \alpha \geq \hat{\alpha}, \text{ and } P \geq \hat{P} \\ 0, \text{ otherwise} \end{cases} .$$

The offering price of stocks in date 0 is given by

$$P = \mu(\alpha, P) = \begin{cases} \mu_2 - \theta\Delta\mu, \text{ if } \alpha \geq \hat{\alpha}, \text{ and } P \geq \hat{P} \\ \mu_1, \text{ otherwise} \end{cases} . \tag{16}$$

The optimal response of outside investors to the existence of low expected value projects among the group of projects in which entrepreneurs retain shares is to lower the equity price offered to that group. As a consequence, realists with high expected value projects underprice the shares sold to outside investors by $\theta\Delta\mu$ and optimists overprice them by $(\mu_2 - \theta\Delta\mu) - \mu_1 = (1-\theta)\Delta\mu$.

In any efficient separating equilibrium, a realist with a low expected value project does not envy an entrepreneur who retains shares:

$$u(w_0 - k + \mu_1) \geq E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_1 + \tilde{x}))]. \tag{17}$$

The left side of (17) is the utility of a realist with a low expected value project who sells his entire project at price μ_1. The right side of (17) represents the expected utility of a realist with a low expected value project who sells fraction $1-\alpha$ of his project at price $\mu_2 - \theta\Delta\mu$ but retains the risk on the remaining fraction α.

Furthermore, an entrepreneur who retains shares does not envy a realist with a low expected value project:

$$E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_2 + \tilde{x}))] \geq u(w_0 - k + \mu_1). \tag{18}$$

The left side of (18) represents the expected utility of a realist with a high expected value project (and the perceived expected utility of an optimist) who sells fraction $1-\alpha$ of his project at price $\mu_2 - \theta\Delta\mu$ but retains the risk on the remaining fraction α. The right side of (18) is the utility of a realist with a high expected value project (and the utility of an optimist) who sells his entire project at price μ_1.

There exists a continuum of separating equilibria parametrized by retained shares α fulfilling (17) and (18).[13] We focus on the least cost separating equilibrium–the one with the lowest level of retained shares–since this is the only one that survives Cho and Krep's (1987) intuitive criterion.[14]

Our first result characterizes the impact of optimism on retained shares under least-cost separation.

Proposition 1: *Assume project i's random cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are independent with mean 0 and variance σ^2, where $0 < \sigma^2 < \infty$, $i = 1, 2$, σ^2 is known to outside investors in date 0, project i's mean is unknown to outside investors in date 0 but is fully revealed in date 1, and entrepreneurs have concave utility.*
(i) If the fraction of optimists among entrepreneurs who signal is not too large, i.e., $\theta < \bar{\theta}$, then condition (17) is binding, condition (18) is slack, $\hat{\alpha}$ satisfies

$$u(w_0 - k + \mu_1) = E[u(w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_1 + \tilde{x}))],$$

and an increase in the fraction of optimists lowers retained shares, i.e., $\partial\hat{\alpha}/\partial\kappa < 0$.
(ii) If the fraction of optimists among entrepreneurs who signal is large enough, i.e., $\theta \geq \bar{\theta}$, and entrepreneurs' absolute risk aversion is either constant or increasing in wealth, then

[13] There exists also a pooling equilibrium where no entrepreneur retains shares and outside investors pay an equity price of $\mu_1 + \pi\Delta\mu$.

[14] The least cost separating equilibrium, by construction, cannot fail the intuitive criterion. Any α that would induce defection of a realist with a high expected value project (or an optimist) alone must impose a lower signaling cost on him. However, the least cost separating equilibrium minimizes signaling cost over all signal levels α that would not induce defection by a realist with a low expected value project.

conditions (17) *and* (18) *are slack,* $\hat{\alpha}$ *satisfies*

$$E\left[u'(w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_2 + \tilde{x}))(\theta\Delta\mu + \tilde{x})\right] = 0,$$

and an increase in the fraction of optimists raises retained shares, i.e., $\partial\hat{\alpha}/\partial\kappa > 0$. *The threshold* $\bar{\theta}$ *satisfies*

$$u(w_0 - k + \mu_1) = E[u(w_0 - k + (1-\bar{\alpha})(\mu_2 - \bar{\theta}\Delta\mu) + \bar{\alpha}(\mu_1 + \tilde{x}))],$$

where $\bar{\alpha}$ *satisfies*

$$E\left[u'(w_0 - k + (1-\bar{\alpha})(\mu_2 - \bar{\theta}\Delta\mu) + \bar{\alpha}(\mu_2 + \tilde{x}))(\bar{\theta}\Delta\mu + \tilde{x})\right] = 0.$$

Proposition 1(i) shows that when the fraction of optimists among entrepreneurs who signal is not too large, the more optimists there are, the lower are retained shares. The intuition behind this result is straightforward. When the fraction of optimists among entrepreneurs who signal is not too large, a realist with a low expected value project is indifferent between full insurance and the partial cover contract intended for entrepreneurs who signal. The more optimists there are, the lower is the stock price of projects where entrepreneurs hold equity and the less attractive signaling becomes to realists with low expected value projects. As a consequence, the more optimists there are, the less is the share of equity holdings needed by an entrepreneur who signals to separate himself from realists with low expected value projects in an incentive compatible manner.

Proposition 1(ii) tell us that if the fraction of optimists among entrepreneurs who signal is large enough and absolute risk aversion is either constant or increasing in wealth, then the more optimists there are, the higher are retained shares. The intuition behind this result is as follows. When the fraction of optimists among entrepreneurs who signal is large enough, the optimal retained shares equate the marginal utility of retaining one more share–selling one less share to outside investors at a discount $\theta\Delta\mu$–to the marginal disutility of retaining one more share–the disutility from the increase in risk exposure. If, in addition, absolute risk aversion is either constant or increasing in wealth, then the more optimists there are, the more attractive it is for entrepreneurs who signal to retain shares because the marginal benefit of selling one less share to outside investors at a discount is higher.

To illustrate these results we assume a project's cash flows are normally distributed and constant absolute risk aversion. In this framework, (17) and (18) become

$$\mu_1 \geq (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha\mu_1 - \frac{\rho}{2}\alpha^2\sigma^2, \tag{19}$$

and

$$(1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma^2 \geq \mu_1, \tag{20}$$

respectively. To be a least cost separating equilibrium $\hat{\alpha}$ must maximize the expected utility of a realist with a high expected value project (and the perceived expected utility of an optimist)

$$-\exp\left\{-\rho\left\{w_0 - k + \mu_2 - \left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right]\right\}\right\},$$

or, equivalently, to minimize his cost of signaling

$$C(\alpha) = (1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2 \tag{21}$$

subject to $\alpha \in [0,1]$ and the incentive constraints (19) and (20). When θ is equal to zero the cost of signaling is given by

$$C(\alpha) = \frac{\rho}{2}\alpha^2\sigma^2. \tag{22}$$

Comparing (21) and (22) we see that, holding retained shares constant, the existence of optimists increases the cost of signaling of a realist with a high expected value project (and the perceived cost of signaling of an optimist). The presence of optimists makes it less profitable for a realist with a high expected value project to sell equity: the stock price of the $(1-\alpha)$ shares sold to outside investors drops by $\theta\Delta\mu$. This increases the cost of signaling by $(1-\alpha)\theta\Delta\mu$. In addition, we see from (21) that the existence of optimists implies that an increase in retained shares has two effects on the cost of signaling. On the one hand, it reduces risk coverage which raises the cost of signaling (like in the standard model). On the other hand, it lowers the number of shares sold to outside investors at a discount of $\theta\Delta\mu$ which lowers the cost of signaling.

Our next result characterizes the least cost separating equilibrium of the specialized model.

Proposition 2: *Assume project i's random cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are independent and normally distributed with mean* 0 *and variance σ^2, where $0 < \sigma^2 < \infty$, $i = 1, 2$, σ^2 is known to outside investors in date* 0, *project i's mean is unknown to outside investors in date* 0 *but is fully revealed in date* 1, *and entrepreneurs have utility $u(w) = -\exp(-\rho w)$.*
(i) If either (a) $\pi > \bar{\pi}$, or (b) $\pi < \bar{\pi}$ and $\kappa < \frac{\pi}{\bar{\pi}} - \pi$, where

$$\bar{\pi} = \sqrt{1 + \left(\frac{\rho\sigma^2}{\Delta\mu}\right)^2} - \frac{\rho\sigma^2}{\Delta\mu}, \tag{23}$$

then

$$\hat{\alpha} = \frac{(1-\theta)\,\Delta\mu}{\rho\sigma^2}\left[\sqrt{1 + 2\frac{\rho\sigma^2}{(1-\theta)\Delta\mu}} - 1\right]; \tag{24}$$

(ii) If $\pi < \bar{\pi}$ and either (a) $1 - \frac{\rho\sigma^2}{\Delta\mu} < \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < 1 - \pi$, or (b) $1 - \frac{\rho\sigma^2}{\Delta\mu} \geq \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < \frac{\pi\Delta\mu}{\Delta\mu - \rho\sigma^2} - \pi$, then

$$\hat{\alpha} = \frac{\theta\Delta\mu}{\rho\sigma^2}. \tag{25}$$

Proposition 2(i) illustrates Proposition 1(i) in the case where project's cash flows are normally distributed and where entrepreneurs have constant absolute risk aversion. When the fraction of optimists among entrepreneurs who signal is not too large, condition (19) is binding and condition (20) is slack under least-cost separation. Hence, the optimal retained shares are equal to the α that solves (19) as an equality. The only way for (19) to be satisfied as an equality when the fraction of optimists κ increases (θ increases) is for α to decrease. Proposition 2(i) also provides a precise meaning to the sentence "the fraction of optimists among entrepreneurs who signal is not too large," namely: either (a) the fraction of realists with high expected value projects is greater than $\bar{\pi}$, or (b) the fraction of realists with high expected value projects is smaller than $\bar{\pi}$ and the fraction of optimists κ is smaller than $\frac{\pi}{\bar{\pi}} - \pi$.

Proposition 2(ii) illustrates Proposition 1(ii). When the fraction of optimists among entrepreneurs who signal is large enough, conditions (19) and (20) are slack under least-cost separation. Hence, the optimal retained shares equate the marginal benefit of retaining one more share–selling one less share to outside investors at a discount $\theta\Delta\mu$–to the marginal cost of retaining one more share–the disutility $\rho\alpha\sigma^2$ from the increase in risk exposure. The more optimists there are, the more attractive it is to retain shares because the marginal benefit of selling one less share to outside investors at a discount $\theta\Delta\mu$ is higher.

To complete this section we turn to the impact of optimism on welfare. From (10), welfare under least cost-separation is equal to:

$$\begin{aligned}\hat{W} \quad = \quad & n\,(1-\pi-\kappa)\,u(w_0-k+\mu_1) \\ & +n\kappa E[u(w_0-k+(1-\hat{\alpha})(\mu_2-\theta\Delta\mu)+\hat{\alpha}(\mu_1+\tilde{x}))] \\ & +n\pi E[u(w_0-k+(1-\hat{\alpha})(\mu_2-\theta\Delta\mu)+\hat{\alpha}(\mu_2+\tilde{x}))], \end{aligned} \tag{26}$$

where $\hat{\alpha}$ is determined according to Proposition 1. We consider each type of entrepreneur separately.

Firstly, a realist with a low expected value project is fully covered and is not affected by the existence of optimists.

Secondly, a realist with a high expected value project is adversely affected by the presence of optimists. When the fraction of optimists among entrepreneurs who signal is not too large, the presence of optimists increases the cost of signaling of a realist with a high expected value project since it reduces by $\theta\Delta\mu$ the stock price he receives for selling $(1-\hat{\alpha})$ shares of his project to outside investors. The higher signaling cost for any given level of retained shares implies that a realist with a high expected value project will attain a lower expected utility in the presence of optimists even if this enables him to reduce his exposure to idiosyncratic risk. When the fraction of optimists among entrepreneurs who signal is large enough and absolute risk aversion is either constant or increasing in wealth, a realist with a high expected value project is adversely affected by an increase in the number of optimists because this lowers his equity price and raises his exposure to idiosyncratic risk.

Lastly, an optimist is either unaffected or adversely affected by an increase in the number of optimists. When the fraction of optimists among entrepreneurs who signal is not too large, an optimist is unaffected by an increase in the number of optimists because his expected utility is the same as the utility of a realist with a low expected value project. When the fraction of optimists among entrepreneurs who signal is large enough and absolute risk aversion is either constant or increasing in wealth, an optimist is adversely affected by an increase in the number of optimists because this lowers his equity price and raises his exposure to idiosyncratic risk. Hence, the existence of optimists leads to a Pareto worsening.

Proposition 3 shows that in the specialized model an increase in the number of optimists lowers welfare.

Proposition 3: *If project i's random cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are independent and normally distributed with mean* 0 *and variance σ^2, where $0 < \sigma^2 < \infty$, $i = 1, 2$, σ^2 is known to outside investors in date* 0*, project i's mean is unknown to outside investors in date* 0 *but is fully revealed in date* 1*, and entrepreneurs have utility $u(w) = -\exp(-\rho w)$, then an increase in the fraction of optimists lowers welfare, i.e., $\partial \hat{W}/\partial \kappa < 0$.*

5 Unknown Mean and Variance

This section describes the impact of optimism on the market for new issues when both the mean and the variance of the project's cash flows are private information of the entrepreneur. This allows us to study the impact of optimism on retained shares and underpricing, and on the primary as well as on the secondary market for assets.

To keep the analysis as close as possible to Grinblatt and Hwang (1989) we assume that both the mean and the variance of the project's cash flows are unknown to outside investors in date 0, they become known to outside investors in date 1 with probability $r \in (0, 1)$, the two types of projects have different variances, i.e., $\sigma_1^2 \neq \sigma_2^2$, the project's cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are normally distributed, and entrepreneurs have constant absolute risk aversion. Under these assumptions (8) becomes

$$\begin{aligned} w_0 - k + \mu_1 \geq w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu - D) + \alpha[r\mu_1 + (1-r)(\mu_2 - \theta\Delta\mu)] \\ -\frac{\rho}{2}\alpha^2(1-\theta)^2 r(1-r)(\Delta\mu)^2 - \frac{\rho}{2}\alpha^2\sigma_1^2. \end{aligned} \quad (27)$$

The left side of (27) represents the utility of a realist with a low expected value project who sells his entire project at price μ_1 in date 0. The right side of (27) represents the expected utility of a realist with a low expected value project who mimics an entrepreneur who signals. In date 0 he receives $(1-\alpha)(\mu_2-\theta\Delta\mu-D)$ for selling $(1-\alpha)$ of his shares, because he signals and outside investors erroneously believe that he is either a realist with a high expected value project or an optimist. With probability r outside investors learn the project's type, and he sells his remaining fraction of the project in date 1 since by doing that he obtains a utility of $\alpha\mu_1$ which is better than the expected utility of not selling $\alpha\mu_1-\frac{\rho}{2}\alpha^2\sigma_1^2$. With probability $1-r$ outside investors do not learn the project's type, and he sells his remaining fraction of the project in date 1 since by doing that he obtains a utility of $\alpha(\mu_2-\theta\Delta\mu)$ which is better than the expected utility of not selling $\alpha\mu_1-\frac{\rho}{2}\alpha^2\sigma_1^2$. The last two terms in the right-hand side of (27) represent his disutility of being exposed to risk by retaining fraction α of shares. The term $\frac{\rho}{2}\alpha^2(1-\theta)^2r(1-r)(\Delta\mu)^2$ represents the disutility from the variability of the equity price in date 1. The term $\frac{\rho}{2}\alpha^2\sigma_1^2$ represents the disutility from the variability of the random cash flow in date 1.

Under the assumptions made at the start of this section (9) becomes

$$\begin{aligned} w_0-k+(1-\alpha)(\mu_2-\theta\Delta\mu-D)+\alpha[r\mu_2+(1-r)(\mu_2-\theta\Delta\mu)] \\ -\frac{\rho}{2}\alpha^2\theta^2r(1-r)(\Delta\mu)^2-\frac{\rho}{2}\alpha^2\sigma_2^2\geq w_0-k+\mu_1. \end{aligned} \tag{28}$$

The left side of (28) represents the expected utility of a realist with a high expected value project (and the perceived expected utility of an optimist) who signals. In date 0 he receives $(1-\alpha)(\mu_2-\theta\Delta\mu-D)$ for selling $(1-\alpha)$ of his shares, because he signals and outside investors correctly believe that he is either a realist with a high expected value project or an optimist. With probability r outside investors learn the project's type, and he sells his remaining fraction of the project in date 1 since by doing that he obtains a utility of $\alpha\mu_2$ which is better than the expected utility of not selling $\alpha\mu_2-\frac{\rho}{2}\alpha^2\sigma_2^2$. With probability $1-r$ outside investors do not learn the project's type, and he compares $\alpha(\mu_2-\theta\Delta\mu)$, the utility of selling the remaining fraction of the project in date 1, to $\alpha\mu_2-\frac{\rho}{2}\alpha^2\sigma_2^2$, the expected utility of not selling.[15] He prefers to sell when the number of optimists is not too high, i.e., $\theta\leq\frac{\rho}{2}\alpha\frac{\sigma_2^2}{\Delta\mu}$. We assume that this condition is satisfied and later on provide

[15]When outside investors do not learn the project's type and there are no optimists, an entrepreneur with a high expected value project sells his remaining fraction of the project at date 1 since by doing that he obtains a utility of $\alpha\mu_2$ which is better than the expected utility of not selling $\alpha\mu_2-\frac{\rho}{2}\alpha^2\sigma_2^2$.

a condition under which that is indeed the case. At the end of this section we explain what happens when the number of optimists is high enough, i.e., $\theta > \frac{\rho}{2}\alpha\frac{\sigma_2^2}{\Delta\mu}$. The last two terms in the left-hand side of (28) represent the disutility of a realist with a high expected value project (and the perceived disutility of an optimist) from being exposed to risk by retaining fraction α of shares. The term $\frac{\rho}{2}\alpha^2\theta^2 r(1-r)(\Delta\mu)^2$ represents the disutility from the variability of the equity price in date 1. The term $\frac{\rho}{2}\alpha^2\sigma_2^2$ represents the disutility from the variability of the random cash flow in date 1. The right side of (28) represents the utility of a realist with a high expected value project (and the utility of an optimist) who mimics a realist with a low expected value project by selling the whole project to outside investors at price μ_1 in date 0.

To be a least cost separating equilibrium $\hat{\alpha}$ and $\hat{D}$ must maximize the date 1 expected utility of a realist with a high expected value project (and the perceived date 1 expected utility of an optimist), or, equivalently, to minimize his cost of signaling

$$C(\alpha, D) = (1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma_2^2 + \frac{\rho}{2}\alpha^2\theta^2 r(1-r)(\Delta\mu)^2 \tag{29}$$

subject to $\alpha \in [0, 1]$, $D \in [0, \mu_2 - \theta\Delta\mu]$, (27), and (28). When θ is equal to zero the model collapses to Grinblatt and Hwang (1989) and the cost of signaling is given by

$$C(\alpha, D) = (1-\alpha)D + \frac{\rho}{2}\alpha^2\sigma_2^2. \tag{30}$$

Comparing (29) to (30) we see that, holding retained shares and the average degree of underpricing per share in date 0 constant, the existence of optimists raises the cost of signaling of a realist with a high expected value project (and the perceived cost of signaling of an optimist). First, the presence of optimists makes it less profitable for a realist with a high expected value project to sell equity in dates 0 and 1. In date 0 a realist with a high expected value project sells $(1-\alpha)$ shares of his project by $(1-\alpha)(\mu_2 - \theta\Delta\mu - D)$ rather than by $(1-\alpha)(\mu_2 - D)$. This increases the cost of signaling by $(1-\alpha)\theta\Delta\mu$. In date 1 with probability $1-r$ outside investors do not learn the project's type, and a realist with a high expected value project sells α shares of his project by $\alpha(\mu_2 - \theta\Delta\mu)$ rather than by $\alpha\mu_2$. This increases the cost of signaling by $\alpha(1-r)\theta\Delta\mu$. The sum of $(1-\alpha)\theta\Delta\mu$ and $\alpha(1-r)\theta\Delta\mu$ is equal to $(1-\alpha r)\theta\Delta\mu$, the second term in the right-hand side of (29).

Second, the existence of optimists makes the date 1 wealth of a realist with a high expected value project more variable. With probability r outside investors learn the project's type, and he sells his remaining fraction of the project by $\alpha\mu_2$. With probability $1-r$ outside investors do not learn the project's type, and he sells his remaining fraction of the project by $\alpha(\mu_2 - \theta\Delta\mu)$. This increases the cost of signaling by $\frac{\rho}{2}\alpha^2\theta^2 r(1-r)(\Delta\mu)^2$ which is the fourth term on the right-hand side of (29).

We start by characterizing the least-cost separating equilibrium of the model when the variance of the high expected value project is not too large.

Proposition 4: *Assume project i's random cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are independent and normally distributed with mean* 0 *and variance σ_i^2, where $0 < \sigma_i^2 < \infty$, $i = 1, 2$, entrepreneurs have utility $u(w) = -\exp(-\rho w)$, and*

$$\sigma_2^2 \leq \sigma_1^2 + (1-2\theta)r(1-r)(\Delta\mu)^2, \tag{31}$$

and $\theta \leq \underline{\theta}$, where $\underline{\theta}$ satisfies

$$\rho = \frac{2\underline{\theta}\Delta\mu}{\sigma_2^2}\left[r + \frac{\underline{\theta}}{1-\underline{\theta}}\frac{\sigma_1^2 + (1-\underline{\theta})^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2}\right], \tag{32}$$

and

$$\rho > \frac{2(1-r)(1-\theta)\Delta\mu}{\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2}, \tag{33}$$

then

$$\hat{\alpha} = \frac{(1-\theta)r\Delta\mu\left[\sqrt{1 + 2\frac{\rho\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right]}{(1-\theta)r^2\Delta\mu}} - 1\right]}{\rho\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right]}, \tag{34}$$

and

$$\hat{D} = 0.$$

Proposition 4 provides conditions under which realists with high expected value projects and optimists retain shares in date 0 and do not, on average, misprice the shares sold to outside investors in date 0. First, the variance of the high expected value project can not be too large, i.e., condition (31) must be satisfied. Second, the fraction of optimists among entrepreneurs who signal cannot be too high, i.e., $\theta \leq \underline{\theta}$ where $\underline{\theta}$ is defined by (32). Third, entrepreneurs' absolute risk aversion must be sufficiently high, i.e., condition (33) must be satisfied. When these conditions hold, the cost of signaling only with retained shares is low and so retained shares offer a more efficient signal vector than retaining shares and underpricing.

Equation (34) displays the least cost separating retained shares. It follows from the derivative of (34) with respect to κ that if $\sigma_1^2 > (<)(1-\theta)^2 r(1-r)(\Delta\mu)^2$, then the more optimists there are, the lower (higher) are retained shares.

It follows from Proposition 4 that a realist with a high expected value project underprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $\theta\Delta\mu$ and, if the project's type is not revealed, the α shares sold in date 1 by $\theta\Delta\mu$. In addition, an optimist overprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $(1-\theta)\Delta\mu$ and, if the project's type is not revealed, the α shares sold in date 1 by $(1-\theta)\Delta\mu$. This result shows that optimism can lead to underpricing as well as overpricing in the primary and secondary market for assets.

When the variance of the high expected value project is large enough, i.e., condition (31) is violated, retained shares are not sufficient by itself to signal. In this case a second signal is needed to infer the variance of the project's returns since the equilibrium signaling schedule is a function of both the variance and retained shares. This second signal is the date 0 average degree of underpricing per share.

Our next result characterizes the least-cost separating equilibrium of the model when the variance of the high expected value project is large enough.

Proposition 5: *If project i's random cash flows $\tilde{x}_i$ and $\tilde{y}_i$ are independent and normally distributed with mean 0 and variance σ_i^2, where $0 < \sigma_i^2 < \infty$, $i = 1, 2$, entrepreneurs have utility $u(w) = -\exp(-\rho w)$, and*

$$\sigma_2^2 > \sigma_1^2 + r(1-r)(\Delta\mu)^2, \tag{35}$$

and

$$\theta \leq \frac{\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2}{4r(1-r)(\Delta\mu)^2} \left\{ \sqrt{1 + \frac{4r^2(1-r)(\Delta\mu)^2\sigma_2^2}{\left[\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2\right]^2}} - 1 \right\}, \tag{36}$$

and

$$\rho > \frac{r\triangle\mu \max\left\{1, r + \frac{r}{2(1-\theta)}\frac{\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2}\right\}}{\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2}, \tag{37}$$

then

$$\hat{\alpha} = \frac{r\triangle\mu}{\rho\left[\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2\right]}, \tag{38}$$

and

$$\hat{D} = (1-\theta)\frac{1-\hat{\alpha}r}{1-\hat{\alpha}}\triangle\mu - \frac{\rho}{2}\frac{\hat{\alpha}^2}{1-\hat{\alpha}}[\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2]. \tag{39}$$

Proposition 5 provides conditions under which realists with high expected value projects and optimists retain shares in date 0 and, on average, underprice the shares sold to outside investors in date 0. First, the variance of the high expected value project is large enough, i.e., (35) is satisfied. Second, the fraction of optimists among entrepreneurs who signal is not too high, i.e., (36) is satisfied. Third, entrepreneurs' absolute risk aversion is sufficiently high, i.e., (37) is satisfied. When these conditions hold, retaining shares and underpricing in date 0 offer a more efficient signal vector than a pure signal of retained shares.[16]

Equation (38) displays the least cost separating retained shares. We see from (38) that an increase in the fraction of optimists lowers retained shares, i.e., $\partial\hat{\alpha}/\partial\kappa < 0$. The presence of optimists makes it less profitable for a realist with a high expected value project to sell equity because the presence of optimists reduces stock prices in dates 0 and 1 and increases the variability in date 1 wealth. This makes signaling less attractive to a realist with a low expected value project. As a consequence, a realist with a high expected value project needs to retain less shares to separate himself from a realist with a low expected value project in an incentive compatible manner than he would have to if there were no optimists.

Equation (39) displays the average degree of underpricing per share in date 0. Optimism has three effects on $\hat{D}$. First, an increase in the fraction of optimists lowers $\hat{D}$ via the first term in (39). Holding retained shares constant, the lower stock price of projects in which entrepreneurs retain shares makes signaling less attractive to a realist with a low expected

[16] The proof of Proposition 5 shows that $\hat{D} > 0$.

value project. Second, an increase in the fraction of optimists raises $\hat{D}$ via the second term in (39). Holding retained shares constant, the presence of optimists makes the date 1 wealth of a realist with a low expected value project who mimics an entrepreneur who signals less variable. With probability r outside investors learn the project's type, and he sells his remaining fraction of the project by $\alpha\mu_1$. With probability $1-r$ outside investors do not learn the project's type, and he sells his remaining fraction of the project by $\alpha(\mu_2 - \theta\Delta\mu)$. As a consequence, the variance of date 1 wealth is reduced by $\frac{\rho}{2}\alpha^2(1-\theta)^2 r(1-r)(\Delta\mu)^2$ which makes signaling more attractive to a realist with a low expected value project. Third, an increase in the fraction of optimists affects $\hat{D}$ via the reduction in retained shares. This effect is ambiguous since a decrease in retained shares leads to a fall in the first term in (39)–via a decrease in $\frac{1-\hat{\alpha}r}{1-\hat{\alpha}}$–, as well as a fall in the second term in (39)–via a decrease in $\frac{\hat{\alpha}^2}{1-\hat{\alpha}}$. Overall, optimism has an ambiguous impact on $\hat{D}$.

It follows from Proposition 5 that a realist with a high expected value project underprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $\theta\Delta\mu + \hat{D}$ and, if the project's type is not revealed, the α shares sold in date 1 by $\theta\Delta\mu$. An optimist misprices the $(1-\alpha)$ shares sold to outside investors in date 0 by $(1-\theta)\Delta\mu - \hat{D}$ and, if the project's type is not revealed, overprices the α shares sold in date 1 by $(1-\theta)\Delta\mu$. Note that since $(1-\theta)\Delta\mu - \hat{D}$ can be either positive or negative it is unclear whether an optimist overprices or underprices in date 0.

We now turn to the impact of optimism on welfare. When realists with high expected value projects and optimists retain shares in date 0 and, on average, do not underprice or overprice the shares sold to outside investors in date 0, the impact of optimism on welfare is the same as in the previous section: entrepreneurs are made worse off and there is no impact on outside investors' welfare. Optimism also makes entrepreneurs worse off when realists with high expected value projects and optimists retain shares in date 0 and, on average, underprice the shares sold to outside investors in date 0. In this case, entrepreneurs' welfare is equal to

$$
\begin{aligned}
\hat{W} &= n\,(1-\pi)\,u(w_0 - k + \mu_1) \\
&\quad + n\pi E[u(w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu - \hat{D}) + \hat{\alpha}(\tilde{\mu}_2 + \tilde{x}_2))].
\end{aligned}
$$

where $\hat{\alpha}$ and $\hat{D}$ are given by (38) and (39), respectively. The impact of optimism on entrepreneurs' welfare is determined by its impact on the expected utility of a realist with a high expected value project. This is, in turn, determined by its impact on the cost of signaling. Substituting (38) and (39) into (29) the cost of signaling of a realist with a high expected value project is equal to

$$C(\hat{\alpha}, \hat{D}) = \triangle\mu - \frac{1}{2}\frac{r^2(\triangle\mu)^2}{\rho\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right]}. \tag{40}$$

We see from (40) that the more optimists there are, the higher is the cost of signaling of a realist with a high expected value project. Hence, an increase in the number of optimists reduces entrepreneurs' welfare given least-cost separation, i.e., $\partial\hat{W}/\partial\kappa < 0$.

Let us now consider the impact of optimism on outside investors' welfare when realists with high expected value projects and optimists retain shares in date 0 and, on average, underprice the shares sold to outside investors in date 0. In this case outside investors' welfare is given by:

$$\hat{I}(\theta) = n\left[\pi\hat{U}(\theta) - \kappa\hat{O}(\theta)\right] = n\pi\left[\hat{U}(\theta) - \frac{\theta}{1-\theta}\hat{O}(\theta)\right], \tag{41}$$

where $\hat{U}(\theta)$ denotes the expected profits from financing a realist with a high expected value project and $\hat{O}(\theta)$ denotes the expected losses from financing an optimist. Outside investors' expected profits from financing a realist with a high expected value project are equivalent to the expected cost of underpricing incurred by such entrepreneur:

$$\hat{U}(\theta) = (1-\hat{\alpha})(\theta\triangle\mu + \hat{D}) + \hat{\alpha}(1-r)\theta\triangle\mu. \tag{42}$$

In date 0 a realist with a high expected value project sells $(1-\hat{\alpha})$ shares of his project at a discount of $\theta\triangle\mu + \hat{D}$ per share. In date 1 outside investors do not learn the project's type with probability $(1-r)$ and he sells the remaining $\hat{\alpha}$ shares of his project at a discount of $\theta\triangle\mu$ per share. Substituting (39) into (42) and simplifying terms we obtain

$$\hat{U}(\theta) = (1-\hat{\alpha}r)\triangle\mu - \frac{\rho}{2}\hat{\alpha}^2[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2]. \tag{43}$$

We see from (43) that an increase in the fraction of optimists raises the expected cost of underpricing incurred by a realist with a high expected value project (recall that $\hat{\alpha}$ decreases with θ). This has a favorable impact on outside investors' welfare.

Outside investors' expected losses from financing an optimist are equivalent to the expected benefit of overpricing attained by such entrepreneur:

$$\hat{O}(\theta) = (1-\hat{\alpha})\left[(1-\theta)\triangle\mu - \hat{D}\right] + \hat{\alpha}(1-r)(1-\theta)\triangle\mu. \tag{44}$$

In date 0 an optimist sells $(1-\hat{\alpha})$ shares of his project at a premium of $(1-\theta)\triangle\mu - \hat{D}$ per share. In date 1 outside investors do not learn the project's type with probability $(1-r)$ and he sells the remaining $\hat{\alpha}$ shares of his project at a premium of $(1-\theta)\triangle\mu$ per share. Substituting (39) into (44) and simplifying terms we obtain

$$\hat{O}(\theta) = \frac{\rho}{2}\hat{\alpha}^2[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2]. \tag{45}$$

It follows directly from (45) that an increase in the fraction of optimists lowers the expected benefit of overpricing attained by an optimist (recall that $\hat{\alpha}$ decreases with θ). However, the more optimists there are, the more entrepreneurs overprice and this has an unfavorable impact on outside investors' welfare.

Our last result shows that optimism can improve the welfare of outside investors given least-cost separation.

Proposition 6: *If the conditions in Proposition 5 are satisfied, and*

$$\frac{5\sigma_2^2}{2} - \sqrt{\frac{25\sigma_2^4}{4} - \sigma_1^4\left(\frac{\sigma_2^2}{\sigma_1^2} - 1\right)} < r(1-r)(\Delta\mu)^2 < \frac{5\sigma_2^2}{2} + \sqrt{\frac{25\sigma_2^4}{4} + \sigma_1^4\left(\frac{\sigma_2^2}{\sigma_1^2} - 1\right)}, \tag{46}$$

and

$$\theta < \frac{\hat{U}(\theta) - \hat{U}(0)}{\hat{U}(\theta) - \hat{U}(0) + \hat{O}(\theta)}, \tag{47}$$

then the existence of optimists makes outside investors better off, i.e., $\hat{I}(\theta) > \hat{I}(0)$.

Optimism has two effects on outside investors' welfare when realists with high expected value projects and optimists retain shares in date 0 and, on average, underprice the shares sold to outside investors in date 0. First, the greater the fraction of optimists, the more outside investors gain from financing realists with high expected value projects. An increase in the fraction of optimists, by decreasing retained shares of realists with high expected value projects, increases their date 0 and lowers their date 1 demand for outside finance. Since the amount of underpricing per share in date 0 is higher than in date 1 this is beneficial to outside investors. Second, the greater the fraction of optimists, the higher the number of projects where outside investors make losses due to overpricing. Proposition 6 shows that if conditions (46) and (47) are satisfied, then the former effect dominates the latter and the existence of optimists makes outside investors better off. This is the case when σ_2^2/σ_1^2 is high, the probability that a project's type becomes known to outside investors in date 1, is not too close to 0 or to 1, and when the fraction of optimists is not too large. The assumption that outside investors are risk neutral is critical to this result. If outside investors were risk averse the existence of optimists increases the risk of financing projects and this could make outside investors worse off.

So far we assumed, as in Grinblatt and Hwang (1989), that entrepreneurs who signal sell $(1-\alpha)$ shares in the primary market and the remaining α shares in the secondary market, where the price either equals to true value μ_i or outside investors expectations $\mu(\alpha, D, P)$. This result holds when the fraction of optimists among entrepreneurs who signal is not too large. However, when the fraction of optimists among entrepreneurs who signal is large enough, realists with high expected value projects and optimists may prefer to retain the remaining α shares until the project's value is realized in date 2. The intuition behind this result is simple. When outside investors do not learn the project's type, realists with high expected value projects and optimists compare $\alpha(\mu_2 - \theta\Delta\mu)$, the utility of selling the remaining α shares in date 1, to $\alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma_2^2$, the expected utility of not selling. Not selling the remaining α shares in the secondary market is better than selling them when the fraction of optimists among entrepreneurs who signal is large enough, i.e., $\theta > \frac{\rho}{2}\alpha\frac{\sigma_2^2}{\Delta\mu}$.

In a separating equilibrium where the fraction of optimists among entrepreneurs who signal is large enough, the incentive condition of a realist with low expected value project is left unchanged and is given by (27). However, the incentive condition of a realist with a high expected value project (and of an optimist) becomes

$$w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu - D) + \alpha r\mu_2 + (1-r)\left(\alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma_2^2\right) - \frac{\rho}{2}\alpha^2\sigma_2^2 \geq w_0 - k + \mu_1. \quad (48)$$

The new terms in (48), by comparison to (28), are $\alpha r\mu_2$ and $(1-r)\left(\alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma_2^2\right)$. They are obtained as follows. With probability r outside investors learn the project's type, and a realist with a high expected value project sells the remaining α shares in date 1 since by doing that he obtains an utility of $\alpha\mu_2$ which is better than the expected utility of not selling $\alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma_2^2$. With probability $1-r$ outside investors do not learn the project's type, and he retains the remaining α shares until the project's value is realized in date 2 since the large fraction of optimists among entrepreneurs who signal implies that the expected utility of not selling the remaining α shares $\alpha\mu_2 - \frac{\rho}{2}\alpha^2\sigma_2^2$ is higher than the utility of selling them $\alpha(\mu_2 - \theta\Delta\mu)$.

To be a least cost separating equilibrium $\hat{\alpha}$ and $\hat{D}$ must maximize the date 2 expected utility of a realist with a high expected value project (and the perceived date 2 expected utility of an optimist), or, equivalently, to minimize his cost of signaling

$$C(\alpha, D) = (1-\alpha)(D + \theta\Delta\mu) + (2-r)\frac{\rho}{2}\alpha^2\sigma_2^2$$

subject to $\alpha \in [0,1]$, $D \in [0, \mu_2 - \theta\Delta\mu]$, (27), and (48). Solving this problem we obtain the following results.[17]

When the variance of the high expected value project is not too large, realists with high expected value projects and optimists retain α shares in date 0 and do not, on average, overprice or underprice the $(1-\alpha)$ shares sold to outside investors in date 0. When the fraction of optimists among entrepreneurs who signal is moderate, i.e., $\underline{\theta} < \theta < \bar{\theta}$, the incentive constraint of realists with low expected value projects binds and retained shares are equal to (34). In this case the more optimists there are, the lower are retained shares. However, when the fraction of optimists among entrepreneurs who signal is high, i.e., $\bar{\theta} < \theta < \rho(2-r)\sigma_2^2/\Delta\mu$, the incentive constraint of realists with low expected value projects does not bind and retained shares are equal to $\hat{\alpha} = \frac{\theta\Delta\mu}{\rho(2-r)\sigma_2^2}$. In this case the more optimists there are, the higher are retained shares.

When the variance of the high expected value project is large enough, the fraction of optimists among entrepreneurs who signal is sufficiently high, and absolute risk aversion is high enough, realists with high expected value projects and optimists retain α shares in date 0 and, on average, underprice the $(1-\alpha)$ shares sold to outside investors in date 0.

[17] The formal results are available upon request.

Retained shares are given by $\hat{\alpha} = \frac{[r+\theta(1-r)]\Delta\mu}{\rho\left[(2-r)\sigma_2^2-\sigma_1^2-(1-\theta)^2 r(1-r)(\Delta\mu)^2\right]}$ and the average degree of underpricing per share is equal to (39).

Overall, we see that the main qualitative findings of this section also extend to the case where the remaining α shares are retained until the project's value is realized in date 2.

6 Conclusion

This paper studies the impact of entrepreneurial optimism on the market for new issues. To do that it extends Grinblatt and Hwang (1989) by including optimists and shows how optimism affects the pricing of new issues, entrepreneurs' equity holdings, and welfare.

We find that the existence of optimists provides a new reason for entrepreneurs to own equity in their firms when outside investors are able to directly observe entrepreneurs' beliefs.

We show that optimism is a natural explanation for why some new issues are underpriced while others are overpriced. We also show that the impact of optimism on entrepreneurs' equity holdings depends on the number of optimists, absolute risk aversion, and cash flow variance.

We find that optimism makes entrepreneurs worse off. In contrast, optimism can make outside investors better off when entrepreneurs signal firm value by retaining shares and, on average, by underpricing the shares sold to outside investors.

Appendix

Proof of Proposition 1: To be a lcse $\hat{\alpha}$ must be the solution to

$$\max_{\alpha\in[0,1]} E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_2 + \tilde{x}))]$$
$$\text{s.t. } u(w_0 - k + \mu_1) \geq E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_1 + \tilde{x}))]$$
$$E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_2 + \tilde{x}))] \geq u(w_0 - k + \mu_1)$$

Let λ_1 and λ_2 denote the Lagrange multipliers of the first and second constraints, respectively. It can never be the case that $\alpha = 0$ since otherwise there would be no separation. We need to consider four possibilities: (1) $\lambda_1 > 0$ and $\lambda_2 = 0$, (2) $\lambda_1 = 0$ and $\lambda_2 = 0$, (3) $\lambda_1 = 0$ and $\lambda_2 > 0$, and (4) $\lambda_1 > 0$ and $\lambda_2 > 0$.

(1) When $\lambda_1 > 0$ and $\lambda_2 = 0$ the lcse retained shares are defined implicitly by

$$u(w_0 - k + \mu_1) = E[u(w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_1 + \tilde{x}))]. \tag{49}$$

Rewrite (49) as

$$F_1 = -u(w_0 - k + \mu_1) + E[u(w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_1 + \tilde{x}))] = 0. \tag{50}$$

Let $\tilde{w}(\mu_1) = w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_1 + \tilde{x})$. Applying the implicit function theorem to (50) we obtain

$$\begin{aligned} \frac{\partial\hat{\alpha}}{\partial\theta} &= -\frac{\partial F_1/\partial\theta}{\partial F_1/\partial\hat{\alpha}} = -\frac{-E\left[u'(\tilde{w}(\mu_1))\right](1-\hat{\alpha})\Delta\mu}{E\left[u'(\tilde{w}(\mu_1))(-\mu_2 + \theta\Delta\mu + \mu_1 + \tilde{x})\right]} \\ &= \frac{E\left[u'(\tilde{w}(\mu_1))\right](1-\hat{\alpha})\Delta\mu}{-E\left[u'(\tilde{w}(\mu_1))\right](1-\theta)\Delta\mu + Cov[u'(\tilde{w}(\mu_1)), \tilde{x}]} < 0, \end{aligned}$$

since concavity of u implies $Cov[u'(\tilde{w}(\mu_1)), \tilde{x}] < 0$ and therefore $\partial\hat{\alpha}/\partial\theta < 0$. Since $\partial\hat{\alpha}/\partial\kappa = \partial\hat{\alpha}/\partial\theta \times \partial\theta/\partial\kappa$ and $\partial\theta/\partial\kappa = \pi/(\pi+\kappa)^2 > 0$ it follows that $\partial\hat{\alpha}/\partial\kappa < 0$. For this solution to be valid we must confirm that second incentive constraint is slack, i.e., $\lambda_2 = 0$. The assumption that entrepreneurs are averse to risk, $\mu_2 > \mu_1$, and the fact that the two projects have the same variance imply $E[u(\tilde{w}(\mu_2))] > E[u(\tilde{w}(\mu_1))]$. This and $\lambda_1 > 0$ imply $E[u(\tilde{w}(\mu_2))] > u(w_0 - k + \mu_1)$, i.e., $\lambda_2 = 0$.

(2) When $\lambda_1 = 0$ and $\lambda_2 = 0$, the lcse retained shares are the solution to

$$\max_{\alpha\in[0,1]} E[u(w_0 - k + (1-\alpha)(\mu_2 - \theta\Delta\mu) + \alpha(\mu_2 + \tilde{x}))].$$

Let $\tilde{w}(\mu_2) = w_0 - k + (1-\hat{\alpha})(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\mu_2 + \tilde{x})$. The first-order condition to this problem is

$$\begin{aligned}\frac{\partial E[u(\tilde{w}(\mu_2))]}{\partial \alpha} &= E\left[u'(\tilde{w}(\mu_2))(\theta\Delta\mu + \tilde{x})\right] \\ &= E\left[u'(\tilde{w}(\mu_2))\right]\theta\Delta\mu + E[u'(\tilde{w}(\mu_2))\tilde{x}] \\ &= E\left[u'(\tilde{w}(\mu_2))\right]\theta\Delta\mu + Cov[u'(\tilde{w}(\mu_2)),\tilde{x}] = 0. \qquad (51)\end{aligned}$$

Note that the first-order condition is well defined since the first term in (51) is positive ($u' > 0$ implies $E\left[u'(\tilde{w}(\mu_2))\right] > 0$) whereas the second term in (51) is negative ($u'' < 0$ implies $Cov[u'(\tilde{w}(\mu_2)),\tilde{x}] < 0$). The second-derivative of the expected utility is

$$\begin{aligned}\frac{\partial^2 E[u(\tilde{w}(\mu_2))]}{\partial \alpha^2} &= E\left[u''(\tilde{w}(\mu_2))(\theta\Delta\mu + \tilde{x})^2\right] \\ &= E\left[u''(\tilde{w}(\mu_2))\right]\theta^2(\Delta\mu)^2 + 2E[u''(\tilde{w}(\mu_2))\tilde{x}]\theta\Delta\mu + E[u''(\tilde{w}(\mu_2))\tilde{x}^2].\end{aligned}$$

Note that risk aversion implies $E\left[u''(\tilde{w}(\mu_2))\right] < 0$ and $E[u''(\tilde{w}(\mu_2))\tilde{x}^2] < 0$. The sign of $E[u''(\tilde{w}(\mu_2))\tilde{x}]$ is positive, zero, or negative as the coefficient of absolute risk aversion $\rho(w) = -u''(w)/u'(w)$ is decreasing, constant, or increasing in wealth–see Varian (1992, pp. 184-186). Hence, the second-order condition is satisfied when the coefficient of absolute risk aversion is nondecreasing in wealth since $E[u''(\tilde{w}(\mu_2))\tilde{x}] \leq 0$. Rewrite (51) as

$$F_2 = E\left[u'(\tilde{w}(\mu_2))\right]\theta\Delta\mu + E[u'(\tilde{w}(\mu_2))\tilde{x}] = 0. \qquad (52)$$

Applying the implicit function theorem to (52) we obtain

$$\begin{aligned}\frac{\partial\hat{\alpha}}{\partial\theta} &= -\frac{\partial F_2/\partial\theta}{\partial F_2/\partial\hat{\alpha}} \\ &= -\frac{E\left[u'(\tilde{w}(\mu_2))\right] - E\left[u''(\tilde{w}(\mu_2))\right]\theta(1-\hat{\alpha})\Delta\mu - E[u''(\tilde{w}(\mu_2))\tilde{x}](1-\hat{\alpha})}{E\left[u''(\tilde{w}(\mu_2))\right]\theta^2(\Delta\mu)^2 + 2E[u''(\tilde{w}(\mu_2))\tilde{x}]\theta\Delta\mu + E[u''(\tilde{w}(\mu_2))\tilde{x}^2)]}\Delta\mu.\end{aligned}$$

The denominator is negative since it is the second-order condition. The first term in the numerator is positive ($u' > 0$ implies $E[u'(\tilde{w}(\mu_2))] > 0$), the second term is positive ($u'' < 0$ implies $-E[u''(\tilde{w}(\mu_2))] > 0$), and the third term is nonnegative (the coefficient of absolute risk aversion being nondecreasing in wealth implies $-E[u''(\tilde{w}(\mu_2))\tilde{x}] \geq 0$). Since the denominator is negative and the numerator is positive it follows that $\partial\hat{\alpha}/\partial\theta > 0$. For this solution to be valid we must confirm that both incentive constraints are slack, i.e., $\lambda_1 = 0$ and $\lambda_2 = 0$. We start by proving a condition under which the first incentive constraint is slack. Denote the solution to (51) as a function of θ by $\hat{\alpha}(\theta)$. Define the threshold $\bar{\theta}$ as the θ that solves

$$u(w_0 - k + \mu_1) = E[u(w_0 - k + (1 - \hat{\alpha}(\theta))(\mu_2 - \theta\Delta\mu) + \hat{\alpha}(\theta)(\mu_1 + \tilde{x}))]. \qquad (53)$$

It follows from the definition of $\bar{\theta}$ that if $\theta > \bar{\theta}$, then the first incentive constraint is slack, i.e., $\lambda_1 = 0$. This happens because the left side of (53) does not depend on θ whereas the right side of (53) is decreasing in θ when $\theta > \bar{\theta}$. To see this note that the right side of (53) is equivalent to

$$E[u(w_0 - k + \mu_1 + (1 - \hat{\alpha}(\theta))(1 - \theta)\Delta\mu + \hat{\alpha}(\theta)\tilde{x})].$$

which is decreasing in θ when $\theta > \bar{\theta}$ since $(1 - \hat{\alpha}(\theta))(1 - \theta)$ is decreasing in θ ($\hat{\alpha}(\theta)$ is increasing in θ) and risk aversion implies that an increase in $\hat{\alpha}(\theta)\tilde{x}$ lowers expected utility. Hence, when the fraction of optimists among entrepreneurs who signal is large, i.e., $\theta > \bar{\theta}$, the first incentive constraint is slack, i.e., $\lambda_1 = 0$. We now show that the second incentive constraint is slack, i.e., $\lambda_2 = 0$. Suppose, by contradiction, that $\lambda_1 = 0$ and $\lambda_2 > 0$ solve the problem. If that were the case, the expected utility of a realist with a high expected value project is equal to $u(w_0 - k + \mu_1)$. However, when $\lambda_1 > 0$ and $\lambda_2 = 0$ the expected utility of a realist with a high expected value project is strictly higher than $u(w_0 - k + \mu_1)$. Hence, $\lambda_1 = 0$ and $\lambda_2 > 0$ is not a solution to the problem because it is not least cost separating. Therefore, in a solution to the problem with $\lambda_1 = 0$ it must be that $\lambda_2 = 0$. Finally, it follows from the definition of $\bar{\theta}$ that if the fraction of optimists among entrepreneurs who signal is small, i.e., $\theta < \bar{\theta}$, then the first incentive constraint is binding, i.e., $\lambda_1 > 0$. This implies that $\lambda_1 > 0$ and $\lambda_2 = 0$ are the solution to the problem if and only if $\theta < \bar{\theta}$.

(3) When $\lambda_1 = 0$ and $\lambda_2 > 0$ the expected utility of a realist with a high expected value

project is equal to $u(w_0 - k + \mu_1)$. However, when $\lambda_1 > 0$ and $\lambda_2 = 0$ the expected utility of a realist with a high expected value project is strictly higher than $u(w_0 - k + \mu_1)$. Hence, $\lambda_1 = 0$ and $\lambda_2 > 0$ is not a solution to the problem because it is not least cost separating. (4) When $\lambda_1 > 0$ and $\lambda_2 > 0$ we have $E[u(\tilde{w}(\mu_2))] = E[u(\tilde{w}(\mu_1))]$. This contradicts the fact that $\mu_2 > \mu_1$ implies $E[u(\tilde{w}(\mu_2))] > E[u(\tilde{w}(\mu_1))]$. Hence, $\lambda_1 > 0$ and $\lambda_2 > 0$ is not a solution to the problem. *Q.E.D.*

Proof of Proposition 2: The expected utility of a realist with a high expected value project is

$$E[u(\tilde{w}(\mu_2))] = -\exp\left\{-\rho\left\{w_0 - k + \mu_2 - \left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right]\right\}\right\}. \tag{54}$$

To be a lcse $\hat{\alpha}$ must be selected to maximize (54) or, equivalently, to minimize the cost of signaling

$$C(\alpha) = (1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2, \tag{55}$$

subject to $\alpha \in [0,1]$ and the incentive compatibility constraints (19) and (20):

$$\begin{aligned} &\min_{\alpha\in[0,1]} \left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right] \\ &\text{s.t. } \mu_1 \geq \mu_2 - \alpha\Delta\mu - \left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right] \\ &\quad \mu_2 - \left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right] \geq \mu_1 \end{aligned}$$

Let λ_1 and λ_2 denote the Lagrange multipliers of the first and second constraints, respectively. It can never be that $\alpha = 0$ since this violates the first incentive constraint. So, it must be that $\alpha > 0$. We have four possibilities: (1) $\lambda_1 > 0$ and $\lambda_2 = 0$, (2) $\lambda_1 = 0$ and $\lambda_2 = 0$, (3) $\lambda_1 = 0$ and $\lambda_2 > 0$, and (4) $\lambda_1 > 0$ and $\lambda_2 > 0$.

(1) When $\lambda_1 > 0$ and $\lambda_2 = 0$, the lcse retained shares are obtained by solving (19) as an equality with respect to α:

$$\frac{\rho\sigma^2}{2}\alpha^2 + (1-\theta)\Delta\mu\alpha - (1-\theta)\Delta\mu = 0. \tag{56}$$

The positive root of this quadratic equation is

$$\hat{\alpha} = \frac{(1-\theta)\Delta\mu}{\rho\sigma^2}\left[\sqrt{1+2\frac{\rho\sigma^2}{(1-\theta)\Delta\mu}}-1\right]. \tag{57}$$

It follows from (57) that $\hat{\alpha} \in (0,1)$. From (55) and (56), the cost of signaling associated with $\hat{\alpha}$ is:

$$C(\hat{\alpha}) = (1-\hat{\alpha})\theta\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2\sigma^2 = (1-\hat{\alpha})\Delta\mu < \Delta\mu. \tag{58}$$

Note that the inequality in (58) implies that (20) is slack, i.e., $\lambda_2 = 0$.
(2) When $\lambda_1 = 0$ and $\lambda_2 = 0$, the lcse retained shares are the solution to

$$\min_{\alpha\in[0,1]}\left[(1-\alpha)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma^2\right].$$

The first-order condition is $-\theta\Delta\mu + \rho\alpha\sigma^2 = 0$, and so

$$\hat{\alpha} = \frac{\theta\Delta\mu}{\rho\sigma^2}. \tag{59}$$

The second-order condition is satisfied since $d^2C(\alpha)/d\alpha^2 = \rho\sigma^2 > 0$. Note that since $\theta \in (0, 1-\pi]$, $\hat{\alpha}$ is well defined (it is less than 1) for all θ when $(1-\pi)\Delta\mu < \rho\sigma^2$. When $(1-\pi)\Delta\mu \geq \rho\sigma^2$, $\hat{\alpha}$ is well defined as long as $\theta < \rho\sigma^2/\Delta\mu$, i.e., as long as the fraction of optimists κ is less than $\frac{\pi\Delta\mu}{\Delta\mu-\rho\sigma_2^2} - \pi$. We now check whether (19) and (20) are slack. Simplifying (19) we obtain

$$\frac{\rho\sigma^2}{2}\hat{\alpha}^2 \geq (1-\hat{\alpha})(1-\theta)\Delta\mu.$$

Substituting (59) into the inequality we obtain

$$\frac{\Delta\mu}{2}\theta^2 - (\Delta\mu + \rho\sigma^2)\theta + \rho\sigma^2 \leq 0.$$

The inequality is satisfied strictly when

$$\theta > 1 - \left[\sqrt{1+\left(\frac{\rho\sigma^2}{\Delta\mu}\right)^2} - \frac{\rho\sigma^2}{\Delta\mu}\right],$$

or

$$\frac{\kappa}{\pi + \kappa} > 1 - \bar{\pi},$$

where

$$\bar{\pi} = \sqrt{1 + \left(\frac{\rho\sigma^2}{\Delta\mu}\right)^2} - \frac{\rho\sigma^2}{\Delta\mu}.$$

The inequality is equivalent to

$$\kappa > \frac{\pi}{\bar{\pi}} - \pi.$$

Since $\kappa \in (0, 1-\pi]$ and $\bar{\pi} \in (0,1)$, the inequality only makes sense when $\pi < \bar{\pi}$. From (55) and (59), the cost of signaling associated with $\hat{\alpha}$ is:

$$C(\hat{\alpha}) = (1-\hat{\alpha})\theta\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2\sigma^2 = \theta\left(1 - \frac{\hat{\alpha}}{2}\right)\Delta\mu < \Delta\mu. \tag{60}$$

The inequality in (60) implies that (20) is slack, i.e., $\lambda_2 = 0$. Hence, if $\pi < \bar{\pi}$ and either (a) $1 - \frac{\rho\sigma^2}{\Delta\mu} < \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < 1 - \pi$, or (b) $1 - \frac{\rho\sigma^2}{\Delta\mu} \geq \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < \frac{\pi\Delta\mu}{\Delta\mu - \rho\sigma_2^2} - \pi$, then the lcse retained shares are given by (59).

(3) When $\lambda_1 = 0$ and $\lambda_2 > 0$ the cost of signaling is $\Delta\mu$. Hence, this case is never a solution to the problem. This and (2) imply that if either (a) $\pi > \bar{\pi}$, or (b) $\pi < \bar{\pi}$ and $\kappa < \frac{\pi}{\bar{\pi}} - \pi$, then the lcse retained shares are given by (57).

(4) It can never be that both constraints bind, i.e., $\lambda_1 > 0$ and $\lambda_2 > 0$. Suppose, by contradiction, that both constraints bind, i.e., $(1-\alpha)(1-\theta)\Delta\mu = \frac{\rho}{2}\alpha^2\sigma^2$ and $[1-(1-\alpha)\theta]\,\Delta\mu = \frac{\rho}{2}\alpha^2\sigma^2$. Substituting the second equality into the first we obtain $(1-\alpha) - (1-\alpha)\theta = 1 - (1-\alpha)\theta$. This equality is only satisfied when $\alpha = 0$ which contradicts $\alpha > 0$.

Finally, undertaking the project without selling equity, i.e. $\alpha = 1$, is not a solution to the problem since it leads to a lower expected utility than undertaking the project and retaining $\hat{\alpha}$ shares. *Q.E.D.*

Proof of Proposition 3: To prove this result we show that an increase in the fraction of optimists κ lowers welfare in each of the two parts of Proposition 2.

(i) Assume that either (a) $\pi > \bar{\pi}$, or (b) $\pi < \bar{\pi}$ and $\kappa < \frac{\pi}{\bar{\pi}} - \pi$. In this case the incentive constraint of a realist with a low expected value project is binding. Hence, from the perspective of an outside observer, the expected utility of an optimist is the same as the utility of a realist with a low expected value project. As a consequence, to show that an increase in κ lowers welfare we only need to show that an increase in κ raises the cost of signaling of a realist with a high expected value project $C(\hat{\alpha})$. We know from Proposition 2 that in each case $C(\hat{\alpha}) = (1 - \hat{\alpha})\Delta\mu$, where $\hat{\alpha}$ is given by (57). Since the right-hand side of (57) decreases with θ it follows that $C(\hat{\alpha}) = (1 - \hat{\alpha})\Delta\mu$ increases with θ. Hence, $\partial\hat{W}/\partial\kappa < 0$.

(ii) Assume that $\pi < \bar{\pi}$ and either (a) $1 - \frac{\rho\sigma^2}{\Delta\mu} < \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < 1 - \pi$, or (b) $1 - \frac{\rho\sigma^2}{\Delta\mu} \geq \pi$ and $\frac{\pi}{\bar{\pi}} - \pi < \kappa < \frac{\pi\Delta\mu}{\Delta\mu - \rho\sigma_2^2} - \pi$. Let us start by considering what is the impact of an increase in the fraction of optimists on the expected utility of a realist with a high expected value project. In this case we know from (60) that the cost of signaling of a realist with a high expected value project is

$$C(\hat{\alpha}) = \theta\left(1 - \frac{\hat{\alpha}}{2}\right)\Delta\mu = \theta\Delta\mu - \frac{\theta^2}{2}\frac{(\Delta\mu)^2}{\rho\sigma^2}.$$

Therefore $\frac{dC(\hat{\alpha})}{d\theta} = \left(1 - \frac{\theta\Delta\mu}{\rho\sigma^2}\right)\Delta\mu = (1 - \hat{\alpha})\Delta\mu > 0$. Hence, an increase in κ raises the cost of signaling and lowers the expected utility of a realist with a high expected value project. Let us now consider what is the impact of an increase in κ on the expected utility of an optimist. From the perspective of an outside observer, the expected utility of an optimist is

$$-\exp\left\{-\rho\left[w_0 - k + \mu_2 - \theta\Delta\mu - \left(\theta - \frac{\theta^2}{2}\right)\frac{(\Delta\mu)^2}{\rho\sigma^2}\right]\right\}.$$

This expression shows that an increase in κ lowers the expected utility of an optimist. Hence, $\partial\hat{W}/\partial\kappa < 0$. *Q.E.D.*

Proof of Proposition 4: The date 1 expected utility of a realist with a high expected value project is equal to

$$\begin{aligned} E[u(\tilde{w}_1(\mu_2, \sigma_2^2))] \;=\; & -\exp\left\{-\rho\left\{w_0 - k + \mu_2 - \left[(1-\alpha)D + (1-\alpha r)\theta\Delta\mu\right.\right.\right. \\ & \left.\left.\left. + \frac{\rho}{2}\alpha^2\sigma_2^2 + \frac{\rho}{2}\alpha^2\theta^2 r(1-r)(\Delta\mu)^2\right]\right\}\right\}. \end{aligned} \tag{61}$$

To be a lcse $\hat{\alpha}$ and $\hat{D}$ must be selected to maximize (61) or, equivalently, to minimize the cost of signaling

$$C(\alpha, D) = (1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\sigma_2^2 + \frac{\rho}{2}\alpha^2\theta^2 r(1-r)(\Delta\mu)^2, \tag{62}$$

subject to the incentive constraints (27) and (28), $D \in [0, \mu_2 - \theta\Delta\mu]$ and $\alpha \in [0, 1]$:

$$\min_{\alpha,D} \left\{(1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]\right\}$$

$$\begin{aligned}
\text{s.t.}\quad & \mu_1 \geq \mu_2 - \alpha r\Delta\mu \\
& \qquad - \left\{(1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right]\right\} \\
& \mu_2 - \left\{(1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\left[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2\right]\right\} \geq \mu_1 \\
& D \geq 0 \\
& \mu_2 - \theta\Delta\mu \geq D \\
& 1 \geq \alpha \\
& \alpha \geq 0
\end{aligned}$$

Let λ_1, λ_2, λ_3, λ_4, λ_5, and λ_6 denote the Lagrange multipliers of the first, second, third, fourth, fifth, and sixth constraints, respectively. Assume that the fourth, fifth, and sixth constraints are slack, i.e., $\lambda_4 = \lambda_5 = \lambda_6 = 0$ (later on we will see that these assumptions are satisfied). Suppose, by contradiction, that $\sigma_2^2 \leq \sigma_1^2 + (1-2\theta)r(1-r)(\Delta\mu)^2$ and that the problem has a solution with $\alpha > 0$ and $D > 0$. If $D > 0$, then the incentive constraint of a realist with a low expected value project must bind, i.e., $\lambda_1 > 0$. If that were not the case, it would be possible for a realist with a high expected value project to reduce D without violating the two incentive constraints. This would lead to a lower cost of signaling which would contradict least cost separation. Hence, the efficient separating equilibrium must minimize C, subject to the first incentive constraint as an equality, i.e.,

$$\min_{\alpha,D} \left\{(1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]\right\}$$

$$\begin{aligned}
\text{s.t.}\quad & \mu_1 = \mu_2 - \alpha r\Delta\mu \\
& \qquad - \left\{(1-\alpha)D + (1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right]\right\}
\end{aligned}$$

The first incentive constraint can be rewritten as

$$\begin{aligned} 0 &= (1-\alpha r)\Delta\mu - (1-\alpha)D - (1-\alpha r)\theta\Delta\mu - \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2] \\ &\quad + \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2] - \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right], \end{aligned}$$

or

$$0 = (1-\alpha r)\Delta\mu - C - \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-2\theta)r(1-r)(\Delta\mu)^2 - \sigma_2^2\right]. \tag{63}$$

If the efficient separating equilibrium has $D > 0$, it must have a lower α and a lower C than the schedule in which $D = 0$ and α equals the lowest α that makes (63) an equality. But then the first incentive constraint is violated since lowering C and lowering α raises the right side of (63). Therefore, when $\sigma_2^2 \leq \sigma_1^2 + (1-2\theta)r(1-r)(\Delta\mu)^2$ we must have $D = 0$. When $D = 0$ the problem becomes

$$\begin{aligned} \min_{\alpha\in[0,1]} \quad & \left\{(1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]\right\} \\ \text{s.t.} \quad & \mu_1 \geq \mu_2 - \alpha r\Delta\mu - (1-\alpha r)\theta\Delta\mu - \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right] \\ & \mu_2 - (1-\alpha r)\theta\Delta\mu - \frac{\rho}{2}\alpha^2\left[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2\right] \geq \mu_1. \end{aligned}$$

It can never be that $\alpha = 0$ since this violates the first incentive constraint. So, it must be that $\alpha > 0$. We have four possibilities: (1) $\lambda_1 > 0$ and $\lambda_2 = 0$, (2) $\lambda_1 = 0$ and $\lambda_2 = 0$, (3) $\lambda_1 = 0$ and $\lambda_2 > 0$, and (4) $\lambda_1 > 0$ and $\lambda_2 > 0$.

(1) Let $\lambda_1 > 0$ and $\lambda_2 = 0$. In this case the first constraint is binding:

$$\mu_1 = \mu_2 - \alpha r\Delta\mu - (1-\alpha r)\theta\Delta\mu - \frac{\rho}{2}\alpha^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right],$$

Solving for α we obtain

$$\alpha = \frac{r(1-\theta)\Delta\mu}{\rho\sigma_I^2}\left[\pm\sqrt{1 + 2\frac{\rho\sigma_I^2}{r^2(1-\theta)\Delta\mu}} - 1\right],$$

where $\sigma_I^2 = \sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2$. We have only one positive root. Hence, the α that makes the first incentive constraint an equality is

$$\hat{\alpha} = \frac{r(1-\theta)\Delta\mu}{\rho\sigma_I^2}\left[\sqrt{1+2\frac{\rho\sigma_I^2}{r^2(1-\theta)\Delta\mu}}-1\right].$$

For this solution to be well defined it must be that $\hat{\alpha} < 1$:

$$\frac{r(1-\theta)\Delta\mu}{\rho\sigma_I^2}\left[\sqrt{1+2\frac{\rho\sigma_I^2}{r^2(1-\theta)\Delta\mu}}-1\right] < 1.$$

This inequality is equivalent to

$$1+2\frac{\rho\sigma_I^2}{r^2(1-\theta)\Delta\mu} < 1+2\frac{\rho\sigma_I^2}{r(1-\theta)\Delta\mu}+\frac{\rho^2\sigma_I^4}{r^2(1-\theta)^2(\Delta\mu)^2},$$

or

$$\rho > \frac{2(1-r)(1-\theta)\Delta\mu}{\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2},$$

which is (33). The cost of signaling of $\hat{\alpha}$ is

$$\begin{aligned}
C(\hat{\alpha}) &= (1-\hat{\alpha}r)\theta\Delta\mu+\frac{\rho}{2}\hat{\alpha}^2\sigma_{II}^2 \\
&= (1-\hat{\alpha}r)\theta\Delta\mu+\frac{\rho}{2}\hat{\alpha}^2\sigma_{II}^2+(1-\hat{\alpha}r)\Delta\mu-(1-\hat{\alpha}r)\theta\Delta\mu-\frac{\rho}{2}\hat{\alpha}^2\sigma_I^2 \\
&= (1-\hat{\alpha}r)\Delta\mu-\frac{\rho}{2}\hat{\alpha}^2\left[\sigma_1^2+(1-2\theta)r(1-r)(\Delta\mu)^2-\sigma_2^2\right] < \Delta\mu,
\end{aligned}$$

with $\sigma_{II}^2 = \sigma_2^2+\theta^2 r(1-r)(\Delta\mu)^2$, and where the inequality follows from the assumption $\sigma_2^2 \leq \sigma_1^2+(1-2\theta)r(1-r)(\Delta\mu)^2$. Since the cost of signaling is less than $\Delta\mu$, we confirm that the second incentive constraint is slack, i.e., $\lambda_2 = 0$. We need to check that, if the project's type is not revealed until date 1, then realists with high expected value projects and optimists sell the remaining α shares in date 1. This is the case when the number of optimists satisfies $\theta \leq \frac{\rho}{2}\hat{\alpha}\frac{\sigma_2^2}{\Delta\mu}$ or $\hat{\alpha} \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2}$. This inequality is equivalent to

$$\frac{r(1-\theta)\Delta\mu}{\rho\sigma_I^2}\left[\sqrt{1+2\frac{\rho\sigma_I^2}{r^2(1-\theta)\Delta\mu}}-1\right] \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2},$$

which can be simplified to

$$\rho \geq \frac{2\theta\Delta\mu}{\sigma_2^2}\left[r + \frac{\theta}{1-\theta}\frac{\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2}\right].$$

The right side is equal to zero when $\theta = 0$ and is increasing in θ. This means that $\theta \leq \frac{\rho}{2}\hat{\alpha}\frac{\sigma_2^2}{\Delta\mu}$ is satisfied when $\theta \leq \underline{\theta}$ where $\underline{\theta}$ satisfies

$$\rho = \frac{2\underline{\theta}\Delta\mu}{\sigma_2^2}\left[r + \frac{\underline{\theta}}{1-\underline{\theta}}\frac{\sigma_1^2 + (1-\underline{\theta})^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2}\right].$$

(2) Let $\lambda_1 = 0$ and $\lambda_2 = 0$. In this case the problem becomes

$$\min_{\alpha\in[0,1]} \left\{(1-\alpha r)\theta\Delta\mu + \frac{\rho}{2}\alpha^2[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]\right\}.$$

The first-order condition is

$$-r\theta\Delta\mu + \rho\alpha[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2] = 0,$$

and we obtain

$$\hat{\alpha} = \frac{r\theta\Delta\mu}{\rho[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]}.$$

The second-order condition is satisfied since $d^2C(\alpha)/d\alpha^2 = \rho[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2] > 0$. We need to check that, if the project's type is not revealed until date 1, then realists with high expected value projects and optimists sell the remaining α shares in date 1. This is the case when the number of optimists satisfies $\theta \leq \frac{\rho}{2}\hat{\alpha}\frac{\sigma_2^2}{\Delta\mu}$ or $\hat{\alpha} \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2}$. This inequality is equivalent to

$$\frac{r\theta\Delta\mu}{\rho[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]} \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2},$$

which is false since the left side is lower than the right side. Hence, $\lambda_1 = 0$ and $\lambda_2 = 0$ is not a solution to the problem.

(3) When $\lambda_1 = 0$ and $\lambda_2 > 0$ the cost of signaling is $\Delta\mu$ so this can never be a solution to the problem.

(4) It can never be that both constraints bind, i.e., $\lambda_1 > 0$ and $\lambda_2 > 0$. Suppose, by contradiction, that both constraints bind, i.e., $\frac{\rho}{2}\alpha^2\sigma_I^2 = (1-\alpha r)(1-\theta)\Delta\mu$ and $\frac{\rho}{2}\alpha^2\sigma_{II}^2 =$

$(1-\theta+\alpha\theta r)\Delta\mu$. Substituting the second equality into the first and rearranging terms we obtain $\sigma_I^2(1-\theta+\alpha\theta r) = \sigma_{II}^2(1-\theta-\alpha r+\alpha\theta r)$. This equality is false since $\sigma_{II}^2 \leq \sigma_I^2$ and $1-\theta-\alpha r+\alpha\theta r < 1-\theta+\alpha\theta r$. *Q.E.D.*

Proof of Proposition 5: We start by showing that underpricing alone, i.e., $\alpha = 0$ and $D > 0$, is never a solution to the cost minimization problem. Suppose, by contradiction, that in the solution to the problem we have $\alpha = 0$ and $D > 0$. The problem becomes:

$$\begin{aligned} &\min_{D\in[0,\mu_2-\theta\Delta\mu]} \quad D \\ &\text{s.t.} \quad \mu_1 \geq \mu_2 - D - \theta\Delta\mu \\ &\qquad\quad \mu_2 - D - \theta\Delta\mu \geq \mu_1 \end{aligned}$$

The unique solution is $D = (1-\theta)\Delta\mu$. Note that $D = 0$ violates the first constraint and $D = \mu_2 - \theta\Delta\mu$ violates the second constraint. However, $D = (1-\theta)\Delta\mu$ cannot be a solution since it implies pooling:

$$P_2 = \mu_2 - \theta\Delta\mu - D = \mu_2 - \theta\Delta\mu - (1-\theta)\Delta\mu = \mu_1.$$

Hence, underpricing alone is never a solution to the cost minimization problem. Let's therefore assume that when conditions (35), (36), and (37) are satisfied the solution is $\alpha > 0$ and $D > 0$. Suppose, by contraction, that there is a solution with $\alpha > 0$ and $D > 0$ where the first constraint is slack, i.e., $\lambda_1 = 0$. This cannot be a solution since it would be possible to reduce D while still satisfying the first constraint but at a lower cost of signaling. So, $D > 0$ implies $\lambda_1 > 0$. Therefore, in a solution with $\alpha > 0$ and $D > 0$ we only need to consider two cases: (1) $\lambda_1 > 0$ and $\lambda_2 = 0$, and (2) $\lambda_1 > 0$ and $\lambda_2 > 0$.

(1) When $\lambda_1 > 0$ and $\lambda_2 = 0$ we have

$$\hat{D} = (1-\theta)\frac{1-\alpha r}{1-\alpha}\triangle\mu - \frac{\rho}{2}\frac{\alpha^2}{1-\alpha}[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2].$$

Substituting $\hat{D}$ in the objective function we obtain:

$$\min_{\alpha\in[0,1]} \left\{(1-\alpha r)\Delta\mu + \frac{\rho}{2}\alpha^2\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right]\right\}.$$

The first-order condition is

$$-r\Delta\mu + \rho\alpha\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right] = 0.$$

The second-order condition is

$$\rho\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right] > 0.$$

The second order condition is satisfied given (35). The solution to the first-order condition is:

$$\hat{\alpha} = \frac{r\triangle\mu}{\rho\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right]}.$$

Note that $\hat{\alpha}$ is well defined by (37). We now show that $\hat{D} > 0$. This is the case as long as

$$(1-\theta)(1-\hat{\alpha}r)\Delta\mu > \frac{\rho}{2}\hat{\alpha}^2\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right],$$

or

$$1 > \hat{\alpha}r\left[1 + \frac{1}{2(1-\theta)}\frac{\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2}\right],$$

or

$$\rho > \frac{r\Delta\mu\left[r + \frac{r}{2(1-\theta)}\frac{\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2}{\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2}\right]}{\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2},$$

which is implied by (37). The cost of signaling of $\hat{\alpha}$ and $\hat{D}$ is

$$\begin{aligned} C(\hat{\alpha}, \hat{D}) &= (1-\hat{\alpha})\hat{D} + (1-\hat{\alpha}r)\theta\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2\sigma_{II}^2 \\ &= (1-r\hat{\alpha})\Delta\mu - \frac{\rho}{2}\hat{\alpha}^2\sigma_I^2 + \frac{\rho}{2}\hat{\alpha}^2\sigma_{II}^2 \\ &= \left(1 - \frac{r\hat{\alpha}}{2}\right)\triangle\mu < \Delta\mu. \end{aligned} \tag{64}$$

This confirms that the second incentive constraint is slack, i.e., $\lambda_2 = 0$. We need to check that, if the project's type is not revealed until date 1, then realists with high expected value projects and optimists sell the remaining α shares in date 1. This is the case when the number of optimists satisfies $\theta \leq \frac{\rho}{2}\hat{\alpha}\frac{\sigma_2^2}{\Delta\mu}$ or $\hat{\alpha} \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2}$. This inequality is equivalent to

$$\frac{r\triangle\mu}{\rho\left[\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2\right]} \geq \frac{2\theta\Delta\mu}{\rho\sigma_2^2},$$

which, after some algebra, is equivalent to

$$4r(1-r)(\Delta\mu)^2\theta^2+2\left[\sigma_2^2-\sigma_1^2-r(1-r)(\Delta\mu)^2\right]\theta-r\sigma_2^2 \leq 0.$$

The roots of the associated quadratic equation are

$$\theta=\frac{\sigma_2^2-\sigma_1^2-r(1-r)(\Delta\mu)^2}{4r(1-r)(\Delta\mu)^2}\left\{\pm\sqrt{1+\frac{4r^2(1-r)(\Delta\mu)^2\sigma_2^2}{\left[\sigma_2^2-\sigma_1^2-r(1-r)(\Delta\mu)^2\right]^2}}-1\right\}.$$

We have only one positive root. Hence, $\theta \leq \frac{\rho}{2}\hat{\alpha}\frac{\sigma_2^2}{\Delta\mu}$ as long as

$$\theta \leq \frac{\sigma_2^2-\sigma_1^2-r(1-r)(\Delta\mu)^2}{4r(1-r)(\Delta\mu)^2}\left\{\sqrt{1+\frac{4r^2(1-r)(\Delta\mu)^2\sigma_2^2}{\left[\sigma_2^2-\sigma_1^2-r(1-r)(\Delta\mu)^2\right]^2}}-1\right\},$$

which is (36).
(2) When $\lambda_1 > 0$ and $\lambda_2 > 0$ the cost of signaling is $\Delta\mu$ so this case can never be a solution to the problem.
To complete the proof we need to show that if conditions (35), (36), and (37) are satisfied, then $\alpha > 0$ and $D = 0$ is not a solution to the problem. We know from Proposition 4 that if $\alpha > 0$ and $D = 0$, then there are two possible solutions: (i) $\lambda_1 > 0$ and $\lambda_2 = 0$, and (ii) $\lambda_1 = 0$ and $\lambda_2 = 0$. We start with case (i). When $\alpha > 0$ and $D = 0$ with $\lambda_1 > 0$ and $\lambda_2 = 0$, the solution is

$$\hat{\alpha}_1=\frac{r(1-\theta)\Delta\mu}{\rho\left[\sigma_1^2+(1-\theta)^2r(1-r)(\Delta\mu)^2\right]}\left[\sqrt{1+2\frac{\rho\left[\sigma_1^2+(1-\theta)^2r(1-r)(\Delta\mu)^2\right]}{r^2(1-\theta)\Delta\mu}}-1\right].$$

The cost of signaling of $\hat{\alpha}_1$ is

$$C(\hat{\alpha}_1)=(1-\hat{\alpha}_1r)\theta\Delta\mu+\frac{\rho}{2}\hat{\alpha}_1^2\sigma_{II}^2=(1-\hat{\alpha}_1r)\Delta\mu+\frac{\rho}{2}\hat{\alpha}_1^2(\sigma_{II}^2-\sigma_I^2), \tag{65}$$

where the second equality comes from the fact that the first incentive constraint is binding. The cost of signaling of $\hat{\alpha}$ and $\hat{D}$, given by (38) and (39), respectively, is

$$C(\hat{\alpha},\hat{D}) = (1-\hat{\alpha})\hat{D} + (1-\hat{\alpha}r)\theta\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2\sigma_{II}^2 = (1-\hat{\alpha}r)\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2(\sigma_{II}^2-\sigma_I^2), \qquad (66)$$

where the second equality comes from the fact that the first incentive constraint is binding. It follows from (65) and (66) that $C(\hat{\alpha},\hat{D})$ is less than $C(\hat{\alpha}_1)$ as long as

$$(1-\hat{\alpha}r)\Delta\mu + \frac{\rho}{2}\hat{\alpha}^2(\sigma_{II}^2-\sigma_I^2) < (1-\hat{\alpha}_1 r)\Delta\mu + \frac{\rho}{2}\hat{\alpha}_1^2(\sigma_{II}^2-\sigma_I^2),$$

or

$$\hat{\alpha}_1 r\Delta\mu - \frac{\rho}{2}\hat{\alpha}_1^2(\sigma_{II}^2-\sigma_I^2) < \hat{\alpha}r\Delta\mu - \frac{\rho}{2}\hat{\alpha}^2(\sigma_{II}^2-\sigma_I^2),$$

or, assuming $\hat{\alpha}_1 > \hat{\alpha}$, $(\hat{\alpha}_1-\hat{\alpha})r\Delta\mu < \frac{\rho}{2}(\hat{\alpha}_1^2-\hat{\alpha}^2)(\sigma_{II}^2-\sigma_I^2)$, or $r\Delta\mu < \frac{\rho}{2}(\hat{\alpha}_1+\hat{\alpha})(\sigma_{II}^2-\sigma_I^2)$, or $\frac{2r\Delta\mu}{\rho(\sigma_{II}^2-\sigma_I^2)} < \hat{\alpha}_1+\hat{\alpha}$, or $2\hat{\alpha} < \hat{\alpha}_1+\hat{\alpha}$, or $\hat{\alpha}_1 > \hat{\alpha}$. We still need to show that $\hat{\alpha}_1 > \hat{\alpha}$. This is the case as long as

$$\frac{r(1-\theta)\Delta\mu}{\rho\left[\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2\right]}\left[\sqrt{1+2\frac{\rho\left[\sigma_1^2+(1-\theta)^2 r(1-r)(\Delta\mu)^2\right]}{r^2(1-\theta)\Delta\mu}}-1\right]$$
$$> \frac{r\Delta\mu}{\rho\left[\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2\right]}.$$

It is easy to show that this inequality is satisfied when

$$\rho > \frac{r\Delta\mu\left[r+\frac{r}{2(1-\theta)}\frac{\sigma_1^2+(1-\theta)^2r(1-r)(\Delta\mu)^2}{\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2}\right]}{\sigma_2^2-\sigma_1^2-(1-2\theta)r(1-r)(\Delta\mu)^2},$$

which is implied by (37). Hence, when conditions (35), (36), and (37) are satisfied, $\alpha > 0$ and $D = 0$ with $\lambda_1 > 0$ and $\lambda_2 = 0$ is not a solution to the problem. Let's now consider case (ii). When $\alpha > 0$ and $D = 0$ with $\lambda_1 = 0$ and $\lambda_2 = 0$, the solution is

$$\hat{\alpha}_2 = \frac{r\theta\Delta\mu}{\rho[\sigma_2^2+\theta^2 r(1-r)(\Delta\mu)^2]}.$$

The cost of signaling of $\hat{\alpha}_2$ is

$$C(\hat{\alpha}_2) = \left(1 - \frac{r\hat{\alpha}_2}{2}\right)\theta\Delta\mu. \tag{67}$$

It follows from (64) and (67) that $C(\hat{\alpha}, \hat{D})$ is less than $C(\hat{\alpha}_2)$ as long as $\left(1 - \frac{r\hat{\alpha}}{2}\right)\triangle\mu < \left(1 - \frac{r\hat{\alpha}_2}{2}\right)\theta\Delta\mu$, or $\hat{\alpha} > \theta\hat{\alpha}_2$, or

$$\frac{r\triangle\mu}{\rho\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right]} > \frac{r\theta^2\Delta\mu}{\rho[\sigma_2^2 + \theta^2 r(1-r)(\Delta\mu)^2]},$$

which is true. Hence, when conditions (35), (36), and (37) are satisfied, $\alpha > 0$ and $D = 0$ with $\lambda_1 = 0$ and $\lambda_2 = 0$ is not a solution to the problem. *Q.E.D.*

Proof of Proposition 6: Outside investors are better off in the presence of optimists when $\hat{I}(\theta) > \hat{I}(0)$ or

$$n\pi\left[\hat{U}(\theta) - \left(\frac{\theta}{1-\theta}\right)\hat{O}(\theta)\right] > n\pi\hat{U}(0),$$

or

$$\theta < \frac{\hat{U}(\theta) - \hat{U}(0)}{\hat{U}(\theta) - \hat{U}(0) + \hat{O}(\theta)} = f(\theta). \tag{68}$$

From (68) it follows that

$$\frac{\partial f(\theta)}{\partial\theta} = \frac{\hat{U}'(\theta)\hat{O}(\theta) - \hat{O}'(\theta)[\hat{U}(\theta) - \hat{U}(0)]}{\left[\hat{U}(\theta) - \hat{U}(0) + \hat{O}(\theta)\right]^2} > 0, \tag{69}$$

since $\hat{U}'(\theta) > 0$, $\hat{O}'(\theta) < 0$, $\hat{U}(\theta) > \hat{U}(0)$, and $\hat{O}(\theta) > 0$. We see that $f(\theta)$ is an increasing and continuous function of θ with $f(0) = 0$ and $f(1) < 1$. Hence, for the inequality (68) to be satisfied for a non-empty set of θ it must be that

$$\left.\frac{\partial f(\theta)}{\partial\theta}\right|_{\theta=0} > 1. \tag{70}$$

From (69) it follows that

$$\left.\frac{\partial f(\theta)}{\partial\theta}\right|_{\theta=0} = \frac{\hat{U}'(0)\hat{O}(0)}{\left[\hat{O}(0)\right]^2} = \frac{\hat{U}'(0)}{\hat{O}(0)}.$$

So, (70) is equivalent to $\hat{U}'(0) > \hat{O}(0)$. We know that

$$\hat{U}(\theta) = (1-\hat{\alpha}r)\triangle\mu - \frac{\rho}{2}\hat{\alpha}^2[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2].$$

The derivative of $\hat{U}(\theta)$ with respect to θ is

$$\hat{U}'(\theta) = -\frac{\partial\hat{\alpha}}{\partial\theta}r\triangle\mu - \rho\hat{\alpha}\frac{\partial\hat{\alpha}}{\partial\theta}\left[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2\right] + \rho\hat{\alpha}^2(1-\theta)r(1-r)(\Delta\mu)^2.$$

From (38) we have

$$\frac{\partial\hat{\alpha}}{\partial\theta} = -\frac{2r^2(1-r)(\Delta\mu)^3}{\rho\left[\sigma_2^2 - \sigma_1^2 - (1-2\theta)r(1-r)(\Delta\mu)^2\right]^2}.$$

Evaluating $\hat{U}'(\theta)$ at $\theta = 0$ we obtain

$$\hat{U}'(0) = \frac{r^2(\triangle\mu)^2[r(1-r)(\Delta\mu)^2]}{\rho\left[\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2\right]^2}\left[3 + 2\frac{\sigma_1^2 + r(1-r)(\Delta\mu)^2}{\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2}\right].$$

We know that

$$\hat{O}(\theta) = \frac{\rho}{2}\hat{\alpha}^2[\sigma_1^2 + (1-\theta)^2 r(1-r)(\Delta\mu)^2].$$

Hence,

$$\hat{O}(0) = \frac{1}{2}\frac{r^2(\triangle\mu)^2[\sigma_1^2 + r(1-r)(\Delta\mu)^2]}{\rho\left[\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2\right]^2}.$$

So, (70) is equivalent to

$$3 + 2\frac{\sigma_1^2 + r(1-r)(\Delta\mu)^2}{\sigma_2^2 - \sigma_1^2 - r(1-r)(\Delta\mu)^2} > \frac{1}{2}\frac{\sigma_1^2 + r(1-r)(\Delta\mu)^2}{r(1-r)(\Delta\mu)^2}.$$

Solving this inequality for $r(1-r)(\Delta\mu)^2$ we obtain (46). *Q.E.D.*

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