Unconventional Monetary Policy in a Currency Union with Segmentation in the Market for Government Debt

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Abstract

The literature on large-scale purchases of government debt emphasises the importance of bond market segmentation along the maturity dimension for their transmission. This study investigates how another form of segmentation that we observe, the segmentation of government bond markets across countries, can be exploited by the central bank of a currency union in which fiscal coordination is not attainable. Under general conditions, government bond purchases which lower bond yields have first-order effects through a fiscal channel, even in the absence of the heterogeneity in investment opportunities found in Chen et al. (2012). The total effect on aggregate demand can be broken down into an "income-from-debt-issuance effect" and a "primary-surplus effect". If there is cross-country segmentation in bond markets and home bias in government spending, the central bank is able to use government bond purchases to control the terms of trade and achieve asymmetric degrees of stimulus across the members of the currency union without a transfer of resources. I characterise the welfare-optimising mix of conventional and unconventional monetary policy in this scenario and give an upper bound on the welfare benefits from using the unconventional tool.

Keywords: Unconventional Monetary Policy, Quantitative Easing, Policy Coordination, Monetary Union, Market Segmentation
JEL-Classification: E50, E52, E58, F45

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1 Introduction

The member countries of a currency union share a common monetary policy institution—the ECB in the case of the euro area. As a result, the widely-discussed tension between a monetary policy stance that is common towards all members and less than perfectly correlated business cycles across member countries arises. While, for example, a strong stimulus might be needed in some parts of the union, other parts could require only weakly expansionary policy, forcing the centralised policy maker to adopt an intermediate strategy that may in fact be individually suboptimal for most if not all member states. It is often argued that close fiscal policy coordination is required to address this issue. Governments, however, are subject to constraints that make doubtful whether fiscal policy can fulfil the role that it would optimally play in smoothing idiosyncratic shocks, especially since, in the absence of a unified institutional framework, fiscal policy authorities are likely to put higher weight on domestic welfare compared to welfare in the union as a whole. This paper takes a lack of fiscal coordination as given and investigates if unconventional monetary policy, more precisely central bank purchases of government bonds, can be used to stabilise idiosyncratic disturbances within a currency union.

This is the case if a) government bond yields across the union respond to central bank purchases of government debt which are targeted at specific countries in an asymmetric way and b) these yield responses lead to asymmetric changes in aggregate demand and inflation. Greenwood and Vayanos (2010) and Krishnamurthy and Vissing-Jorgensen (2012) show that, in accordance with the preferred habitat theory of the term structure, investor clienteles exist that have a preference for bonds of particular maturities, which implies that bond yields respond to demand and supply effects local to individual maturities. This paper starts by demonstrating that the market for government debt is segmented not only along the maturity dimension but also across country borders and that, as a result, country-specific demand and supply for government bonds affect their yields, validating a) independent from the effects of bond purchases on sovereign default risk premia.

It then uses a model of a currency union to derive general conditions under which an unconventional intervention by the central bank that lowers government bond yields is expansionary. These conditions do not rely on the financial market imperfections assumed in Chen et al. (2012) but rather on explicit modelling of government finances. The negative effect of government bond yields on aggregate demand is transmitted through a fiscal channel, which can be disaggregated into an “income-from-debt-issuance effect” and a “primary-surplus effect”. Intuitively, government bond purchases increase their scarcity, lower their yield and increase their price provided that bond issuance remains sufficiently passive. Thus, the government budget constraint is relaxed, which allows taxes to fall and spending to increase. Since higher spending leads to inflation that deteriorates a potential real primary surplus, a government that has targets for real taxes and spending may increase spending further. Taking the empirical fact that governments are subject to home-bias in spending as given, part b) from above also holds.

A second aim of the paper is to determine the desirability of introducing government bond purchases as an asymmetric stabilisation tool in the euro area. Note that none of the analysis is crisis-specific. In fact, many important features of the Financial Crisis of 2007-09 and the ensuing sovereign debt crisis are not included in the model. A key ingredient of the model, segmentation in the market for government debt, was exacerbated by the events surrounding the crisis though. Influences such as time-varying degrees of fiscal cooperation and market segmentation are difficult to capture reliably. The strategy employed here is therefore to estimate an upper bound on the benefits obtainable from asymmetric bond purchases, that is, the benefits when there is optimal coordination of monetary policy tools, a minimum of fiscal cooperation and a maximum of government bond market segmentation across member countries. The characteristics of the optimal mix of conventional and unconventional monetary policy are also discussed in detail. The estimated welfare gains, expressed in consumption-equivalent terms of average union-wide utility gains, are generally small but become significant in times of diverging productivity and demand disturbances that cause higher variation in the natural terms of trade. Under these circumstances, controlling the relative price level between the member countries is of particular importance which is not possible using the conventional instrument alone.

The paper is related to the literature on large-scale asset purchases and to that on policy coordination in a currency union. Wallace (1981) and Eggertsson and Woodford (2003) show that, under certain conditions, the size and composition of the central bank's balance sheet is irrelevant. This result equally holds in the special case of the model outlined below in which open market operations are not permitted to affect yield spreads. When this link is allowed to exist, asset purchases by the central bank have real effects through a fiscal channel as in the model laid out by Auerbach and Obstfeld (2005), although the mechanisms differ. In Auerbach and Obstfeld (2005), welfare gains arise from the fact that bond purchases reduce future debt servicing costs and thus the need to raise income through distortionary taxation, while here welfare gains result from asymmetric effects on inflation that lead to reductions in the misallocation of productive activity across countries.[1] Policy coordination in a currency union has been studied predominantly from the perspective of fiscal and standard short-term interest rate policy. Examples include Beetsma and Jensen (2005), Gali and Monacelli (2008) and Ferrero (2009). Reis (2013) discusses the central bank's ability to re-distribute resources between the members of the euro area. Corsetti et al. (2014) show that bond purchases which involve risk-sharing can increase the set of fiscal policies consistent with equilibrium determinacy in a currency area. In accordance with the legal framework of the euro area, the unconventional policy discussed below is constructed to avoid this issue.

[1] Since asset purchases are financed by money creation, the analysis is also related to the recent debate on monetary-financed fiscal stimulus. Gali (2014) shows that an increase in wasteful government spending that is financed directly through seigniorage is welfare-enhancing in a New Keynesian model with a sufficiently-distorted steady state. The unconventional policy discussed below does not fall into this category though, since a spending response emerges endogenously in general equilibrium caused by changes in government bond yields rather than as a result of a coordinated intervention of the central bank and the treasury.

The remainder is organised as follows. Section 2 discusses the evidence on bond market segmentation. Section 3 introduces the model. Section 4 shows a Linear-Quadratic (LQ) approximation to the central bank's optimal policy problem and the calibration used for simulations. Section 5 characterises the optimal policy mix. Section 6 gives estimates of the gains obtainable from using the unconventional tool. The final section concludes. Proofs and derivations are relegated to a short appendix included in the paper and a detailed online appendix.[2]

2 Two Dimensions of Government Bond Market Segmentation

The adoption of unconventional tools by several central banks in the wake of the Financial Crisis has spawned interest in research on segmentation in the market for government debt. This section first reviews findings on segmentation along the maturity dimension and its role in the transmission of unconventional monetary policy. It then presents evidence for a second form of segmentation—imperfect substitutability of sovereign debt across country borders.

Using the UK pension reform of 2004 and the US Treasury buybacks of 2000 to 2002 as examples, Greenwood and Vayanos (2010) argue that government bond demand and supply changes local to a specific maturity have strong effects on the yield at that maturity. While this fact is at odds with standard Euler-equation based theory, it is in accordance with the "preferred habitat" view of the term structure, which dates back at least to Modigliano and Sutch (1966). According to the preferred habitat theory, investor clienteles exist that have preferences for bonds of a particular maturity. Thus, the yield of a bond with a given maturity is influenced by its supply and the demand of the corresponding investor clientele. Evidence in favour of this view is given also in Krishnamurthy and Vissing-Jorgensen (2012), who show that the price of US long-term Treasuries contains a premium that is supply-dependent and can therefore be distinguished from the risk premium predicted, for example, by a standard consumption-based Capital Asset Pricing Model (CAPM).

A preferred habitat channel has been shown to play a significant role in the transmission of unconventional monetary policy interventions aimed at lowering long-term government bond yields through purchases, i.e. reductions in the available supply, of long-term government debt. D'Amico et al. (2012) estimate that preferred habitat effects are responsible for a fall of 18 to 20 basis points in the 10-year yield in response to the Federal Reserve's initial Treasury purchase programme completed in September 2009. Krishnamurthy and Vissing-Jorgensen (2011) give a lower bound of 160 basis points for the aggregate preferred habitat effect of all asset purchases under the Federal Reserve's first Quantitative Easing package (QE1) on the 10-year Treasury yield.[3] McLaren et al. (2014) attribute about half of the total impact of the Bank of England's Quantitative Easing programme in 2009-12 on gilt yields to local supply effects.

[2] The online appendix is available at https://sites.google.com/site/tischbirekandreas/research.

[3] QE1 was announced in 2008 and extended in 2009. It comprises purchases of long-term Treasuries, agency securities and agency-guaranteed mortgage backed securities.

Specific maturities are not the only bond characteristic for which investors are willing to pay a premium. Another such characteristic is the country of issuance. An indication for this type of preference is the large degree of home bias observed in bond markets. Fidora et al. (2007) calculate that between 2001 and 2003 the domestic share of all debt securities held was on average 65.2% in France, 74.3% in Germany and 80.0% in Italy. Different explanations for the existence of home bias in financial asset holdings have been given in the literature, for example, transaction costs, information asymmetries, the quality of institutions and real exchange rate volatility (Fidora et al., 2007). These obstacles to perfect cross-country bond substitutability imply that investor clienteles exist that are willing to pay a premium for bonds issued in a particular country, comparable to those willing to pay a premium for bonds with a particular maturity. As a result, not only maturity-specific but also country-specific demand and supply changes affect bond yields. The following example illustrates this point.

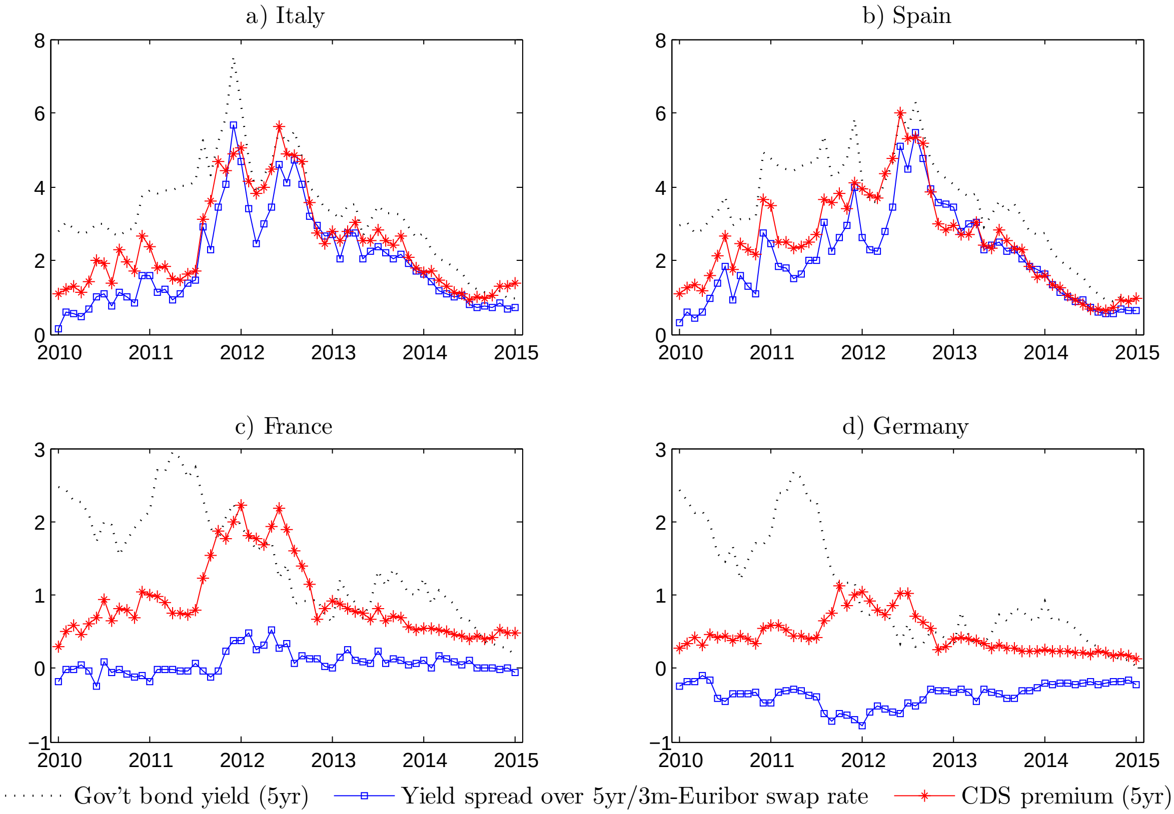

Figure 1: Government bond yields, yield spread over swap rate and CDS premia for selected countries of the euro area (all at 5-year maturity)

Figure 1 plots the 5-year government bond yield, the credit default swap (CDS) premium at 5-year maturity and the spread of the respective government bond yield over a 5-year interest rate swap rate for Italy, Spain, France and Germany at monthly frequency from January 2010 until December 2014. The interest rate swap has the 3-month Euribor as the floating leg. The swap rate is generally viewed as a reliable indicator for what the market considers to be the prevailing risk-free rate.[4] Thus, the yield spread depends positively on sovereign default risk and, if existent, negatively on the local scarcity of government debt.

Increased sovereign default risk caused the CDS premium to rise sharply in all four countries in mid-2011. The yield spread follows the CDS premium closely in Italy and Spain and to a lesser degree in France, suggesting that it is driven in large part by default risk in these countries. In Germany, on the other hand, there is a decline instead of a rise in the yield spread at the time at which the CDS premium rises. This decline can be explained by a "flight to safety" (Battistini et al., 2014). The demand for German government debt rose sharply as the bonds issued by a group of other countries in the euro area ceased to satisfy investors' risk preferences. With increased demand and roughly constant supply, German government bond yields contained a scarcity premium that was large enough to overcompensate the effect of increased sovereign default risk on the yield spread. Thus, changes in the relative scarcity of bonds across countries led to changes in the relative size of their yields.

3 Model

Two countries, "Home" (H) and "Foreign" (F), form a currency union with a common central bank but separate fiscal policy institutions. The countries are indexed by i with $i \in \{H, F\}$, while the agents that inhabit the two countries are indexed by j. For simplicity it is assumed that agents are both producers of a single differentiated good and consumers of a basket of all goods produced in the union. Agents are uniformly distributed on the interval $[0, 1]$, where agents with $j \in [0, n)$ reside in H and those indexed by $j \in [n, 1]$ are located in F.

3.1 Consumer Problem

The preferences of a generic household j in country i are described by the utility function

$$U_t^j = \mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ U(C_T^j) + L\left(\frac{M_T^j}{P_T}\right) - V\left[y_T(j), z_T^i\right] \right\} \tag{1}$$

where $i = H$ if $j \in [0, n)$ and $i = F$ if $j \in [n, 1]$. $\beta \in (0, 1)$ is the discount factor. U and L are increasing and concave functions of consumption and real money holdings, respectively.[5] V is an increasing and convex function of agent j's output $y_T(j)$ and a country-specific supply shock z_T^i. A low draw of z_T^i reflects diminished production costs, which is interpreted as a positive technology shock below. Each household consumes all goods produced in the entire union. C_t^j

[4] Since the principal is not traded in an interest rate swap, counterparty risk premia only concern the interest payments and are therefore small.

[5] U, L and V are assumed to be sufficiently differentiable and U satisfies the standard Inada conditions $\lim_{C_T^j \to 0} U_C(C_T^j) = \infty$ and $\lim_{C_T^j \to \infty} U_C(C_T^j) = 0$.

is a CES aggregate of a basket of goods produced in H, $C_t^{H,j}$, and one produced in F, $C_t^{F,j}$. The elasticity of substitution between the two baskets approaches one, giving rise to Cobb-Douglas preferences over consumption goods from H and F.[6] The weights on domestically and foreign produced goods are set equal to the respective country size, implying

$$C_t^j \equiv \frac{(C_t^{H,j})^n (C_t^{F,j})^{1-n}}{n^n (1-n)^{1-n}} \tag{2}$$

The baskets $C_t^{H,j}$ and $C_t^{F,j}$ are composed as follows

$$C_t^{H,j} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

$$C_t^{F,j} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

$\sigma > 1$ is the elasticity of substitution between goods from the same country. Agent j's demand for generic goods h and f with $h \in [0, n)$ and $f \in [n, 1]$ is

$$c_t^j(h) = \left[\frac{p_t(h)}{P_t^H} \right]^{-\sigma} T_t^{1-n} C_t^j \tag{5}$$

$$c_t^j(f) = \left[\frac{p_t(f)}{P_t^F} \right]^{-\sigma} T_t^{-n} C_t^j \tag{6}$$

where $T_t \equiv P_t^F / P_t^H$ are the terms of trade of country F or, alternatively, the competitiveness of H. The law of one price holds, $p_t^H(j) = p_t^F(j)$ for all $j \in [0, 1]$, which together with the fact that the composition of the basket of goods consumed in both countries is identical implies that purchasing power parity holds. The aggregate price index, found as the expenditure minimising price of one unit of the basket C_t^j, is given by

$$P_t = (P_t^H)^n (P_t^F)^{1-n} \tag{7}$$

The price indices P_t^H and P_t^F that correspond to the basket of home goods and the basket of foreign goods defined in (3) and (4), respectively, are

$$P_t^H = \left[\left(\frac{1}{n} \right) \int_0^n p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \tag{8}$$

$$P_t^F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} \tag{9}$$

[6] The assumption of a unitary elasticity of substitution is made to ensure equilibrium determinacy despite of incomplete financial markets. See Section 3.6 for more details.

Equation (10) is the budget constraint of an agent j in units of the consumption basket that includes goods from both H and F.

$$q_t^i B_t^{i,j} + \frac{B_t^j}{P_t(1+i_t)} + \frac{(1+\xi_t^i)Q_t^{i,j}}{P_t(1+i_t^{Q,i})} + \frac{M_t^j}{P_t} + C_t^j + \frac{T_t^i}{P_t} \leq$$
$$\mathbb{1}_{1\times S} B_{t-1}^{i,j} + \frac{B_{t-1}^j}{P_t} + \frac{Q_{t-1}^{i,j}}{P_t} + \frac{M_{t-1}^j}{P_t} + (1-\tau)\frac{p_t(j)y_t(j)}{P_t} + \frac{\mathcal{D}_t^i}{P_t} + \frac{T_t^{\tau,j}}{P_t} \tag{10}$$

$B_t^{i,j}$ is an $S \times 1$ vector of state-contingent real securities, issued and traded only in the country i in which j resides. There exists a complete set of such state-contingent securities in H and F, where S is the number of possible states in period $t+1$.[7] q_t^i denotes the corresponding vector of security prices. B_t^j are holdings of a non-contingent nominal bond that is traded within and across the two countries. By investing the amount $B_t^j/(1+i_t)$ in period t, agent j secures a payment of B_t^j in $t+1$, where i_t is the one-period nominal interest rate assumed to be directly controlled by the union's central bank. i_t is safe and known in t but not in advance.

$Q_t^{i,j}$ are holdings of a non-contingent nominal government bond that is traded only within j's country of residence i. Cross-border segmentation in the market for government debt implies that the interest paid on this bond $i_t^{Q,i}$ may differ across H and F. As in Chen et al. (2012), agents have to pay a transaction fee of size ξ_t^i for each unit of the bond purchased to a financial intermediary. Each country possesses a, possibly public, institution of this type that returns its profits as dividends in equal parts to the households of the country in which it is located. The nominal amount of dividends received by a household residing in country i is $\mathcal{D}_t^i$.

In addition, agent j has nominal money holdings M_t^j, consumes C_t^j units of the basket defined above, pays lump-sum taxes at the nominal value of T_t^i and receives income from the investments made in the previous period and from production. The latter is subject to a proportional tax τ. As in Rotemberg and Woodford (1998), τ is set in a way to ensure that output in the long-run equilibrium of the economy is at its efficient level, which will be shown below to imply that $\tau < 0$. To neutralise the effect of this subsidy on the households' and the governments' budget constraints, households are assumed to receive a lump-sum transfer of size $T_t^{\tau,j} = \tau p_t(j)y_t(j)$ from their local government.

As initial endowments, agents hold the assets supplied in the long-run equilibrium of the economy in equal proportions. This means $B_{-1}^{i,j} = B_{-1}^j = 0$, $Q_{-1}^{i,j} = \bar{Q}^i/n^i$ with $n^i = n$ if $i = H$ and $n^i = 1-n$ if $i = F$ and $M_{-1}^j = \bar{M}$, where $\bar{Q}^i$ and $\bar{M}$ are the quantity of government debt issued by country i and money supply in the non-stochastic steady state, respectively. Since all agents of a country have identical initial wealth and identical preferences and financial markets are complete within each country, there is perfect risk sharing in consumption within H and F so that, in all periods $t \geq 0$, $C_t^j = C_t^H$ for all $j \in [0,n)$ and $C_t^j = C_t^F$ for all $j \in [n,1]$. The

[7] The state-contingent securities are assumed to be in zero net supply, which means that $\int_0^n B_t^{H,j} dj$ and $\int_n^1 B_t^{F,j} dj$ are equal to an $S \times 1$ vector of zeros.

consumer problem can thus be analysed for representative agents from H and F. An optimum in the agents' utility-maximisation problem, where agents are now indexed by $i \in \{H, F\}$, is characterised by

$$1 = \beta \mathrm{E}_t \left[\frac{U_C(C^i_{t+1})}{U_C(C^i_t)} \frac{1}{\Pi_{t+1}} \right] (1 + i_t) \tag{11}$$

$$1 + \xi^i_t = \beta \mathrm{E}_t \left[\frac{U_C(C^i_{t+1})}{U_C(C^i_t)} \frac{1}{\Pi_{t+1}} \right] \left(1 + i^{Q,i}_t\right) \tag{12}$$

$$\frac{M^i_t}{P_t} = \min \left\{ L^{-1}_{M/P} \left[\frac{i_t}{1 + i_t} U_C(C^i_t) \right], \frac{\bar{M}^i_t}{n^i P_t} \right\} \tag{13}$$

$\Pi_{t+1} \equiv P_{t+1}/P_t$ denotes inflation. Equation (11) is the Euler equation associated with the non-contingent security that is traded between all agents of the union. Equation (12) is the corresponding condition used to price government debt. Equation (13) shows that in an interior optimum, money demand is determined by the interest rate, the marginal utility of consumption and the sub-utility function $L(\cdot)$. If money demand in the interior optimum exceeds the real per-capita quantity of money that the central bank supplies to country i, $\bar{M}^i_t/(n^i P_t)$, the representative household of i demands the maximum amount available to it. The conditions (11) to (13) as well as the way in which ξ^i_t is determined play an important role in the transmission mechanism of unconventional monetary policy, which is discussed in detail in Section 3.5.

3.2 Producer Problem

Only a fraction $1 - \alpha^i$ of the firms resident in country $i \in \{H, F\}$ can change the price of their respective good (Calvo, 1983). All firms that reside in the same country and are able to adjust their price in a given period face the same profit maximisation problem. Let $p^*_t(h)$ be the price that a firm $h \in [0, n)$ chooses if it can re-optimise its price in t and let $p^*_t(f)$ be equivalently defined for a firm $f \in [n, 1]$. Then P^H_t and P^F_t evolve according to

$$P^H_t = \left[\alpha^H \left(P_{H,t-1}\right)^{1-\sigma} + \left(1 - \alpha^H\right) p^*_t(h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{14}$$

$$P^F_t = \left[\alpha^F \left(P_{F,t-1}\right)^{1-\sigma} + \left(1 - \alpha^F\right) p^*_t(f)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{15}$$

A generic firm j that is able to adjust its price in period t sets $p_t(j)$ to maximise the expected discounted stream of future profits given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\alpha^i \beta\right)^{T-t} \left\{ \frac{U_C(C_T)}{P_T} (1 - \tau) p_t(j) y_{t,T}(j) - V\left[y_{t,T}(j), z^i_T\right] \right\} \tag{16}$$

Revenues are weighted by the marginal utility of consumption. $y_{t,T}(j)$ denotes the firm's output in period T when it could last change its price in period t. The optimisation is subject to the relevant demand constraint, that is

$$y_{t,T}(j) \;=\; \left[\frac{p_t(j)}{P_T^i}\right]^{-\sigma} Y_T^i \tag{17}$$

where Y_T^i is aggregate demand for the goods produced in country i. The first-order condition of this problem is

$$\begin{aligned} \mathrm{E}_t \sum_{T=t}^{\infty} \left(\alpha^i \beta\right)^{T-t} \Bigg\{ (1-\sigma)(1-\tau)\frac{U_C(C_T)}{P_T}\left[\frac{p_t^*(j)}{P_T^i}\right]^{-\sigma} Y_T^i \\ +\sigma V_y\left[y_{t,T}(j), z_T^i\right]\frac{p_t^*(j)^{-\sigma-1}}{\left(P_T^i\right)^{-\sigma}} Y_T^i \Bigg\} = 0 \end{aligned} \tag{18}$$

Making appropriate substitutions, one can solve for the optimal re-set price $p_t^*(j)$:

$$p_t^*(j) = \frac{\sigma}{(\sigma-1)(1-\tau)} \frac{\mathrm{E}_t \sum_{T=t}^{\infty} \left(\alpha^i\beta\right)^{T-t} V_y\left[y_{t,T}(j), z_T^i\right] y_{t,T}(j)}{\mathrm{E}_t \sum_{T=t}^{\infty} \left(\alpha^i\beta\right)^{T-t} \frac{U_C(C_T)}{P_T} y_{t,T}(j)} \tag{19}$$

The marginal tax rate τ is assumed to equal $1/(1-\sigma)$, which, in steady state, precisely eliminates the mark-up that firms charge as a result of their market power.[8]

3.3 Central Bank

The central bank sets the interest rate on the union-wide traded securities, controls the supply of money and, in addition, is permitted to purchase government debt from both countries.

Its real surplus in period t is given by

$$\frac{\Delta_t}{P_t} = \frac{\bar{M}_t}{P_t} - \frac{\bar{M}_{t-1}}{P_t} + \frac{Q^H_{CB,t-1}}{P_t} - \frac{Q^H_{CB,t}}{P_t(1+i_t^{Q,H})} + \frac{Q^F_{CB,t-1}}{P_t} - \frac{Q^F_{CB,t}}{P_t(1+i_t^{Q,F})} \tag{20}$$

$\bar{M}_t$ is the nominal amount of money supplied to the union and $Q^i_{CB,t}$ is the nominal amount of government debt that the central bank purchases from country $i \in \{H, F\}$ in period t. Central bank profits are made up of increases in money supply, interpreted as seigniorage here, and net revenues from asset purchases in both countries. The central bank surplus is divided and transferred to the local governments of H and F. The real amount received by country i is

$$\frac{\Delta_t^i}{P_t} = \frac{\bar{M}_t^i}{P_t} - \frac{\bar{M}_{t-1}^i}{P_t} + \frac{Q^i_{CB,t-1}}{P_t} - \frac{Q^i_{CB,t}}{P_t(1+i_t^{Q,i})} \tag{21}$$

As in Benigno (2004), seigniorage is assumed to be returned to the two countries according to its source. $\bar{M}_t^i - \bar{M}_{t-1}^i$ is the part of seigniorage which originates from country i, where

[8] Note that, as mentioned above, $\tau < 0$ since $\sigma > 1$.

$\bar{M}_t^H + \bar{M}_t^F = \bar{M}_t$. The net return of bond purchases by the central bank is equally transferred back to the country from which it was obtained. Equation (21) implies that no resources are shifted between countries. It is shown below that bond purchases by the central bank work through their effect on government-bond prices. If the revenues from unconventional asset purchases in one country were shared between H and F, then the central bank could act essentially as a central fiscal policy maker and re-distribute resources between both countries. It is doubtful however if this role of the central bank were in accordance with its mandate. The rule for dividing central bank surpluses assumed here leaves this problem for future discussion.[9]

Government-bond purchases are financed by increases in money supply, that is[10]

$$\frac{Q^i_{CB,t}}{P_t(1+i_t^{Q,i})} = \frac{\bar{M}_t^i}{P_t} - \frac{\bar{M}_{t-1}^i}{P_t} \tag{22}$$

The surplus transferred to each country can thus be re-written as

$$\frac{\Delta_t^i}{P_t} = \frac{Q^i_{CB,t-1}}{P_t} \tag{23}$$

which is the gross return obtained from asset purchases in that country in the previous period.

3.4 Local Governments

The respective budget constraints of the governments of H and F are

$$\int_0^n \frac{T_t^H}{P_t} dj + \frac{1}{1+i_t^{Q,H}} \left(\int_0^n \frac{Q_t^H}{P_t} dj + \frac{Q^H_{CB,t}}{P_t} \right) + \frac{\Delta_t^H}{P_t} =$$
$$\int_0^n \frac{p_t(j) g_t(j)}{P_t} dj + \int_0^n \frac{Q^H_{t-1}}{P_t} dj + \frac{Q^H_{CB,t-1}}{P_t} \tag{24}$$
$$\int_n^1 \frac{T_t^F}{P_t} dj + \frac{1}{1+i_t^{Q,F}} \left(\int_n^1 \frac{Q_t^F}{P_t} dj + \frac{Q^F_{CB,t}}{P_t} \right) + \frac{\Delta_t^F}{P_t} =$$
$$\int_n^1 \frac{p_t(j) g_t(j)}{P_t} dj + \int_n^1 \frac{Q^F_{t-1}}{P_t} dj + \frac{Q^F_{CB,t-1}}{P_t} \tag{25}$$

where sales tax revenues have been cancelled with the transfer of equal size. Q_t^i is the nominal quantity of government debt held by the representative agent of country i and $g(j)$ is real government spending on the good produced by j. The fiscal policy institutions of both countries

[9] A union in which the central bank transfers the returns from debt purchases back to their source country is isomorphic to one with a coordinated system of national central banks that purchase and hold debt issued by their respective local governments. The assumption made here about the division of central bank profits is therefore in line with the operational modalities of the ECB's Expanded Asset Purchase Programme announced in January 2015 under which only as small fraction of the assets purchased are subject to risk sharing.

[10] This assumption, made to simplify the exposition, is not essential to the transmission mechanism of bond purchases, as will become clear in Section 3.5.

finance spending and debt repayments to the private sector and the central bank through lump-sum taxes, newly-issued debt and their share of the central bank surplus.

Real government demand is given by

$$g_t(j) = \left[\frac{p_t(j)}{P_t^H}\right]^{-\sigma} G_t^H \quad \forall j \in [0, n) \tag{26}$$

$$g_t(j) = \left[\frac{p_t(j)}{P_t^F}\right]^{-\sigma} G_t^F \quad \forall j \in [n, 1] \tag{27}$$

where G_t^H and G_t^F represent aggregate real spending by the governments of H and F. Note that as in Benigno (2004), for example, the fiscal policy makers allocate their expenditures only to domestically produced goods.

Government bond market clearing implies

$$\bar{Q}_t^H = \int_0^n Q_t^H dj + Q_{CB,t}^H \tag{28}$$

$$\bar{Q}_t^F = \int_n^1 Q_t^F dj + Q_{CB,t}^F \tag{29}$$

where $\bar{Q}_t^i$ is the nominal amount of debt supplied by the government of country i. Debt issuance, expressed as the income in per-capita terms received from issuing bonds in a given period, follows a rule given by

$$\frac{1}{1+i_t^{Q,i}} \frac{\bar{Q}_t^i}{n^i} = b_0 \frac{1}{1+i_t^{Q,i}} + b_1 Q_{t-1}^i - b_2(T_t^i - P_t^i G_t^i) \tag{30}$$

It is allowed to depend on the inverse of the gross bond yield which just equals the bond price, the value of debt held by the private sector in the previous period and the primary surplus. This rule is flexible enough to capture a variety of different debt management regimes. Note that it permits a response of debt issuance to contemporaneous bond purchases by the central bank through the bond yield and the primary surplus as well as to bond purchases carried out in the previous period through the amount of debt held by the households.

To illustrate the effects of unconventional monetary policy on the fiscal sector, it is instructive to compare the budget of government i when the central bank intervenes in debt markets in period t with its budget in the hypothetical scenario that it does not intervene *in period* t, i.e. $Q_{CB,t}^i = 0$. Government spending in this scenario, $G_t^{i,0}$, is assumed to follow an exogenous stochastic process. The level of lump-sum taxation $T^{i,0}$ is then chosen to satisfy the government's budget constraint. The budget constraints of H and F for the case that $Q_{CB,t}^i = 0$ are

$$\int_0^n \frac{T_t^{H,0}}{P_t^0} dj + \frac{\bar{Q}_t^{H,0}}{P_t^0(1+i_t^{Q,H,0})} + \frac{\Delta_t^H}{P_t^0} = \int_0^n \frac{p_t^0(j) g_t^0(j)}{P_t^0} dj + \frac{\bar{Q}_{t-1}^H}{P_t^0} \tag{31}$$

$$\int_n^1 \frac{T_t^{F,0}}{P_t^0} dj + \frac{\bar{Q}_t^{F,0}}{P_t^0(1+i_t^{Q,F,0})} + \frac{\Delta_t^F}{P_t^0} = \int_n^1 \frac{p_t^0(j) g_t^0(j)}{P_t^0} dj + \frac{\bar{Q}_{t-1}^F}{P_t^0} \tag{32}$$

where all variables indexed by "0" take a value that is specific to the case that $Q^i_{CB,t} = 0$ in period t.[11] Differencing i's budget constraint in the laissez-faire scenario, (31) for H or (32) for F, with the general one, (24) or (25), and taking integrals yields

$$\frac{1}{1+i_t^{Q,i}} \frac{\bar{Q}_t^i}{n^i} - \frac{1}{1+i_t^{Q,i,0}} \frac{\bar{Q}_t^{i,0}}{n^i} = T_t^{i,0} - T_t^i + P_t^i G_t^i - P_t^{i,0} G_t^{i,0} \tag{33}$$

The change in income from debt issuance due to bond purchases by the central bank has to equal the sum of adjustments in taxation and spending. Thus, if bond purchases increase the income from debt issuance, the government of the country in which the purchases were carried out can raise nominal spending and lower lump-sum taxes. In the following section, it is shown that unconventional monetary policy increases aggregate demand through this mechanism for plausible values of the parameters.

In choosing lump-sum taxes and spending, the governments of H and F are assumed to minimise the simple loss function given below subject to their respective budget constraint.

$$L^i_{G,t} = \left(G_t^i - G_t^{i,0}\right)^2 + \omega_T \left(\frac{T_t^i}{P_t^i} - \frac{T_t^{i,0}}{P_t^{i,0}}\right)^2, \quad \omega_T \geq 0 \tag{34}$$

To guarantee that the fiscal response to unconventional monetary policy is not overstated, it is assumed that the fiscal authorities aim to replicate the policy choices from the reference scenario in which no bond purchases by the central bank are carried out as closely as possible. This specification is flexible enough to allow for the case that the income change on the left-hand side of Equation (33) is accounted for entirely at the tax margin ($\omega_T = 0$) or entirely at the spending margin ($\omega_T \to \infty$). The first-order condition of government i's optimal policy problem is given by

$$\omega_T = \frac{G_t^i - G_t^{i,0}}{\frac{T_t^{i,0}}{P_t^{i,0}} - \frac{T_t^i}{P_t^i}} \tag{35}$$

In optimum, the governments of H and F set the ratio of real spending to lump-sum tax adjustments equal to ω_T.

3.5 Transmission Mechanism of Unconventional Monetary Policy

Consistent with the empirical evidence in D'Amico et al. (2012) and Krishnamurthy and Vissing-Jorgensen (2011), purchases of government bonds by the central bank are allowed to affect government bond yields through a premium paid on government debt. As in Chen et al. (2012), this premium arises in equilibrium to compensate investors for the transaction cost of size ξ_t^i.[12]

[11] Equation (23) implies that Δ_t^i is independent of $Q^i_{CB,t}$.

[12] Government debt can be thought of as taking the form of long-term bonds (whose entire stock has to be sold at the end of each period here) and the premium generated by the fee ξ_t^i as a scarcity or liquidity premium.

The pricing equations (11) and (12) imply that investors demand an excess return of size

$$i_t^{Q,i} - i_t = \frac{\xi_t^i}{\beta \mathrm{E}_t \left[\frac{U_C(C_{t+1}^i)}{U_C(C_t^i)} \frac{1}{\Pi_{t+1}} \right]} \tag{36}$$

in response to the transaction cost paid on each unit of the government bond. The interest rate spread between government debt and the union-wide traded security is equal to the transaction cost ξ_t^i adjusted for the expectation of the stochastic discount factor. Bond purchases by the central bank reduce the availability of government debt for private investors and increase their scarcity. Households are therefore willing to pay a higher price and accept a lower yield on government debt which, following Chen et al. (2012), is modelled here as a reduction in ξ_t^i, leading to a decrease in the yield spread $i_t^{Q,i} - i_t$.[13] The assumptions made about ξ_t^i are summarised more formally in Assumption 1.

Assumption 1 $\xi_t^i = \xi(\bar{Q}_t^i - Q_{CB,t}^i)$ *with* $\partial\xi/\partial(\bar{Q}_t^i - Q_{CB,t}^i) > 0$

ξ_t^i and thus the interest rate spread depends on the net amount of government debt available to the private sector. Chen et al. (2012) note that up to a first-order approximation this specification is equivalent to the mechanism in Andrés et al. (2004) which relies on ideas from Tobin (1969). In their model, households have a preference for holding financial assets in a particular proportion. If households are forced to absorb a large amount of long-term government debt relative to more liquid assets like money, they have to be compensated through a higher return. Hence, a reduction in the net supply of government debt reduces the liquidity premium that has to be paid on it as a result of agents' desire to back debt holdings with liquidity.[14]

As outlined above, central bank purchases of government debt are assumed to be financed by seigniorage. The central bank is able to extend its balance sheet only if households are willing to hold the additional amount of money created to fund bond purchases. The following assumption on the utility that households derive from liquidity services of real money holdings guarantees that this is the case, making the size of the balance sheet a choice variable from the perspective of the central bank.

Assumption 2 *Let* $A^{RE} = \{a_t\}_{t=0}^{\infty}$ *be the sequence of allocations in the unique stationary equilibrium of the model, then* $L(\cdot)$ *has the property that* $L_{M/P}\left[\bar{M}_t^i/(n^i P_t)\right] > i_t/(1+i_t)U_C(C_t^i)$ *for all* $a_t \in A^{RE}$.

Assumption 2 ensures that utility maximisation gives rise to the corner solution with respect to money holdings described in connection with Equation (13) at all times. Agents hold the entire

[13] If government bonds are interpreted as long-term debt, a reduction in ξ_t^i in response to central bank purchases of government bonds leads to a "flattening of the yield curve".

[14] The interpretation of a scarcity premium is favoured here over that of a liquidity premium, because it is closer in line with the empirical evidence in D'Amico et al. (2012), for example.

supply of money available to them, which allows the central bank to raise seigniorage revenues in order to purchase government debt.[15]

In general, the effects of government bond purchases by central banks can be broken down into those arising from the withdrawal of government debt from the hands of the public and those arising from the accompanying increase in the money supply. In this model, bond purchases are effective only as a result of the former type of effects, the ones associated with the re-allocation of government debt. To demonstrate this fact, bond purchases can be shown to be neutral if the effects from the decrease in the net supply of government debt are muted by assuming, in contrast to Assumption 1, that ξ_t^i is constant and thus independent of $\bar{Q}_t^i - Q_{CB,t}^i$.[16]

Proposition 1 *If* $\xi_t^i = \xi^i$, *then* $\left(C_t^i, Y_t^i, i_t^{Q,i}, P_t^i\right) = \left(C_t^{i,0}, Y_t^{i,0}, i_t^{Q,i,0}, P_t^{i,0}\right)$ *for all* $t \geq 0$ *and* $i \in \{H, F\}$.

Proof: See Section A.1 in the appendix.

If debt purchases by the central bank do not affect the interest rate spread, they merely result in lump-sum flows between the central bank, the local governments and the households that leave the resource constraint of both countries unchanged. As a result, consumption and government spending remain unaffected, which implies that output prices are equally unchanged. Comparable to Chen et al. (2012), I therefore make use of Assumption 1 below to allow bond purchases to have the empirically observed effects on government bond yields and thus to have "a chance to work" (Chen et al., 2012, p. F299).

Since bond purchases reduce the yield of government bonds, they affect the government budget constraint and consequently government spending. To gain intuition for the precise effects, recall Equation (33) which is restated here for convenience.

$$\frac{1}{1+i_t^{Q,i}}\frac{\bar{Q}_t^i}{n^i} - \frac{1}{1+i_t^{Q,i,0}}\frac{\bar{Q}_t^{i,0}}{n^i} = T_t^{i,0} - T_t^i + P_t^i G_t^i - P_t^{i,0} G_t^{i,0} \tag{37}$$

Using the optimality condition for government spending and taxation (35), one can express this equation as

$$G_t^i - G_t^{i,0} = \frac{\omega_T}{(1+\omega_T)P_t^{i,0}}\left[\frac{1}{1+i_t^{Q,i}}\frac{\bar{Q}_t^i}{n^i} - \frac{1}{1+i_t^{Q,i,0}}\frac{\bar{Q}_t^{i,0}}{n^i} + \left(P_t^i - P_t^{i,0}\right)\left(\frac{T_t^i}{P_t^i} - G_t^i\right)\right] \tag{38}$$

The total change in real government spending in response to bond purchases in period t can be

[15] In the interior solution also shown in (13), money demand is tied to i_t. Thus, the central bank is unable to extend the money supply for a given value of i_t. Assumption 2 can be viewed as a formalisation of the "decoupling principle" described by Borio and Disyatat (2009).

[16] This result confirms the irrelevance propositions contained in Eggertsson and Woodford (2003) and Wallace (1981) in a currency-union framework.

attributed to an "income-from-debt-issuance" effect

$$\bar{\mathcal{P}}_t^{ID,i} = \frac{\omega_T}{1+\omega_T}\frac{1}{P_t^{i,0}}\left(\frac{1}{1+i_t^{Q,i}}\frac{\bar{Q}_t^i}{n^i} - \frac{1}{1+i_t^{Q,i,0}}\frac{\bar{Q}_t^{i,0}}{n^i}\right) \tag{39}$$

and a "primary-surplus" effect

$$\bar{\mathcal{P}}_t^{PS,i} = \frac{\omega_T}{1+\omega_T}\frac{P_t^i - P_t^{i,0}}{P_t^{i,0}}\left(\frac{T_t^i}{P_t^i} - G_t^i\right) \tag{40}$$

where $\bar{\mathcal{P}}_t^{ID,i} + \bar{\mathcal{P}}_t^{PS,i} = G_t^i - G_t^{i,0} \equiv \bar{\mathcal{P}}_t^i$.

The income-from-debt-issuance effect reflects the change in both the price and the quantity of debt issued. For a given value of nominal debt issuance, bond purchases by the central bank increase the price at which the government can sell bonds, $(1+i_t^{Q,i})^{-1} > (1+i_t^{Q,i,0})^{-1}$, and thus raise its income. If the government subsequently increases bond supply, the effect on the bond price may be partly offset, because higher supply reduces the scarcity of government bonds. For $\omega_T > 0$, the resulting change in government income leads to adjustments in spending, which in turn affect aggregate demand and the price level.

A rise in the price level, for example, erodes the real primary surplus or deficit, which triggers the primary-surplus effect. The government loss function (34) implies that spending and taxes are adjusted such that the impact of the elevated price level on the real values of spending and lump-sum taxes is minimised. In case of a surplus, this implies that real spending is increased, while in case of a deficit it is decreased. The relative intensity of adjustment at the tax and at the spending margin is again governed by ω_T.

The sign of both effects is ambiguous. Proposition 2 gives general conditions that are sufficient for the total effect of bond purchases to be expansionary.

Proposition 2 *$Q_{CB,t}^i > 0$ implies $\bar{\mathcal{P}}_t^{ID,i} > 0$ and $\bar{\mathcal{P}}_t^{PS,i} > 0$ if $\omega_T > 0$, $b_0 > 0$, $b_2 < \bar{b}_2$ and country $i \in \{H, F\}$ runs a primary surplus, i.e. $T_t^i/P_t^i - G_t^i > 0$.*

Proof: See Section A.2 in the appendix.

The parameters b_0 and b_2 govern how bond issuance responds to purchases by the central bank. $b_0 > 0$ implies that, all else equal, an increase in the bond price leads to an increase in the discounted value of bond supply. $b_2 < \bar{b}_2$ implies that purchases of government debt by the central bank make government bonds more scarce from the point of view of private investors, that is, it rules out the scenario that local governments react to bond purchases by increasing the supply by more than the amount purchased by the central bank. As a result, debt purchases by the central bank reduce the government bond yield. $\omega_T > 0$ implies that governments adjust their budgets in response to changes in the bond yield not only at the debt and at the tax margin, but also at the spending margin. Under these three conditions, government bond purchases by the central bank are unambiguously expansionary in a country with a primary surplus.

According to the loss function (34), an increase in the price level induces the local governments to adjust their budgets such that the effect on real spending and taxes is minimised and their budget constraint is satisfied. If a country runs a primary surplus, an elevated price level allows it to let nominal taxes increase enough to meet the budget constraint, yet minimise its loss by decreasing real taxes and raising real spending. A country that runs a primary deficit on the other hand would have to decrease real spending in a comparable situation to meet its budget constraint. Lemma 1 gives a condition that has to be met in period t for a primary surplus to occur in this model.

Lemma 1 $T_t^i/P_t^i - G_t^i > 0$ *if* $b_0/(1-b_1) < \left(1 + i_t^{Q,i}\right) Q_{t-1}^i$.

The inequality above can be viewed as a constraint on b_1, the responsiveness of debt issuance in period t to debt holdings by the private sector in the previous period.

3.6 Aggregate Demand

Aggregate demand for each individual good from H and F is given by

$$y_t(h) = \int_0^1 c_t^j(h)dj + g_t(h) = \left[\frac{p_t(h)}{P_t^H}\right]^{-\sigma} \left(T_t^{1-n} C_t^U + G_t^H\right) \tag{41}$$

$$y_t(f) = \int_0^1 c_t^j(f)dj + g_t(f) = \left[\frac{p_t(f)}{P_t^F}\right]^{-\sigma} \left(T_t^{-n} C_t^U + G_t^F\right) \tag{42}$$

where $C_t^U \equiv \int_0^1 C_t^j dj$ is consumption aggregated over the entire union. The demand for the goods baskets from both countries is

$$Y_t^H = \left[\frac{1}{n}\int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}} = T_t^{1-n} C_t^U + G_t^H \tag{43}$$

$$Y_t^F = \left[\frac{1}{1-n}\int_n^1 y_t(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}} = T_t^{-n} C_t^U + G_t^F \tag{44}$$

Aggregate demand depends on the terms of trade, private union-wide consumption and government spending.

Equilibrium consumption and net foreign bond holdings can be characterised further.[17]

Lemma 2

a) All households select the same quantity of consumption.
$C_t^i = C_t^U \equiv C_t \quad \forall t \geq 0, \; i \in \{H, F\}$

b) The non-contingent security traded between all households of the union is redundant.
$B_t^i = 0 \quad \forall t \geq 0, \; i \in \{H, F\}$

[17] See Benigno (2004), Corsetti and Pesenti (2001) and Obstfeld and Rogoff (1998) for analogous results.

Despite of incomplete financial markets at the union-level, there is perfect consumption risk-sharing across regions and the non-contingent security is not traded. Therefore, monetary policy can induce stationarity of consumption implying that the equilibrium indeterminacy issues of open economy models with incomplete asset markets described in Schmitt-Grohe and Uribe (2003) do not play a role here. Preferences over the baskets from H and F are Cobb-Douglas (CES with an elasticity of substitution equal to one). Each country therefore earns a constant share of real union-wide income. Since the Cobb-Douglas weighting parameters are chosen to equal the country sizes of H and F, real per capita income is equal across countries leading to identical per capita consumption.[18]

Aggregate demand in H and F is thus given by

$$Y_t^H = T_t^{1-n} C_t + G_t^H = T_t^{1-n} C_t + G_t^{H,0} + \bar{\mathcal{P}}_t^H \tag{45}$$

$$Y_t^F = T_t^{-n} C_t + G_t^F = T_t^{-n} C_t + G_t^{F,0} + \bar{\mathcal{P}}_t^F \tag{46}$$

with $\bar{\mathcal{P}}_t^i = \bar{\mathcal{P}}_t^{ID,i} + \bar{\mathcal{P}}_t^{PS,i} = G_t^i - G_t^{i,0}$. It is the sum of private consumption, government spending in the absence of unconventional monetary policy and the additional demand resulting from central bank asset purchases.

4 Model Approximation

This section describes the Linear-Quadratic (LQ) approximation of the optimisation problem faced by the central bank and the calibration of the model.

4.1 Steady State

The model is approximated around a symmetric non-stochastic steady state. In the absence of shocks, the central bank does not intervene in the bond market, i.e. $\bar{\mathcal{P}}^H = \bar{\mathcal{P}}^F = \bar{\mathcal{P}} = 0$.

The first-order condition of the producers' profit maximisation problem in steady state becomes

$$U_C(C) = \frac{\sigma}{(\sigma-1)(1-\tau)} V_y\left[y(j), 0\right] \tag{47}$$

When the proportional tax rate τ is chosen such that the mark-up that producers are able to charge due to market power is just offset, the marginal utility of consuming an additional unit of the consumption basket equals the marginal disutility from producing it.

4.2 Dynamics

Below, for each variable X, $\tilde{X}_t \equiv \ln X_t - \ln X$ denotes log-deviations from steady state under flexible prices. $\hat{X}_t \equiv \ln X_t - \ln X$ is defined equivalently for the case of sticky output prices

[18] See Section A.3 of the appendix for more details on the proof.

outlined above.[19] Variables indexed by "U" are union-wide aggregates defined as $X^U \equiv nX^H + (1-n)X^F$ and those with a superscript "R" are relative values given by $X^R \equiv X^F - X^H$.

4.2.1 Dynamics under Flexible Prices

In the hypothetical case of flexible output prices, the central bank is equally assumed to refrain from interventions in the asset markets of both countries so that $\bar{\mathcal{P}}_t^H = \bar{\mathcal{P}}_t^F = 0$ for all t.

Profit maximisation then implies

$$p_t(j) = \frac{\sigma}{(\sigma-1)(1-\tau)} V_y \left[y_t(j), z_t^i \right] \tag{48}$$

Prices are set at a mark-up over marginal cost. Since this condition is identical for all producers that are located within the same country, it implies that

$$P_t^i = \frac{\sigma}{(\sigma-1)(1-\tau)} V_y \left(Y_t^i, z_t^i \right) \tag{49}$$

By log-linearising and combining the consumer and fiscal policy blocks of the model with a log-linear version of (49), union-wide aggregates can be expressed as functions of the exogenous disturbances only, yielding[20]

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(g_t^{R,0} - \zeta_t^R \right) \tag{50}$$

$$\tilde{C}_t = \frac{\eta}{\rho+\eta}\left(\zeta_t^U - g_t^{U,0} \right) \tag{51}$$

$$\tilde{Y}_t^U = \frac{\eta}{\rho+\eta}\zeta_t^U + \frac{\rho}{\rho+\eta} g_t^{U,0} \tag{52}$$

$$\tilde{i}_t - \mathrm{E}_t \tilde{\Pi}_{t+1}^U = \frac{\rho\eta}{\rho+\eta} \mathrm{E}_t \left[\zeta_{t+1}^U - \zeta_t^U - \left(g_{t+1}^{U,0} - g_t^{U,0} \right) \right] \tag{53}$$

where $\eta \equiv \frac{V_{yy}(C,0)}{V_y(C,0)} C$, $\rho \equiv -\frac{U_{CC}(C)}{U_C(C)} C$ and the shocks are normalised as follows

$$g_t^{i,0} \equiv \frac{G_t^{i,0} - G^{i,0}}{Y^i} \tag{54}$$

$$\zeta_t^i \equiv -\frac{V_{yz}(C,0)}{C V_{yy}(C,0)} z_t^i \tag{55}$$

$g_t^{i,0}$ is the exogenous government spending process expressed in deviations from its mean and normalised by steady-state output. ζ_t^i is the technology disturbance multiplied by a negative scaling factor. Recall that an increase in z_t^i raises the production costs in country i. Since ζ_t^i is negatively correlated with z_t^i, a high value of ζ_t^i has a positive effect on production, affecting supply in i in a positive way.

[19] Interest rates and the transaction fee are in "gross-log-deviation form", so e.g. $\hat{i}_t \equiv \ln(1+i_t) - \ln(1+i)$.
[20] The derivations are shown in the online appendix.

Positive values of both shocks increase natural world output. However, a positive government spending shock does not raise output one-for-one but partly crowds out natural consumption. Domestic demand shocks raise the relative price level of domestic to foreign goods and domestic supply shocks have the opposite effect. The natural real rate of interest depends positively on expected improvements in technology and negatively on expected increases in the exogenous component of demand.

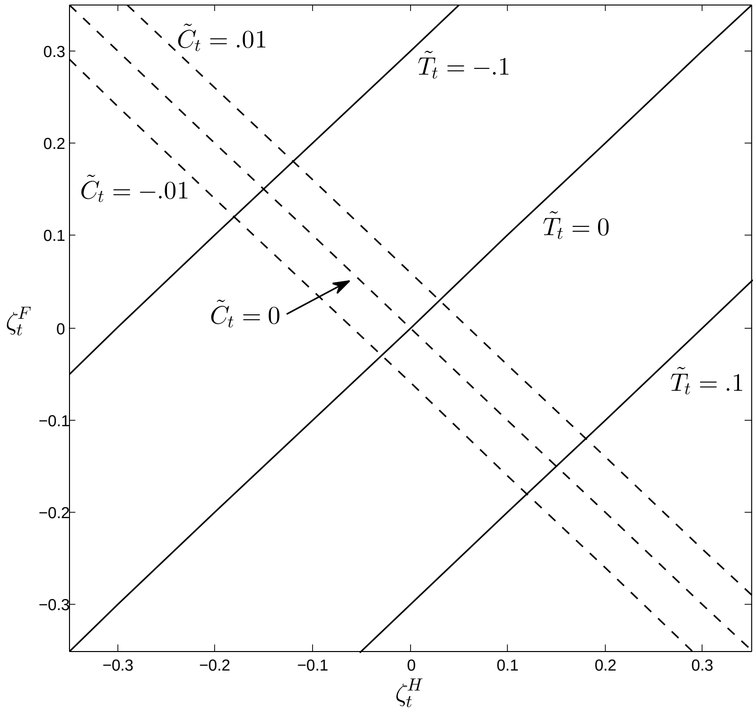

Figure 2: Natural consumption and natural terms of trade as a function of supply shocks

In a currency union, shocks are likely to be correlated. However, we have no reason to believe that $\zeta_t^H = \zeta_t^F$ and $g_t^{H,0} = g_t^{F,0}$ at all times. A central aim of this paper is to investigate the role that central bank asset purchases can play in stabilising imperfectly correlated shocks. Figure 2 shows selected loci of the natural terms of trade and natural consumption in ζ_t^H,ζ_t^F-space in the absence of demand shocks.[21] When discussing the dynamic properties of the model, it is convenient to think of shocks as exogenous shifts in $\tilde{T}_t$ and $\tilde{C}_t$. The figure above shows how these can result from asymmetric supply disturbances. A situation, in which $\tilde{T}_t = 0.1$ and $\tilde{C}_t = 0$, for example, can be viewed as a combination of a positive supply shock in H and a negative supply shock in F of magnitudes fully determined by the intersection of the corresponding loci. An analogous diagram could be constructed for demand shocks. Both types of disturbances together yield infinitely many possible combinations of shocks underlying each $\left(\tilde{T}_t, \tilde{C}_t\right)$-pair.

[21] Figure 1 is drawn using the calibration described later.

4.2.2 LQ Approximation of the Policy Problem

Aggregate demand is described by the approximated country-specific demand schedules, (45) and (46).[22]

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t + g_t^{H,0} + \mathcal{P}_t^H \tag{56}$$
$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^{F,0} + \mathcal{P}_t^F \tag{57}$$

$\mathcal{P}_t^H$ and $\mathcal{P}_t^F$ are terms capturing the effects of government bond purchases in H and F defined as

$$\mathcal{P}_t^i \equiv \frac{\bar{\mathcal{P}}_t^i - \bar{\mathcal{P}}^i}{Y^i} \tag{58}$$

and assumed to be under direct control of the central bank.[23] Demand in each country depends positively on the competitiveness of domestically produced goods, private consumption, the exogenous component of government spending and unconventional monetary policy.

Private consumption satisfies the log-linearised Euler equations

$$\hat{C}_t = \mathrm{E}_t\hat{C}_{t+1} - \frac{1}{\rho}\left(\hat{i}_t - \mathrm{E}_t\hat{\Pi}_{t+1}^U\right) \tag{59}$$
$$\hat{C}_t = \mathrm{E}_t\hat{C}_{t+1} - \frac{1}{\rho}\left(\hat{i}_t^{Q,i} - \hat{\xi}_t^i - \mathrm{E}_t\hat{\Pi}_{t+1}^U\right) \tag{60}$$

which imply that

$$\hat{i}_t^{Q,i} - \hat{i}_t = \hat{\xi}_t^i \tag{61}$$

The yield spread between government bonds and privately exchanged bonds is given by $\hat{\xi}_t^i$ and is thus influenced by the scarcity of government debt from the viewpoint of the private investors.

Inflation in the price of the basket of goods produced in H and F is given by two Phillips curves.

$$\hat{\Pi}_t^H = \beta\mathrm{E}_t\hat{\Pi}_{t+1}^H + (1-n)a_T^H\left(\hat{T}_t - \tilde{T}_t\right) + a_C^H\left(\hat{C}_t - \tilde{C}_t\right) + a_{\mathcal{P}}^H\mathcal{P}_t^H \tag{62}$$
$$\hat{\Pi}_t^F = \beta\mathrm{E}_t\hat{\Pi}_{t+1}^F - na_T^F\left(\hat{T}_t - \tilde{T}_t\right) + a_C^F\left(\hat{C}_t - \tilde{C}_t\right) + a_{\mathcal{P}}^F\mathcal{P}_t^F \tag{63}$$

It depends on expected inflation in the following period, the consumption gap and deviations of the terms of trade from their natural rate.[24] Improvements in competitiveness of country H increase output and inflation in H and have the opposite effect in F. Additionally, inflation rises in response to central bank purchases of government debt. The terms of trade evolve according to the following law of motion

$$\hat{T}_t = \hat{T}_{t-1} + \hat{\Pi}_t^F - \hat{\Pi}_t^H \tag{64}$$

[22] The online appendix contains all derivations for the LQ approximation of the optimal policy problem.

[23] Since $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$ are zero in steady state, they are expressed in linear-deviation form from steady state rather than log-deviations.

[24] The parameters a_T^i, a_C^i and $a_{\mathcal{P}}^i$ with $i \in \{H, F\}$ are defined in the online appendix.

The motivation for employing the unconventional tool may be twofold. First, bond purchases can be helpful in stabilising disturbances to the *relative* price level of H and F. To see this, note that $\hat{T}_t$ depends on $\hat{\Pi}_t^F - \hat{\Pi}_t^H$. Subtracting (62) from (63) shows that the inflation differential itself depends on $a_{\mathcal{P}}^F \mathcal{P}_t^F - a_{\mathcal{P}}^H \mathcal{P}_t^H$. Thus, the central bank is able to address the terms of trade gap directly through $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$. Second, bond purchases can be helpful in stimulating or depressing economic activity in the union *in its entirety*. Taking the weighted average of (56) and (57) gives

$$\hat{Y}_t^U = \hat{C}_t + g_t^{U,0} + \mathcal{P}_t^U \tag{65}$$

Union output can be boosted by choosing $\mathcal{P}_t^U > 0$. Results from a number of recent contributions obtained in closed-economy models suggest that this type of intervention may be beneficial under certain conditions.[25] However, the aim of this paper is to evaluate how well bond purchases are suited to address the challenges specific to a monetary union. To separate the two issues described above, I restrict my attention to policies that give rise to paths of the unconventional monetary policy tool with

$$\mathcal{P}_t^U = 0 \tag{66}$$

for all t. In each period, the average intervention across the entire union is required to equal zero, which implies that asset purchases in one country have to be met with sales in the other, eliminating the central bank's ability to affect union-wide output through asset purchases. Additionally, (66) ensures that long-run affects on the central bank's balance sheet are small, $\mathcal{P}_t^i$ is centred around the approximation point at which $\mathcal{P}^i = 0$ and the appropriate transversality conditions are not violated.

Policy is chosen to maximise welfare, measured as

$$W_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t w_t \tag{67}$$

where per-period union-wide welfare w_t is defined as $w_t \equiv n w_t^H + (1-n) w_t^F$ and per-period welfare in H and F is given by

$$w_t^H \equiv \frac{1}{n} \int_0^n \left\{ U(C_t) - V\left[y_t(h), z_t^H\right] - W\left(\mathcal{P}_t^H\right) \right\} dh \tag{68}$$

$$w_t^F \equiv \frac{1}{1-n} \int_n^1 \left\{ U(C_t) - V\left[y_t(f), z_t^F\right] - W\left(\mathcal{P}_t^F\right) \right\} df \tag{69}$$

As in Benigno (2004) and Woodford (2003), the welfare measure abstracts from the utility derived from liquidity services. Liquidity consequently does not enter the approximated model, which, similar to the model outlined in Woodford (2003), can be viewed as the cashless limit of an economy with money.

[25] See, for example, Auerbach and Obstfeld (2005) and Ellison and Tischbirek (2014).

Large-scale open-market interventions by the central bank may be associated with welfare costs beyond those captured by the model. Important concerns have to do, for example, with the re-distribution of wealth from the lower end of the wealth distribution towards equity holders who are more likely to be found in the upper end, and with lowered incentives to implement structural reforms that improve competitiveness. Formal research on these topics is still in the development phase.[26] A comprehensive account of all costs is beyond the scope of the paper. Nonetheless, as a robustness exercise, I additionally present results below that are based on a welfare criterion for which a cost function W with $W(0) = 0$, $W_{\mathcal{P}}\left(\left|\mathcal{P}_t^i\right|\right) > 0$ and $W_{\mathcal{P}\mathcal{P}}\left(\left|\mathcal{P}_t^i\right|\right) \geq 0$ is added to household utility.[27] It will become clear that the influence of W is muted if its second derivative vanishes in steady state. This case is seen as the benchmark below. Strictly positive values of the second derivative add an additional dimension to the optimal policy trade-off.

A second-order approximation to the welfare criterion is

$$W_0 = -\Omega \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{70}$$

The parameter Ω is defined in the online appendix. When the Arrow-Pratt coefficient of relative risk aversion in consumption ρ equals one, per-period loss is given by

$$\begin{aligned} L_t \;\; = \;\; & \Lambda_C \left(\hat{C}_t - \tilde{C}_t\right)^2 + \Lambda_C n(1-n)\left(\hat{T}_t - \tilde{T}_t\right)^2 + \gamma\left(\hat{\Pi}_t^H\right)^2 + (1-\gamma)\left(\hat{\Pi}_t^F\right)^2 \\ & + \Lambda_{\mathcal{P}}\left\{\frac{1}{2}\left(1+\frac{\kappa}{\eta}\right)\left[n\left(\mathcal{P}_t^H\right)^2 + (1-n)\left(\mathcal{P}_t^F\right)^2\right] - n(1-n)\mathcal{P}_t^R\left(\hat{T}_t - \delta\tilde{T}_t\right)\right\} \\ & + \text{tip} + O(3) \end{aligned} \tag{71}$$

with $\delta \equiv (\eta+1)/\eta$.[28] In the absence of bond purchases, per-period loss is standard. It increases quadratically with deviations of consumption and the terms of trade from their respective natural rates and with inflation. When the central bank makes use of the unconventional tool, additional welfare costs arise. These losses depend on the squared size of the intervention in both countries and on the interaction of the relative size of interventions with a modified terms of trade gap.

Sticky prices and the potential welfare costs of bond purchases are the sole sources of inefficiency in the model. Welfare losses from market power are eliminated by the subsidy that ensures that long-run output is at its efficient level.[29] Price rigidities cause distortions in relative prices, which lead to an inefficient allocation of productive activity within and across countries.

[26] See Muellbauer (2016) for a discussion of possible costs associated with QE.

[27] W can be viewed as capturing in reduced form the costs arising from bond purchases in a model that is large enough to include all sources of welfare loss from QE.

[28] The definitions of all other parameters can also be found in the online appendix. "tip" represents terms that are independent of policy.

[29] Rotemberg and Woodford (1998) distinguish between welfare losses arising from relatively stable sources of inefficiency like market power and those resulting from inefficient short-run fluctuations. They argue that monetary policy should be responsible for minimising the latter type of losses, while other kinds of policy are more suitable for addressing the former. Since the focus is on short-term stabilisation policy here, I make use of their strategy to eliminate the negative long-run effects of market power with an appropriate subsidy.

All producers of a given country produce output with the same technology. Price dispersion within H and F is inefficient as it implies that producers operate at different marginal cost. Aggregate cost from production within both countries could therefore be lowered by re-allocating productive activity such that production is equally distributed among all producers located in the same country. Price rigidities furthermore distort the relative price level between H and F away from the optimal value implied by differences in technology. As a result, union-wide production costs could be lowered by re-allocating productive activity between the two countries.

From the Phillips curves (62) and (63), it can be seen that $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$ have a direct effect on inflation. The strength of this effect is governed by $a_{\mathcal{P}}^H$ and $a_{\mathcal{P}}^F$. The parameter $\Lambda_{\mathcal{P}}$ that determines the size of welfare losses from unconventional monetary policy is zero if $a_{\mathcal{P}}^H$ and $a_{\mathcal{P}}^F$ are zero, which shows that the welfare losses of the unconventional instrument arise from its effect on inflation. When expected inflation, the consumption gap and the terms of trade gap are zero, positive or negative values of $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$ steer inflation away from zero and therefore cause distortions in relative prices, which is reflected in the quadratic welfare costs from $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$. The final term implies that dead-weight losses from the unconventional tool are lowered when it is employed in a country with an inefficiently low level of inflation and vice versa. Welfare losses are shown by contrasting the dynamics under nominal price rigidities with counter-factual dynamics that would arise under fully flexible prices and complete central bank inactivity in government bond markets. This second feature of the flexible price equilibrium used as a reference however implies that it could be improved upon by adequate asset purchases resulting in the small modification of the terms of trade gap in the final term of the loss function.[30] The cost function W enters welfare only through the parameter κ, which is defined as $\kappa \equiv \frac{W_{\mathcal{P}\mathcal{P}}(\mathcal{P})}{U_C(C)C}$. Welfare costs of bond purchases that arise independent of their effects on relative prices therefore depend on the convexity of W. If W is linear, for example, κ equals zero and W plays no role.

The welfare criterion is purely quadratic. This is achieved through a combination of the steady state subsidy described before and appropriate substitutions. The methods used here are described in detail in Benigno and Woodford (2012). As a result, welfare is correctly approximated to the second order despite of the policy derived from the LQ problem necessarily being of first order only. Furthermore, the policy rule obtained from the LQ problem is a correct first-order approximation of the optimal policy in the non-linear problem.

4.3 Calibration

The calibration of the model is summarised in Table 1. It is mostly standard. A period is assumed to be a quarter. β is set to 0.99, giving rise to an annualised interest rate of about

[30] Beetsma and Jensen (2005) analyse the optimal monetary and fiscal policy mix for a currency union in a comparable model. They avoid a similar modification of the terms of trade gap in a cross term with the fiscal instrument by deriving natural government spending levels that permit writing the fiscal instrument in gap form. This route is not followed here to avoid a dependency of loss on the unintuitive concept of "natural unconventional monetary policy".

4 per cent in steady state. As in Benigno (2004), the intratemporal elasticity of substitution between consumption goods of the same country is set to 7.66, implying a steady state mark-up of about 15 per cent. For the coefficient of relative risk aversion, I choose a value of 1, consistent with logarithmic utility from consumption.

Parameter	Value	Description
β	0.99	Households discount factor
σ	7.66	Intratemporal elasticity of substitution between consumption goods
ρ	1	Coefficient of relative risk aversion
η	0.47	Elasticity of production
α_H	0.75	Price stickiness in H
α_F	0.75	Price stickiness in F
κ	$\{0, 1, 2\}$	Normalised convexity of $W(\cdot)$ at the approximation point
n	$\{0.5, 0.8\}$	Size of country H

Table 1: Calibration

Assuming that output at the firm-level is produced using a simple CRS production function with labour as the only input, η is the inverse Frisch elasticity of labour supply.[31] Ferrero (2009) sets this value equal to 0.47, which I follow here. The degree of price stickiness is assumed to be equal in both countries with a Calvo paramter of 0.75, which implies that prices are changed every year on average. κ is defined as $\kappa \equiv \frac{W_{\mathcal{PP}}(\mathcal{P})}{U_C(C)C}$. If $U(\cdot)$ and $V(\cdot)$ are of the CES form, equation (47) implies that $C = 1$ and $U_C(C) = 1$. Since $\mathcal{P} = 0$, κ in this case equals $W_{\mathcal{PP}}(0)$, giving the convexity of the cost function $W(\cdot)$ at the steady state. It was assumed before that the marginal costs of intervening in asset markets are non-decreasing, which implies that κ is non-negative. If $\kappa = 2$, the non-allocative marginal costs of bond purchases increase at twice the rate of the benefits from consumption in steady state. It seems plausible for the true costs to lie well below those implied by a κ-value of 2. Below, results are shown for the case in which W is muted ($\kappa = 0$), of intermediate size ($\kappa = 1$) and likely over-stated ($\kappa = 2$). Furthermore, two scenarios will be considered, one in which H and F are of equal size and one in which one country, here H, is significantly larger than the other, containing 80 per cent of all productive capacity and consumers of the union. H and F can equally be thought of as two sets of countries, corresponding for example to the "core" and the "periphery" of the euro area. For the calibration of the shock processes shown in Section 6, the former interpretation is used.

[31] Suppose that the disutility from production $V\left[y_t(j), z_t^i\right]$ results from supplying $l_t(j)$ units of labour where $y_t(j) = \exp(-z_t^i)l_t(j)$ so that V can be expressed as $V\left[l(j)\right]$ in steady state. Since utility is additively separable in consumption and labour supply, the inverse Frisch elasticity is given by $\frac{V_{ll}}{V_l}l(j)$ and can be written as $\frac{V_{yy}}{V_y}y(j)$ which is precisely equal to η given that $y(j) = C$.

5 Optimal Policy Coordination

This section starts by characterising the optimal mix of conventional and unconventional policy analytically before discussing the impulse responses to asymmetric shocks that shift the first-best relative price level between H and F.

The central bank is assumed to maximise welfare given by (70) subject to the constraints posed by the structure of the economy.[32] It possess a credible commitment device and chooses policy from the "timeless perspective".[33]

Lemma 3 *If $\alpha_H = \alpha_F$, then $\hat{T}_t$ is independent of $\hat{i}_t$.*

The terms of trade evolve according to

$$\hat{T}_t = \hat{T}_{t-1} + \hat{\Pi}_t^F - \hat{\Pi}_t^H \tag{72}$$

where $\hat{\Pi}_t^F - \hat{\Pi}_t^H$ can be found by subtracting the Phillips curve for H from that for F,

$$\hat{\Pi}_t^F - \hat{\Pi}_t^H = \beta \mathrm{E}_t \left(\hat{\Pi}_{t+1}^F - \hat{\Pi}_{t+1}^H \right) - a \left(\hat{T}_t - \tilde{T}_t \right) + a_{\mathcal{P}} \left(\mathcal{P}_t^F - \mathcal{P}_t^H \right) \tag{73}$$

with $a \equiv a_C^H = a_C^F = a_T^H = a_T^T$ and $a_{\mathcal{P}} \equiv a_{\mathcal{P}}^H = a_{\mathcal{P}}^F$.[34] Since the consumption gap has a symmetric effect on inflation in both countries when price stickiness is identical, it does not enter the *relative* inflation in producer prices. As changes in the nominal interest rate only feed into the rest of the economy through changes to the households' consumption and savings behaviour, the inflation rate differential and thus the terms of trade are independent of $\hat{i}_t$ in this case. Using the conventional tool alone, the central bank can therefore not actively close the terms of trade gap in response to disturbances that move the natural terms of trade away from their steady state level.[35] It was shown before that $\hat{\Pi}_t^F - \hat{\Pi}_t^H$ depends on the unconventional tool, allowing the central bank to affect the terms of trade through purchases and sales of government bonds in H and F. As a result, all policy induced smoothing of terms of trade shocks is entirely due to unconventional policy in the special case considered here.

Proposition 3 *Under the optimal commitment policy chosen from the timeless perspective, a transitory shift in the natural terms of trade is addressed by persistent interventions in the market for government debt.*

Combining (72) and (73) gives

$$\hat{T}_t = \frac{1}{1+\beta+a} \left[\hat{T}_{t-1} + \beta \mathrm{E}_t \hat{T}_{t+1} + a_{\mathcal{P}} \left(\mathcal{P}_t^F - \mathcal{P}_t^H \right) + a \tilde{T}_t \right] \tag{74}$$

[32] The policy optimisation problem is described in detail in the online appendix.
[33] See Woodford (1999) and Woodford (2003) for details on optimal policy from the timeless perspective.
[34] These relationships hold for $\alpha_H = \alpha_F$ and $\rho = 1$.
[35] This result goes back to Benigno (2004).

The terms of trade in period t depend on a lag, their natural rate, expectations formed in t about the terms of trade in $t+1$ and the relative strength of unconventional policy interventions. When $\tilde{T}_t > 0$ and $\hat{T}_{t-1} = 0$, for example, which can result from a more favourable development of technology in H compared to F, the central bank aims to create the expectation in t that $\hat{T}_{t+1}$ is positive to close the terms of trade gap. Since a quadratic cost is attached to all three margins at which the shock can be accommodated, current and future deviations of the terms of trade from its natural rate, current and future deviations of inflation from target in both countries and unconventional interventions, generally a mix of all three minimises welfare losses, giving rise to a persistent unconventional policy response along the terms of trade and inflation adjustment path. The proposition below characterises the optimal policy further.

Proposition 4 *For $\alpha_H = \alpha_F$, in the equilibrium generated by the optimal commitment policy chosen from the timeless perspective, the following is true for all $t \geq 0$.*

a) Average inflation is entirely stabilised.
$n\hat{\Pi}_t^H + (1-n)\hat{\Pi}_t^F = 0$

b) The short-term interest rate only depends on the natural rate of consumption.
$\hat{i}_t = -\rho\tilde{C}_t$

c) $\mathcal{P}_t^R$ is made dependent on $\mathcal{P}_{t-1}^R$, $\mathrm{E}_t\mathcal{P}_{t+1}^R$, past, present and expected future values of the terms of trade as well as current and lagged values of its natural rate.
$\mathcal{P}_t^R = \mathrm{E}_t f\left(\mathcal{P}_{t-1}^R, \mathcal{P}_{t+1}^R, \hat{T}_{t-1}, \hat{T}_t, \hat{T}_{t+1}, \tilde{T}_{t-1}, \tilde{T}_t\right)$

Proof: See Section A.4 in the appendix.

Analogous to a result by Benigno (2004), a central bank that is equipped with the conventional tool only should stabilise average union-wide inflation. Part a) of the proposition above shows that this is still true when the unconventional tool is at its disposal as well. Note that this implies that small deviations of inflation from target in a large country are accompanied by large deviations in a small country. For example, if inflation is 50 basis points below target in 80 per cent of the union, it should be 200 basis points above target in the rest of the currency union. Large inflation differentials are therefore not necessarily an indicator for suboptimal conventional or unconventional monetary policy. It has been argued before that conventional monetary policy is unable to affect the relative price level between H and F. Since parallel shifts in inflation in response to shocks to the natural terms of trade are not welfare-improving, $\hat{i}_t$ is adjusted to close the consumption gap at all times, which results in the simple rule for $\hat{i}_t$ given in b). The final part shows that $\mathcal{P}_t^R$ has an autoregressive component, leading to persistence in the unconventional tool and that the path of the terms of trade only depends on the the paths of the corresponding natural rate and the relative strength of unconventional policy in both countries. The results described in b) and c) give insights for the question of how to design simple policy rules that approximate the optimal policy as closely as possible.

5.1 Countries of Equal Size

Figure 3 shows the impulse responses of an unexpected 10% one-off increase in the natural terms of trade in period $t = 1$. Recall that such a disturbance can result from demand or supply shocks of asymmetric size.[36] The two countries are assumed to be equal sized.

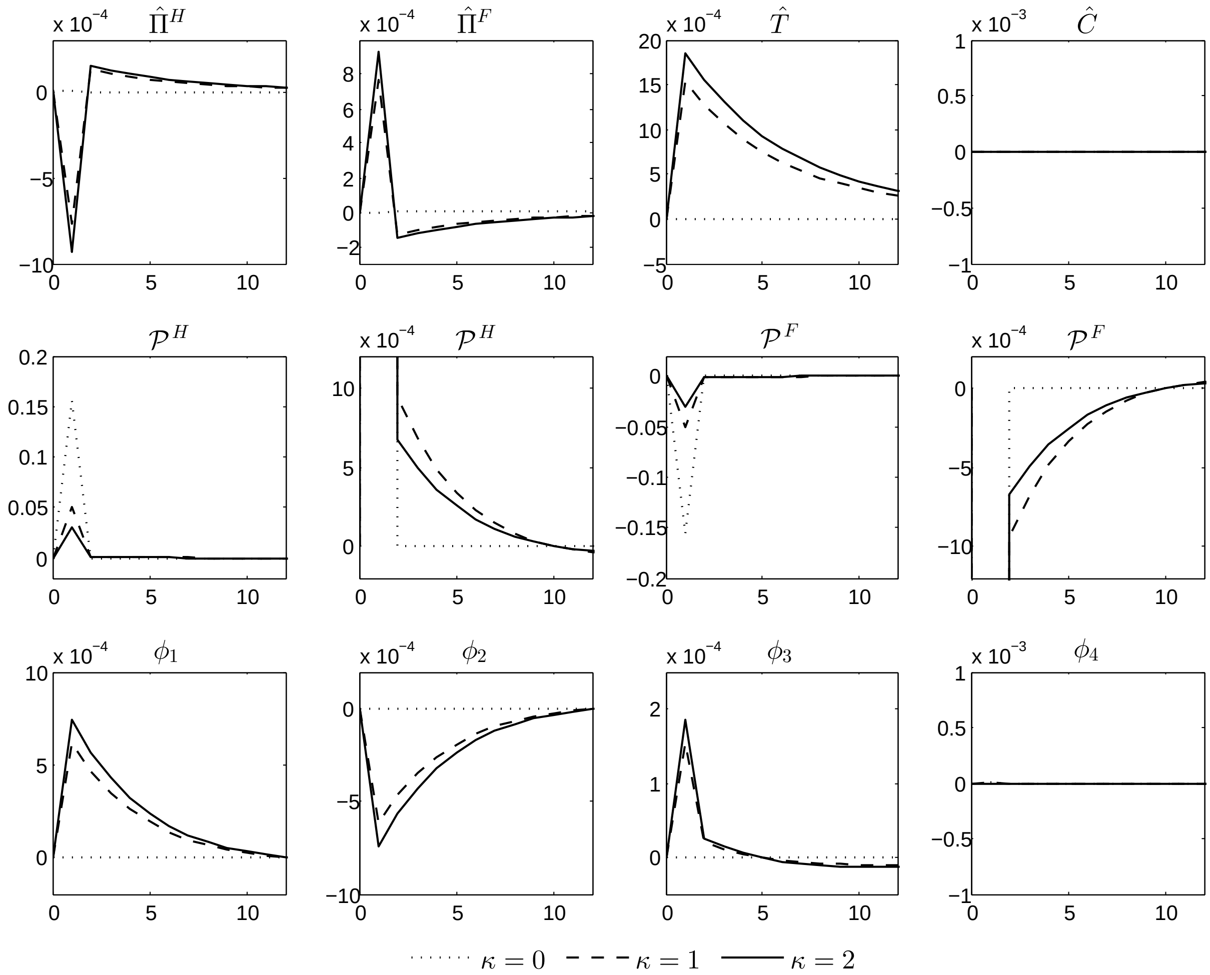

Figure 3: Impulse responses to a 10% increase in natural terms of trade for different values of κ

When confronted with a shock to the natural terms of trade, the monetary policy maker faces a trade-off with respect to the optimal adjustment of the terms of trade. By letting $\hat{T}_t$ rise upon impact of the shock, contemporaneous losses from an increase in the terms of trade gap are decreased. To achieve the increase in $\hat{T}_t$, inflation has to be allowed to fall below target in H and to rise above target in F, which leads to contemporaneous welfare costs. Since the terms of trade are elevated entering into the next period in this case, additional welfare costs arise from

[36] In fact, the discussion here generalises to all sources of efficient flucutations that shift the relative price level.

steering $\hat{T}_t$ back to target. The additional costs result from a succession of positive terms of trade gaps, increased inflation in H, decreased inflation in F and unconventional interventions along the adjustment path. In contrast, when the terms of trade are not permitted to rise in $t = 1$, large contemporaneous costs resulting from the terms of trade gap and unconventional policy necessary to prevent inflation from falling in H and rising in F have to be incurred. However, the shock is fully stabilised upon impact, which means that future losses are avoided.

The degree to which it is optimal to let $\hat{T}_t$ initially increase depends on the costs associated with unconventional policy. If $\kappa = 0$, unconventional policy is sufficiently uncostly to make it optimal to stabilise inflation and thus the terms of trade entirely. Large asset purchases in H limit the deflationary effect of the shock and large sales in F limit the inflationary pressure that it causes there. As κ increases, larger initial reactions of $\hat{T}_t$ are permitted, since stabilising $\hat{T}_t$ becomes more costly. Even for large values of κ, the terms of trade increase in $t = 1$ is moderate, however. This is due to the fact that deviations of inflation from target are significantly more costly than all other components of the welfare loss function.

Unconventional policy is used in $t = 1$ to dampen the impact of the shock on inflation in both countries and, in the subsequent periods, to steer the terms of trade back to target. Since it is optimal to prevent large movements in $\hat{T}_t$ due to the reasons outlined above, the unconventional tool is more heavily used at the time at which the unexpected shift in the natural terms of trade occurs compared to the following periods. The impulse responses of $\mathcal{P}_t^H$ and $\mathcal{P}_t^F$ are each plotted twice using different scales to be able to illustrate both the initial reaction to the shock and the subsequent adjustment process. For the calibration chosen here, unconventional policy in $t = 1$ is applied at about 16% of steady state output for $\kappa = 0$, 5% for $\kappa = 1$ and 3% for $\kappa = 2$. In $t = 2$ these shares fall to about 0.00%, 0.09% and 0.07% and then gradually decline as the economy converges to back towards its steady state.

Conventional monetary policy is inactive in response to the terms of trade shock. This can be seen from the fact that consumption remains entirely unaffected by it. The bigger κ is, the more restrictive are the constraints posed by the Phillips curves and the law of motion of the terms of trade resulting in more volatile reactions of the corresponding multipliers $\phi_{1,t}$, $\phi_{2,t}$ and $\phi_{3,t}$. The constraint associated with the requirement that the average unconventional intervention be zero at all times does no bind, since it has been imposed through appropriate substitution into the welfare criterion, thus $\phi_{4,t} = 0$.

Figure 4 compares the impulse responses for the case that unconventional monetary policy can be used without restrictions and the case that central bank purchases of government debt are restricted to be zero at all times. The responses that pertain to the former scenario are depicted for the intermediate value of the cost parameter. Bond market interventions by the central bank stabilise the terms of trade and inflation in both countries significantly. In particular, the unconventional tool causes the initial impact on inflation in both countries and the terms of trade to be significantly less pronounced. The additional monetary policy tool also works to reduce the tightness of the restrictions posed by the Euler equation as can be seen from the response of the corresponding Lagrange multipliers. The comparison between policy regimes in which unconventional monetary policy can and cannot be used is continued in Section 6 which derives an estimate of the welfare gains of having access to the unconventional tool.

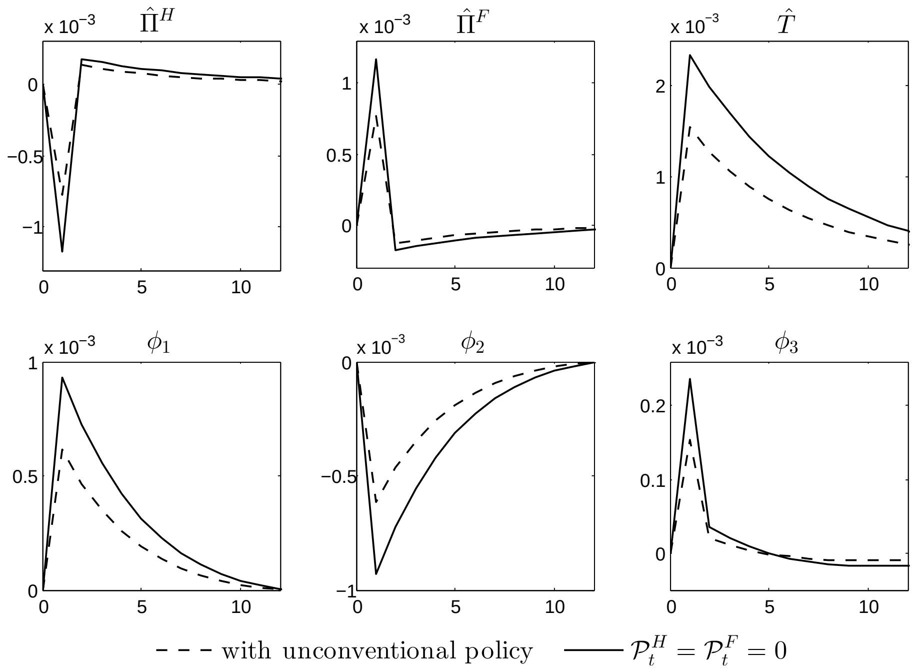

Figure 4: Comparison to the case without unconventional policy ($\kappa = 1$)

5.2 Countries of Unequal Size

When H and F are of unequal size, optimal unconventional policy is asymmetric. The impulse responses to a 10% increase in the natural terms of trade for the case that country H contains 80 per cent of all consumers and productive capacity are shown in Figure 5.

For each κ, the same initial increase in the terms of trade as in the case of identical countries is permitted. However, since the contribution of F to aggregate welfare is smaller than that of H, higher inflation volatility is allowed there. A large share of the goods consumed in the small country is imported from H while only a small fraction of the goods produced in H are imported from F, which implies that the terms of trade shock increases inflation in F more than in H as can be seen from the two Phillips curves. To dampen the initial impact of the disturbance and to steer inflation in both countries and the terms of trade back to target from period $t = 2$ onwards, the unconventional tool is therefore used more heavily in F than in H. Shocks that affect the relative price level between two countries of a currency union thus result not only in more inflation volatility but also in stronger unconventional interventions in the smaller of the two countries if policy is chosen optimally.

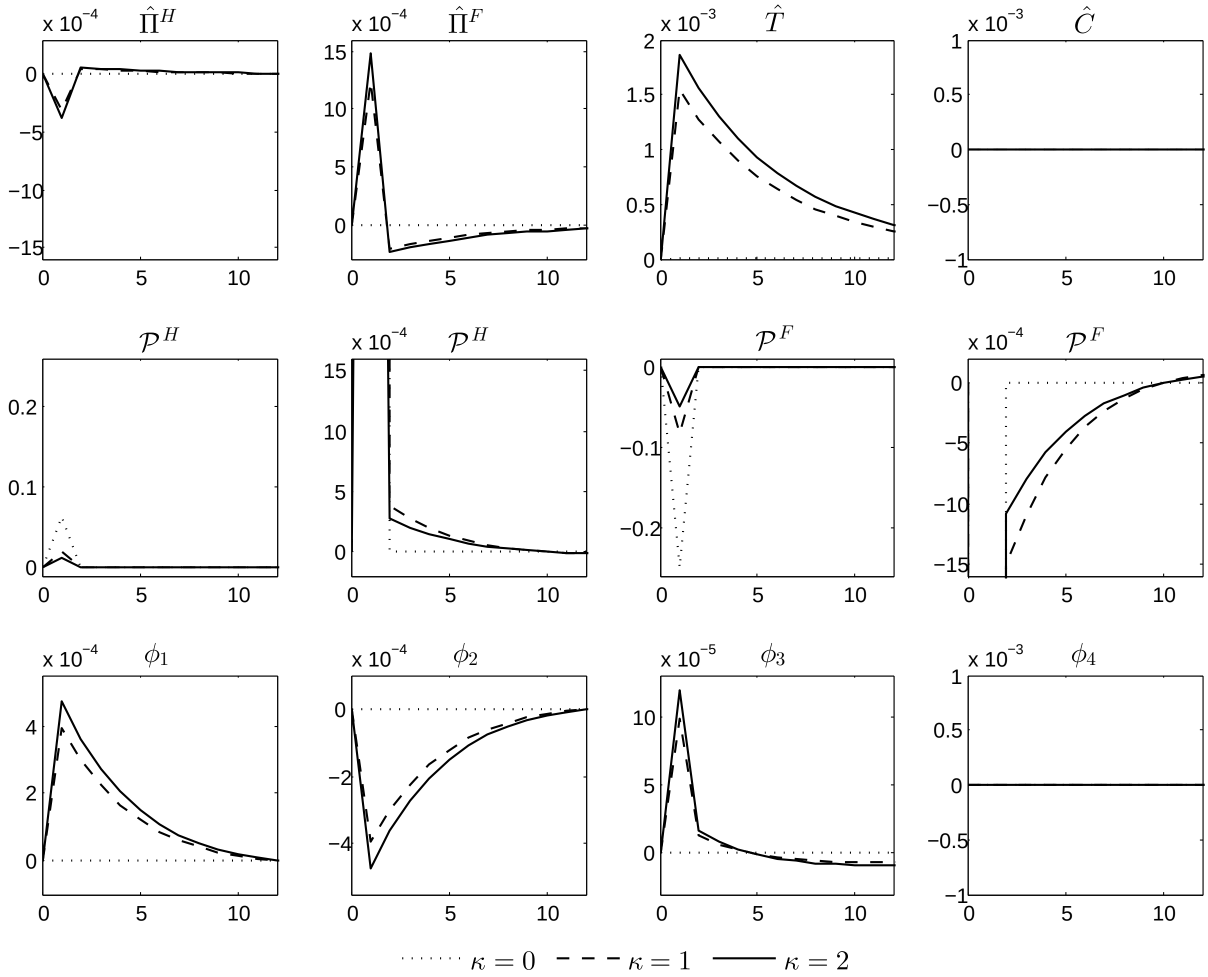

Figure 5: Impulse responses to a terms of trade shock in a union of a large and a small country

6 Welfare Implications

It was shown above that the central bank employs the conventional policy tool to close the consumption gap at all times when the degrees of price stickiness are equal in both countries. I continue to consider the case $\alpha^H = \alpha^F$ here to isolate the welfare effects of unconventional bond purchases specific to a currency union and therefore only describe the calibration of the stochastic process followed by $\tilde{T}_t$. If the underlying exogenous components of $\tilde{T}_t$, that is, technology and government spending shocks, are assumed to follow AR(1) processes, then, according to Granger and Morris (1976), $\tilde{T}_t$ follows a higher order ARMA process. To be able to derive a closed form expression for welfare, the strategy chosen here is to approximate the micro-founded process of $\tilde{T}_t$ using an AR(1) process and to calculate welfare differentials based on the approximated time series.

The law of motion of the natural terms of trade is approximated by

$$\tilde{T}_t^a = \rho_T \tilde{T}_{t-1}^a + u_{T,t} \tag{75}$$

where $u_{T,t} \sim N(0, \sigma_T^2)$. To derive ρ_T and σ_T^2, recall that $\tilde{T}_t$ is defined as

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(g_t^{P,R} - \zeta_t^R\right) \tag{76}$$

with $g_t^{P,i} \equiv \frac{G_t^{P,i} - G^{P,i}}{Y^i}$ and $\zeta_t^i \equiv -\frac{V_{yz}(C,0)}{CV_{yy}(C,0)} z_t^i$. The disutility from production V can be thought of as arising from labour exerted in the production process. Assuming that the production function is $y_t(j) = \exp(-z_t^i) l_t(j)$ where $l_t(j)$ is labour and that $V(\cdot)$ is a standard CES function of $l_t(j)$, given by $V\left[l_t(j)\right] = \frac{l_t(j)^{1+\eta}}{1+\eta}$, the supply side shock is given by $\zeta_t^i = -\frac{1+\eta}{\eta} z_t^i$. Both types of "deep" shocks evolve according to AR(1) processes, that is

$$z_t^i = \rho_z z_{t-1}^i + \varepsilon_t^{z,i} \tag{77}$$

$$\frac{G_t^{P,i}}{Y^i} = \rho_G \frac{G_{t-1}^{P,i}}{Y^i} + \varepsilon_t^{G,i} \tag{78}$$

where $\left(\varepsilon_t^{z,H}, \varepsilon_t^{z,F}\right) \sim N(0, \Sigma^z)$, $\left(\varepsilon_t^{G,H}, \varepsilon_t^{G,F}\right) \sim N(0, \Sigma^G)$ and

$$\Sigma^z \equiv \begin{bmatrix} \sigma_{z,H}^2 & \sigma_{z,HF}^2 \\ \sigma_{z,FH}^2 & \sigma_{z,F}^2 \end{bmatrix}, \quad \Sigma^G \equiv \begin{bmatrix} \sigma_{G,H}^2 & \sigma_{G,HF}^2 \\ \sigma_{G,FH}^2 & \sigma_{G,F}^2 \end{bmatrix} \tag{79}$$

This specification allows for correlation between the evolution of technology in both countries and for correlated government spending shocks.[37] Demand and supply side shocks are assumed to be independent. Using equation (76), one can derive the variance of $\tilde{T}_t$ as a function of parameters. The variance of $\tilde{T}_t^a$ is chosen to match this variance, more precisely

$$\begin{aligned} \text{Var}\left(\tilde{T}_t^a\right) = \text{Var}\left(\tilde{T}_t\right) &= \left(\frac{\eta}{1+\eta}\right)^2 \text{Var}\left[g_t^{P,F} - g_t^{P,H} - \left(\zeta_t^F - \zeta_t^H\right)\right] \\ &= \left(\frac{\eta}{1+\eta}\right)^2 \left[\text{Var}\left(g_t^{P,F}\right) + \text{Var}\left(g_t^{P,H}\right) + \text{Var}\left(\zeta_t^F\right) + \text{Var}\left(\zeta_t^H\right)\right. \\ &\quad \left. -2\text{Cov}\left(g_t^{P,H}, g_t^{P,F}\right) - 2\text{Cov}\left(\zeta_t^H, \zeta_t^F\right)\right] \\ &= \left(\frac{\eta}{1+\eta}\right)^2 \left[\frac{\sigma_{G,F}^2}{1-\rho_G^2} + \frac{\sigma_{G,H}^2}{1-\rho_G^2} + \left(\frac{1+\eta}{\eta}\right)^2 \frac{\sigma_{z,F}^2}{1-\rho_z^2} + \left(\frac{1+\eta}{\eta}\right)^2 \frac{\sigma_{z,H}^2}{1-\rho_z^2}\right. \\ &\quad \left. -2\frac{\sigma_{G,HF}^2}{1-\rho_G^2} - 2\left(\frac{1+\eta}{\eta}\right)^2 \frac{\sigma_{z,HF}^2}{1-\rho_z^2}\right] \end{aligned} \tag{80}$$

[37] Government spending is normalised by steady state output to facilitate the comparability with the previous literature.

The persistence parameter is found numerically. To do so, I simulate $\tilde{T}_t$ for one million periods and calculate ρ_T using the OLS/ML estimator, so $\rho_T = \sum \tilde{T}_t \tilde{T}_{t-1} / \sum \tilde{T}_{t-1}^2$.

Parameter	Value	Description
ρ_z	0.94	Persistence of technology shocks
ρ_G	0.98	Persistence of government spending shock
$\sigma^2_{z,H}$, $\sigma^2_{z,F}$	$0.16 * 10^{-3}$	Variance of technology shock
$\sigma^2_{G,H}$, $\sigma^2_{G,F}$	$0.21 * 10^{-3}$	Variance of government spending shock

Table 2: Additional Calibration

Table 2 summarises the calibration of the demand and supply shock sequences. The variance and persistence of the shock processes are assumed to be equal in H and F. The estimates of the persistence parameters are taken from Smets and Wouters (2005), which includes an estimation of a DSGE model for the euro area based on quarterly data from 1983 to mid-2002. The values for the variances are estimates reported in Duarte and Wolman (2008). Duarte and Wolman estimate the laws of motion of technology shocks in the traded goods sector and the government spending share of GDP using French and German quarterly data from the 1990s and early 2000s.[38] In addition, consumption utility is now assumed to be logarithmic, consistent with the previous assumption that $\rho = 1$.

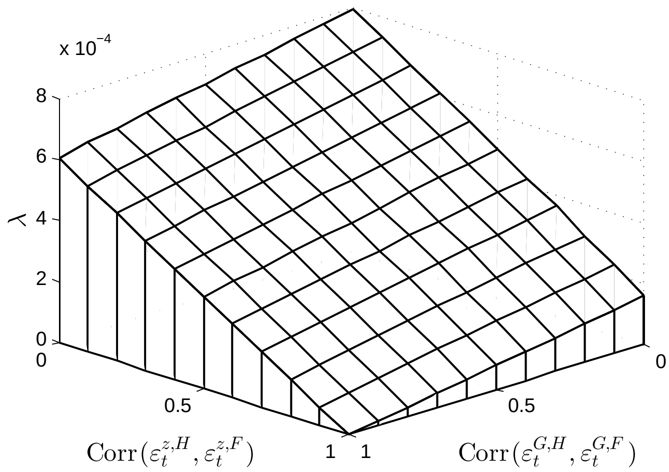

Figure 6: Welfare gains as a function of shock correlations (for $\kappa = 1$)

Figure 6 shows the welfare gains from employing the unconventional tool as a function of the respective correlation of demand and supply shocks across the two countries. The welfare

[38] The variance estimates reported in Smets and Wouters (2005) are significantly larger than those from Duarte and Wolman (2008). The smaller set of variances is used here, which is also in line with the calibrations from Gali (2008) and others.

measure λ is given by

$$\lambda = 1 - e^{(1-\beta)(W^a - W)} \tag{81}$$

W and W^a are the unconditional expectation of welfare under the policy that is optimal from the timeless perspective when the central bank is able to credibly commit and under an alternative policy, respectively. Here, the alternative policy is optimal subject to the same qualifications as W if the central bank does not have access to the unconventional tool. λ is defined as the amount of consumption that an individual would have to give up in each period under the optimal policy to be as well off as under the alternative policy.[39]

If both types of disturbances are perfectly correlated, no welfare gains are obtainable. A perfect correlation of demand and supply shocks implies that $\tilde{T}_t = \tilde{T} = 0$ for all t. To see this, note that for $\sigma^2_{z,H} = \sigma^2_{z,F} = \sigma^2_z$ and $\sigma^2_{G,H} = \sigma^2_{G,F} = \sigma^2_G$, the variance of $\tilde{T}_t$ can be written as

$$\begin{aligned} \text{Var}\left(\tilde{T}_t\right) = \left(\frac{\eta}{1+\eta}\right)^2 \Bigg\{ & \frac{2}{1-\rho_G^2}\sigma_G^2 \left[1 - \text{Corr}(\varepsilon_t^{G,H}, \varepsilon_t^{G,F})\right] + \\ & \frac{2}{1-\rho_z^2}\left(\frac{1+\eta}{\eta}\right)^2 \sigma_z^2 \left[1 - \text{Corr}(\varepsilon_t^{z,H}, \varepsilon_t^{z,F})\right] \Bigg\} \end{aligned} \tag{82}$$

where $\text{Corr}(\cdot,\cdot) \in [-1, 1]$ is Pearson's correlation coefficient. Thus, if the shocks within the currency union are perfectly correlated across countries, the natural terms of trade remain at their steady state level and there are no gains from employing unconventional monetary policy. The lower is the correlation of the disturbances, the larger becomes the variance of $\tilde{T}_t$ and the higher are the welfare gains associated with central bank asset purchases. In addition, the variances of the individual shocks, their persistence and the inverse elasticity of substitution in production matter for the variance of $\tilde{T}_t$ and thus for the gains obtainable from employing asset purchases.

The welfare effects of the unconventional tool depend on κ, the parameter that reflects the costs associated with unconventional interventions. Figure 7 contrasts the gains obtainable for the different values of κ. Shown are isoquants of λ with a step size of 0.01 percentage points. When starting at the point of perfect correlation $(1, 1)$ and moving into the direction of lower correlation of both disturbances, i.e. to the south-west, welfare increases faster for lower values of κ. The calibration chosen here implies that welfare gains increase faster with declining correlation in supply shocks than in demand shocks, emphasising the importance for the central bank to monitor productivity differences within the currency union.

It should be emphasised here again that the values shown above have to be interpreted as an upper bound on the currency-union-specific gains obtainable from government bond purchases. The fraction of these gains that the central bank is able to secure depends on the, likely time-varying, elasticity of substitution between government bonds from the member countries, the

[39] See Sections A.5 and A.6 of the appendix for the details.

degree of fiscal coordination and the ability to implement optimal policy. Yet, this upper bound is meaningful. For example, at average household consumption of €28,500 as in Germany in 2014 and $\text{Corr}\left(\varepsilon_t^{z,H}, \varepsilon_t^{z,F}\right) = \text{Corr}\left(\varepsilon_t^{G,H}, \varepsilon_t^{G,F}\right) = 0.8$, the model implies that a household's willingness to pay for the central bank to engage in bond purchases is less than €12.95 for $\kappa = 0$, €4.53 for $\kappa = 1$ and €2.77 for $\kappa = 2$. Based on these relatively small numbers, the ECB's decision not to engage in asymmetric large-scale bond purchases in normal times seems justified. However, in a crisis scenario in which the evolution of technology across countries becomes uncoupled, i.e. $\text{Corr}\left(\varepsilon_t^{z,H}, \varepsilon_t^{z,F}\right) = 0$ and $\text{Corr}\left(\varepsilon_t^{G,H}, \varepsilon_t^{G,F}\right) = 0.8$, the analogous values for λ increase to €51.70 ($\kappa = 0$), €18.40 ($\kappa = 1$) and €11.20 ($\kappa = 2$). As a situation with strongly asymmetric technology shocks may also involve higher degrees of segmentation in asset markets, government bond purchases with the goal of closing the terms of trade gap can be helpful in mitigating the effects of the disturbances in such a scenario.

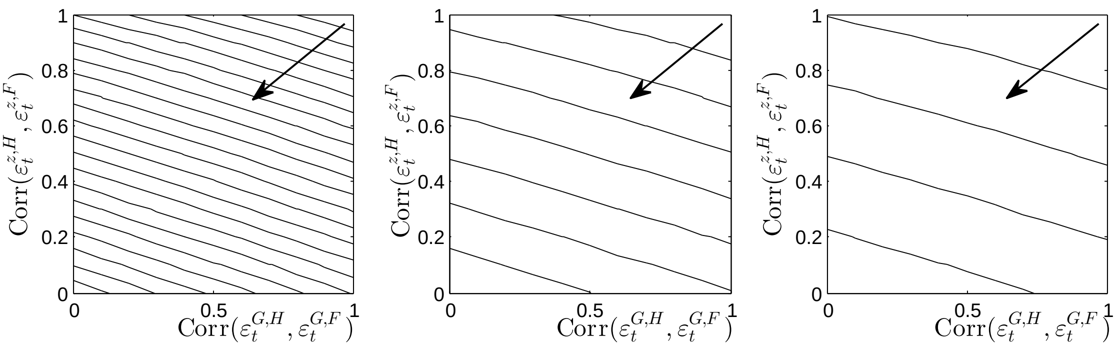

Figure 7: Welfare gains for $\kappa = 0$ (left), $\kappa = 1$ (centre), $\kappa = 2$ (right)

7 Conclusion

In a currency union, not only aggregate output and inflation, but also the relative price level between countries may be temporarily driven away from its optimal value. This fact poses a challenge to the monetary policy authority, because conventional interest rate policy is not suitable for steering the terms of trade close to their natural rate. It has been widely argued that establishing a fiscal union is the first-best solution to this issue in the case of the euro area. Independent of this claim, the question arises whether there are policy tools that can substitute for coordinated fiscal responses to asymmetric disturbances until a potential future fiscal union has been established or support it following its creation. The candidate tool considered in this paper involves purchases of government debt, comparable to those carried out as a part of the Large-Scale Asset Purchases by the Federal Reserve or the Quantitative Easing programme of the Bank of England.

To establish that the central bank of a currency union is able to affect the terms of trade using government debt purchases, it is shown first that market segmentation across country borders—comparable to the segmentation along the maturity dimension vital to the preferred habitat theory of the term structure—leads bond prices to respond to local demand and supply effects. Local yield changes in turn cause local expansions or contractions and thus affect the relative price level. This paper derives sufficient conditions for the link between government bond yields and the domestic price level to be negative in a model with fully forward-looking agents that are not subject to within-country restrictions regarding their investment opportunities but with endogenous government spending. A decline in government bond yields and the accompanying rise in bond prices have expansionary effects if bond supply is well-behaved, debt management is sufficiently passive and, in case the fiscal authorities have targets for real spending and taxation, the country runs a primary surplus.

For countries of similar structure, the optimal commitment policy mix stabilises average union-wide inflation, addresses movements in natural consumption using the conventional tool and involves persistent interventions in debt markets in reaction to a transitory shift in the natural terms of trade. In response to less than perfectly correlated demand and supply disturbances that cause an unexpected increase in the natural terms of trade, government bonds are bought in country H and sold in country F. To be able to implement this policy, the central bank would have to hold a buffer of assets that could be increased or run down as required. It could therefore only be used to address mean-reverting cyclical fluctuations, not sustained structural differences between countries.

The potential gains from using asset purchases to close the terms of trade gap are small in normal times but significant in times of diverging technological progress among the member countries. While with highly correlated technology shocks my model suggests that these gains are no larger than the equivalent of €12.95 of annual household consumption, this figure is about four times as large when technology shocks cease to be correlated. The fact that periods of high volatility in the natural terms of trade can be expected to coincide with periods of strong market segmentation implies that bond purchases are most effective when their potential welfare impact is largest.

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Appendix

A.1 Proof of Proposition 1

C_t^i and $i_t^{Q,i}$ are jointly determined by the household optimality conditions (11) and (12) as well as the intertemporal resource constraint, which is derived first. The derivations are shown for country H only, analogous steps lead to the result for country F.

The part of central bank surplus transferred to H is given by

$$\frac{\Delta_t^H}{P_t} = \frac{\bar{M}_t^H}{P_t} - \frac{\bar{M}_{t-1}^H}{P_t} + \frac{Q_{CB,t-1}^H}{P_t} - \frac{Q_{CB,t}^H}{P_t(1+i_t^{Q,H})} \tag{83}$$

Substituting for $\frac{\Delta_t^H}{P_t}$ in the government budget constraint and solving for aggregate real taxes gives

$$\int_0^n \frac{T_t^H}{P_t} dj = \int_0^n \frac{p_t(j)g_t(j)}{P_t} dj - \left[\int_0^n \frac{Q_t^H}{P_t(1+i_t^{Q,H})} dj - \int_0^n \frac{Q_{t-1}^H}{P_t} dj \right] - \left(\frac{\bar{M}_t^H}{P_t} - \frac{\bar{M}_{t-1}^H}{P_t} \right) \tag{84}$$

By aggregating the budget constraints for all agents $j \in [0, n)$ and employing the fact that $\mathcal{D}_t^i = \xi_t^i Q_t^i / (1 + i_t^{Q,i})$ and $T_t^{\tau,j} = \tau p_t(j) y_t(j)$, one obtains

$$\begin{aligned} &q_t^i \int_0^n B_t^{i,j} dj + \int_0^n \frac{B_t^H}{P_t(1+i_t)} dj + \int_0^n \frac{Q_t^H}{P_t(1+i_t^{Q,H})} dj + \int_0^n \frac{M_t^H}{P_t} dj + \int_0^n C_t^H dj + \int_0^n \frac{T_t^H}{P_t} dj = \\ &\mathbb{1}_{1\times S} \int_0^n B_{t-1}^{i,j} dj + \int_0^n \frac{B_{t-1}^H}{P_t} dj + \int_0^n \frac{Q_{t-1}^H}{P_t} dj + \int_0^n \frac{M_{t-1}^H}{P_t} dj + \int_0^n \frac{p_t(j) y_t(j)}{P_t} dj \end{aligned} \tag{85}$$

Entering (84) into the equation above, using that state-contingent securities are in zero net supply and simplifying yields

$$\begin{aligned} &\int_0^n \frac{B_t^H}{P_t(1+i_t)} dj + \int_0^n \frac{M_t^H}{P_t} dj + \int_0^n C_t^H dj - \left(\frac{\bar{M}_t^H}{P_t} - \frac{\bar{M}_{t-1}^H}{P_t} \right) = \\ &\int_0^n \frac{B_{t-1}^H}{P_t} dj + \int_0^n \frac{M_{t-1}^H}{P_t} dj + \int_0^n \frac{p_t(j)\left[y_t(j) - g_t(j)\right]}{P_t} dj \end{aligned} \tag{86}$$

The money market clears in equilibrium, $\bar{M}_t^H = \int_0^n M_t^H dj$, thus (86) becomes

$$\int_0^n \frac{B_t^H}{P_t(1+i_t)} dj + \int_0^n C_t^H dj = \int_0^n \frac{B_{t-1}^H}{P_t} dj + \int_0^n \frac{p_t(j)\left[y_t(j) - g_t(j)\right]}{P_t} dj \tag{87}$$

which is the intertemporal resource constraint for country H. Neither the quantity of bonds purchased by the central bank nor money holdings enter this equation. The same is true for the analogous constraint of country F, which is not shown here for brevity, and, since $\xi_t^i = \xi^i$, for the optimality conditions (11) to (13).

Suppose now that $P_t^i = P_t^{i,0}\ \forall t \geq 0$. Then $P_t = P_t^0$, (87) and (11) to (13) imply that $C_t^i = C_t^{i,0}$ and $i_t^{Q,i} = i_t^{Q,i,0}$ for all $t \geq 0$. Using government bond market clearing and the central bank's financing rule (22), Equation (84) can be re-written as

$$n\frac{T_t^H}{P_t^0} = n\frac{P_t^{H,0}G_t^H}{P_t^0} - \left[\frac{\bar{Q}_t^H}{P_t^0\left(1+i_t^{Q,H,0}\right)} - \frac{\bar{Q}_{t-1}^H}{P_t^0}\right] - \frac{Q_{CB,t-1}^H}{P_t^0} \tag{88}$$

In the absence of bond purchases in t, we have

$$n\frac{T_t^{H,0}}{P_t^0} = n\frac{P_t^{H,0}G_t^{H,0}}{P_t^0} - \left[\frac{\bar{Q}_t^{H,0}}{P_t^0\left(1+i_t^{Q,H,0}\right)} - \frac{\bar{Q}_{t-1}^H}{P_t^0}\right] - \frac{Q_{CB,t-1}^H}{P_t^0} \tag{89}$$

Suppose also that $\bar{Q}_t^H = \bar{Q}_t^{H,0}$. Then the two equations above imply that $T_t^H = T_t^{H,0}$ if $G_t^H = G_t^{H,0}$. Since this is the bliss point of the government's loss function (34), the government chooses this tax-spending combination. According to the debt issuance rule (30), we then have $\bar{Q}_t^H = \bar{Q}_t^{H,0}$ which confirms the initial supposition. Note that since $T_{t'}^H$ does not depend on $Q_{CB,t'}^H$, it must be possible to choose the bliss point also in all periods $t' > t$. Periods prior to the intervention are equally unaffected. A symmetric argument applies to F.

Since $C_t^i = C_t^{i,0}$, $G_t^i = G_t^{i,0}$ and $P_t^i = P_t^{i,0}$ at all times, it must also be true that $Y_t^i = Y_t^{i,0}\ \forall t \geq 0$. As a result, the optimal re-set price (19) is unaffected by bond purchases and the initial assumption about the price level in both countries being unchanged is correct.

A.2 Proof of Proposition 2

From (39) and (40) it is clear that, given $\omega_T \geq 0$, $\omega_T > 0$ is a necessary condition for $\bar{\mathcal{P}}_t^{ID,i} > 0$ and $\bar{\mathcal{P}}_t^{PS,i} > 0$.

Then $\bar{\mathcal{P}}_t^{ID,i} > 0$ if

$$D^i \equiv \frac{1}{1+i_t^{Q,i}}\frac{\bar{Q}_t^i}{n^i} - \frac{1}{1+i_t^{Q,i,0}}\frac{\bar{Q}_t^{i,0}}{n^i} > 0 \tag{90}$$

D^i can be re-written as follows. Plugging in from (30) yields

$$D^i = b_0\left(\frac{1}{1+i_t^{Q,i}} - \frac{1}{1+i_t^{Q,i,0}}\right) + b_2\left(T_t^{i,0} - T_t^i + P_t^iG_t^i - P_t^{i,0}G_t^{i,0}\right) \tag{91}$$

which, using (37), becomes

$$D^i = b_0\left(\frac{1}{1+i_t^{Q,i}} - \frac{1}{1+i_t^{Q,i,0}}\right) + b_2D^i \tag{92}$$

$$= \frac{b_0}{1-b_2}\left(\frac{1}{1+i_t^{Q,i}} - \frac{1}{1+i_t^{Q,i,0}}\right) \tag{93}$$

Thus, $\bar{\mathcal{P}}_t^{ID,i} > 0$ if $b_0 > 0$, $b_2 < 1$ and $i_t^{Q,i} < i_t^{Q,i,0}$.

For $i_t^{Q,i} < i_t^{Q,i,0}$ to hold, we must have that

$$\bar{Q}_t^i - Q_{CB,t}^i < \bar{Q}_t^{i,0} \tag{94}$$

or

$$\frac{\bar{Q}_t^i}{n^i} - \frac{\bar{Q}_t^{i,0}}{n^i} < \frac{Q_{CB,t}^i}{n^i} \tag{95}$$

Note that (30) implies

$$\frac{\bar{Q}_t^i}{n^i} = b_0 + b_1\left(1 + i_t^{Q,i}\right)Q_{t-1}^i - b_2\left(1 + i_t^{Q,i}\right)(T_t^i - P_t^i G_t^i) \tag{96}$$

and

$$\frac{\bar{Q}_t^{i,0}}{n^i} = b_0 + b_1\left(1 + i_t^{Q,i,0}\right)Q_{t-1}^i - b_2\left(1 + i_t^{Q,i,0}\right)(T_t^{i,0} - P_t^{i,0} G_t^{i,0}) \tag{97}$$

Plugging the two equations above into (95) and solving for b_2 gives

$$b_2 < \frac{\frac{Q_{CB,t}^i}{n^i} + b_1 Q_{t-1}^i\left(i_t^{Q,i,0} - i_t^{Q,i}\right)}{\left(1 + i_t^{Q,i,0}\right)\left(T_t^{i,0} - P_t^{i,0} G_t^{i,0}\right) - \left(1 + i_t^{Q,i}\right)(T_t^i - P_t^i G_t^i)} \equiv \tilde{b}_2 \tag{98}$$

so that, in summary, $\bar{\mathcal{P}}_t^{ID,i} > 0$ for $\omega_T > 0$ and $b_2 < \bar{b}_2 \equiv \min\left\{1, \tilde{b}_2\right\}$. From the expression above, it is obvious that for realistic values of the elasticity of ξ_t^i to $\bar{Q}_t^i - Q_{CB,t}^i$, $\tilde{b}_2 > 1$ so that the constraint on b_2 becomes $b_2 < 1$.

Suppose that $P_t^i > P_t^{i,0}$, then $T_t^i / P_t^i - G_t^i$ and $\omega_T > 0$ immediately imply $\bar{\mathcal{P}}_t^{PS,i} > 0$. For $\bar{\mathcal{P}}_t^{ID,i} > 0$ and $\bar{\mathcal{P}}_t^{PS,i} > 0$, $G_t^i > G_t^{i,0}$ so that aggregate demand and thus the price level in equilibrium is increased, verifying the initial supposition regarding the price level.

A.3 Equality of Private Consumption Across Countries

The real central bank profit share transferred to H is given by

$$\frac{\Delta_t^H}{P_t} = n\left(\frac{M_t^S}{P_t} - \frac{M_{t-1}^S}{P_t}\right) + \frac{Q_{CB,t-1}^H}{P_t} - \frac{P_t^{QH} Q_{CB,t}^H}{P_t} \tag{99}$$

where $P_t^{QH} \equiv [(1 + i_t^Q)(1 - \epsilon_t^H Q_{CB,t}^H)]^{-1}$. Substituting for $\frac{\Delta_t^H}{P_t}$ in the budget constraint of the government of H and solving for aggregate real taxes gives

$$\int_0^n \frac{T_t^H}{P_t} dj = \int_0^n \frac{p_t(j) g_t(j)}{P_t} dj - \tau \int_0^n \frac{p_t(j) y_t(j)}{P_t} dj \\ - \left[\int_0^n \frac{P_t^{QH} Q_t^H}{P_t} dj - \int_0^n \frac{Q_{t-1}^H}{P_t} dj\right] - n\left(\frac{M_t^S}{P_t} - \frac{M_{t-1}^S}{P_t}\right) \tag{100}$$

The budget constraint of an agent $j \in [0, n)$, integrated over all agents of H, is given by

$$\int_0^n q_t^H B_t^{H,j} dj + \int_0^n \frac{B_t^j}{P_t(1+i_t)} dj + \int_0^n \frac{P_t^{QH} Q_t^H}{P_t} dj + \int_0^n \frac{M_t^j}{P_t} dj + \int_0^n C_t^j dj + \int_0^n \frac{T_t^H}{P_t} dj$$
$$= \int_0^n \mathbb{1}_{1\times S} B_{t-1}^{H,j} dj + \int_0^n \frac{B_{t-1}^j}{P_t} dj + \int_0^n \frac{Q_{t-1}^H}{P_t} dj + \int_0^n \frac{M_{t-1}^j}{P_t} dj + (1-\tau) \int_0^n \frac{p_t(j) y_t(j)}{P_t} dj \quad (101)$$

Together with the previous equation and using the fact that the state-contingent securities are in zero net supply within each country, this implies that the resource constraint for H is

$$\int_0^n \frac{B_t^j}{P_t(1+i_t)} dj + \int_0^n \frac{M_t^j}{P_t} dj + \int_0^n C_t^j dj + \int_0^n \frac{p_t(j) g_t(j)}{P_t} dj - n \left(\frac{M_t^S}{P_t} - \frac{M_{t-1}^S}{P_t} \right)$$
$$= \int_0^n \frac{B_{t-1}^j}{P_t} dj + \int_0^n \frac{M_{t-1}^j}{P_t} dj + \int_0^n \frac{p_t(j) y_t(j)}{P_t} dj \quad (102)$$

We now observe that if $C_t^H = C_t^F = C_t$, then the optimality condition for money demand (13) implies that $M_t^H = M_t^F = M_t$. Using these relationships, which will be verified later, together with the fact that the money market clears in equilibrium and expressing integrals in terms of quantities pertaining to the representative agent of H where possible, one can re-write this equation as

$$n \frac{B_t^H}{P_t(1+i_t)} - n \frac{B_{t-1}^H}{P_t} + n C_t^H = \int_0^n \frac{p_t(j) \left[y_t(j) - g_t(j) \right]}{P_t} dj \quad (103)$$

Given (41), this can be expressed as

$$n \frac{B_t^H}{P_t(1+i_t)} - n \frac{B_{t-1}^H}{P_t} + n C_t^H = \int_0^n \frac{p_t(j)}{P_t} \left(\frac{p_t(j)}{P_t^H} \right)^{-\sigma} T_t^{1-n} C_t^U dj \quad (104)$$

Since $\int_0^n p_t(j)^{1-\sigma} dj = n \left(P_t^H \right)^{1-\sigma}$ and $T_t^{1-n} = \frac{P_t}{P_t^H}$, we get

$$\frac{B_t^H}{P_t(1+i_t)} - \frac{B_{t-1}^H}{P_t} + C_t^H = C_t^U \quad (105)$$

Analogous steps lead to a symmetric condition for country F. The proof proceeds from here as shown in Benigno (2004).[40] Benigno shows that given the resource constraints for both countries, the optimality conditions from the agents' utility maximisation problem and the assumption of symmetric initial wealth, B_t^i is zero for all $t \geq 0$, which immediately implies that $C_t^i = C_t \ \ \forall t \geq 0$.

[40] See the online appendix to Benigno (2004), pp. i – ii.

A.4 Proof of Proposition 4

The optimal policy problem is to maximise (70) subject to (62) to (64), (66), suitable initial conditions that reflect that policy is assumed to be chosen “from the timeless perspective” and transversality conditions. The first-order conditions are

$$2\gamma\hat{\Pi}_t^H + \phi_{1,t} + \phi_{3,t} - \phi_{1,t-1} = 0 \tag{106}$$

$$2(1-\gamma)\hat{\Pi}_t^F + \phi_{2,t} - \phi_{3,t} - \phi_{2,t-1} = 0 \tag{107}$$

$$\begin{aligned} 2\Lambda_C n(1-n)\left(\hat{T}_t - \tilde{T}_t\right) - \Lambda_{\mathcal{P}} n(1-n)\left(\mathcal{P}_t^F - \mathcal{P}_t^H\right) - \\ a_T^H(1-n)\phi_{1,t} + a_T^F n\phi_{2,t} + \phi_{3,t} - \beta \mathrm{E}_t\phi_{3,t+1} = 0 \end{aligned} \tag{108}$$

$$2\Lambda_C\left(\hat{C}_t - \tilde{C}_t\right) - a_C^H\phi_{1,t} - a_C^F\phi_{2,t} = 0 \tag{109}$$

$$\Lambda_{\mathcal{P}}\left(1+\frac{\kappa}{\eta}\right)n\mathcal{P}_t^H + \Lambda_{\mathcal{P}} n(1-n)\left(\hat{T}_t - \delta\tilde{T}_t\right) - a_{\mathcal{P}}^H\phi_{1,t} + n\phi_{4,t} = 0 \tag{110}$$

$$\Lambda_{\mathcal{P}}\left(1+\frac{\kappa}{\eta}\right)(1-n)\mathcal{P}_t^F - \Lambda_{\mathcal{P}} n(1-n)\left(\hat{T}_t - \delta\tilde{T}_t\right) - a_{\mathcal{P}}^F\phi_{2,t} + (1-n)\phi_{4,t} = 0 \tag{111}$$

for all $t \geq 0$, where $\phi_{1,t}$ to $\phi_{4,t}$ are the multipliers associated with the constraints.

For $\alpha_H = \alpha_F$ together with $\rho = 1$, one can define $a \equiv a_C^H = a_C^F = a_T^H = a_T^T$ and $a_{\mathcal{P}} \equiv a_{\mathcal{P}}^H = a_{\mathcal{P}}^F$. It is argued in the text that $\hat{C}_t$ is independent of $\left\{\tilde{T}_t\right\}$. Deviations of $\hat{C}_t$ from $\tilde{C}_t$ are costly, which implies that $\hat{C}_t - \tilde{C}_t = 0$.[41] In this case, (109) implies

$$\phi_{1,t} + \phi_{2,t} = 0 \tag{112}$$

Since $\gamma = n$ for $\alpha_H = \alpha_F$, (106) and (107) then immediately give

$$n\hat{\Pi}_t^H + (1-n)\hat{\Pi}_t^F = 0 \tag{113}$$

verifying part a) of the proposition.

Given that average inflation is zero and that $\hat{C}_t = \tilde{C}_t$ for all t, the Euler equation immediately implies

$$\hat{i}_t = -\rho\tilde{C}_t \tag{114}$$

which is part b).

Using (106) to eliminate $\phi_{3,t}$ from (108) gives

$$\begin{aligned} \beta \mathrm{E}_t\Pi_{t+1}^H = & -\frac{\Lambda_C n(1-n)}{\gamma}(\hat{T}_t - \tilde{T}_t) + \frac{\Lambda_{\mathcal{P}} n(1-n)}{2\gamma}\mathcal{P}_t^R + \Pi_t^H \\ & + \frac{1}{2\gamma}\left[a(1-n)\phi_{1,t} - an\phi_{2,t} + \phi_{1,t} - \phi_{1,t-1} - \beta \mathrm{E}_t(\phi_{1,t+1} - \phi_{1,t})\right] \end{aligned} \tag{115}$$

[41] It is straightforward to show this more formally using a proof by contradiction.

By solving the Phillips curve for H for $\beta \mathrm{E}_t \Pi^H_{t+1}$ and substituting, one obtains

$$\begin{aligned} a_{\mathcal{P}} \mathcal{P}^H_t =& - \left[(1-n)a - \frac{\Lambda_C n(1-n)}{\gamma} \right] (\hat{T}_t - \tilde{T}_t) - \frac{\Lambda_{\mathcal{P}} n(1-n)}{2\gamma} \mathcal{P}^R_t \\ & - \frac{1}{2\gamma} \left[a(1-n)\phi_{1,t} - an\phi_{2,t} + \phi_{1,t} - \phi_{1,t-1} - \beta \mathrm{E}_t(\phi_{1,t+1} - \phi_{1,t}) \right] \end{aligned} \tag{116}$$

Analogous steps for F yield

$$\begin{aligned} a_{\mathcal{P}} \mathcal{P}^F_t =& - \left[\frac{\Lambda_C n(1-n)}{1-\gamma} - na \right] (\hat{T}_t - \tilde{T}_t) + \frac{\Lambda_{\mathcal{P}} n(1-n)}{2(1-\gamma)} \mathcal{P}^R_t \\ & - \frac{1}{2(1-\gamma)} \left[-a(1-n)\phi_{1,t} + an\phi_{2,t} + \phi_{2,t} - \phi_{2,t-1} - \beta \mathrm{E}_t(\phi_{2,t+1} - \phi_{2,t}) \right] \end{aligned} \tag{117}$$

Taking the difference of these two equations and simplifying gives

$$\begin{aligned} a_{\mathcal{P}} \mathcal{P}^R_t =& - (\Lambda_C - a)(\hat{T}_t - \tilde{T}_t) + \frac{\Lambda_{\mathcal{P}}}{2} \mathcal{P}^R_t \\ & - \frac{1}{2(1-\gamma)} \left[-a(1-n)\phi_{1,t} + an\phi_{2,t} + \phi_{2,t} - \phi_{2,t-1} - \beta \mathrm{E}_t(\phi_{2,t+1} - \phi_{2,t}) \right] \\ & + \frac{1}{2\gamma} \left[a(1-n)\phi_{1,t} - an\phi_{2,t} + \phi_{1,t} - \phi_{1,t-1} - \beta \mathrm{E}_t(\phi_{1,t+1} - \phi_{1,t}) \right] \end{aligned} \tag{118}$$

Equations (72) and (73) can be combined to

$$\hat{T}_t - \hat{T}_{t-1} = \beta \mathrm{E}_t(\hat{T}_{t+1} - \hat{T}_t) - a(\hat{T}_t - \tilde{T}_t) + a_{\mathcal{P}} \mathcal{P}^R_t \tag{119}$$

Together with (118) this equation can be written as

$$\begin{aligned} \hat{T}_t - \hat{T}_{t-1} =& \beta \mathrm{E}_t(\hat{T}_{t+1} - \hat{T}_t) - a(\hat{T}_t - \tilde{T}_t) - (\Lambda_C - a)(\hat{T}_t - \tilde{T}_t) + \frac{\Lambda_{\mathcal{P}}}{2} \mathcal{P}^R_t \\ & + \frac{a}{2\gamma(1-\gamma)} \left[(1-n)\phi_{1,t} - n\phi_{2,t} \right] + \frac{1+\beta}{2} \left(\frac{\phi_{1,t}}{n} - \frac{\phi_{2,t}}{1-n} \right) \\ & - \frac{1}{2} \left(\frac{\phi_{1,t-1}}{n} - \frac{\phi_{2,t-1}}{1-n} \right) - \frac{\beta}{2} \mathrm{E}_t \left(\frac{\phi_{1,t+1}}{n} - \frac{\phi_{2,t+1}}{1-n} \right) \end{aligned} \tag{120}$$

To substitute out the multipliers, turn to (110) and (111). Adding both equations yields

$$\Lambda_{\mathcal{P}} \left(1 + \frac{\kappa}{\eta} \right) \left[n\mathcal{P}^H_t + (1-n)\mathcal{P}^F_t \right] - a(\phi_{1,t} + \phi_{2,t}) + \phi_{4,t} = 0 \tag{121}$$

The first two components of the sum are zero due to (112) and (66). This implies that $\phi_{4,t} = 0$. Using this fact, one can combine (110) and (111) to obtain

$$(1-n)\phi_{1,t} - n\phi_{2,t} = \frac{1}{a_{\mathcal{P}}} \left[-\Lambda_{\mathcal{P}} n(1-n) \left(1 + \frac{\kappa}{\eta} \right) \mathcal{P}^R_t + \Lambda_{\mathcal{P}} n(1-n)(\hat{T}_t - \delta \tilde{T}_t) \right] \tag{122}$$

and

$$\frac{\phi_{1,t}}{n} - \frac{\phi_{2,t}}{1-n} = \frac{1}{a_{\mathcal{P}}}\left[-\Lambda_{\mathcal{P}}\left(1+\frac{\kappa}{\eta}\right)\mathcal{P}_t^R + \Lambda_{\mathcal{P}}(\hat{T}_t - \delta\tilde{T}_t)\right] \tag{123}$$

Inserting these two expressions into (120) and simplifying yields

$$\begin{aligned}\mathcal{P}_t^R =& \Psi\left\{\left[1+\beta+\Lambda_C - \frac{(1+\beta+a)\Lambda_{\mathcal{P}}}{2a_{\mathcal{P}}}\right]\hat{T}_t - \left(1-\frac{\Lambda_P}{2a_{\mathcal{P}}}\right)\hat{T}_{t-1} - \left(1-\frac{\Lambda_P}{2a_{\mathcal{P}}}\right)\beta \mathrm{E}_t\hat{T}_{t+1} - \right.\\ &\left.\left[\Lambda_C - \frac{\delta(1+\beta+a)\Lambda_{\mathcal{P}}}{2a_{\mathcal{P}}}\right]\tilde{T}_t - \frac{\delta\Lambda_{\mathcal{P}}}{2a_{\mathcal{P}}}\tilde{T}_{t-1} - \frac{\Lambda_P}{2a_{\mathcal{P}}}\left(1+\frac{\kappa}{\eta}\right)\mathcal{P}_{t-1}^R - \frac{\Lambda_P}{2a_{\mathcal{P}}}\left(1+\frac{\kappa}{\eta}\right)\beta\mathrm{E}_t\mathcal{P}_{t+1}^R\right\}\end{aligned} \tag{124}$$

with

$$\Psi \equiv \frac{2a_{\mathcal{P}}}{\Lambda_{\mathcal{P}}a_{\mathcal{P}} - \Lambda_{\mathcal{P}}\left(1+\frac{\kappa}{\eta}\right)(1+\beta+a)} \tag{125}$$

which can be written as in part c) of Proposition 4.

A.5 Unconditional Expectation of Welfare

To evaluate welfare, the strategy adopted here is to follow a number of recent contributions in considering the unconditional expectation of the welfare criterion in period $t=0$.The second-order approximation to welfare derived above can be written as

$$-\frac{1-\beta}{\Omega}W_0 = (1-\beta)\,\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t L_t \tag{126}$$

Taking the unconditional expectation based on the probability distribution of the exogenous variables in $t=0$ on both sides and using the law of iterated expectations yields

$$-\frac{1-\beta}{\Omega}W = (1-\beta)\,\mathrm{E}\sum_{t=0}^{\infty}\beta^t L_t \tag{127}$$

where $W \equiv \mathrm{E}(W_0)$.

In order to find a simple expression for per period loss L_t, let the vector of all endogenous variables x_t be defined as $x_t \equiv \left(\hat{\Pi}_t^H, \hat{\Pi}_t^F, \hat{C}_t, \phi_{3,t}, \phi_{4,t}, \phi_{5,t}, \phi_{6,t}, \mathcal{P}_t^H, \mathcal{P}_t^F, \hat{T}_t, \phi_{1,t}, \phi_{2,t}, \tilde{T}_t^a, \tilde{C}_t\right)'$. The equilibrium laws of motion derived using Klein's method for the case of AR(1)-shocks can then be re-written as

$$x_t = Kx_{t-1} + Nu_t \tag{128}$$

with $u_t \equiv (u_{T,t}, u_{C,t})'$. K is a 14×14 matrix whose first nine elements of each row are zeroes and N is 14×2. Now define $y_t \equiv \left(\hat{\Pi}_t^H, \hat{\Pi}_t^F, \hat{C}_t - \tilde{C}_t, \hat{T}_t - \tilde{T}_t^a, \hat{T}_t - \delta\tilde{T}_t^a, \mathcal{P}_t^H, \mathcal{P}_t^F, \mathcal{P}_t^F - \mathcal{P}_t^H\right)'$ and note that one can find a matrix M such that

$$y_t = Mx_t \tag{129}$$

M is a simple 8×14 matrix that is not shown here for conciseness. y_t is constructed so that it contains the elements of the per period loss function. L_t is therefore given by the quadratic expression

$$L_t = y_t' \Gamma y_t \tag{130}$$

where

$$\Gamma \equiv \begin{bmatrix} \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & n(1-n)\Lambda_C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Lambda_{\mathcal{P}} n(1-n) \\ 0 & 0 & 0 & 0 & 0 & \Lambda_{\mathcal{P}} \frac{1}{2} n \left(1+\frac{\kappa}{\eta}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Lambda_{\mathcal{P}} \frac{1}{2}(1-n)\left(1+\frac{\kappa}{\eta}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the definitions above, L_t becomes

$$\begin{aligned} L_t &= x_t' M' \Gamma M x_t \\ &= (K x_{t-1} + N u_t)' M' \Gamma M (K x_{t-1} + N u_t) \\ &= x_{t-1}' K' M' \Gamma M K x_{t-1} + x_{t-1}' K' M' \Gamma M N u_t + u_t' N' M' \Gamma M K x_{t-1} + u_t' N' M' \Gamma M N u_t \\ &= x_{t-1}' K' M' \Gamma M K x_{t-1} + 2 u_t' N' M' \Gamma M K x_{t-1} + u_t' N' M' \Gamma M N u_t \end{aligned} \tag{131}$$

Substituting back into (127) yields

$$-\frac{1-\beta}{\Omega} W = (1-\beta)\,\mathrm{E} \sum_{t=0}^{\infty} \beta^t \left(x_{t-1}' K' M' \Gamma M K x_{t-1} + 2 u_t' N' M' \Gamma M K x_{t-1} + u_t' N' M' \Gamma M N u_t \right) \tag{132}$$

The individual components of the right-hand-side of this equation are now considered in turn starting with the last one.

$$\begin{aligned} (1-\beta)\,\mathrm{E} \sum_{t=0}^{\infty} \beta^t u_t' N' M' \Gamma M N u_t &= (1-\beta) \sum_{t=0}^{\infty} \beta^t \mathrm{E} \left[\mathrm{tr}(u_t' N' M' \Gamma M N u_t) \right] \\ &= (1-\beta) \sum_{t=0}^{\infty} \beta^t \mathrm{tr} \left[\mathrm{E}(u_t u_t') N' M' \Gamma M N \right] \\ &= \mathrm{tr} \left(\Sigma N' M' \Gamma M N \right) \end{aligned} \tag{133}$$

where $\Sigma \equiv \mathrm{E}(u_t u_t')$ is the covariance matrix associated with the white noise components of the disturbances.

$$
\begin{aligned}
(1-\beta)\,\mathrm{E}\sum_{t=0}^{\infty}\beta^t 2u_t' N' M' \Gamma M K x_{t-1} &= (1-\beta)\sum_{t=0}^{\infty}\beta^t 2\mathrm{E}\left[\mathrm{tr}(u_t' N' M' \Gamma M K x_{t-1})\right] \\
&= (1-\beta)\sum_{t=0}^{\infty}\beta^t 2\mathrm{tr}\left[\mathrm{E}(x_{t-1}u_t')N' M' \Gamma M K\right] \\
&= 0 \qquad (134)
\end{aligned}
$$

since $\mathrm{E}(x_{t-1}u_t') = 0$.

$$
\begin{aligned}
(1-\beta)\,\mathrm{E}\sum_{t=0}^{\infty}\beta^t x_{t-1}' K' M' \Gamma M K x_{t-1} &= (1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left[\mathrm{tr}(x_{t-1}' K' M' \Gamma M K x_{t-1})\right] \\
&= \mathrm{tr}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t K' M' \Gamma M K \mathrm{E}\left(x_{t-1}x_{t-1}'\right)\right] \\
&= \mathrm{tr}\left(K' M' \Gamma M K J\right) \qquad (135)
\end{aligned}
$$

where J is defined as $J \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left(x_{t-1}x_{t-1}'\right)$. By making use of the starting conditions $x_{-1} = \mathbf{0}$, J can be re-arranged as follows.

$$
\begin{aligned}
J &\equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left(x_{t-1}x_{t-1}'\right) \\
&= \beta\,(1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left(x_t x_t'\right) \\
&= \beta\,(1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left[(Kx_{t-1}+Nu_t)(Kx_{t-1}+Nu_t)'\right] \\
&= \beta\,(1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}\left[(Kx_{t-1}x_{t-1}'K' + Kx_{t-1}u_t'N' + Nu_t x_{t-1}'K' + Nu_t u_t' N')\right] \\
&= \beta\left(KJK' + N\Sigma N'\right) \qquad (136)
\end{aligned}
$$

This expression can be solved for J using the fact that for any matrices A, B, C, $\mathrm{vec}(ABC) = (C' \otimes A)\mathrm{vec}(B)$. Hence,

$$
\begin{aligned}
\mathrm{vec}(J) &= \beta\mathrm{vec}(KJK') + \beta\mathrm{vec}(N\Sigma N') \\
&= \beta(K\otimes K)\mathrm{vec}(J) + \beta\mathrm{vec}(N\Sigma N') \qquad (137) \\
\mathrm{vec}(J)\left[\mathrm{I} - \beta(K\otimes K)\right] &= \beta\mathrm{vec}(N\Sigma N') \qquad (138) \\
\mathrm{vec}(J) &= \beta\left[\mathrm{I} - \beta(K\otimes K)\right]^{-1}\mathrm{vec}(N\Sigma N') \qquad (139)
\end{aligned}
$$

By combining the equations (132) through (135), one obtains

$$
W = -\frac{\Omega}{1-\beta}\left[\mathrm{tr}\left(\Sigma N' M' \Gamma M N\right) + \mathrm{tr}\left(K' M' \Gamma M K J\right)\right] \qquad (140)
$$

where J is implicitly given by (139).

A.6 Consumption equivalent of welfare differentials

The value associated with the Ramsey policy is given by

$$V^r \equiv \mathrm{E}\sum_{t=0}^{\infty}\beta^t\left\{U(C_t^r) - \int_0^n V\left[y_t^r(h), z_t^H\right]dh - \int_n^1 V\left[y_t^r(f), z_t^F\right]df \right.$$
$$\left. -nW\left(\mathcal{P}_t^{H,r}\right) - (1-n)W\left(\mathcal{P}_t^{F,r}\right)\right\} \tag{141}$$

Similarly, the value associated with an alternative policy is

$$V^a \equiv \mathrm{E}\sum_{t=0}^{\infty}\beta^t\left\{U(C_t^a) - \int_0^n V\left[y_t^a(h), z_t^H\right]dh - \int_n^1 V\left[y_t^a(f), z_t^F\right]df \right.$$
$$\left. -nW\left(\mathcal{P}_t^{H,a}\right) - (1-n)W\left(\mathcal{P}_t^{F,a}\right)\right\} \tag{142}$$

λ is then defined as the fraction of consumption under the Ramsey policy that a household would have to give up to be as well off under the Ramsey policy as under the alternative policy.

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\left\{U\left[C_t^r(1-\lambda)\right] - \int_0^n V\left[y_t^r(h), z_t^H\right]dh - \int_n^1 V\left[y_t^r(f), z_t^F\right]df \right.$$
$$\left. -nW\left(\mathcal{P}_t^{H,r}\right) - (1-n)W\left(\mathcal{P}_t^{F,r}\right)\right\} = V^a \tag{143}$$

To be able to solve this condition for λ, consistent with the previous assumption $\rho = 1$ it is assumed that $U(\cdot) = \ln(\cdot)$. Therefore,

$$\begin{aligned} V^a =& \mathrm{E}\sum_{t=0}^{\infty}\beta^t\left\{\ln\left(C_t^r\right) + \ln(1-\lambda) - \int_0^n V\left[y_t^r(h), z_t^H\right]dh - \int_n^1 V\left[y_t^r(f), z_t^F\right]df \right. \\ & \left. -nW\left(\mathcal{P}_t^{H,r}\right) - (1-n)W\left(\mathcal{P}_t^{F,r}\right)\right\} \\ =& \frac{1}{1-\beta}\ln(1-\lambda) + V^r & (144) \\ \ln(1-\lambda) =& (1-\beta)\left(V^a - V^r\right) & (145) \\ \lambda =& 1 - e^{(1-\beta)(V^a - V^r)} & (146) \end{aligned}$$

In this equation, V^r is approximated to the second order by W given in (140). V^a is approximated by W^a which can be derived in an analogous way. The consumption equivalent is then given by

$$\lambda = 1 - e^{(1-\beta)(W^a - W)} \tag{147}$$

which is the measure used in the text.