Long-Term Government Debt and Household Portfolio Composition

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Abstract

Formal dynamic analyses of household portfolio choice in the literature focus on holdings of equity and a risk-free asset or bonds of different maturities, neglecting the interdependence of the decisions to invest in equity, short-term and long-term bonds made by households. Data from the Survey of Consumer Finances is used to derive stylised facts about participation in the long-term government-debt market and conditional portfolio shares. These facts are explained with the help of a portfolio-choice model in which investors have access to three assets—equity, long-term debt and a riskless short-term bond—and are exposed to uninsurable idiosyncratic risk through non-financial income, retirement and longevity, as well as aggregate risk through the asset returns. An application shows that the low Treasury returns observed in the US between 2009 and 2013 have quantitatively significant yet transitory effects on the composition of household portfolios. In combination with the observed rise in stock returns, they lead to persistent changes in the participation rate, the conditional portfolio shares and the distribution of wealth.

Keywords: Dynamic Portfolio Choice, Long-Term Government Debt, Asset-Market Participation, Survey of Consumer Finances
JEL-Classification: E21, D91, G11

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1 Introduction

Analyses of portfolio choice over the life cycle generally focus on holdings of stock and a risk-free asset, not taking into account the significant positions of long-term government debt that can be found in household portfolios. With short-term nominal interest rates close to the zero lower bound in Europe and the US more or less since the the Financial Crisis of 2007 to 2009, central banks have purchased large amounts of long-term debt as part of their unconventional monetary policy programmes. In this context, the role that long-term bonds play in household portfolios and the motives for rebalancing portfolios in response to return shocks have become of considerable interest. Using a portfolio-choice model in which agents can invest in three financial assets—stock, long-term government debt and a riskless asset—this paper studies the decision to participate in financial markets and the composition of household portfolios over the course of the life cycle with a focus on the role of long-term government debt.

Stylised facts are derived based on a data set constructed from seven consecutive waves of the Survey of Consumer Finances (SCF). The joint existence of birth cohort, time and age influences on the participation rate and the respective portfolio shares conditional on participation result in a well-known identification problem.[1] Using three different identification strategies, two from the literature and one novel, a latent variable model with a participation equation is estimated. The standard approach based on cohort restrictions performs the least well. The remaining two, although distinctive, give nearly identical results. Similar to participation in the stock market, participation in the market for long-term debt takes an inverse U-shape. While the conditional portfolio share of stock is declining with age, the conditional share of long-term government debt is weakly increasing until the age of around 65 and somewhat declining thereafter.[2] Furthermore, long-term bond holdings and holdings of the riskless asset differ with respect to their elasticities of substitution with equity, suggesting that the shares of long-term debt and cash are rebalanced in distinctive ways in response to wealth and return shocks.

The theoretical analysis is based on a model in which agents adjust consumption and holdings of the three financial assets facing uninsurable labour or retirement income risk, random stock and long-term debt returns, as well as stochastic retirement and death ages as in Viceira (2001). Fixed participation costs prevent agents from investing in stock and government debt at young age. The participation rate first rises as they accumulate wealth and later mildly declines due to agents retiring and running down their savings. The average long-term debt and stock share conditional on participation respectively increases and decreases with age in line with the data. This is the case, since the portfolio income of market participants grows on average, implying that the ratio of portfolio to labour income rises. As a result, agents with CRRA utility rebalance their portfolios to reduce their risk exposure. Long-term debt plays an important part in this

[1] Browning et al. (2012) give a detailed description of the "age-period-cohort" problem. According to them, the problem can be traced back at least to Ryder (1965). See also Ameriks and Zeldes (2004).

[2] The results regarding equity holdings are in line with findings by Fagereng et al. (2016) and Gomes and Michaelides (2005)

process, because its return is less volatile than that of stock but higher in expectation than that of the risk-free asset. Incompleteness of financial markets gives rise to a non-degenerate distribution of wealth. Agents that consistently participate in the markets for equity and debt at a young age accumulate wealth faster than those that enter later or remain stuck below the participation threshold, implying that the wealth distribution among employed investors shows the characteristic positive skew found in the data and that inequality increases with age.

Finally, the model is used to study the period of negative real 5-year Treasury returns and elevated real stock returns that followed the recession of 2007 to 2009 in the US. In the model, the Treasury-return shocks observed between 2009 and 2013, when considered in isolation, lead to a significant rebalancing of household portfolios towards stock holdings. The adjustments are transitory though. This is the case, because the negative effect of the debt-return shocks on the portfolio return are compensated by the higher stock share so that wealth and hence the participation rate are nearly unaffected. Consequently, the initial adjustments are undone when the return on debt rises again. The observed shocks to the returns of both assets jointly have an even larger effect on the stock share upon impact and lead to a decline in holdings of the safe asset. Since average wealth rises, the participation rate increases and there are persistent effects on the holdings of all three assets. The distribution of wealth among agents that are hit by the shocks at a young age is more unequal for nearly their entire working lives.

A large literature is concerned with portfolio choice over the life cycle. Early contributions by Merton (1969, 1971) and Samuelson (1969) analyse optimal portfolio choice neglecting the asset market participation decision. More recent examples include, but are not limited to, Alan (2006), Bonaparte et al. (2012), Campanale et al. (2015), Cocco et al. (2005), Fagereng et al. (2016), Gomes and Michaelides (2005) and Haliassos and Michaelides (2003).[3] In these papers, households are restricted to holdings of stock and a riskless asset. I relax this constraint by adding a long-term bond to the portfolio choice problem. Campbell and Viceira (2001) and Wachter (2003) also consider asset allocation problems with long-term bonds but do not study life-cycle effects. In the model used here, long-term debt is only partially liquid, reflecting the fact that long-term debt like US savings bonds can be sold only at a substantial cost in the first years after they have been issued. As in Campanale et al. (2015), the composition of financial wealth therefore becomes an important state variable in the portfolio choice problem. While they assume that only holdings of the risk-free asset can be transformed costlessly into consumption, the portfolio composition matters here because of a maturity-specific liquidity constraint.

The paper is organised as follows. Section 2 presents the empirical results. It starts by describing the data set, then gives a detailed discussion of the identification strategies employed and finally shows estimation results. Section 3 contains a description of the model, its calibration and the resulting policy functions. In Section 4, model simulations are confronted with the data and the model is applied to the 2009-2013 period in the US. Section 5 concludes.

[3] The model follows this literature in abstracting from informational frictions or incentive problems that may arise if households delegate the portfolio-choice decision to a portfolio manager.

2 Empirical Results

This section presents the stylised facts on long-term government bond and stock holdings of US households which inform and provide a benchmark for the model-based analysis that follows.

2.1 Data

The data set employed is constructed from the seven consecutive waves of the SCF collected between 1989 and 2007.[4] Since the data consists of repeated cross-sections rather than a panel, I am not able to track *individuals* over time. However, due to the large amount of households included in each survey wave, one can track *cohorts* of individuals defined by their birth year over the sample period.[5]

It should be noted that there is an intentional oversampling of wealthy households in the SCF relative to the US population. This is done to allow for more precise estimates of financial asset holdings, which are highly concentrated among households in the upper tail of the wealth distribution, and to correct for the fact that the non-response rate is positively correlated with wealth (Kennickell, 2007). The benefit of much improved estimation precision comes at the cost of being able to make inferences for wealthier households only. Nonetheless, I believe that uncovering the life-cycle patterns in asset holdings among those that are the likely holders of the assets in question is of considerable interest.[6]

The SCF contains information on a large variety of assets held by households, which I divide into three categories. These categories are long-term government debt, stocks and a residual category that mainly includes cash/liquidity, short-term sovereign debt and corporate bonds. Most assets that appear in household balance sheets can be fully attributed to one of these three categories. When this is not the case, a careful partial assignment is done based on additional information about the institutions that issue the asset in question.

According to the "Monthly Statement of the Public Debt of the United States" from December 2007, 22.1% of total marketable debt held by the public took the form of Bills (maturity of one year or less), while the remaining 77.9% were issued in the form of Notes, Bonds and TIPS (maturity of two years or more). Assuming that agents are homogeneous in regards to the maturity composition of government debt in their portfolios, 77.9% of the marketable US government debt held by a household is assigned to long-term government debt and the rest to the residual category. Savings bonds and tax-exempt bonds, for example, are fully assigned to the long-term debt category, since they typically have a maturity of several years.

Funds held in individual retirement accounts (IRAs) are also divided into more than one category. An IRA is a tax-advantaged retirement savings plan. Funds transferred into an IRA

[4] Data from 2010 and 2013 is also available; it is not used here to avoid bias introduced by crisis-specific effects. Data from before 1989 is not used due to changes in the availability of a subset of variables.

[5] This fact is emphasised also in Deaton and Paxson (1994).

[6] Descriptive statistics of the sample are shown in Table A.1 in the appendix.

can be requested to be allocated to a large variety of financial assets. The Employee Benefit Research Institute (EBRI) collects data on the allocation of assets in IRAs.[7] Based on this data, I attribute 45.8% of the funds held in an IRA to stocks, 18.4% to long-term government debt and the remainder to the last category. The assignment of all assets into the three broad categories is described in detail in Section B in the appendix and summarised in Table A.3.

The figures 1-3 illustrate the average shares of the portfolio categories constructed in this way. Each line represents the average portfolio share of a given birth-year cohort at a particular age. Since data points are available only every three years, both respondent age and cohort (birth year) are divided into three-year intervals. For example, the earliest data available is from 1989. The youngest age group considered includes households with a "household head" aged 26-28.[8] Individuals that are 26-28 of age in 1989 belong to the birth-year cohort 1961-63. The 1961-63 cohort is sampled seven times between 1989 and 2007. Its members are aged 29-31 in 1992, 32-34 in 1995 and so on. In Figures 1-3, the blue line at the very left represents the average portfolio shares held by the 1961-63 cohort. Similarly, the green line starting with age group 29-31 represents the average portfolio shares of the 1958-60 cohort. Altogether, the sample contains the eleven cohorts born between 1931-33 and 1961-63, each observed at seven consecutive age groups between the ages of 26-28 and 74-76.[9]

The average portfolio share of stocks is increasing in household portfolios with a peak in the late fifties or early sixties of the household head. Average long-term government debt holdings behave in a somewhat similar way, although the pattern is less well pronounced. The average portfolio share of the residual category follows a pattern that is markedly distinct from that of the average long-term government bond share. However, it is a well-known fact the averages computed in Figures 1-3 provide a biased picture of the composition of household portfolios. The reasons are twofold.

First, the adjustments visible in the figures can be due to changes at the internal or external margin. A number of papers report that the rate of participation in stock markets first increases and later decreases significantly over the life cycle.[10] This suggests that the inverse U-shape in Figure 2 largely results from households entering and exiting the stock market rather than adjustments at the internal margin. An important question explored below is whether or not the same is true for holdings of long-term government debt.

[7] See EBRI Note "IRA Asset Allocation and Characteristics of the CDHP Population, 2005-2010" from May 2011, available at www.ebri.org/publications/notes.

[8] In the SCF, the "household head" is defined as the single economically-dominant individual in a household without a core couple, the male in a household with a mixed-sex core couple and the older individual in the case of a same-sex core couple.

[9] Only cohorts that fall inside this age interval at all seven survey waves are considered. Younger and older age groups are not examined due to a lack of sufficient data. Table A.2 in the appendix shows the number of observations for each cohort-age pair. Note that the data contains multiple imputations as is explained in more detail in the table notes.

[10] See Ameriks and Zeldes (2004), Fagereng et al. (2016) and Guiso et al. (2003). Haliassos (2008) contains a more general summary of the literature on limited participation in asset markets.

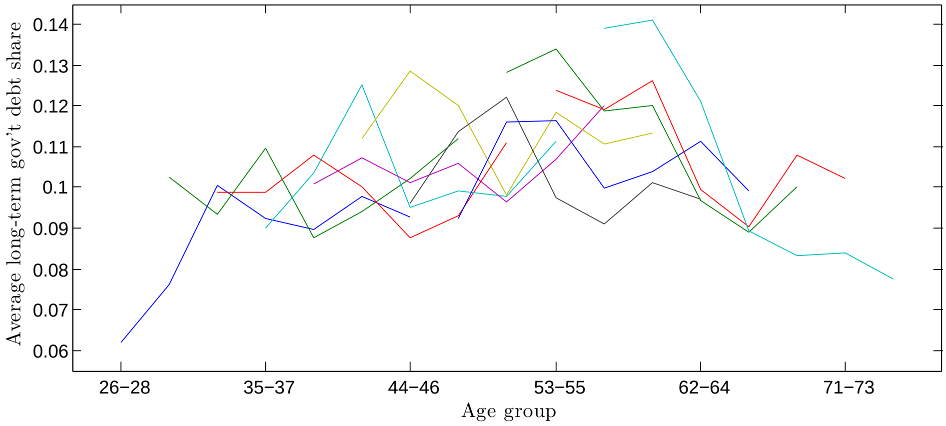

Figure 1: Average portfolio share of long-term government debt

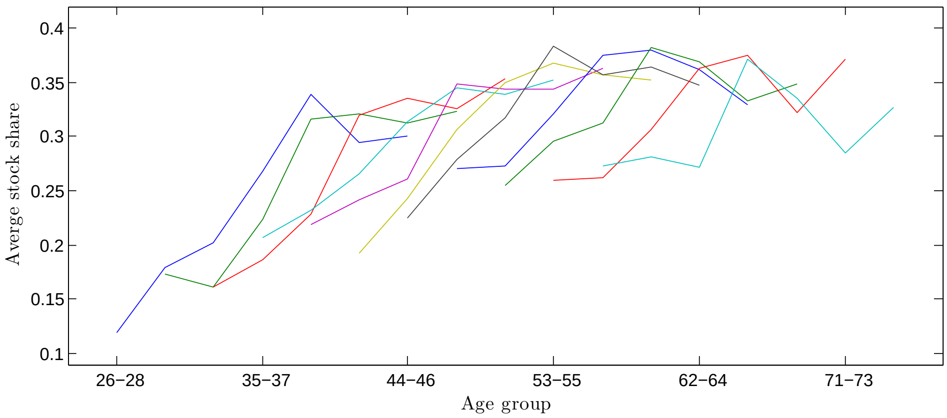

Figure 2: Average portfolio share of equity

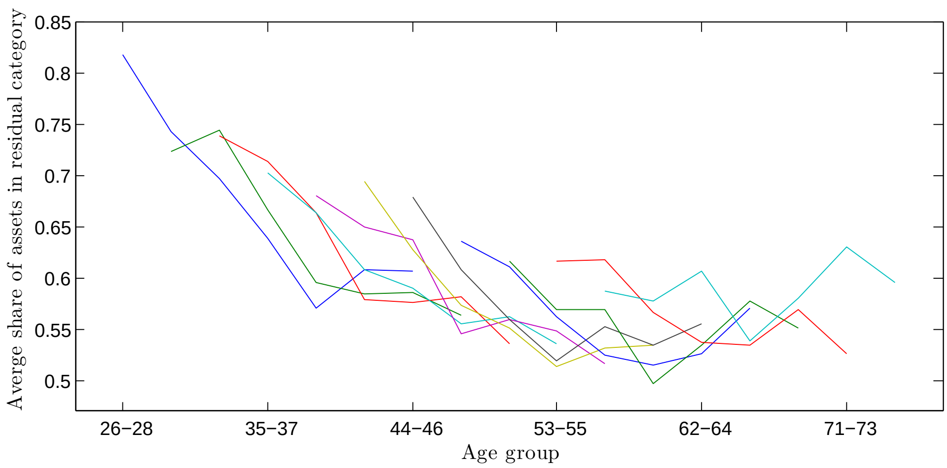

Figure 3: Average portfolio share of cash/liquidity, corporate bonds and other financial assets

Second, as discussed in Ameriks and Zeldes (2004), it is not possible from the figures to disentangle the effects of age, observation period and cohort. Age effects are related to education, family formation and retirement. Period effects result from events that occur at the time of data collection. For example, the dot-com-bubble and its bursting is reflected in the survey waves from 2001 and 2004. Cohort effects include cohort-specific experiences like growing up during war time as was the case for the oldest cohorts in the sample. Even if, for example, we were to observe a figure of the same kind as Figures 1-3 in which all lines were perfectly aligned such that they formed one single upward-sloping line, we could not say whether this was due to pure age effects or a combination of time and cohort effects.[11]

2.2 Identification

As outlined above, identification problems arise from sample selection and perfect multicollinearity of a respondent's cohort, the age at which they are sampled and the year in which the survey is conducted (birth year + age when sampled = observation period). An unsolved issue in the literature on equity holdings over the life cycle is that estimation results are somewhat dependent on the underlying identifying assumptions.[12] I therefore present the results obtained under three different identification strategies. Two are borrowed from the literature and one is novel. A number of robust findings emerge. To be able to motivate the strategies employed below, the nature of the identification problem is laid out before in detail.

2.2.1 Sample Selection

The self-selection of agents into participants and non-participants in the market for a given financial asset results in a sample selection problem. To address this issue, I employ a standard latent variable model with a Probit selection equation. Formally, the model is given by

$$s_i = x'_{2,i}\beta_2 + \sigma_{12}\lambda\left(x'_{1,i}\hat{\beta}_1\right) + e_{2,i} \tag{1}$$

$\forall i$ with $s_i > 0$, where s_i is the portfolio share of the asset in question, $\lambda \equiv \phi\left(x'_{1,i}\hat{\beta}_1\right)/\Phi\left(x'_{1,i}\hat{\beta}_1\right)$ is the Inverse Mills Ratio and $\hat{\beta}_1$ is obtained from estimating the first-stage Probit model

$$\begin{aligned}\Pr(P_i = 1|x_{1,i}) &= \Pr\left(x'_{1,i}\beta_1 + e_{1,i} > 0\right)\\ &= \Phi\left(x'_{1,i}\beta_1\right)\end{aligned} \tag{2}$$

$P_i = 1$ if i is a participant in the market for the asset under consideration and $P_i = 0$ otherwise. The error terms are normal, $e_{1,i} \sim N(0,1)$ and $e_{2,i} \sim N(0,\sigma^2)$, with $\text{Cov}(e_{1,i}, e_{2,i}) = \sigma_{12}$.

[11] Time effects could cause each individual line to be sloped upwards and cohort effects of increasing size could result in all lines aligning precisely in the way previously mentioned.
[12] See Ameriks and Zeldes (2004) and Gomes and Michaelides (2005) for detailed discussions.

2.2.2 The "Age-Period-Cohort Problem"

Due to the multicollinearity described above, a simple linear model that aims to separate age, period and cohort effects is under-identified. To see this, consider the following example.[13] Let a_i denote the age of respondents, t_i the time period in which they are sampled and c_i their year of birth. Suppose that data is available for two consecutive time periods, $t_i \in \{t^1, t^2\}$, and that three consecutive cohorts are sampled in both periods, $c_i \in \{c^1, c^2, c^3\}$. Age can then take on four distinct values, $a_i \in \{a^1, a^2, a^3, a^4\}$.[14]

A linear regression of some variable of interest y_i on a constant and complete sets of age, period and cohort dummies is given by

$$\begin{aligned} y_i = \beta_0 &+ \alpha_1 a_i^1 + \alpha_2 a_i^2 + \alpha_3 a_i^3 + \alpha_4 a_i^4 \\ &+ \theta_1 t_i^1 + \theta_2 t_i^2 \\ &+ \gamma_1 c_i^1 + \gamma_2 c_i^2 + \gamma_3 c_i^3 + e_i \end{aligned} \tag{3}$$

where $x_i^n = 1$ if $x_i = x^n$ and $x_i^n = 0$ otherwise for $x \in \{a, t, c\}$ and $n \in \{1, 2, 3, 4\}$. Of course, the sum of each set of binary variables is equal to the constant. This can be easily avoided by making use of the normalisation $\beta_0 = \theta_1 = \gamma_1 = 0$ and re-interpreting the remaining parameters accordingly. The equation above becomes

$$\begin{aligned} y_i = &\alpha_1 a_i^1 + \alpha_2 a_i^2 + \alpha_3 a_i^3 + \alpha_4 a_i^4 \\ &+ \theta_2 t_i^2 \\ &+ \gamma_2 c_i^2 + \gamma_3 c_i^3 + e_i \end{aligned} \tag{4}$$

However, the fact that there exists a linear relationship between the age, observation period and cohort of each respondent implies that the data matrix corresponding to regression equation (4) is not invertible and parameter estimates cannot be computed. More precisely, the linear relationship between age, period and cohort implies[15]

$$2a_i^1 + a_i^2 - a_i^4 + t_i^2 = c_i^2 + 2c_i^3 \tag{5}$$

Inserting (5) into (4) yields

$$\begin{aligned} y_i = &\tilde{\alpha}_1 a_i^1 + \tilde{\alpha}_2 a_i^2 + \tilde{\alpha}_3 a_i^3 + \tilde{\alpha}_4 a_i^4 \\ &+ \tilde{\gamma}_2 c_i^2 + \tilde{\gamma}_3 c_i^3 + e_i \end{aligned} \tag{6}$$

[13] See also Browning et al. (2012).

[14] For example, if $t_i \in \{2000, 2001\}$ and $c_i \in \{1950, 1951, 1952\}$ then $a_i \in \{48, 49, 50, 51\}$.

[15] Continuing the previous example, a person that is born say in 1952 and surveyed in 2001 is aged 49 when surveyed; thus $t_i^2 = c_i^3 = a_i^2 = 1$ and $a_i^1 = a_i^4 = c_i^2 = 0$. It is straight forward to verify that (5) holds for all six such combinations of binary-variable values for which birth year and age sum to observation period.

where

$$\begin{pmatrix} \tilde{\alpha}_1 \\ \tilde{\alpha}_2 \\ \tilde{\alpha}_3 \\ \tilde{\alpha}_4 \\ \tilde{\gamma}_2 \\ \tilde{\gamma}_3 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \theta_2 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \tag{7}$$

Using equation (6), the six reduced form parameters $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4, \tilde{\gamma}_2, \tilde{\gamma}_3$ can be estimated. From (7) it is clear though that it is not possible to back out the seven structural parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \theta_2, \gamma_2, \gamma_3$ from knowledge of the reduced form parameters. The structural parameters are under-identified, unless at least *one* parameter restriction is imposed. It can be easily shown that this result generalises to scenarios with more observation periods and cohorts.

To be able to judge the robustness of the estimation results, I pursue three distinct identification strategies. The first and most standard is to impose an equality restriction on neighbouring cohort effects, i.e. to impose

$$\gamma_n = \gamma_{n+1} \tag{8}$$

for some n. This restriction formally reduces the generality of the model, yet the bias it introduces should be expected to be small if two neighbouring cohorts can be identified that have a sufficiently similar history.

The second strategy was suggested by Deaton and Paxson (1994).[16] The idea is to attribute cyclical fluctuations to time effects and trends to age and cohort effects. This is achieved by requiring time effects to sum to zero and to be orthogonal to a linear trend, that is

$$g'\theta = 0 \tag{9}$$

where $g = (0, 1, 2, ..., T-1)'$ is the trend, θ is the vector of coefficients on the time dummies and T is the number of observation periods. This set of assumptions correctly identifies all effects if indeed only age and cohort effects are trending. In the context here, one cannot be sure however that there is no trend in time effects. In particular, in the time period examined (1989-2007), stocks became a more widely-used mode of saving. Imposing (9) when a trend in time effects is present in the data could cause the coefficients on the age and cohort variables to pick up this trend and therefore to be biased. In the second-stage regression, I therefore de-trend the dependent variable, the portfolio share of a given asset, by subtracting its cross-sectional average at each time period. Since this is not feasible in a binary dependent variable model, I add a linear time trend as an explanatory variable at the first stage. Note that this implies that one of the age dummies has to be excluded from the Probit model.

[16] See also Deaton (1997).

Under the final identification strategy, the time dummies are replaced with the first p principal components of a large set of stationary macroeconomic time series ranging from the first until the last sampling period. This resolves the linear dependence of the independent variables. As before, the asset share is de-trended and a trend is added to the selection equation. To the extent to which the principal components contain the effects otherwise picked up by the time dummies, this modification allows estimating age, period and cohort effects without parameter restrictions. In particular, institutional and regulatory changes regarding the usage of different savings instruments can be expected to be reflected in asset prices and interest rates.

2.3 Estimation and Results

Beginning with the second-stage regression, the equations estimated in case of the first identification strategy (parameter restriction on cohort effects) are

$$s_i = a_i'\alpha_2 + t_i'\theta_2 + c_i'\gamma_2 + \varsigma\lambda_i + z_{2,i}'\delta_2 + e_{2,i} \tag{10}$$

$$\Pr(P_i = 1|x_{1,i}) = \Phi\left(a_i'\alpha_1 + t_i'\theta_1 + c_i'\gamma_1 + z_{1,i}'\delta_1\right) \tag{11}$$

$a_i = \left(a_i^{26-28}, a_i^{29-31}, \dots, a_i^{74-76}\right)'$ is a complete set of age dummies for seventeen age groups, $t_i = \left(t_i^{1992}, t_i^{1995}, \dots, t_i^{2007}\right)'$ is a vector of six year dummies and $c_i = \left(c_i^{1934-36}, c_i^{1937-39}, \dots, c_i^{1961-63}\right)'$ contains a dummy for each of ten cohorts. $z_{2,i}$ and $z_{1,i}$ are additional household-specific controls and $x_{1,i} \equiv (a_i, t_i, c_i, z_{1,i})'$. In the case of the other two strategies, the equations are modified as explained in the previous section.[17]

Figure 4 plots the estimation results for all three identification strategies outlined before including separate sets of results for two different cohort restrictions. The first cohort restriction equates the effects of the two oldest cohorts in the sample, 1934-36 and 1937-39, the second one those from the first two post-war cohorts, 1946-48 and 1949-51. The cohort effects of the oldest respondents are equated, since it seems likely for any differences between them to wash out over the years until the sampling period and the second restriction may appear reasonable from a historical perspective. In the model in which the time dummies are dropped, the first three principal components of a large set of macroeconomic aggregates from the US are used. Panels a) and c) show the marginal values, the average predicted probabilities, of being a stockholder and a long-term government debt holder, respectively, for each age group. Since the second-stage regression of each model is linear, it is possible to isolate the marginal age effects on the conditional asset shares without having to fix the remaining covariates. Consequently, the panels b) and d) graph the estimated coefficients on the binary age indicators from the second stage. Note that the size of the effects necessarily differ between those based on cohort restrictions and those based on the remaining two identification strategies as the dependent variable is de-trended in the latter two models and all models have to be estimated without a constant.

[17] More details on the estimation are given in Appendix C.

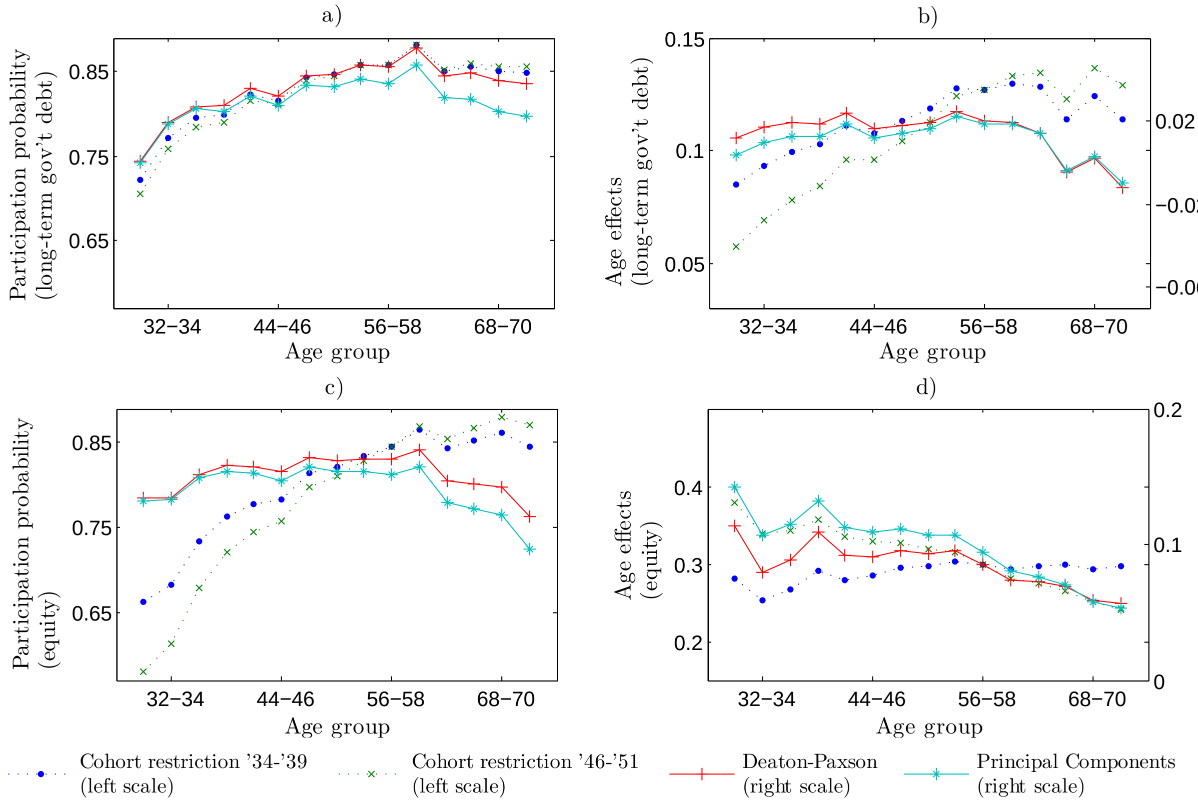

Figure 4: Predicted participation probabilities and estimated age effects on conditional asset shares

From the figure it becomes obvious that the standard identification approach based on cohort restrictions is only partially successful here. While all estimates for long-term debt market participation are of similar shape, the estimates obtained under cohort restrictions deviate significantly from each other and from the results obtained under the remaining two identification schemes in the panels b) to d). Experimenting with different cohort restrictions showed that the discrepancies are even more severe for other pairs of economically plausible restrictions, likely because trends in the cohort effects not accounted for by the model are forced into the estimates of the age effects. However, the results obtained using Deaton-Paxson restrictions and principal components nearly coincide despite of their distinctive way of accounting for time influences and are consistent with previous findings about equity holdings from the literature.

Several stylised facts emerge from the estimations that make use of Deaton-Paxson restrictions or principal components. The profile of participation in the market for long-term government debt shows a pronounced hump shape. Participation rates rise over the course of nearly the entire working life and then begin to decline at the age of 62-64 as retirement approaches. The age effects on the conditional portfolio share of long-term government debt are mildly increasing at first and roughly constant from the mid-forties until retirement. A significant decline is not observable until after the age of 65. Overall, the results suggest that there is a clear inverse U-shape in participation rates and that the conditional portfolio share is non-decreasing until retirement, but potentially falls thereafter. Stock market participation takes an almost identical shape to participation in the market for long-term government debt. The conditional stock share is monotone declining from 39-41 onwards.

The life-cycle dynamics of stock-market participation and the conditional share of stocks have been a topic of debate in the literature. In summarising the existing empirical evidence, Gomes and Michaelides (2005) state that 1) stock-market participation increases over the working life, 2) there is some evidence which suggests that participation rates decline after retirement and 3) there is "no clear pattern of equity holdings over the life cycle". I interpret the results presented here as support for 1) and 2). In recent work, Fagereng et al. (2016) find evidence for the conditional stock share to decline over the life cycle using administrative panel data from the Norwegian Tax Registry.[18] Regarding 3), my estimates are more in line with their findings.

	Deaton-Paxson		Principal Comp.		Cohort Restr. 1		Cohort Restr. 2	
	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.
λ		0.065^{**}		0.066^{**}		0.065^{**}		0.065^{**}
ρ		0.445		0.449		0.445		0.445
$\min(N_{\text{imp}})$	17,202	12,673	17,202	12,673	17,202	12,673	17,202	12,673
Total N	86,030	63,437	86,030	63,437	86,030	63,437	86,030	63,437
Sign. tests								
Age Eff's	96.4^{***}	27.6^{**}	82.4^{***}	25.9^{*}	96.8^{***}	106.1^{***}	99.2^{***}	108.5^{***}
(d.o.f.)	(16)	(17)	(16)	(17)	(17)	(17)	(17)	(17)
Time Eff's	12.7^{**}	5.9			12.8^{**}	43.3^{***}	12.8^{**}	44.5^{***}
(d.o.f.)	(5)	(5)			(6)	(6)	(6)	(6)
Cohort Eff's	11.6	26.0^{***}	11.7	16.9^{*}	11.5	15.1^{*}	11.5	15.1^{*}
(d.o.f.)	(10)	(10)	(10)	(10)	(9)	(9)	(9)	(9)
Pri. Comp's			9.5^{**}	5.4				
(d.o.f.)			(3)	(3)				

Notes: Results of first and second stage estimation shown for four models—Deaton-Paxson restrictions, principal components of macroeconomic variables replacing time dummies, cohort effects equated for '34-'36 and '37-'39 (Cohort Restr. 1), cohort effects equated for '46-'48 and '49-'51 (Cohort Restr. 2). Models estimated using two-step estimator (Heckit). Data is multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors are adjusted in an appropriate way (*p<0.1, **p<0.05, ***p<0.01). N_{imp} is number of obs. for imputation imp $\in \{1, 2, ..., 5\}$. For joint significant tests, avrg. Wald test stat. shown (each $\sim \chi^2$), degrees of freedom in parentheses.

Table 1: Estimation results for long-term government debt share

Table 1 provides more detailed information about the estimations for the long-term debt share. Results from the models with cohort restrictions are included for comparison purposes. The coefficient on the inverse Mills ratio λ is significant and the correlation of first-stage and second-stage residuals ρ is estimated to be positive, confirming the need to address the selection problem. SCF data is multiply imputed. The size of the smallest imputation group $\min(N_{\text{imp}})$ therefore gives a more accurate picture of the number of respondents in the sample than the

[18] Considering cross-sectional data only, other studies conclude that the conditional equity share may be mildly increasing or also mildly hump-shaped. See Campanale et al. (2015) for a short discussion.

total amount of observations N. With Deaton-Paxson restrictions and principal components, all age effects are significant. The significance level is slightly higher in the selection equations than in the conditional share equations, in line with the estimated size of the slope coefficients. Time effects play an important role at the first stage but cease to do so at the second stage. Thus, the participation decision is strongly influenced by time effects even after controlling for a linear trend. Demeaning the conditional long-term debt share successfully eliminates time effects at the second stage. Cohort effects are jointly significant only for the conditional asset share.

	Deaton-Paxson		Deaton-Paxson		Principal Comp.		Principal Comp.	
	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.	1^{st} st.	2^{nd} st.
Long-t. debt	3.305^{***}	-0.5161^{***}			3.306^{***}	-0.5157^{***}		
Cash			-5.241^{***}	-0.8840^{***}			-5.239^{***}	-0.8838^{***}
Wealth	0.0066^{***}	0.0004^{***}	0.0041^{***}	-8.7e-6	0.0066^{***}	0.0004^{***}	0.0041^{***}	-8.7e-6
(in millions)								
$\min(N_{\text{imp}})$	17,202	12,750	17,202	12,750	17,202	12,750	17,202	12,750
Total N	86,030	63,799	86,030	63,799	86,030	63,799	86,030	63,799
Sign. tests								
Age Eff's	51.2^{***}	176.0^{***}	77.8^{***}	49.7^{***}	43.9^{***}	139.4^{***}	50.7^{***}	41.1^{***}
(d.o.f.)	(16)	(17)	(16)	(17)	(16)	(17)	(16)	(17)
Time Eff's	26.0^{***}	9.6^{*}	73.5^{***}	12.5^{**}				
(d.o.f.)	(5)	(5)	(5)	(5)				
Cohort Eff's	11.4	91.7^{***}	13.0	19.7^{**}	12.9	46.7^{***}	9.8	13.0
(d.o.f.)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)
Pri. Comp's					24.9^{***}	9.4^{**}	72.8^{***}	10.1^{**}
(d.o.f.)					(3)	(3)	(3)	(3)

Notes: Results of first and second stage shown for four models estimated using two-step estimator (Heckit). Data is multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors are adjusted in an appropriate way (*p<0.1, **p<0.05, ***p<0.01). N_{imp} is number of obs. for imputation imp $\in \{1, 2, ..., 5\}$. For joint significant tests, avrg. Wald test stat. shown (each $\sim \chi^2$), degrees of freedom in parentheses.

Table 2: Substitution of long-term government debt and cash with equity

To uncover the interdependence between the different portfolio components, I additionally estimate the models for the stock share with either the long-term bond share or the share of the residual category "cash" as independent variables. The results are shown in Table 2. Conditional on (financial and non-financial) wealth, a higher long-term debt share is correlated with a higher probability of being a stockholder, while the opposite is true for the portfolio share of cash, as one would expect. The estimates from the second stage suggest that the elasticities of substitution between long-term debt and equity and between cash and equity differ, reflected in coefficients of -0.52 and -0.88, respectively.[19] Additional cash holdings are associated with a larger reduction in the stock share than additional long-term debt holdings. In addition to differing age profiles, this suggests that long-term debt plays a significant and distinctive role in the dynamic rebalancing of household portfolios. The model outlined in the following section allows studying these relationships in more detail.

[19] Significantly differing values also emerge from a naive OLS regression among stock and long-term debt holders. See Table A.4 in the appendix.

3 Model

A large number of agents is faced with an asset market participation decision and, conditional on participation, an asset allocation problem in each period of their lives. I refer to model agents as households or investors interchangeably below.[20] Investors are born employed. They retire and subsequently die with fixed probabilities in every period, providing them with a motive to save for retirement and to deplete their asset stock once retired. Asset market participation is costly, but allows an investor to hold stocks *and* long-term government debt. A non-participant is able to save only through a riskless and low-interest bearing asset that is comparable to short-term bonds or cash.[21] Investors are subject to idiosyncratic and aggregate risk. Idiosyncratic risk, which is assumed to be uninsurable, arises from non-financial income as well as the stochastic nature of retirement and longevity. Aggregate risk results from the returns on stocks and long-term government debt.

3.1 Life Cycle Stages

Each investor $i \in I$ is born in period $t = 0$ and, over the course of their life, goes through an employment stage and a retirement stage. Both of these stages are of stochastic length. Note that the model describes the decisions of a large number of agents belonging to the *same* generation and that, as a result, there is no interaction between different, potentially overlapping, generations. Investors are born employed and supply labour inelastically as long as they remain employed, which entitles them to an exogenous income stream given by

$$Y_{i,t} = P_{i,t}U_{i,t}, \qquad \ln U_{i,t} \sim N(-0.5\sigma_u^2, \sigma_u^2) \tag{12}$$

$$P_{i,t} = GP_{i,t-1}N_{i,t}, \quad \ln N_{i,t} \sim N(-0.5\sigma_n^2, \sigma_n^2) \tag{13}$$

Labour income has a transitory component $U_{i,t}$ and a persistent component $P_{i,t}$.[22] The logarithm of $P_{i,t}$ follows a random walk with drift. The expectation of the shock to the persistent component of income $N_{i,t}$ and the expectation of the transitory shock $U_{i,t}$ equal one, so that, in expectation, the labour income of all agents grows at the common rate $G-1$.[23] Retired investors receive a pension $\Omega_{i,t} = \omega P_{i,t}$, which is a fraction of the persistent income that they would obtain if they remained employed. This specification captures the empirical fact that differences in income that develop over the course of the working life persist among retired investors.

[20] A model agent can be viewed as a household that either is in direct control of the consumption-savings and the portfolio-choice decision or delegates the latter decision to a portfolio manager that is informed about the risks faced by the household and its preferences towards them.

[21] I do not distinguish between stock market participation and long-term government-debt market participation in the model to reduce the dimensionality of the portfolio choice problem. Participation in both markets is highly correlated in the sample with a coefficient of 0.80 and Figure 4 suggests that this simplification yields a good approximation of observed household behaviour.

[22] This income process is frequently used in the literature and originally due to Carroll et al. (1992). They refer to P somewhat ambiguously as "permanent labour income".

[23] In general, if $\ln x \sim N(\mu, \sigma)$, then $\mathrm{E}x = \exp\left(\mu + 0.5\sigma^2\right)$, therefore $\mathrm{E}U_{i,t} = \mathrm{E}N_{i,t} = 1$.

As in Viceira (2001), an individual that is employed in the current period retires with probability π^r at the beginning of the next period and, once retired, dies in each period with probability π^d. By assuming stochastic working and retirement spells, life-cycle effects are introduced to the model without relinquishing the computational benefits of an infinite-horizon model (Viceira, 2001).[24] A more detailed discussion of this assumption is deferred to Section 3.4, to allow the reader to become familiar with the remaining components of the model first. For the numerical calculations below, π^r and π^d will be calibrated to obtain realistic average lengths of the employment and retirement stages.

3.2 Investment Opportunities

There are three types of assets available to the investors: a one-period bond, stocks and long-term government debt. Long-term government debt has a maturity of δ periods. A strategy frequently adopted in the literature is to assume that long-term bonds are entirely illiquid, or more precisely, that they have to be held until maturity. Aside from understating the liquidity of long-term government debt, this assumption leads to a big inflation of the state space as δ becomes large, causing exact solutions to portfolio choice problems to be computationally burdensome. A specification is proposed here that, in accordance with the US long-term bond market, allows investors to access some of the funds held in the form of long-term debt in each period and that makes the portfolio optimisation computationally feasible.

An investor that has purchased long-term government bonds in period $t-1$ at the amount of $Q_{i,t-1}$ receives a Calvo-type signal for each infinitesimal unit of $r_{q,t}Q_{i,t-1}$ in period t indicating whether it can be sold or not. $r_{q,t}$ is the annual gross return on the long-term government bond. A positive signal is received with probability δ^{-1}, implying that each infinitesimal unit has to be held on average for δ periods. Thus, portfolios are chosen in all periods subject to the constraint

$$Q_{i,t} \geq (1-\delta^{-1})r_{q,t}Q_{i,t-1} \tag{14}$$

Comparable to the case in which long-term bonds have to be held until maturity, the minimum expected holding period of the entire stock is equal to its maturity, but a fraction of this stock can be accessed in each period. Since the probability of being able to sell a given unit of long-term debt is time-constant, all long-term debt held by i can be summarised by one single state variable. Modelling long-term government bonds as a perpetuity as in Woodford (2001) would equally permit all long-term debt to be represented by a single state variable. However, the specification chosen here emphasises the imperfect liquidity of long-term government debt, which is an important characteristic of assets such as US savings bonds and tax-exempt bonds.[25]

[24] Gertler (1999) follows a comparable strategy in an OLG framework. Blanchard (1985) works with a constant death hazard and labour income that is a monotone function of age.

[25] The two specifications are similar though. For a perpetuity, the pay-off stream from a one-dollar investment is $\rho, \rho^2, \rho^3, \ldots$ for some $\rho \in \left[0, \beta^{-1}\right)$. Here, if government debt is run down at the fastest possible rate, this stream is $(1-\delta^{-1})r_{q,t+1}, (1-\delta^{-1})^2 r_{q,t+1}r_{q,t+2}, (1-\delta^{-1})^3 r_{q,t+1}r_{q,t+2}r_{q,t+3}, \ldots$ with $(1-\delta^{-1}) \in [0,1)$.

The one-period bond yields the riskless gross return r_b. Following Bonaparte et al. (2012), the gross stock return $r_{s,t}$ evolves according to a two-state Markov process, $r_{s,t} \in \{r_s^l, r_s^h\}$, with mean r_s and standard deviation σ_{rs}. Accounting for capital gains and dividends, Bonaparte et al. cannot reject that the annual stock return in the US is serially uncorrelated. $r_{s,t}$ is therefore assumed to be i.i.d. across periods with probabilities of a half for both return states. The return on long-term government debt equally follows a two-state Markov process. The mean, standard deviation and transition matrix are given by r_q, σ_{rq} and Γ_{rq}, respectively. No restrictions are placed on Γ_{rq}, allowing for persistence in the government bond return process.

Holdings of the short-term bond $B_{i,t}$ are costless. Investments in stocks $S_{i,t}$ and long-term government debt $Q_{i,t}$ are associated with a cost $\Psi_{i,t} = \psi P_{i,t}$ that has to be paid in each period of active participation in the markets for stocks or long-term government debt. $\Psi_{i,t}$ represents costs associated with the acquisition of information about financial markets and is scaled to the persistent component of income in order to capture the opportunity cost of time.[26] Investors are not considered active participants in financial markets if they hold no stocks and allow potential previously acquired holdings of long-term bonds to mature at the fastest possible rate. Thus, if the investor chooses not to pay $\Psi_{i,t}$, $S_{i,t} = 0$ and (14) holds with equality.[27] In addition, stock holdings are subject to a variable cost $\psi_s S_{i,t}$, reflecting the monetary costs of maintaining a stock portfolio. The role played by the two types of costs is re-visited below in more detail.

3.3 Optimisation Problem

The optimal plan of investor $i \in I$ solves the problem described in this section in each period $t = 0, 1, 2, \ldots$. The indices i and t are suppressed below for notational clarity. Employed investors take the retirement stage into account. The problem is therefore solved backwards.

3.3.1 Retired Investors

The budget constraint of a retired investor that participates in financial markets is given by

$$C + S(1 + \psi_s) + B + Q + \Psi = r_s S_{-1} + r_b B_{-1} + r_q Q_{-1} + \Omega \tag{15}$$

The sum of expenditures on consumption, stocks, short-term bonds, long-term government debt and all costs incurred must be equal to income, which is given by the gross return on last period's

[26] In Alan (2006), the cost of stock-market participation is equally made dependent on the persistent component of labour income, however, it is incurred only the first time an agent enters the market and not, for example, at a later re-entry.

[27] If non-participants were able to reduce long-term bond holdings at a faster rate, there would be liquidity gains associated with not acquiring information about financial markets. If they were able to reduce long-term bond holdings at a slower rate, the expected average holding period of long-term bonds would be larger than the maturity of the bond. Therefore, the assumption that (14) must hold with equality for non-participants seems most plausible.

investments and pension income. Defining "cash on hand" of a retired investor as as

$$X_r \equiv r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + \Omega \tag{16}$$

and illiquid assets as

$$Z \equiv (1-\delta^{-1}) r_q Q_{-1}, \tag{17}$$

one can express the budget constraint as

$$C + S(1+\psi_s) + B + Q + \Psi = X_r + Z \tag{18}$$

In Equation (18), income is divided into liquid funds X_r that can be freely allocated towards all types of expenditures and illiquid funds Z which are tied to a re-investment in long-term government debt. Using this notation, the liquidity constraint on long-term government debt becomes

$$Q \geq Z \tag{19}$$

requiring investors to carry an amount of long-term debt forward into the next period that is at least as large as the amount of illiquid assets brought into the period.

The budget constraint of a retired investor that does not participate in financial markets is

$$\begin{aligned} C + B + Q &= r_s S_{-1} + r_b B_{-1} + r_q Q_{-1} + \Omega \\ &= X_r + Z \end{aligned} \tag{20}$$

As discussed before, for a non-participant

$$Q = Z \tag{21}$$

which implies that the budget constraint can be written as

$$C + B = X_r \tag{22}$$

The equation above is independent of Q, reflecting the fact that the only choice that a non-participant faces is how to allocate cash on hand towards consumption and savings at the risk-free rate.

The value of the problem of a retired investor is given by

$$v_r\left(X_r, Z, r_q, P\right) = \max\left\{v_{r,p}\left(X_r, Z, r_q, P\right), v_{r,n}\left(X_r, Z, r_q, P\right)\right\} \tag{23}$$

where $v_{r,k} : \mathbb{R}^2 \times \left(\mathbb{R}^+\right)^2 \to \mathbb{R}$ with $k \in \{p, n\}$ are the value functions pertaining to a financial market participant and a non-participants, respectively. For each point in the state space, the investor decides to participate in financial markets if the value from participating is higher than that from not participating. In the event of participation in the current period, the optimal allocation satisfies the Bellman equation

$$v_{r,p}\left(X_r, Z, r_q, P\right) = \max_{C,S,B,Q} \quad u(C) + \beta\left(1-\pi^d\right)\mathrm{E}_{P',r_s',r_q'|P,r_q} v_r\left(X_r', Z', r_q', P'\right) \tag{24}$$

together with (16)-(19) and borrowing constraints $(S,B,Q) \geq (\underline{S},\underline{B},\underline{Q})$. $u:\mathbb{R}^+ \rightarrow \mathbb{R}$ is the period utility function with $u'(C)>0$ and $u''(C)<0$ for all $C\in\mathbb{R}^+$. For a non-participant, the optimum must satisfy

$$v_{r,n}\left(X_r, Z, r_q, P\right) = \max_{C,B} \quad u(C) + \beta\left(1-\pi^d\right)\mathrm{E}_{P',r_q'|P,r_q} v_r\left(X_r', Z', r_q', P'\right) \tag{25}$$

as well as (16), (17), (21), (22) and $B \geq \underline{B}$.

In each period, investors are faced with a participation problem, a consumption-savings problem and, in case of participation, a portfolio choice problem. The probability of dying π^d increases the rate at which future consumption utility is discounted, effectively making retired agents more impatient than employed agents.

3.3.2 Employed Investors

Employed investors differ from retired ones in two ways. They receive labour rather than pension income and have to account for retirement in addition to longevity risk.

The budget constraint of an employed financial market participant is

$$C + S(1+\psi_s) + B + Q + \Psi = X_e + Z \tag{26}$$

where cash on hand is now given by

$$X_e \equiv r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + Y \tag{27}$$

Since $S=0$ and $Q=Z$ if the participation fee Ψ is not paid, the budget constraint of a non-participant simplifies to

$$C + B = X_e \tag{28}$$

As above, an investor decides whether or not to participate in each period and for each state by comparing the value from participating with that from not participating. The value of the problem for an employed investor therefore is

$$v_e\left(X_e, Z, r_q, P\right) = \max\left\{v_{e,p}\left(X_e, Z, r_q, P\right), v_{e,n}\left(X_e, Z, r_q, P\right)\right\} \tag{29}$$

with $v_{e,k}:\mathbb{R}^2\times\left(\mathbb{R}^+\right)^2 \rightarrow \mathbb{R}$ for $k\in\{p,n\}$. In forming expectations about the value of the problem in the next period, employed investors have to evaluate the implications of their decisions taken today in both possible future employment states. If the agent participates in the current period, the optimal path is characterised by

$$v_{e,p}\left(X_e, Z, r_q, P\right) = \max_{C,S,B,Q} \quad u(C) + \beta\left(1-\pi^r\right)\mathrm{E}_{P',r'_s,r'_q|P,r_q} v_e\left(X'_e, Z', r'_q, P'\right) \\ + \beta\pi^r \mathrm{E}_{P',r'_s,r'_q|P,r_q} v_r\left(X'_r, Z', r'_q, P'\right) \tag{30}$$

together with the definitions (16), (17) and (27), the liquidity constraint (19), the budget constraint (26) and the borrowing constraints. If the agent decides not to become a financial market participant in the current period, the corresponding equation is

$$v_{e,n}\left(X_e, Z, r_q, P\right) = \max_{C,B} \quad u(C) + \beta\left(1-\pi^r\right)\mathrm{E}_{P',r'_q|P,r_q} v_e\left(X'_e, Z', r'_q, P'\right) \\ + \beta\pi^r \mathrm{E}_{P',r'_q|P,r_q} v_r\left(X'_r, Z', r'_q, P'\right) \tag{31}$$

subject to the same definitions, the binding constraint on long-term debt (21), the budget constraint (28) and $B \geq \underline{B}$.

3.4 Computation

The model has to be solved numerically. One state variable can be eliminated from the problem by dividing all endogenous variables by the persistent component of labour income P. Below, lower case letters denote normalised variables, e.g. $x_e \equiv X_e/P$. Period utility is assumed to be of CRRA form with coefficient of relative risk aversion γ. This implies that all value functions introduced above are homogeneous of degree $1-\gamma$ (Haliassos and Michaelides, 2002).[28] For example, the Bellman equation of an employed investor that participates in financial markets in the current period can be expressed as

$$v_{e,p}\left(x_e, z, r_q\right) = \max_{c,s,b,q} \quad u(c) + \beta\left(1-\pi^r\right)\mathrm{E}_{U',N',r'_q,r'_s|r_q}\left(GN'\right)^{1-\gamma} v_e\left(x'_e, z', r'_q\right) \\ + \beta\pi^r \mathrm{E}_{N',r'_q,r'_s|r_q}\left(GN'\right)^{1-\gamma} v_r\left(x'_r, z', r'_q\right) \tag{32}$$

Note that the stochastic growth rate of the persistent component of income $P'/P = GN'$ raised to $1-\gamma$ now pre-multiplies v_e and v_r in the expected values, accounting for uncertainty resulting from persistent income shocks. All normalised model equations together with more details on their derivation are listed in Section D of the appendix.

The fact that agents are able to invest in a third financial asset with a persistent return increases the dimensionality of the problem significantly in comparison to other recent models of portfolio choice over the life cycle.[29] Having eliminated P as a state variable, there remain

[28] See also Alan (2006) and Gomes and Michaelides (2005).

[29] Examples of models with two assets include Bonaparte et al. (2012), Campanale et al. (2015) and Cocco et al. (2005). In Davis et al. (2006), there are three financial assets, although only one of them has a stochastic return.

two continuous states in addition to the prevailing level of the return on long-term debt *in each period* and the optimisation involves four continuous controls. The strategy employed here to keep the model tractable is to assume stochastic rather than deterministic lengths of the working life and the retirement phase, which implies that the additional state variable "time" can be summarised using a binary employment indicator. While this is clearly a simplification, it allows me to study how households that can be viewed as representative for each of the two employment stages wish to rebalance equity, long-term debt and cash in their portfolios depending on current asset holdings and prices. It is shown below that the model is able to capture key characteristics of the data.

3.5 Calibration

Table 3 shows the calibration used for the simulations presented in the next section. Note that the model is written entirely in real terms. A period corresponds to a year. Since the age group 26-28 is the youngest contained in the estimations described in Section 2, it is assumed that agents are born at the age of 27. The retirement and death probabilities are calibrated to yield an average retirement age of 62 and a life expectancy of of 80 years, respectively, consistent with US data. δ is chosen such that the long-term bond approximates 5-year government debt. This is in line with the average maturity of all outstanding marketable securities issued by the US Treasury which averaged 59.7 months between January 2000 and December 2007.[30]

As in Bonaparte et al. (2012) and Campanale et al. (2015), the risk-free rate is set to 1.02, a value that is commonly used in the literature. Bonaparte et al. further estimate the average net return of stock in the US, inclusive of dividend payments, to be 6.33% with a a standard deviation of 0.155 and no serial correlation, which I also adopt here. The mean excess return of 5-year government debt over the risk-free rate is set to 0.6%, the standard deviation of 5-year US debt over the sampling period was about 0.011. The 5-year government debt return is modelled using a two-state Markov process whose transition matrix is found by first fitting an AR(1) process to the data and then finding the transition probabilities that best describe the estimated AR(1) process as proposed in Hussey and Tauchen (1991). r_q remains in state $k \in \{l, h\}$ with probability 0.663 and switches with the converse probability.

Following Haliassos and Michaelides (2003) and Viceira (2001) among others, annual income growth is set to 3%. The standard deviations of the shocks to the transitory and the persistent component of income are chosen based on the estimates reported in the seminal contribution by Carroll et al. (1992). The replacement ratio employed is 0.6 in line with Campanale et al. (2015) and Munnell and Soto (2005).[31]

[30] Between January 2000 and March 2016, the longest period for which the data is currently published, the average debt maturity was 60.7 months.

[31] Munnell and Soto (2005) find that, in the US, the replacement rate ranges from about 0.6 to 0.8 for households covered by a pension plan and from about 0.45 to 0.6 for those without pension coverage depending on the precise definitions used.

A number of papers that examine stock holdings over the life cycle contain estimates of the discount factor and the coefficient of relative risk aversion. Although results vary considerably, generally a relatively high degree of discounting and risk aversion is required to match the data. The estimates reported in Bonaparte et al. (2012) are $\beta = 0.69$ and $\gamma = 7.24$. Alan (2006) estimates $\beta = 0.92$ and $\gamma = 1.6$. Cagetti (2003) shows that both variables strongly depend on education with estimates for groups of different education levels ranging from 0.78 to 1.14 for β and from 2.40 to 8.13 for γ. I use $\beta = 0.94$ and $\gamma = 5$, again following Campanale et al. (2015).

Parameter	Value	Description
π_r	1/35	Retirement probability for employed investors
π_d	1/18	Death probability for retired investors
δ	5	Maturity of government debt
β	0.94	Discount factor
γ	5	Coefficient of relative risk aversion
G	1.03	Drift in persistent component of income
σ_u	0.16	Standard deviation of log of transitory income shock
σ_n	0.12	Standard deviation of log of persistent income shock
ω	0.6	Pension income parameter
r_b	1.02	Gross return on riskless bond
r_q	1.026	Mean gross return of δ-year government bond
r_s	1.0633	Mean gross stock return
σ_{rq}	0.011	Standard deviation of δ-year government bond return
σ_{rs}	0.155	Standard deviation of stock return
$\Gamma_{rq,kk}$	0.663	Probability for long-term bond return to remain in state $k \in \{l, h\}$
ψ	0.02	Participation cost parameter
ψ_s	0.015	Proportional cost of stock holdings
$\underline{B}$, $\underline{Q}$, $\underline{S}$	0	Borrowing limits

Table 3: Calibration

The high structural estimates for the discount rate and the coefficient of relative risk aversion are related to a puzzle that poses a challenge to the literature on life-cycle portfolio choice. According to standard models, it is optimal for households to invest a bigger share into high-risk and high-return assets like stocks than they do according to survey data. A number of strategies have been employed to align model-based predictions more closely with the empirical evidence. Calibrations with high, occasionally extreme levels of discounting and risk aversion respectively lower the benefits from the high average return and increase the sensitivity of households towards the high volatility of stocks. Transaction costs which imply that investors cannot costlessly convert stock holdings into consumption equally reduce the value of stock holdings for households (Campanale et al., 2015). Disastrous labour income shocks occurring with a small probability make total income, ceteris paribus, more risky and therefore lead households to reduce portfolio risk by lowering the stock share. Finally, an isolated decrease in the elasticity of intertemporal substitution, implemented by generalising CRRA utility to the Epstein-Zin-Weil recursive form, causes households to increase savings and thus financial income which also implies a lower optimal stock share.[32]

The focus of this paper lies on the dynamic patterns at which liquidity, long-term debt and stocks are substituted as wealth is accumulated and de-accumulated with age rather than a description of the effect that the addition of long-term debt to the portfolio choice problem of households has on the mean level of stock holdings. To keep the analysis as clean as possible, therefore merely two types of realistically-calibrated costs whose effects are easily understood are included in the model. The fixed cost of asset-market participation ψ gives rise to a meaningful participation decision and the variable cost of stock holdings ψ_s to an interior solution to the conditional asset allocation problem. Alan (2006) estimates stock-market entry costs to be about 2.1% of the persistent component of labour income, Gomes and Michaelides (2005) employ a value of 2.5%. In the model of Bonaparte et al. (2012), agents incur a transaction cost in each period in which stock holdings are adjusted. Their estimate for these costs is 1.2% of total labour income.[33] In line with these values, ψ is set to 2%. The Investment Company Institute (ICI) publishes data on the fees associated with investments in equity funds, which I use as an approximation of the marginal cost of maintaining a stock portfolio. The average expense ratio, the ratio of annual fees to the total size of the investment, fell from 1.6% in 2000 to 1.5% in 2007 and 1.3% in 2015.[34] Based on these figures, a value of 1.5% is chosen for ψ_s. Finally, note that following the literature the possibility of borrowing and short sales is excluded in the model.[35]

3.6 Portfolio Rebalancing

The policy functions associated with the optimisation problem laid out above show how investors rebalance their portfolios as financial wealth increases. In Figures 5 and 6, the optimal choice of financial assets is plotted against cash on hand for employed and retired investors, respectively. Long-term bond holdings are set to zero for now and the long-term interest rate to its low state. All variables are normalised by the trending persistent component of income as described before.

The solution is highly non-linear. At very low levels of cash on hand, investors in both employment states do not participate in the markets for equity and long-term debt. All savings are done using the risk-free asset, since the higher expected returns obtainable when they participate do not compensate investors for the additional risks born and the cost from participation ψ incurred. Note that in the non-participation region, consumption of an employed investor is

[32] See Cocco et al. (2005) for more details on the effects of disastrous labour income shocks and changes in the elasticity of intertemporal substitution.

[33] Campanale et al. (2015) consider transaction costs that are comparable to those in Bonaparte et al. (2012) which they calibrate to 0.3% in a low-cost scenario and to 4%-7%, depending on educational level, in a high-cost scenario.

[34] See www.icifactbook.org. Asset-weighted averages are slightly smaller, however, the expense ratio does not include costs like portfolio transaction fees, brokerage costs or sales charges.

[35] Cocco et al. (2005) provide a detailed discussion of this assumption.

higher than that of a retired investor with an equal amount of wealth, as can be seen from the higher distance between the line representing holdings of the riskless asset and the 45-degree line. Employed investors have an incentive to consume more, because labour income exceeds pension income. They also have an incentive to consume less, since retired investors die with probability π_d in the next period, effectively making retired agents less patient. For the parameter values chosen here, the first effect dominates the second.

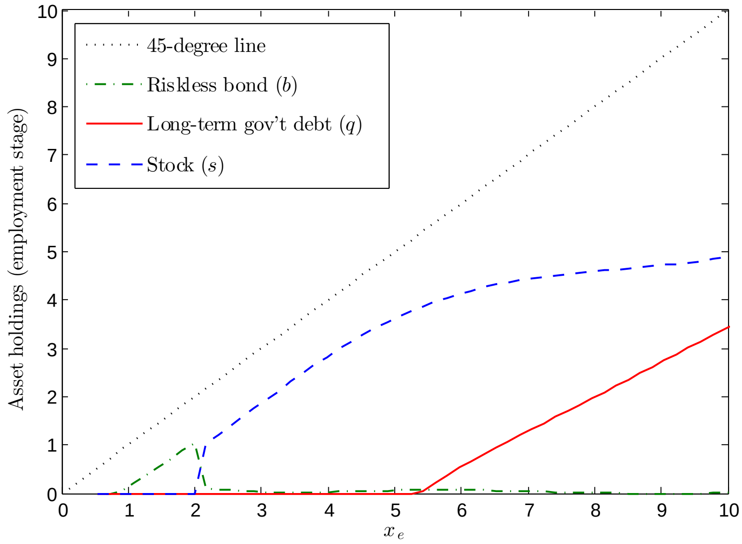

Figure 5: Policy functions of employed investors (for $z = 0$, $r_q = r_q^l$)

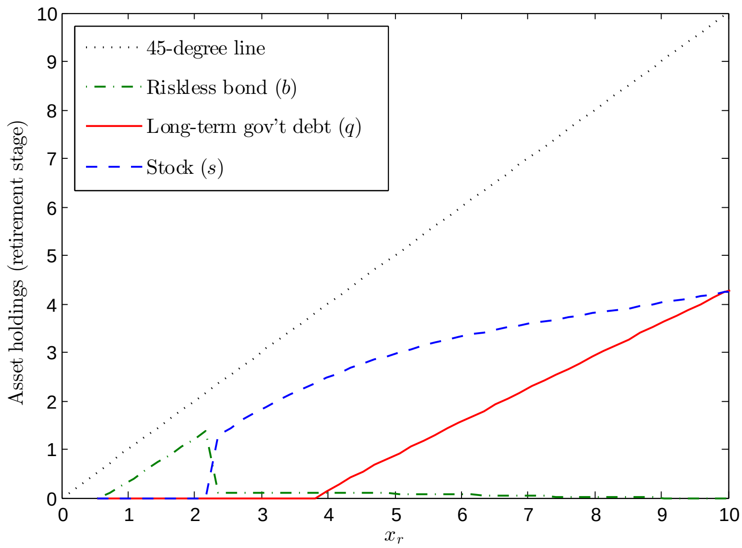

Figure 6: Policy functions of retired investors (for $z = 0$, $r_q = r_q^l$)

As cash on hand increases, the benefits from participating in financial markets outweigh the costs. Holdings of the riskless asset then play only a very minor role.[36] The threshold value at which equity and long-term debt market participation becomes attractive is slightly smaller for employed investors, reflecting the fact that employed agents invest a higher fraction of cash on hand in stocks and therefore have a higher expected benefit from participating. At the employment stage, investors choose to be exposed to higher risk through their portfolio than at the retirement stage, since labour provides them with a stream of income that is higher than non-financial income at retirement, allowing them to better smooth consumption in response to volatile portfolio returns. Additionally, employed investors aim to build up wealth in anticipation of retirement, whereas retired agents aim to run down wealth before they die.

Just above the participation threshold, agents allocate nearly all their investments towards stocks. However, as cash on hand increases, investors reduce the risk that they are exposed to *through their financial portfolio* by rebalancing their portfolios away from stock and towards government debt. To understand why portfolios are adjusted in this way, suppose first that labour and retirement income were non-stochastic. At low levels of cash on hand, labour and retirement income make up a large part of overall income. Thus, investments in financial assets could be comparably risky without giving rise to much risk exposure on aggregate. At high levels of cash on hand, labour and retirement income make up only a small fraction of overall income. Thus, investors would be exposed to more risk on aggregate if the same share of investments were made in the form of stocks. As a result, given that relative risk aversion is assumed to be constant, an equal or even increasing portfolio share of stocks across cash-on-hand levels could not be part of the solution to the investors' portfolio choice problem.[37]

Viceira (2001) shows that investors with stochastic labour and retirement income behave in an analogous way. Provided that the non-financial income is uncorrelated with the asset returns, agents choose their portfolios as if this income were generated by an investment in a safe asset of a size below the expected discounted value of its future payment stream.[38] Consequently, they also reduce the stock share in their financial portfolio as wealth is accumulated. A comparable effect can be observed in models in which investors have the choice between stock and a safe asset only.[39] In this class of models, it is the share of the riskless asset that increases with cash on hand as the stock share is reduced. At high levels of wealth, these models therefore assign a role to money holdings as a savings instrument that the model discussed here assigns to long-term debt holdings.

[36] There is no intrinsic value of holding the riskless asset in the model. A more significant portfolio share would be obtained for financial market participants, for example, if this asset were interpreted as "money" and a cash-in-advance constraint were introduced.

[37] The initial increase in the risk exposure of asset market participants at cash on hand levels beyond the "kink" in holdings of the riskless asset is a result of the lower bound on government debt and riskless bond holdings. Without borrowing constraints, investors would hold negative amounts of at least one of the two assets initially.

[38] This is shown formally in Proposition 3 of Viceira (2001).

[39] See, for example, Alan (2006) and Fagereng et al. (2016).

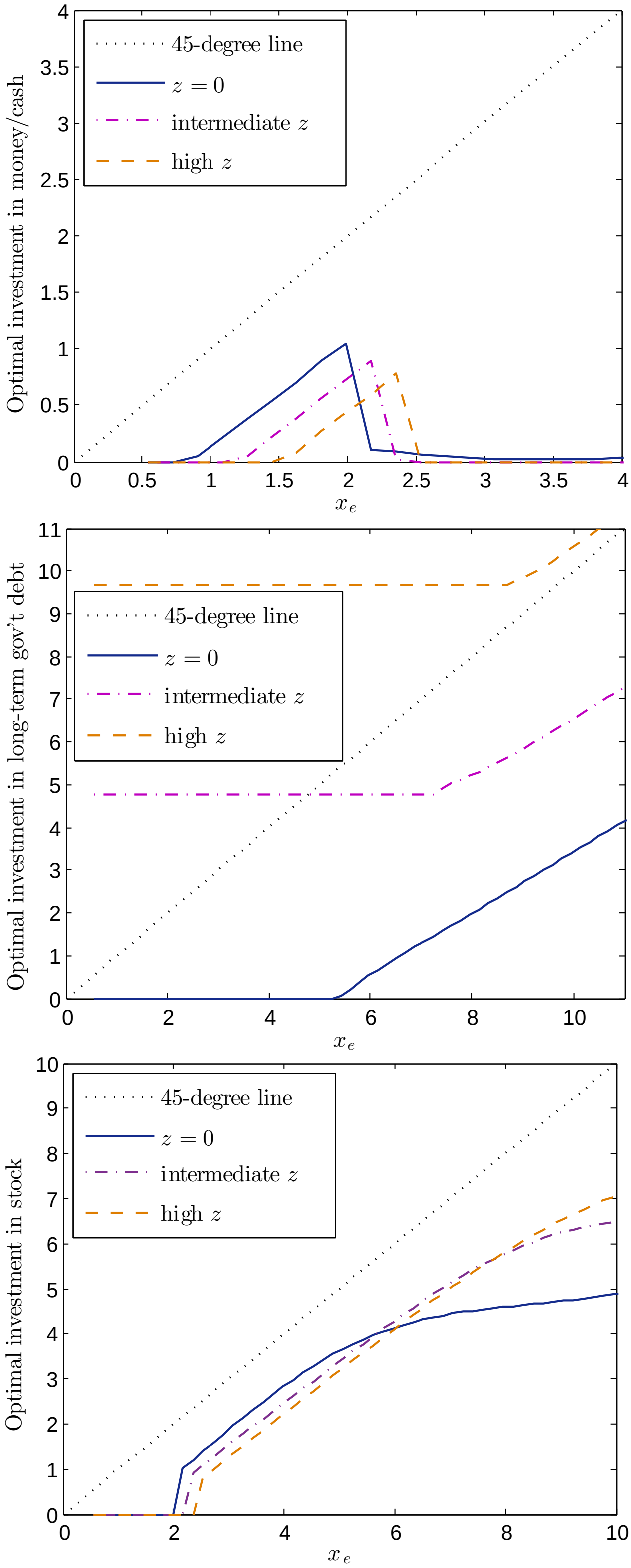

Figure 7: Optimal holdings of the riskless asset (top), long-term debt (middle) and equity (bottom) as a function of liquid and illiquid financial wealth (employed investor, $r_q = r_q^l$)

Figure 7 shows how the investment decision is influenced by existing long-term bond positions. It plots the optimal level of investment in all three financial assets for increasing values of illiquid wealth z. Results are only shown for employed investors for conciseness and $r_q = r_q^l$ as before. The higher the illiquid wealth of an investor, the less they invest in the riskless asset conditional on not participating in government-debt and equity markets and the more liquid wealth is required to make full asset market participation worthwhile. Intuitively, the more funds are currently locked up in previous investments but will become available in the future, the less willing investors are to pay the participation fee to make new investments in stock or long-term bonds. The optimal amount of new long-term bonds purchased is constant as long as the liquidity constraint is binding. The more illiquid wealth investors possess, the higher is the level of liquid assets at which they wish to hold long-term bonds in excess of the required amount. For levels of cash on hand at which the liquidity constraint binds, stock holdings decline with illiquid wealth. Once the liquidity constraint ceases to bind, this relationship is reversed.

4 Simulations

This section reports results from simulations of the "baseline" model outlined above and an application to the period of historically low US Treasury returns between 2009 and 2013.

4.1 Baseline Results

I simulate the model for $T = 80 - 27 = 53$ years, for I individuals with idiosyncratic shocks $\left\{U_{i,t}, N_{i,t}, \mathbb{1}^e_{i,t}, \mathbb{1}^r_{i,t}\right\}_{t=0}^T$ where $\mathbb{1}^e_{i,t}$ and $\mathbb{1}^r_{i,t}$ are indicators that equal one respectively if i is employed and retired in t, and for J aggregate shock sequences $\{r_{s,t}, r_{q,t}\}_{t=0}^T$. I and J are chosen sufficiently large to guarantee full convergence of all model aggregates presented below.

Figure 8: Fraction of initial population alive at t, fraction of initial population employed at t and fraction of current population participating in financial markets at t

The line most to the top in Figure 8 gives the fraction of the initial population that is alive in period t, the dashed line beneath it the fraction of the initial population that is employed in t. The difference between the two represents the fraction of retired agents. Participation in financial markets as a share of the population that is alive in t is given by the solid line. It is zero at first, since initial wealth is too low for it to be profitable to pay the participation cost ψ and then increases sharply as agents accumulate wealth. As more and more individuals retire and fall back to income levels at which participation is not profitable, the participation rate declines. This decline is slower than the retirement rate, since a number of individuals die without having run down their wealth to the level at which participating ceases to be optimal.

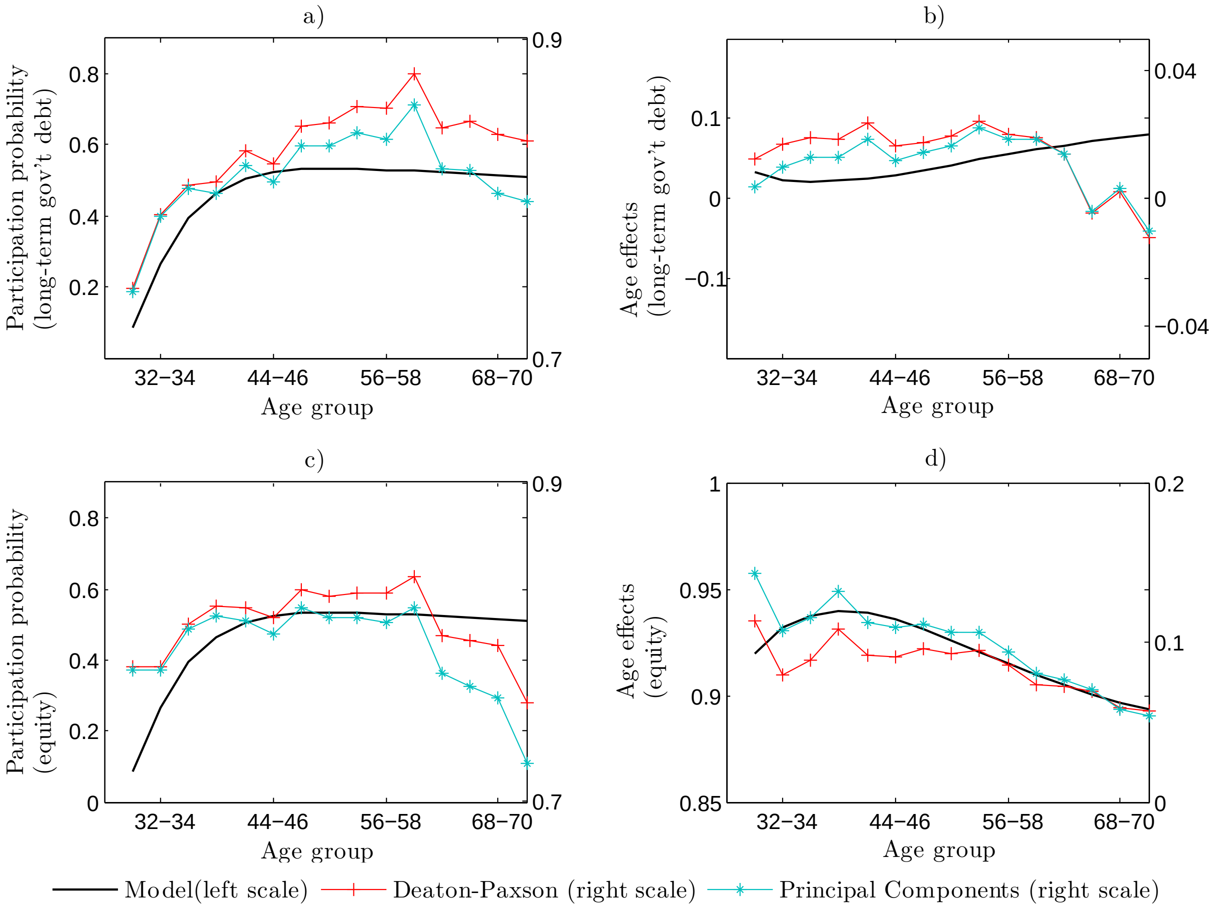

Figure 9: Participation and conditional portfolio shares

Figure 9 plots the participation rate and the asset shares conditional on participation implied by the model against the estimates based on SCF data. The estimates shown use the Deaton-Paxson identifying assumptions and principal components to capture time effects, respectively. The dynamics of asset market participation in the model mirror the patterns of participation in the markets for equity and long-term debt found in the data. In particular, the gradual increase in participation rates until around age 50-52 is successfully captured by the model. On average, asset-market participation is smaller in the model than in the sample, which does not constitute a shortcoming of the model though given that wealthier households are deliberately oversampled in the SCF. Alan (2006) estimates the life-cycle pattern of stock-market participation based on data from the Panel Study of Income Dynamics (PSID).[40] The simulation results shown here are closely in line with her finding that the participation rate is hump shaped with a peak value of about 0.6. The data suggests that households largely continue to hold long-term debt even after retirement, while stock market participation declines. Given that the model does not consider the participation decision for both markets separately and the black line in panels a) and c) should thus be interpreted as participation in at least one of the two markets, the fact that the model follows the flatter profile from a) seems reasonable.

In panels b) and d), the simulated conditional asset shares are plotted against the estimated marginal age effects. The conditional government debt share in the model is first flat and then mildly increasing. Until age 59-61 this model prediction is in line with the data. As explained above, it results from investors rebalancing their portfolios towards safer assets as they accumulate wealth. Accordingly, after a short increase for young age groups, the model predicts a decline in the conditional stock share, which is also present in the data. Both the long-term debt share and the stock share found in the data decline for older age groups, implying that the share of the residual category increases. For the oldest age groups, the model thus slightly under-states the importance of money holdings in favour of long-term debt holdings, which points to a weakness of the model. Because of the constant longevity risk, households have an incentive to invest in long-term bonds even at a relatively old age. While one might expect a bequest motive to have similar effects, the data suggests that bequests are not primarily made in the form of the two asset classes depicted here.

Household behaviour in the model gives rise to a plausible evolution of the wealth distribution. Figure 10 compares the distribution of wealth for each age group among employed investors in the model to that in the SCF. For the model and the SCF data, kernel density estimates based on a Gaussian kernel are reported.[41] The model successfully generates the characteristic positively-skewed distribution of wealth found in the data. The mass of wealth bunched at very low levels declines with age, giving rise to an increasingly fatter right tail and thus an increase in average wealth but also in inequality. In the model, average wealth rises in response to the accumulation of income and the associated rise in equity and debt-market participation. The distribution becomes more skewed, because agents that escape from the non-participation threshold at a young age accumulate wealth at a faster rate than those that have to exit asset markets occasionally and those that begin to participate persistently at a later stage, despite of rebalancing their portfolios towards safer assets as financial wealth increases.

[40] The PSID contains less information on financial asset holdings than the SCF, which implies that the estimates are based on supplements collected in 1985 and 1989.

[41] The distributions from the data are obtained by pooling the survey waves contained in the sample.

Interestingly, the model therefore provides a simple explanation for both the disproportionately high share of wealth held by financial market participants discussed in Guvenen (2009) and the skew of the wealth distribution described in detail by Benhabib and Bisin (2016).

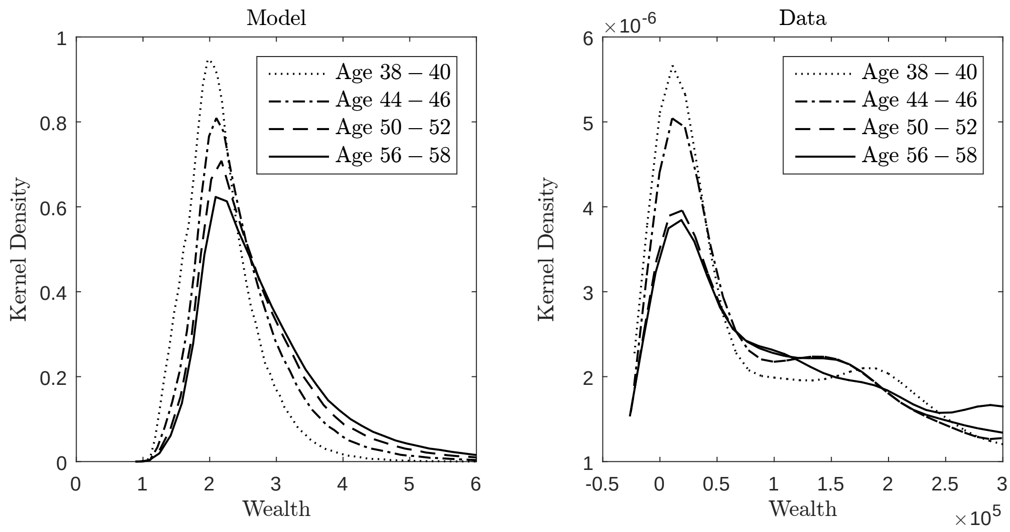

Figure 10: Wealth distribution by age

4.2 Application – Historically low US Treasury yields 2009-2013

The years following the US recession of 2007 to 2009 saw historically low levels of government bond returns and very high, although not unprecedented, levels of stock returns. Figure 11 shows the 5-year Treasury constant maturity rate in real terms between 2000 and 2014 in the left panel and the real return of the S&P 500 in the right panel.[42] In 2009, the real 5-year Treasury return became negative, recovered somewhat in the following year, and turned negative again for the following three years. At the same time, the real return on US equity, initially driven by stock price gains then also by dividend payments, reached double digits in four out of five years between 2009 and 2013, breaking the 20% mark in the first and the last year of the interval.

The effects of these conditions on household portfolios can be studied through the lens of the model. To do so, I conduct the following experiments. The stock and the long-term debt return follow the same stochastic processes as outlined above in all years except for a five year window, in which the returns fed into the model are matched precisely to those observed in the data between 2009 and 2013 (depicted by blue stars in Figure 11). Results are shown below for the case that the observed return shocks occur at the age of 35 to 39, that is, towards the beginning of wealth accumulation through financial markets. A set of corresponding results for an older cohort, affected at the age of 50 to 54, is relegated to Appendix E. The returns of

[42] The data on real US stock returns is taken from Robert Shiller's online database, which is freely accessible at www.econ.yale.edu/∼shiller/data.htm

2009-2013 are treated as exceptional draws from the same distributions as those that govern the asset returns before and after this window, since the period is too short to reliably estimate a separate set of return processes. As a result, assumptions have to be made about the underlying state of the long-term debt return process. In the first and the last year of the time period examined, the population of model agents is split in half between those forming expectations about the 5-year bond return in the following period based on the high and the low state, so that on aggregate both states are seen as equally likely, reflecting uncertainty about the persistence of the observed shocks. During the intermediate years, agents form expectations based on the probability distribution pertaining to the low return state. To be able to distinguish the effects caused by the return shocks to each asset, I also report results from the same experiments when the long-term debt return is matched to the 2009-2013 period as described above but the stock return evolves as in the baseline model.

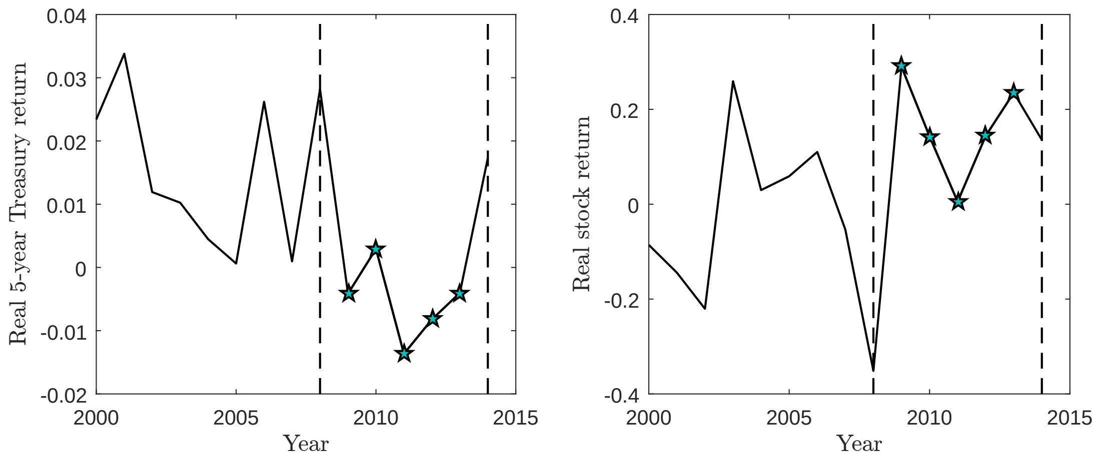

Figure 11: Real return on 5-year Treasuries and the US stock market (S&P 500)

The aggregate dynamics of financial-market participation and the conditional asset shares in response to the historical return shocks are plotted in Figure 12. Dashed vertical lines are used to indicate the last and the first period, respectively, in which all asset returns follow the baseline specification. First, consider the case that only the long-term bond yields are matched to the five-year period in the data, depicted by the red dashed lines. On average, agents react to the decline in bond yields by rebalancing their portfolios towards stock holdings following an initial period of adjustments in expectations. In the three intermediate shock years, the average conditional long-term bond share is lowered by 1.8-1.9 percentage points (pps) and the conditional stock share is increased by about the same amount compared to the baseline outcome. All adjustments take place between the two risky assets, leaving the share of the safe asset unaffected. The decline in the Treasury return is compensated by the increased stock share, implying that the resulting change in the overall portfolio return and wealth is not large enough to alter the participation rate in a significant way. Consequently, while quantitatively significant on impact, the effects on the composition of household portfolios are of temporary nature. Two years after the final negative government debt return shock has occurred, all adjustments are nearly undone.

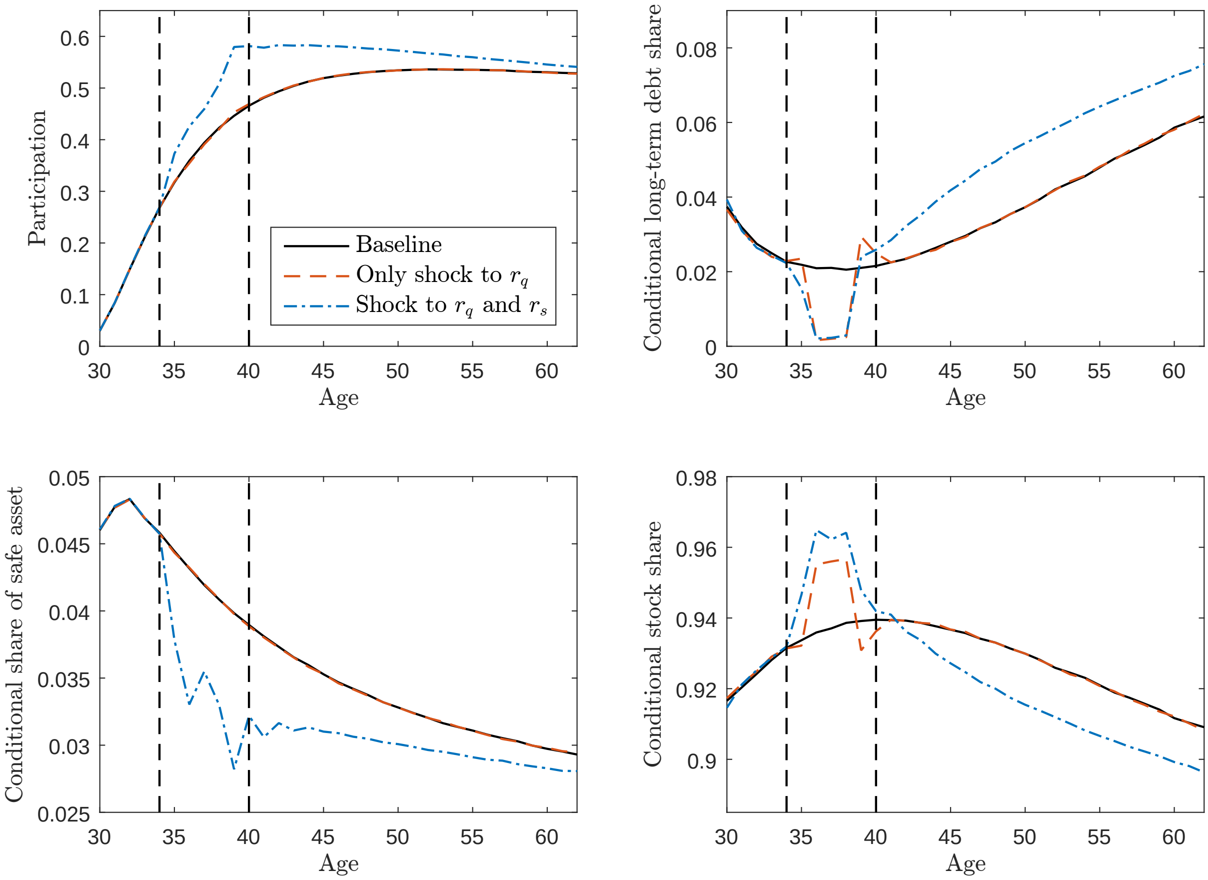

Figure 12: Participation and conditional asset shares in response to return shocks

When both historical shock sequences are fed into the model, the immediate impact on the conditional stock share is magnified, as can be seen from the corresponding blue dash-dot line. Compared to the baseline scenario, the stock share is elevated by 2.5-2.9 pps for the three intermediate shock years. The reduction in the average long-term debt share is of similar magnitude as that in the case in which only the long-term debt return is matched, giving rise to a significant drop in the conditional share of the safe asset. Since stock holdings make up a large part of the portfolio, positive stock return shocks increase the benefits from participating in asset markets markedly. As a result, in the first year of historical stock returns, the participation rate is 5.6 pps higher compared to the baseline simulation. This difference increases to 13.3 pps in the last year. In contrast to the previous case, average wealth is significantly raised, which leads to persistent deviations of the participation rate and the portfolio shares from their respective baseline paths even after the period of extreme returns. Average participation is 3.8 pps above the baseline at the age of 50 and 1.5 pps at the age of 60. Similarly, long-term debt and stock holdings deviate from the baseline by 1.7 pps and -1.5 pps at the age of 50, and 1.4 pps and -1.2 pps at the age of 60, respectively. The results are qualitatively similar for investors that are confronted with the return shocks at the age of 50 to 54 (See Figure A.1 in Appendix E). As the fraction of financial income in total income is larger at a higher age, the over-shooting of asset shares due to wealth effects are even larger in this case.

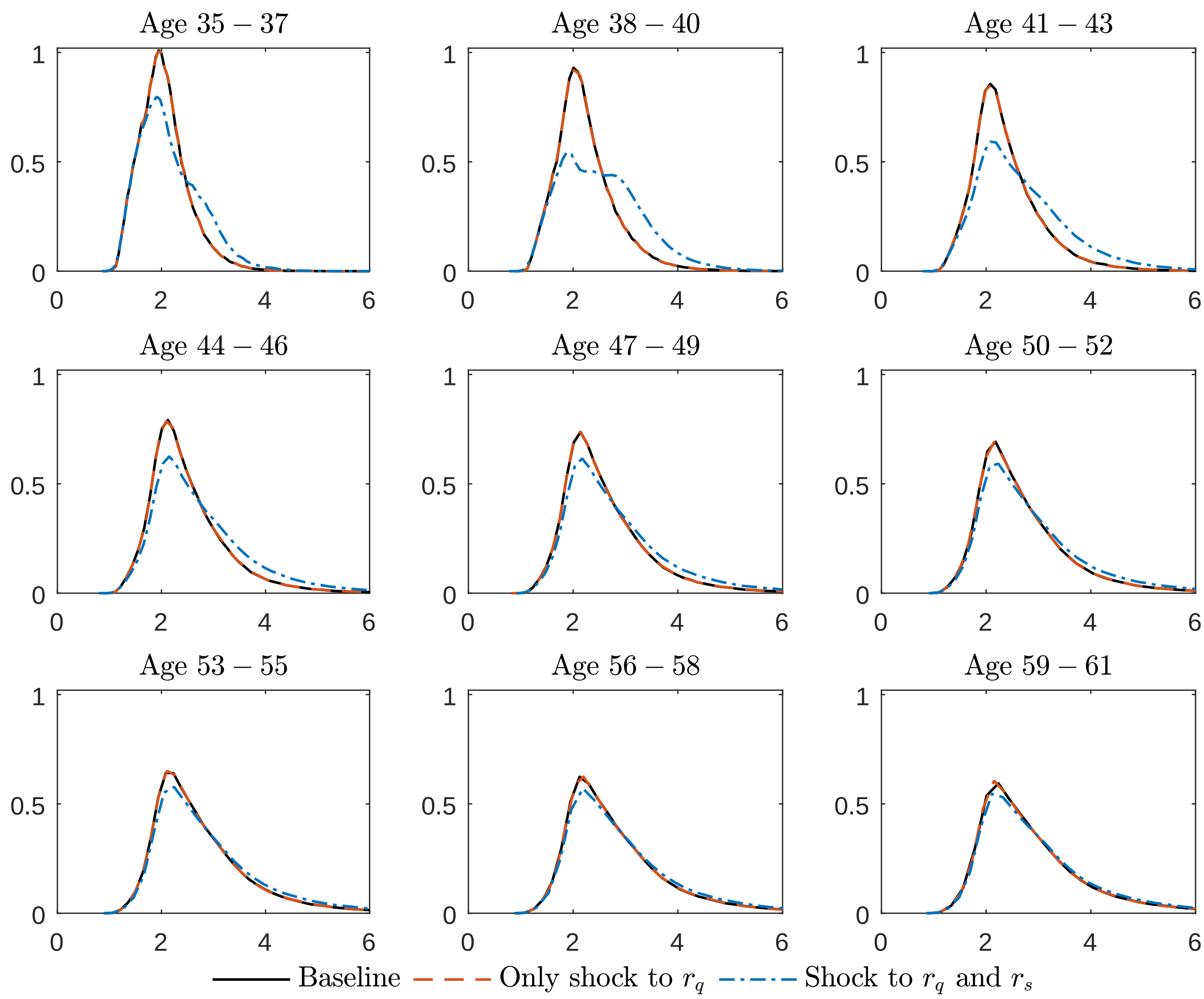

Figure 13: Evolution of the wealth distribution following return shocks at age 35 – 39

Figure 13 shows the predicted evolution of the wealth distribution among employed investors following the return shocks of 2009 to 2013. The shocks to the stock return significantly raise average wealth by increasing the mass in the right tail of the distribution. Investors that are able to afford significant stock holdings when the shocks first hit experience a large increase in their wealth levels, while the rest is "left behind". Deviations from the baseline take until the end of the working life to disappear, suggesting that booms in the stock market lead to persistent increases in inequality. For generations that experience the shocks at an older age, the wealth distribution continues to be strongly affected when agents enter retirement, as can be seen from Figure A.2 in the appendix.

5 Conclusion

Households systematically rebalance their financial portfolios as they accumulate wealth. Nearly riskless assets like cash or bank deposits, long-term bonds like Treasury notes or Treasury bonds, and equity play distinctive roles in this process. At low levels of liquid wealth, the expected benefits from participating in the markets for long-term debt and equity do not outweigh the costs, implying that households save through low-cost assets with a low return. As they accumulate more cash on hand and save more, entering the stock market becomes profitable. Cash holdings due to a pure savings motive markedly decline at this point. At levels of liquid wealth just above the participation threshold, investment income is small compared to labour income. Agents can therefore afford to invest a large share of their financial portfolio in equity without being exposed to an excessive amount of risk on aggregate. Additional accumulation of liquid wealth implies that investment income as a fraction of total income increases. As a result, the average asset-market participant reduces the equity share and increases the share of safer assets to limit their risk exposure. My model suggests that long-term debt rather than cash is used by households to readjust the risk properties of their portfolios in this way.

In accordance with debt instruments like US savings bonds, which can be cashed by households only at a significant cost in the first five years after they have been issued, government debt is assumed to be partially illiquid in the model. Consequently, the portfolio-choice problem of households in a given period is dependent on past investments in long-term debt. The higher the illiquid wealth of a household, the less it invests in the riskless asset conditional on not acquiring new government debt and stock, the more liquid wealth is required for full asset-market participation to be beneficial, the less it invests in stock as long as the liquidity constraint is binding, but the more stock it acquires at sufficiently high levels of cash on hand.

This behaviour at the micro-level gives rise to aggregate dynamics that are in line with the data. The asset-market participation rate rises until the age of about 50 to 52, the average stock share briefly increases then monotonically decreases, and the average long-term government-bond share mildly increases with age. The wealth distribution is increasingly skewed to the right, because households that begin to consistently invest in equity at a young age accumulate wealth faster than those that join later and those that are forced to exit temporarily.

A model-based decomposition of the effects resulting from the fluctuations in asset returns observed in the US between 2009 and 2013 shows that the observed decline in long-term bond yields has sizeable effects on the composition of household portfolios. Since the average participation rate and wealth remain nearly unaffected, these effects are of temporary nature. The observed stock-return shocks give households an additional motive to rebalance their portfolios in the short-run, lead the asset-market participation rate to increase and shift additional mass into the right tail of the wealth distribution, which leads to persistent changes in the conditional asset shares. Naturally, the decomposition used here is hypothetical. An important question relevant, for example, for evaluating the macroeconomic effects of large-scale purchases of long-term bonds by central banks is to what degree lowered bond yields exert upward pressure on stock prices. In the presence of a strong link between the two, the model suggests that unconventional monetary policy in the form of government-debt purchases causes inequality to increase for several decades.

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A Descriptive Statistics

Variable	Mean	Variance	Min	Max
Age of household (h.h.) head	50.32	10.04	26	76
Marital Status of h.h. head (=1 if married)	1.26	0.44	1	2
Number of children living in h.h.	0.98	1.20	0	10
Overall wage income of h.h. (in thousands)	210.04	1,143.30	-86.92	88,630.81
Highest education of h.h. head				
None	0.07	0.26	0	1
School diploma	0.24	0.42	0	1
Some college education	0.16	0.36	0	1
College diploma	0.53	0.50	0	1
Ethnic background of h.h. head				
White and non-Hispanic	0.84	0.36	0	1
Black	0.08	0.27	0	1
Hispanic	0.04	0.20	0	1
Other	0.04	0.19	0	1
Occupational category of h.h. head				
Managerial and professional	0.45	0.50	0	1
Technical, sales and services	0.21	0.41	0	1
Other	0.18	0.39	0	1
Not working	0.15	0.36	0	1

Notes: Nominal variables in 2010 US dollar.

Table A.1: Selected descriptive statistics

Age	Cohort (birth year)										
group	'31-'33	'34-'36	'37-'39	'40-'42	'43-'45	'46-'48	'49-'51	'52-'54	'55-'57	'58-'60	'61-'63
26-28	0	0	0	0	0	0	0	0	0	0	495
29-31	0	0	0	0	0	0	0	0	0	510	858
32-34	0	0	0	0	0	0	0	0	822	1,009	1,047
35-37	0	0	0	0	0	0	0	850	1,102	1,200	1,123
38-40	0	0	0	0	0	0	906	1,155	1,227	1,305	1,296
41-43	0	0	0	0	0	1,125	1,125	1,328	1,447	1,601	1,362
44-46	0	0	0	0	890	1,229	1,462	1,386	1,517	1,365	1,230
47-49	0	0	0	810	1,162	1,265	1,411	1,690	1,608	1,511	0
50-52	0	0	834	1,035	1,286	1,519	1,327	1,725	1,535	0	0
53-55	0	916	1,000	1,086	1,206	1,480	1,321	1,566	0	0	0
56-58	825	800	937	1,117	1,180	1,595	1,295	0	0	0	0
59-61	855	945	1,010	1,016	1,292	1,437	0	0	0	0	0
62-64	943	782	905	1,097	1,235	0	0	0	0	0	0
65-67	739	756	867	1,020	0	0	0	0	0	0	0
68-70	733	711	760	0	0	0	0	0	0	0	0
71-73	690	705	0	0	0	0	0	0	0	0	0
74-76	518	0	0	0	0	0	0	0	0	0	0
Σ	5,303	5,615	6,313	7,181	8,251	9,650	8,847	9,700	9,258	8,501	7,411

Notes: N=86,030. SCF data is multiply imputed. For each respondent there are five observations in the data. To approximate the number of respondents in each field in the table, the number shown therefore has to be divided by five. Age groups and cohorts are chosen such that there is a sufficient number of responses in each age-cohort pairing. Figures are not multiples of five if asset shares are missing for some but not all imputations.

Table A.2: Distribution of observations across age groups and cohorts

B Asset Categorisation

This section shows how the set of financial assets sampled in the SCF is divided into the three broad categories stocks, long-term government debt and residual assets. I begin by discussing assignments that require explanation. The categorisation of all assets is summarised in Table A.3.

Savings bonds and tax-exempt bonds are issued by the Department of the Treasury and municipal, county or state governments, respectively. They typically have a maturity of at least five years and are therefore treated as long-term government debt.

US government (agency) debt contains Bills, Notes and Bonds. As explained in the text, Bills are classified as short-term and Notes/Bonds as long-term. According to the "Monthly Statement of the Public Debt of the United States" from December 2007, 22.1% of total marketable debt held by the public were Bills and the remainder Notes/Bonds. Households are assumed to hold long-term and short-term debt at this proportion.

Certificates of deposit are a fixed-term savings product issued by banks, credit unions and thrift institutions; they are allocated to the residual category.

Government bond investment funds are allocated in the same way as direct holdings of US government debt.

Hybrid investment funds are allocated equally to stocks and government debt. Government debt is again split into long-term and short-term debt as described above.

Individual Retirement Accounts (IRAs) and Keogh Plans can include a variety of different financial assets. The Employee Benefit Research Institute (EBRI) estimates that, in 2008, a total of 45.8% of funds in IRAs were held in the form of equity (38.5% in equity mutual funds, directly held individual stocks, and other 100%-equity investment vehicles; they add to this 60% of balanced, lifestyle/lifecycle, target-date funds, and any other funds that have a partial investment in equities and bonds, which make up 12.1% of all funds in IRAs).[43] I assign 18.4% to long-term government debt (the remaining 40% of hybrid equity-bond funds and all bond investments). Money and other assets make up 35.8% and are classified as "residual" for the purposes of this paper. Keogh Plans are similar to IRAs but designed for self-employed individuals. I assume that funds held in Keogh Plans are invested as in IRAs.

Life insurance and annuities are assigned to the residual category, since life insurances and annuities are claims to a safe future payoff (stream).

Other managed assets include trusts and managed investment accounts which are counted towards stocks here.

[43] See "IRA Asset Allocation and Characteristics of the CDHP Population, 2005-2010", May 2011, available at www.ebri.org/publications/notes.

Asset Type	Asset Sub-Type	Stocks	Long-Term Gov't Debt	Residual
Stocks		1	0	0
Bonds				
	Savings bonds	0	1	0
	Tax-exempt bonds	0	1	0
	US gov't (agency) debt	0	0.779	0.221
	Corporate and foreign bonds	0	0	1
	Mortgage-backed bonds	0	0	1
Transaction accounts				
	Checking accounts	0	0	1
	Saving accounts	0	0	1
	Money market deposit accounts	0	0	1
	Money market mutual funds	0	0	1
	Call accounts at brokerages	0	0	1
Certificates of deposit		0	0	1
Pooled investment funds				
	Stock funds	1	0	0
	Tax-free bond funds	0	1	0
	Government bond funds	0	0.779	0.221
	Other bond funds	0	0	1
	Hybrid funds	0.5	0.389	0.111
	Other funds	0	0	1
Retirement accounts				
	IRAs and Keogh plans	0.458	0.184	0.358
	Thrift plans	0.458	0.184	0.358
	Current pensions	0	0	1
Life insurance, annuities		0	0	1
Other managed assets		1	0	0

Table A.3: Asset Categorisation

C Estimation

Controls included in the estimation are binary variables indicating the marital status, education, race, type of employment as well as information on income, assets, debt and pension plans.

An exclusion restriction is required to avoid identification based on functional form. In the model of the equity share, the assumption maintained is that, at low levels of education, a marginal amount of additional schooling increases the likelihood of stock market participation but not the portfolio share conditional on participation. Compared to some years of high-school attendance, a completed high-school education may provide individuals with knowledge about investment opportunities and lower the costs of access to the stock market, but should have no direct effect on the stock share conditional on participation when other characteristics such as the occupation of the individual are controlled for. In the data, having earned a high-school diploma and visited college for some time without graduating has a strong positive effect on the participation probability. It is insignificant at the 5% level in a regression of the second-stage residuals on all explanatory variables.

The variable excluded from the second stage of the model for long-term government debt is a binary variable indicating whether the household head is "not doing any work for pay at the present time". As we would expect, it has a highly significant negative effect on the participation decision. The hypothesis is that it does not directly affect the portfolio share of long-term government debt among those who hold the asset. The average age of those responding positively is significantly larger than that of individuals responding negatively (57.9 compared to 48.9) suggesting that positive responses come, at a disproportionate amount, from retired individuals. Age effects are controlled for in the second stage regression so that the hypothesis holds true for this group unless there are effects from being retired that the age dummies do not pick up. Positive responses also come from individuals that have not yet reached retirement age. These cases are unproblematic if there are other household members that provide the family with labour income, since the source of family income should not affect portfolio shares. To address cases where labour income is diminished as a result of one or more household members being involuntarily unemployed and where the family participates in the market for long-term debt nevertheless, a set of educational and occupational variables are used to control for potential common drivers of unemployment and the long-term government debt share. In a regression of the residuals from the second stage on all explanatory variables including the employment indicator, the instrument is insignificant well beyond the 10% level.

	OLS	
	(1)	(2)
Long-term debt	-0.6381***	
Cash		-0.7542***
Wealth (in millions)	0.0006***	-0.0001**
min(N_{imp})	4,299	4,299
Total N	21,531	21,531
Sign. tests		
Time Eff's	4.4***	5.9***
(d.o.f.)	(6)	(6)

Notes: Dependent variable: Stock share. Data is multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors are adjusted in an appropriate way (*p<0.1, **p<0.05, ***p<0.01). N_{imp} is number of obs. for imputation imp $\in \{1, 2, ..., 5\}$. For joint significant tests, avrg. F-test stat. shown, degrees of freedom in parentheses.

Table A.4: Conditional correlations of long-term debt and cash share with stock share

D Normalisation

Consider the amount of funds brought into the period for $j \in \{e, r\}$ first. Cash on hand of an employed investor can be normalised as follows

$$X_e \equiv r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + Y \tag{33}$$

$$\frac{X_e}{P} = r_s \frac{S_{-1} P_{-1}}{P_{-1} P} + r_b \frac{B_{-1} P_{-1}}{P_{-1} P} + \delta^{-1} r_q \frac{Q_{-1} P_{-1}}{P_{-1} P} + \frac{PU}{P} \tag{34}$$

$$x_e = \frac{P_{-1}}{P} \left(r_s s_{-1} + r_b b_{-1} + \delta^{-1} r_q q_{-1} \right) + U \tag{35}$$

$$= \frac{r_s s_{-1} + r_b b_{-1} + \delta^{-1} r_q q_{-1}}{GN} + U \tag{36}$$

For a retired investor, analogous steps yield

$$x_r = \frac{r_s s_{-1} + r_b b_{-1} + \delta^{-1} r_q q_{-1}}{GN} + \omega \tag{37}$$

and normalised illiquid assets brought forward are

$$z = \frac{(1 - \delta^{-1}) r_q q_{-1}}{GN} \tag{38}$$

The four Bellman equations can be transformed in a similar way. Starting with an employed financial-market participant, we have

$$\begin{aligned} v_{e,p}\left(X_e, Z, r_q, P\right) = \max_{C,S,B,Q} \quad & u(C) + \beta\left(1 - \pi^r\right) \mathrm{E}_{P', r_s', r_q' | P, r_q} v_e\left(X_e', Z', r_q', P'\right) \\ & + \beta \pi^r \mathrm{E}_{P', r_s', r_q' | P, r_q} v_r\left(X_r', Z', r_q', P'\right) \end{aligned} \tag{39}$$

$$\begin{aligned} \frac{v_{e,p}\left(X_e, Z, r_q, P\right)}{P^{1-\gamma}} = \max_{C,S,B,Q} \quad & \frac{u(C)}{P^{1-\gamma}} + \beta\left(1 - \pi^r\right) \mathrm{E}_{P', r_s', r_q' | P, r_q} \frac{v_e\left(X_e', Z', r_q', P'\right)}{P^{1-\gamma}} \\ & + \beta \pi^r \mathrm{E}_{P', r_s', r_q' | P, r_q} \frac{v_r\left(X_r', Z', r_q', P'\right)}{P^{1-\gamma}} \end{aligned} \tag{40}$$

$$\begin{aligned} v_{e,p}\left(\frac{X_e}{P}, \frac{Z}{P}, r_q, 1\right) = \max_{C,S,B,Q} \quad & u\left(\frac{C}{P}\right) + \beta\left(1 - \pi^r\right) \mathrm{E}_{P', r_s', r_q' | P, r_q} v_e\left(\frac{X_e'}{P}, \frac{Z'}{P}, r_q', \frac{P'}{P}\right) \\ & + \beta \pi^r \mathrm{E}_{P', r_s', r_q' | P, r_q} v_r\left(\frac{X_r'}{P}, \frac{Z'}{P}, r_q', \frac{P'}{P}\right) \end{aligned} \tag{41}$$

$$\begin{aligned} v_{e,p}\left(x_e, z, r_q\right) = \max_{c,s,b,q} \quad & u\left(c\right) + \beta\left(1 - \pi^r\right) \mathrm{E}_{P', r_s', r_q' | P, r_q} v_e\left(x_e' \frac{P'}{P}, z' \frac{P'}{P}, r_q', \frac{P'}{P}\right) \\ & + \beta \pi^r \mathrm{E}_{P', r_s', r_q' | P, r_q} v_r\left(x_r' \frac{P'}{P}, z' \frac{P'}{P}, r_q', \frac{P'}{P}\right) \end{aligned} \tag{42}$$

$$\begin{aligned} v_{e,p}\left(x_e, z, r_q\right) = \max_{c,s,b,q} \quad & u(c) + \beta\left(1 - \pi^r\right) \mathrm{E}_{U', N', r_s', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_e\left(x_e', z', r_q'\right) \\ & + \beta \pi^r \mathrm{E}_{N', r_s', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_r\left(x_r', z', r_q'\right) \end{aligned} \tag{43}$$

The remaining equations are

$$
\begin{aligned}
v_{e,n}\left(x_e, z, r_q\right) = \max_{c,b} \quad & u(c) + \beta\left(1 - \pi^r\right) \mathrm{E}_{U', N', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_e\left(x_e', z', r_q'\right) \\
& + \beta \pi^r \mathrm{E}_{N', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_r\left(x_r', z', r_q'\right) \qquad (44)
\end{aligned}
$$

$$
v_{r,p}\left(x_r, z, r_q\right) = \max_{c,s,b,q} \quad u(c) + \beta\left(1 - \pi^d\right) \mathrm{E}_{N', r_s', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_r\left(x_r', z', r_q'\right) \qquad (45)
$$

$$
v_{r,n}\left(x_r, z, r_q\right) = \max_{c,b} \quad u(c) + \beta\left(1 - \pi^d\right) \mathrm{E}_{N', r_q' | r_q} \left(GN'\right)^{1-\gamma} v_r\left(x_r', z', r_q'\right) \qquad (46)
$$

where

$$
v_e\left(x_e, z, r_q\right) = \max\left\{v_{e,p}\left(x_e, z, r_q\right), v_{e,n}\left(x_e, z, r_q\right)\right\} \qquad (47)
$$

$$
v_r\left(x_r, z, r_q\right) = \max\left\{v_{r,p}\left(x_r, z, r_q\right), v_{r,n}\left(x_r, z, r_q\right)\right\} \qquad (48)
$$

The corresponding budget constraints in normalised form are given by

$$
c + s(1 + \psi_s) + b + q + \psi = x_e + z \qquad (49)
$$

$$
c + b = x_e \qquad (50)
$$

$$
c + s(1 + \psi_s) + b + q + \psi = x_r + z \qquad (51)
$$

$$
c + b = x_r \qquad (52)
$$

E Additional Simulations

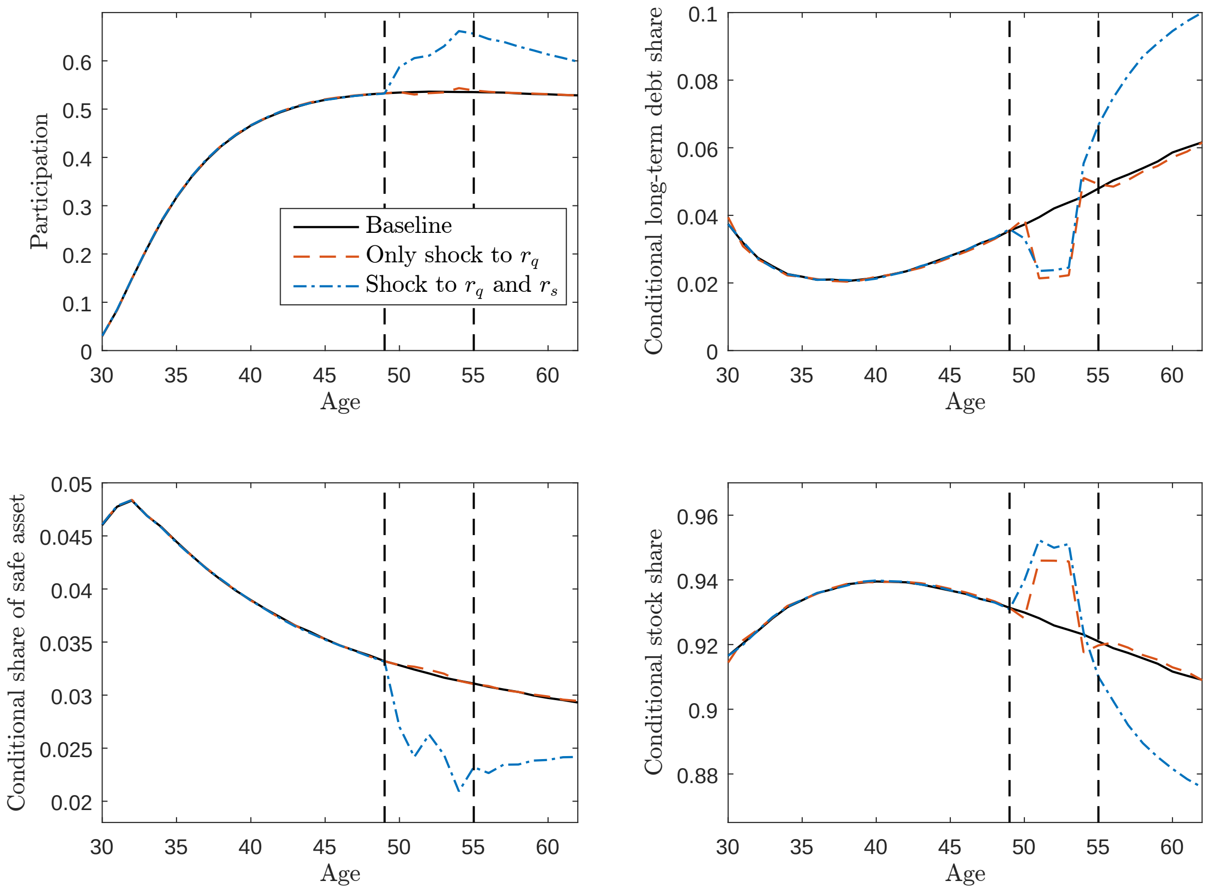

Figure A.1: Participation, conditional asset shares in response to the return shocks of 2009-13 impacting at age $50 - 54$

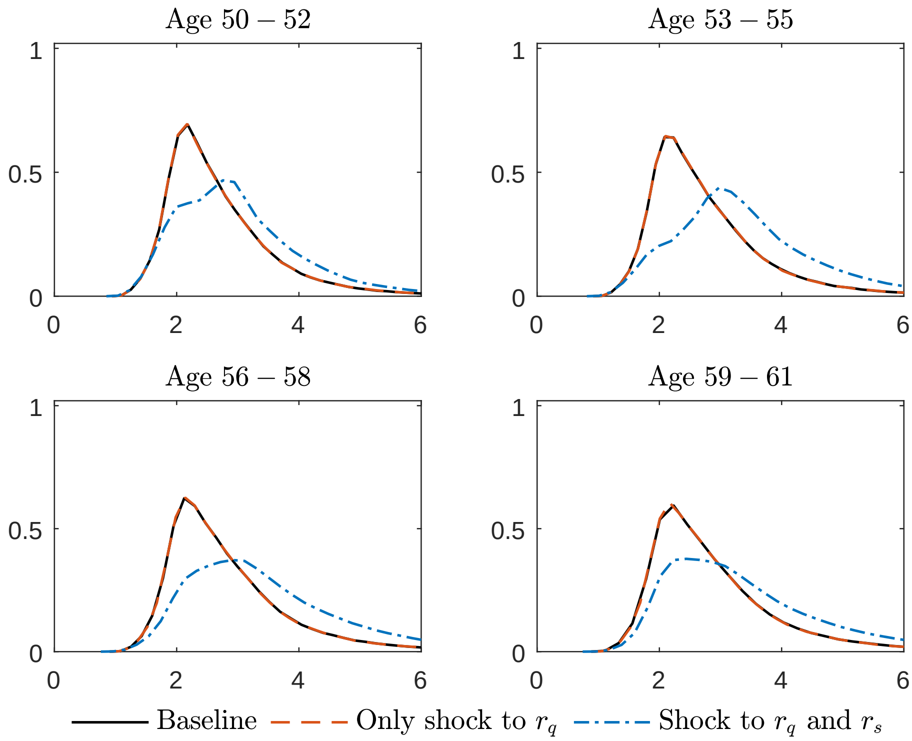

Figure A.2: Evolution of the wealth distribution following the return shocks of 2009-13 impacting at age $50-54$