Optimal Monetary Policy when Information is Market-Generated*

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July 2017

Abstract

Endogenous - i.e. market-generated - signals observed by firms have crucial implications for monetary policy. When information is endogenous, firms gather a demand signal from their market that is both real and nominal. As a result, the traditional "surprise channel" of monetary policy is absent. Instead, monetary policy works through a "signaling channel", as it affects firms' information through the demand signal. The optimal policy is then the "signaling policy", i.e. the policy that maximizes the information content of the demand signal. In our setup, the signaling policy targets a positive correlation between money supply and prices, which emphasizes the natural response of prices to real shocks. On the contrary, in the more traditional case of exogenous information, optimal monetary policy would stabilize prices as it acts through the "surprise channel". We show that the signaling policy is optimal regardless of the amount of attention that firms pay to central bank communication.

Keywords: Optimal monetary policy, information frictions, expectations, central bank communication.

JEL codes: D83, E32, E52.

*We would like to thank Philippe Bacchetta, Gaetano Gaballo, Luisa Lambertini, Leonardo Melosi, Céline Poilly, seminar participants at the University of Lausanne, participants to the T2M conference in Lausanne for helpful comments. We gratefully acknowledge financial support from he ERC Advanced Grant #269573. This research was funded by FNS research grant, Project # 100018_150068.

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1 Introduction

It is widely admitted that agents cannot perfectly observe the state of the economic fundamentals (e.g. Lucas, 1972). Given the important effects of imperfect information on economic dynamics and policies, central banks devote a lot of resources to economic analysis, forecasting and communication. However, there is growing evidence that private agents pay little attention to publicly available data.[1] At the same time, several empirical analyses show that prices do change, but mostly because of sectoral conditions, rather than aggregate conditions.[2] These two sets of observations are compatible with a world in which agents' decisions are mostly due to the attention paid to local market conditions.[3] If central bank communication has little impact on the economy, how can the central bank optimally use its information in order to influence economic outcomes? Our answer is that the central bank should affect the local conditions to which agents pay attention, in order to influence their decisions.

Using a model with monopolistically competitive firms and imperfect information on aggregate shocks, we show that endogenous - i.e. market-generated - signals observed by firms have crucial implications for monetary policy. When information is endogenous, firms gather a demand signal from their local market that is both a real and nominal signal. Because the demand signal contains nominal information, the traditional "surprise channel" of monetary policy is absent. Indeed, firms adjust their prices to offset any nominal change in aggregate demand, including those driven by monetary policy. But, because the demand signal is sensitive to monetary policy, the central bank can rely on a "signaling channel", as it affects firms' information through the demand signal. The optimal policy is then the "signaling policy", i.e. the policy that maximizes the information content of the demand signal.

In our setup, the demand signal is increasing in both nominal demand and the aggregate price. Because firms have private information on the aggregate real shocks that they use to set their individual price, the aggregate price contains valuable information, that is not fully revealed to firms through the demand signal because of nominal shocks. To maximize the information content of the demand signal, the signaling policy targets a positive correlation between nominal demand and prices. This policy therefore results in emphasizing the natural response of prices to the real shocks. For instance, consider a labor

[1] See, among others, Mankiw et al. (2003), Carroll (2003), Golosov and Lucas Jr. (2007), Andrade and Le Bihan (2013), Coibion and Gorodnichenko (2012), Coibion et al. (2015).

[2] See, among others, Bils and Klenow (2004), Boivin et al. (2009) and Klenow and Kryvtsov (2008).

[3] Mackowiak and Wiederholt (2009) explain how these facts can arise from endogenous attention.

supply shock that decreases the marginal cost. As this shock is naturally deflationary, the signaling policy is counter-cyclical. Indeed, for a given firm, a reduction in aggregate prices reduces the demand for her individual good. By observing a decline in her demand signal, she infers that other firms have decreased their price as a response to a positive aggregate supply shock. So a decline in the demand signal is good news for a firm. When the central bank gets a positive signal on the supply shock, it then implements a counter-cyclical policy that reduces nominal demand, which further reduces the demand signal. The demand signal is then a better signal of the supply shock for firms, which are then able to set prices more accurately.

This contrasts with the more traditional case of exogenous information, where optimal monetary policy stabilizes prices as it acts through the "surprise channel". As firms do not use the demand signal to offset policy-driven changes in nominal demand, the optimal policy of an informed central bank is to close the output gap by implementing a "demand management" policy, aimed at boosting demand and at directly steering the economy towards the efficient allocation. By stimulating demand, the central bank reduces the need for firms to adjust their price, which results in price stabilization.

We establish that the signaling policy is a substitute to direct communication by the central bank. In fact, the signaling policy consists in making the demand signal coincide with the signal firms would build if they had a direct access to central bank's information. This has important implications when we consider, as has been shown empirically, that firms pay little attention to public information. When firms pay limited attention to central bank communication, the signaling policy is still optimal. Indeed, in that case, by replicating the perfect communication outcome, the signaling policy improves price-setters' information, even for an arbitrarily high degree of attention.

We show that the degree of competition has important implications on the information revealed in local markets and on policy. When markets are more competitive (i.e., when the elasticity of substitution between goods is higher), the demand signal is more sensitive to the aggregate price, and thus it reveals firms' private information better. Central bank's interventions through the signaling policy are less needed. Imperfect competition is inefficient here not only because of the standard deadweight losses that is generates, but also because it prevents the revelation of aggregate information through local markets.

When the central bank receives information not only on real shocks but on nominal shocks as well, the central bank does not need to make the demand signal more responsive to them. On the contrary, it must counteract their effect by implementing a counter-cyclical policy, hence limiting the response of nominal demand to nominal shocks. This policy, contrary to the case of real shocks, results in price stabilization. So, when the central bank has information on both real and nominal shocks, it is still optimal to make the demand signal a better signal of real shocks. This result comes from a form of "divine coincidence" that is due to the demand signal. Indeed, as the demand signal is driven by both real shocks and nominal demand, improving information on real shocks also improves information on nominal demand.

Finally, our main focus is on shocks that drive efficient fluctuations. In that case, the central bank objective is to maximize the information content of endogenous variables. However, as Angeletos and Pavan (2007) stress, in the case of shocks that drive inefficient fluctuations, like mark-up shocks, more information can be detrimental to the economy. Consistently, we show in an extension that optimal policy in that case must reduce the information content of the demand signal.

Our paper confers an information role to equilibrium variables. This approach dates back to Phelps (1969) and Lucas (1972). Recent papers analyze how endogenous information affects economic outcomes, without looking at the monetary policy implications.[4] An earlier literature, including King (1982), Dotsey and King (1986) and Weiss (1980) examines how monetary policy affects the information content of prices. However, contrary to us, they focus on optimal feedback rules, as they assume that the central bank has no private information on the current state.

The analysis of optimal monetary policy has been done mostly in the context of exogenous information. In that context, the general result is that optimal monetary policy targets price stability, at least for shocks driving efficient fluctuations.[5] In this literature, all the signals received by agents are exogenous, contrary to our approach, where some signals are endogenous.[6] In this standard framework, monetary policy should accommodate these shocks and aim for price stability.[7][8] We show that endogenous signals have the

[4]See Angeletos and Werning, (2006), Amador and Weill (2010, 2012), Benhima (2014), Hellwig and Venkateswaran (2009) and Gaballo (2015)

[5]Ball et al. (2005) find this in a sticky-information à la Mankiw and Reis (2002), while Adam (2007) finds this in a rational inattention model à la Sims (2003). Paciello and Wiederholt (2014) show that, when inattention is endogenous, price stabilization also generalizes to shocks that drive inefficient fluctuations, like mark-up shocks. Lorenzoni (2010) studies optimal policy when information is dispersed and the central bank has no private information.

[6]Paciello and Wiederholt (2014) study an extension where decision-makers choose the optimal linear combination of shocks to observe. In our case, the signals observed by agents come from the market they are involved in, it is therefore not necessarily optimal, and this is what drives our results.

[7]This extends the results of the sticky-price New-Keynesian literature such as Gali (2008) and Woodford (2003), which has also established the benefits of price stability.

[8]An exception is Angeletos, Iovino and La'o (2016) who find, like us, that price stabilization is not optimal, and that policy should target a negative correlation between prices and economic activity. It is

potential to reverse the traditional results in terms of optimal policy.

Our approach is also linked to the literature on central bank's information. In our paper, we assume that the central bank receives signals that are distinct from the ones received by private agents. This is backed by empirical findings that support the idea the central bank might have some information that is not common knowledge, and that can therefore be communicated through public statements and monetary policy actions.[9] Some papers have explored theoretically the consequences of the signaling channel of monetary policy through monetary instruments, as Tang (2015), Baeryswil and Cornand (2010) or Berkelmans (2011). Our signaling channel differs from theirs as we assume that the agents' main source of information is their local market. The central bank can also choose to communicate directly its information to the public, with more or less precision.[10] We do not rule out direct central bank communication and consider it jointly with our market signaling channel.

The structure of the paper is the following. Section 2 presents our baseline model with firms and productivity shocks. In section 3 we study the equilibrium of the model. Section 4 studies optimal policy. In Section 5 we compare the results of our model with the ones that we would obtain with central bank' s communication. Section 6 presents several extensions to the model. Section 7 concludes.

2 The model

We consider a one-period model with flexible prices. There is a representative household, a continuum of firms and a central bank. The household consumes a bundle of goods Y and supplies competitively a quantity N of labor to firms. Firms, which are owned by the household, are indexed by $i \in [0,1]$. Each firm i produces a quantity Y_i of a differentiated good using a quantity N_i of labor, and sets prices monopolistically. The central bank conducts monetary policy with the goal of maximizing the household' utility.

Firms do not directly observe aggregate conditions, but infer them from private signals and from their local market conditions. The central bank, on its side, collects imperfect signals on real shocks, and defines its policy on the basis of its own information. Since we

important to note, however, that our result depends on the presence of endogenous information, while theirs originates from the assumption of both nominal and real frictions.

[9]See Romer and Romer (2000), Justiniano et al. (2012), Nakamura and Steinsson (2015) and Melosi (2013).

[10]See for example Morris and Shin (2002), Hellwig (2005), Woodford (2005), Angeletos and Pavan (2007), Amador and Weill (2010), who study the optimal level of central bank "transparency".

focus on firms' information extraction issues, we assume that the household knows all the shocks in the economy.

2.1 The household and the demand for individual goods

The utility function of the representative household depends on his consumption bundle, Y, on his labor, N, and on a labor supply shock, Z:

$$u(Y, N, Z) = \frac{Y^{1-\phi}}{1-\phi} - Z^{-1}\frac{N^{1+\eta}}{1+\eta}, \tag{1}$$

Y, the consumption bundle, is defined as $Y = \left(\int_0^1 C_i^{\frac{\varrho-1}{\varrho}} di\right)^{\frac{\varrho}{\varrho-1}}$, where C_i is the consumption of good i and $\varrho > 1$ is the elasticity of substitution between goods. $\phi > 0$ is the inverse of the elasticity of intertemporal substitution and $\eta > 0$ is the inverse of the Frisch elasticity of labor supply.

Z acts as a labor supply shifter, so we refer to it as the "supply shock".[11] As we will show, Z moves the perfect-information level of output, so it is a real shock. $z = \log(Z)$ has a Gaussian distribution with mean zero and variance σ_z^2. The budget constraint of the household is

$$\int_0^1 P_i C_i di = \int_0^1 \Pi_i di + WN \tag{2}$$

where $\int_0^1 \Pi_i di$ is the sum of profits distributed by firms, P_i is the price of good i and W is the nominal wage. Profits and labor income are then used to finance the consumption of individual goods.

The household shops the differentiated goods. He observes the prices and the quantities purchased. The individual good demand equation is then given by:

$$C_i = Y\left(\frac{P_i}{P}\right)^{-\varrho}, \tag{3}$$

where the consumer price index, P, is an average of the the individual prices:

$$P = \left(\int_0^1 P_i^{1-\varrho} di\right)^{\frac{1}{1-\varrho}}, \tag{4}$$

[11] It is a shock that moves prices and quantities in opposite directions.

2.2 Firms and price-setting

Each good i is produced and sold by firm i. More precisely, firm i produces the good using the following linear technology:

$$Y_i = N_i, \tag{5}$$

Firm i is a price-setter. She chooses P_i monopolistically in order to maximize her expected profits, $\Pi_i = P_i Y_i - W N_i$, subject to the individual good demand equation, (3), the production technology, (5), and equilibrium in the goods market, $C_i = Y_i$, $i \in [0, 1]$.

Denote by I_i the information set of firm i when she decides the price. We denote by $E_i(.) = E(.|I_i)$ the individual expectations and by $\bar{E}(.) = \int_0^1 E_i(.)di$ their cross-individual average. We denote variables in logs by lower-case letters and neglect constant terms.

The optimal price set by the individual firm i is equal to the expected nominal marginal cost, which corresponds to the expected nominal wage in our setup: $p_i = E_i(w)$. The nominal wage is equal to the households' nominal marginal rate of substitution between consumption and labor so, up to a fist-order approximation, the optimal price can be written as follows:[12]

$$p_i = \chi E_i(p) + (1 - \chi)[E_i(q) - \delta E_i(z)], \tag{6}$$

where z is the supply shock, p is the aggregate price, q is nominal aggregate demand, defined as

$$q = y + p, \tag{7}$$

and δ and χ are functions of the parameters of the model. We have

$$\begin{aligned} \delta = & \ \tfrac{1}{\eta+\phi}; \\ \chi = & \ 1 - (\eta + \phi). \end{aligned}$$

The price-setting equation (6) states that the optimal price of each good i is equal to the expected household's nominal marginal rate of substitution between consumption and labor and is hence related positively to the nominal aggregate demand q and negatively to the labor supply shock z. We refer to these terms as the *nominal* determinant and the *real* determinant of prices. If $0 < \chi < 1$, there are some strategic complementarities in price-setting. This is the case as long as $\eta + \phi < 1$. We maintain this assumption in order to be in line with the literature, although our results do not hinge on it.

The model is closed simply with a quantity equation. That is, we assume a cash-in-advance constraint that implies that nominal spending q depends on money supply and

[12] The details can be found in Appendix A.1.

on a velocity shock:

$$q = m + v, \tag{8}$$

where m is the log of money supply set by the central bank and v is money velocity. Velocity v is a gaussian i.i.d. shock with mean zero and variance σ_v^2. Nominal aggregate demand is thus partially controlled by the central bank, up to the velocity shock, v. This assumption captures the fact that there are exogenous shifts in aggregate demand that policy cannot control. Because v only affects prices under perfect information, we consider it as a nominal shock.

The demand y_i for the individual good i can also be written in log-linear form:

$$y_i = y - \varrho(p_i - p), \tag{9}$$

Equations (6)-(9), along with the approximations $\int_0^1 p_i di = p$ and $\int_0^1 y_i di = y$, compose the reduced-form log-linear model. To close the model, we still need to specify the information set of firms I_i, $i \in [0, 1]$, as well as monetary policy m.

2.3 Information Structure

The household and the firms have different information sets. We assume, without loss of generality, that the household knows all the shocks.[13] Firms do not know z and participate only to the market for good i, so they have a more limited information set.

Exogenous signals Firm i does not observe z. Instead, she observes a private exogenous signal on z:

$$z_i = z + \varepsilon_i$$

where ε_i is a gaussian i.i.d. shock with mean zero and variance σ_ε^2. It averages out in the aggregate so $\int_0^1 \varepsilon_i di = 0$.

Firms do not observe the nominal shock v either. This assumption is important as it will prevent firms from learning z from endogenous variables. In an extension, we will allow firms to observe private exogenous signals on v.

Importantly, we assume that firms cannot observe the money supply, m nor nominal aggregate demand, q, nor any other aggregate variable. This assumption hinges on the

[13] Even if the household did not directly observe all shocks, he would have all the relevant information. Indeed, the household perfectly observes the labor supply shock z, as well as the set of prices and quantities, because he participates to all markets.

idea that private agents do not pay attention to aggregate variables, but also on the fact that aggregate information is typically not *contemporaneously* available to private agents.

Market timing and demand signal We assume that the labor market opens after the goods market. In the beginning of period, each firm i sets its price p_i and receives an order y_i from the household. At the end of period, firms go on the competitive labor market and observe the nominal wage w. Therefore, when firm i sets her price p_i, she observes the demand for her good, but she does not know the nominal wage. As a result, the price is conditional on a limited information set. This assumption is crucial because observing the nominal wage would make the firms' nominal marginal cost inference problem trivial, as the firms would observe it directly.[14] It is well known that marginal costs are difficult to measure.[15] In an extension, we relax this hypothesis and we consider the case in which firms observe the marginal cost with a noise while fixing their price.

When setting her price, firm i observes her own individual demand y_i. By combining her individual demand y_i and her price p_i, the firm can easily construct an endogenous signal $\tilde{y}$ that is independent of idiosyncratic shocks:

$$\tilde{y} = y_i + \varrho p_i = q + (\varrho - 1)p, \tag{10}$$

where we used Equation (9) and the quantity Equation (8). $\tilde{y}$ can be interpreted as an adjusted measure of the demand for goods that is invariant across goods. We refer to $\tilde{y}$ as the "demand signal".

To understand, consider the case where $\varrho = 1$, which corresponds to the Cobb-Douglas case with unitary elasticity of substitution, where $\tilde{y} = y_i + p_i = q$. In this special case $\tilde{y}$ corresponds to the total spending on good i. Households consume a smaller quantity of more expensive goods, but they still spend equal amounts across goods. Thus, spending on individual goods is proportional to total spending q. The nominal demand for good i, observed by firm i, is therefore a good indicator of total nominal demand q.

On the opposite, when $\varrho \to +\infty$, goods become perfect substitutes and the endogenous

[14] As shown in Hellwig and Venkateswaran (2009).

[15] The difficulties associated to the measurement of the marginal cost have been emphasized in a different strand of literature by Rotemberg and Woodford (1999). They underline that the crucial measurement issue is to infer the marginal cost as opposed to the average cost. They also find that the ratio between prices and different measures of the marginal cost changes over time. This suggests that, on the one hand, firms have to constantly re-evaluate their marginal cost; on the other, the calculation of the marginal cost would be a very difficult task for the random firm. Other studies belonging to the same literature include Bils (1987) and Bils and Kahan (1996).

signal becomes driven exclusively by the price level, so firms observe the signal $p_i = p$.[16] Under perfect substitutability between goods, the household wants to consume the cheapest goods. The goods market becomes perfectly competitive, and prices must equalize. The price that households are willing to pay for the individual good i therefore reveals the competitors price, which is p.

In general, when ϱ is at an intermediate level, the demand signal is a combination of the state of aggregate nominal demand, q, and of the aggregate price, p, with a larger weight on prices when ϱ is large. Intuitively, for a given individual price, p_i, a higher demand y_i can be driven either by higher aggregate demand q or by a higher competitors' price p. The role of prices becomes larger as the elasticity of substitution increases.

As we will see, prices partly aggregate the firms' private information on z. Since the demand signal $\tilde{y}$ depends on the average price p, it provides information on the real shock. But $\tilde{y}$ is also driven by nominal demand q, and hence by nominal shocks, which will prevent firms from fully identifying the real shock. However, $\tilde{y}$ depends on monetary policy, m, through nominal demand q. Monetary policy can then potentially affect the information set of agents through the demand signal.[17]

Information of the central bank The monetary authority does not observe either the real shock, z, nor the nominal shock, v. Similarly to firms, it receives a noisy signal z^{cb}:

$$z^{cb} = z + \xi$$

where ξ is the central bank noise. It is a gaussian i.i.d. shock with mean zero and variance σ_ξ^2. The central bank does not observe firms' private signals, z_i.

The central bank does not have access to endogenous variables, such as prices. This assumption is made for simplicity, as it is the most convenient way to represent the central bank's imperfect information. In an extension, we assume that the central bank instead observes a noisy measure of prices. Besides, we abstract for the moment from central bank's direct communication of its signal z^{cb}, by assuming that firms do not observe z^{cb}. This assumption is relaxed in Section 5 where we allow the central bank to communicate its information.

[16] This can be seen by writing the endogenous signal as $\tilde{y}/(\varrho-1) = (y_i+p_i)/(\varrho-1)+p_i = q/(\varrho-1)+p$. With $\varrho \to +\infty$, the endogenous signal $\tilde{y}/(\varrho-1)$ converges to $p_i = p$.

[17] Note that if the firms' information were only public, firms would be able to infer p from their public signals. $\tilde{y}$ would then only reveal q. Private information is therefore crucial to generate confusion between the real and nominal determinants.

2.4 Monetary Policy

The goal of the central bank (CB) is to choose money supply, m, in order to maximize the welfare of the representative agent. In Appendix A.2, we show that this is akin to minimizing the loss function L whose arguments are the volatility of the price gap and the dispersion of individual prices:

$$L = V(p - p^*) + \Phi V(p_i - p) \tag{11}$$

with p^* the optimal price level that would hold under perfect information:

$$p^* = q - \delta z$$

This optimal price depends on the nominal aggregate demand, q, and on the perfect information output $y^* = \delta z$. Under perfect information, firms would decrease their price in response to a positive supply shock in order to generate an increase in demand. On the contrary, they would increase their price in response to an increase in nominal aggregate demand. The parameter Φ is a function of the deep parameters of the model: $\Phi = \varrho/(1-\chi)$.

Note that the price gap is tightly linked to the output gap $y - y^*$, as $y - y^* = -(p - p^*)$. Therefore, $V(y - y^*) = V(p - p^*)$. The loss function thus represents the central bank's dual goal of stabilizing the output gap and limiting price dispersion. Helping firms set their individual price at the optimal level p^* suffices to achieve that dual goal. We therefore focus our analysis on the individual price gap $p_i - p^* = p_i - p + p - p^*$ and distinguish between the price gap $p - p^*$ and the individual price deviation from the mean $p_i - p$ only when necessary.

Money supply is assumed to react linearly to the central bank signal: $m = \beta z^{cb}$. The nominal aggregate demand defined in equation (8) therefore boils down to:

$$q = \beta(z + \xi) + v = \beta z + \nu, \tag{12}$$

where $\nu = \beta\xi + v$ is the total nominal disturbance, which is composed of the central bank noise $\beta\xi$ and the nominal shock v. We assume that the central bank commits to β before the realization of the shocks.

An equilibrium is a set of quantities $\{y_i\}_{i\in[0,1]}$, and prices $\{p_i\}_{i\in[0,1]}$ such that price-setting follows (6), aggregate demand follows (7), monetary policy follows (12), $p = \int_0^1 p_i di$ and $y = \int_0^1 y_i di$, and the information set I_i of price setter i includes z_i and y_i, for all $i \in [0, 1]$.

3 Equilibrium

In this section, we study the equilibrium for a given policy parameter β. We focus especially on how monetary policy influences equilibrium outcomes.

We first present a simple version of the model. We show how the introduction of endogenous information leads us from a situation in which monetary policy can exploit the traditional "surprise channel", to a new one in which the central bank can exploit the "signaling channel" that we emphasize in this paper. In the second part of the section, we will further explore our novel mechanism by analyzing the full model.

3.1 A simple model

In this special version of the model, we assume that there are no strategic complementarities ($\chi = 0$). We also assume that ϱ equals 1. As a result, the demand signal $\tilde{y}$, described in equation (10) coincides with nominal demand q. Finally, we assume that the central bank is perfectly informed ($\xi = 0$), so that $z^{cb} = z$, and that there is no nominal shock ($\upsilon = 0$). This means that there is no nominal demand disturbance ($\nu = 0$) and that nominal demand depends only on the response of the monetary instrument to z: $q = \beta z$.

Individual price gap Under the simplifying assumption that that there are no strategic complementarities ($\chi = 0$), the equilibrium of the economy should satisfy the following equation

$$p_i - p^* = [E_i(q) - q] - \delta[E_i(z) - z], \tag{13}$$

describing the individual price gap. In the absence of strategic complementarities, the individual price gap is simply a function of the errors made by firms regarding q and z. Therefore, the distance from the optimum depends on the accuracy of agents' expectations on both q and z.

Exogenous signal only Let's first assume that the firms do not observe the demand signal. In that case, firms use their exogenous signal on the real shock, z_i, to evaluate the real and nominal components of aggregate demand: $E_i(z) = \gamma(z + \epsilon_i)$, where γ is the Bayesian weight associated to the private signal, and $E_i(q) = \beta E_i(z) = \beta\gamma(z + \epsilon_i)$. Equation (13), becomes

$$p_i - p^* = (\beta - \delta)[E_i(z) - z]$$

By setting $\beta = \delta$, the central bank can reach the first best by completely closing the price gap. In fact, by doing so, it makes the informational problem of firms irrelevant. The monetary authority nails the optimal price down to zero by setting nominal demand in response to the real shock. Namely, it sets $q = \delta z$, so that the nominal component offsets the real one in the optimal price. In practice, this policy consists in moving aggregate demand in line with the optimal supply.

Therefore, under exogenous information, monetary policy is non-neutral as it produces *real effects* through the management of nominal demand. As a result, the economy reaches the fist-best. We say that in this case monetary policy acts through the "surprise channel".

Endogenous signal and cursed firms Now, we assume that firms observe not only the exogenous signal, z_i, but also the demand signal $\tilde{y}$, that perfectly coincides with q in this simple model. Nevertheless we assume that our agents do not extract any information from q regarding the state of the real shock, i.e. they neglect the reasons why nominal demand changes. In other words, firms are *cursed* in the sense of Eyster and Rabin (2005). They do not see the link between nominal demand movements and the real shock. Firms then adjust the nominal component of their price perfectly: $E_i q = q$, but they do not use the endogenous signal to infer the real component: $E_i z = \gamma(z + \epsilon_i)$. As a result, Equation (13) becomes

$$p_i - p^* = -\delta[E_i(z) - z]$$

which is now independent of the monetary policy parameter β. As cursed firms are able to see the change in money supply, the monetary authority loses power. The reason is that firms react to a change in the supply of money by changing proportionally their prices so that money has no real effect.

In this case, monetary policy has lost the capacity to surprise agents that generated monetary non-neutrality in the absence of endogenous signal. As firms have access to information on nominal demand, monetary policy has become neutral.

Endogenous signal and rational firms We now suppose that firms are not cursed, i.e. they use the demand signal to infer z. They understand the monetary policy reaction function and that the demand signal gives information regarding the state of the real shock.

In the absence of nominal shocks and central bank noise, nominal demand perfectly reflects the real shock. As a result, when agents observe variations in q, they understand that they are due to the real shock ($q = \beta z$) and can thus infer it, as long as $\beta \neq 0$:

$E_i z = z$. As firms know both the real and the nominal components of the marginal cost, we now have $p_i = p^*$.

Here, the reason why the fist best is achieved is not that the central bank is implementing a monetary policy that perfectly offsets the real shock, as in the exogenous information case ($\beta = \delta$). Now, the first best is achieved because agents can perfectly infer the real shock, for any $\beta \neq 0$. In other words, the "surprise channel" that allowed monetary policy to produce real effects is completely absent. It is instead replaced by a new channel, a market signaling channel, which we call for short the "signaling channel". Through market variables, the central bank transmits its information to the public that is now able to behave optimally.

Discussion In this simple model, the signaling channel that appears with endogenous information is equivalent to the communication by the central bank of its policy instrument (equivalently, its signal) to the public. In that respect, it implies two things: on the one hand, it suppresses the standard role of monetary policy by making money neutral; on the other hand, it plays an informational role by supplementing the information set of firms with the central bank's information.

However, this is a simple model. In general, the demand signal is not strictly equivalent to central bank communication, since it does not exactly mirror the central bank signal. There are two main reasons for that. First, nominal shocks introduce noise in the demand signal. In our model, the nominal shock v makes q an imperfect signal of the central bank's information z^{cb}: $q = \beta z^{cb} + v$. Second, the structure of the economy shapes the demand signal. In particular, when $\varrho > 1$, $\tilde{y}$ depends not only on q but also on the price level p, as described in Equation (10). So it reflects not only the decisions of the central bank but also those of the firms. The demand signal therefore contains also the "natural" response of prices to the firms' private information.

The endogenous nature of information thus differentiates the signaling channel from the communication channel. Two questions arise naturally. The first question is: can we generalize the result that the demand signal suppresses the surprise channel of monetary policy? The second question is: given the endogenous nature of the demand signal, how can the central bank best exploit the signaling channel? The first question is analyzed in the remaining part of the section by studying price-setting in equilibrium, while the second question is the object of the next section on optimal monetary policy.

But before turning to the full model, we need to discuss the role of two additional assumptions: the presence of noise in the central bank signal (ξ) and strategic complementarities in price-setting ($\chi > 0$). The central bank noise makes the central bank signal imperfect, which prevents the central bank from reaching the first best, and introduces relevant policy trade-offs. Strategic complementarities on the opposite are not central in our setup. With strategic complementarities, firms are concerned about the price-setting behavior of others. In order to form expectations on others' price-setting, they need to form expectations about others' expectations, so they form higher-order expectations. These higher-order expectations distort the use of private information, as is well-known. While these effects are present in our setting, first-order expectations (on q and z) will be the main concern of the policy, because the central bank can use its information to help firm form more accurate expectations.

3.2 The full model

We now go back to the general case with strategic complementarities ($\chi \geq 0$). The individual price gap can now be written as

$$p_i - p^* = \chi[E_i(p) - p^*] + (1-\chi)\left\{[E_i(q) - q] - \delta[E_i(z) - z]\right\} \tag{14}$$

We also suppose the existence of central bank noise ($\sigma_\xi > 0$), nominal shocks ($\sigma_v > 0$) and that $\varrho \geq 1$. We thus have $q = \beta z^{cb} + v = \beta z + \nu$, with $\nu = \beta z + v$. In this more general case, the endogenous signal has to be determined in equilibrium.

Endogenous signal and signal extraction Following the literature on noisy rational expectations, we restrict ourselves to analyze linear equilibria. We guess that, by combining their individual signal z_i and their individual demand y_i, firms can extract an endogenous signal on z of the following form:

$$\tilde{z} = z + \kappa^{-1}\nu,$$

where κ is the elasticity of the endogenous signal to z relative to ν. We will show that our guess is verified and we will characterize the solution for κ, by deriving the equilibrium demand signal $\tilde{y}$. The endogenous signal $\tilde{z}$ is then simply a normalization of $\tilde{y}$. Similarly to what happens in Lucas' economy, firms cannot precisely understand whether changes in their individual demand are due to the nominal disturbance, represented by ν, or to a change in the real shock, represented by z.

Because $\nu = \beta\xi + v$, the precision of the endogenous signal $\tilde{z}$ is $\kappa^2(\sigma_v^2 + \beta^2\sigma_\xi^2)^{-1}$. This precision depends on β through two channels. First, κ is a function of the parameters of the model and, therefore, of the policy parameter β. Second, β inflates the contribution of the central bank noise to the nominal disturbance ν. We thus denote this precision as $P(\beta)$.

Agents use their exogenous signal z_i and the endogenous one $\tilde{z}$ in order to formulate their expectations:

$$E_i[z|\tilde{z}, z_i] = \gamma z_i + \tilde{\gamma}\tilde{z}, \tag{15}$$

where γ and $\tilde{\gamma}$ are Bayesian weights defined as a function of the precisions of the signals:

$$\begin{aligned} \gamma &= \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_z^{-2}+P(\beta)} \\ \tilde{\gamma} &= \frac{P(\beta)}{\sigma_\varepsilon^{-2}+\sigma_z^{-2}+P(\beta)}. \end{aligned} \tag{16}$$

Equilibrium endogenous signal and equilibrium price gaps In equilibrium, the resulting κ is described in the following Lemma (see proof in Appendix A.3):

Lemma 1 *For a given policy parameter β, κ is characterized in equilibrium by*

$$\kappa = \beta - \lambda. \tag{17}$$

where γ is defined in Equation (16) *and λ is given by*

$$\lambda = \frac{(\varrho - 1)(1-\chi)\gamma\delta}{1 + [\varrho(1-\chi) - 1]\gamma} \tag{18}$$

A solution for $\kappa(\beta)$ always exists and, when $\beta < 0$, it is unique.

Remember that the demand signal $\tilde{y}$ depends positively on both nominal demand and prices. κ, the total response of the signal to the real shock, can be decomposed into two terms: β, the policy-induced response of nominal demand to z, and $-\lambda$, which comes from the "natural" response of prices to z. Indeed, using the definition of the endogenous signal with Equations (12) and (17), we can decompose $\tilde{z}$ into two parts: nominal demand q and $-\lambda z$, that is due to the adjustment in prices.

$$\tilde{z} = \kappa^{-1}(q - \lambda z) \tag{19}$$

The "natural" response of the signal to z is negative because it is optimal for firms to decrease their price if they expect a positive supply shock. As a result, λ is proportional to δ, the optimal response of prices to z, and to γ, the Bayesian weight of the private signal in the firms' expectations of z. Indeed, prices reveal information to the agents through the endogenous signal because firms have a private source of information on z.

The more informative the private signal is, the larger γ, and the more informative prices are. Information is also revealed better if χ, the degree of strategic complementarities, is small, because firms will rely more on their private signal. The elasticity of substitution ϱ also plays a role, because the larger it is, the larger the degree of competition is, and the more reactive the demand signal will be to aggregate prices p.

Thanks to the endogenous signal, as described in Equation (19), firms can form expectations on nominal demand q. Crucially, the expectations on q will be related to the expectations on z, since $E(q) = \kappa\tilde{z} + \lambda E(z)$. If firms expect a high z, they expect a high nominal demand, for a given endogenous signal. This is because when they expect a high z, they expect low aggregate prices. Low prices in turn should lead to a low endogenous signal. If firms yet do not witness a decrease in the endogenous signal, then they will infer that nominal aggregate demand has increased, leaving the endogenous signal unchanged.

Importantly, this means that the expectation error on q is related to the expectation error on z: $E_i(q) - q = \lambda[E_i(z) - z]$. The more accurate the expectation on the real component of prices z, the more accurate the expectation on the nominal component q. As the demand signal is both a real and a nominal signal, firms are able to better assess the nominal component when they have better information on the real component. The distance to the optimal price, described in Equation (14), then depends only on errors about the real shock z. Indeed, as shown in Appendix A.4, we can rewrite Equation (14) as $p_i - p^* = (p - p^*) + (p_i - p)$ with:

$$\begin{aligned} p\_p^* &= (\lambda - \delta) \quad (1 + \gamma\tilde{\chi}) \quad \left[\bar{E}(z) - z\right] \\ p_i - p &= (\lambda - \delta) \quad [1 - (1 - \gamma)\tilde{\chi}] \quad \left[E_i(z) - \bar{E}(z)\right] \end{aligned} \tag{20}$$

where $\tilde{\chi} = \chi/(1 - \chi\gamma)$, γ and $\tilde{\gamma}$ are defined in Equation (16), $E_i(z)$ follows Equation (15), with $\bar{E}(z) = \int_0^1 E_i(z)di$.

The average price gap (the individual price deviation from the mean) thus depends on the average error on z (the individual deviation of the expectation of z from the mean) through two terms: $-\delta$, which is the optimal response of prices to the expectations on z, and λ, because a higher expectation on z leads to higher expectations on q. $\lambda - \delta$ therefore reflects the firms' incentive to react to their forecasts of z when setting their price. Strategic complementarities, which is reflected through a positive $\tilde{\chi}$, increase the incentives to react to public signals and lowers the incentives to react to private signals, as is well-known. They hence increase the volatility of the price gap and reduce the cross-sectional dispersion.[18]

[18]Note that in the absence of strategic complementarities ($\chi = \tilde{\chi} = 0$), the individual price gap boils

Finally, as in our simple model, monetary policy does not affect the economy through surprise. To see that, note that the monetary policy parameter, β, cannot bring the term $\lambda - \delta$ to zero. Lemma 1 implies that in equilibrium:

$$\lambda - \delta = -\frac{\delta(1-\gamma\chi)}{1+[\varrho(1-\chi)-1]\gamma},$$

As $\gamma \in [0,1]$, the absolute value of the term $|\lambda - \delta|$ is larger than $\max\{\delta/\varrho, \delta(1-\chi)\}$, which is strictly positive. Indeed, given that the demand signal depends on nominal demand, any change in nominal demand due to monetary policy is perceived by agents through their market information. As a result, they adjust their price accordingly, thus offsetting the change in nominal aggregate demand. Monetary policy is thus neutral in the traditional sense.

Our analysis shows that the surprise channel is not working. Nevertheless, as we show in the following section, monetary policy can help improve the quality of agents' information, thus exploiting the signaling channel.

4 Optimal monetary policy

The goal of the central bank is to set β in order to minimize the loss function (11). Contrary to common wisdom, optimal policy does not entail price stabilization. On the opposite, the purpose of monetary policy is to follow the natural movement of prices in order to maximize the information content of the demand signal.

The following Lemma establishes that the optimal policy maximizes the information on z contained in the endogenous signal (see proof in Appendix A.5):

Lemma 2 *Denote by β^* the β that minimizes L under the constraint* (20) *with $\kappa = \kappa(\beta)$ as defined by Equation* (17). *β^* is also the value that maximizes the precision of the endogenous signal $P(\beta)$.*

First, this Lemma shows that information is the only concern of policy. Given that the surprise channel of monetary policy disappears, the central bank can only rely on the signaling channel. We thus label optimal policy the "signaling policy". Second, it shows that only information on the *real* shock matters. As the endogenous signal is both real and nominal, improving information on the real component of prices helps firms assess the

down to $p_i - p^* = (\lambda - \delta)[E_i(z) - z]$. Higher-order expectations vanish, but first-order expectations still matter.

nominal component. This generates a form of "divine coincidence", since the policy-maker does not need to choose between improving information on one or the other components.

In what follows, we use this Lemma to characterize the optimal β. Remember that the equilibrium precision of the endogenous signal is $P(\beta) = \kappa(\beta)^2(\sigma_v^2 + \beta^2\sigma_\xi^2)^{-1}$, with $\kappa(\beta)$ defined in Equation (17). The parameter β affects the precision of the signal in opposite ways. As β goes up in absolute value, the sensitivity of the signal to the supply-side shock ($\kappa(\beta)$) rises as well, thus increasing its precision. On the other hand, the same increase in β inflates the noise of the signal. The signaling policy trades off these two effects.

The following Proposition then characterizes the signaling policy along these lines (see proof in Appendix A.6):

Proposition 1 *The equilibrium precision of the endogenous signal $P(\beta)$ is maximized for β^*, where β^* is the unique solution to*

$$\beta^* = -\frac{\sigma_v^2}{\lambda(\beta^*)\sigma_\xi^2}, \tag{21}$$

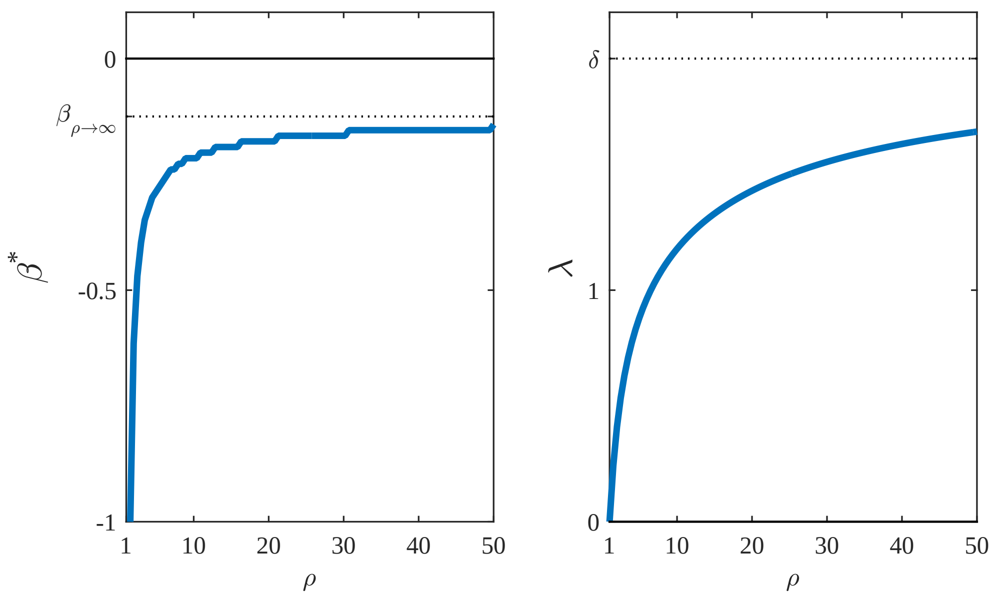
where $\lambda(\beta) = \kappa(\beta) - \beta$. Optimal policy is therefore achieved for $\beta = \beta^*$.

As implied by Equation (21), the value of β^* is negative. This derives from the fact that $\kappa = \beta - \lambda$. For $\tilde{z}$ to be a good signal, κ needs to be large in absolute value. As compared to setting a positive β, setting a negative β generates a larger κ in absolute value, while adding the same amount of noise, so a negative β is preferable.

The central bank in fact uses a counter-cyclical monetary policy to emphasize the natural effect of the real shock on the demand signal. Given that an increase in z would naturally decrease the the demand signal through lower prices, the monetary authority emphasizes that movement by reducing nominal demand, which further lowers the demand signal. In doing so, the monetary authority targets a positive correlation between money supply and the price level. This implies that the signaling policy emphasizes the response of prices to real shocks, contrary to standard policy.

Comparative statics on Equation (21) show that the absolute value of β^* is decreasing in the variance of the central bank noise σ_ξ, as a more reactive policy is more costly in terms of noise. At the same time, the absolute value of β^* is increasing in the variance of the nominal shock, σ_v^2, as the benefits of an active policy are greater when the endogenous signal is made more by nominal shocks. In the limit case where there are no nominal shocks ($\sigma_v = 0$), β^* goes to zero, as the endogenous signal fully reveals z. Finally, it is decreasing in the term λ, the "natural" elasticity of the demand signal to z. Indeed, a strong policy response is less necessary when the non-policy part of the endogenous signal is relatively more informative.

Figure 1: Optimal policy



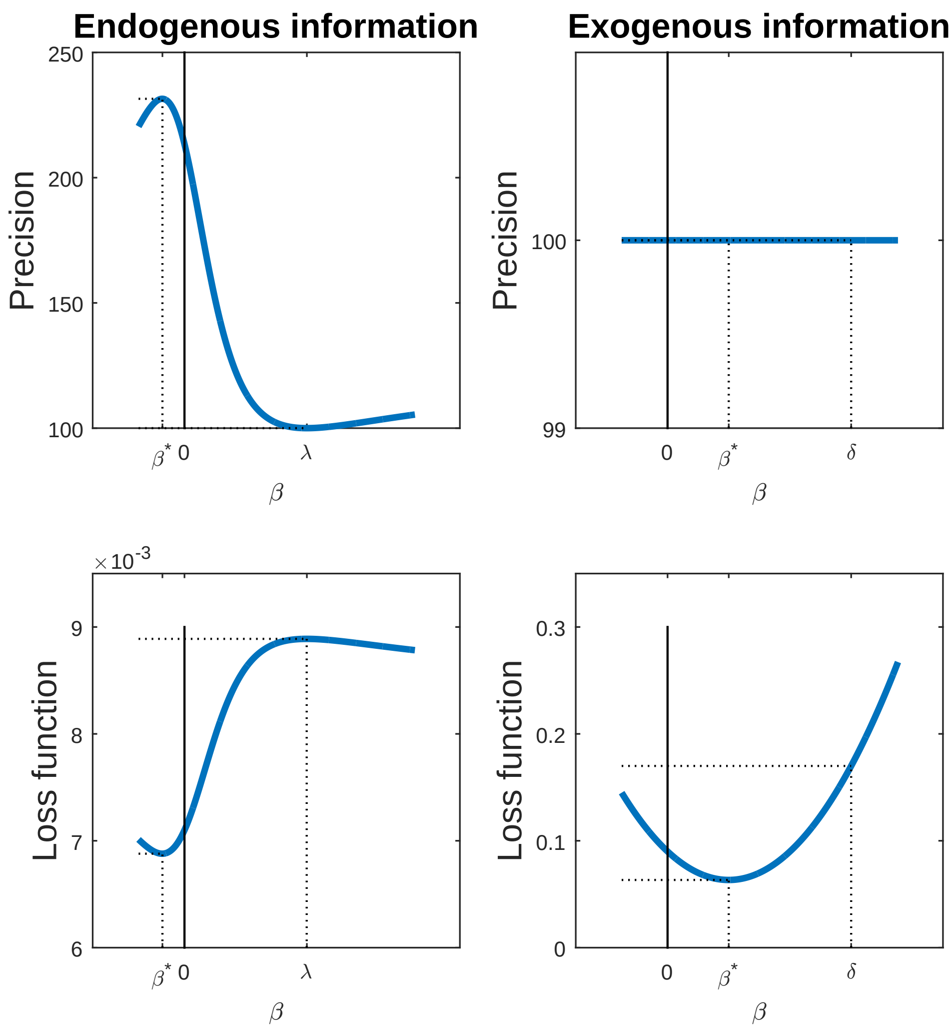
Note: We set $\phi + \eta = .5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$.

Competition and optimal policy Consider more specifically the role of ϱ (through λ), which measures the degree of competition between goods. When ϱ increases, the endogenous signal reveals p better. As p aggregates the private information of firms on z, $\tilde{y}$ becomes a better signal of z. Then introducing noise through central bank intervention is more costly, because it blurs the demand signal, which is now a better "natural" signal of z.

Figure 1 shows how the elasticity of substitution ϱ affects λ and the optimal β. We take $\phi + \eta = .5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$. As ϱ increases, the degree of competition increases and λ increases, which reflects the fact that the endogenous signal becomes a better real signal. Through prices, the private information of firms is better aggregated in the endogenous signal, and the role of policy diminishes.

This analysis shows that the negative effects of imperfect competition go beyond the standard deadweight loss. Imperfect competition also limits the amount of information revelation through demand signals, by making those signals more sensitive to nominal demand than to prices. The necessity for the central bank to intervene through a signaling policy is therefore stronger with weaker competition.

Figure 2: Optimal policy



Note: We set $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$.

Comparison with exogenous information Our results differ from the literature studying optimal monetary policy with exogenous information, which finds that it is optimal to stabilize prices through a procyclical policy. To illustrate this, we represent in Figure 2 the case with and without endogenous signal, where firms would observe only the exogenous signal z_i. We use the same parameter values as before, with $\varrho = 7$. In the exogenous information case, the loss function of the central bank is minimized for a positive policy parameter β. Here the policy-maker trades off shutting down the firms' incentives to respond to their expectation of z by setting β as close as possible to δ, with the extra noise it introduces by responding to its noisy signal, so that $0 < \beta < \delta$. Optimal policy is procyclical so that prices need to respond less to the supply shock. In that case the precision of the exogenous signal is constant and does not vary with β.

5 Central Bank Communication

So far we have assumed that the central bank did not explicitly communicate with the public. The central bank communicated only implicitly through its monetary policy. Therefore agents could observe the demand signal influenced by the action of the central bank, and react to it. Here we examine whether the possibility of explicit communication by the central bank affects our main results. In what follows, we will analyze two alternative scenarios: an ideal one, in which the central bank can completely transfer its signal to the public (perfect communication), and a more realistic one, in which the central bank communicates its signal to the public with a noise (noisy communication), to convey the idea that the understanding of the public might be affected by interpretation errors or inattention.

Moreover, by comparing our baseline model with communication, we can better understand what the signaling policy does. We show that the signaling policy (i.e., optimal implicit communication through market signals) is a substitute to explicit communication. In fact, monetary policy achieves the signaling policy by *mimicking* the information structure under explicit communication. Namely, the endogenous signal gathered by agents is similar to the compound signal that they would build under explicit communication.

This has powerful implications. In particular, under the signaling policy, explicit communication is irrelevant. One could argue that conversely, if the central bank can perfectly communicate its signal, the signaling policy is irrelevant. However, if the central bank can only communicate with noise (even with an arbitrarily low level of noise), the signaling policy is necessary to reach the optimum. In fact, the signaling policy is a way to overcome the limits of explicit communication.

5.1 Perfect Central Bank Communication

Suppose that the central bank can perfectly communicate its own signal about the real shock, $z^{cb} = z + \xi$, to the public. As a result, firms receive now two exogenous signals, z^{cb} and z_i. Additionally, they can still use the demand signal $\tilde{y}$ as a source of information.

Using z^{cb}, they can perfectly infer the monetary policy component $m = \beta z^{cb}$ of nominal demand q. We guess that they can therefore extract an endogenous signal of the form $\bar{z} = z - \lambda_c^{-1} v$, that is built using the demand signal $\tilde{y}$ and the central bank signal z^{cb}. Given this new information set, they can clearly identify the effect of money supply, m, on nominal aggregate demand, q, but they still cannot perfectly see the real shock because of the presence of the nominal shock, v.

We can show that the equilibrium λ_c is similar to λ in the baseline case with no communication, as summarized in the following Lemma (see proof in Appendix A.7):

Lemma 3 *For a given policy parameter β, λ_c is characterized in equilibrium by*

$$\lambda_c = \frac{(\varrho - 1)(1 - \chi)\gamma_c \delta}{1 + [\varrho(1 - \chi) - 1]\gamma_c}, \tag{22}$$

with

$$\gamma_c = \frac{\sigma_\varepsilon^{-2}}{\sigma_z^{-2} + \sigma_\varepsilon^{-2} + P_c}$$

where P_c is the combined precision of z^{cb} and $\bar{z}$.

The endogenous signal $\bar{z}$ therefore reflects only the "natural" reaction of the demand signal to z, and its only source of noise is the nominal shock that moves q.

Consistently with our guess, policy is irrelevant for the precision of the endogenous signal. Differently from before, the central bank cannot affect the information of the firms by using a monetary policy that is more responsive to the central bank signal, because this signal is already in firms' information set. On the other hand, similarly to what we found in the no communication case, the endogenous signal is still affected by the "natural" reaction of prices to the supply shock, z, and reveals z partly through that channel.

Even if they observe its policy component, firms still do not fully observe nominal aggregate demand q, so they rely on the demand signal as a nominal source of information, as before. The normalized demand signal can be constructed as $\tilde{z}_c = \beta z^{cb} - \lambda_c \bar{z}$. The firms can therefore take into account the nominal component of their price, as $E(q) = \kappa_c \tilde{z}_c + \lambda_c E(z)$. The equilibrium is again characterized by the price gap and individual price deviation from the mean that is similar to (20) (see proof in Appendix A.8):

$$\begin{array}{rcl} p\_p^* & = & (\lambda_c - \delta) \quad (1+\gamma_c\tilde{\chi}_c) \quad \left[\bar{E}(z) - z\right] \\ p_i - p & = & (\lambda_c - \delta) \quad [1-(1-\gamma_c)\tilde{\chi}_c] \quad \left[E_i(z) - \bar{E}(z)\right] \end{array} \tag{23}$$

where $\tilde{\chi}_c = \chi/(1-\chi\gamma_c)$. As before, firms react to their assessment of z when setting their price. $-\delta$ corresponds to the optimal response of prices to the supply shock, while λ_c describes how their expectation on z is used to correct their expectation on q.

From the expressions of the price gap and individual deviation, it appears that the only potential difference between the case with communication and the baseline case lies in the structure of information about z. In what follows, we show that, since agents benefit from an additional signal in the communication case, the precision of the information about z is higher. But, even without communication, when monetary policy is optimal, the central bank can achieve the same level of precision of information as with communication.

Equilibrium precision of information We can now compare the precision with which firms can estimate the real shock z. As already underlined, in the baseline case agents see an endogenous signal $\tilde{z}$ that is affected at the same time by the central bank noise, ξ, and by the nominal shock, v. Monetary policy directly affects the signal precision:

$$P(\beta) = \kappa(\beta)^2(\sigma_v^2 + \beta^2\sigma_\xi^2)^{-1}.$$

Instead, in the case with perfect communication, agents' information boils down to an endogenous signal ($\bar{z}$) that is independent of policy and the central bank signal (z^{cb}). As z^{cb} and $\bar{z}$ are independent, they can be combined into a new signal $z^* = E(z|z^{cb}, \bar{z})$, whose precision is given by

$$P_c = \lambda_c^2\sigma_v^{-2} + \sigma_\xi^{-2}$$

which is independent of policy.

We can prove that the precision with communication is strictly larger than without communication, the only exception being when policy is optimal: $\beta = \beta^*$. In the latter case, the precision is exactly the same. This is formalized in the following Proposition (see proof in Appendix A.9):

Proposition 2 *For all $\beta \neq \beta^*$, we have $P(\beta) < P_c$. For $\beta = \beta^*$, we have $P(\beta) = P_c$.*

The intuition behind this result is as follows. When firms share the same information as the central bank, they can combine the central bank signal z^{cb} with their endogenous signal,

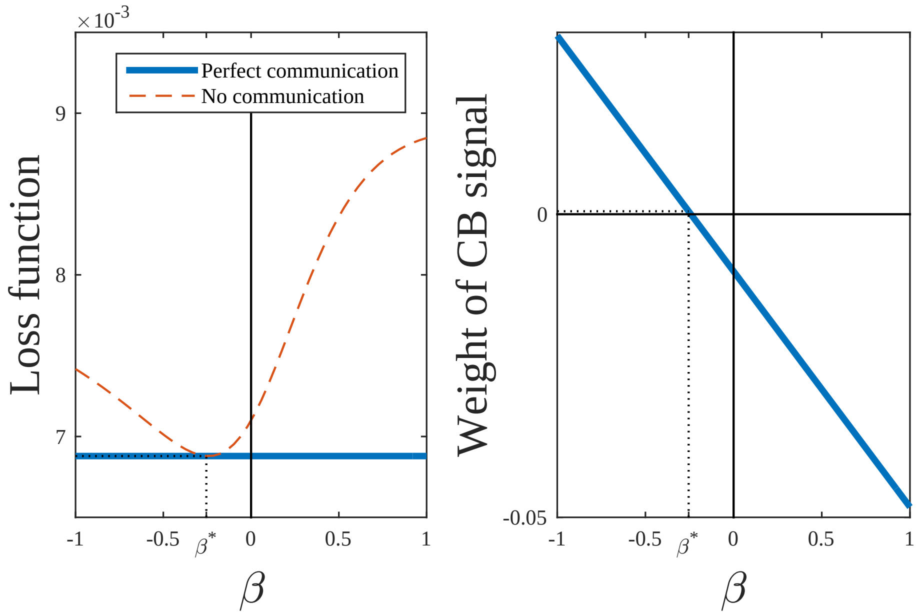
$\bar{z}$ to optimally extract information about z. To do this, they solve a trade-off between the informational content of the central bank signal and the noise it introduces in their expectations. On the opposite, when there is no communication between the central bank and the public, firms can use only their endogenous signal, $\tilde{z}$. This implies that whenever the central bank does not behave optimally, it actually increases the uncertainty faced by firms. So, in general, it is easier for firms to infer the real shock in the communication case. The only exception is when the central bank implements the signaling policy. In that case, the monetary authority itself trades-off the informational value of monetary policy with the additional noise it introduces. Therefore, the optimal endogenous signal in the no communication case is exactly equal to the signal that optimally combines the central bank signal and the endogenous signal in the communication case. In other terms, $\tilde{z}(\beta^*) = z^*$. The signaling policy is therefore a substitute to communication. This is illustrated in the left panel of Figure 3, where the loss function in the no communication case is minimized for $\beta = \beta^*$, where it becomes exactly equal to the loss function in the perfect communication case.

We can show that the optimal monetary policy is the one that makes the central bank signal irrelevant, because the endogenous signal $\tilde{z}$ is a sufficient statistics to set individual prices.[19] This is illustrated in the right panel of Figure 3, which shows that, if firms can use z^{cb} when setting their price, along with their private signal z_i and the demand signal $\tilde{y}$, the weight of z^{cb} depends on β. The weight given to z^{cb} equals zero for $\beta = \beta^*$. In that latter case, whether z^{cb} is observed or not, does not make any difference.

To understand, it is useful to consider how firms use $\tilde{y}$ and z^{cb} in their price-setting. When $\beta = 0$, $\tilde{y}$ and z^{cb} are independent signals of z, because $\tilde{y}$ does not depend on z^{cb}. $\tilde{y}$ has then a positive weight in the individual price, because it depends negatively on the supply shock, and z^{cb} has a negative weight, as shown in the figure, because it depends positively on z. When β starts to become negative, $\tilde{y}$ itself starts to depend negatively on z^{cb}, so the weight of z^{cb} has to decline, until it becomes equal to zero. By modifying β, the central bank then suppresses the necessity for firms to use z^{cb}. Hence, the optimal no-communication pricing policy is the one that does not even need communication, because the endogenous signal already summarizes all the relevant information.

[19] See Appendix A.10.

Figure 3: Optimal policy with perfect communication



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$. The weight of CB signal corresponds to α^{cb} when the optimal pricing equation is written as a function of signals: $p_i = \alpha z_i + \tilde{\alpha}\tilde{z} + \alpha^{cb} z^{cb}$.

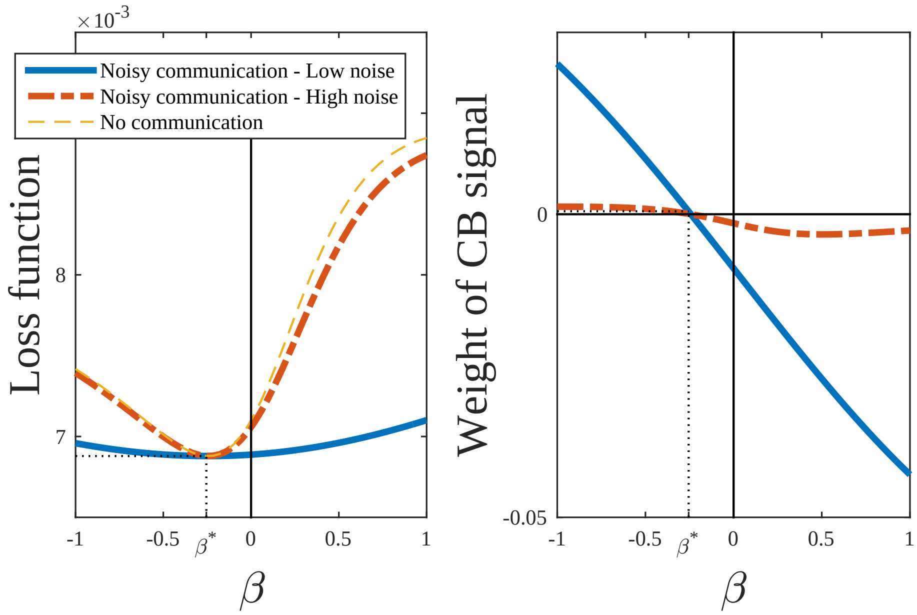
5.2 Noisy Central bank Communication

Our analysis with perfect central bank communication shows that, in terms of welfare, communication in general dominates the absence of communication, and that policy becomes irrelevant in the presence of communication. Therefore, it would suffice for the central bank to communicate its signal to achieve a better outcome, without consideration for the conduct of monetary policy. However, as we show in what follows, this result is not robust to the introduction of noise in central bank communication. In that case, the signaling policy improves firms' information. As a consequence, the signaling policy remains optimal.

We thus assume, once again, that the central bank can communicate its assessment on the state of the economy z^{cb} to the public. However, the central bank signal is processed with a cost by the agents, so that they receive the communication signal $\bar{z}_i = z + \xi + u_i$ where u_i is a gaussian i.i.d. idiosyncratic noise with mean zero and variance σ_u^2.

As illustrated in the left panel of Figure 4, the simulations show that, in the presence of noisy communication, the optimal β^* is equal to the value that holds in the absence of

Figure 4: Optimal policy with noisy communication



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$. Low noise corresponds to $\sigma_u = 0.1$. High noise corresponds to $\sigma_u = 1$. The weight of CB signal corresponds to α^{cb} when the optimal pricing equation is written as a function of signals: $p_i = \alpha z_i + \tilde{\alpha}\tilde{z} + \alpha^{cb} z^{cb}$.

communication and is hence independent of σ_u. The precision of the communication signal has therefore no effect on optimal policy. Moreover, the figure shows that the loss functions are all identical for $\beta = \beta^*$, whatever the level of communication noise. This means that, by using the signaling policy, the central bank can reach the same level of welfare as under perfect communication, even when communication is noisy. As in the absence of communication, the signaling policy makes the endogenous signal the best summary of the central bank's information and of the firms' private information, thus overcoming the potential limits of direct communication. Even with an arbitrarily low level of noise, the signaling policy is still optimal.[20]

We can further understand this point by looking at the right panel of the figure. The weight of the central bank signal is zero in the presence of noisy communication, when $\beta = \beta^*$. As we already underlined, the signaling policy makes the central bank's signal redundant. And since firms are unwilling to use of the central bank signal under perfect

[20]Note though that a lower level of noise σ_u makes the loss function less steep, which means that marginal welfare losses from deviating form β^* are lower.

communication, they are unwilling to use of a noisy version of it either.

6 Extensions

Here we consider extensions to our model. As information is at the core of our paper, most of our extensions examine alternative information sets. First, we examine how our results carry through if the demand signal is noisy, and if we allow firms to receive a noisy signal about the marginal cost. We also examine alternative information sets for the central bank. In particular, we assume that the central bank uses a noisy measure of prices and we study the implications of a more traditional price-targeting policy. We additionally examine the possibility for the central bank and firms to receive signals on the nominal (velocity) shock v. In our baseline model, we have considered "efficient" shocks, as they moved the social optimum in the same direction as the private optimum. We thus consider also "inefficient" shocks, that is, shocks that move the private optimum but not the social optimum. Finally, we take a fist pass at extending our framework to a dynamic setting.

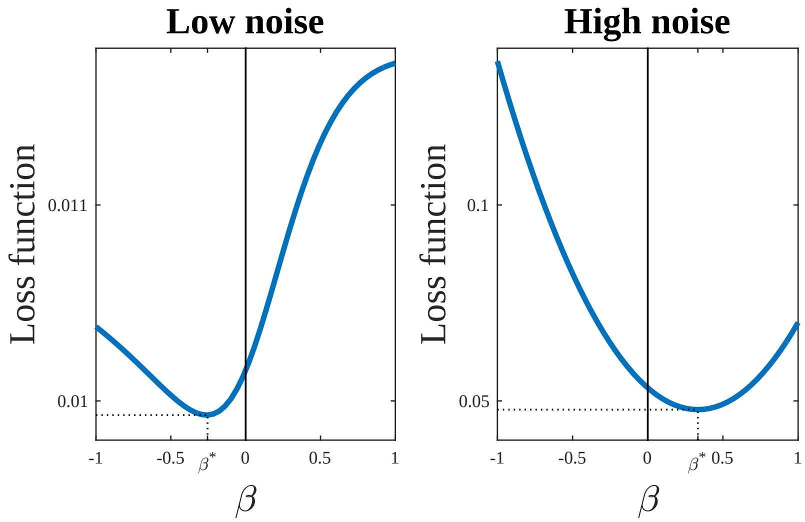
6.1 Noisy endogenous signal

In the baseline model, we assumed that the observed individual demand could be perfectly backed out to an endogenous signal that depends only on aggregate shocks. However, it is reasonable to assume that either cognitive limits, or some individual demand shocks could blur the endogenous signal. We therefore allow the individual demand y_i to be observed with an idiosyncratic noise. Agents then observe $y_i + x_i$, where x_i is an idiosyncratic gaussian i.i.d. shock with mean zero and variance σ_x^2. The endogenous signal extracted from this observation is therefore perturbed both by the demand shock ν and by the idiosyncratic noise x_i: $\tilde{z}_i = z + \kappa^{-1}\nu + \tau^{-1}x_i$, where κ and τ are endogenously determined.

The results here are simulated. Consider Figure 5, which represents the loss function with low noise in endogenous signal ($\sigma_x = 0.01$) and high noise in endogenous signal ($\sigma_x = 0.1$) as a function of β. Note that with low noise, the optimal β is negative, as in our baseline. With higher noise, optimal policy can consist in setting a positive β. Indeed, as market signals are less powerful, the central bank renounces to its signaling channel in order to make use of the more traditional surprise channel.[21]

[21] We can compare our results with Paciello and Wiederholt (2014). In a world where the central bank is perfectly informed about the economic fundamentals, they find that price stabilization and lower attention maximize welfare. On the contrary, we find that with an imperfectly informed central bank, more attention (that is to say, a more precise endogenous signal) and a policy that emphasizes the natural movement of

Figure 5: Noisy endogenous signal



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$. Low noise corresponds to $\sigma_x = 0.1$. High noise corresponds to $\sigma_x = 1$.

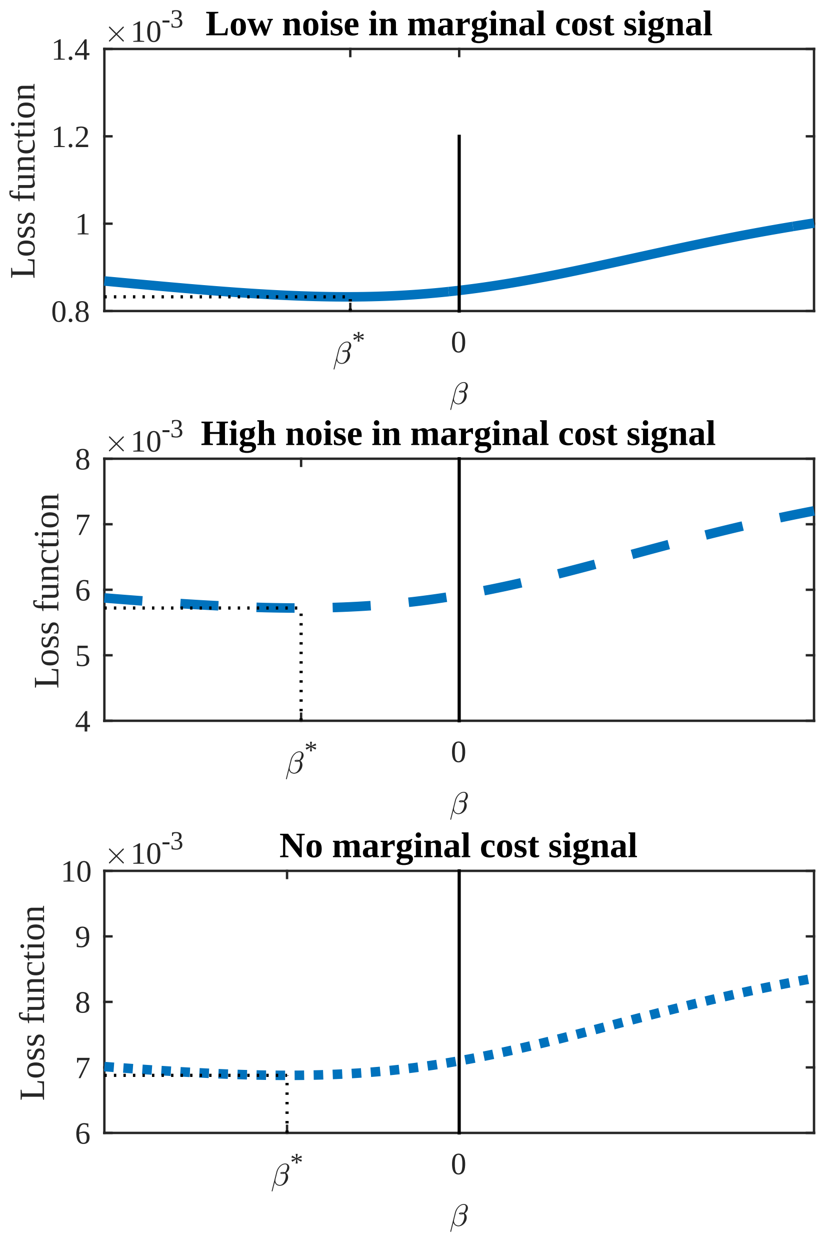
This numerical exercise suggests that, for β^* to be negative, the idiosyncratic noise associated to the endogenous signal must not be too large. This assumption is backed by the literature on rational inattention, and especially Mackowiak and Wiederholt (2009), who show that firms must pay almost all their attention to idiosyncratic shocks, as opposed to aggregate shocks, because they estimate that idiosyncratic volatility is almost one order of magnitude larger than aggregate volatility. This implies that agents must be particularly attentive to their local source of information, represented here by y_i.

6.2 Marginal cost signal

The firms' information problem is to infer their marginal cost. In our simple framework, this marginal cost corresponds to the wage, which is observed only at the end of period. If the wage were known at the price-setting stage, then the signal extraction problem of firms would become trivial, as it would be optimal to simply set $p_i = w$. As argued earlier, in our view, it is reasonable to assume that firms do not observe their marginal cost directly. However, it is also reasonable to assume that firms have some information on their marginal cost. We introduce this idea by assuming that firms observe a signal on

prices, actually lowers the loss function. This can be seen from Figure 5, where the loss function is higher when the endogenous signal is observed with higher noise.

Figure 6: Marginal cost signal



Note: We set $\varrho = 7$, $\phi+\eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$. Low noise corresponds to $\sigma_e = 0.01$. High noise corresponds to $\sigma_e = 0.1$.

the wage $w_i = w + e_i$, where e_i is an idiosyncratic noise with mean zero and standard error σ_e. We then examine how this assumption affects the equilibrium outcome and optimal policy.

This extension is solved numerically, with different levels of σ_e. The results are shown in Figure 6. For a low level of noise ($\sigma_e = 0.01$), the loss function is lower than with a high level of noise ($\sigma_e = 0.1$), and even lower than the limit case with no marginal cost signal. This is expected as firms are able to set prices more accurately when they have more precise information. However, it appears that the optimal β is still negative in all cases, which means that our signaling channel is still present. The accuracy of the marginal cost signal decreases the magnitude of the optimal β though: it appears to get closer to zero as the level of noise σ_e diminishes. This has the same effect as a greater accuracy of the private signal on z (a lower σ_ϵ). A greater accuracy of private information in general makes the endogenous signal a better signal of the real shock, which renders central bank intervention less necessary.

6.3 Price target

We have assumed so far that the central bank actions were not conditional on any endogenous information, which does not correspond to how monetary policy is implemented in practice. We assume here that instead of observing a noisy signal of the real shock, the central bank observes the price level with noise, which is a more realistic assumption. It is also a natural way to introduce central bank noise. We then examine how optimal policy would translate into a standard Taylor-type rule, where money supply reacts to its measure of the price level. We find that the signaling policy described in Section 4 is still optimal. As in our baseline model, the central bank targets a positive correlation between money supply and prices.

Denote by ξ_p the monetary policy noise, so that the central bank observes $p - \xi_p$. ξ_p has mean zero and standard error σ_{ξ_p}.

The monetary policy rule is $m = -\beta_p(p - \xi_p)$. A positive β_p would correspond to a standard price-stabilization policy. Nominal demand then follows:

$$q = -\beta_p p + \beta_p \xi_p + v = -\beta_p p + \nu_p \tag{24}$$

where $\nu_p = \beta_p \xi_p + v$ is the total demand disturbance. Now denote $\tilde{z}_p = z + \kappa_p^{-1} \nu_p$. The precision of the signal is given by $P_p(\beta_p) = [\kappa_p(\beta_p)]^2 (\sigma_v^2 + \beta_p^2 \sigma_{\xi_p}^2)^{-1}$.

The analysis remains similar to our baseline. To see this, consider the endogenous signal $\tilde{y}$:

$$\tilde{y} = q + (\varrho - 1)p = (\varrho - 1 - \beta_p)p + \beta_p \xi_p + v$$

As p respond to the real shock z, the central bank can make the endogenous signal more sensitive to it by setting a large β_p in absolute value. However, as the endogenous signal reacts positively to both prices and nominal demand, it is more efficient to make nominal demand react in the same way as prices. This is achieved by setting a negative β_p.

We show indeed (see Appendix A.11 for details), that in equilibrium,

$$\kappa_p = \frac{(1-\chi)\delta\gamma\beta_p}{1 - \gamma[\varrho(1-\chi) - 1]} - \lambda \tag{25}$$

where λ is defined as in (18). As before, κ_p, the sensitivity of the endogenous signal to z, depends on the policy-induced response of nominal demand and on the natural response of prices. The natural response is the same as before (λ), while now the reaction of the nominal demand is a function of β_p but also of the natural response of prices, because policy reacts to prices and not directly to the shocks.

Again, optimal policy maximizes the precision of the endogenous signal. We show that this is achieved by setting $\beta_p = \beta_p^*$, where

$$\beta_p^* = \frac{-\sigma_v^2}{(\varrho - 1)\sigma_{\xi_p}^2} \tag{26}$$

The optimal β_p still depends on the relative variance of v and ξ_p. The only difference with the baseline case is that it does not depend on λ but only on $\varrho - 1$, the elasticity of the endogenous signal to prices.

It is optimal to make nominal demand respond positively to the central bank's signal on prices. This gives more information on the real shock z to prise-setters, through the following mechanism. Prices react negatively to z, because firms receive private signals on z. As a result, the central bank measures a decrease in prices, and reacts by decreasing money supply. The endogenous signal then decreases through two channels. First, because of the price decrease, and second, because of the policy-induced decrease in nominal demand. This decrease in endogenous signal constitutes a positive signal on z, which leads firms to set even lower prices, etc... This policy design thus enables firms to react more accurately to the real shock z.

The optimal response of the money supply to the price level measure is therefore positive to emphasize the response of the endogenous signal to the real shock. The response of prices to a supply shock tends to be negative, so the optimal rule makes the money supply respond negatively to a supply shock, which would make monetary policy counter-cyclical, as in our baseline model.

6.4 Signal on the nominal shock

In our baseline model, we have assumed that the central bank and firms received information only on the real shock (z), but not on the nominal or velocity shock (v). We relax this assumption here, by assuming that both the central bank and firms receive noisy signals on v.

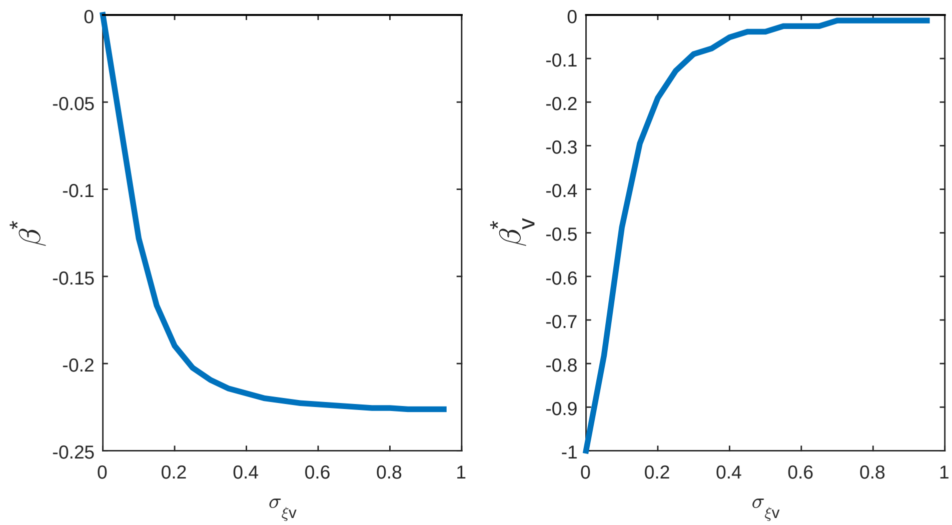
We show that, while the central bank effectively emphasizes the price response to real shocks, it stabilizes prices as a response to nominal shocks. As the nominal shock is inflationary, the central bank reduces money supply following a positive signal on the nominal shock, in order to limit its impact on the endogenous signal, which stabilizing the response of price to the nominal shock. The objective of the central bank is still to make the endogenous signal the best signal possible of the real shock. It has therefore to minimize the impact of nominal shocks on the endogenous signal. This is an implication of the divine coincidence through which a better information on the real shock implies a better information on nominal demand.

We assume that the central bank gets a noisy signal on v: $v^{cb} = v + \xi_v$, where ξ_v is a Gaussian i.i.d. shock with mean zero and standard deviation σ_{ξ_v}. Each firm also receives a private signal on v: $v_i = v + \epsilon_{vi}$, where ϵ_{vi} is an idiosyncratic Gaussian shock with mean zero and standard deviation σ_{ϵ_v}.

Figure (7) represents the optimal β and β_v under different values of σ_{ξ_v}. The optimal response to both real and nominal central bank signals is negative. As the real shock is inflationary while the nominal shock is deflationary, this implies that it is still optimal to make money supply comove positively with prices in the case of real shocks and negatively in the case of nominal shocks, whatever the level of σ_{ϵ_v} and σ_{ξ_v}.

Consider now the effect of σ_{ξ_v}. When the central bank is perfectly informed on v (σ_{ξ_v} goes to 0), β_v^* goes to -1 and β^* goes to zero: it is enough to shut down nominal shocks to make the endogenous variable reveal z to firms perfectly. As the central bank becomes less informed on v, (σ_{ξ_v} increases), then it reacts less and less to its signal on v (β_v^* is less negative) and more and more to its signal on z (β^* is more negative).

Figure 7: Optimal policy with information on the nominal shock



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set in the baseline $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$, $\sigma_{\epsilon_v} = 1$ and $\sigma_\xi = 0.2$.

6.5 "Inefficient" shocks

We introduce shocks to the elasticity of substitution between goods ϱ, which is now time-varying. Mark-ups are inversely related to ϱ, so we denote ρ the opposite of the log-deviation of ϱ from its steady state, and interpret ρ as shocks to the mark-up. By introducing mark-up shocks, we introduce shocks that drive inefficient fluctuations. In that case, the central bank does not necessarily want to improve the information of firms.

We assume that ρ is a gaussian shock with mean zero and variance σ_ρ^2. The optimal price-setting equation now becomes

$$p_i = \chi E_i p + (1 - \chi)[E_i q + \delta_\rho E_i \rho], \tag{27}$$

with $\delta_\rho = 1/[(\eta + \phi)(\varrho - 1)]$. The price responds to the firm's expectation of the mark-up shock. Notice that δ_ρ is positive, which means that a mark-up shock tends to be inflationary, contrary to the supply shock, which is deflationary.

Firms have the same information set as before, but they also receive an individual signal ρ_i on ρ: $\rho_i = \rho + \omega_i$, where ω_i is a gaussian iid idiosyncratic shock with mean zero and variance σ_ω^2.

The central bank observes a signal ρ^{cb} on the mark-up shock ρ: $\rho^{cb} = \rho + \xi_\rho$ where

ξ_ρ is a gaussian iid shock with mean zero and variance $\sigma^2_{\xi_\rho}$. It then sets money supply as $m = \beta_\rho \rho^{cb}$, so that the nominal demand becomes

$$q = \beta_\rho \rho + \nu_\rho,$$

with $\nu_\rho = v + \beta_\rho \xi_\rho$.

All the analysis of Section 3 holds, except for the definition of the price gap $p - p^*$ and for Proposition 1 which defined optimal policy. Indeed, since mark-up shocks drive inefficient output fluctuations, the socially optimal price is $p^* = q$. The spread between the realized and optimal price is then

$$p_i - p^* = \chi[E_i(p) - p^*] + (1 - \chi)\left\{[E_i(q) - q] + \delta_\rho[E_i(\rho) - \rho]\right\} + \delta_\rho \rho \tag{28}$$

As before, the expectation errors can be reduced by making the endogenous signal as informative as possible. However, this would not be necessarily optimal here because it would make agents respond better to mark-up shocks, which would drive inefficient fluctuations. Indeed, the price gap would still be equal to $\delta_\rho \rho$ if the firms made no expectation errors. It could then be optimal for the central bank to make the endogenous signal less precise.

In fact, the optimal monetary policy *minimizes* the precision. The optimal policy is then given by β^*_ρ defined as follows (the proof is available in Appendix A.12):

$$\beta^*_\rho = -\lambda_\rho(\beta^*_\rho) = \frac{-(\varrho - 1)(1 - \chi)\delta_\rho \sigma_\omega^{-2}}{\sigma_\rho^{-2} + \varrho(1 - \chi)\sigma_\omega^{-2}} \tag{29}$$

Now, the purpose of the central bank is to reduce the information content of the endogenous signal by counteracting the effect of mark-up shocks on prices. Since a mark-up shock is inflationary, monetary policy is restrictive following a mark-up shock, so that the endogenous signal does not reveal anything about that shock to the agents. This contributes to lower the responsiveness of output to the inefficient mark-up shocks.[22]

[22] Our results are consistent with Angeletos and Pavan (2007) and Angeletos et al. (2016). They find that when the business cycle is driven by non-distortionary forces (e.g., productivity shocks), welfare always increases with information's precision. When the business cycle is driven by distortionary forces (e.g., mark-up shocks), welfare decreases with information.

6.6 Dynamic extension

Here we examine how our simple static framework can be extended to a dynamic one. The purpose is not to provide a full-fledged dynamic extension, but to show how such a dynamic model could be developed. We therefore present the simplest dynamic extension possible, and show that it can be mapped into our simple static framework, with some slight changes.

We keep the same structure as in our baseline model, and consider now that time is infinite and discreet. We also now focus on a cashless economy, as defined by Woodford (2003), where monetary policy is defined in terms of nominal interest rate, and money is not introduced explicitly.

The household is infinitely lived and has the following lifetime utility:

$$U_t = \sum_{s=0}^{\infty} \beta^s u(Y_{t+s}, N_{t+s}, Z_{t+s})$$

where u, Y, N and Z are defined as in (1). The household has the same budget constraint (2) as before, except that she also holds nominal bonds B that yield the nominal interest rate i. We also introduce transfers T from the government. We thus have

$$\int_0^1 P_{it} C_{it} di + B_t = \int_0^1 \Pi_{it} di + W_t N_t + (1 + i_{t-1}) B_{t-1} + T_t$$

The budget constraint of the government being $B_{t+1} = (1 + i_t)B_t + T_t$, the resource constraint of the economy boils down to Equation (2).

The central bank sets the nominal policy rate i_t^{cb}, and the effective interest rate faced by the household is then

$$i_t = i_t^{cb} - v_t \tag{30}$$

v_t is a nominal interest rate shock that is not under the control of the central bank. It plays the same role as the velocity shock in our baseline model.

For simplicity, we assume that $z = \log(Z)$ and v are i.i.d. shocks and follow the same distribution as before. Regarding information, all past shocks and past variables are known to all agents. The dynamic problem then reduces to a repeated static problem, as in our baseline. The information about the current realization of shocks follows the same structure as in our baseline model. All our key equations hold, especially the pricing equation (6). The endogenous signal is still $\tilde{y} = y + \varrho p$. We define nominal spending as before $q = y + p$, so that $\tilde{y} = q + (\varrho - 1)p$. The monetary policy rule is now defined as

$$i_t^{cb} = \beta z^{cb} \tag{31}$$

The only difference with our static model is the way nominal demand q is determined. From the model's Euler equation, we can show that

$$(\phi y_t + p_t) = E_t(\phi y_{t+1} + p_{t+1}) - i_t$$

where $E_t(.)$ represents the household's expectations. Since all our shocks are i.i.d., $E_t(y_{t+1}) = E_t(p_{t+1}) = 0$, which enables us to write, using $q = y + p$, and the monetary policy rule (31), along with the interest rate equation (30)

$$q_t = \frac{1}{\phi}(\beta z^{cb} + v_t) - \frac{1-\phi}{\phi} p_t = \frac{1}{\phi}(\beta z + \nu_t) - \frac{1-\phi}{\phi} p_t \tag{32}$$

where $\nu = \beta\xi + v$ as before. This equation is the counterpart of Equation (12). Nominal demand is related to the policy rule through the elasticity of intertemporal substitution $1/\phi$. Importantly, nominal demand does not depend on policy only. The price now plays a role as well, as a higher price today means a higher expected real interest rate, which has a negative impact on demand.

We can show that the optimal β is still the one that maximizes the precision of the endogenous signal (see Appendix A.13). It is now defined as follows:

$$\beta^* = -\frac{\sigma_v^2}{\lambda \sigma_\xi^2} \tag{33}$$

where λ is given by

$$\lambda = \frac{(\phi\varrho - 1)(1-\chi)\gamma\delta}{1 + [\phi\varrho(1-\chi) - 1]\gamma} \tag{34}$$

Notice that now, the sign of λ can be negative, depending on the sign of $\phi\varrho - 1$. This comes from the fact that the effect of aggregate prices on the endogenous signal is now ambiguous. On the one hand, higher prices increase the demand for the individual good, for a given level of nominal demand. This effect was present in the static model and is governed by the elasticity of substitution ϱ. However, now higher prices decrease nominal demand, which decreases the demand for the individual good. This effect was absent in the static model and is governed by the elasticity of intertemporal substitution $1/\phi$. When $\phi\varrho < 1$, the latter effect dominates, which is reflected in a negative λ.

In that case, it would be optimal to set a positive β. Indeed, a positive supply shock, by decreasing prices, will have a positive effect on the endogenous signal. An increase in the endogenous signal is therefore good news about z for firms. When the central bank itself gets a good news about z, it can reinforce the information content of the endogenous signal by decreasing the nominal interest rate and hence further stimulating nominal demand, which will reinforce the positive effect of z on the endogenous signal.

Can $\phi\varrho$ be lower than 1 for realistic parameter values? The admissible values for $1/\phi$ go from 0.1 to 2.[23] The value for the elasticity of substitution between goods ϱ has been estimated in the range of 1.5-10, where lower values correspond to estimations on aggregate data and higher values to estimations on sectoral data.[24] Whereas we cannot exclude this case completely, $\phi\varrho$ is lower than 1 only for extremely low values of ϱ, and extremely large values of $1/\phi$. Our main predictions are therefore most likely to carry through in this simple dynamic framework.

Discussion How could the dynamic analysis be extended further and how would that change our results? First, more persistence in shocks could be added. With more persistent shocks, and under the assumption that past shocks are not known, it would be relevant to make the policy rate depend on past signals as well, which would give rise to some interest rate smoothing.

Second, the structure of the model could be enriched. In particular, what appears to be crucial is the structure of the endogenous signal, and especially, how that signal depends respectively on aggregate prices and on policy-related demand. In our baseline model, aggregate prices affect positively the endogenous signal. Since the real shock has a negative effect on prices, this led to a counter-cyclical monetary policy. If aggregate prices affected the endogenous signal negatively, then the signaling policy would be pro-cyclical. This result could be obtained if prices had a sufficiently negative effect on aggregate demand – and hence on the individual demand for good. The introduction of a Fisherian debt-deflation mechanism, for instance, because it makes aggregate demand react more negatively to the price level, could make a pro-cyclical policy a better way to reveal information to agents.

[23] See for instance Hall (1988), Barro (2009), and, for a meta-analysis, Havranek et al. (2015).

[24] See Imbs and Méjean (2015).

7 Conclusion

In this paper we show that optimal monetary policy is deeply affected by the assumptions concerning the information structure of the economy. In more traditional settings with exogenous information, the central bank uses its policy to directly influence the economic outcomes. In that case, monetary policy uses the "surprise channel" to produce real effects. When signals comes from local markets, the central bank cannot exploit the surprise channel. Its policy actions are reflected in the market variables and thus anticipated by the agents. This implies that the central bank has to renounce to that channel. Nevertheless it can activate the "signaling channel", thus influencing the endogenous signals observed by agents. Through endogenous signals, it can maximize agents' ability to infer the state of the fundamentals. This constitutes what we called the signaling policy.

In our model with endogenous information, we show that when a positive supply shock hits the economy, the signaling policy must be conter-cyclical, which emphasizes the natural movement in prices. The assumption of exogenous information would instead lead to price stabilization. While we show that the signaling policy can starkly differ from standard policy recommendations, our dynamic extension suggests though that the precise shape of the signaling policy can depend more generally on the structure of the economy and on parameters. An avenue of research will consist in evaluating the signaling policy in richer environments.

References

Adam, Klaus. 2007. "Optimal monetary policy with imperfect common knowledge", *Journal of Monetary Economics*, Volume 54, Issue 2, Pages 267-301

Amador, Manuel and Pierre-Olivier Weill. 2010. "Learning from Prices: Public Communication and Welfare", *Journal of Political Economy*, vol. 118, no. 5.

Amador, Manuel and Pierre-Olivier Weill. 2012. "Learning from private and public observations of others' actions," *Journal of Economic Theory*, Elsevier, vol. 147(3), pages 910-940.

Andrade, Philippe, and Herve Le Bihan. 2013. "Inattentive Professional Forecasters." *Journal of Monetary Economics*, 60(8), pages 967-982.

Angeletos, George-Marios and Jenifer La'o. 2013, "Optimal Monetary Policy with Informational Frictions," mimeo.

Angeletos, George-Marios and Jenifer La'o. 2008. "Real Rigidity, Nominal Rigidity, and the Social Value of Information," *American Economic Review* 106:1.

Angeletos, George-Marios and Alessandro Pavan. 2007. "Socially Optimal Coordination: Characterization and Policy Implications," *Journal of the European Economic Association*, MIT Press, vol. 5(2-3), pages 585-593, 04-05.

Angeletos, George-Marios, and Ivàn Werning. 2006. "Crises and Prices: Information Aggregation, Multiplicity, and Volatility." *American Economic Review*, 96(5): 1720-1736.

Baeriswyl, Romain and Camille Cornand. 2010, "The signaling role of policy actions", *Journal of Monetary Economics*, Volume 57, Issue 6, September 2010, Pages 682-695.

Ball, Laurence, Gregory Mankiw, N. and Ricardo Reis. 2005. "Monetary policy for inattentive economies," *Journal of Monetary Economics*, Elsevier, vol. 52(4), pages 703-725, May.

Benhima, Kenza. 2014. "Booms and Busts with Dispersed Information," mimeo.

Berkelmans, Leon. 2011. "Imperfect information, multiple shocks, and policy's signaling role", *Journal of Monetary Economics*, Volume 58, Issue 4, Pages 373-386.

Bils, Mark. 1987. "The Cyclical Behavior of Marginal Cost and Price,"*The American Economic Review* Volume 77, Issue 5, Pages 838-855.

Bils, Mark and James A. Kahn. 2000. "What Inventory Behavior Tells Us about Business Cycles,"*American Economic Review* Volume 90, Issue 3.

Bils, Mark and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy* Volume 112, Issue 5.

Boivin, Jean, Marc P. Giannoni and Ilian Mihov. 2009. "Sticky Prices and Monetary Policy: Evidence from Disaggregated US Data,"*American Economic Review* Volume 99, Issue 1.

Carroll, Christopher D. 2003. "Macroeconomic Expectations of Households and Professional Forecasters." *Quarterly Journal of Economics* 118, 269-298.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012. "What Can Survey Forecasts Tell Us About Informational Rigidities?" *Journal of Political Economy* 120, 116-159.

Coibion, Olivier, Yuriy Gorodnichenko and Saten Kumar. 2015. "How Do Firms Form their Expectations? New Survey Evidence." mimeo.

Dotsey, Michael, and Robert G. King. 1986. "Informational Implications of Interest Rate Rules". *The American Economic Review* 76.1: 33-42.

Eyster, Erik and Matthew Rabin. 2005. "Cursed Equilibrium". *Econometrica*, 73. 5: 1623-1672.

Gaballo, Gaetano. 2015. "Price Dispersion, Private Uncertainty and Endogenous Nominal Rigidities," mimeo.

Galì, Jordi. 2008. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*, Second Edition, Princeton Press.

Golosov, Mikhail and Robert E. Lucas Jr. 2007. "What Can Survey Forecasts Tell Us About Informational Rigidities?" *Journal of Political Economy* 115, 21.

Hellwig, Christian. 2005. "Heterogeneous Information and the Benefits of Public Information Disclosures (October 2005)," UCLA Economics Online Papers 283, UCLA Department of Economics.

Hellwig, Christian and Venky Venkateswaran. 2009. "Setting the right prices for the wrong reasons," *Journal of Monetary Economics*, Volume 56, Supplement, 15 October 2009, Pages S57-S77.

Justiniano, Alejandro, Chalres L. Evans, Jeffrey R. Campbell and Jonas D.M. Fisher. 2012. "Macroeconomic Effects of FOMC Forward Guidance", *Brookings Papers on Economic Activity*, Spring 2012.

King, Robert G.. 1982. "Monetary Policy and the Information Content of Prices". *Journal of Political Economy* 90.2 (1982): 247-279.

Klenow, Peter J. and OlekSiy Krystov. 2008. "State-Dependent or Time-Dependent Pricing: Does It Matter For Recent U.S. Inflation?", *The Quarterly Journal of Economics*, Vol. 123, Issue 3.

Lorenzoni, Guido. 2010. "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information," *The Review of Economic Studies*, 77(1), P.305-338.

Lucas, R.E.. 1972. "Expectations and the neutrality of money". *Journal of Economic Theory* 4, 103-124.

Mackowiak, Bartosz, and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review*, 99(3): 769-803.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *The Quarterly Journal of Economics*, Vol. 117, Issue 4, P.1295-1328.

Mankiw, N. Gregory, Ricardo Reis and Justin Wolfers. 2003. "Disagreement about Inflation Expectations." *NBER Macroeconomics Annual 2003*, Cambridge, MIT Press, 18: 209-248, 2004.

Melosi, Leonardo. 2013. "Signaling Effects of Monetary Policy," PIER Working Paper Archive 13-029, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.

Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information ." *American Economic Review*, 92(5): 1521-1534.

Nakamura, Emi and Jón Steinsson. 2015. "High Frequency Identification of Monetary Non-Neutrality", mimeo.

Paciello, Luigi and Mirko Wiederholt. 2014. "Exogenous Information, Endogenous Information, and Optimal Monetary Policy," *The Review of Economic Studies*, 81(1), P.356-388.

Phelps, Edmund S. 1969. "The New Microeconomics in Inflation and Employment Theory," *American Economic Review*, American Economic Association, vol. 59(2), pages 147-60, May.

Romer, Christina D., and David H. Romer. 2000. "Federal Reserve Information and the Behavior of Interest Rates." *American Economic Review*, 90(3): 429-457.

Rotemberg, Julio J., and Michael Woodford. "The Cyclical Behavior of Prices and Costs." In *Handbook of Macroeconomics*, edited by J. B. Taylor and M. Woodford. North Holland: Elsevier Science, 1999.

Sims, Christopher A.. 2003. "Implications of rational inattention," *Journal of Monetary Economics*, Elsevier, vol. 50(3), pages 665-690, April.

Tang, Jenny. 2015. "Uncertainty and the Signaling Channel of Monetary Policy", Working Paper No 15-8, Boston Fed.

Weiss, Laurence. 1980. "The Role for Active Monetary Policy in a Rational Expectations Model". *Journal of Political Economy* 88.2 (1980): 221-233.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Woodford, Michael. 2005. "Central bank communication and policy effectiveness," *Proceedings - Economic Policy Symposium - Jackson Hole*, Federal Reserve Bank of Kansas City, issue Aug, pages 399-474.

A Proofs

A.1 The Price-setting Equation

The profits of firm i are $\Pi_i = P_i Y_i - W N_i$. Using the individual good demand equation, (3), the production technology, (5), we can write these profits as

$$\Pi_i = (P_i - W) Y \left(\frac{P_i}{P}\right)^{-\varrho}$$

The optimality condition with respect to P_i, resulting from the model specified in section 2 is then

$$E_i \left\{ Y \left(\frac{P_i}{P}\right)^{-\varrho} \right\} = E_i \left\{ \varrho \frac{(P_i - W)Y}{P} \left(\frac{P_i}{P}\right)^{-(\varrho+1)} \right\}. \tag{35}$$

The log-linear version of equation (35) is:

$$p_i = E_i(w).$$

where we discard constant terms.

In order to find w, we use the household's optimality conditions and equation (3)

$$\frac{W}{P} = \frac{N^\eta}{ZY^{-\phi}},$$

which yields in log-linear terms:

$$w = p + \eta n + \phi y - z.$$

Using the approximation $y = \int_0^1 y_i di$, and $n = \int_0^1 n_i di$, we get $n = y$ as $y_i = n_i$, which implies that the individual price is set as follows:

$$p_i = E_i(p) + (\eta + \phi)E_i(y) - E_i(z).$$

Finally, replacing y with $q - p$, we obtain the price-setting equation (6).

A.2 The Welfare Approximation

The efficient level of output is defined as the one that emerges under a social planner in the symmetric equilibrium, where $P_i = P$, $W = P$ and $Y_i = Y$ for all i. In that case, the marginal rate of substitution between labor and consumption is equal to the marginal product of labor. With $N = Y$, this gives

$$Y^{\phi+\eta} = Z$$

that in logs becomes

$$y = \frac{1}{\phi + \eta} z. \tag{36}$$

Efficient output is therefore $y^* = \delta z$. Besides, (36) should be valid in the steady state as well so $\bar{y} = \frac{1}{\phi+\eta}\bar{z}$.

Using the full employment condition and the production function for each individual good i, we can rewrite the utility function as

$$u(Y, N, Z) = \frac{e^{(1-\phi)y}}{1 - \phi} - e^{-z} \frac{\left(\int_0^1 e^{y_i} di\right)^{1+\eta}}{1 + \eta}. \tag{37}$$

Note that $y_i = y + y_i - y$. The second-order approximation of the utility function is then

$$\begin{aligned} u(Y,N,Z) \;\approx\;& e^{(1-\phi)\bar{y}}\left(y + \tfrac{1-\phi}{2}y^2\right) \\ & -e^{(1+\eta)\bar{y}-\bar{z}}\left(y + \tfrac{1+\eta}{2}y^2 + \int_0^1 (y_i - y)di + \int_0^1 (y_i-y)^2 di - zy - z\int_0^1 (y_i - y)di\right) \\ & +t.i.p.,, \end{aligned} \tag{38}$$

where $t.i.p.$ stands for "terms that are independent of policy" and includes the terms that are completely exogenous, like productivity.

Defining the cross sectional mean as $E_i(y_i) = \int_0^1 y_i di$, the cross sectional variance as $Var_i(y_i - y) = \int_0^1 (y_i - y)^2 di$, and taking into account that $(1-\phi)\bar{y} = (1+\eta)\bar{y} - \bar{z}$, Equation (38) becomes

$$u(Y,N,Z) \approx e^{(1-\phi)\bar{y}}\left(-\frac{\eta+\phi}{2}y^2 - [E_i(y_i) - y] - Var_i(y_i - y) + zy + zVar_i(y_i - y)\right) + t.i.p.,, \tag{39}$$

Using the second-order approximation of the consumption bundle

$$y = E_i(y_i) + \frac{1-\varrho^{-1}}{2}Var_i(y_i - y) \tag{40}$$

we can rewrite the approximation in (39) as

$$\begin{aligned} u(Y,N,Z) \;&\approx\; -e^{(1-\phi)\bar{y}}\left(\frac{\eta+\phi}{2}y^2 - zy - \frac{1}{2\varrho}Var_i(y_i - y) - zVar_i(y_i - y)\right) + t.i.p., \\ &\approx\; -e^{(1-\phi)\bar{y}}\left(\frac{\eta+\phi}{2}\left[y - \frac{1}{\eta+\phi}z\right]^2 - \frac{1}{2\varrho}Var_i(y_i - y) - zVar_i(y_i - y)\right) + t.i.p. \end{aligned}$$

Given that $\delta = 1/(\phi + \eta)$ and $\chi = 1 - (\eta + \phi)$, this becomes

$$u(Y,N,Z) \approx -e^{(1-\phi)\bar{y}}\frac{\eta+\phi}{2}\left([y - \delta z]^2 - \frac{1}{\varrho(1-\chi)}Var_i(y_i - y) - 2\delta z Var_i(y_i - y)\right) + t.i.p., \tag{41}$$

Dropping the proportionality factor and substituting in the second term the full information production $y^* = \delta z$, gives:

$$\begin{aligned} -Eu(Y,N,Z) \;&\approx\; E(y - y^*)^2 + \tfrac{1}{\varrho(1-\chi)}Var_i(y_i - y) + t.i.p. \\ &\approx\; Var(y - y^*) + \tfrac{1}{\varrho(1-\chi)}Var_i(y_i - y) + t.i.p., \end{aligned} \tag{42}$$

Finally, note that, by defining $p^* = q - \delta z$ and by using the aggregate demand equation (7), we get $y - y^* = -(p - p^*)$. Besides, the individual demand equation (9) yields $y_i - y = -\varrho(p_i - p)$, so (42) rewrites as

$$-Eu(Y, N, Z) \approx Var(p - p^*) + \frac{\varrho}{(1-\chi)} Var_i(p_i - p) + t.i.p.$$

Then simply define $\Phi = \varrho/(1-\chi)$ to obtain the central bank loss function (11).

A.3 Proof of Lemma 1

We first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\tilde{z} = z + \kappa^{-1}\nu$, then characterize κ, and finally study existence and unicity of the solution.

Solving the pricing equation Replacing the monetary policy rule (12) in the price-setting equation (6), we obtain

$$\begin{aligned} p_i &= \chi E_i(p) + (1-\chi)E_i[(\beta - \delta)z + \nu] \\ &= \chi E_i(p) + (1-\chi)E_i[(\beta - \delta - \kappa)z + \kappa\tilde{z}] \end{aligned} \tag{43}$$

We make the following guess: $p_i = \alpha z_i + \tilde{\alpha}\tilde{z}$. This yields $p = \alpha z + \tilde{\alpha}\tilde{z}$. Taking expectations of p and z and replacing in (43), we get

$$\begin{aligned} p_i \;=\; & \Big\{\chi\alpha\gamma \quad +(1-\chi)(\beta-\delta-\kappa)\gamma \Big\} z_i \\ + \; & \Big\{\chi\Big[\alpha\tilde{\gamma} + \tilde{\alpha}\Big] \quad +(1-\chi)\Big[(\beta-\delta-\kappa)\tilde{\gamma} + \kappa\Big] \Big\} \tilde{z} \end{aligned}$$

After identifying the coefficients and re-arranging we find

$$\alpha = \frac{(1-\chi)(\beta-\delta-\kappa)\gamma}{1-\chi\gamma} \qquad\qquad \tilde{\alpha} = \kappa + \frac{(\beta-\delta-\kappa)\tilde{\gamma}}{1-\chi\gamma} \tag{44}$$

Characterization Firms observe the adjusted demand $\tilde{y} = y + \varrho p = q + (\varrho - 1)p = \beta z + \nu + (\varrho - 1)p = \kappa\tilde{z} + (\beta - \kappa)z + (\varrho - 1)p$. Using our guess-and-verify solution for p_i, we infer that $p = \alpha z + \tilde{\alpha}\tilde{z}$ and then replace in $\tilde{y}$, so we can write $\tilde{y} = [\kappa\tilde{z} + (\varrho-1)\tilde{\alpha}] + [\beta - \kappa + (\varrho - 1)\alpha]z$.

$\tilde{y}$ depends linearly on z and ν. For our guess to be true, we need that $\tilde{y} = f\tilde{z}$ for some constant f. Identifying the coefficients, we get

$$\beta - \kappa + (\varrho - 1)\alpha = 0$$

replacing α using (44) and rearranging, we get (17).

Existence and unicity Equation (17) can be rewritten as $X(\kappa) = \beta$ with

$$X(\kappa) = \kappa + \frac{(\varrho - 1)(1-\chi)\sigma_\varepsilon^{-2}\delta}{\sigma_z^{-2} + \kappa^2\sigma_\nu^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{45}$$

X is continuous on $\mathbb{R}$ with values between $-\infty$ and $+\infty$, so a solution $\kappa(\beta)$ to (17) exists.

Suppose $\beta < 0$. According to (51), $\kappa < X(\kappa)$ as $\delta > 0$, so necessarily a solution $\kappa(\beta) < \beta < 0$. For the solution to be unique, it is then sufficient that X is strictly increasing for all $\kappa < 0$. X is continuously differentiable with

$$X'(\kappa) \quad = \quad 1 - \frac{2\kappa(\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\sigma_\nu^{-2}\delta}{[\sigma_z^{-2}+\kappa^2\sigma_\nu^{-2}+\varrho(1-\chi)\sigma_\varepsilon^{-2}]^2}$$

This is positive if $\kappa < 0$, as $\delta > 0$. So X is strictly increasing in κ for $\kappa \in \mathbb{R}-$. Therefore, for $\beta < 0$, the solution $\kappa(\beta)$ of $X(\kappa) = \beta$ is unique.

When $\beta > 0$, there could be multiple solutions.

A.4 Proof of Equations (20)

Note that we can write $p^* = (\beta - \delta)z + \nu = (\beta - \delta - \kappa)z + \kappa\tilde{z}$. Then combining with $p = \alpha z + \tilde{\alpha}\tilde{z}$, where α and $\tilde{\alpha}$ are defined by (44), we obtain

$$p - p^* = \frac{(\beta - \delta - \kappa)}{1 - \chi\gamma}[(1 - \gamma - \tilde{\gamma})z - \tilde{\gamma}\kappa^{-1}\nu]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{(1-\chi)(\beta - \delta - \kappa)}{1 - \chi\gamma}\gamma\varepsilon_i$$

Finally, note that

$$\begin{aligned} \bar{E}z - z \quad &= \quad -(1 - \gamma - \tilde{\gamma})z + \tilde{\gamma}\kappa^{-1}\nu \\ E_i z - \bar{E}z \quad &= \quad \gamma\varepsilon_i \end{aligned}$$

and that $\beta - \delta - \kappa = \lambda - \delta$, which yields (20).

A.5 Proof of Lemma 2

We first write more explicitly the loss function:

$$L = \delta^2 \frac{\sigma_z^{-2} + P(\beta) + \Phi(1-\chi)^2\sigma_\varepsilon^{-2}}{[\sigma_z^{-2} + P(\beta) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2} \tag{46}$$

where $P(\beta) = \kappa(\beta)^2[\beta^2\sigma_\xi^2 + \sigma_v^2]^{-1}$. This expression is obtained by replacing the output gap and the individual deviations using (20), then replacing κ using (17), and finally by replacing γ, $\tilde{\gamma}$ and $\tilde{\chi}$ by their expressions.

We then replace $\Phi = \varrho/(1-\chi)$ and obtain

$$L = \frac{\delta^2}{\sigma_z^{-2} + P(\beta) + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{47}$$

L depends on β only through the precision $P(\beta)$. L decreases monotonously with the precision $P(\beta)$. Therefore, the level of β that maximizes the precision also minimizes L.

A.6 Proof of Proposition 1

Take the log of $P(\beta)$ to obtain $\log(P(\beta)) = 2\log(\kappa(\beta)) - \log(\beta^2\sigma_\xi^2 + \sigma_v^2)$. The precision has a component due to the sensitivity of the signal (first part) and a component due to the policy-induced noise (second part). The optimal β is then defined by:

$$\frac{\partial \log(P)}{\partial \beta} = \frac{2}{P(\beta)} \left[\underbrace{\frac{\beta\kappa'(\beta)}{\kappa(\beta)}}_{\text{Elasticity of } \kappa \text{ to } \beta} - \underbrace{\frac{\beta^2\sigma_\xi^2}{\beta^2\sigma_\xi^2 + \sigma_v^2}}_{\text{Share of policy-induced noise in demand disturbance}} \right] = 0 \tag{48}$$

The equilibrium β therefore equalizes the elasticity of κ to β to the share of policy-induced noise in the endogenous signal's noise.

We can show that at the optimum, the elasticity is equal to the share of policy in the signal sensitivity β/κ. To establish this result on the elasticity, suppose there exists β^* such that $\partial \log(P)(\beta^*)/\partial\beta = 0$. This implies that $\kappa'(\beta^*) = 1$, since, according to (17) and (18),

$$\kappa'(\beta) = 1 + \frac{(\varrho - 1)(1-\chi)\delta(\sigma_\varepsilon^2)^{-1}P(\beta)\partial\log(P)/\partial\beta}{[\sigma_z^{-2} + P(\beta) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2}$$

Therefore, $\beta\kappa'(\beta)/\kappa(\beta) = \beta/\kappa$ at the optimum.

Replacing in $\partial \log(P)/\partial\beta$ and rearranging, we find that β^* must satisfy

$$\frac{\beta^*}{\kappa(\beta^*)} = \frac{(\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^2 + \sigma_v^2},$$

which characterizes β^* uniquely. This equation can be rewritten using $\kappa = \beta + \lambda$ to obtain Equation (21).

A.7 Proof of Lemma 3

As for Lemma 1, we first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\bar{z} = z - \lambda_c^{-1} v$, then characterize λ_c, and finally study existence and unicity of the solution.

Solving the pricing equation The price-setting equation (43) can be re-written as

$$p_i = \chi E_i(p) + (1-\chi)E_i[(\lambda_c - \delta)z + \beta z^{cb} - \lambda_c \bar{z}] \tag{49}$$

We make the following guess: $p_i = \alpha z_i + \bar{\alpha}\bar{z} + \alpha^{cb} z^{cb}$. This yields $p = \alpha z + \bar{\alpha}\bar{z} + \alpha^{cb} z^{cb}$. Taking expectations of p and z and replacing in (49), we get

$$\begin{aligned} p_i = & \left\{\chi\alpha\gamma \quad +(1-\chi)(\lambda_c-\delta)\gamma \right\} z_i \\ + & \left\{\chi\left[\alpha\bar{\gamma} + \bar{\alpha}\right] \quad +(1-\chi)\left[(\lambda_c - \delta)\bar{\gamma} - \lambda_c\right] \right\} \bar{z} \\ + & \left\{\chi\left[\alpha\gamma^{cb} + \alpha^{cb}\right] \quad +(1-\chi)\left[(\lambda_c-\delta)\gamma^{cb} + \beta\right] \right\} z^{cb} \end{aligned}$$

where γ, $\bar{\gamma}$ and γ^{cb} are the Bayesian weights associated to z_i, $\bar{z}$ and z^{cb} respectively, so that $E_i(z) = \gamma z_i + \bar{\gamma}\bar{z} + \gamma^{cb}\bar{z}$. After identifying the coefficients and re-arranging we find

$$\alpha = \frac{(1-\chi)(\lambda_c-\delta)\gamma}{1-\chi\gamma} \qquad \bar{\alpha} = -\lambda_c + \frac{(\lambda_c - \delta)\bar{\gamma}}{1-\chi\gamma} \qquad \alpha^{cb} = \beta + \frac{(\lambda_c-\delta)\gamma^{cb}}{1-\chi\gamma} \tag{50}$$

Characterization Firms observe the demand index: $\bar{y} = q + (\varrho - 1)p = \beta z^{cb} + v + (\varrho - 1)(\alpha z + \bar{\alpha}\bar{z} + \alpha^{cb} z^{cb})$, where we have used $p = \alpha z + \bar{\alpha}\bar{y} + \alpha^{cb} z^{cb}$. $\tilde{y}$ can be a function of z^{cb} and $\bar{z}$ only. Therefore, the term $v + (\varrho - 1)\alpha z$ must be proportional to $\bar{z} = z - \lambda^{-1} v$.

Identifying the coefficients, we get

$$\lambda = -(\varrho - 1)\alpha$$

using the solution for α given by (50), this yields (22).

Existence and unicity Equation (22) can be rewritten as $x(\lambda_c) = \lambda_c$ with

$$x(\lambda_c) = \frac{(\varrho - 1)(1-\chi)\sigma_\varepsilon^{-2}\delta}{\sigma_z^{-2} + \lambda_c^2\sigma_v^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{51}$$

x is strictly positive so λ_c cannot be negative. On $\mathbb{R}+$, x is continuously decreasing with bounded values in $(0, \bar{\lambda}]$, $\bar{\lambda} = (\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\delta/[\sigma_z^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}]$, so a unique solution λ_c to (22) exists.

A.8 Proof of Equations (23)

Since we can write $p^* = (\beta - \delta)z + \nu = \beta z^{cb} + (\lambda_c - \delta)z - \lambda_c\bar{z}$, we have

$$p - p^* = \frac{(\lambda_c - \delta)}{1 - \chi\gamma_c}[(1 - \gamma_c - \bar{\gamma}_c - \gamma_c^{cb})z - \bar{\gamma}_c\lambda_c^{-1}v - \gamma_c^{cb}\xi]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{(1-\chi)(\lambda_c - \delta)}{1 - \chi\gamma_c}\gamma_c\varepsilon_i$$

Finally, note that

$$\begin{aligned} \bar{E}z - z &= (1 - \gamma_c - \bar{\gamma}_c - \gamma_c^{cb})z - \bar{\gamma}_c\lambda_c^{-1}v - \gamma_c^{cb}\xi \\ E_i z - \bar{E}z &= \gamma_c\varepsilon_i \end{aligned}$$

which yields (23).

A.9 Proof of Proposition 2

Here we prove that the precision under communication is strictly larger than in the absence of communication, except when $\beta = \beta^*$, in which case they are equal.

Suppose $P(\beta^*) = P_c$. This would imply:

$$\kappa(\beta^*)^2(\sigma_v^2 + (\beta^*)^2\sigma_\xi^2)^{-1} = \lambda_c^2\sigma_v^{-2} + \sigma_\xi^{-2} \tag{52}$$

Besides, according to (18) and (22), it would imply that:

$$\lambda_c = \lambda(\beta^*)$$

Using the characterization of β^* given by Equations (A.6) and (21), we know that

$$\begin{aligned} \kappa(\beta^*) &= \beta^* \tfrac{\sigma_v^2 + (\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^2} \\ \lambda(\beta^*) &= \tfrac{\beta^*\sigma_v^2}{(\beta^*)^2\sigma_\xi^2} \end{aligned}$$

Replacing λ_c and κ in (52), then replacing λ, we get

$$\begin{aligned} \tfrac{(\beta^*)^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^{-1}}{[(\beta^*)^2\sigma_\xi^2]^2} &= \tfrac{(\beta^*)^2\sigma_v^2}{[(\beta^*)^2\sigma_\xi^2]^2} + \sigma_\xi^{-2} \\ \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^{-1}}{(\beta^*)^2\sigma_\xi^4} &= \tfrac{\sigma_v^4\sigma_v^{-2}}{(\beta^*)^2\sigma_\xi^4} + \tfrac{(\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^4} \\ \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]}{(\beta^*)^2\sigma_\xi^4} &= \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]}{(\beta^*)^2\sigma_\xi^4} \end{aligned}$$

which is always true.

Therefore, $P(\beta^*) = P_c$. Since $P(\beta^*) > P(\beta)$ for all $\beta \neq \beta^*$, then $P(\beta) < P_c$ for all $\beta \neq \beta^*$.

A.10 Weight of z^{cb} in the pricing equation

We consider the case when $\beta = \beta^*$. Note that in that case $\kappa_c = \kappa$, $\lambda_c = \lambda$, $\gamma_c = \gamma$ and $\tilde{z}_c = \tilde{z}$. We therefore discard in what follows the subscript c.

Consider now the optimal pricing equation $p_i = \alpha z_i + \bar{\alpha}\bar{z} + \alpha^{cb}z^{cb}$, where α, $\bar{\alpha}$,α^{cb}, are defined in (50). We now show that $\bar{\alpha}\bar{z} + \alpha^{cb}z^{cb} = \tilde{\alpha}\tilde{z}$, where $\tilde{\alpha}$ is given by (44).

We begin by writing

$$\begin{aligned} \bar{\alpha}\bar{z} + \alpha^{cb}z^{cb} &= \left[\kappa + \tfrac{(\lambda-\delta)(\bar{\gamma}+\gamma^{cb})}{1-\chi\gamma}\right] z \\ &\quad - \left[\lambda + \tfrac{(\lambda-\delta)\bar{\gamma}}{1-\chi\gamma}\right] \lambda^{-1} v \\ &= \left[\beta + \tfrac{(\lambda-\delta)\gamma^{cb}}{1-\chi\gamma}\right] \xi \end{aligned}$$

From Equation (18) in Proposition 1, we have at the optimum

$$\frac{-\lambda}{\beta} = \frac{\bar{\gamma}}{\gamma^{cb}}$$

Hence

$$\frac{\kappa}{\beta} = \frac{\bar{\gamma} + \gamma^{cb}}{\gamma^{cb}} \qquad \frac{\kappa}{-\lambda} = \frac{\bar{\gamma} + \gamma^{cb}}{\bar{\gamma}}$$

Replacing β and $-\lambda$ using these expressions and rearranging, we obtain:

$$\begin{aligned} \bar{\alpha}\bar{z} + \alpha^{cb} z^{cb} &= \left[\kappa + \tfrac{(\lambda-\delta)(\bar{\gamma}+\gamma^{cb})}{1-\chi\gamma}\right] [z + \kappa^{-1}(v + \beta\xi)] \\ &= \tilde{\alpha}\tilde{z} \end{aligned}$$

This means that, once we use the endogenous signal $\tilde{z}$, the weight of z^{cb} in the optimal pricing equation is zero.

A.11 Solving the model with a price target

Here we sketch the proofs, as they follow the same steps as for the baseline case. We first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\tilde{z}_p = z + \kappa_p^{-1}\nu_p$, with $\nu_p = \beta_p \xi_p + v$, then characterize κ_p. Finally, we solve for the optimal β_p.

Solving the pricing equation The price-setting equation (43) can be re-written as

$$p_i = \chi E_i(p) + (1-\chi) E_i[\kappa_p \tilde{z}_p - \beta_p p - \kappa_p z - \delta z] \tag{53}$$

We make the following guess: $p_i = \alpha z_i + \tilde{\alpha}_p \tilde{z}_p$. This yields $p = \alpha z + \tilde{\alpha}_p \tilde{z}_p$. Taking expectations of p and z and replacing in (53), then after some calculations, we can identify the coefficients and obtain after rearranging:

$$\alpha = \frac{-(1-\chi)(\kappa_p+\delta)\gamma}{(1+\beta_p)[1-\chi\gamma+(1-\chi)\gamma\beta_p]} \qquad \tilde{\alpha}_p = \frac{\kappa_p}{1+\beta_p} - \frac{(\kappa_p+\delta)\tilde{\gamma}}{(1+\beta_p)[1-\chi\gamma+(1-\chi)\gamma\beta_p]} \tag{54}$$

Characterization of κ_p Firms observe the demand index: $\tilde{y} = q + (\varrho - 1)p = (-\beta_p + \varrho - 1)p + \kappa_p \kappa_p^{-1} \nu_p$. Using our guess-and-verify solution for p_i, we obtain p and then replace in $\tilde{y}$, which yields:

$$\tilde{y} = (-\beta_p + \varrho - 1)(\alpha z + \tilde{\alpha}_p \tilde{z}_p) + \kappa_p \kappa_p^{-1} \nu_p$$

$\tilde{y}$ depends linearly on z, v and $\tilde{z}_p$. For our guess to be true, we need that $\tilde{y} - (-\beta_p + \varrho - 1)\tilde{\alpha}_p \tilde{z}_p = f\bar{z}$ for some constant f. Identifying the coefficients, we get

$$(-\beta_p + \varrho - 1)\alpha = \kappa_p$$

Replacing α and rearranging, we obtain

$$\kappa_p = \frac{-(1-\chi)[\varrho - 1 - \beta_p]\gamma\delta}{1 + \gamma[\varrho(1-\chi) - 1]}$$

which yields (25).

Optimal β_p Our proof follows the lines of the proof of lemma 2 and Proposition 1. The steps are very similar because the optimal β_p maximizes the precision of $\tilde{z}_p$, $P_p(\beta_p) = \kappa_p^2[\beta_p^2\sigma_\xi^2 + \sigma_v^2]^{-1}$, which is of the same form as P.

We first determine the price gaps. Since we can write $p = \alpha z + \tilde{\alpha}_p \tilde{z}_p$ and $p^* = \kappa_p \tilde{z}_p - \beta_p p - (\kappa_p + \delta)z = (\kappa_p - \beta_p \tilde{\alpha}_p)\tilde{z}_p - (\kappa_p + \beta_p \alpha + \delta)z$, we get

$$p - p^* = \frac{\kappa_p + \delta}{1 - \chi\gamma + (1-\chi)\gamma\beta}[(1 - \gamma - \tilde{\gamma})z - \tilde{\gamma}\kappa_p^{-1}\nu]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{-(1-\chi)(\kappa_p + \delta)}{1 - \chi\gamma + (1-\chi)\gamma\beta}\gamma\varepsilon_i$$

We can then write more explicitly the loss function:

$$L = \delta^2 \frac{\sigma_z^{-2} + P_p(\beta_p) + \Phi(1-\chi)^2\sigma_\varepsilon^{-2}}{[\sigma_z^{-2} + P_p(\beta_p) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2} \tag{55}$$

This expression is obtained by replacing the above price gap and the individual deviations in the loss function, then replacing κ_p using (25), and finally by replacing γ and $\tilde{\gamma}$ and by their expressions.

The expression for the loss function is exactly identical to (46), suggesting that, as in the baseline, the optimal β_p maximizes the precision of the endogenous signal P_p.

As P_p has the same expression as P, the optimal β_p, denoted β_p^*, must satisfy the same optimality condition (48). However, now the expression for κ_p, (25), is different from (17), yielding

$$\kappa_p'(\beta_p^*) = \frac{(1-\chi)\gamma\delta\beta_p}{1+\gamma[\varrho(1-\chi)-1]}$$

where we use the fact that $\partial log(P_p)(\beta_p)/\partial\beta_p = 0$ for $\beta_p = \beta_p^*$.

Replacing in $\partial \log(P)/\partial\beta$ and rearranging, we find that β_p^* must satisfy Equation (26), which characterizes β_p^* uniquely.

A.12 Solving the model with mark-up shocks

We proceed as in the baseline case to derive the average price gap and the individual deviations from the mean:

$$\begin{array}{rcllll} p\_p^* & = & -(\lambda_\rho-\delta_\rho) & (1+\gamma_\rho\tilde{\chi}_\rho) & \left[\bar{E}(\rho)-\rho\right]+\delta_\rho\rho \\ p_i-p & = & -(\lambda_\rho-\delta_\rho) & [1-(1-\gamma_\rho)\tilde{\chi}_\rho] & \left[E_i(\rho)-\bar{E}(\rho)\right] \end{array} \tag{56}$$

where $\tilde{\chi}_\rho = \chi/(1-\chi\gamma_\rho)$, γ_ρ and $\tilde{\gamma}_\rho$ are defined as in (16), $E_i(\rho)$ follows (15) and $\bar{E}(\rho) = \int_0^1 E_i(\rho)di$. λ_ρ is

$$\lambda_\rho = \frac{-(\varrho-1)(1-\chi)\delta_\rho\gamma_\rho}{1+[\varrho(1-\chi)-1]\gamma_\rho} \tag{57}$$

and $\kappa_\rho = \beta_\rho + \lambda_\rho$.

To determine optimal policy, we proceed as for the proof of Lemma 2 and write more explicitly the loss function:

$$L = \delta_\rho^2 \frac{[P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}]^2\sigma_\rho^2 + P_\rho(\beta_\rho) + \Phi(1-\chi)^2\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\rho(1-\chi)\sigma_\omega^{-2}\right]^2} \tag{58}$$

where $P_\rho(\beta_\rho) = \kappa_\rho(\beta_\rho)^2(\sigma_v^2+\beta_\rho^2\sigma_{\xi_\rho}^2)^{-1}$. This expression is obtained by replacing the output gap and the individual deviations using (56), then replacing λ_ρ using (57) and finally by replacing γ_ρ, $\tilde{\gamma}_\rho$ and $\tilde{\chi}_\rho$ by their expressions.

Replacing $\Phi = \varrho/(1-\chi)$, we obtain

$$\begin{array}{rcl} L & = & \delta_\rho^2 \frac{[P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}]^2\sigma_\rho^2+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}\right]^2} \\ & = & \delta_\rho^2\sigma_\rho^2[P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}]\frac{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}\right]^2} \\ & = & \delta_\rho^2\sigma_\rho^2\frac{P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}} \end{array} \tag{59}$$

L depends on β_ρ only through the precision $P_\rho(\beta_\rho)$. L increases monotonously with the precision $P_\rho(\beta_\rho)$. Therefore, the level of β_ρ that minimizes the precision also minimizes L. That level is such that $\kappa_\rho = 0$, which implies $\beta_\rho = -\lambda_\rho$. This yields expression (29)

A.13 Solving the dynamic model

Combining the expression for nominal demand (32) along with the endogenous signal $\tilde{y} = q + (\varrho - 1)p$, we determine that the endogenous signal can be written as follows

$$\tilde{y}_t = \frac{1}{\phi}(\beta z_t + \nu_t) + \left(\varrho - \frac{1}{\phi}\right) p_t$$

rescaling $\tilde{y}$, we get

$$\phi \tilde{y}_t = \beta z_t + \nu_t + (\phi\varrho - 1)p_t$$

$\phi\tilde{y}_t$ is exactly of the same form as $\tilde{y}$ in the static case, except that ϱ has been replaced with $\phi\varrho$. The solutions and optimal policy are then adjusted accordingly.