Are Overconfident Players More Likely to Win Tournaments and Contests?

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Abstract

This paper investigates whether an overconfident player is more likely to win a competition against a rational player. The two players are identical, except that the overconfident player overestimates his productivity of effort and, as a consequence, his probability of winning. The competition can take the form of either a tournament or a contest. The paper shows that the overconfident player is the Nash winner (loser) of a tournament with monotonic best responses when his effort and overconfidence are complements (substitutes). The overconfident player is the Nash winner (loser) of a tournament with non-monotonic best responses when he is slightly (significantly) overconfident. In contrast, the overconfident player is always the Nash loser of a contest. The paper also discusses the welfare implications of overconfidence.

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1 Introduction

This paper investigates whether overconfident players are more likely to win competitions. This question is of relevance since evidence from psychology and economics shows that humans tend to be overconfident. A majority of people believe they are better than others in a wide variety of positive traits and skills (Myers 1996, Santos-Pinto and Sobel 2005). Examples include car drivers (Svenson 1981), entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006), currency traders (Oberlechner and Osler 2008), poker and chess players (Parker and Santos-Pinto 2010), CFOs (Ben-David et al. 2013), marathon runners (Krawczyk and Wilamowski 2017), free-divers (Lackner and Sonnabend 2020), and truck drivers (Hoffman and Burks 2020).

Competitions often take the form of tournaments and contests. For example, sports like soccer, tennis, chess, and poker are organized as tournaments. In labor markets, firms often use tournaments to incentivize effort provision–a bonus or free vacation for the top salesperson–and to promote employees–workers compete to become managers and managers to become CEOs (Malcomson 1986, Gibbons and Murphy 1990, Baker et al. 1994, Murphy et al. 2004, Harbring and Lünser 2008). An R&D race to be the first to develop or get a patent in new product or technology, election campaigns, rent-seeking games, competitions for monopolies, litigation, and wars, are examples of contests.[1] Overconfidence matters for entry and performance in competitions and for labor markets (Camerer and Lovallo 1999, Niederle and Vesterlund 2007, Moore and Healy 2008, Malmendier and Taylor 2015, Huffman et al. 2019, Santos-Pinto and de la Rosa 2020). Overconfidence also seems to play a

[1]The main difference between a tournament and a contest is that the winning probabilities in a contest are defined by a contest success function. For example, in a standard two player Tullock (1980) contest, player i's winning probability is $P_i(a_i, a_j) = a_i^r/(a_i^r + a_j^r)$, with $r > 0$. The parameter r captures the degree of noise in the Tullock contest. The higher is r, the more sensitive is the success probability to effort. When $r = 0$ effort plays no role and each player always has a success probability of 1/2. The most popular versions of the Tullock contest are the lottery ($r = 1$) and the first-price all-pay auction ($r = \infty$).

role in mate competition and acquisition (Waldman 1994, Murphy et al. 2015).

To determine whether overconfident players are more likely to win competitions I consider tournaments and contests where an overconfident player competes against a rational player. Each player chooses an effort level independently and simultaneously. Effort plus random factors determine who produces the highest output or who attains the best performance. The overconfident player overestimates his productivity of effort and, as a consequence, his winning probability. The rational player knows about the overconfident player's bias and optimally reacts to it. Both players have identical productivity, preferences, outside options, and face identical random shocks. These symmetry assumptions allow me to focus exclusively on the role that the heterogeneity in beliefs plays in determining the winner of the competition. Moreover, they imply that the player who exerts the highest effort has the highest winning probability. I define as the Nash winner (loser) the player with the highest (lowest) probability of winning at the pure-strategy equilibrium.

I distinguish two kind of tournaments: monotone and non-monotone. The former describe situations where best responses are monotonic. For example, when efforts are strategic complements, each player best responds in a monotone increasing way to an increase in the rival's effort (e.g., Nalebuff and Stiglitz 1983, Santos-Pinto 2010). Similarly, when efforts are strategic substitutes, each player best responds in a monotone decreasing way to an increase in the rival's effort. Non-monotone tournaments describe situations where best responses are non-monotonic. For example, given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort but, given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort (e.g., Lazear and Rosen 1981, Green and Stokey 1983, Akerlof and Holden 2012).

I start by considering monotone tournaments. Proposition 1 shows that the overconfident player is the Nash winner of a monotone tournament when his effort and overconfidence are complements. In this case, the overconfident player overestimates his *marginal* probability of winning the tournament and hence exerts higher effort than the rational player. In contrast, Proposition 2 shows that the overconfident player is the Nash loser of a monotone tournament when his effort and overconfidence are substitutes. In this case, the overconfident player underestimates his *marginal* probability of winning the tournament and hence exerts lower effort than the rational player. These two results hold regardless of whether players' efforts are strategic complements or substitutes.

Propositions 1 and 2 carry welfare implications. Overconfidence makes the principal better off in monotone tournaments where efforts are strategic complements and where there is complementarity between effort and overconfidence: both players exert more effort than if both were rational. In this case, the principal would not want to de-bias the overconfident player. In contrast, overconfidence makes the principal worse off in monotone tournaments where efforts are strategic complements and where there is substitutability between effort and overconfidence: both players exert less effort than if both were rational. In this case, the principal would want to de-bias the overconfident player. Overconfidence can make the overconfident player better off when it lowers the effort of the rational player but always makes the overconfident player worse off when it raises the effort of the rational player. The rational player is worse off (better off) when there is complementarity (substitutability) between effort and overconfidence.

Next, I consider non-monotone tournaments. Proposition 3 shows the overconfident player is the Nash winner of a non-monotone tournament when he is slightly overconfident and the Nash loser when he is significantly overconfident. The intuition behind this result is as follows. If the overconfident player is slightly overconfident, he overestimates his *marginal* probability of winning the tournament. This raises the effort of the overconfident player and lowers the effort of the rational player. In contrast, if the overconfident player is significantly overconfident, he underestimates his *marginal* probability of winning the tournament. This lowers the effort of both players but more so that of the overconfident player. I also show that a significantly overconfident player thinks, mistakenly, he is the Nash winner of a non-monotone tournament even if he anticipates he will exert less effort than his rival.

Proposition 3 has welfare implications. The principal is better off (worse off) with a slightly overconfident when his increase in effort is greater (smaller) than the decrease in effort of the rational player. The principal is worse off with a significantly overconfident since both players exert less effort than if both were rational. Overconfidence can make a slightly overconfident player better off but always makes a significantly overconfident player worse off. The rational player is worse off (better off) when the overconfident player is slightly (significantly) overconfident.

Finally, I consider contests. I study a generalized Tullock contest where player i's winning probability is $P_i(a_i, a_j) = q(a_i)/[q(a_i)+q(a_j)]$, with $q'(a_i) > 0$. The function $q(a_i)$ is the impact function. I assume the overconfident player 1 thinks, mistakenly, his impact function is $\lambda q(a_1)$, where $\lambda > 1$, and thinks, correctly, his rival's impact function is $q(a_2)$. Hence, the overconfident player's perceived winning probability is $P_1(a_1, a_2, \lambda) = \lambda q(a_1)/[\lambda q(a_1) + q(a_2)]$.[2] Proposition 4 shows that the overconfident player is always the Nash loser of a generalized Tullock contest. I also show that an overconfident player thinks, mistakenly, he is the Nash winner of the contest even if he anticipates he will exert less effort than his rival. In addition, I show that the overconfident player makes the contest less competitive since both players exert less effort than if both were rational. Hence, the contest organizer would prefer to de-bias the overconfident player. Overconfidence has an ambiguous effect of the overconfident player's welfare. Overconfidence makes the rational player better off since she exerts less effort and has a higher winning probability than if both players were rational.

The paper is organized as follows. Section 2 discusses related literature. Section 3 sets-up the tournament model. Sections 4 and 5 derive the results for monotone and non-monotone tournaments, respectively. Section 6 sets-up and derives the results for contests. Section 7 concludes the paper. All proofs are in the Appendix.

[2]Section 6 shows that this way of modeling overconfidence in a contest satisfies four desirable properties.

2 Related Literature

This study relates to several strands of literature. First, it contributes to the growing literature on overconfidence and competitions. This literature shows that overconfidence matters for entry and performance in competitions. Camerer and Lovallo (1999) show that participants in market entry experiments who overplace themselves are more likely to self-select into markets where rewards depend on relative performance. Niederle and Vesterlund (2007) show experimentally that gender differences in overplacement can lead to gender differences in choice of compensation scheme. Despite there being no gender differences in performance, 73 percent of the men select to enter a tournament but only 35 percent of the women make this choice. The gender gap in tournament entry is driven by two factors. First, men have a stronger preference for competing than women. Second, men are substantially more overconfident about their relative performance than women. Dohmen and Falk (2011) find that experimental participants are more likely to select into tournaments the more they overplace themselves. Huffman et al. (2019) find that managers of a chain of food-and-beverage stores who compete repeatedly in high-stakes tournaments overplace themselves relative to a range of different predictors obtained from past tournament outcomes.

Second, this study contributes to the literature on tournaments. Several studies have analyzed tournaments with heterogeneous competitors. Lazear and Rosen (1981) show that heterogeneity in effort costs leads to inefficient tournament outcomes. Shotter and Weigelt (1992) study the impact of equal opportunity laws and affirmative actions on effort provision. They find that policies that increase the probability of winning for disadvantaged (high cost) players reduce the effort they exert when heterogeneity is low but increase the effort exerted by both advantaged and disadvantaged players when heterogeneity is high. Harbring and Lünser (2008) show that an increase in the price spread raises effort provision in a tournament with heterogeneous competitors and that, for larger prize spreads, weaker competitors exert higher effort than in a tournament with identical competitors. Gürtler and

Kräkel (2010) show that inefficiencies can arise if the principal sets uniform prizes (i.e., prizes that are independent from players' identity) in a tournament with heterogeneous competitors while efficient effort provision can be induced if the principal sets individualized prizes. In this paper I consider tournaments where competitors differ in their beliefs about their productivity of effort. More precisely, I focus on two player tournaments where an overconfident player competes against a rational player.

The most closely related studies in the literate on tournaments are Goel and Thakor (2008) and Santos-Pinto (2010). Goel and Thakor (2008) study promotion tournaments where overconfident and rational managers compete against each other. They assume that overconfident managers underestimate the risk of their projects. They find that overconfident managers have a higher likelihood of being promoted to CEO than rational ones. My results differ from those in Goel and Thakor (2008) due to two reasons. First, I assume an overconfident manager overestimates his productivity of effort instead of underestimating the risk of his project. Second, I assume managers compete by choosing effort instead of risk exposure. My results imply that an overconfident manager has a higher (lower) probability of being promoted to CEO than a rational manager in a monotone tournament when his effort and overconfidence are complements (substitutes). I also find that a slightly (significantly) overconfident manager has a higher (lower) probability of being promoted to CEO than a rational manager in a non-monotone tournament.

Santos-Pinto (2010) studies tournaments where all workers equally overestimate their productivity of effort. The main finding is that firms can be better off with an overconfident workforce if they wisely structure tournament prizes. If workers' effort and overconfidence are complements, the firm can implement the same effort with lower prizes or obtain a higher outcome for the same prizes. If workers' effort and overconfidence are substitutes, the firm needs to raise the power of incentives to implement the same effort level in a tournament with an overconfident workforce as in one with a workforce with correct beliefs. My results extend and go beyond those in Santos-Pinto (2010) building on the same definition of overconfidence in a tournament but allowing for heterogeneity in beliefs.

Third, this study contributes to the literature on contests. Several studies have analyzed contests with heterogeneous competitors. Baik (1994) studies two player contests where the players differ in their valuation of the prize and in their marginal productivity of effort. He provides conditions under which the player who values the prize the most or the most productive player is the Nash winner or loser of the contest. Singh and Wittman (2001) show that when players differ in marginal productivity of effort, output increases in ability, and effort provision decreases in effort costs. In this paper I consider contests where players differ in their beliefs about their productivity of effort. Stein (2002) considers a contest with N heterogeneous players and determines the number of active players, that is, those who exert positive effort. In this paper I consider contests where the contestants have heterogeneous beliefs about their winning probabilities. More precisely, I focus on two player contests where an overconfident player competes against a rational player.

The most closely related studies in the literature on contests are Ando (2004), Krähmer (2007), and Ludwig et al. (2011). Ando (2004) studies a two player contest where players are uncertain about their types which represent the monetary value of winning the contest. All players are overconfident and two definitions of overconfidence are considered. An overconfident player can either overestimate his own type or, alternatively, underestimate his rival's type. Ando (2004) finds that an overconfident player who overestimates his own type always exerts more effort. In contrast, an overconfident player who underestimates his rival's type might exert less effort. My results on contests differ from those in Ando (2004) since I define an overconfident player as someone who overestimates his winning probability instead of his monetary value of winning the contest. My definition is adequate in contests where the monetary value is known before entry.

Krähmer (2007) considers a repeated contest where players are uncertain about their true relative abilities but learn them over time. players can exert either high or low effort and effort and ability are complements. Krähmer (2007) shows that whenever the players' belief that one of them is the less able player is sufficiently large, this player chooses low effort and the other player chooses high effort. A player who overestimates his relative ability will, due to complementarity, tends to choose high effort, resulting in a higher number of actual successes. Increased successes, in turn, promotes the player's self-confidence further. At the same time, a player who underestimates his relative ability becomes discouraged and, as a result, ever less self-confident.

Ludwig et al. (2011) analyze a Tullock contest where an overconfident player competes against a rational player. They assume the overconfident player underestimates his cost of effort. They find that the overconfident player exerts more effort and the rational player exerts less effort than if both were rational. They also find that the bias makes the principal better off since the overconfident player's increase in effort more than compensates the rational player's decrease in effort. My results on contests differ from those in Ludwig et al. (2011) since I define an overconfident player has someone who overestimates his winning probability instead of underestimating his cost of effort. My definition is adequate in contests where the cost of effort is known before entry.

3 Set-up

Consider two players, 1 and 2, competing in a tournament. The player who produces the highest output receives the winner's prize y_W and the other player receives the loser's prize y_L, with $0 < y_L < y_W$. Winning or losing the tournament, and thus individual prizes, depend on the relative performance of the players and not on their absolute performance.

The two players have an identical productivity of effort, utility function, and outside option. However, they differ from one another in terms of the perception of their own productivity. player 1 is overconfident as he overestimates his productivity of effort. player 2 is rational as she has an accurate perception of her productivity of effort. player 1 is not aware of being overconfident while player 2 is aware that player 1 is overconfident. Finally, both players correctly assess their utility functions and their outside options.

The players are weakly risk averse and expected utility maximizers and have von Neumann-Morgenstern utility functions that are separable in income (y_i) and effort (a_i):

$$U_i(y_i, a_i) = u(y_i) - c(a_i),$$

for $i = 1, 2$. I assume u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' > 0$, $c(0) = 0$, $c'(0) = 0$, and $c(a_i) = \infty$, for $a_i \to \infty$, where the last two conditions ensure that equilibrium effort is strictly positive but finite. The two players have outside options which guarantee each $\bar{u}$; so unless the perceived expected utility from participation is at least equal to $\bar{u}$, players will not be willing to participate. Income y_i is equal to y_W if player i wins the tournament and to y_L if player i loses the tournament.

The output of player i is a stochastic function of effort. Each level of effort of player i induces a distribution over output given by

$$F_i(q_i|e_i(a_i, \omega)),$$

for $i = 1, 2$. Here $e_i(a_i, \omega)$ defines player i's productivity as a function of effort a_i and the common shock ω. Individual productivity strictly increases in effort i.e., $e_i' > 0$, and marginal productivity is subject to diminishing returns to effort.

As mentioned, player 1 is overconfident and overestimates his productivity whereas player 2 is rational. player 1's perceived productivity of effort is

$$e_1 = e_1(a_1, \omega, \lambda)$$

where $\lambda > 1$ is a parameter that captures player 1's overconfidence. Given player 1's perceived productivity of effort, his perceived distribution over output is

$$F_1(q_1|e_1(a_1, \omega, \lambda)).$$

In the case of player 1, who is overconfident, $F_1(q_1|e_1(a_1,\omega,\lambda))$ first order stochastically dominates $F_1(q_1|e_1(a_1,\omega))$ for all levels of effort a_1: for each level of effort exerted, player 1 believes that she is more likely to produce a higher level of output than he actually does. player 2 has an accurate perception of her own productivity $e_2 = e_2(a_2,\omega)$ and thus her perceived and actual distribution over output coincide at $F_2(q_2|e_2(a_2,\omega))$. This implies that $F_1(q_1|e_1(a_1,\omega,\lambda))$ first order stochastically dominates $F_2(q_2|e_2(a_2,\omega))$ when $a_1 = a_2$. That is, player 1 believes that he is more likely to produce a higher level of output than player 2 when the two players exert the same effort.

player 1's perceived probability of winning the tournament is

$$\Pr(\tilde{Q}_1 \geq q_2) = 1 - \Pr(\tilde{Q}_1 \leq q_2) = 1 - F_1(q_2|e_1(a_1,\omega,\lambda)),$$

and his unconditional perceived probability of winning the tournament is

$$P_1(a_1,a_2,\lambda) = \Pr(\tilde{Q}_1 \geq Q_2) = \int [1 - F_1(q_2|e_1(a_1,\omega,\lambda))] f_2(q_2|e_2(a_2,\omega)) \partial q_2.$$

player 2's probability of winning the tournament is

$$\Pr(Q_2 \geq q_1) = 1 - \Pr(Q_2 \leq q_1) = 1 - F_2(q_1|e_2(a_2,\omega)),$$

and her unconditional probability of winning the tournament is

$$P_2(a_1,a_2) = \Pr(Q_2 \geq Q_1) = \int [1 - F_2(q_1|e_2(a_2,\omega))] f_1(q_1|e_1(a_1,\omega)) \partial q_1.$$

player 1's perceived expected utility is

$$E[U_1(a_1,a_2,\lambda)] = u(y_L) + P_1(a_1,a_2,\lambda)\Delta u - c(a_1),$$

and player 2's objective expected utility is

$$E[U_2(a_1,a_2)] = u(y_L) + P_2(a_1,a_2)\Delta u - c(a_2),$$

where $\Delta u = u(y_W) - u(y_L)$.

The principal is risk neutral and correctly assesses the players' productivity. The principal's profits are the difference between expected benefits and compensation costs:

$$E[\pi] = E[Q_1 + Q_2] - (y_L + y_W).$$

The timing of the events is as follows. The principal commits to a prize schedule. The players decide whether or not to participate. All players who agree to participate observe the realization of a common shock and then simultaneously and independently choose their effort levels. The principal observes the players' output realizations and awards the prizes according to the prize schedule.

4 Monotone Tournaments

This section studies monotone tournaments where an overconfident player competes against a rational player. To understand whether an overconfident player has a higher or a lower probability of winning a tournament we need to determine the impact of overconfidence on the pure-strategy equilibrium efforts of the overconfident player and the rational rival.

The pure-strategy equilibrium efforts are the solution to the first-order conditions:

$$\frac{\partial E[U_1(a_1, a_2, \lambda)]}{\partial a_1} = \frac{\partial P_1(a_1, a_2, \lambda)}{\partial a_1}\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1, a_2)]}{\partial a_2} = \frac{\partial P_2(a_1, a_2)}{\partial a_2}\Delta u - c'(a_2) = 0.$$

The second-order conditions are

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda)]}{\partial a_1^2} < 0 \text{ and } \frac{\partial^2 E[U_2(a_1, a_2)]}{\partial a_1^2} < 0. \tag{1}$$

I assume the second-order conditions are satisfied. I also assume the tournament has a unique pure-strategy Nash equilibrium. A sufficient condition for this to hold is that the derivatives of the players' best responses are less than 1 in absolute value over the relevant range.[3] Thus,

$$\left|\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2}\right| > \left|\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2}\right| \text{ and } \left|\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2}\right| > \left|\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2}\right|. \tag{2}$$

is a sufficient condition for uniqueness. Finally, I assume the tournament has monotonic best responses, that is, players' efforts are either strategic complements or substitutes over all effort levels. The assumption that efforts are strategic complements represents tournaments where a player's increase in effort makes it more desirable for the rival to increase effort too. This happens when higher effort by a player raises the rival's *marginal* expected utility. In this case we have

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_1(a_1,a_2,\lambda)}{\partial a_1 \partial a_2} > 0 \text{ and } \frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_2(a_1,a_2)}{\partial a_1 \partial a_2} > 0.$$

The assumption that efforts are strategic substitutes represents tournaments where a player's increase in effort makes it more desirable for the rival to lower effort. This happens when higher effort by a player lowers the rival's *marginal* expected utility. In this case we have

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_1(a_1,a_2,\lambda)}{\partial a_1 \partial a_2} < 0 \text{ and } \frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_2(a_1,a_2)}{\partial a_1 \partial a_2} < 0.$$

The unique pure-strategy equilibrium (a_1^*, a_2^*) satisfies the first-order conditions simultaneously and is given by:

$$\frac{\partial P_1(a_1^*,a_2^*,\lambda)}{\partial a_1}\Delta u = c'(a_1^*), \tag{3}$$

and

$$\frac{\partial P_2(a_1^*,a_2^*)}{\partial a_2}\Delta u = c'(a_2^*). \tag{4}$$

[3] A sufficient condition for the pure strategy Nash equilibrium to be unique is that best responses intersect only once.

The impact of overconfidence on the pure-strategy equilibrium efforts can be obtained from (3) and (4). Differentiation of (3) and (4) gives us[4]

$$\frac{\partial a_1^*}{\partial \lambda} = -\frac{1}{D^*}\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)\right]\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\Delta u, \tag{5}$$

and

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{1}{D^*}\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2}\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}(\Delta u)^2, \tag{6}$$

where

$$D^* = \left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\Delta u - c''(a_1^*)\right]\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)\right] - \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2}(\Delta u)^2. \tag{7}$$

The sign of the two terms inside square brackets in (7) is strictly negative given the second-order conditions. Note that (2) and (7) imply $D^* > 0$.

It follows from (1), (2), (5), and (7) that the relation between player 1's equilibrium effort and overconfidence only depends on the sign of $\partial^2 P_1(a_1^*, a_2^*, \lambda)/\partial a_1 \partial \lambda$, that is, how overconfidence influences player 1's perceived marginal probability of winning the tournament. If effort and overconfidence are complements, that is, $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda > 0$, then an increase in overconfidence raises player 1's perceived marginal probability of winning the tournament and player 1's effort. If player 1's effort and overconfidence are substitutes, that is, $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda < 0$, then an increase in overconfidence lowers player 1's perceived marginal probability of winning the tournament and player 1's effort.

It follows from (1), (2), (6), and (7) that the relation between player 2's equilibrium effort and player 1's overconfidence depends on the signs of $\partial^2 P_1(a_1^*, a_2^*, \lambda)/\partial a_1 \partial \lambda$ and $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$. The sign of $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$ is determined by the nature of the strategic relation between the players' efforts. When efforts are strategic complements (substitutes), the sign of $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$ is positive (negative).

[4] The derivation can be found in the Appendix.

My first result characterizes the impact of overconfidence on the pure-strategy equilibrium winning probabilities and efforts in a monotone tournament when player 1's effort and overconfidence are complements. I define as the Nash winner (loser) the player with the higher (lower) probability of winning the tournament at the pure-strategy equilibrium.

Proposition 1: *The overconfident player is the Nash winner of a monotone tournament when his effort and overconfidence are complements, i.e.,* $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$ *when* $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda > 0$. *If efforts are strategic complements, then both players exert more effort than if both were rational, with the overconfident player exerting the greatest effort. Furthermore, an increase in overconfidence raises the effort of both players and more so that of the overconfident player, i.e.,* $\partial a_1^*/\partial \lambda > \partial a_2^*/\partial \lambda > 0$. *If efforts are strategic substitutes, then the overconfident player exerts more effort and the rational player exerts less effort than if both were rational. Furthermore, an increase in overconfidence raises the effort of the overconfident player and lowers that of the rational player, i.e.,* $\partial a_1^*/\partial \lambda > 0 > \partial a_2^*/\partial \lambda$.

Proposition 1 tells us that an overconfident player has a higher probability of winning a monotone tournament when his effort and overconfidence are complements. The intuition behind this result is as follows. When effort and overconfidence are complements, an increase in overconfidence raises the overconfident player's perceived marginal probability of winning the tournament. This, in turn, raises the overconfident player's effort. When efforts are strategic complements, the rational player's optimal response to the higher effort of the overconfident player is to raise her effort. However, the increase in effort of the rational player is less pronounced than that of the overconfident player. When efforts are strategic substitutes, the rational player's optimal response to the higher effort of the overconfident player is to lower her effort. One way or the other, the overconfident player exerts higher effort than the rational player and therefore has a higher probability of winning the tournament.

When effort and overconfidence are complements, individuals with higher beliefs about their abilities work harder. Chen and Schildberg-Hörisch (2019) find experimental support for this assumption using a real effort task. They also show that informing individuals about their true abilities lowers effort provision which further reinforces the idea that overconfidence and effort are complements. However, whether this assumption holds generally is unclear. It might just as well the case that individuals with higher beliefs about their abilities exert less effort, that is, effort and overconfidence are substitutes.

My second result characterizes the impact of overconfidence on the pure-strategy equilibrium winning probabilities and efforts in a monotone tournament when player 1's effort and overconfidence are substitutes.

Proposition 2: *The overconfident player is the Nash loser of a monotone tournament when his effort and overconfidence are substitutes, i.e.,* $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$ *when* $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda < 0$. *If efforts are strategic complements, then both players exert less effort than if both were rational, with the overconfident player exerting the least effort. Furthermore, an increase in overconfidence lowers the effort of both players and more so that of the overconfident player, i.e.,* $\partial a_1^*/\partial \lambda < \partial a_2^*/\partial \lambda < 0$. *If efforts are strategic substitutes, then the overconfident player exerts less effort and the rational player exerts more effort than if both were rational. Furthermore, an increase in overconfidence lowers the effort of the overconfident player and raises that of the rational player, i.e.,* $\partial a_1^*/\partial \lambda < 0 < \partial a_2^*/\partial \lambda$.

Proposition 2 shows that an overconfident player has a lower probability of winning a monotone tournament when his effort and overconfidence are substitutes. In this case, an increase in overconfidence lowers the overconfident player's perceived marginal probability of winning the tournament. This, in turn, lowers the overconfident player's effort. When efforts are strategic complements, the rational player's optimal response to the lower effort of the overconfident player is to lower her effort. However, the decrease in effort of the rational player is less pronounced than that of the overconfident player. When efforts are strategic substitutes, the rational player's optimal response to the lower effort of the overconfident player is to increase her effort. Since the overconfident player exerts lower effort than the rational player in either case, he has a lower probability of winning the tournament.

Propositions 1 and 2 carry welfare implications. Overconfidence makes the principal better off when efforts are strategic complements and there is complementarity between effort and overconfidence: both players exert more effort than if both were rational. In this case, the principal would not want to de-bias the overconfident player. In contrast, overconfidence makes the principal worse off when efforts are strategic complements and there is substitutability between effort and overconfidence: both players exert less effort than if both were rational. In this case, the principal would want to de-bias the overconfident player. When efforts are strategic substitutes, one player exerts more effort and the other player exerts less effort so the impact of overconfidence on the principal's welfare is ambiguous.

To evaluate the welfare implications of overconfidence for the overconfident player I take the perspective of an outside observer who knows the overconfident player's true productivity (knows that $\lambda = 1$). I consider how the overconfident player's equilibrium objective expected utility $E\left[U_1(a_1^*, a_2^*)\right] = u(y_L) + P_1(a_1^*, a_2^*)\Delta u - c(a_1^*)$ changes with λ:

$$\begin{aligned}\frac{\partial E\left[U_1(a_1^*, a_2^*)\right]}{\partial \lambda} &= \left[\frac{\partial P_1(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial P_1(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial \lambda}\right]\Delta u - c'(a_1^*)\frac{\partial a_1^*}{\partial \lambda} \\ &= \left[\frac{\partial P_1(a_1^*, a_2^*)}{\partial a_1} - \frac{\partial P_1(a_1^*, a_2^*, \lambda)}{\partial a_1}\right]\frac{\partial a_1^*}{\partial \lambda}\Delta u + \frac{\partial P_1(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial \lambda}\Delta u, \qquad (8)\end{aligned}$$

where the second equality follows from the first-order condition of the overconfident player. The first term on the right-hand side of (8) is the direct effect and the second term is the strategic effect. The direct effect is always negative because the overconfident player fails to play a best response against his rival.[5] Given that

[5] When effort and overconfidence are complements, player 1's marginal perceived probability of winning the tournament is higher than his actual marginal probability. Hence, the term inside square brakets in (8) is negative. Furthermore, an increase in λ raises the effort of the overconfident player, i.e., $\partial a_1^*/\partial \lambda > 0$. Hence, an increase in λ has an unfavorable direct effect when effort and overconfidence are complements. When effort and overconfidence are substitutes, player 1's marginal perceived probability of winning the tournament is lower than his actual marginal probability.

$\partial P_1(a_1^*, a_2^*)/\partial a_2 < 0$, the sign of the strategic effect is negative when $\partial a_2^*/\partial\lambda > 0$ and positive when $\partial a_2^*/\partial\lambda < 0$. Hence, an increase in overconfidence always makes the overconfident player worse off when it raises the effort of the rational player. However, an increase in overconfidence can make the overconfident player better off when it lowers the effort of the rational player. This happens when the strategic effect dominates the direct effect. These welfare results for the overconfident player are in line with Heifetz et al. (2007).

To evaluate the welfare implications of overconfidence for the rational player I consider how her equilibrium expected utility $E[U_2(a_1^*, a_2^*)]$ changes with λ:

$$\begin{aligned}\frac{\partial E[U_2(a_1^*, a_2^*)]}{\partial\lambda} &= \left[\frac{\partial P_2(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial\lambda} + \frac{\partial P_2(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial\lambda}\right]\Delta u - c'(a_2^*)\frac{\partial a_2^*}{\partial\lambda} \\ &= \frac{\partial P_2(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial\lambda}\Delta u,\end{aligned}$$

where the second equality follows from the first-order condition of the rational player. The sign of the derivative $\partial P_2(a_1^*, a_2^*)/\partial a_1$ is negative since an increase in the effort of the overconfident player lowers the winning probability of the rational player. Hence, an increase in overconfidence makes the rational player worse off when the sign of $\partial a_1^*/\partial\lambda$ is positive, i.e., when overconfidence raises the effort of the overconfident player. This happens when there is complementarity between effort and overconfidence. In contrast, an increase in overconfidence makes the rational player better off when the sign of $\partial a_1^*/\partial\lambda$ is negative, i.e., when overconfidence lowers the effort of the overconfident player. This happens when there is substitutability between effort and overconfidence.

Hence, the term inside square brakets in (8) is positive. Furthermore, an increase in λ lowers the effort of the overconfident player, i.e., $\partial a_1^*/\partial\lambda < 0$. Hence, an increase in λ also has an unfavorable direct effect when effort and overconfidence are substitutes.

5 Non-Monotone Tournaments

This section studies non-monotone tournaments where an overconfident player competes against a rational player. I assume output is linearly additive in effort, an idiosyncratic shock, and a common shock. That is, if player i exerts effort a_i his output is given by $Q_i = a_i + \varepsilon_i + \omega$, where ε_i and ω are random variables with zero mean. The random variables ε_1 and ε_2 are identically and independently distributed. Additionally, ε_1 and ε_2 are independent of ω. This specification for output is chosen for its analytical simplicity and is often used in the tournament literature (see Lazear and Rosen 1981, Green and Stokey 1983, Akerlof and Holden 2012).

The overconfident player mistakenly perceives his stochastic production function to be equal to $\tilde{Q}_1 = \lambda a_1 + \varepsilon_1 + \omega$, with $\lambda > 1$. The rational player correctly perceives her stochastic production function to be equal to $Q_2 = a_2 + \varepsilon_2 + \omega$. Under this specification, player 1's overconfidence and effort are complements in generating output, that is, $\partial^2 Q_1 / \partial a_1 \partial \lambda > 0$. In other words, under this specification an overconfident player overestimates his total as well as his marginal productivity of effort.[6]

The overconfident player chooses the optimal effort level that maximizes his perceived expected utility:

$$\begin{aligned} E[U_1(a_1, a_2, \lambda)] &= u(y_L) + P_1(a_1, a_2, \lambda)\Delta u - c(a_1) \\ &= u(y_L) + \Pr(\tilde{Q}_1 \geq Q_2)\Delta u - c(a_1) \\ &= u(y_L) + \Pr(\varepsilon_2 - \varepsilon_1 \leq a_1\lambda - a_2)\Delta u - c(a_1) \\ &= u(y_L) + G(\lambda a_1 - a_2)\Delta u - c(a_1). \end{aligned} \tag{9}$$

The rational player chooses the optimal effort level that maximizes her objective

[6] An alternative specification would be $\tilde{Q}_1 = \lambda + a_1 + \varepsilon_1 + \omega$. Under this specification, player 1 overestimates his total productivity of effort while holding a correct assessment of his marginal productivity of effort.

expected utility:

$$\begin{aligned}
E[U_2(a_1,a_2)] &= u(y_L) + P_2(a_1,a_2)\Delta u - c(a_2) \\
&= u(y_L) + \Pr(Q_2 \geq Q_1)\Delta u - c(a_2) \\
&= u(y_L) + \Pr(\varepsilon_2 - \varepsilon_1 \geq a_1 - a_2)\Delta u - c(a_1) \\
&= u(y_L) + [1 - G(a_1 - a_2)]\Delta u - c(a_2). \qquad (10)
\end{aligned}$$

Since the difference between the random variables ε_1 and ε_2 will be crucial, I define the random variable $x = \varepsilon_2 - \varepsilon_1$ with cumulative distribution function $G(x)$ and density $g(x)$. I assume $G(x)$ is continuous and twice differentiable. Because ε_1 and ε_2 are identically distributed, $g(x)$ is symmetric around zero. Additionally, $g(x)$ satisfies $g'(x) > 0$ for $x < 0$, and $g'(x) < 0$ for $x > 0$.[7]

The first-order conditions are

$$\frac{\partial E[U_1(a_1,a_2,\lambda)]}{\partial a_1} = \lambda g(\lambda a_1 - a_2)\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1,a_2)]}{\partial a_2} = g(a_1 - a_2)\Delta u - c'(a_2) = 0.$$

The second-order conditions are

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2} = \lambda^2 g'(\lambda a_1 - a_2)\Delta u - c''(a_1) < 0, \qquad (11)$$

and

$$\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2} = -g'(a_1 - a_2)\Delta u - c''(a_2) < 0, \qquad (12)$$

and

$$\begin{aligned}
D = {} & \frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2}\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2} - \frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2}\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} \\
= {} & \left[\lambda^2 g'(\lambda a_1 - a_2)\Delta u - c''(a_1)\right]\left[-g'(a_1 - a_2)\Delta u - c''(a_2)\right] \\
& + \lambda g'(\lambda a_1 - a_2) g'(a_1 - a_2)(\Delta u)^2 > 0. \qquad (13)
\end{aligned}$$

[7]For example, when ε_1 and ε_2 are normally distributed with mean 0 and variance σ^2, then x is normally distributed with mean 0 and variance $2\sigma^2$. When ε_1 and ε_2 are uniformly distributed with mean 0, then x follows a triangular distribution with mean 0. See, e.g., Drago et al. (1996), Hvide (2002), Chen (2003), among others.

I assume throughout the second-order conditions (11), (12), and (13) are satisfied. The pure-strategy Nash equilibrium (a_1^*, a_2^*) satisfies the two first-order conditions simultaneously and is given by

$$\lambda g(\lambda a_1^* - a_2^*)\Delta u = c'(a_1^*), \tag{14}$$

and

$$g(a_1^* - a_2^*)\Delta u = c'(a_2^*). \tag{15}$$

The impact of overconfidence on the pure-strategy equilibrium efforts is obtained from total differentiation of (14) and (15):[8]

$$\frac{\partial a_1^*}{\partial \lambda} = \frac{1}{D^*}\left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u, \tag{16}$$

and

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{1}{D^*} g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right](\Delta u)^2, \tag{17}$$

where D^* denotes D at (a_1^*, a_2^*). As we will see, the impact of overconfidence on the pure-strategy equilibrium efforts depends on the size of player 1's bias. The following definition will prove helpful to characterize the magnitude of player 1's bias.

Definition 1: player 1 is said to be *slightly overconfident* if

$$\lambda < \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)}.$$

Conversely, player 1 is said to be *significantly overconfident* if

$$\lambda > \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)}.$$

I denote the value of the threshold that determines whether player 1 is slightly or significantly overconfident by $\bar{\lambda}$. A necessary condition for $\bar{\lambda}$ to be greater than 0 is that $g'(\lambda a_1^* - a_2^*) < 0$, or, equivalently, $\lambda a_1^* > a_2^*$. Furthermore, a necessary condition for $\bar{\lambda}$ to be greater than 1 is that $g(\lambda a_1^* - a_2^*) + a_1^* g'(\lambda a_1^* - a_2^*) > 0$.

[8] The derivation can be found in the Appendix.

Proposition 3:

(i) The overconfident player is the Nash winner of a non-monotone tournament when he is slightly overconfident, i.e., $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$ *when* $\lambda \in (1, \bar{\lambda}]$*. In this case, the overconfident player exerts more effort and the rational player exerts less effort than if both were rational. Furthermore, an increase in overconfidence raises the effort of the overconfident player and lowers that of the rational player, i.e.,* $\partial a_1^*/\partial \lambda > 0 > \partial a_2^*/\partial \lambda$.

(ii) The overconfident player is the Nash loser a non-monotone tournament when he is significantly overconfident, i.e., $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$ *when* $\lambda > \bar{\lambda}$*. In this case, both players exert less effort than if both were rational, with the overconfident player exerting the least effort. Furthermore, an increase in overconfidence lowers the efforts of both players and more so that of the overconfident player, i.e.,* $\partial a_1^*/\partial \lambda < \partial a_2^*/\partial \lambda < 0$.

In the pure-strategy Nash equilibrium, the overconfident player wins the tournament with probability $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ and the rational player with probability $P_2(a_1^*, a_2^*) = 1 - G(a_1^* - a_2^*)$. When both players are rational ($\lambda = 1$), the tournament is symmetric and the pure-strategy Nash equilibrium is $a_1^* = a_2^* = a^*$ where a^* solves $g(0)\Delta u = c'(a^*)$. Symmetry of $g(x)$ implies $P_1(a_1^*, a_2^*) = P_2(a_1^*, a_2^*) = G(0) = 1/2$. Hence, when both players are rational, each is equally likely to win the tournament (i.e., the winner is purely random).

Proposition 3 shows that in a non-monotone tournament the identity of the Nash winner depends critically on the size of overconfident player's bias. Part (i) tells us that a slightly overconfident player exerts more effort than the rational player and therefore is the Nash winner. In this case, the overconfident player believes, mistakenly, he is slightly more productive than the rational player. This raises the overconfident player's perception of his marginal productivity of effort and leads him to exert more effort. The rational player, knowing that the overconfident player believes he has a slight advantage, decides to lower her effort in response. Note that a slightly overconfident player anticipates, correctly, he will be the Nash winner but overestimates his winning probability. In fact, the overconfident player's perceived probability of winning $P_1(a_1^*, a_2^*, \lambda) = G(\lambda a_1^* - a_2^*)$ is greater than his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ since $\lambda > 1$.

Part (ii) tells us that a significantly overconfident player exerts less effort than the rational player and therefore is the Nash loser. In this case, the overconfident player believes, mistakenly, he is significantly more productive than the rational player. As a consequence, the overconfident player decides to lower his effort since he perceives to have a large productivity advantage. The rational player, knowing that the overconfident player will lower his effort also decides to lower her effort but not as much as the overconfident player. Interestingly, even though a significantly overconfident player anticipates, correctly, he will exert less effort than the rational player, he anticipates, incorrectly, he will be the Nash winner. This happens because the overconfident player's perceived probability of winning the tournament $P_1(a_1^*, a_2^*, \lambda) = G(\lambda a_1^* - a_2^*)$ is greater than 1/2 (in equilibrium $\lambda a_1^* > a_2^*$) whereas his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ is less than 1/2 (in equilibrium $a_2^* > a_1^*$).

Proposition 3 has welfare implications. The principal is better off (worse off) with a slightly overconfident when his increase in effort is greater (smaller) than the decrease in effort of the rational player. The principal is always worse off with a significantly overconfident player since both players exert less effort than if both were rational.

To evaluate the welfare implications for the overconfident player I consider how his equilibrium objective expected utility $E\left[U_1(a_1^*, a_2^*)\right] = u(y_L) + G(a_1^* - a_2^*)\Delta u - c(a_1^*)$ changes with λ:

$$\begin{aligned} \frac{\partial E\left[U_1(a_1^*, a_2^*)\right]}{\partial \lambda} &= g(a_1^* - a_2^*)\Delta u \left(\frac{\partial a_1^*}{\partial \lambda} - \frac{\partial a_2^*}{\partial \lambda}\right) - c'(a_1^*)\frac{\partial a_1^*}{\partial \lambda} \\ &= \left[g(a_1^* - a_2^*) - \lambda g(\lambda a_1^* - a_2^*)\right]\Delta u \frac{\partial a_1^*}{\partial \lambda} - g(a_1^* - a_2^*)\Delta u \frac{\partial a_2^*}{\partial \lambda}, \quad (18) \end{aligned}$$

where the second equality follows from the first-order condition of the overconfident player. The first term on the right-hand side of (18) is the direct effect and the second term is the strategic effect. The direct effect is always negative because the overconfident player fails to play a best response against his rival.[9] The sign of the strategic effect is negative when $\partial a_2^*/\partial\lambda > 0$ (player 1 is significantly overconfident) and positive when $\partial a_2^*/\partial\lambda < 0$ (player 1 is slightly overconfident). Hence, when player 1 is significantly overconfident, the direct and the strategic effects are both negative and an increase in overconfidence always makes a significantly overconfident player worse off. However, when player 1 is slightly overconfident, the direct effect is negative and the strategic effect is positive. Therefore, an increase in overconfidence can make a slightly overconfident player better off. This happens when the strategic effect dominates the direct effect.

To evaluate the welfare implications for the rational player I consider how her equilibrium objective expected utility $E\left[U_2(a_1^*, a_2^*)\right]$ changes with λ:

$$\begin{aligned}\frac{\partial E\left[U_2(a_1^*, a_2^*)\right]}{\partial\lambda} &= -g(a_1^* - a_2^*)\Delta u\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right) - c'(a_2^*)\frac{\partial a_2^*}{\partial\lambda} \\ &= -g(a_1^* - a_2^*)\frac{\partial a_1^*}{\partial\lambda}\Delta u,\end{aligned}$$

where the second equality follows from the first-order condition of the rational player. Hence, an increase in overconfidence makes the rational player worse off when overconfidence raises the effort of the overconfident player. This is the case when the rival is slightly overconfident. In contrast, an increase in overconfidence makes the rational player better off when overconfidence lowers the effort of the overconfident player. This is the case when the rival is significantly overconfident: the rational player has a higher probability of winning the tournament and exerts less effort than if both players were rational.

[9] When player 1 is slightly overconfident $a_1^* > a_2^*$ and $\partial a_1^*/\partial\lambda > 0$. The first-order conditions and $a_1^* > a_2^*$ imply $g(a_1^* - a_2^*) < \lambda g(\lambda a_1^* - a_2^*)$. Hence, when player 1 is slightly overconfident, the direct effect is negative. When player 1 is significantly overconfident $a_1^* < a_2^*$ and $\partial a_1^*/\partial\lambda < 0$. The first-order conditions and $a_1^* < a_2^*$ imply $g(a_1^* - a_2^*) > \lambda g(\lambda a_1^* - a_2^*)$. Hence, when player 1 is significantly overconfident, the direct effect is also negative.

6 Contests

This section studies contests where an overconfident player competes against a rational player. In a standard two player Tullock (1980) contest with linear effort costs the players compete for the winner prize V. player i chooses an effort level a_i to maximize $E[U_i] = P_i(a_i, a_j)V - a_i$, where $P_i(a_i, a_j)$ is the probability player i wins the contest–the contest success function (CSF). Tullock (1980) assumes the CSF is:

$$P_i(a_i, a_j) = \begin{cases} a_i^r/(a_i^r + a_j^r) & \text{if } \ a_i + a_j > 0 \\ 1/2 & \text{if } \ a_i + a_j = 0 \end{cases},$$

where $r \geq 0$. Note that under the a Tullock contest, as in a tournament, the player who exerts higher effort does not necessarily win the contest. However, unlike in a tournament, a player who exerts zero effort has a zero probability of winning if the other player exerts some positive amount of effort no matter how small.[10]

To study contests where an overconfident player competes against a rational player I consider a generalized Tullock contest. The utility of the monetary prize V is $v = u(V)$ with $u' > 0$. The effort cost is $c(a_i)$ with $c(0) = 0, c' > 0$ and $c'' \geq 0$. Following Baik (1994) I assume the CSF is:

$$P_i(a_i, a_j) = \begin{cases} q(a_i)/[q(a_i) + q(a_j)] & \text{if } \ q(a_i) + q(a_j) > 0 \\ 1/2 & \text{if } \ q(a_i) + q(a_j) = 0 \end{cases},$$

where $q(0) \geq 0$ and $q(a_i)$ is increasing in a_i.[11] The function $q(a)$ is often referred to as the impact function (Ewerhart 2015). The overconfident player mistakenly

[10] There are at least three reasons why Tullock contests are widely employed. First, a number of studies have provided axiomatic justification for it (Skaperdas 1996, Clark and Riis 1998). Second, a variety of rent-seeking contests, innovation tournaments, and patent-race games are strategically equivalent to the Tullock contest (Baye and Hoppe 2003). Third, its tractability. The drawback of Tullock contests is that they do not separate the degree to which luck as opposed to effort affects behavior (Amegashie 2006).

[11] When $q(0) = 0$ the player who spends zero effort has a zero probability of winning if the other player spends some positive effort no matter how small. This is no longer the case when $q(0) > 0$.

perceives his impact function to be $\lambda q(a_1)$, with $\lambda > 1$, and correctly perceives the rational player's impact function to be $q(a_2)$. This way of modeling overconfidence in a contest implies that the overconfident player's perceived winning probability is equal to

$$P_1(a_1, a_2, \lambda) = \begin{cases} \lambda q(a_1)/[\lambda q(a_1) + q(a_2)] & \text{if } \; q(a_1) + q(a_2) > 0 \\ 1/2 & \text{if } \; q(a_1) + q(a_2) = 0 \end{cases} .$$

This specification of overconfidence in a contest satisfies four desirable properties. First, contests where players have heterogeneous productivity of effort are modeled similarly: players are assumed to have heterogeneous impact functions (Baik 1994, Singh and Wittman 2001, Stein 2002). Second, the overconfident player's perceived winning probability is well defined for any value of $\lambda > 1$.[12] Third, the overconfident player's perceived winning probability is increasing in λ. Fourth, overestimating one's impact function is equivalent to underestimating the rival's impact function: $\lambda q(a_1)/[\lambda q(a_1) + q(a_2)] = q(a_1)/[q(a_1) + q(a_2)/\lambda]$.

The overconfident player chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_1(a_1, a_2, \lambda)] = P_1(a_1, a_2, \lambda)v - c(a_1) = \frac{\lambda q(a_1)}{\lambda q(a_1) + q(a_2)} v - c(a_1).$$

The rational player chooses the optimal effort level that maximizes her objective expected utility:

$$E[U_2(a_1, a_2)] = P_2(a_1, a_2)v - c(a_2) = \frac{q(a_2)}{q(a_1) + q(a_2)} v - c(a_2).$$

The first-order conditions are

$$\frac{\partial E[U_1(a_1, a_2, \lambda)]}{\partial a_1} = \frac{\lambda q'(a_1) q(a_2)}{[\lambda q(a_1) + q(a_2)]^2} v - c'(a_1) = 0, \tag{19}$$

[12] This is not the case with alternative specifications. For example, if one assumes the overconfident player's perceived winning probability is $P_1(a_1, a_2, \lambda) = \lambda q(a_1)/[q(a_1) + q(a_2)]$, with $\lambda > 1$, then $P_1(a_1, a_2, \lambda)$ is not a well defined probability for any value of $\lambda > 1$. Note that this specification for overconfidence is equivalent to assuming the overconfident player overestimates the winning prize.

and

$$\frac{\partial E[U_2(a_1,a_2)]}{\partial a_2} = \frac{q'(a_2)q(a_1)}{[q(a_1)+q(a_2)]^2}v - c'(a_2) = 0. \tag{20}$$

The second-order conditions are

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2} = \frac{q''(a_1)[\lambda q(a_1)+q(a_2)] - 2\lambda[q'(a_1)]^2}{[\lambda q(a_1)+q(a_2)]^3}\lambda q(a_2)v - c''(a_1) < 0, \tag{21}$$

and

$$\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2} = \frac{q''(a_2)[q(a_1)+q(a_2)] - 2[q'(a_2)]^2}{[q(a_1)+q(a_2)]^3}q(a_1)v - c''(a_2) < 0, \tag{22}$$

and

$$D = \frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2}\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2} - \frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2}\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} > 0. \tag{23}$$

I assume throughout the second-order conditions (21), (22), and (23) are satisfied. Let $a_1 = R_1(a_2)$ denote player 1's best response obtained from (19). Along player 1's best response we have

$$\lambda q'(a_1)q(a_2)v = c'(a_1)\left[\lambda q(a_1)+q(a_2)\right]^2.$$

Let $a_2 = R_2(a_1)$ denote player 2's best response obtained from (20). Along player 2's best response we have

$$q'(a_2)q(a_1)v = c'(a_2)\left[q(a_1)+q(a_2)\right]^2.$$

Lemma 1 describes the shapes of the players' best responses.

Lemma 1: *As a_2 increases from zero, the overconfident player's best response lies below curve $q(a_2) = \lambda q(a_1)$ and increases in a_2, lies on curve $q(a_2) = \lambda q(a_1)$ and reaches the maximum, and then lies above curve $q(a_2) = \lambda q(a_1)$ and decreases in a_2. As a_1 increases from zero, the rational player's best response lies above line $a_2 = a_1$ and increases in a_1, lies on line $a_2 = a_1$ and reaches the maximum, and then lies below line $a_2 = a_1$ and decreases in a_1.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort. Lemma 2 describes how the overconfident player's best response changes with λ.

Lemma 2: *When λ increases from λ_1 to λ_2, above curve $q(a_2) = \lambda_2 q(a_1)$ the overconfident player's best response shifts to the right but below curve $q(a_2) = \lambda_1 q(a_1)$ the overconfident player's best response shifts to the left, and the maximum value of the overconfident player's best response remains constant.*

Lemma 2 tells us how an increase in overconfidence shifts the best response of the overconfident player. Given high effort of the rational rival, an increase in overconfidence raises the overconfident player's effort level; given low effort of the rational rival, an increase in overconfidence lowers the overconfident player's effort level. This result is driven by player 1's marginal perceived probability of winning the contest which, using (19), is given by:

$$\frac{\partial^2 P_1(a_1, a_2, \lambda)}{\partial a_1 \partial \lambda} = \frac{q(a_2) - \lambda q(a_1)}{[\lambda q(a_1) + q(a_2)]^3} q'(a_1) q(a_2) v.$$

Lemma 2 also tells us that the maximum value of the overconfident player's best response does not depend on his degree of overconfidence.

The pure-strategy Nash equilibrium (a_1^*, a_2^*) satisfies the two first-order conditions simultaneously and is given by

$$\lambda q'(a_1^*) q(a_2^*) v = c'(a_1^*) \left[\lambda q(a_1^*) + q(a_2^*)\right]^2, \tag{24}$$

and

$$q(a_1^*) q'(a_2^*) v = c'(a_2^*) \left[q(a_1^*) + q(a_2^*)\right]^2. \tag{25}$$

From (24) and (25) it follows that

$$\lambda \frac{q'(a_1^*) q(a_2^*)}{q(a_1^*) q'(a_2^*)} = \frac{c'(a_1^*)}{c'(a_2^*)} \left[\frac{\lambda q(a_1^*) + q(a_2^*)}{q(a_1^*) + q(a_2^*)}\right]^2. \tag{26}$$

Proposition 4: *The overconfident player is the Nash loser of a generalized Tullock contest, i.e.,* $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$*. Both players exert less effort than if both were rational, with the overconfident player exerting the least effort. Furthermore, an increase in overconfidence lowers the efforts of both players and more so that of the overconfident player, i.e.,* $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

Proposition 4 shows that an overconfident player always has a lower probability of winning a generalized Tullock contest. This happens because overconfidence lowers the effort level of the overconfident player more than that of the rational player. Interestingly, even though the overconfident player knows he will exert less effort than his rational rival, he anticipates being the Nash winner. This happens because in the pure-strategy equilibrium we have $\lambda q(a_1^*) > q(a_2^*)$. This implies that the overconfident player has a perceived probability of winning the tournament $P_1(a_1^*, a_2^*, \lambda) = \lambda q(a_1^*)/[\lambda q(a_1^*) + q(a_2^*)]$ greater than 1/2.

To illustrate this result consider a Tullock contest. In this case, the impact function is $q(a_i) = a_i^r$ and the cost of effort is $c(a_i) = a_i$. This implies $q'(a_i) = ra_i^{r-1}$ and $c'(a_i) = 1$. Hence, expression (26) becomes

$$\lambda \frac{a_2^*}{a_1^*} = \left[\frac{\lambda(a_1^*)^r + (a_2^*)^r}{(a_1^*)^r + (a_2^*)^r}\right]^2. \tag{27}$$

It is easy to see that setting $a_1^* = a_2^*$ in (27) implies that the left-hand side is equal to λ and the right-hand side to λ^2 and hence the equality does not hold. Furthermore, setting $a_1^* > a_2^*$ in (27) lowers the left-hand side below λ and raises the right-hand side above λ^2 and hence the equality also does not hold. Therefore, for (27) to hold it must be that $a_1^* < a_2^*$, that is, in a Tullock contest, the overconfident player exerts less effort than the rational player.

Proposition 4 carries welfare implications. Since both players exert less effort the principal is worse off than if both players were rational. To evaluate the welfare implications for the overconfident player I consider how his equilibrium objective expected utility $E[U_1(a_1^*, a_2^*)] = vq(a_1^*)/[q(a_1^*) + q(a_2^*)] - c(a_1^*)$ changes with λ:

$$\frac{\partial E[U_1(a_1^*, a_2^*)]}{\partial \lambda} = \left[\frac{vq'(a_1^*)q(a_2^*)}{[q(a_1^*) + q(a_2^*)]^2} - c'(a_1^*)\right]\frac{\partial a_1^*}{\partial \lambda} - \frac{vq(a_1^*)}{[q(a_1^*) + q(a_2^*)]^2}\frac{\partial a_2^*}{\partial \lambda}, \tag{28}$$

The first term on the right-hand side of (28) is the direct effect and is negative since the overconfident player fails to best respond against his rival. The second term on the right-hand side of (28) is the strategic effect and is positive since $\partial a_2^*/\partial \lambda < 0$. Hence, an increase in overconfidence has an ambiguous impact on the welfare of the overconfident player.

To evaluate the welfare implications for the rational player I consider how her equilibrium objective expected utility $E[U_2(a_1^*, a_2^*)]$ changes with λ:

$$\begin{aligned}\frac{\partial E[U_2(a_1^*, a_2^*)]}{\partial \lambda} &= \left[\frac{vq(a_1^*)q'(a_2^*)}{[q(a_1^*) + q(a_2^*)]^2} - c'(a_2^*)\right]\frac{\partial a_2^*}{\partial \lambda} - \frac{vq(a_2^*)}{[q(a_1^*) + q(a_2^*)]^2}\frac{\partial a_1^*}{\partial \lambda} \\ &= -\frac{vq(a_2^*)}{[q(a_1^*) + q(a_2^*)]^2}\frac{\partial a_1^*}{\partial \lambda},\end{aligned}$$

where the second equality comes from the first-order condition of the rational player. Hence, an increase in overconfidence makes the rational player better off since overconfidence always lowers the effort of the overconfident player. Note that the rational player has a higher equilibrium probability of winning the contest and exerts less effort than in a benchmark contest with two rational players

7 Conclusion

This paper studies the impact of overconfidence on tournaments and contests where an overconfident player competes against a rational player. The overconfident player overestimates his productivity of effort and, as a consequence, his probability of winning. The paper provides conditions under which the overconfident player is either the Nash winner or loser of a tournament. The overconfidence player is the Nash winner (loser) of a monotone tournament when his effort and overconfidence are complements (substitutes). The overconfident player is the Nash winner (loser) of a non-monotone tournament when he is slightly (significantly) overconfident. The overconfidence player is always the Nash loser of a contest.

The results hold under the assumption that an overconfident player overestimates his productivity of effort. This way of modeling overconfidence is often used in principal-agent settings (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). Alternatively, an overconfident player might underestimate the cost of effort. This alternative specification for overconfidence can lead to different results as shown by previous studies on contests (Ando 2004, Ludwig et al. 2011).

The results were derived under the assumption that tournament and contest prizes are exogenously specified. However, if the principal is aware of the asymmetry in beliefs of the players he will adapt the prizes to the players' characteristics. This is an interesting avenue for future research.

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8 Appendix

Derivation of Equations (5) and (6): Total differentiation of the first-order conditions (3) and (4) gives:

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\partial a_1^* + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\partial a_2^* + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\partial \lambda\right]\Delta u = c''(a_1^*)\partial a_1^*.$$

and

$$\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\partial a_1^* + \frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\partial a_2^*\right]\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial \lambda$ we obtain

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\frac{\partial a_2^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial \lambda}, \quad (29)$$

and

$$\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\frac{\partial a_2^*}{\partial \lambda}\right]\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial \lambda}. \quad (30)$$

Solving (30) for $\partial a_2^*/\partial \lambda$ we have

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda}. \quad (31)$$

Substituting (31) into (29) we obtain

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\frac{\partial a_1^*}{\partial \lambda} - \frac{\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial \lambda}.$$

Solving this equation for $\partial a_1^*/\partial \lambda$ we find (5). Substituting (5) into (31) we obtain (6).

Proof of Proposition 1:

i) When player 1's overconfidence and effort are complements

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} > 0.$$

In addition, if efforts are strategic complements, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} > 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (5) and (6) imply

$$\frac{\partial a_1^*}{\partial \lambda} > 0 \text{ and } \frac{\partial a_2^*}{\partial \lambda} > 0.$$

We know from (31) that

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1} \Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2} \Delta u - c''(a_2^*)} \frac{\partial a_1^*}{\partial \lambda}. \tag{32}$$

Assumptions (1), and (2) imply that the first term on the right-hand side of (32) is greater than 0 and less than 1. Hence, it follows that

$$\frac{\partial a_1^*}{\partial \lambda} > \frac{\partial a_2^*}{\partial \lambda} > 0. \tag{33}$$

If (33) holds and players have identical utility functions, then $a_1^* > a_2^*$. If $a_1^* > a_2^*$ and players have identical productivity of effort, then $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$.
ii) When player 1's overconfidence and effort are complements

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} > 0.$$

In addition, if efforts are strategic substitutes, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} < 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (5) and (6) imply

$$\frac{\partial a_1^*}{\partial \lambda} > 0 > \frac{\partial a_2^*}{\partial \lambda}. \tag{34}$$

If (34) holds and players have identical utility functions, then $a_1^* > a_2^*$. If $a_1^* > a_2^*$ and players have identical productivity of effort, then $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$.

Proof of Proposition 2:

i) When player 1's overconfidence and effort are substitutes

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} < 0.$$

In addition, if efforts are strategic complements, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} > 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (5) and (6) imply

$$\frac{\partial a_1^*}{\partial \lambda} < 0 \text{ and } \frac{\partial a_2^*}{\partial \lambda} < 0.$$

We know from (31) that

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1} \Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2} \Delta u - c''(a_2^*)} \frac{\partial a_1^*}{\partial \lambda}.$$

Assumptions (1), and (2) imply that the first term on the right-hand side of (32) is greater than 0 and less than 1. Hence, it follows that

$$\frac{\partial a_1^*}{\partial \lambda} < \frac{\partial a_2^*}{\partial \lambda} < 0. \tag{35}$$

If (35) holds and players have identical utility functions, then $a_2^* > a_1^*$. If $a_2^* > a_1^*$ and players have identical productivity of effort, then $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.

ii) When player 1's overconfidence and effort are substitutes

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} < 0.$$

In addition, if efforts are strategic substitutes, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} < 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (5) and (6) imply

$$\frac{\partial a_2^*}{\partial \lambda} > 0 > \frac{\partial a_1^*}{\partial \lambda}. \tag{36}$$

If (36) holds and players have identical utility functions, then $a_2^* > a_1^*$. If $a_2^* > a_1^*$ and players have identical productivity of effort, then $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.

Derivation of Equations (16) and (17): Total differentiation of the first-order conditions (14) and (15) gives us:

$$\partial\lambda g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)(a_1^*\partial\lambda + \lambda\partial a_1^* - \partial a_2^*)\Delta u = c''(a_1^*)\partial a_1^*$$

and

$$g'(a_1^* - a_2^*)(\partial a_1^* - \partial a_2^*)\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial\lambda$ we obtain

$$g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)\left(a_1^* + \lambda\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}, \tag{37}$$

and

$$g'(a_1^* - a_2^*)\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}. \tag{38}$$

Solving (38) for $\partial a_2^*/\partial\lambda$ we have

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}. \tag{39}$$

Substituting (39) into (37) we obtain

$$g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)\left[a_1^* + \lambda\frac{\partial a_1^*}{\partial\lambda} - \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}.$$

Solving this equation for $\partial a_1^*/\partial\lambda$ we obtain (16). Substituting (16) into (39) we obtain (17).

Proof of Proposition 3: To prove this result I consider the four possible ways an increase in player 1's overconfidence can change the pure-strategy Nash equilibrium efforts and show that only two of them are feasible.

(i) Assume $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda > 0$. If $\partial a_1^*/\partial\lambda > 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$. If $\partial a_2^*/\partial\lambda > 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$, then $g'(a_1^* - a_2^*) > 0$. Now, $g'(a_1^* - a_2^*) > 0$ and $g'(x) > 0$ for $x < 0$ implies $a_2^* > a_1^*$. This, in turn, implies $\partial a_2^*/\partial\lambda > \partial a_1^*/\partial\lambda$ or

$$\begin{aligned} & g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right](\Delta u)^2 \\ > \; & \left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u, \end{aligned}$$

or

$$g'(a_1^* - a_2^*)\Delta u > g'(a_1^* - a_2^*)\Delta u + c''(a_2^*),$$

or

$$c''(a_2^*) < 0,$$

which contradicts $c'' > 0$. Hence, $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda > 0$ do not characterize the impact of player 1's overconfidence on the Nash equilibrium efforts.

(ii) Assume $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$. This implies $a_1^* < a_2^*$. This, in turn, implies $g'(a_1^* - a_2^*) > 0$. If $\partial a_1^*/\partial\lambda < 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$. If $\partial a_2^*/\partial\lambda > 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$, then $g'(a_1^* - a_2^*) < 0$. But $g'(a_1^* - a_2^*) < 0$ contradicts $g'(a_1^* - a_2^*) > 0$. Hence, $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$ do not characterize the impact of player 1's overconfidence on the Nash equilibrium efforts.

(iii) Assume $\partial a_2^*/\partial\lambda < 0 < \partial a_1^*/\partial\lambda$. This implies $a_1^* > a_2^*$. This, in turn, implies $g'(a_1^* - a_2^*) < 0$. Furthermore, $a_1^* > a_2^*$ and $\lambda > 1$ imply $\lambda a_1^* > a_2^*$. This, in turn, implies $g'(\lambda a_1^* - a_2^*) < 0$. Since $D^* > 0$, then for $\partial a_1^*/\partial\lambda > 0$ it must be that

$$g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0,$$

or

$$\lambda < \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)} = \bar{\lambda}.$$

Note that $g(\lambda a_1^* - a_2^*) > 0$ and $g'(\lambda a_1^* - a_2^*) < 0$ imply that $\bar{\lambda}$ is strictly positive. Furthermore, $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$ implies $\bar{\lambda} > 1$. Hence, $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda < 0$ characterize the impact of player 1's overconfidence on the Nash equilibrium efforts when $\lambda \in (1, \bar{\lambda})$. Let us now check whether the pure-strategy Nash equilibrium with a slightly overconfident player satisfies the second-order conditions (11), (12), and (13). The second-order conditions at (a_1^*, a_2^*) are

$$\lambda^2 g'(\lambda a_1^* - a_2^*)\Delta u - c''(a_1^*) < 0,$$

and

$$-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*) < 0,$$

and

$$D^* = \left[\lambda^2 g'(\lambda a_1^* - a_2^*)\Delta u - c''(a_1^*)\right]\left[-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*)\right] + \lambda g'(\lambda a_1^* - a_2^*) g'(a_1^* - a_2^*)(\Delta u)^2 > 0.$$

The first second-order condition is satisfied since $g'(\lambda a_1^* - a_2^*) < 0$. The second second-order condition might not be satisfied since $g'(a_1^* - a_2^*) < 0$. The third second-order condition is satisfied since $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) < 0$ imply $g'(\lambda a_1^* - a_2^*) g'(a_1^* - a_2^*) > 0$. Hence, the pure-strategy Nash equilibrium with a slightly overconfident player satisfies the second-order conditions provided that $-g'(a_1^* - a_2^*)\Delta u < c''(a_2^*)$.
(iv) Assume $\partial a_1^*/\partial\lambda < 0$ and $\partial a_2^*/\partial\lambda < 0$. If $\partial a_1^*/\partial\lambda < 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$. A necessary (but not sufficient) condition for $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$ is that $g'(\lambda a_1^* - a_2^*) < 0$. If $g'(\lambda a_1^* - a_2^*) < 0$, then $\lambda a_1^* > a_2^*$. If $\partial a_2^*/\partial\lambda < 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$, then $g'(a_1^* - a_2^*) > 0$. Furthermore, if $g'(a_1^* - a_2^*) > 0$, then $a_2^* > a_1^*$. Hence, we have $\lambda a_1^* > a_2^* > a_1^*$. This, in turn, implies $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda$ or

$$\begin{aligned} & \left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u \\ < \; & g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^3)\right](\Delta u)^2, \end{aligned}$$

or

$$g'(a_1^* - a_2^*)\Delta u + c''(a_2^*) > g'(a_1^* - a_2^*)\Delta u,$$

or

$$c''(a_2^*) > 0,$$

which is true. Hence, $\partial a_1^*/\partial\lambda < 0$ and $\partial a_2^*/\partial\lambda < 0$ characterize the impact of player 1's overconfidence on the Nash equilibrium efforts when $\lambda > \bar{\lambda}$. Let us now check whether the pure-strategy Nash equilibrium with a significantly overconfident player satisfies the second-order conditions (11), (12), and (13). The second-order conditions at (a_1^*, a_2^*) are

$$\lambda^2 g'(\lambda a_1^* - a_2^*)\Delta u - c''(a_1^*) < 0,$$

and

$$-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*) < 0,$$

and

$$D^* = \left[\lambda^2 g'(\lambda a_1^* - a_2^*)\Delta u - c''(a_1^*)\right]\left[-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*)\right] + \lambda g'(\lambda a_1^* - a_2^*) g'(a_1^* - a_2^*)(\Delta u)^2 > 0.$$

The first second-order condition is satisfied since $g'(\lambda a_1^* - a_2^*) < 0$. The second second-order condition is also satisfied since $g'(a_1^* - a_2^*) > 0$. Finally, to see that the third second-order condition is also satisfied note that

$$\begin{aligned} D^* &= \lambda(1-\lambda) g'(\lambda a_1^* - a_2^*) g'(a_1^* - a_2^*)(\Delta u)^2 - \lambda^2 g'(\lambda a_1^* - a_2^*) c''(a_2^*)\Delta u \\ &\quad + g'(a_1^* - a_2^*) c''(a_1^*)\Delta u + c''(a_1^*) c''(a_2^*). \end{aligned} \tag{40}$$

When $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, the first term in (40) is positive since $\lambda > 1$, $g'(\lambda a_1^* - a_2^*) < 0$, and $g'(a_1^* - a_2^*) > 0$. The second and third terms in (40) are also positive since $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, respectively. Finally, the fourth term in (40) also is positive since $c'' > 0$. Hence, the pure-strategy Nash equilibrium with a significantly overconfident player satisfies the second-order conditions.

I now show that player 1 has a higher (lower) probability of winning if he is slightly (significantly) overconfident. player 1's probability of winning is $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$. player 2's probability of winning is $P_2(a_1^*, a_2^*) = 1 - G(a_1^* - a_2^*)$. We have $G(0) = 0$ and $G' > 0$. When $\lambda = 1$ the tournament is symmetric and the pure-strategy Nash equilibrium is $a_1^* = a_2^* = a$. Symmetry of $g(x)$ implies that $P_1(a_1^*, a_2^*) = P_2(a_1^*, a_2^*) = G(0) = 1/2$. If player 1 is slightly overconfident, then $\partial a_2^*/\partial\lambda < 0 < \partial a_1^*/\partial\lambda$ which implies $a_1^* > a_2^*$. Hence, if player 1 is slightly overconfident, then

$$P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*) > 1/2 > 1 - G(a_1^* - a_2^*) = P_2(a_1^*, a_2^*).$$

In contrast, if player 1 is significantly overconfident, then $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$ which implies $a_1^* < a_2^*$. Hence, if player 1 is significantly overconfident, then

$$P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*) < 1/2 < 1 - G(a_1^* - a_2^*) = P_2(a_1^*, a_2^*).$$

Proof of Lemma 1: The best response of player 1 is defined implicitly by (19). Hence, the slope of the best response of player 1 is

$$R_1'(a_2) = -\frac{\partial R_1/\partial a_2}{\partial R_1/\partial a_1} = -\frac{\frac{\partial^2 E[U_1]}{\partial a_1 \partial a_2}}{\frac{\partial^2 E[U_1]}{\partial a_1^2}} = -\frac{\frac{\lambda q(a_1) - q(a_2)}{[\lambda q(a_1) + q(a_2)]^3}\lambda q'(a_1) q'(a_2) v}{\frac{q''(a_1)[\lambda q(a_1) + q(a_2)] - 2\lambda[q'(a_1)]^2}{[\lambda q(a_1) + q(a_2)]^3}\lambda q(a_2) v - c''(a_1)}.$$

The denominator is player 1's second-order condition and so it is negative. Therefore, the sign of the slope of player 1's best response is only determined by the sign of the numerator which only depends on $\lambda q(a_1) - q(a_2)$. Hence, $R_1'(a_2)$ is positive for $\lambda q(a_1) > q(a_2)$, zero for $\lambda q(a_1) = q(a_2)$, and negative for $\lambda q(a_1) < q(a_2)$. This implies that $R_1(a_2)$ increases in a_2 for $\lambda q(a_1) > q(a_2)$, reaches the maximum at $\lambda q(a_1) = q(a_2)$, and decreases in a_2 for $\lambda q(a_1) < q(a_2)$.

The best response of player 2 is defined implicitly by (20). Hence, the slope of the best response of player 2 is

$$R_2'(a_1) = -\frac{\partial R_2/\partial a_1}{\partial R_2/\partial a_2} = -\frac{\frac{\partial^2 E[U_2]}{\partial a_2 \partial a_1}}{\frac{\partial^2 E[U_2]}{\partial a_2^2}} = -\frac{\frac{q(a_2) - q(a_1)}{[q(a_1) + q(a_2)]^3} q'(a_1) q'(a_2) v}{\frac{q''(a_2)[q(a_1) + q(a_2)] - 2\lambda[q'(a_2)]^2}{[q(a_1) + q(a_2)]^3} q(a_1) v - c''(a_2)}.$$

The denominator is player 2's second-order condition and so it is negative. Therefore, the sign of the slope of player 2's best response is only determined by the sign of the numerator which only depends on $q(a_2) - q(a_1)$ or, equivalently, on $a_2 - a_1$. Hence,

$R_2'(a_1)$ is positive for $a_2 > a_1$, zero for $a_2 = a_1$, and negative for $a_2 < a_1$. This implies that $R_2(a_1)$ increases in a_1 for $a_2 > a_1$, reaches the maximum at $a_1 = a_2$, and decreases in a_1 for $a_2 < a_1$.

Proof of Lemma 2: (This proof follows Baik 1994) The overconfident player's best response is defined by (19):

$$\frac{\lambda q'(a_1)q(a_2)}{[\lambda q(a_1) + q(a_2)]^2}v - c'(a_1) = 0.$$

Hence, we have

$$\frac{\partial R_1(a_2)}{\partial \lambda} = \frac{q(a_2) - \lambda q(a_1)}{[\lambda q(a_1) + q(a_2)]^3}q'(a_1)q(a_2)v.$$

We see that when λ increases from λ_1 to λ_2, $\partial R_1(a_2)/\partial\lambda > 0$ holds at the points above curve $q(a_2) = \lambda_2 q(a_1)$ but $\partial R_1(a_2)/\partial\lambda < 0$ holds at the points below curve $q(a_2) = \lambda_1 q(a_1)$. This follows from the fact that $q(a_2) > \lambda_2 q(a_1)$ holds at point above curve $q(a_2) = \lambda_2 q(a_1)$ but $q(a_2) < \lambda_1 q(a_1)$ holds at points below curve $q(a_2) = \lambda_1 q(a_1)$. We also know, from the second-order condition of player 1, that $\partial R_1(a_2)/\partial a_1 < 0$ and $\partial R_1(a_2)/\partial\lambda > 0$ holds at the points above curve $q(a_2) = \lambda_2 q(a_1)$ but $\partial R_1(a_2)/\partial a_1 < 0$ and $\partial R_1(a_2)/\partial\lambda < 0$ hold at points below curve $q(a_2) = \lambda q(a_1)$. Therefore, given player 2's effort level, when λ increases from λ_1 to λ_2, above curve $q(a_2) = \lambda_2 q(a_1)$ player 1's effort must increase but below curve $q(a_2) = \lambda_1 q(a_1)$ player 1's effort must decrease, in order to satisfy the first-order condition. Furthermore, as λ increases from λ_1 to λ_2, the maximum value of the overconfident player's best response remains unchanged. We know from Lemma 1 that the maximum point of the overconfident player's best response satisfies the first-order condition (19) and $q(a_2) = \lambda q(a_1)$. Substituting $q(a_2) = \lambda q(a_1)$ into the first-order condition of player 1 we obtain

$$\frac{\lambda q'(a_1)\lambda q(a_1)}{[\lambda q(a_1) + \lambda q(a_1)]^2}v = c'(a_1),$$

or

$$\frac{\lambda^2 q'(a_1)q(a_1)}{4\lambda^2 [q(a_1)]^2}v = c'(a_1),$$

or

$$\frac{q'(a_1)}{4q(a_1)}v = c'(a_1).$$

This implies that the value of a_1 corresponding to the maximum value of the overconfident player's best response does not depend on λ.

Proof of Proposition 4: The proof has three steps. First, I show that for the Nash equilibrium (a_1^*, a_2^*) to satisfy (26) it must be located below curve $q(a_2) = \lambda q(a_1)$. Second, I show that for the Nash equilibrium (a_1^*, a_2^*) to satisfy (26) it must be located above line $a_2 = a_1$. Note that the second step implies that the overconfident player exerts less effort than the rational player. Third, I show that both players exert less effort than if both were rational.

Step 1: Assume, by contradiction, the Nash equilibrium (a_1^*, a_2^*) is located on curve $q(a_2) = kq(a_1)$, with $k \geq \lambda$. If that is the case, then $q(a_2^*) = kq(a_1^*)$ and $q'(a_2^*) = kq'(a_1^*)$. In addition, $k \geq \lambda > 1$ and $q(a_2^*) = kq(a_1^*)$ imply $a_2^* > a_1^*$ which, given convexity of $c(a_i)$, implies $c'(a_2^*) > c'(a_1^*)$. However, on curve $q(a_2^*) = kq(a_1^*)$, we have

$$\lambda \frac{q'(a_1^*)q(a_2^*)}{q(a_1^*)q'(a_2^*)} > \frac{c'(a_1^*)}{c'(a_2^*)} \left[\frac{\lambda q(a_1^*) + q(a_2^*)}{q(a_1^*) + q(a_2^*)} \right]^2,$$

since

$$\lambda \frac{q'(a_1^*)kq(a_1^*)}{q(a_1^*)kq'(a_1^*)} > \frac{c'(a_1^*)}{c'(a_2^*)} \left[\frac{\lambda q(a_1^*) + kq(a_1^*)}{q(a_1^*) + kq(a_1^*)} \right]^2,$$

or

$$\lambda > \frac{c'(a_1^*)}{c'(a_2^*)} \left(\frac{\lambda + k}{1 + k} \right)^2,$$

which holds given that $c'(a_2^*) > c'(a_1^*)$ and $k \geq \lambda > 1$. This shows that (26) is not satisfied on the curve $q(a_2) = kq(a_1)$ with $k \geq \lambda$. Hence, for the Nash equilibrium (a_1^*, a_2^*) to satisfy (26) it must be located below curve $q(a_2) = \lambda q(a_1)$. If the Nash equilibrium (a_1^*, a_2^*) is located below curve $q(a_2) = \lambda q(a_1)$, then $q(a_2^*) < \lambda q(a_1^*)$.

Step 2: Assume, by contradiction, that the Nash equilibrium (a_1^*, a_2^*) is located on curve $q(a_2) = mq(a_1)$, with $m \in (0, 1]$. If that is the case, then $q(a_2^*) = mq(a_1^*)$ and $q'(a_2^*) = mq'(a_1^*)$. In addition, $m \leq 1$ and $q(a_2^*) = mq(a_1^*)$ imply $a_2^* \leq a_1^*$ which, given convexity of $c(a_i)$, implies $c'(a_2^*) \leq c'(a_1^*)$. Hence, on curve $q(a_2^*) = mq(a_1^*)$, we have

$$\lambda \frac{q'(a_1^*)q(a_2^*)}{q(a_1^*)q'(a_2^*)} < \frac{c'(a_1^*)}{c'(a_2^*)} \left[\frac{\lambda q(a_1^*) + q(a_2^*)}{q(a_1^*) + q(a_2^*)} \right]^2,$$

since

$$\lambda \frac{q'(a_1^*)}{q(a_1^*)} \frac{mq(a_1^*)}{mq'(a_1^*)} < \frac{c'(a_1^*)}{c'(a_2^*)} \left[\frac{\lambda q(a_1^*) + mq(a_1^*)}{q(a_1^*) + mq(a_1^*)} \right]^2,$$

or

$$\lambda < \frac{c'(a_1^*)}{c'(a_2^*)} \left(\frac{\lambda + m}{1 + m} \right)^2,$$

which holds given that $c'(a_2^*) \leq c'(a_1^*)$ and $\lambda > 1 > m$. This shows that (26) is not satisfied on the curve $q(a_2) = mq(a_1)$ with $m \in (0,1]$. Hence, for the Nash equilibrium (a_1^*, a_2^*) to satisfy (26) it must be located above line $a_2 = a_1$. If the Nash equilibrium (a_1^*, a_2^*) is located above line $a_2 = a_1$, then $a_2^* > a_1^*$. Hence, in the Nash equilibrium of a generalized Tullock contest the rational player exerts more effort than the overconfident player. Since both players are equally productive, this implies the overconfident player is the Nash loser of a generalized Tullock contest.

Step 3: We know from step 2 that $a_2^* > a_1^*$. We know for Lemma 2 that the rational player's best response reaches its maximum on line $a_2 = a_1$. Hence, it must be that the rational player exerts a lower effort than in the case where $\lambda = 1$. This shows that player's 1 overconfidence lowers the efforts of both players compared to the case where $\lambda = 1$. Moreover, the higher is the overconfidence of player 1, the lower are the efforts of both players. The impact of player 1's overconfidence on the pure-strategy equilibrium efforts is obtained from total differentiation of (24) and (25):

$$\begin{aligned}
&[q'(a_1^*)q(a_2^*)\partial\lambda + \lambda q''(a_1^*)q(a_2^*)\partial a_1^* + \lambda q'(a_1^*)q'(a_2^*)\partial a_2^*]\, v \\
&\qquad = c''(a_1^*) \left[\lambda q(a_1^*) + q(a_2^*)\right]^2 \partial a_1^* \\
&\qquad + 2c'(a_1^*) \left[\lambda q(a_1^*) + q(a_2^*)\right] \left[q(a_1^*)\partial\lambda + \lambda q'(a_1^*)\partial a_1^* + q'(a_2^*)\partial a_2^*\right],
\end{aligned}$$

and

$$\begin{aligned}
&[q'(a_1^*)q'(a_2^*)\partial a_1^* + q(a_1^*)q''(a_2^*)\partial a_2^*]\, v = c''(a_2^*) \left[q(a_1^*) + q(a_2^*)\right]^2 \partial a_2^* \\
&\qquad + 2c'(a_2^*) \left[q(a_1^*) + q(a_2^*)\right] \left[q'(a_1^*)\partial a_1^* + q'(a_2^*)\partial a_2^*\right].
\end{aligned}$$

Diving both equations by $\partial\lambda$ we obtain

$$\left[q'(a_1^*)q(a_2^*) + \lambda q''(a_1^*)q(a_2^*)\frac{\partial a_1^*}{\partial\lambda} + \lambda q'(a_1^*)q'(a_2^*)\frac{\partial a_2^*}{\partial\lambda}\right]v$$
$$= c''(a_1^*)\left[\lambda q(a_1^*) + q(a_2^*)\right]^2\frac{\partial a_1^*}{\partial\lambda}$$
$$+ 2c'(a_1^*)\left[\lambda q(a_1^*) + q(a_2^*)\right]\left[q(a_1^*) + \lambda q'(a_1^*)\frac{\partial a_1^*}{\partial\lambda} + q'(a_2^*)\frac{\partial a_2^*}{\partial\lambda}\right],$$

and

$$\left[q'(a_1^*)q'(a_2^*)\frac{\partial a_1^*}{\partial\lambda} + q(a_1^*)q''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}\right]v$$
$$= c''(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]^2\frac{\partial a_2^*}{\partial\lambda}$$
$$+ 2c'(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]\left[q'(a_1^*)\frac{\partial a_1^*}{\partial\lambda} + q'(a_2^*)\frac{\partial a_2^*}{\partial\lambda}\right].$$

Solving the second equation for $\partial a_2^*/\partial\lambda$ we obtain

$$q'(a_1^*)q'(a_2^*)\frac{\partial a_1^*}{\partial\lambda}\Delta u + q(a_1^*)q''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}v$$
$$= c''(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]^2\frac{\partial a_2^*}{\partial\lambda} + 2c'(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]q'(a_1^*)\frac{\partial a_1^*}{\partial\lambda}$$
$$+ 2c'(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]q'(a_2^*)\frac{\partial a_2^*}{\partial\lambda},$$

or

$$\left[q(a_1^*)q''(a_2^*)\Delta u - 2c'(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]q'(a_2^*) - c''(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]^2\right]\frac{\partial a_2^*}{\partial\lambda}$$
$$= \left[2c'(a_2^*)\left[q(a_1^*) + q(a_2^*)\right]q'(a_1^*) - q'(a_1^*)q'(a_2^*)v\right]\frac{\partial a_1^*}{\partial\lambda},$$

or

$$
\begin{aligned}
\frac{\partial a_2^*}{\partial \lambda} &= \frac{2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]q'(a_1^*) - q'(a_1^*)q'(a_2^*)v}{q(a_1^*)q''(a_2^*)\Delta u - 2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]q'(a_2^*) - c''(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]^2}\frac{\partial a_1^*}{\partial \lambda} \\
&= \frac{\frac{2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]-q'(a_2^*)v}{\left[q(a_1^*)+q(a_2^*)\right]^2}q'(a_1^*)}{\frac{q(a_1^*)q''(a_2^*)\Delta u-2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]q'(a_2^*)}{\left[q(a_1^*)+q(a_2^*)\right]^2} - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} \\
&= \frac{\frac{2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]-q'(a_2^*)v}{\left[q(a_1^*)+q(a_2^*)\right]^2}q'(a_1^*)}{\frac{q(a_1^*)q''(a_2^*)[q(a_1^*)+q(a_2^*)]v-2c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]^2q'(a_2^*)}{\left[q(a_1^*)+q(a_2^*)\right]^3} - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} \\
&= \frac{\frac{2c'(a_2^*)q(a_1^*)\left[q(a_1^*)+q(a_2^*)\right]-q(a_1^*)q'(a_2^*)v}{\left[q(a_1^*)+q(a_2^*)\right]^2}\frac{q'(a_1^*)}{q(a_1^*)}}{\frac{q(a_1^*)q''(a_2^*)[q(a_1^*)+q(a_2^*)]v-2\frac{q(a_1^*)q'(a_2^*)v}{\left[q(a_1^*)+q(a_2^*)\right]^2}\left[q(a_1^*)+q(a_2^*)\right]^2q'(a_2^*)}{\left[q(a_1^*)+q(a_2^*)\right]^3} - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} \\
&= \frac{\frac{2c'(a_2^*)q(a_1^*)-c'(a_2^*)\left[q(a_1^*)+q(a_2^*)\right]}{q(a_1^*)+q(a_2^*)}\frac{q'(a_1^*)}{q(a_1^*)}}{\frac{q''(a_2^*)[q(a_1^*)+q(a_2^*)]-2[q'(a_2^*)]^2}{\left[q(a_1^*)+q(a_2^*)\right]^3}q(a_1^*)v - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} \\
&= \frac{\frac{q(a_1^*)-q(a_2^*)}{q(a_1^*)+q(a_2^*)}\frac{c'(a_2^*)q'(a_1^*)}{q(a_1^*)}}{\frac{q''(a_2^*)[q(a_1^*)+q(a_2^*)]-2[q'(a_2^*)]^2}{\left[q(a_1^*)+q(a_2^*)\right]^3}q(a_1^*)v - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda}.
\end{aligned}
$$

The denominator is negative from the second-order condition of player 2. The numerator is negative as long as $a_2^* > a_1^*$. Hence, this expression implies that if $a_2^* > a_1^*$, then the sign of $\partial a_2^*/\partial\lambda$ is the same as the sign of $\partial a_1^*/\partial\lambda$. This shows that an increase in overconfidence lowers the efforts of the two players and more so that of the overconfident player.