Asymmetric Information, Bank Lending and Implicit Contracts: The Winner's Curse*

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Abstract

The purpose of this note is to point out an error in a widely cited paper by Sharpe (1990) on long-term bank-firm relationships and to provide a correct analysis of the problem. The model studies repeated lending under asymmetric information which leads to winner's-curse type distortions of competition. Contrary to the claims in Sharpe (1990), this game only has an equilibriuim in mixed strategies, which features a partial informational lock-in by firms and random termination of lending relationships.

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In a widely cited paper, Sharpe (1990) has formulated a model of corporate borrowing under asymmetric information which provides a theoretical explanation of long-term bank-firm relationships. While the model is conceptually important and makes a main feature of actual lending relationships amenable to theoretical analysis, the analysis offered in the paper is not correct. The purpose of this note is to point out the error and to provide a correct analysis of the problem. Furthermore, I want to draw attention to the work by Fischer (1990), who has studied a simpler version of the same problem independently and correctly, though not with full mathematical rigour.

Sharpe (1990) and Fischer (1990) consider a model of repeated corporate borrowing under adverse selection, in which lenders obtain inside information about their borrowers' quality. This inside information gives existing lenders an informational advantage over potential competitors at the refinancing stage and reduces ex-post competition. Hence, an initial situation of competition between symmetrically informed lenders turns into one of asymmetric information once one lender has attracted the business and dealt with the customer for some time. Since borrowers and lenders rationally anticipate that the borrower will be "informationally captured" in the relationship in the future, initial finance is offered at a discount which reflects the expected mark up on future terms of finance.

In the absence of binding long-term contracts, these future terms of finance are determined at the refinancing stage by competition between the inside lender and potential outside competitors. The analysis of this interaction – a contract offer game under asymmetric information – provided by Sharpe (1990) is incorrect. In Proposition 1, he identifies two pure-strategy Nash equilibria for one version of this game, in Proposition 2 one pure-strategy Nash equilibrium for another version. However, these games do not have pure-strategy equilibria at all.

The principal reason for the non-existence of pure-strategy equilibria in this situation is a "winner's curse" type phenomenon, known from the theory of competitive bidding (see, e.g., Wilson (1967), Milgrom and Weber (1982)).[1] Under asymmetric information about the common value of an object – here: the profitability of a lending contract – the fact that a bid wins

[1] The mathematical theory of bidding under asymmetric information relevant for the competition problem in Sharpe (1990) has been developed by Engelbrecht-Wiggans, Milgrom and Weber (1983).

contains information about the value of the object. In particular, the higher the bid by an individual bidder – here: the lower the interest rate offered by a single lender – the higher is not only the probability of obtaining the object, but also that the object, if obtained, is estimated by others to be of lower value. Therefore, bidding in such situations must not only take individual private information into account, but also the information that would be revealed by the fact that the bid wins over the others. This limits the viability of standard overbidding (undercutting) strategies à la Bertrand, without completely eliminating the incentives to use them. As a consequence, pure strategies, which are directly vulnerable to overbidding (undercutting), cannot constitute an equilibrium. However, mixed-strategy equilibria exist, because through optimal randomization competitors can balance the gains from increasing the bid's success probability and the losses from decreasing the expected value of the object conditional on winning.

The similarities between banking competition for corporate customers and bidding in auctions with common values have first been explored by Broecker (1990) in a model of interbank competition under imperfect information acquisition. In particular, Broecker (1990) studies the nature of the resulting mixed-strategy equilibria if the number of competing banks becomes large. Rajan (1992), which enriches the Fischer-Sharpe model by the possibility of long-term lending, finds a mixed-strategy equilibrium in a situation similar to that of Sharpe (1990).

Equilibrium in mixed strategies is the appropriate formalization of the intuition of "uneven competition" between inside and outside lenders for the provision of continuation finance. While informed lenders can be expected to capture some informational rent in this game, it is implausible that they can completely dictate the terms of the contract. Yet, any deterministic counteroffer to such dictatorial insider offers by an outsider would, in turn, fall prey to selective undercutting by the insider, leaving the outsider with losses. However, randomization of offers allows the outsider to keep the rents extracted by the insider in check, without exposing herself too directly to the thrust of the insider's superior information.[2] Empirically, therefore, the model predicts a *limited informational capture* of borrowers in bank-firm relationships, with interest rates charged above the market rate and occasional

[2]Since contract offers by competing lenders are not announced publicly, there is no incentive to change the random contract offers once competing offers have been observed. Hence, the standard objection to the plausibility of mixed strategies in the case of posted pricing (cf. Tirole (1988), p. 215) does not apply in this context.

switching of borrowers in equilibrium. Although firm-level studies of pricing and termination in lending relationships are still rare and sometimes difficult to interpret, these predictions seem to correspond to observed behaviour in bank-firm relationships (see, e.g., Petersen and Rajan (1994), Angelini, Di Salvo, Ferri (1998), Ongena and Smith (1997), and Degryse and van Cayseele (2000).

The remainder of this note is organized as follows. Section I summarizes the model formulated by Sharpe, using the terminology of the paper as much as possible. Section II describes and briefly discusses the mixed-strategy equilibrium for a slightly simplified version of the model. The appendix contains the proof, which may be of some more general interest, because it derives the equilibrium properties directly, instead of adapting the general results of Engelbrecht-Wiggans, Milgrom and Weber (1983).

1 The Model

There is a continuum of firms (not necessarily risk-neutral) who all want to carry out a sequence of two investment projects. For each firm, the project at time $t = 1, 2$ transforms an investment of I^t at the beginning of the period (I^t is chosen by the firm) into a random return at the end of the period. This return depends on the firm's quality, q, and is given by

$$\begin{cases} X^t = g(I^t)I^t & \text{with probability } p_q \\ 0 & \text{with probability } 1 - p_q, \end{cases}$$

where g is strictly decreasing and concave.[3] For each firm, returns for project 1 and 2 are stochastically independent, and the same is true across firms. There are two possible qualities of firms, $q = L, H$, with $p_L < p_H$. The proportion of high quality firms, H, is $\theta \in (0, 1)$, and this is common knowledge. Firms do not know their own quality.

Because the key issue in Sharpe's (1990) paper is the problem of informational capture in relationship lending and because the variable investment case is a trivial extension, I assume from now on that project sizes I^1 and I^2 are fixed. Furthermore, to save on unnecessary indices, I consider the borrowing problem of one given firm, randomly drawn from the pool described

[3] Clearly, one needs that $g(I) \geq 1 + \overline{r}$ for some I.

above.[4]

The firm has no own funds, but can borrow from competing banks. Banks are risk-neutral, compete à la Bertrand, and have unlimited access to funds at the net interest rate $\overline{r}$ per period. Like the firm, banks do not know the firm's quality at the beginning of period 1. However, if a bank finances the firm's first project, it perfectly observes the outcome of the project, which provides information about the firm's quality. Denote by

$$\gamma = \begin{cases} S & \text{if first period result is } X^1 \\ F & \text{if first period result is } 0 \end{cases}$$

the firm's performance in the first project. It is assumed that the first project is financed by at most one bank,[5] which becomes the "inside bank". "Outside banks", who have not provided first round finance, each get an identical, costless noisy signal of γ, $\widetilde{\gamma}$, defined by

$$\Pr\left(\widetilde{\gamma} = \widetilde{S}\,|S\right) = \Pr\left(\widetilde{\gamma} = \widetilde{F}\,|F\right) = \frac{1+\Phi}{2},$$

with $0 \leq \Phi < 1$. In the limiting case of $\Phi = 1$ inside and outside banks are both perfectly informed about the first-period outcome; this case is trivial. If $\Phi = 0$, the outside banks do not observe anything. Sharpe (1990) considers both the case of $\widetilde{\gamma}$ being observed by the inside bank (his Proposition 2) and of $\widetilde{\gamma}$ not being observed by the inside bank (Proposition 1). Both analyses are similar and contain a similar error; to economise on space and because the case of unobserved $\widetilde{\gamma}$ may be less intuitive, I focus here on the latter case.

The key assumption concerning the strategic interaction among the players is the absence of binding long-term contracting possibilities.As forcefully argued by Sharpe (1990), this absence of long-term contracts is the interesting scenario to consider: without it the analysis would reduce to standard competitive pricing and miss the important point in bank relationships. The dynamic game played between the firm and the banks then has the following structure:

[4] This is effectively as in Sharpe (1990). Investment size and the description of the continuum are rightly ignored in most of his argument.

[5] For an analysis of funding by several banks, see von Thadden (1992) and Detragiache, Garella, and Guiso (2000).

$t = 1$	1. Each bank j announces a short-term lending rate r_j^1.
	2. The firm chooses one bank, borrows and invests I^1.
	3. The firm repays $(1 + r^1)I^1$ iff $\gamma = S$. Outside banks observe $\widetilde{\gamma}$.
$t = 2$	4. Simultaneously, the inside bank offers a second-period interest rate $r_i^2 = r_i^2(\gamma)$ and each outside bank h offers a second-period interest rate $r_h^2 = r_h^2(\widetilde{\gamma})$.
	5. The firm chooses an offer and invests I^2. If indifferent, the firm stays with the inside bank.
	6. The firm repays $(1 + r^2)I^2$ iff the second project has been successful.

The presentation of this game is slightly different from the one in Sharpe (1990), but both games are identical (with the restriction to one firm in my version).[6] Apart from the absence of long-term contracting possibilities, two other assumptions of the model are

1. The firm consumes any profit after the first period.

2. Outstanding debt after a failure of the first project is forgiven.

The first assumption excludes the possibility of using retained earnings for investment and signaling purposes in the second period. The second eliminates all contractual links between the two periods, in particular, firms can switch freely from one bank to another despite their credit history.[7] While

[6] The reader may suspect a difference in substance in stage 4, where Sharpe (1990) assumes that "each bank j makes offers of credit $r_j^2(\gamma)$ to its previous customers, contingent upon the observed outcomes γ. Each bank j observes a signal $\widetilde{\gamma}(f)$ of the first-period performance of those firms that borrowed from other banks. It also observes the lending policies $(r_h^2(\gamma))$ of their banks, but not individual offers. It then makes credit offers $(r_{jh}^2(\widetilde{\gamma}))$ to customers of each bank $h \neq j$." Despite the wording, since outsiders do not observe the inside offers, insiders and outsiders effectively move simultanously. The fact that outsiders observe "lending policies" is simply the Nash assumption.

[7] The second assumption is innocuous as long as X^2 is sufficiently large relative to X^1. Then, here as in practice, firms can switch banks even when in financial distress, if the new bank rolls over the old debt. The more restrictive assumption is the first one, because a successful firm has an interest to put up $X^1 - (1 + r^1)I^1$ as a contribution towards the second project, thereby signaling its type (by construction, a $\gamma = F$ firm cannot do that).

The problem disappears if one assumes that X^1 is sufficiently small relative to I^1 (which is reasonable, because the model wants to explain the lock-in of once unprofitable but now successful firms). An alternative model in which the first assumption can be relaxed would be to assume that the inside bank observes the firm's quality q, whereas the firm and the outside banks only observe the project outcome γ. Then the informational asymmetry is preserved, but signaling is ruled out by construction.

these assumptions are somewhat extreme, they are useful simplifications to highlight the role of intertemporal informational constraints in bank competition.[8]

Before analysing the model, it is useful to introduce, just as Sharpe (1990), some benchmark loan rates and notation. Let

$$p = \theta p_H + (1-\theta) p_L \tag{1}$$

$$p(S) = \frac{\Pr\ (\gamma = S \ \&\ \text{success in t=2})}{\Pr\ (\gamma = S)} = \frac{\theta p_H^2 + (1-\theta) p_L^2}{p} \tag{2}$$

$$p(F) = \frac{\Pr\ (\gamma = F \ \&\ \text{success in t=2})}{\Pr\ (\gamma = F)} = \frac{\theta (1-p_H) p_H + (1-\theta)(1-p_L) p_L}{1-p} \tag{3}$$

denote the success probabilities of the firm's second project, if there is, respectively, no information about first-period performance (equation (1)), if the first-period outcome has been observed to be good (equation (2)), and if the first-period outcome has been observed to be bad (equation (3)).

Similarly, by Bayes' rule, the success probabilities conditional on the noisy observation $\widetilde{\gamma}$ are given by

$$\begin{aligned} p(\widetilde{S}) &= \Pr\ (\widetilde{\gamma} = \widetilde{S} \ \&\ X^2 \Big| \widetilde{\gamma} = \widetilde{S}) \\ &= \frac{\Pr\ (\widetilde{\gamma} = \widetilde{S} \ \&\ X^2)}{\Pr\ (\widetilde{\gamma} = \widetilde{S})} \\ &= \frac{\theta(p_H \frac{1+\Phi}{2} + (1-p_H)\frac{1-\Phi}{2}) p_H + (1-\theta)(p_L \frac{1+\Phi}{2} + (1-p_L)\frac{1-\Phi}{2}) p_L}{\theta(p_H \frac{1+\Phi}{2} + (1-p_H)\frac{1-\Phi}{2}) + (1-\theta)(p_L \frac{1+\Phi}{2} + (1-p_L)\frac{1-\Phi}{2})} \\ &= \frac{(1-\Phi)p + 2\Phi p(S) p}{(1-\Phi) + 2\Phi p} \end{aligned}$$

and

$$\begin{aligned} p(\widetilde{F}) &= \Pr\ (\widetilde{\gamma} = \widetilde{F} \ \&\ X^2 \Big| \widetilde{\gamma} = \widetilde{F}) \\ &= \frac{\Pr\ (\widetilde{\gamma} = \widetilde{F} \ \&\ X^2)}{\Pr\ (\widetilde{\gamma} = \widetilde{F})} \\ &= \frac{(1-\Phi)p + 2\Phi p(F)(1-p)}{(1-\Phi) + 2\Phi(1-p)} \end{aligned}$$

[8] For an analysis of long-term contractual links in this type of problem, see von Thadden (1995).

Using these probabilities, one can define hypothetical zero-profit loan rates in each of these five situations:

$$1 + r_p = \frac{1 + \overline{r}}{p} \tag{4}$$

$$1 + r_S = \frac{1 + \overline{r}}{p(S)} \tag{5}$$

$$1 + r_F = \frac{1 + \overline{r}}{p(F)} \tag{6}$$

$$1 + r_{\widetilde{S}} = \frac{1 + \overline{r}}{p(\widetilde{S})} \tag{7}$$

$$1 + r_{\widetilde{F}} = \frac{1 + \overline{r}}{p(\widetilde{F})} \tag{8}$$

Clearly,

$$r_S < r_{\widetilde{S}} < r_p < r_{\widetilde{F}} < r_F. \tag{9}$$

The final assumption is that $(1 + r_F)I^2 \leq X^2$, i.e. that second-period lending is profitable even if the firm is known to have failed in the first period.[9]

The natural solution concept for this game is Perfect Bayesian Nash equilibrium (see, e.g., Fudenberg and Tirole (1991)).[10] The interesting part of the analysis of this game is the bidding competition between banks in the second period (Propositions 1 and 2 in Sharpe (1990)). The firm's response to competing bids is completely mechanic, and bank competition in stage 1 is standard bidding under symmetric information for the informational rent to be reaped in $t = 2$.

The bidding game in the second period is a Bayesian game whose information structure (i.e. players' types and priors) has been determined in period 1. Denote pure strategies of outside banks by $r_h = r_h(\widetilde{\gamma})$, let $r_o = r_o(\widetilde{\gamma}) = \min_h r_h(\widetilde{\gamma})$, and denote a pure strategy of the inside bank by $r_i = r_i(\gamma)$.

Proposition 1: *The bidding game in stage 4 has no Bayesian Nash equilibrium in pure strategies.*

Proof: The proof is by contradiction.

[9] It is straightforward to analyse the model without this assumption, in which case bad performers are excluded from continuation finance by the inside bank.

[10] This is as in Sharpe (1990), although he does not state this explicitly.

1) Suppose that $r_o(\tilde{\gamma}) < r_{\tilde{\gamma}}$ for $\tilde{\gamma} = \tilde{S}$ or $\tilde{\gamma} = \tilde{F}$. Such an offer attracts at best (if $r_o(\tilde{\gamma}) < r_i(S)$) the S- and the F- type firm. In this case, $\tilde{\gamma}$ is an unbiased estimator of the firm's γ and the winning outside bank would make a strictly positive expected loss. Contradiction.

2) Suppose that $r_i(S) > \max(r_o(\tilde{S}), r_o(\tilde{F}))$. By 1) and (9), a deviation by the inside bank to $\max(r_o(\tilde{S}), r_o(\tilde{F}))$ would raise expected profits on the S-type strictly above zero. Contradiction.

3) Suppose that $r_o(\tilde{\gamma}) < r_i(S) < r_F$ for $\tilde{\gamma} = \tilde{S}$ or $\tilde{\gamma} = \tilde{F}$. By 2), $\min(r_o(\tilde{S}), r_o(\tilde{F})) < r_i(S) \leq \max(r_o(\tilde{S}), r_o(\tilde{F}))$. By the optimality of $r_i(S)$, we must have $r_i(S) = \max(r_o(\tilde{S}), r_o(\tilde{F}))$. Because $r_i(F) \leq r_i(S)$ (which is smaller than r_F) is impossible, $\max(r_o(\tilde{S}), r_o(\tilde{F}))$ attracts exactly the F-type firm in equilibrium. Contradiction to $\max(r_o(\tilde{S}), r_o(\tilde{F})) < r_F$.

4) Suppose that $r_i(S) \leq r_o(\tilde{\gamma}) < r_F$ for $\tilde{\gamma} = \tilde{S}$ or $\tilde{\gamma} = \tilde{F}$. Then the outside banks' bid attracts at most the F-type firm as a customer. Clearly, $r_i(F) \geq r_F (> r_o(\tilde{\gamma}))$ (otherwise, the inside bank would make an expected loss on the F-firm). Hence, the outside offer attracts exactly the F-firm and makes a strictly positive expected loss because $r_o(\tilde{\gamma}) < r_F$.

5) Points 3 and 4 imply either directly that $r_i(S) \geq r_F$ or that $r_o(\tilde{\gamma}) \geq r_F$ for $\tilde{\gamma} = \tilde{S}$ and $\tilde{\gamma} = \tilde{F}$. If the latter is true, the optimality of $r_i(S)$ again implies $r_i(S) \geq r_F$. Clearly, also $r_i(F) \geq r_F$.

If $\min(r_o(\tilde{S}), r_o(\tilde{F})) > r_i(S)$, then the inside bank would do better with a bid of $r_i(S) + \varepsilon$ for ε sufficiently small, because this would allow it to realize higher profits per loan without loosing customers.

If $\min(r_o(\tilde{S}), r_o(\tilde{F})) = r_i(S)$, then any of the winning outside banks would do better with a bid of $r_i(S) - \varepsilon$ for ε sufficiently small, because this would allow it to attract the S-firm, on which it makes a strictly positive expected profit given its information.

Suppose, therefore, finally that $\min(r_o(\tilde{S}), r_o(\tilde{F})) < r_i(S)$. By 2), $\max(r_o(\tilde{S}), r_o(\tilde{F})) \geq r_i(S)$. Because of competition from the inside bank, $r_i(F) \leq \max(r_o(\tilde{S}), r_o(\tilde{F}))$ or $\max(r_o(\tilde{S}), r_o(\tilde{F})) = r_F$. Hence, the winning outside banks for the signal $\tilde{\gamma}$ with $r_o(\tilde{\gamma}) = \max(r_o(\tilde{S}), r_o(\tilde{F}))$ make no profit on their offer. Because $r_{\tilde{F}} < r_F \leq r_i(S)$, they would be strictly better off undercutting $r_i(S)$ slightly, thus attracting both types of the firm.

■

Proposition 1 shows that Sharpe's (1990) Proposition 1, in which he proposes two pure-strategiy equilibria as the solutions of the bidding game, is wrong. The problem with his proof is that he correctly rules out a number of pure-strategy combinations, but not all of them, and then concludes that what is left must be an equilibrium.

The outcome described in Proposition 1 is a classical example of the winner's curse familiar from Bertrand competition and auction theory: if an outside bank wins the bidding contest, it must take into account that its success is due to its bid being attractive, but also to the fact that the inside bank did not want to bid more aggressively. Hence, the very fact of winning contains information that a rational player must take into account. Typically, in such situations pure-strategy equilibria do not exist.

The proof of Proposition 1 can easily be adapted to the case of discrete action spaces (where interest rates must be expressed in terms of a smallest unit), as long as the interest rate grid is not too coarse. The non-existence problem is, therefore, more fundamental than the simple open-set problem which causes non-existence in Bertrand competition or first-price auctions under complete information and which disappears with discretization.

A similar argument to the one given above shows that Sharpe's (1990) Proposition 2, which deals with the case of $\tilde{\gamma}$ being observable by the inside bank, is incorrect, as well. Intuitively, the case of observable $\tilde{\gamma}$ may be easier to understand than the one of unobservable $\tilde{\gamma}$, which is why the explicit proof given here considers the latter case. If $\tilde{\gamma}$ is unobservable to the inside bank, inside and outside banks all observe a signal unknown to the other. Hence, it may seem that all banks can condition deterministically on their signal and thereby still obtain sufficient randomness to rule out deviations, as, for example, in Gibbons and Katz (1991) in the context of labor markets. Proposition 1 shows that this intuition is not correct: the fact that one competitor's action cannot be predicted by the others is not enough, the competitors need to randomize actively. In the case of $\tilde{\gamma}$ being observable to the inside bank, however, the inside bank faces no noise, and randomization is even more plausible a priori.

2 Equilibrium

To simplify notation and some of the calculations, I consider here the case of extreme informational asymmetry $\Phi = 0$, in which the outside banks have no information. The analysis of the more general case introduced above is a relatively straightforward extension. Furthermore, I suppose that there is only one outside bank, which simplifies the analysis, but still conveys the full intuition.

Proposition 2: *The Bayesian game between the inside and one outside bank in stage 4 of the dynamic bank competition game with* $\Phi = 0$ *has a unique mixed-strategy equilibrium. The inside bank's equilibrium strategy is to offer* $r(F) = r_F$ *with certainty and is an atomless distribution on* $[r_p, r_F]$ *for* $\gamma = S$*, with density*

$$h_i^S(r) = \frac{p(S)(1 + r_p) - (1 + \overline{r})}{(p(S)(1 + r) - (1 + \overline{r}))^2}. \tag{10}$$

The outside bank's equilibrium strategy has a point mass of $1-p(S)$ *at* $r = r_F$ *and an atomless distribution on* $[r_p, r_F)$ *with density*

$$h_o(r) = p(S)h_i^S(r). \tag{11}$$

The proof of the proposition is given in the appendix. The strategy of the proof is to first characterise equilibrium strategies assuming that they exist, which yields a unique characterization, and then verify that the behaviour found constitutes indeed a Nash equilibrium. It is therefore not necessary to invoke the existence theorem of Dasgupta and Maskin (1986).

Proposition 2 shows that firms switch banks in equilibrium. In fact, a bad firm switches whenever it receives an offer, which occurs with probability $p(S)$, but even good firms switch occasionally, namely with probability $\int_{r_p}^{r_F}(1 - H_i^S(r))h_o(r)dr$. Proposition 2 also shows that in equilibrium, full competition is only effective for bad firms. In particular, the inside bank always offers the zero-profit interest rate r_F to bad firms. Yet, with probability $p(S)$ bad firms are even getting excessively favorable terms on their loan, which happens when the outside bank underbids the inside bank. In this case, the outside bank makes a strict loss on the loan. Yet, the bank is neither acting irrationally nor recklessly: in order to put up a limited degree of competition for the good risks it must optimally take into account the occasional flop. On average, the outside bank puts up the maximum competition possible and makes zero expected profits.

By the same token, since competition by the outside bank is limited, the inside bank makes positive expected profits on the good risks. Since the inside bank is indifferent between the interest rates in the interval $[r_p, r_F]$, these profits are proportional to $(r_p - r_S)$, which can be interpreted as a measure of adverse selection in the market. In fact, as seen following equation (22) in the appendix, overall the inside bank's expected profits on good risks (expectation taken over the outside bank's randomization) are given by $p(S)(r_p - r_S)$. Hence, they are proportional to the success probability of the good firm. This is reasonable, although Proposition 2 also exhibits a counterveiling effect: the higher $p(S)$, the tougher the competition by the outside bank. Moreover, the inside bank's pricing strategy, as given by (10), is quite intuitive: because h_i^S is decreasing, most of the pricing occurs at moderate profit levels (slightly above r_p), with occasional attempts to really squeeze the firm (prices up to r_F).

Concerning the robustness of the model, Proposition 2 can be easily adapted to cover the case $(1 + r_F)I^2 > X^2$, in which second-period lending is not profitable if the firm is known to have failed in the first period.[11] In this case, the inside bank does not continue financing a failed firm and randomizes atomlessly over the range $[r_p, X^2)$ with a point mass at X^2 for the successful firm. The outside bank does not bid at all with some probability μ, and with probability $1 - \mu$ it bids and randomizes atomlessly over the whole range $[r_p, X^2]$.

[11] This case has been considered by Fischer (1990) and Rajan (1992).

Appendix: Proof of Proposition 2

Let H_i^γ, $\gamma \in \{S, F\}$, denote the c.d.f. of the equilibrium mixed strategy of the inside bank, given its information γ, and H_o the c.d.f. of the equilibrium mixed strategy of the outside bank. As usual, H_i^γ and H_o are weakly monotone and continuous from the right, i.e. $H(\widehat{r}) = \Pr\ (r \leq \widehat{r})$ for each of the three mixed strategies. Define $H(r^-) = \lim_{t \nearrow r} H(t)$. Finally, let

$$
\begin{aligned}
\ell_i^\gamma &= \inf\{r; H_i^\gamma(r) > 0\}, \gamma \in \{S, F\} && (12)\\
u_i^\gamma &= \sup\{r; H_i^\gamma(r) < 1\}, \gamma \in \{S, F\} && (13)\\
\ell_o &= \inf\{r; H_o(r) > 0\} && (14)\\
u_o &= \sup\{r; H_o(r) < 1\}. && (15)
\end{aligned}
$$

denote the lower and upper end point, respectively, of the supports of the three mixed strategies. Without loss of generality we can assume that $[\ell, u] \subseteq [0, \frac{X^2}{I^2} - 1]$ for the three different distributions considered (no repayment can be greater than what is available in the good state).

It useful to write out and label each player's expected payoff for any interest rate quoted, given the mixed strategy of the other:

$$
\begin{aligned}
P_i^\gamma(r) &= (1 - H_o(r^-))[p(\gamma)(1 + r) - (1 + \overline{r})], \gamma \in \{S, F\} && (16)\\
P_o(r) &= p(1 - H_i^S(r))[p(S)(1 + r) - (1 + \overline{r})] + && (17)\\
&\quad (1 - p)(1 - H_i^F(r))[p(F)(1 + r) - (1 + \overline{r})].
\end{aligned}
$$

The proof now characterizes the distributions H_i^γ and H_o in a sequence of several, more or less simple steps.

Step 1: $\ell_i^\gamma \geq r_\gamma$ for $\gamma \in \{S, F\}$.
Proof. Obvious.

Step 2: $\ell_o \geq r_p$.
Proof. Because $r_F > r_p$, any offer $r < r_p$ attracts, by Step 1, the F-firm. Therefore, the pool of applicants has at best success probability p.

Step 3: $\ell_i^S \geq r_p$.
Proof. By Step 2, putting mass to the left of r_p cannot be optimal.

Step 4: $u_o \geq u_i^S$.

Proof. Suppose that $u_o < u_i^S$. Then the inside bank makes zero expected profits on all offers $r(S) \in (u_o, u_i^S]$. However, by Step 3, the inside bank makes strictly positive expected profits on the S-firm.

Step 5: H_i^S is continuous on $[\ell_i^S, u_i^S)$.

Proof. Suppose that there is a $\widehat{r} \in [\ell_i^S, u_i^S)$ at which H_i^S is discontinuous, i.e. with $H_i^S(\widehat{r}^-) < H_i^S(\widehat{r})$. Then, by (17), $P_o(\widehat{r}^-) > P_o(\widehat{r})$, because $p(S)(1+r) - (1+\overline{r}) > 0$ on $[\ell_i^S, u_i^S)$ by Step 3.

By the right-hand continuity of $H_i^\gamma, \gamma \in \{S, F\}$, there is an $\varepsilon > 0$ such that $H_o(\widehat{r}^-) = H_o(r) =$ constant on $[\widehat{r}, \widehat{r} + \varepsilon]$. Therefore, P_i^S is continuous at $\widehat{r}$ and strictly increasing on $[\widehat{r}, \widehat{r} + \varepsilon]$. Hence, H_i^S can have no mass on $[\widehat{r}, \widehat{r} + \varepsilon]$, which implies that $H_i^S(\widehat{r}^-) = H_i^S(\widehat{r})$. Contradiction.

Note that the proof of Step 5 does not apply to H_i^F, because we do not know whether the inside bank makes strictly positive profits on the F-firm.

Step 6: $u_i^S \geq \ell_i^F$.

Proof. Suppose that $u_i^S < \ell_i^F$. This implies that the inside bank never makes an offer $r \in (u_i^S, \ell_i^F)$.

(a) Suppose that $u_i^S < u_o$. Then H_o can have no mass on $[u_i^S, \ell_i^F)$, because for every offer $r \in [u_i^S, \ell_i^F)$ the offer $\frac{1}{2}\left(r + \ell_i^F\right)$ would be strictly better for the outside banks. Then the (positive) mass of H_o on $[u_i^S, u_o]$ lies on $[\ell_i^F, u_o]$. In particular, H_o is continuous at u_i^S.

Consider the following deviation from H_i^S: Let $\delta > 0$ and $\varepsilon > 0$ be given and small. Let M_ε be the mass of H_i^S on $[u_i^S - \varepsilon, u_i^S]$. The deviation strategy is identical to H_i^S on $[\ell_i^S, u_i^S - \varepsilon)$, has zero mass on $[u_i^S - \varepsilon, \ell_i^F - \delta)$ and point mass M_ε on $\ell_i^F - \delta$.

The expected net gain (given $\gamma = S$) from this deviation is not smaller than

$$M_\varepsilon \left[(1 - H_o(u_i^S))(\ell_i^F - u_i^S - \delta) - (H_o(u_i^S) - H_o(u_i^S - \varepsilon^-))u_i^S\right]. \qquad (18)$$

The first of the two terms in (18) (which corresponds to the total gain from the deviation) is strictly positive for δ sufficiently small. The second term (corresponding to the total loss from the deviation) tends to 0 for $\varepsilon \to 0$

by the continuity of H_o at u_i^S. Hence, the deviation is strictly profitable for δ and ε small enough.

(b) Suppose that $u_i^S = u_o$. Consider the following deviation from H_o: Let $\delta > 0$ and $\varepsilon > 0$ be given and small. Let N_ε be the mass of H_o on $[u_o - \varepsilon, u_o]$. Move all mass of $[u_o - \varepsilon, u_o)$ to $\ell_i^F - \delta$. Then the expected net gain from this deviation is not smaller than

$$N_\varepsilon \left[(1-p)(\ell_i^F - u_o - \delta) - p(H_i^S(u_o) - H_i^S(u_o - \varepsilon))(p(S)(1+u_o) - (1+\overline{r}))\right],$$

where the second term now tends to 0 for $\varepsilon \to 0$ by Step 5.

Step 7: $u_i^F \leq u_o$.
Proof. Suppose that $u_i^F > u_o$. Then the inside bank makes zero expected profits on the F-firm (otherwise $u_i^F = \frac{X^2}{I^2} - 1$, and, by Steps 4 and 5, the outside bank would obtain a jump increase in expected profits by shifting the mass of $[u_o - \varepsilon, u_o]$ for any small ε to a rate strictly above u_o). Hence, $\ell_i^F \geq u_o$. By Step 4 and 6, $\ell_i^F \geq u_o$. This and the zero expected profits imply $\ell_i^F = r_F$.

Consider the following deviation from H_o: Let $\varepsilon > 0$ be given and small. Let L_ε be the mass of H_o on $[u_o - \varepsilon, u_o]$. The deviation strategy is identical to H_o on $[\ell_o, u_o - \varepsilon)$ and concentrates all the remaining mass L_ε on $\frac{1}{2}(r_F + u_i^F) =: \alpha$. By the definition of u_i^F we have $H_i^F(\alpha) < 1$.

The expected net gain from this deviation is not smaller than

$$L_\varepsilon \left[(1-p)(1 - H_i^F(\alpha))p(F)(\alpha - r_F) - p \underbrace{(1 - H_i^S(r_F - \varepsilon))}_{\to 0 \text{ for } \varepsilon \to 0 \text{ by Step 5}} (p(S)(1 + r_F) - (1 + \overline{r}))\right]. \tag{19}$$

Intuitively, since mass is taken away below r_F, the outside bank only gains if $\gamma = F$.

Remark: If one wants to prove $u_i^F = r_F$ directly, one needs to have $\ell_o \leq r_F$.

Step 8: $u_i^F = r_F$.
Proof. Clearly, $u_i^F \geq r_F$. Suppose that $u_i^F > r_F$. Since the outside bank can obtain strictly positive expected profits by choosing $r \equiv \frac{1}{2}(r_F + u_i^F)$, it must make strictly positive profits also with H_o. By Step 7, $u_o > r_F$; hence, also H_i^F must make strictly positive expected profits. By Steps 4 and 7, then,

$$
\begin{aligned}
& P_o(u_o) = 0 \\
\Rightarrow \quad & H_o(u_o^-) = 1 \\
\Rightarrow \quad & P_i^S(u_o) = P_i^F(u_o) = 0 \\
\Rightarrow \quad & H_i^S(u_o^-) = H_i^F(u_o^-) = 1 \\
\Rightarrow \quad & P_o(u_o^-) = 0,
\end{aligned}
$$

which is a contradiction to the finding that H_o makes strictly positive expected profits.

Step 8 implies that $r_i(F) = r_F$ with probability 1; in particular, the inside bank makes zero expected profits on the F-firm.

Step 9: $u_o = u_i^S = r_F$.
Proof. Suppose that $u_o > r_F$. Then choosing $r_i(F) = \frac{1}{2}(r_F + u_o)$ with probability 1 would yield the inside bank strictly positive expected profits on the F-firm. The equality for u_i^S follows from Steps 4 and 6.

Step 10: The outside bank makes zero expected profits.
Proof. By Steps 8 and 9, (17) simplifies to

$$
\begin{aligned}
P_o(r) \quad = \quad & p(1 - H_i^S(r))[p(S)(1 + r) - (1 + \overline{r})] + \\
& (1 - p)[p(F)(1 + r) - (1 + \overline{r})]
\end{aligned}
\tag{20}
$$

on $[\ell_o, u_o)$. By Step 5, P_o is continuous on $[\ell_o, u_o)$, and by Step 9 we have $P_o(u_o^-) = 0$.

Step 11: $\ell_o = \ell_i^S = r_p$.
Proof. It is impossible that $\ell_o > \ell_i^S$, because then the inside bank would make strictly higher profits if it placed the mass of $[\ell_i^S, \ell_o]$ on ℓ_o. By a similar argument for the outside bank, $\ell_o < \ell_i^S$ is impossible. Finally, if $\ell_o > r_p$, the outside bank would make strictly positive expected profits, contradicting Step 10.

Step 12: H_o is continuous on $[r_p, r_F)$.
Proof. Suppose that $H_o(\widehat{r}^-) < H_o(\widehat{r})$ for some $\widehat{r} \in [r_p, r_F)$.

Then $P_i^S(\hat{r}) > P_i^S(r)$ for $r \in (\hat{r}, \hat{r}+\varepsilon)$, $\varepsilon > 0$ sufficiently small, by the right-hand continuity of H_o. Hence, H_i^S is constant on $(\hat{r}, \hat{r}+\varepsilon)$. Therefore, by (20), P_o is strictly increasing on $(\hat{r}, \hat{r}+\varepsilon)$. By the continuity of P_o (which follows from Step 5 and (20)), $P_o(\hat{r}) < P_o(\hat{r}+\varepsilon)$. Hence, H_o can have no mass on $\hat{r}$.

Step 13: H_i^S and H_o are strictly increasing on $[r_p, r_F]$.
Proof. Suppose that H_i^S is constant on some interval $[\alpha, b] \subset [r_p, r_F]$. Let $[a, b] \supseteq [\alpha, b]$ be the maximal such interval. By Step 5 and the definition of ℓ_i^S, $a > r_p$. Then P_o is strictly increasing on $[a, b]$, hence, H_o constant on $[a, b)$. By the continuity of H_o, P_i^S is strictly increasing on $[a, b]$, a contradiction to the maximality of $[a, b]$.

The last step has completed the characterization of the mixed strategies, because it implies that P_i^S and P_o are constant on $[r_p, r_F]$. By the continuity of H_o and H_i^S on $[r_p, r_F)$, we obtain therefore from (16) and (20) for $r \in [r_p, r_F)$

$$(1 - H_o(r))[p(S)(1+r) - (1+\overline{r})] = c \tag{21}$$

$$\begin{aligned} p(1 - H_i^S(r))[p(S)(1+r) - (1+\overline{r})&] \\ +(1-p)[p(F)(1+r) - (1+\overline{r})&] = 0. \end{aligned} \tag{22}$$

The constant c in (21) can be determined by substituting any value $r \in [r_p, r_F)$ into (21); for $r = r_p$ one obtains $c = p(S)(r_p - r_S)$. Straightforward manipulations of (21) and (22) then yield

$$H_i^S(r) = \frac{r - r_p}{p(S)(1+r) - (1+\overline{r})} \tag{23}$$

$$\begin{aligned} H_o(r) &= p(S)\frac{r - r_p}{p(S)(1+r) - (1+\overline{r})} \\ &= p(S)H_i^S(r) \end{aligned} \tag{24}$$

for $r \in [r_p, r_F)$. One easily checks that $H_i^S(r_F^-) = 1$, hence, H_i^S is continuous on $[r_p, r_F]$ and given by (23) on all of its domain. On the other hand, (24) shows that H_o is discontinuous at $r = r_F$ with jump $1 - p(S)$.

This identifies a unique mixed strategy profile. Because both players randomize over the whole of $[r_p, r_F]$, there are no profitable deviations from these strategies for either player. Proposition 2 is therefore proved.

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