A Silver Rule for Financing Local Transport Facilities

Robert Bichsel[*][†]
Université de Lausanne - Ecole des HEC
Département d'Econométrie et Economie Politique

February 1999

Abstract

According to the Henry George Theorem (HGT), the cost of a pure local public good can be covered through a tax on related land–rents. As we show in this paper, this general proposition does not apply to transport facilities.

Nonetheless, even if the 'Golden Rule' does not apply in this context, land value and transport facilities are related. We show that (i) an improvement of the transport facilities does have a positive effect onto land value when taking into account the effect on the equilibrium city size; (ii) a simple relationship, similar to the HGT, does exist between the cost of optimal transport facilities and aggregate land rents; (iii) any exogenous shock reducing travel costs leads to higher optimal spending in transport facilities and higher land value. This suggests that associated changes in land value could, in a way that we define, subsidize optimal improvements in transport facilities land rents are related to spending in transport facilities.

Keywords: Land rents, Transport facilities, Henry George Theorem.

JEL Classification: H71, R53, R42

[*]Département d'Econométrie et d'Economie Politique, Université de Lausanne, CH-1015 Lausanne, Switzerland. e-mail: rbichsel@hec.unil.ch.

[†]I would like to thank Thomas von Ungern-Sternberg, Claude Henry and Ralph Turvey, for very helpful comments and discussions.

1 Introduction

According to the Henry George Theorem[1] (HGT), the cost of an optimally supplied pure local public good could (should) be …nanced through a tax on the land value it creates. This is an attractive result, as it combines equity considerations – landlords are the …nal bene…ciaries of the public good – with the e¢ciency of the land tax. However, this theorem does not apply to all kind of local public goods. In particular, we will see that it does not apply to transport facilities[2]. As a consequence one should not expect *a priori* a speci…c tax on related land rents to cover the entire cost of e¢cient transport facilities.

In spite of the similarities between a classic local public good and transport facilities, di¤erent mechanisms than those described by the HGT are involved when analyzing how they a¤ect land value. Two reasons are at the origin of this di¤erence.

First, transport facilities do not provide utility *per se* to their users. The existence of well–developed transport facilities alone cannot explain agglomeration of economic activities and thus cannot be the exclusive source of land rents. Transport facilities can only a¤ect land rents through a synergy with some source of economic agglomeration.

Second, transport facilities a¤ect the travel costs, the core element of the relationship between a local public good and land value; *ceteris paribus*, better transport facilities imply lower travel costs and thus a decrease in land value.

Nonetheless, even though the 'Golden Rule of Local Public Finance' does not apply in this context, land value and transport facilities are related. We show that: (i) an improvement of the transport facilities does have a positive e¤ect onto land value when taking into account the e¤ect on the equilibrium city size; (ii) a simple relationship, similar to the HGT, does exist between

---

[1] A …rst version of this theorem was proposed by Flatters, Henderson and Mieszkowski (1974).

See in particular Arnott et al. (1979) for an extended description of the conditions under which the theorem holds.

[2] With roughly 7% of total direct expenditures in 1994, spending in transport facilities (highway and mass transit) would represent the second most important category of spending at the local level; though far behind spending in education (36%).

Source: US Bureau of the Census, June 1997, Table No 478, "All Governments–Detailed Finances: 1994" and Table No 480, "All Governments–Direct Expenditures for Public Works: 1980 to 1994".

the cost of optimal transport facilities and aggregate land rents; (iii) any exogenous shock reducing travel costs leads to higher optimal spending in transport facilities and higher land value. This suggests that associated changes in land value could, in a way that we de…ne, subsidize optimal improvements in transport facilities.

The relationship between land–value and transport cost has been studied previously. However most authors only consider free improvements of transport. Among them, Haig (1926), Getz (1975), Vickrey (1977) and Arnott et al. (1981). Others, like Lee et al. (1973) or Kanemoto (1980), consider the endogenous property of travel cost but do not study how aggregate land rents and optimal spending in transport facilities are related.

As a consequence, none of these papers provide useful information in a public …nance perspective in the context of transport facilities. The aim of our analysis is to provide elements that could help …ll that gap. This is done using a simple general equilibrium model – the size of the cities is endogenous – of an urban economy with travel costs depending on transport facilities.

The paper is organized as follows. In section 2, we present the model of spatial organization that includes a measure of the performance of the transport facilities. Section 3 provides the analysis of the relationship between spending in transport facilities and land value. We discuss our results and conclude in section 4. Section 5 contains the technical appendix.

2 The model

We develop a model of spatial organization of an economy, based on the approach pioneered by von Thünen (1826) and adapted by Isard (1956) and Alonso (1964).

2.1 Characteristics

We consider an urban economy populated by an exogenous number of identical agents. Land is homogeneous and cities form because of the existence of some kind of Marshallian agglomeration economies. The size of the cities is endogenous. The economic activities take place at the central business district (CBD) of the monocentric cities. The agents live in the cities, on land lots of unit size, and travel to the CBD, work one unit of time and buy a composite consumption good. The …rms, located at the CBD, consume no geographical space. Finally, the firms produce the same composite good in every city, and there is no inter–city trade.

2.1.1 Government

Every city is being led by a – democratically elected – government. This government owns the land occupied by the city, rents it competitively to the citizens and distributes the income of this activity uniformly to the population[3].

Additionally, the government affects the unitary travel cost, through it's choice of the lump–sum financed transport facilities.

Travel costs The unitary travel cost[4], particularly it's time component, is largely dependant on the quality of the transport facilities. We consider that the cost of a trip from distance t to the CBD[5] and back is given by the following constant elasticity function:

$$f_{i,t} := \alpha(D_i, N_i)t^{\tau} := \left(\alpha_0 D_i^{-\beta} N_i^{\mu}\right) t^{\tau} = \left(\alpha_0 d_i^{-\beta} N_i^{\mu-\beta}\right) t^{\tau} \qquad (1)$$

where, N_i is the size (population) of city i, D_i is the total – and d_i the per capita – cost of the transport facilities. As any variable in our model, D and d are flows. They measure the cost per unit of time, including the opportunity cost of capital, associated to transport facilities of a given quality level; the higher D_i or d_i, the better the facilities and the lower the unitary travel cost.

α_0 is a positive scale parameter. $-\beta$ is the elasticity of unitary travel cost with respect to spending in transport facilities; we can reasonably expect decreasing returns to scale in this context and thus $0 < \beta < 1$. μ is the elasticity of unitary travel cost with respect to the size of the city. If total spending in transport facilities are constant, the bigger the city size and the higher the unitary travel cost; thus: $\mu > 0$. If $\mu = \beta$, the travel cost from distance t is a zero degree homogeneous function and thus, the travel cost

[3] This is a convenient way to represent the fact that the land the city is built on, globally belongs to it's citizens, without having to cope with the problems of heterogeneity that would arise if we suppose that every citizen is the owner of his own land lot. In this case, identical individuals would have different incomes (or wealth).

[4] We use travel cost rather than transport cost in order to avoid the possible confusion between the transport facilities cost and the commuting cost.

[5] Suppose that the unitary period corresponds to one working day, with a trip to work (CBD) and a trip back home.

from t to the CBD will be the same in two cities of different size but with the same per capita spending in transport facilities. However, economies or deseconomies of scale in the construction of transport facilities may be related to city size (e.g. fixed costs and network effects related to a metro). Thus, given d, the absolute size of a city can affect the unitary transport cost positively ($\mu > \beta$) or negatively ($\mu < \beta$).

2.1.2 Inhabitants–consumers–workers–shareholders

The citizens of the economy produce and consume the products of the city they are living in and are shareholders of the firms. Their utility is given by:

$$U_{i,t} := \begin{cases} u(c_{i,t}) & \text{if } \ell \geq 1 \\ 0 & \text{else} \end{cases} \tag{2}$$

where $c_{i,t}$ measures the consumption for an individual located at distance t from de CBD of city i, ℓ is a measure of land consumption, and $u(\cdot)$ is a strictly increasing function. Demand for land at the individual level will be inelastic as land consumption enters the function only in a limitative way. Additionally, we suppose an inelastic supply of labor, normalized to 1. The individual gross income is $I_i := w_i + ALR_i/N_i - D_i/N_i$ where w_i is the salary and ALR_i is the aggregate land rent of the city. This income will be used to buy consumption goods, pay the land rent and the travel costs to the CBD. The objective function of a consumer is given by:

$$\begin{aligned} &\max_{\{i,t\}} u(c_{i,t}) \\ &\text{s.t.}: I_i := w_i + ALR_i/N_i - D_i/N_i \geq r_{t,i} + f_t + c_{t,i} \end{aligned} \tag{3}$$

with f_t the travel costs, $r_{t,i}$ the renting price of a land lot of unitary size and $c_{i,t}$ the consumption of the composite good, which price is normalized to 1.

2.1.3 Firms

The composite good is produced competitively and there is free entry in the market. Thus any individual firm is a price taker, in particular with respect to salaries.

The production function of a representative firm in city i is $y = g(N_i)L$, where L is the labor force used by the firm. The production function at the

…rm level is characterized by constant returns to scale; the unitary cost of production is thus independent of the number of (identical)[6] …rms that can be represented by an arbitrary number of (one) representative …rm(s) that behave(s) competitively.

The existence of Marshallian type economies of agglomeration, imply that productivity of a …rm is positively related to the size of (the labor force of) the city. Formally, $g(N_i) > 0$ and $g'(\cdot) > 0$. These economies of agglomeration act as a centripetal force and are su¢cient to justify the concentration of economic activities. We consider:

$$g(N_i) := N_i^{\gamma - 1} \tag{4}$$

with $\gamma > 1$:

2.2 Equilibrium

2.2.1 Labor market

The individual labor–supply is …xed at one unit of labor per unit of time. Thus, in a city of a given size, the wage rate is directly derivable from the demand function for labor. Every single …rm chooses L; the labor force it uses, and takes the labor force in the city as given. Given the market wage w_i; it's objective function is:

$$\Pi_i := g(N_i) L - w_i L$$

Under the constant returns to scale hypothesis, the pro…t function is linear in its argument. The …rst order condition of its maximization does thus provide no information describing the …rm's optimal behavior. The pro…t function is maximized for $L = +\infty$ if $g(N_i) > w_i$; $L \in [0; +\infty[$ if $g(N_i) = w_i$ and $L = 0$ if $g(N_i) < w_i$: In other words, the demand for labor is a perfectly elastic function of the wage, located at $g(N_i) = w_i$. Thus, the aggregate and individual demand function for labor are identical. The free entry of …rms will lead to an equilibrium wage in the labor market that re‡ects the average productivity of the working force and secures the absence of pro…ts. Using (4), we have:

$$g(N_i) := N_i^{\gamma - 1} = w_i \tag{5}$$

[6] The free entry hypothesis ensures that all the surviving …rms will be identical and e¢cient.

2.2.2 Land market

The quest of the highest utility level, through the choice the city and the localization within the city will lead to inter– and intra–city migration of people. These movements will in turn lead to an equilibrium situation in which every opportunity of utility increase through a move from one location to another must have been exploited; at the equilibrium:

$$c_{i,t} = c^*, \qquad \forall t, i$$

Formation of cities The migration mechanism is at the source of the formation of cities. The existence of some level of economies of agglomeration makes a homogeneous distribution of the population over the whole territory – the absence of cities – an unstable equilibrium. A shock leading two people to agglomerate will initialize the global agglomeration dynamics. The average utility level in the embryonic city being higher than elsewhere in the economy, this attracts new residents. The city will grow. But as a consequence of the increase of the number of residents, the average length of trips to the CBD – thus the average quantity of resources dedicated to travel – will grow. This fact will actuate as a centrifugal force, limiting the equilibrium size of the city.

We will consider that the citizens (or the government) can limit the size of the cities. As a consequence, the equilibrium city size will be optimal[7]. By extension, as total population of the economy is exogenous and lives in cities, the optimality of the size of the representative city implies the optimality of the geographical distribution of the economy[8]. Migrations will stop once the

[7] The optimality of the equilibrium can appear surprising in the presence of externalities: at the aggregate level, the marginal productivity of labor exceeds the wage level: $g(N) + Ng'(N) > g(N) = w$.

Nonetheless, because the land rent is returned to the citizens, this externality is implicitly internalized by the supply side of the labor market. The government of each city will welcome new residents only if they increase the average utility level of the inhabitants and so, they take into account the effect on the average productivity, and thus the externality.

However, if for some reason a government could not limit the size of his city, the spatial equilibrium organization of the economy will not necessarily be efficient; a system of over populated cities is also an equilibirium.

See Kanemoto (1980), p 64–66, for an additional discussion.

[8] We suppose that the total population is big enough and neglect the problem associated to the integer condition for the number of cities.

after–tax income net of travel costs and land rent is maximized given that the transport facilities are set at the optimal level.

If the opportunity cost of non–urban land is zero[9], $r_T = 0$. Given (3) the objective function of the consumers, the equilibrium consumption level c^* in any city of the economy will solve:

$$\max_{\{N,T,D\}} \left[N^{\gamma - 1} + ALR/N - D/N - f_T \right]$$
$$s.t.: \int_0^T \phi_t dt = N$$

where, T is the radius of the city and ϕ_t measures the land density at a distance t from the CBD. The constraint states that the number of citizens cannot exceed the surface occupied by the city[10]. The equality will hold, as the unoccupancy of some land is costly in terms of consumption within a city. At equal population, the radius of a city is positively related to it's rate of unoccupied land, which means higher average transport cost for identical average production and thus, lower average consumption.

Indifferently we will consider the central planner's perspective, and maximize the distance between the average production level and the average quantity of resources allocated to transport (traveling and facilities)[11]:

$$\max_{\{N,T,D\}} c(N,D) := \frac{N^{\gamma} - ATC(T,D) - D}{N} \qquad (6)$$
$$s.t.: \int_0^T \phi_t dt = N$$

Measure of the land rent Even though non–urban land is abundant within an economy, land becomes a scarce resource within the cities. Utility maximization and positive travel costs will lead to intense competition for land located close to the CBD of the various cities. The supply of land at one precise localization being perfectly inelastic, the only effect of competition

---

[9] We are only interested in the land–value associated to urban development and transport facilities. If the non–urban land has a positive opportunity cost the results would not be affecte provided the differential rather than the aggregate land–rent is being considered.

[10] The individual demand for land is inelastic and has been normalized to one. In a two dimensional city for example, this unitary demand can be measured by any surface unity; e.g. one square–foot, etc.

[11] The resources spent on land rents are only a transfer.

will be to lead to relatively high prices – and associated rents – for land located close to the CBD.

One standard result of the basic spatial model[12] is that the price of land is a decreasing function of the distance from the CBD and that it's slope is given by the slope of the travel cost function. The land price di¤erential exactly re‡ects the travel cost di¤erential:

$$\frac{@r_t}{@t} = \text{i}\,\frac{@f_t}{@t} \qquad (7)$$

If the opportunity cost of non urban land is zero, the price of the land located at the city center will exactly re‡ect the travel cost of the people living at the frontier of the city. For a city of given size it is thus possible to compute the aggregate land rent (ALR) and aggregate travel cost (ATC) using (7) and the travel cost function (1). Taking the integral of the land rent function over the entire surface of the city, and rearranging,yields:

$$alr_l = \frac{¿}{¿ + 1}®_0 2^{\text{i}¿} d^{\text{i}¯} N^{1+¿\,\text{i}¯} = ¿\,(atc_l) \qquad (8)$$

the per capita ALR (alr) and per capita ATC (atc) in a linear city[13], and:

$$alr_c = \frac{¿}{¿ + 2}®_0 ¼^{\text{i}¿=2} z^{\text{i}¯} N^{\text{i}¯+1+\frac{1}{2}¿} = \frac{¿}{2}\,(atc_c) \qquad (9)$$

in a circular city.

Mohring (1961) and Arnott et al. (1979) give an early statement of this result, showing that in a linear (circular) city, when travel costs are proportional to distance ($¿ = 1$), the aggregate land rent is equal to (the half part of) the aggregate travel cost[14].

---

[12] Computation is given at appendix A: Also see, among others, Arnott et al. (1979).

[13] In a linear city, at any point located at a distance t of the CBD; there exists two pieces of land of unitary size (same unit as t) : $Á_t = 2$; thus: $N = 2T$.

[14] For a given ¿; the per capita land rent represents a bigger proportion of the per capita travel costs in a linear city than in a circular city.

This di¤erence is due to the fact that in the circular city, the number of people living at any given distance from the CBD increases with this distance; thus high land rents (people located close to the CBD) are relatively poorly weighted in comparison to high travel costs (people located far from the CBD). In oposition, in a linear city, the number of people living at any distance from the CBD is constant and so is the weight of high and low land rent and high and low travel costs.

Optimal city size Substituting the ATC given by (8) in the linear case and by (9) in the circular case, into the objective function (6) and solving the FOC of the maximization of this objective function with respect to N yields:

$$N_l^*(d) = \left(\frac{(\gamma - 1)(\tau + 1)\,2^{\tau} d^{\beta}}{\alpha_0 (\mu + \tau - \beta)} \right)^{\frac{1}{1+\mu+\tau-\beta-\gamma}} \quad (10)$$

the equilibrium–size linear city and:

$$N_c^*(d) = \left(\frac{(\gamma - 1)(\tau + 2)\,\pi^{\frac{\tau}{2}} d^{\beta}}{\alpha_0 (2\mu + \tau - 2\beta)} \right)^{\frac{1}{1+\mu+\tau/2-\beta-\gamma}} \quad (11)$$

the equilibrium–size circular city for given per capita spending in transport facilities.

3 Analysis

3.1 Impact on land value of an improvement of transport facilities

An improvement of transport facilities has a 'price–effect' and a 'quantity effect' on land value. The price effect is measured by considering cities of given size. In this case, an improvement of transport facilities – an increase of D or d – reduces the unitary travel cost and, through the relationship between travel costs and land value given by (8) and (9), reduces aggregate and per capita land value in the representative city and in the whole economy[15]. The magnitude of the relation is given by β; the elasticity of travel costs with respect to transport facilities as can be seen in (1).

However, better transport facilities for a city of a given size imply lower unitary travel costs. This in turn lowers the marginal cost of increasing the city–size but does not affect the marginal benefit (the increase in per capita production due to the existence of agglomeration economies) of doing so. Thus, the equilibrium size of the cities grows in response to more spending in transport facilities; this has a positive effect on land value. This is the

[15] The average land value of the entire economy is identical to the average land value of the representative city.

'quantity effect' derived from (10) and (11); formally, the elasticities of the equilibrium city size with respect to transport facilities are given by:

$$\varepsilon_{N_l^*,d} = \frac{\beta}{1 + \mu + \tau - \beta - \gamma} > 0$$

and

$$\varepsilon_{N_c^*,d} = \frac{\beta}{1 + \mu + \tau/2 - \beta - \gamma} > 0$$

According to the second order condition of the parameters[16], both are positive. The higher β; the productivity of transport facilities and γ; the magnitude of the agglomeration economy, the bigger the effect on equilibrium city size of a change in d. On the other hand, the higher the parameters reflecting the cost of an agglomeration (τ; the elasticity of the unitary travel cost and μ; the deseconomies of scale parameter) the weaker the effect of a change in d:

Combining the price and the quantity effect yields:

$$\varepsilon_{alr_l;d} = (\gamma - 1)\,\varepsilon_{N_l^*;d} > 0 \qquad \text{and} \qquad \varepsilon_{alr_c;d} = (\gamma - 1)\,\varepsilon_{N_c^*;d} > 0$$

The quantity effect dominates the price effect even in a model with inelastic individual demand for land[17]; an increase in per capita spending in transport facilities increases the per capita land value (alr) once we allow city–size to adjust. This suggests that landowners as a whole eventually benefit from better transport facilities, through the effect on global economic activity, even if the first (short term) effect of improved transport facilities is a decrease in land value. This result is valid at the level of the entire economy; even if cities (and thus the associated land–value) disappear in the process. The per capita land–rent paid in the representative city increases and so does the per capita land rent in every remaining city in the economy; as total population is exogenous, this implies an increase of the aggregate land–value of the economy.

[16] See appendix A for a computation and discussion of a workable version of the SOC of the model.

[17] If demand for land were elastic and demand for travel remains inelastic, a decrease in travel cost would imply more demand for land and thus a bigger quantity effect.

Note that though land value and transport facilities are positively related; the effect on unitary travel cost is uncertain. Using (1), the elasticity of the unitary travel cost with respect to d, when we allow for an adjustment of the city size is:

$$\varepsilon_{\alpha_l,d} = (\gamma - 1 - \tau)\,\varepsilon_{N_l^*,d} \gtrless 0 \quad \text{and} \quad \varepsilon_{\alpha_c,d} = (\gamma - 1 - \tau/2)\,\varepsilon_{N_c^*,d} \gtrless 0$$

For example, if (i) travel costs are less than proportionally related to distance ($\tau < 1$) and (ii) agglomeration economies (γ) are relatively important, one may well have $\gamma - 1 - \tau > 0$ (linear cities) or $\gamma - 1 - \tau/2 > 0$ (circular cities) and still have a model that is well behaved[18]: In these cases, after city–size adjustment, the unitary travel cost would increase in response to an increase in per capita spending in transport facilities.

This means that an increase in unitary travel costs within the cities may reflect the economy's best answer to better transport facilities rather than the consequence of some source of inefficiency. Given the necessary conditions on the parameters, this situation is more likely to occur in circular cities than in linear ones.

3.2 Ratio between land rents and the cost of efficient transport facilities

The relationship between land rents and the cost of efficient transport facilities in a city of given size can be derived from the FOC of the maximization of (6) with respect to D:

$$-ATC_D\left(N, D\right) - 1 = 0$$

The marginal benefit of increasing D is $-ATC_D$; the aggregate reduction of the travel costs for the city. The marginal cost of increasing D is one. As can be seen from (8) and (9), if the travel cost is related to distance through a constant elasticity function, we have:

$$ATC = h\left(N, \tau, \mu, \alpha_0\right) D^{-\beta}$$

[18] The model still requires that $1 + \mu + \tau - \beta - \gamma > 0$ (linear case) or $1 + \mu + \tau/2 - \beta - \gamma > 0$ (circular case). Thus, if $\gamma - 1 - \tau > 0$ (linear case) or $\gamma - 1 - \tau/2 > 0$ (circular case), the diseconomies of scale in transport utilities related to the city size would have to be relatively high ($\mu > \beta$) for the model to behave well.

This implies that the marginal benefit exceeds the marginal cost as long as $D < \beta ATC$.

Using (8) and (9) we can also write:

$$ALR = g(\tau)\, ATC$$

This implies that, at the efficient level, the cost of the transport facilities has to satisfy:

$$d^{*} = \beta \frac{alr}{g(\tau)}$$

Or, identically:

$$\frac{D^{*}}{ALR} = \frac{d^{*}}{alr} = \frac{\beta}{g(\tau)} \tag{12}$$

where $g(\tau)$ is τ and $\tau/2$ for a linear and circular cities respectively.

This simple relationship is similar to the 'Golden Rule' between aggregate land rents and the cost of – or spending on – an optimal pure local public good. However, this result is closer to a 'Silver Rule' in the sense that land rents will match the cost of optimal transport facilities only under particular parameters value. Only two parameters appear in the rule: β, the elasticity of travel cost with respect to spending in transport facilities and τ, the elasticity of travel cost with respect to distance. In a linear city, considering $\beta < 1$ (decreasing returns for spending in transport facilities) and linear travel costs ($\tau = 1$), the aggregate land rent will always exceed the cost of efficient transport facilities. Still for linear travel costs, but in a circular city, if $1/2 < \beta < 1$, the cost will exceed the aggregate land rent. Remember that all the variables are flows; thus, if $0 < \beta < 1/2$, the per period land value in a circular city, could integrally finance the per period spending in transport facilities (capital and maintenance costs).

3.3 Effects on land value of optimal improvements of transport facilities

Using the whole land rent to finance or subsidize the transport facilities is questionable, in particular in a model with pure multiplicative effects. One cannot distinguish the part of the aggregate land rent that is attributable to the (exogenous) existence of agglomeration economies, from the part directly attributable to transport facilities. Only the synergy between both elements is being measured. This is problematic if the agglomeration economies are not due to a purely exogenous phenomenon. If they are due to the existence of a local public good, or to economies of scale at the level of the firm[19], the determination of which part of the aggregate land rent should be used to finance the former or to subsidize the latter, and which part should be used to finance the transport facilities is debatable.

One way to deal with this problem could be the following: suppose one starts from an optimal situation where a given proportion of transport facilities is financed by land rents. Suppose now that parameter α_0, the exogenous component of the unitary travel cost, decreases in response to a technological progress in the transport sector or decrease in the price of energy. This decrease in α_0 will positively affect the equilibrium city size, the optimal per capita spending in facilities and the equilibrium per capita land rent. The effect on land rents can be split into the following effects: (i) a direct effect from a change in α_0, leaving both the city size and per capita facilities unchanged, (ii) the effect of a change in the equilibrium city size leaving the per capita facilities unchanged, (iii) the effect of a change in the optimal per capita spending in facilities.

Changes in land value associated to points (i) and (ii) are not induced by any change in facilities. Land rents grow but per capita spending in transport facilities remains constant. As such, there would be no direct justification to use this increase in per capita land rents to subsidize facilities: On the other side, changes in land value associated to point (iii) are directly related to changes in transport facilities and could, in the philosophy of the 'Golden Rule', be taxed away specifically to subsidize the related cost.

Formally, a measure of point (i) and (ii) is given by the elasticity of (8) and (9) with respect to α_0, where N is set at it's optimal level given by (10) and (11) respectively, and d is left unchanged:

$$\left.\frac{\partial alr_l}{\partial \alpha_0}\frac{\alpha_0}{alr_l}\right|_{\substack{d=\bar{d};\\ N=N_l^*(\bar{d})}} = -\frac{(\gamma - 1)}{1+\mu+\tau-\beta-\gamma} < 0 \qquad (13)$$

[19] See Vickrey (1977).

and:

$$\left.\frac{\partial alr_c}{\partial \alpha_0}\frac{\alpha_0}{alr_c}\right|_{\substack{d=\bar{d};\\ N=N_c^*(\bar{d})}} = -\frac{(\gamma-1)}{1+\mu+\tau/2-\beta-\gamma} < 0 \qquad (14)$$

Based on the SOC of the maximization, both expressions are negative. Leaving the per capita spending unchanged, a decrease in α_0 induces, through the effect on the equilibrium size of the cities, an increase of the average land rent in the economy.

Point (iii) requires the knowledge of how the optimal d is being affected by a change in α_0: The optimal d, computed at the spatial equilibrium of the economy, maximizes (6) the per capita consumption in the representative city, when the latter is set at it's equilibrium level given by (10) and (11) respectively. We observe that the cost of optimal transport facilities is a constant (independent of α_0) proportion of aggregate production of the equilibrium city size[20]:

$$\frac{d_l^* N_l^*}{F(N_l^*)} = \frac{\beta(\gamma-1)}{\mu+\tau-\beta} \quad \text{and} \quad \frac{d_c^* N_c^*}{F(N_c^*)} = \frac{\beta(\gamma-1)}{\mu+\tau/2-\beta} \qquad (15)$$

Optimal spending in transport facilities is proportionally related to production. Thus, as the production grows more than proportionally to the size of the cities (through the agglomeration economies), we know that the cost of optimal transport facilities grows with the equilibrium size of the cities. An economy with greater equilibrium – and optimal – city–size should spend the same proportion of its production in transport facilities as an economy with smaller equilibrium city size; however per capita spending in transport facilities will be greater, the greater the city size.

Using (22) and (23), we have:

$$\frac{\partial alr_l}{\partial \alpha_0}\frac{\alpha_0}{alr_l} = -\frac{(\gamma-1)}{1+\mu+\tau-\gamma-\beta\gamma} < 0 \quad \text{and} \qquad (16)$$

$$\frac{\partial alr_c}{\partial \alpha_0}\frac{\alpha_0}{alr_c} = -\frac{(\gamma-1)}{1+\mu+\tau/2-\gamma-\beta\gamma} < 0 \qquad (17)$$

[20] See appendix B for computation.

Both expressions are negative and of greater magnitude than (13) and (14) respectively: Note that, for given parameter values, the impact on land value is bigger if the city shape is circular rather than linear.

Using (13) and (14) as well as the elasticity of the optimal level of transport facilities with respect to a change in α_0 given by:

$$\varepsilon_{d_l^*,\alpha_0} = -\frac{\gamma - 1}{1 + \mu + \tau - \gamma - \beta\gamma} < 0 \quad \text{and} \quad \varepsilon_{d_c^*,\alpha_0} = -\frac{\gamma + \gamma\beta - 1 - \beta}{1 + \mu + \tau/2 - \gamma - \beta\gamma} < 0$$

we can compute the part of a desired improvement of transport facilities that could be subsidized by the attributable change in land value:

$$\frac{d\,(alr_l)}{d\,(d_l^*)} = \frac{\left(\varepsilon_{alr_l,\alpha_0} - \varepsilon_{alr_l,\alpha_0}\big|_{d=\bar{d}}\right) alr_l}{\left(\varepsilon_{d_l^*,\alpha_0}\right) d_l^*} = \frac{\tau}{\beta} \underbrace{\left(\frac{\beta(\gamma - 1)}{1 + \mu + \tau - \beta - \gamma}\right)}_{\zeta_l} > 0 \qquad (18)$$

for a linear city, and:

$$\frac{d\,(alr_c)}{d\,(d_c^*)} = \frac{\left(\varepsilon_{alr_c,\alpha_0} - \varepsilon_{alr_c,\alpha_0}\big|_{d=\bar{d}}\right) alr_c}{\left(\varepsilon_{d_c^*,\alpha_0}\right) d_c^*} = \frac{\tau}{2\beta} \underbrace{\left(\frac{\beta(\gamma - 1)}{(1 + \beta)(1 + \mu + \tau/2 - \beta - \gamma)}\right)}_{\zeta_c} > 0 \qquad (19)$$

for a circular city, where $0 < \zeta_l < 1$[21] and $0 < \zeta_c < 1$[22].

According to (12), the ratio between optimal transport facilities and land value is $\beta/g(\tau)$ and does not depend on α_0; as a consequence, any change in the optimal level of transport facilities will be accompanied by a proportional change in land value. The inverse of this ratios give us the dollar change in

[21] The SOC (appendix A) on the parameters are such that $1 + \mu + \tau - \gamma - \beta\gamma > 0$ and $\gamma > 1$. This implies that $1 + \mu + \tau - \beta - \gamma > 0$ and that $1 + \mu + \tau - \beta - \gamma > \beta(\gamma - 1)$ and thus that $0 < \zeta < 1$.

[22] The SOC (appendix A) on the parameters in the circular case are such that $1 + \mu + \tau/2 - \gamma - \beta\gamma > 0$, $\beta > 0$ and $\gamma > 1$. This implies that $1 + \mu + \tau/2 - \beta - \gamma > 0$ and that $1 + \mu + \tau/2 - \beta - \gamma > \beta(\gamma - 1)$ and thus that $0 < \zeta < 1$.

land value induced by any optimal dollar change in facilities. Thus, when ¯ is smaller (greater) than g (¿) ; one additional dollar spent in transport facilities, that results from an exogenous decrease in the parameter $®_0$, will generate a land value increase of more (less) than one dollar: However, according to (18) and (19), only a part $³_l$ and $³_c$ of this change in land value can justi…ably be allocated to subsidize transport facilities: As a consequence the ratio between the increase in land value that can be attributed to improvements in transport facilities is strictly[23] less than ¿=¯ in linear cities and ¿=(2¯) in circular cities.

In summary, any desired improvement in transport facilities that result from an exogenous decrease of the scale parameter of the travel cost function (a decrease in the price of energy for example), has a positive e¤ect on the value of per capita land rents of the economy. This e¤ect on land value can be greater or smaller than the own change in optimal spending in transport facilities: However one can argue that only a fraction of this change in land value should be used to subsidize the related cost. This fraction will depend on the parameter values of the model. Most parameters have an ambiguous e¤ect, but the ratio is greater the greater the scale economies (°) and the smaller the deseconomies of scale in transport facilities associated to city size (¹):

4 Discussion and conclusion

In an ideal world, the pricing of public facilities should be independent of the capacity cost. The pricing scheme should only be set in order to lead to an e¢cient use of the facilities. In the case of a local public facilities, a tax on the related land rents would, according to the Henry George Theorem, exactly cover the capacity cost. Using such a tax on land rents presents the advantage of creating few (no) distortions and of meeting some social justice goals in the sense that the landowners pay the bene…ts they derive from the existence of the public facilities.

In this paper we saw that in the particular context of transport facilities, the Henry George Theorem does not apply. However land rents are strongly related to spending in transport facilities, though in a di¤erent way than

[23] Reaching these upper limits, ¿=¯ in the linear case and ¿=(2¯) in the circular case, would require land value changes to only depend on changes of d or formally, that (13) or (14) equal 0: However this would require ° = 1 and in this case the equilibrium city size would be zero.

they are related to a classic local public good. The ratio between optimal per capita spending in transport facilities and per capita land rents, only depends on two parameters: the elasticity of the travel cost with respect to distance and the elasticity of travel cost with respect to spending in transport facilities.

Additionally, we saw that land value is positively related to improvements in transport facilities, and discussed to what extend this relationship can lead to the former in order to subsidize the latter.

These results give some theoretical support to the use of taxes on property value to ...nance spending in transport facilities. However, they also show that a priori, neither the cost of optimal transport facilities will be exactly covered by the aggregated land rent of a city, nor the cost of an optimal improvement of transport facilities, will be exactly covered by a tax on the change in land value directly related to it.

This paper gives a ...rst theoretical support to the use of taxes on land value to ...nance spending in transport facilities. It would be interesting to see how the results are being altered if a more realistic pattern for the individual demands for land and travelling is being considered; however, this would require the use of numerical methods to solve the model as analytical solutions are unlikely in this context. One could also consider explicit land consumption for transport facilities and transport congestion's problems with or without user's charge, following Kanemoto (1980) or Fujita (1986).

5 Appendix

5.0.1 Appendix A

Rent gradient we derive the price structure of land within a city using the properties of the indirect utility functions. The indirect utility of a person living at t units of distance from the center is:

$$V_t := v(Y_t; p; r_t)$$

where Y_t is the gross income net of the travel costs, p is the price of the consumption good and r_t the unitary land rent.

At the equilibrium, the utility levels at any location are identical. Any utility di¤erential induce an emigration of people from locations providing a relatively low level of utility; this will lead to a decrease of the land value of these locations and thus improve their relative attractiveness.

Formally:

$$\frac{\partial V_t}{\partial t} = 0 = \frac{\partial V}{\partial Y_t}\frac{\partial Y_t}{\partial t} + \frac{\partial V}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial V}{\partial r_t}\frac{\partial r_t}{\partial t} \tag{20}$$

Using Roy's identity in order to express the demand for land, set to unity by hypothesis, we have:

$$\lambda = -\frac{\partial V_t / \partial r_t}{\partial V_t / \partial Y_t} = 1 \Rightarrow \frac{\partial V_t}{\partial Y_t} = -\frac{\partial V_t}{\partial r_t}$$

A marginal decrease of the price of land will induce an increase of the individual's utility corresponding to the product between the disposable income generated by this decrease and the marginal utility of income.

Both, the price of the consumption good produced in the CBD and the gross income, are independent of t. Thus, (20) becomes:

$$\frac{\partial r_t}{\partial t} := r_t' = -f_t' := \frac{\partial f_t}{\partial t} \tag{21}$$

The price of land is a decreasing function of the distance from the CBD. It's slope is given by the travel cost function. In other words, the land price differential exactly reflects the travel cost differential. And as the opportunity cost of non urban land is zero, the price of the land located at the city center will exactly reflect the travel cost of the people living at the frontier of the city.

5.0.2 Appendix B

Second order condition in the linear city case The exact SOC on our parameters, based on the conditions on the Hessian matrix are cumbersome and unworkable. We thus base our analysis on an intuitive condition required to ensure our model is well behaved. We consider the restriction on the parameters that ensure:

$$\frac{\partial N_l^*(\delta)}{\partial \gamma} > 0$$

where $\delta := dN^{1-\gamma}$ is the proportion of aggregate production that is spent into transport facilities. In other words, we consider that the parameters have to be such that higher agglomeration economies (higher γ) lead to bigger cities at the equilibrium, in particular if the proportion of the aggregate production dedicated to transport facilities (δ) remains constant. This leads to the following condition on the parameters in the linear case:

$$1 + \mu + \tau - \gamma - \beta\gamma > 0$$

Proof: The city size that maximizes the per capita consumption given by (6) and using (8) is:

$$N_l^*(\delta) = \left(\underbrace{\frac{(1-\delta)(\gamma-1)(\tau+1)\,2^{\tau}\delta^{\beta}}{\alpha_0(\tau+\mu-\beta\gamma)}}_{A_l} \right)^{\frac{1}{1+\mu+\tau-\gamma(1+\beta)}}$$

the sign of $\partial N_l^*(\delta)/\partial\gamma$ depends in particular on the sign of:

$$\frac{\partial A_l}{\partial \gamma} = (1-\delta)(\tau+1)\,2^{\tau}\delta^{\beta}\,\frac{\mu+\tau-\beta}{\alpha_0(\tau+\mu-\beta\gamma)^2}$$

Considering that by definition $0 < \delta < 1$ and $\alpha_0, \beta, \gamma\mu, \tau > 0$, the sign of $\partial A_l/\partial\gamma$ depends on the sign of $\mu+\tau-\beta$. For $N_l^*(\delta)$ to be an interior solution, A_l is to be positive. Given $\gamma > 1$ (presence of agglomeration economies), we need $(\mu+\tau-\beta\gamma) > 0$, and thus, $\partial A/\partial\gamma > 0$.

As a consequence, $\partial N_l^*(\delta)/\partial\gamma > 0$ only if:

$$1 + \mu + \tau - \gamma - \beta\gamma > 0$$

which represents the intuitive second order condition in the linear case.

Second order condition in the circular city case We are looking for the restriction on the parameters that ensure:

$$\frac{\partial N_c^*(\delta)}{\partial\gamma} > 0$$

This leads to the following condition on the parameters in the circular case:

$$1 + \mu + \tau/2 - \gamma - \beta\gamma > 0$$

Proof: The city size that maximizes the per capita consumption given by (6) and using (9) is:

$$N_c^*(\delta) = \left(\underbrace{\frac{\pi^{\tau/2}\delta^{\beta}(1-\delta)(\gamma-1)(\tau+2)}{\alpha_0(2\mu+\tau-2\beta\gamma)}}_{A_c} \right)^{\frac{1}{1+\mu+\tau/2-\gamma-\beta\gamma}}$$

the sign of $\partial N_c^*(\delta)/\partial\gamma$ depends in particular on the sign of:

$$\frac{\partial A_c}{\partial \gamma} = \pi^{\frac{1}{2}\tau}\delta^{\beta}(1-\delta)(\tau+2)\frac{2\mu+\tau-2\beta}{\alpha_0(2\mu+\tau-2\beta\gamma)^2}$$

Considering that by definition $0<\delta<1$ and $\alpha_0;\beta;\gamma\,\mu;\tau>0$; the sign of $\partial A_c/\partial\gamma$ depends on the sign of $2\mu+\tau-2\beta$: For N_δ to be an interior solution, A_c is to be positive. Given $\gamma>1$ (presence of agglomeration economies), we need $(2\mu+\tau-2\beta\gamma)>0$, and thus, $\partial A_c/\partial\gamma>0$.

As a consequence, $\partial N_c^*(\delta)/\partial\gamma>0$ only if:

$$1+\mu+\tau/2-\gamma-\beta\gamma>0$$

which represents our intuitive second order condition in the circular case.

5.0.3 Appendix C

Optimal spending in transport facilities in the linear city The optimal spending in transport facilities solves the FOC of the maximization of (6) with respect to d; and is given by:

$$d_l^*(N) = \left(\frac{\alpha_0\beta}{(\tau+1)2^{\tau}}N^{\mu+\tau-\beta}\right)^{\frac{1}{1+\beta}}$$

Using (10) and rearranging yields:

$$d_l^* = \left(\beta^{1-\gamma}\left(\frac{\beta(\gamma-1)}{(\mu+\tau-\beta)}\right)^{\mu+\tau-\beta}\left(\frac{2^{\tau}(\tau+1)}{\alpha_0}\right)^{\gamma-1}\right)^{\frac{1}{1+\mu+\tau-\gamma-\beta\gamma}} \qquad (22)$$

Implying:

$$\frac{D_l^*}{F(N_l^*)} = d_l^*(N_l^*)^{1-\gamma} = \frac{\beta(\gamma-1)}{\mu+\tau-\beta}$$

Optimal spending in transport facilities in the circular city The optimal spending in transport facilities solves the FOC of the maximization of (6) with respect to d; and is given by:

$$d_c^*(N) = \left(\frac{2\alpha_0\beta}{(\tau + 2)\pi^{\tau/2}} N^{\tau/2+\mu-\beta} \right)^{\frac{1}{(1+\beta)}} \tag{23}$$

Using (11) and rearranging after simple but tedious algebra, we get:

$$\frac{D_c^*}{F(N_c^*)} = z_c^*(N_c^*)^{1-\gamma} = \frac{\beta(\gamma - 1)}{\mu + \tau/2 - \beta}$$

I would like to thank Thomas von Ungern-Sternberg, Claude Henry and Ralph Turvey, for very helpful comments and discussions.

6 References

References

[1] Alonso, W., 1964. Location and Land Use. Harvard University Press, Cambridge (Mass.).

[2] Arnott, R.J., Stiglitz, J.E., 1979. Aggregate Land Rents, Expenditure on Public Goods, and Optimal City Size. Quarterly Journal of Economics 93, 471–500.

[3] Arnott, R.J., Stiglitz, J.E., 1981. Aggregate Land Rents and Aggregate Transport Costs. The Economic Journal 91, 331–347.

[4] Flatters, F., Henderson, V., Mieszkowski, P., 1974. Public Goods, E¢-ciency and Regional Fiscal Equalization. Journal of Public Economics 3, 99–112.

[5] Fujita, M., 1986. Urban Land Use Theory. In Arnott, R. (Ed), Location Theory. Harwood Academic Publishers, Chur, pp. 73–149.

[6] Getz, M., 1975. A model of the Impact of Transportation Investment on Land Rents. Journal of Public Economics 4, 57–74.

[7] Haig, R.M., 1926. Toward an Understanding of the Metropolis: II. The Assignment of Activities to Areas in Urban Regions'. Quarterly Journal of Economics 40, 402–434.

[8] Isard, W., 1956. Location and Space-economy : a general theory relating to industrial location, market areas, land use, trade and urban structure. MIT Press, Cambridge (Mass.).

[9] Kanemoto, Y., 1980. Theories of urban externalities. North-Holland Publishing Company, Amsterdam.

[10] Lee, D.B. Jr., Averous, C.P., 1973. Land Use and Transportation: Basic Theory'. Environment and Planning 5, 491–502.

[11] Marshall, A., 1906. Principes d'économie politique, V. Giard & E.Brière, Paris.

[12] US Bureau of the Census, June 1997. Table No 478. All Governments–Detailed Finances: 1994. Table No 480. All Governments–Direct Expenditures for Public Works: 1980 to 1994.

[13] Vickrey, W., 1977. The City as a Firm. In: Feldstein, M.S., Inman, R.P. (Eds), The Economics of Public Services. McMillan, London, pp. 334–343.

[14] Von Thünen, J.H., 1826. Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie. Wiegandt Hempel & Parey, Berlin. Traduction: Perthes, A., 1966. The Isolated State. Pergammon Press, Oxford.