Introduction

In most western economies, the housing market is subject to some kind of regulation ; rents have to follow the evolution of the inflation rate, mortgage rates or some other economic index ; minimum quality standards apply for new constructions and the landlord-tenant relationship may be regulated. The housing market remains at the centre of a long standing political and economical debate ; should this market be regulated in some way? If yes, what is the appropriate form of the intervention and how does the regulated market outcome compare to a free market ? To understand the widespread existence of some form of regulation on this market, a short historical review will be helpful.

Governments often introduced regulation during wartime years : in some European countries during WWI and in the USA during WWII. In order to guarantee affordable rents, regulation often took the form of a nominal rent freeze on the existing housing stock. This lead to a decrease in real rents because of inflation and an important rent difference with the newly built stock, when the latter was not subject to regulation. If the newly built stock also was subject to rent control, it would simply depress new construction. Economists tend to agree that a nominal rent freeze will cause abandonment, retard maintenance and thus decrease the quality of housing. Preventing the market mechanism from working properly will usually result in welfare losses and is often not even in the long run interest of tenants. This type of control is referred to as first-generation rent controls in the literature.

After WWII and from the early 1950's, rent controls quickly disappeared in the USA, except for New York City. In Europe, decontrol was delayed and instead of disappearing, rent control was adapted : the hard first-generation programs were replaced by softer second-generation programs. Nominal rent freezes were replaced by a combination of a large variety of instruments : rents may adjust to inflation, construction costs, the costs of maintenance or mortgage rates, to mention but a few. A flat may be decontrolled and its price adjusted between two tenants or when it is vacated. Second-generation rent controls also typically contain provisions on maintenance or landlord-tenant relations. Whereas economists have been clearly opposed to first-generation rent controls, there is no clear consensus on second-generation programs. Many economists now argue that a well designed package of regulation may be welfare enhancing. Why is it so? The purpose of this paper is to develop a theoretical model to obtain some further insights into this kind of question.

It is obvious that the housing market is "different", in the sense that it is only a very distant parent of the textbook perfect competition model. The question then is what would be a more reasonable model within which to study problems of rent regulation. In our model we will emphasize the following deviations from the standard world of perfect competition.

First, houses are differentiated in a large number of dimensions, including the number of rooms, the size of rooms, the age of the building, location, isolation, etc... It thus seems natural to model the housing market within a setting of monopolistic competition. We shall model housing heterogeneity by using the circular road model of monopolistic competition first developed by Salop (1979). In the model, housing heterogeneity is limited to one single dimension, differences in location. We assume that tenants are differentiated by only one dimension, their place of work, so the transportation costs along the circle can be conveniently interpreted as travel costs from the working place to the place of residence. A more realistic approach might well be to permit differentiation according to more than one dimension, but for reasons of tractability we restrict ourselves to a one dimensional framework.

Second, it seems to us that informational problems play an exceptionally important role on the housing market : often attractive flats are passed on between friends without even being advertised ; the descriptions of apartments that do appear in advertisements are usually so summary that one has to visit them to have a clear idea just what they are like ; and some apartments are usually advertised quite early while others appear just shortly before they are vacated. We model the opacity of the rental market by introducing search costs. Again this is of course a strong simplification of the real world, but it is still an improvement over the assumption of complete information that is frequently made.

Third, and most important, tenants usually do not know, at the time they rent an apartment, how long they will stay there. As a result, there is often a strong discrepancy between the duration of the initial rental contract and the length of time the tenant actually stays in the flat. The rental contract has to be renewed or renegotiated periodically. The key element of our model is the fact (or assumption) that the mechanism of price setting is different at the time a tenant first enters an apartment and when he renegotiates his rental contract. When a tenant is looking for an apartment there is competition between the different owners to attract him. When the contract is being renegotiated, the owner is in a much better bargaining situation. The tenant has large fixed costs associated with moving. Not only the costs of searching for a new apartment and the fixed cost of moving his furniture, but also the more psychological costs of loosing neighbors and having to give up a social environment and habits he may have taken. When a rental contract is renegotiated, there is a classical ”hold-up” problem. The owner may take advantage of the tenant’s fixed costs of moving to extract a higher rent. While it may be true that the owner may also have some fixed costs when taking a new tenant, it seems to us that on the whole it is the tenant who is in a more difficult bargaining position. We will model the process of renegotiation as one where the owner can make the tenant a ”take it or leave it” offer.

Finally the landlord may learn something about the tenant’s characteristics once he has moved in. This opens the possibility for ex post price discrimination. The landlord may substantially increase the rent when he finds out that his tenant really likes the apartment.

Our model thus studies the interaction among elements of monopolistic competition, search, bargaining with one-sided sunk costs and price discrimination. Regulation in our model takes the form of specifying the time and rules of rent renegotiation, once the initial rental contract has expired. More specifically, regulation fixes the number of years over which the landlord cannot increase (renegotiate) the rent once a new tenant has moved in. We study how the price level, search costs, the number of apartments and social welfare are affected as the duration over which the initial rent has to be maintained changes. This approach to modelling tenant protection is inspired by the rental regulation as it is applied in Germany or Switzerland (but not the USA). In both of these countries the standard rental agreement is a one year contract, which is then renewed. Rent control essentially specifies rules setting upper limits for the amount by which rents can be increased at the moment the contract is renewed.

Our main results are as follows : even with rational expectations and perfect foresight, it is not true that the level of rents and social welfare are unaffected by rent regulation. Quite to the contrary. In our model, where initial rents are freely set by the landlords, there is a strong positive relation between rent regulation and social welfare. The longer the period for which the landlord has to maintain the rent he has initially agreed upon, the lower will be the expected rent and the higher social welfare.

The rest of our paper is structured as follows : Section 2 formally sets out the basic assumptions of our model and solves it for the case where we have a ”Free Market”, i.e. the landlord may renegotiate the rental contract at the end of every year. Section 3 studies the ”Fully Regulated market”, which is the situation where the landlord has to maintain the rent initially agreed upon as long as the tenant stays in. Section 4 compares the outcomes of the two models i.e. Free Market vs Full Regulation. In Section 5 we address the question of what would happen in intermediate situations, i.e. what happens to prices when landlords only can renegotiate rental contracts after T years? This more general model admits the Free Market model ($T = 1$) and the Fully Regulated market model ($T = \infty$) as special cases. In Section 6 we make a welfare analysis of this general model. Finally Section 7 concludes.

Model I : The Free Market

Landlords

The basic model we use is the ”circular road” model of monopolistic competition developed by Salop (1979). N risk neutral landlords, each possessing a apartments, enter the market and locate uniformly on the circular road of unit circumference. Thus there are aN apartments on the market and the density of apartments on the circle is also aN, whereas the number of competitors is N. The flats are identical in their characteristics : the only source of product differentiation is their location.

Each apartment causes per year fixed cost f and zero marginal cost. Owners will compete in prices to attract tenants and entry occurs until their expected profits are equal to zero.

Tenants

There is a population of L tenants, with $L < aN$. Each tenant has a place where he works, his most preferred location, and a place of residence. Workplaces are uniformly distributed on the circle (as are flats).

At the end of each year, each tenant has a probability λ of dying. The life expectation is thus $1/\lambda$ at any point in the consumer's life (the process is memoryless). In each year λL tenants die and λL new tenants, drawn from a uniform distribution, are born. Thus the population is a constant L in any year, consisting of λL ”new” tenants and $(1-\lambda)L$ ”old” tenants.

As information about available flats is costly for tenants, it will not usually be worth their while to search for an apartment right next to their place of work. Rather they will be satisfied if they find a flat which is sufficiently close to their most preferred location. The distance from the working place to the place of residence is denoted by d. The unit transport cost along the circle is denoted by t. U is the utility derived from occupying a flat for one year. We assume that U is sufficiently high so that each tenant ”consumes” a flat in each year. When a tenant wants to switch apartments, he incurs fixed moving costs F.

Search

In the real world it is more difficult to find a flat which corresponds to ones' preferences when there are relatively few vacancies, i.e. when the market vacancy rate is low. The higher the vacancy rate, the easier it is to find a flat. When modelling the tenants' search process we wish to capture this crucial element. There are two simple ways of achieving this result : the first is to have the tenants search not just among the set of all empty flats, but among the set of all flats (including the ones that are occupied). The cost of searching for an empty flat then increases as the vacancy rate decreases. The second alternative is to let the tenants search only among the set of empty flats, but to assume that the (per unit) search costs are inversely proportional to the vacancy rate. For technical reasons we shall adopt this second approach. If we denote the vacancy rate by V, the per unit search costs are thus equal to s/V.

The search process is as follows : we assume that each tenant randomly contacts one of the N landlords. The landlord then randomly selects one of his a apartments and proposes it to the tenant footnote . The information the landlord transmits the tenant is the rental price he charges and the location of the flat. As tenants want to live in a location as close as possible to their workplace they are willing to search ; but since search is costly, there is a trade-off between searching more for a flat closer to the most preferred location and bearing extra unit search cost s/V. As Lippman and McCall (1976) have shown, the optimal search behavior in such a setting is a sequential search. This optimal strategy can be stated as a stopping rule : accept the flat if it yields an expected utility level greater than some critical value, otherwise keep on searching. In equilibrium, distance will be the only source of product differentiation, so it is the only reservation variable we need. We define reservation variables with a bar : $\overline{d}_i$ is tenant i's distance of reservation i.e. the distance within which he would accept a flat on both sides of his most preferred location (his workplace). Tenant i's interval of reservation is thus of length $2\overline{d}_i$.

Prices

There are two prices in the Free Market model : new tenants pay the ”first year” price p_1 and after renegotiation, the ”second year” price is denoted by p_2.

In the market for new tenants, landlords compete in prices to attract them. The equilibrium concept is the standard Nash Equilibrium in prices. Each owner sets the rent as an optimal reply to the rents charged by other owners.

In the Free Market model, where rent protection is inexistent, rents can be renegotiated at the end of the ”first year”. We assume the renegotiation of rents takes the following form : the owner makes the tenant a ”take it or leave it” offer. In doing so he knows the tenant’s expected lifetime, moving costs and workplace. The ”second year” price is influenced by the following factors : first, the fixed moving costs that a tenant would have to incur, were he to move, allows for a rent increase known as the ”hold-up”. Second, the owner can observe his tenant’s workplace in the ”first year” : this is an important piece of information as it tells the owner the intensity of his tenants’ preferences. This will allow the owner to price discriminate between the tenants during renegotiation, something he could not do (by assumption) with new tenants. The ”second year” price is thus a function of the tenant’s distance from his workplace ; $p_2 = p_2(d)$.

Equilibrium

The model analysed is a four stage game. We use the concept of Perfect Nash Equilibrium. We are interested in the properties of the steady state, symmetric, rational expectations equilibrium.

In the first stage of the game, the number of owners, N (each owning a apartments) is determined by a zero profit condition. In the second stage the owners compete in prices to attract tenants by setting their (Nash Equilibrium) ”first year” rents, p_1. In the third stage, new tenants (and those who should decide whether to leave their apartment) search for an apartment. In the fourth and last stage, the old tenant and the landlord renegotiate the rental contract and set $p_2(d)$.

As usual, the model will be solved by backward induction. For simplicity we ignore discounting.

Solving the model

The fourth stage : Renegotiation

Note first of all that the probability of a tenant dying at the end of any given year is by assumption constant and equal to λ. In a steady state, the incentives for a tenant to leave an apartment at the end of any year in time are thus always the same. Similarly for an owner, the incentives he faces to increase the rent at any end of year are also constant. As a result, the conditions of renewal negotiated at the end of the ”first year” will be maintained for all the following years. Both the owner’s and the tenant’s outside options do not change over time (by construction) : tomorrow’s problem is as today’s. In essence the Free Market model is thus a two period model. The only reason why we do not limit the tenant’s lifetime to two years is because in Section 5 the longer lifetime allows us to do comparative statics with respect to the duration of rental protection.

The problem the landlord solves during renegotiation at the end of the ”first year”, for each of his tenants, is the following : maximize the rent increase given the tenant’s alternative to move. At the end of the ”first year”, if the tenant stays in his flat, located at distance d of his most preferred location, he has the following expected (lifetime) surplus :

$$\frac{1}{\lambda}(U - td - p_2(d)) \qquad \#$$

In words he gets the utility of occupying a flat minus transport costs from his workplace to his place of residence minus the rent paid, multiplied by his expected lifetime.

The tenant’s alternative to staying in his apartment is moving : this is his outside option. If he does so, the costs he incurs are of four types : (a) expected rent paid per year $E(P)$, (b) expected travel costs per year from his place of residence to his workplace $E(Tr)$, (c) expected search costs $E(S)$ and (d) certain moving cost F. The utility of occupying a flat U, expected rent $E(P)$, and expected transport costs $E(Tr)$ are incurred in each year, whereas expected search costs $E(S)$ and certain moving cost F are incurred only once. Thus if the tenant moves at the end of the ”first year” his expected utility is writing, valued at the moment of the renegotiation

$$\frac{1}{\lambda}(U - E(P) - E(Tr)) \ - E(S) - F \qquad \#$$

The tenant will stay in his current flat if and only if the surplus derived from staying is at least as large as the outside option of moving. The landlord, knowing this, will set $p_2(d)$ so that the tenant is just indifferent between staying and moving : this is the "take it or leave it" offer. Thus $p_2(d)$ is defined by the following condition, $(1) = (2)$:

$$\frac{1}{\lambda}(U - td - p_2(d)) \;=\; \frac{1}{\lambda}(U - E(P) - E(Tr)) \;- E(S) - F \qquad \#$$

Solving for $p_2(d)$ yields

$$p_2(d) = E(P) + E(Tr) + \lambda E(S) + \lambda F - td \qquad \#$$

We note that $p_2(d)$ is a decreasing function of d, and the slope is $(-t)$. As of the "second year" the owner will extract all the surplus a tenant gets from living in a flat closer to his place of work. This is, of course, a form of price discrimination.

The fact that the owner can practice price discrimination as of the "second year", has consequences for the tenant's search behavior. He will rationally anticipate that finding a flat close to his place of work will yield him a surplus only for the "first year" before the contract can be renegotiated.

The third stage : **Search**

Tenants can (potentially) search for a new flat for two different reasons. Because they have only just entered the market and need a flat, or because they might decide to leave their current flat and move to a new one. Given the assumption of a constant probability of dying, the search problems of these two "types" of tenants are the same. As we look for a symmetric equilibrium in our model, we take the price distribution of "first year" rents as degenerate : all prices are equal, and tenants rationally anticipate this. This type of search model, where tenants know the exact distribution of both prices and "quality" of the good, has been widely studied in the literature and Lippman and McCall (1976) have shown that the optimal search rule in such a setting is a sequential search. footnote In our model tenant i will define a distance of reservation $\bar{d}_i$, on both sides of where he is working and will accept a flat if it lies within this interval $2\bar{d}_i$. If $\bar{d}_i$ is high then the expected amount spent on travel is high. But if $\bar{d}_i$ is high the expected number of searches is low. This trade-off defines $\bar{d}_i$.

If the tenant is willing to accept any flat within an interval of length $2\bar{d}_i$ around his workplace, the probability of finding an acceptable flat with any given draw is equal to $2\bar{d}_i$. (Remember that the circumference of the circle is just equal to 1). The expected number of searches can be written as a sum :

$$1 * 2\bar{d}_i + 2(1-2\bar{d}_i)2\bar{d}_i + 3(1-2\bar{d}_i)^2 2\bar{d}_i + 4(1-2\bar{d}_i)^3 2\bar{d}_i + \ldots = \frac{1}{2\bar{d}_i} \qquad \#$$

As the unit search cost is s/V, total expected search cost is $\frac{s}{2\bar{d}_i V}$. Now tenant i's problem is to choose $\bar{d}_i$ that maximizes his intertemporal expected utility $E\widetilde{U}_i$, as seen at the moment he starts searching. We shall see that the landlords' optimal "second year" pricing behavior is such that a tenant who has once moved in will not have an incentive to move (until he dies). Tenant i's intertemporal expected utility $E\widetilde{U}_i$ can thus be written

$$E\widetilde{U}_i = \frac{1}{\lambda}U - p_1^e - (\frac{1}{\lambda} - 1)p_2^e(d) - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \qquad \#$$

The first term is the utility derived from living in the flat. The second and third are the expected rents paid during lifetime. The fourth term is expected travel cost and the last term is expected search cost.

The tenant undertaking his search is forward-looking in the sense that he rationally anticipates the fourth stage renegotiation : he knows how rents will be set in the next years. In other words he anticipates the price discrimination that takes place from the "second year". As the tenant maximises his expected utility, he expects, on average, to be located in the middle of his interval of reservation, at $\frac{\bar{d}_i}{2}$. He will take this into account in his search behaviour. Because $p_2(d)$ is linear in d, and that the expectation of a linear function is this function evaluated at its expected value, i.e. at

$\frac{\bar{d}_i}{2}$, we have $p_2^e(d) = p_2(\frac{\bar{d}_i}{2})$. After substitution of $p_2(d)$ of equation (4), evaluated at $\frac{\bar{d}_i}{2}$, into (6) we have :

$$E\widetilde{U}_i = \frac{1}{\lambda}U - p_1^e - [\frac{1}{\lambda} - 1][E(P) + E(Tr) + \lambda E(S) + \lambda F - \frac{t\bar{d}_i}{2}] - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \qquad \#$$

Tenant i maximises his intertemporal expected utility with respect to $\bar{d}_i$, which is his choice variable in the search process. Deriving $E\widetilde{U}_i$ with respect to $\bar{d}_i$ yields the following first-order condition :

$$\frac{\partial E\widetilde{U}_i}{\partial \bar{d}_i} = (\frac{1}{\lambda} - 1)\frac{t}{2} - \frac{1}{\lambda}\frac{t}{2} + \frac{s}{2\bar{d}_i^2 V} = 0 \qquad \#$$

Solving (8), we define

$$\bar{d}_i = \sqrt{\frac{s}{tV}} \qquad \#$$

tenant i's optimal interval of reservation. footnote This simple formula for $\bar{d}_i$ is consistent with our expectations ; if the unit search cost s/V increases, searching becomes less attractive and the tenant is willing to accept a flat in a larger interval. If the unit transportation cost t increases, travelling is more costly (or the intensity of preferences increases) which leads the tenant to accept a flat in a smaller interval. Note that $\bar{d}_i$ is independent of λ : this is due to the fact that the owner extracts all the surplus the tenant gets from staying in the flat from the "second year", after the contract is renegotiated (c.f. Section 2.6.1).

Implications for $p_2(d)$:

We are now in a position to be more precise about $p_2(d)$. We previously defined it in equation (4) as follows :

$$p_2(d) = E(P) + E(Tr) + \lambda E(S) + \lambda F - td \qquad \#$$

where $E(P)$ is expected rent paid per year, $E(Tr)$ expected transport costs per year, $E(S)$ expected search costs and F fixed moving cost. All those costs refer to the situation where the tenant is moving.

If the tenant is moving he will pay an expected rent p_1^e for the "first year" and $p_2^e(d)$ for the subsequent years, i.e. for $(\frac{1}{\lambda} - 1)$ years. Thus, expected rent paid per year writes $E(P) = \frac{p_1^e + (\frac{1}{\lambda} - 1)p_2^e(d)}{\frac{1}{\lambda}}$. Expected travel costs per year are $E(Tr) = \frac{t\bar{d}^e}{2}$. If moving, the tenant bears expected search costs of $E(S) = \frac{s}{2\bar{d}^e V}$ and certain fixed moving cost F. Substituting these different expressions into (10) we obtain :

$$p_2(d) = \lambda p_1^e + (1 - \lambda)p_2^e(d) + \frac{t\bar{d}^e}{2} + \lambda\frac{s}{2\bar{d}^e V} + \lambda F - td \qquad \#$$

This expression relates $p_2(d)$ to $p_2^e(d)$. We can simplify the $p_2^e(d)$ term, the expected "second year" price ; because $p_2(d)$ is linear in d, and that the expectation of a linear function is this function evaluated at its expected value $\frac{\bar{d}^e}{2}$, we have $p_2^e(d) = p_2(\frac{\bar{d}^e}{2})$. After simplification we get :

$$p_2(d) = p_1^e + \frac{t\bar{d}^e}{2} + \frac{s}{2\bar{d}^e V} + F - td \qquad \#$$

This equation relates $p_2(d)$ to the "first year" price p_1^e. Before we can say more about $p_2(d)$, we need to solve for the "first year" rent p_1^e, to which we now turn.

The second stage : "**first year**" **rental contracts**

To attract tenants, landlords compete in prices. They maximize profits with respect to p_1, the "first year" rental price, given the tenants' search behavior. As we look for a (symmetric) Nash Equilibrium in p_1, we consider a deviant firm setting p_{1i}, given the other firms set p_1.

When a deviant firm reduces its price from p_1 to p_{1i}, tenants at the margin (i.e. just beyond $\bar{d}^e$, the tenant's interval of reservation as anticipated by the landlord), are willing to accept this flat : in exchange of the higher travelling costs they get a lower rent.

The question then is : for a given unilateral price decrease Δp by firm i, what is the interval Δd from which firm i will attract new tenants? We know that from the "second year" onwards, the owner extracts the entire tenant's surplus through price discrimination. Thus it is only in the "first year" that a tenant can get a utility higher than some critical value, denoted by $\widetilde{U}$. The marginal tenant working at distance $\overline{d}^e$ from his residence place will accept to buy from firm i only if $U - t\overline{d}^e - p_1 \geq \widetilde{U}$. If firm i now decreases its price to $p_{1i} = p_1 - \Delta p$, some tenants beyond $\overline{d}^e$ will now accept firm i's apartment if $U - td_i - p_{1i} \geq \widetilde{U}$. Thus by decreasing its price to p_{1i}, producer i will gain customers from an interval of length $d_i - \overline{d}^e = \frac{p_1 - p_{1i}}{t}$, or $\Delta d = \frac{-\Delta p}{t}$, on each side. Rewriting this expression yields $d_i = \overline{d}^e + \frac{p_1 - p_{1i}}{t}$ and we denote it by $d_i(p_{1i})$.

We now have all the elements to write down deviant firm i's expected profit function (per year). From Section 8.1 of the Appendix, we know that the demand function can be written as $2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N} \mathrm{d}d$ (equation (75)). The expected profit function combines the demand function and the expected rent paid per period. It reads :

$$E\pi_i = 2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N} \frac{p_{1i} + (\frac{1}{\lambda} - 1)p_2(d)}{\frac{1}{\lambda}} \mathrm{d}d - af =$$

$$= 2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N} [\lambda p_{1i} + (1-\lambda)p_2(d)]\mathrm{d}d - af \qquad \#$$

Differentiating $E\pi_i$ with respect to p_{1i} (using Leibniz's theorem of derivation) yields the first-order condition :

$$\frac{dE\pi_i}{dp_{1i}} = 2\frac{L}{2\overline{d}_i N}[\lambda p_{1i} + (1-\lambda)p_2(d_i(p_{1i}))]\frac{dd_i(p_{1i})}{dp_{1i}} + \qquad \#$$

$$2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N}\lambda \mathrm{d}d = 0$$

As we look for a symmetric Nash Equilibrium, we have $p_{1i} = p_1$ and thus $d_i(p_{1i} = p_1) = \overline{d}^e$. Evaluating the first order condition at this point, we get :

$$\left.\frac{dE\pi_i}{dp_{1i}}\right|_{p_{1i}=p_1} = 2\frac{L}{2\overline{d}_i N}[\lambda p_1 + (1-\lambda)p_2(\overline{d}^e)]\frac{-1}{t} + 2\frac{L}{2\overline{d}_i N}\lambda\overline{d}^e = 0 \qquad \#$$

Substituting $p_2(\overline{d}^e)$ from (12) into (15) and remembering that expectations are rational ($p_1^e = p_1$), we can solve for p_1:

$$p_1 = \frac{1+\lambda}{2}t\overline{d}^e - (1-\lambda)(F + \frac{s}{2\overline{d}^e V}) \qquad \#$$

This equation depends on $\overline{d}^e$, the expected distance of reservation. But $\overline{d}^e = \overline{d}_i$ as the landlord has rational expectations on the tenant's search behaviour. For notational ease, we define $\overline{d} = \overline{d}^e = \overline{d}_i$. We thus can substitute the expression of $\overline{d}_i$ given in (9) into p_1 and $p_2(d)$. This yields rents as functions of the structural parameters of the model (for a given number of firms N, solved in the next Section) :

$$p_1 = \lambda\sqrt{\frac{st}{V}} - (1-\lambda)F \qquad \#$$

$$p_2(d) = (1+\lambda)\sqrt{\frac{st}{V}} + \lambda F - td \qquad \#$$

Interpretation

a) Expected rent paid :

As a useful benchmark let us start-off by studying the per-year expected rent a tenant will pay, denoted by $\overline{p}(d)$.

$$\bar{p}(d) = \frac{p_1 + (\frac{1}{\lambda} - 1)p_2(d)}{\frac{1}{\lambda}} = \sqrt{\frac{st}{V}} - (1-\lambda)td \qquad \#$$

One first notes that $\bar{p}(d)$ is independent of F, the fixed moving cost. In spite of the fact that the ”second year” rent increases with the fixed moving cost F, the expected (per year or lifetime) rent does not. This can be explained as follows: Owners anticipate that they can increase their rent form the ”second year” onward. This in turn will intensify ”first year” competition, i.e. competition for new tenants. The resulting rent reduction in the ”first year” is such that the increase in ”second year” rents just compensates the reduction in the ”first year”.

Second, what affects the level of the expected rent are the structural parameters of the model. The higher search costs s/V or transport costs t, the higher the landlord’s monopoly power and thus the higher the average price level. Third, expected rent decreases with distance but only by a factor $(1-\lambda)$. This is due to the fact that the owner can price discriminate only from the second year and onward.

On average, a consumer can expect to be located in the middle of his interval of reservation i.e. at $\frac{\bar{d}}{2}$. Evaluating the expected paid rent $\bar{p}(d)$ at this point yields the average expected paid rent $\bar{p}(\frac{\bar{d}}{2})$:

$$\bar{p}(\frac{\bar{d}}{2}) = \frac{p_1 + (\frac{1}{\lambda} - 1)p_2(\frac{\bar{d}}{2})}{\frac{1}{\lambda}} = \sqrt{\frac{st}{V}}\,\frac{1+\lambda}{2} \qquad \#$$

The lower λ, the higher the life expectation and the lower the average rent paid per year, because landlords will compete tougher to attract tenants if they live longer, as there are more years to extract tenant surplus. To understand (19) and (20) in greater detail, it is useful to turn to the discussion of the ”second year” and ”first year” rents taken individually.

b) ”Second year” rents :

”Second year” rents are given by equation (18) and are an increasing function of the moving cost F. This is for obvious reasons: The greater the moving cost, the more expensive it is for the tenant to move at the end of the ”first year” and the more the owner will be able to increase rents. F enters the equation for ”second year” rents with a coefficient λ. This can be explained as follows : the lower is the probability of dying λ, the less important is the hold-up problem per period, because the fixed costs of moving can be spread over a longer expected lifetime.

Slightly less obvious, ”second year” rents are also an increasing function of search costs s/V. The reason is the same as in the preceding paragraph. The higher search costs, the more expensive it is to move at the end of the ”first year”, and the greater the hold-up problem i.e. the rent increase the owner can obtain at the end of year one.

Transport cost t enters the price for two reasons. First, the higher transport cost t, the higher the owner’s monopoly power as the tenant’s intensity of preferences is larger, which means higher prices. Second, $p_2(d)$ decreases with transport costs $(-td)$; in the presence of fixed costs for moving the owners can practice ”second year” price discrimination when renegotiating their contracts. This insight is often neglected in the discussion about tenant protection. In practice it seems plausible that the owners can learn quite a lot about the tenants once they have moved in : whether they like the flat, whether they like the schools in the area, whether they have made friends with neighbours etc... It is hard to see why in an unregulated housing market the owners could not make use of this information to extract some of the tenant’s surplus.

c) ”First year” rent :

As was to be expected, the ”first year” rent p_1 given by (17) is an increasing function of search costs. In models of monopolistic competition, an increase in search costs leads to less competition and higher equilibrium prices. This is a well known result (see von Ungern-Sternberg (1982)). One notes however, that the term in search costs is an increasing function of λ. The greater is the probability of dying, the higher the influence of search costs on ”first year” prices. This can be explained as follows. In the present model search costs have two effects : first, as mentioned, they reduce the intensity of ”first year” competition, and second they act as a fixed cost of moving house at the end of the ”first year”. This allows the owner to increase ”second year” prices (just like the fixed cost of moving F). Since the owners anticipate the increase in ”second year” prices through search costs, this increases the intensity of ”first year” price competition. As a result of these two effects going in opposite directions, first period prices still do increase with search costs, but this effect gets weaker as life expectancy increases.

The $-(1-\lambda)F$ term has already been discussed previously : it is due to competition to attract new tenants as owners know they will be able to increase prices from the second year and onwards by a fraction λF.

To get some further insights in the mechanisms that arise in the model, we define the mark-up between the first and the ”second year” rents.

d) The mark-up :

The mark-up $m(d)$ is defined as follows :

$$m(d) = p_2(d) - p_1 = \sqrt{\frac{st}{V}} + F - td \qquad \#$$

It represents the extra price paid by a tenant living at distance d of his working place, from year two and onwards. It is increasing in F, which is the hold-up mechanism. s/V and t, that can be interpreted as the landlord’s monopoly power increase the mark-up. The $-td$ term is the price discrimination mechanism. The mark-up is independent of λ : whatever the tenant’s life expectation, he faces the same mark-up.

Evaluating the mark-up at $\bar{d}$, i.e. for the most far away situated tenant at equilibrium yields the lowest rent increase a tenant can get :

$$m(\bar{d}) = F \qquad \#$$

As F is the fixed cost of moving, this means that the mark-up is strictly positive for all tenants. This is the hold-up in action ; all tenants pay a higher rent (of at least F) from the ”second year” on. Tenants living closer than $\bar{d}$ to their workplace experience an even higher rent increase because of the price discrimination mechanism. The higher is F, the more important is the price increase after renegotiation. Recall however that if F is higher, average rents will not be higher (c.f. a)).

Finally, one can ask whether a landlord having a tenant located at $\bar{d}$ would have an incentive to increase the rental price such that this tenant would leave. The answer is no because if the tenant were to leave, the landlord would simply have one more vacant apartment. This strategy would not increase his expected revenue as the probability of getting another tenant is determined by the first period rent p_1 and not the number of vacant apartments (recall that by assumption each landlord always has some vacant apartments). Thus, even if an owner has a tenant living as far away as possible at equilibrium from his place of work, he earns more by keeping him as long as $p_2(\bar{d}) > 0$. This relationship clearly always holds as $p_2(\bar{d}) = \lambda\sqrt{\frac{st}{V}} + \lambda F > 0$.

The first stage : **Entry**

We now take the last step in solving the model : the determination of N, the number of owners, with a free entry condition. As long as $E\pi > 0$, it is profitable for a landlord to enter the market. But entry will increase competition among landlords, lower prices and ultimately lead to zero profits. Entry then stops and the market reaches an equilibrium.

In the symmetric, steady state, rational expectations equilibrium we have characterized, the market vacancy rate is $\frac{aN-L}{aN}$ and the occupancy rate is $\frac{L}{aN}$. The expected revenue a landlord gets from a tenant is a weighted average of first and ”second year” prices. Thus expected profit per year is :

$$E\pi = a\left[\frac{aN-L}{aN} * 0 + \frac{L}{aN}\left[\lambda p_1 + (1-\lambda)p_2(\frac{\bar{d}}{2})\right] - f\right] = 0 \qquad \#$$

where f is the fixed cost of one apartment per year and a the number of apartments of a landlord. $p_2(d)$ is evaluated at $\frac{\bar{d}}{2}$ as it is the expected location of a tenant. This condition defines aN, or equivalently $\frac{aN}{L}$. Substituting p_1 and $p_2(\frac{\bar{d}}{2})$ given by (17) and (18) yields :

$$\frac{aN}{L} = \frac{\lambda+1}{2f}\sqrt{\frac{st}{V}} \qquad \#$$

But the market vacancy rate V (as seen at the moment of search) is a function of N : $V \equiv \frac{aN-(1-\lambda)L}{aN}$. After substitution of V, we can rearrange (24) and write it as an equation of the second degree in $\frac{aN}{L}$

$$(\frac{aN}{L})^2 - (1-\lambda)\frac{aN}{L} - \frac{st(1+\lambda)^2}{4f^2} = 0 \qquad \#$$

that has two solutions :

$$(\frac{aN}{L})' = \frac{(1-\lambda) - \sqrt{(1-\lambda)^2 + \frac{st(1+\lambda)^2}{f^2}}}{2} \qquad \#$$

$$(\frac{aN}{L})'' = \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{st(1+\lambda)^2}{f^2}}}{2} \qquad \#$$

The first solution is always negative and thus uninteresting for our model. The second one is always positive as λ is a probability. But for the solution to make sense we require more than non-negativity : we require $(\frac{aN}{L})'' > 1$. This condition is equivalent to having $\frac{st}{f^2} > \frac{4\lambda}{(1+\lambda)^2}$. This holds for some values of the parameters since $\frac{4\lambda}{(1+\lambda)^2}$ fluctuates between 0 and 1 for all $\lambda \in]0,1]$. Thus, the condition $\frac{st}{f^2} > 1$ is sufficient to insure that $(\frac{aN}{L})'' > 1$. We denote by V^F the vacancy rate in the Free Market model defined as $V^F = \frac{aN-(1-\lambda)L}{aN}$ where aN is given by (27).

We are now in a position to derive some comparative statics results.

Comparative statics results :

aN increases in L (size of the market), search and transport costs (monopoly power). N decreases with the fixed cost f. λ has an ambiguous effect on entry : a high λ increases prices and thus entry, but a high λ means a short live expectation and thus few years of tenant surplus extraction, which depresses entry.

Model II : The Fully Regulated market

In this section, we develop the same story in another institutional setting. The market is now Fully Regulated in the sense that no renegotiation between the owner and the old tenant is possible, at any point in time : rental prices must stay constant over time. We denote this price by p_0.

The structure of the game is the same as before except that there is now only one interaction to set prices ; in the first stage N firms enter the market until their expected profits are zero. In the second stage they compete in prices and set a rental price p_0 that will hold for ever. In the third and last stage tenants search for an apartment. The equilibrium concept we use is the same as before : we look for a symmetric, steady-state, rational expectations Subgame Perfect Equilibrium. The model is again solved by backward induction.

The third stage : Search

The search problem is essentially the same as in the previous model. We can apply the exact same arguments here as in Section 2.6.2. Note however that in this Fully Regulated set-up, a tenant never has any incentive to move: moving would induce both search and fixed moving costs, without any compensating benefit (like a lower ”first year” rent as in the Free Market model).

The problem the tenant faces is again to adopt a search behavior that maximizes his intertemporal expected utility $E\widetilde{U}_i$. This reads, as seen from the moment of birth :

$$E\widetilde{U}_i = \frac{1}{\lambda}U - \frac{1}{\lambda}p_0^e - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \qquad \#$$

The only difference with (6) of the Free Market model is the $\frac{1}{\lambda}p_0^e$ term. Instead of a weigthed average of first and ”second year” rents, the rent is now constant. Maximizing $E\widetilde{U}_i$ with respect to

$\bar{d}_i$ yields the following first order condition :

$$\frac{\partial E\widetilde{U}_i}{\partial \bar{d}_i} = -\frac{t}{2\lambda} + \frac{s}{2\bar{d}_i^2 V} = 0 \qquad \#$$

Solving (29), we get footnote :

$$\bar{d}_i = \sqrt{\frac{\lambda s}{tV}} \qquad \#$$

Search and transport costs play the same roles as in the previous model (Section 2.6.2). The difference is that λ now enters $\bar{d}_i$. This is because a tenant having found an apartment close to his place of work now has a high surplus each year, because there is no possibility for the owner to renegotiate after some years and extract his surplus. If λ is low, the life expectation is large, which means the tenant will incur travelling costs that directly affect his surplus for many years (whereas search costs are incurred only once), thus decreasing his interval of acceptance.

The second stage : Rental Contracts

Given the tenants' search behaviour, landlords will compete in prices to attract tenants, knowing that rents cannot be raised in the future. To solve for the rent p_0, we look for a symmetric Nash Equilibrium in prices and consider a deviant firm setting p_{0i}, given that the other firms set p_0. By unilaterally decreasing its price to p_{0i}, producer i will gain customers from an interval of length $d_i - \bar{d}^e = \frac{p_0 - p_{0i}}{t}$ on each side. Rewriting this expression yields $d_i = \bar{d}^e + \frac{p_0 - p_{0i}}{t}$ and we denote it by $d_i(p_{0i})$. We adopt the same approach as in Section 2.6.3. Firm i's expected profit function thus writes :

$$E\pi_i = 2\int_0^{d_i(p_{0i})} \frac{L}{2\bar{d}_i N} p_{0i} \mathrm{d}d - af \qquad \#$$

Maximizing $E\pi_i$ with respect to p_{0i} we obtain the first order condition :

$$\frac{dE\pi_i}{dp_{0i}} = 2\frac{L}{2\bar{d}_i N} p_{0i} \frac{dd_i(p_{0i})}{dp_{0i}} + 2\int_0^{d_i(p_{0i})} \frac{L}{2\bar{d}_i N} \mathrm{d}d = 0 \qquad \#$$

In the symmetric Nash Equilibrium we have $p_{0i} = p_0$ and thus $d_i(p_{0i}) = \bar{d}^e$. (32) thus becomes :

$$\left.\frac{dE\pi_i}{dp_{0i}}\right|_{p_{0i}=p_0} = 2\frac{L}{2\bar{d}_i N} p_0 \frac{-1}{t} + 2\frac{L}{2\bar{d}_i N} \bar{d}^e = 0 \qquad \#$$

simplifying (33), we get :

$$p_0 = \bar{d}^e t \qquad \#$$

Since landlords' expectations about tenants' search behavior are rational, $\bar{d}^e = \bar{d}_i$. Substituting $\bar{d}_i$ (30) into p_0 yields

$$p_0 = \sqrt{\frac{\lambda st}{V}} \qquad \#$$

Interpretation :

The higher s/V and t, the higher are the firms' monopoly power and the price they charge. Prices do not fall to zero even though $aN > L$, which is a usual result in monopolistic competition models. Price competition will be tougher the longer the tenant's life expectation, i.e. the lower λ.

We are now in a position to move to the entry stage of the game.

The first stage : Entry

We complete the solving of the Fully Regulated model with a free entry condition, $E\pi = 0$.

The expected profit function is as in the Free Market model, except that the price is now a constant p_0 :

$$E\pi = a\left[\frac{aN - L}{aN} * 0 + \frac{L}{aN} p_0 - f\right] = 0 \qquad \#$$

This yields after substitution of p_0 from (35) :

$$\frac{aN}{L} = \frac{\sqrt{\frac{\lambda st}{V}}}{f} \quad \#$$

Because the market vacancy rate (as seen at the moment of search) V is a function of N, we substitute $V \equiv \frac{aN-(1-\lambda)L}{aN}$ to solve for aN or equivalently for $\frac{aN}{L}$. After simplification we get an equation of the second degree in $\frac{aN}{L}$:

$$(\frac{aN}{L})^2 - (1-\lambda)\frac{aN}{L} - \frac{\lambda st}{f^2} = 0 \quad \#$$

This equation has the two following solutions :

$$(\frac{aN}{L})' = \frac{(1-\lambda) - \sqrt{(1-\lambda)^2 + \frac{4\lambda st}{f^2}}}{2} \quad \#$$

$$(\frac{aN}{L})'' = \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{4\lambda st}{f^2}}}{2} \quad \#$$

The first solution is always negative, so we eliminate it. The second solution is always positive, but we require it to be larger than one. This condition is equivalent to having $\frac{st}{f^2} > 1$, which is the same condition as in the Free Market model. We will thus be able to directly compare entry in the two models, which is the object of Section 4. We denote by V^R the vacancy rate in the Fully Regulated model defined as $V^R = \frac{aN-(1-\lambda)L}{aN}$ where aN is given by (40). We now turn to some comparative statics results of the Fully Regulated model.

Comparative statics results :

aN increases in L (market size), search and transport costs (degree of monopoly power) and decreases in f (fixed cost). λ no longer has an ambiguous effect on entry ; if λ is low p_0 is low and this depresses entry. As prices cannot increase from the ”second year”, the possibility of extracting surplus disappears. Thus a low λ (or a long live expectation) leads to low prices in each year and reduces entry.

We just went through two models of rent setting : in the first model, landlords were unconstrained in their rent setting behaviour while in the second one they were totally constrained. Two natural questions then arise: how do the results of these two polar cases compare? And what happens to the market outcome for intermediate levels of rent protection? These questions are addressed in Sections 4 and 5 respectively.

Free Market vs Complete Regulation

We define superscripts F and R to denote the Free Market and the Fully Regulated models respectively. The comparisons we are interested in are the number of entrants and the rents.

Entry

We first show that the Free Market generates more entrants than the Fully Regulated market :

$$(\frac{aN}{L})^F = \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{st(1+\lambda)^2}{f^2}}}{2} > \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{4\lambda st}{f^2}}}{2} = (\frac{aN}{L})^R \quad \#$$

After simplification this inequality reduces to $(1-\lambda)^2 > 0$ which always holds.

Note that in the limiting case where $\lambda = 1$, the two models are equivalent and yield the same number of entrants $\frac{aN}{L} = \frac{\sqrt{st}}{f}$. This is because the hold-up problem and the price discrimination effect disappear in the Free Market if all tenants die at the end of the ”first year” : the landlord’s problem on the Free Market is to set a constant price (because for the landlord today’s problem is the same as tomorrow’s) that maximizes profits. But this is exactly the problem solved under Full Regulation.

The vacancy rates in the two models can also be compared : as $V \equiv \frac{aN-(1-\lambda)L}{aN}$, it follows that $V^F > V^R$. In the comparisons that follow, we shall adopt a two step procedure: first we compare the two models for any given level of vacancy rate, and then we show that the (qualitative) results obtained still hold when we take into account the endogeneity of the vacancy rate.

"First year" Rents

We examine the condition under which $p_1^F < p_0^R$:

$$p_1^F = \lambda\sqrt{\frac{st}{V}} - (1-\lambda)F < \sqrt{\frac{\lambda st}{V}} = p_0^R \qquad \#$$

This inequality always holds as $\sqrt{\frac{st}{V}}(\lambda - \sqrt{\lambda}) < (1-\lambda)F$ holds for all $\lambda \in]0,1]$. In words, the "first year" Free Market price is always lower than in a Regulated Market. The reason is that because prices can be increased from the "second year" in the Free Market, reducing the price in the "first year" to attract a tenant is a profitable strategy. In the Regulated Market, price competition is not as strong as in the competitive market, because a price decrease in the "first year" cannot be balanced by a price increase in the future.

Note that in the limiting case where $\lambda = 1$, the two models are equivalent and yield the same "first year" rents $\sqrt{st}$.

If we take into account that $V^F > V^R$, inequality (42) is (obviously) reinforced.

"Second year" Rents

For the Free Market model we take the average price paid by a tenant (i.e. living at a distance $\frac{\bar{d}}{2}$) that we compare to the constant price p_0^R :

$$p_2^F(\frac{\bar{d}}{2}) = (\lambda + \frac{1}{2})\sqrt{\frac{st}{V}} + \lambda F > \sqrt{\frac{\lambda st}{V}} = p_0^R \qquad \#$$

This inequality always holds as $\sqrt{\frac{st}{V}}(\lambda - \sqrt{\lambda} + \frac{1}{2}) > -\lambda F$ is true for all $\lambda \in]0,1]$. This means that on average, from year two and onward, the Free Market yields higher rents than a Fully Regulated market. This is the hold-up at work. Because there are fixed moving costs and that the landlord can learn about his tenants preferences when renegotiating in the Free Market, this result is not surprising.

Taking into account that $V^F > V^R$, we note that the inequality is weakened. Does it still hold? The answer is yes. From the landlords profit functions in the entry conditions (23) and (36), we can rewrite the profit functions as $\bar{p}^{F,R} = f * (\frac{aN}{L})^{F,R}$, where $\bar{p}$ is average expected rent per period. Since $(\frac{aN}{L})^F > (\frac{aN}{L})^R$, we also know that average rents will be higher in the Free Market (c.f. Section 4.4, below). Thus, as "first year" rents are lower in the Free Market, "second year" rents must necessarily be strictly higher than the constant rent of the Fully Regulated market.

We also note that the lowest possible "second year" price in the Free Market is $p_2^F(\bar{d})$:

$$p_2^F(\bar{d}) = \lambda\sqrt{\frac{st}{V}} + \lambda F \gtrless \sqrt{\frac{\lambda st}{V}} = p_0^R \qquad \#$$

which rewrites $\sqrt{\frac{st}{V}}(\lambda - \sqrt{\lambda}) > -\lambda F$. As $\lambda \in]0,1]$, both the LHS and the RHS are negative and the comparison is ambiguous. This is the result of two effects going in opposite directions in the Free Market : the hold-up effect and the price discrimination effect. The hold-up effect increases the "second year" price. In contrast, the price discrimination effect tends to decrease the "second year" price as d increases.

Taking into account that $V^F > V^R$, $p_2^F(\bar{d})$ is lowered but still could be larger than p_0^R.

The question then is : in which market setting is the tenant paying higher rents on average?

Average Rents

We take a weighted average of rents in the Free Market model and compare it to the constant rent under complete regulation :

$$\bar{p}^F(\frac{\bar{d}}{2}) = \frac{p_1^F + (\frac{1}{\lambda} - 1)p_2^F(\frac{\bar{d}}{2})}{\frac{1}{\lambda}} = \sqrt{\frac{st}{V}}\,\frac{1+\lambda}{2} > \sqrt{\frac{\lambda st}{V}} = p_0^R \qquad \#$$

This inequality always holds as $\frac{1+\lambda}{2} - \sqrt{\lambda} > 0$ for all $\lambda \in]0,1]$. This last result is important : the Free Market generates average rents per year that are strictly higher than rents in a Fully Regulated setting. The reason is the hold-up that takes place at the end of the "first year" when the landlord and the tenant renegotiate. Even though rent control in our model does not set a maximum price on rents but is merely a rule for renegotiation, regulation still leads to lower equilibrium rents. As landlords make zero profits in both institutional settings, the higher average rents paid by tenants in the Free Market finance the fixed costs ($a * f$) of the extra owners that enter the market compared to a Regulated world.

This result also holds if we take into account the fact that $V^F > V^R$. Recall the landlords profit functions in the free entry conditions (23) and (36), can be rewritten in the form $\bar{p}^{F,R} = f * (\frac{aN}{L})^{F,R}$. As $(\frac{aN}{L})^F > (\frac{aN}{L})^R$, it directly follows that average rents are higher in the Free Market.

Note, however, that this result is not sufficient to claim that Regulation is socially profitable. Indeed, the lower rents may be more than compensated by the higher search costs in the Regulated market. We postpone welfare issues to Section 6.

Up to here we have developed two extreme cases of the possible organisation of the housing market ; Free Market or complete rent control. We now develop a more general model allowing for T years of rent control.

The General Model

The central idea of this General Model is to analyse the market outcome when renegotiation is allowed only after T years. We introduce a new parameter T, the duration of rent protection, or, in other words the number of years before the landlord can increase the rental price. When $T = 1,$ we have the Free Market model of Section 2. When $T \to \infty$ we have the Completely Regulated model of Section 3. Within this General Model, we will be able to analyse what happens to rents and the number of entrants in between these two polar cases and derive some comparative statics results.

The structure of the model is as before : N firms enter the market and locate uniformly on the circular road in the first stage. Firms compete in prices in the second stage, which yields p_1, the price for the T "first years". In the third stage, tenants search for an apartment. In the fourth and last stage, at the end of T years, rent renegotiation takes place : the landlord makes the tenant a "take it or leave it offer". We use the same equilibrium concept as before and solve the model by backward induction.

Given the similarity of the structure of this General Model with the Free Market model of Section 2, we will often refer to the latter model in this Section, as the same line of arguments will apply.

The fourth stage : Renegotiation

After T years, renegotiation is allowed. The tenant either stays in the flat or moves to another one and incurs search costs and a fixed moving cost.

As the probability of dying is λ in each year, a tenant's life expectation is $\frac{1}{\lambda}$ after T years (recall the process is memoryless). In any year after the T "first years" the renegotiation problem is the same for both the tenant and the landlord. Thus the renegotiation realized after T years will hold forever after. However, the model is now no longer a two year model (in essence) since we can now do comparative statics with respect to the duration of rent control T.

The problem the landlord faces at the moment of renegotiation is to maximize the rent increase given the tenant's outside option to move. Just as in the Free Market model, he will extract all the surplus the tenant derives from having an apartment close to his place of work, from period T onwards. Thus $p_2(d)$ is again given by (4).

The fact that the owner can practice price discrimination from the T^{th} year has consequences for the tenant's search behavior. He will rationally anticipate that finding a flat close to his place of work will yield him a surplus only for the T "first years" before the contract can be renegotiated.

The third stage : Search

The search problem the tenant faces in the General Model is in essence the same as in the Free Market model (Section 2.6.2). Tenant i will maximise his intertemporal expected utility, as seen from the moment when he starts searching. By assumption, the landlord's pricing behaviour during rent renegotiation is such that the tenant has no incentive to move. His intertemporal expected utility, $E\widetilde{U}_i$ thus writes :

$$E\widetilde{U}_i = \frac{1}{\lambda}U - [(\frac{1}{\lambda} - \frac{(1-\lambda)^T}{\lambda})p_1^e + \frac{(1-\lambda)^T}{\lambda}p_2^e(d)] - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \qquad \#$$

The only difference with the Free Market model (6) is that tenant i will pay p_1^e for the T "first" years" and $p_2^e(d)$ forever after. However he will pay the "first year" rent p_1^e or the second year rent $p_2^e(d)$ only if he is still alive ; this is the reason for the coefficients in front of p_1^e and $p_2^e(d)$.

The tenant who searches is forward-looking : he rationally anticipates the renegotiation stage after T years. He knows how landlords will set rents after the renegotiation. As the tenant maximises his expected utility, he expects, on average, to be located in the middle of his interval of reservation, at $\frac{\bar{d}_i}{2}$. Because $p_2(d)$ is linear in d, and that the expectation of a linear function is this function evaluated at its expected value, i.e. at $\frac{\bar{d}_i}{2}$, we have $p_2^e(d) = p_2(\frac{\bar{d}_i}{2})$. After substitution into (46) of equation (4) (evaluated at $\frac{\bar{d}_i}{2}$) we have :

$$E\widetilde{U}_i = \frac{1}{\lambda}U - (\frac{1}{\lambda} - \frac{(1-\lambda)^T}{\lambda})p_1^e$$

$$- \frac{(1-\lambda)^T}{\lambda}[E(P) + E(Tr) + \lambda E(S) + \lambda F - \frac{t\bar{d}_i}{2}] - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \qquad \#$$

Tenant i maximises his intertemporal expected utility $E\widetilde{U}_i$ by choosing the optimal $\bar{d}_i$ (his choice variable in the search process). Deriving $E\widetilde{U}_i$ with respect to $\bar{d}_i$ yields the following first-order condition :

$$\frac{\partial E\widetilde{U}_i}{\partial \bar{d}_i} = \frac{(1-\lambda)^T}{\lambda}\frac{t}{2} - \frac{1}{\lambda}\frac{t}{2} + \frac{s}{2\bar{d}_i^2 V} = 0 \qquad \#$$

Solving for $\bar{d}_i$ we get :

$$\bar{d}_i = \sqrt{\frac{\lambda s}{tV(1-(1-\lambda)^T)}} \qquad \#$$

tenant i's optimal interval of reservation. footnote Search and transport costs play the same roles as in the previous models. Note $\bar{d}_i$ is a function of λ as the tenant gets a positive surplus from staying in the flat up to the T^{th} year (c.f. Section 5.1).

We also note that when $T = 1,$ we have $\bar{d}_i = \sqrt{\frac{s}{tV}}$ (9), which is the result in the Free Market. When $T \to \infty,$ we have $\bar{d}_i = \sqrt{\frac{\lambda s}{tV}}$ (30), which is the result in the Fully Regulated market.

Implications for $p_2(d)$

To determine $p_2(d)$ we apply the exact same procedure here as in Section 2.6.2 of the Free Market model. We thus get for $p_2(d)$:

$$p_2(d) = p_1^e + \frac{t\bar{d}^e}{2} + \frac{\lambda}{1-(1-\lambda)^T}(F + \frac{s}{2\bar{d}^e V}) - td \qquad \#$$

This equation relates $p_2(d)$ to the T "first year" prices p_1^e. Thus, before we can say more about rents after T years, we need to solve for p_1^e, to which we now turn.

The second stage : T "first years" rental contract

Landlords compete in prices to attract tenants. We look for a (symmetric) Nash Equilibrium in prices and will thus consider a deviant firm setting p_{1i} given the others set p_1.

The problem is again essentially the same as in the Free Market model. By decreasing its price firm i will attract tenants at the margin from an interval of length $d_i - \overline{d}^e = \frac{p_1 - p_{1i}}{t}$ on each side. Rewriting this expression yields $d_i = \overline{d}^e + \frac{p_1 - p_{1i}}{t}$ and we denote it by $d_i(p_{1i})$.

To write down landlord i's maximisation problem, we adopt the same approach as in Section 2.6.3. The expected profit function thus reads :

$$E\pi_i = 2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N} \frac{(\frac{1}{\lambda} - \frac{(1-\lambda)^T}{\lambda})p_{1i} + \frac{(1-\lambda)^T}{\lambda} p_2(d)}{\frac{1}{\lambda}} \mathrm{d}d - af =$$

$$= 2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N}[(1-(1-\lambda)^T)p_{1i} + (1-\lambda)^T p_2(d)]\mathrm{d}d - af \qquad \#$$

Differentiating $E\pi_i$ with respect to p_{1i} yields the following first-order condition, using Leibniz's theorem of derivation :

$$\frac{dE\pi_i}{dp_{1i}} = 2\frac{L}{2\overline{d}_i N}[(1-(1-\lambda)^T)p_{1i} + (1-\lambda)^T p_2(d_i(p_{1i}))]\frac{dd_i(p_{1i})}{dp_{1i}} +$$

$$2\int_0^{d_i(p_{1i})} \frac{L}{2\overline{d}_i N}(1-(1-\lambda)^T)\mathrm{d}d = 0 \qquad \#$$

As we look for a symmetric Nash Equilibrium, we have $p_{1i} = p_1$ and thus $d_i(p_{1i} = p_1) = \overline{d}^e$. Evaluating the first order condition at this point, we get :

$$\left.\frac{dE\pi_i}{dp_{1i}}\right|_{p_{1i}=p_1} = 2\frac{L}{2\overline{d}_i N}[(1-(1-\lambda)^T)p_1 + (1-\lambda)^T p_2(\overline{d}^e)]\frac{-1}{t} +$$

$$2\frac{L}{2\overline{d}_i N}(1-(1-\lambda)^T)\overline{d}^e = 0 \qquad \#$$

Substituting $p_2(\overline{d}^e)$ from (50) into (53) and remembering that expectations are rational ($p_1^e = p_1$), we can solve for p_1:

$$p_1 = t\overline{d}^e(1 - \frac{(1-\lambda)^T}{2}) - \frac{(1-\lambda)^T\lambda}{1-(1-\lambda)^T}(F + \frac{s}{2\overline{d}^e V}) \qquad \#$$

This equation depends on $\overline{d}^e$, the expected distance of reservation. But $\overline{d}^e = \overline{d}_i$ as the landlord has rational expectations on the tenant's search behaviour. We define $\overline{d} = \overline{d}^e = \overline{d}_i$. We thus substitute the expression of $\overline{d}_i$ (49) into p_1 and $p_2(d)$. This yields rents as functions of the structural parameters of the model (for a given number of firms N, solved in the next Section) :

$$p_1 = \sqrt{\frac{\lambda st}{V}(1-(1-\lambda)^T)} - \frac{(1-\lambda)^T\lambda}{1-(1-\lambda)^T}F \qquad \#$$

$$p_2(d) = \sqrt{\frac{\lambda st}{V}}[\frac{1}{\sqrt{1-(1-\lambda)^T}} + \sqrt{1-(1-\lambda)^T}] + \lambda F - td \qquad \#$$

Interpretation

The interpretation of the "first" and "second year" rents, here before T and after T, is basically the same as in the Free Market model of Section 2.6.3. The same mechanisms are at work. For that reason we will not repeat them here. We will focus our discussion on the differences with the Free Market rents. Obviously, (55) and (56) are more complex than (17) and (18). But this complexity is only apparent. The new terms here are of the type $1-(1-\lambda)^T$, and they arise because rents are renegotiated only after T years. We will thus be interested in the effects of the duration of rent protection on rents before and after renegotiation.

a) "first period" rents or rents before T :

As T increases, p_1 increases footnote : when the duration of rent protection increases, renegotiation is postponed. As life expectation is constant ($\frac{1}{\lambda}$) there will be less periods for tenant surplus extraction, thus decreasing competition to attract new tenants, which leads to higher prices.

b) "second period" rents or rents after T :

As T increases $p_2(d)$ decreases footnote . This is because when T increases the tenant's outside option improves : if he moves, there will be more periods with "first year" rents, which disciplines the landlord's pricing behaviour during renegotiation.

c) The mark-up :

The mark-up is defined as the difference in rents before and after renegotiation. Formally, we have :

$$m(d) = p_2(d) - p_1 = \sqrt{\frac{\lambda st}{V(1-(1-\lambda)^T)}} + \frac{\lambda}{1-(1-\lambda)^T}F - td \qquad \#$$

$m(d)$ decreases as the duration of rent protection increases. This is the result of two effects going in the same direction. First, as T increases, "first year" rents increase through less competition to attract new tenants. Second, rents after renegotiation decrease. The difference can thus only be decreasing.

If we evaluate the mark-up at $\bar{d}$, i.e. the lowest possible mark-up at equilibrium :

$$m(\bar{d}) = p_2(\bar{d}) - p_1 = \frac{\lambda}{1-(1-\lambda)^T}F > 0 \qquad \#$$

This inequality holds for all $\lambda \in]0,1]$. It is the hold-up in action : at the end of the T years, all tenants get a rent increase of at least $\frac{\lambda}{1-(1-\lambda)^T}F$. If the tenant is located closer than $\bar{d}$ of his place of work, he will get an even higher rent increase through the price discrimination mechanism.

Finally, as in the Free Market model, one can ask the question whether a landlord having a tenant located at $\bar{d}$ would have an incentive to increase the rental price such that this tenant would leave. The answer is no because if the tenant were to leave, the landlord would simply have one more vacant apartment. The reason is the same as in the Free Market model (c.f. Section 2.6.3). The landlord earns more by keeping a tenant located at $\bar{d}$ as long as $p_2(\bar{d}) > 0$. This relationship clearly always holds as $p_2(\bar{d}) = \sqrt{\frac{\lambda st(1-(1-\lambda)^T)}{V}} + \lambda F > 0$.

The first stage : Entry

We finally solve the first stage of the game with a free entry condition. In the steady-state equilibrium we characterize, we can write the revenue as being the average price paid per year by a tenant multiplied by the occupancy rate, $\frac{L}{aN}$. Thus, expected profits write :

$$E\pi = a\left[\frac{aN-L}{aN} * 0 + \frac{L}{aN}\left[\frac{(\frac{1}{\lambda} - \frac{(1-\lambda)^T}{\lambda})p_1 + \frac{(1-\lambda)^T}{\lambda}p_2(\frac{\bar{d}}{2})}{\frac{1}{\lambda}}\right] - f\right] = 0 \qquad \#$$

This is solved for aN or equivalently for $\frac{aN}{L}$. After substitution of p_1 and $p_2(\frac{\bar{d}}{2})$ from (55) and (56), where (56) is evaluated at $\frac{\bar{d}}{2}$ we get :

$$\frac{aN}{L} = \sqrt{\frac{\lambda st}{V(1-(1-\lambda)^T)}}\,\frac{1-\frac{1}{2}(1-\lambda)^T}{f} \qquad \#$$

To solve for $\frac{aN}{L}$, we substitute $V \equiv \frac{aN-(1-\lambda)L}{aN}$, the market vacancy rate as seen at the moment of search. This yields an equation of the second degree in $\frac{aN}{L}$.

$$(\frac{aN}{L})^2 - (1-\lambda)\frac{aN}{L} - \frac{\lambda st(2-(1-\lambda)^T)^2}{4f^2(1-(1-\lambda)^T)} = 0 \qquad \#$$

This equation has two solutions :

$$(\frac{aN}{L})' = \frac{(1-\lambda) - \sqrt{(1-\lambda)^2 + \frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}}{2} \qquad \#$$

$$(\frac{aN}{L})'' = \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}}{2} \qquad \#$$

The first solution is always negative, so we neglect it. The second solution is always positive and is larger than one if and only if $\frac{st}{f^2} > \frac{4(1-(1-\lambda)^T)}{(2-(1-\lambda)^T)^2}$. footnote

Comparative statics results

aN augments in *L* (market size), but also in *t* because higher transport costs increase prices and thus the number of viable entrants. *aN* increases in search costs as they induce less search, higher prices and more entry. Higher fixed costs *f* decrease the number of entrants. λ, the probability of dying has an ambiguous effect on entry : a high λ increases prices and thus entry, but a high λ means a short live expectation an thus fewer expected years of surplus extraction, which depresses entry.

The last variable we are interested in for a comparative static result is *T*. It can be shown that $\frac{\partial \bar{p}(\frac{\bar{d}}{2})}{\partial T} < 0$ (see Appendix, Section 8.2), where $\bar{p}(\frac{\bar{d}}{2})$ is the average expected rent paid per period by a tenant. This means that the average price paid by a tenant decreases as the duration of rent protection increases. As a corollary, the longer the duration of rent protection, the lower the number of apartments ($\frac{\partial(\frac{aN}{L})}{\partial T} < 0$). This is because rent increases have to be delayed, thus decreasing average prices paid by tenants. Because this relationship is monotonic, the stronger the rent protection, the better the situation of the tenant from the rental price point of view. The situation of a landlord is unaffected as his expected profit is always zero, whatever the duration of rent protection : it is only the number of landlords who enter that is affected by *T*.

But this is only part of the story, as it exclusively takes the aspect of pricing. We know that prices alone do not determine the tenant's well-being: it also depends on the amount of search and on the transport costs incurred. If more firms enter, there is more "choice" and the tenant faces higher average prices, but lower search costs. This naturally leads us to a welfare analysis, which is the object of the next Section.

Welfare Analysis

What would an omnipotent social-planner do if he had to decide on how to regulate the housing market described in our General Model? We assume the social planner minimizes social cost. Social costs do not include prices paid by tenants to producers as they constitute a transfer with no effects on welfare. There are three components in our social cost function : firms' fixed costs, tenants' transport and search costs. In each year, each firm incurs fixed cost *af*. *L* tenants have expected travel costs $\frac{t\bar{d}}{2}$ and a fraction λL, the "new" tenants, also support search costs of $\frac{s}{2\bar{d}V}$. Mathematically, the social cost function per year, denoted by *SC*, reads :

$$SC = aNf + L\frac{t\bar{d}}{2} + \lambda L\frac{s}{2\bar{d}V} \qquad \#$$

The question we want to answer is the following : how is social cost affected by a change in *T*, the duration of rent protection? Does this function have a minimum at some *T*? It is shown in the Appendix (Section 8.3) that $\frac{\partial SC}{\partial T} < 0$. This result is strictly monotonic. Thus an increase in the duration of rent protection *T* leads to a decrease in social costs. In other words, more protection raises social welfare. This means that a social planner would choose an institutional setting for the housing market with an infinite number of years of rent protection, which is exactly the Fully Regulated model we developed in Section 3. The result of this welfare analysis is a strong argument in favor of complete rent protection : rent renegotiation should simply not be allowed. If we translate this result into economic policy terms, the optimal policy a government should adopt is to impose constant real rents, or in other words, to allow the indexation of rents on the inflation rate as long as a tenant decides to stay in an apartment or until he dies.

Conclusion

In order to understand the effects of state intervention in a market economy, it is necessary to have models which capture the most important characteristics influencing the working of the market under examination. In the present paper we have studied the effects of regulation in a simple model of the housing market. Our emphasis has been on the following special features of this market : monopolistic competition (product heterogeneity), imperfect information (tenant search), switching costs (the ”hold-up” problem) and ex post price discrimination. We have shown that government intervention which prevents the landlords from fully exploiting the potential of the ”hold-up” problem and price discrimination would lead to lower equilibrium rents and higher social welfare. Our model was one in which inflation did not exist (by assumption). A natural real world interpretation of our analysis would be a regulation which indexes rents by the consumer price index.

It is of course possible that other drawbacks of regulation would more than compensate the advantages analysed in our model. It would be useful if these advantages were made more explicit within the framework of a formal model. Such an approach would be more productive than the calls one sometimes hears for liberalising the housing market as a question of principle, without any attempt to take into account the special features of this market.

Appendix

Derivation of the Demand function

We know the tenant will stay in the apartment after the renegotiation, whatever his location in the interval $\left[0, \bar{d}\right]$. The demand function we derive here is the one the landlord faces when he tries to attract a new tenant i.e. when he is setting p_1, the price before renegotiation. The search process is the same in all three models previously discussed. This means that the demand the landlord faces in all three problems is the same. Each of the N landlords puts a flats on the market. A landlord's demand function can thus be interpreted as the vacancy rate in his building (i.e. among his a apartments) when he modifies his price, given the price of the other firms. In particular, we will be interested in the demand function of firm i when all the others set the Nash Equilibrium price p_1.

We show that the demand function of firm i is linear down to its vacancy rate of zero (or occupancy rate of one). When its vacancy rate becomes zero, a further decrease in price cannot alter that rate anymore : the demand becomes perfectly inelastic. We denote by $D_i(p_{1i}, p_1)$ landlord i's demand function when his is setting p_{1i} and all the others set p_1.

The demand function is derived as follows. There are N landlords competing in prices to attract tenants. We assume a landlord never has a completely occupied building in the sense that there are always N competing landlords in the market. L consumers search for an apartment. footnote We denote by $d_i(p_{1i})$ the interval from which firm i attracts tenants (on both sides) when it is setting p_{1i} and all the others set p_1. As L consumers search and N landlords compete, landlord i is sampled with probability $\frac{L}{N}$, and consumers from an interval of $2d_i(p_{1i})$ would accept this flat. Thus firm i's occupancy rate after all consumers sampled one flat is (recall the circumference of the circle is one) :

$$2d_i(p_{1i})\frac{L}{N} \qquad \#$$

Among those L consumers, and because all other firms set p_1, a proportion of $2\bar{d}$ accepts the sampled apartment whereas $(1 - 2\bar{d})$ rejects it. Thus firm i may increase its occupancy rate as the remaining consumers keep on searching, by the amount :

$$2d_i(p_{1i})\frac{L}{N}(1 - 2\bar{d}) \qquad \#$$

Again, among those $(1 - 2\bar{d})L$ searching consumers, a proportion $(1 - 2\bar{d})$ rejects the sampled apartment and keeps on searching. Firm i thus increases its occupancy rate by :

$$2d_i(p_{1i})\frac{L}{N}(1 - 2\bar{d})^2 \qquad \#$$

It is now obvious that the demand function is derived from the geometric series when we add the successive increases in firm i's occupancy rate. This occupancy rate thus writes :

$$D_i(p_{1i},p_1) = 2d_i(p_{1i})\frac{L}{N} + 2d_i(p_{1i})\frac{L}{N}(1-2\bar{d}) + 2d_i(p_{1i})\frac{L}{N}(1-2\bar{d})^2 + ...$$
$$= \frac{L}{N}\frac{d_i(p_{1i})}{\bar{d}} \quad \#$$

Note we can also write the demand function in the following way (recall $\bar{d} = \bar{d}_i$) :

$$D_i(p_{1i},p_1) = 2\int_0^{d_i(p_{1i})} \frac{L}{2\bar{d}_i N}\mathrm{d}d \quad \#$$

The two in front of the integral is because of the symmetry of the problem in the circular road model. In the symmetric equilibrium we characterized ($p_{1i} = p_1$) and $d_i(p_{1i} = p_1) = \bar{d} = \bar{d}_i$. At that point, the demand function takes the following value :

$$D_i(p_{1i} = p_1, p_1) = \frac{L}{N} \quad \#$$

From the analysis in Section 2.6.3, we know that $\triangle d = -\frac{\triangle p}{t}$ or infinitesimally, $\mathrm{d}d = -\frac{dp}{t}$. Integrating, we get a linear relation between $d_i(p_{1i})$ and p_{1i}, up to a constant c :

$$d_i(p_{1i}) = -\frac{p_{1i}}{t} + c \quad \#$$

and thus :

$$\frac{\partial d_i(p_{1i})}{\partial p_{1i}} = -\frac{1}{t} \quad \#$$

This is all the information we need to solve our problems.

Comparative Statics : $\bar{p}(\frac{\bar{d}}{2})$

We will show that $\frac{\partial \bar{p}(\frac{\bar{d}}{2})}{\partial T} < 0$. Note this is equivalent to showing that $\frac{\partial(\frac{aN}{L})}{\partial T} < 0$, because of equation (59), that rewrites $\bar{p}(\frac{\bar{d}}{2}) = f * \frac{aN}{L}$. $\frac{aN}{L}$ is given by equation (63) :

$$\frac{aN}{L} = \frac{(1-\lambda) + \sqrt{(1-\lambda)^2 + \frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}}{2} \quad \#$$

Deriving this expression with respect to T yields after simplification :

$$\frac{\partial(\frac{aN}{L})}{\partial T} = \frac{\lambda st}{4f^2\sqrt{(1-\lambda)^2 + \frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}} * \quad \#$$
$$\frac{(2-(1-\lambda)^T)(1-\lambda)^{2T}\ln(1-\lambda)}{(1-(1-\lambda)^T)^2} < 0$$

This expression is easy to sign : as $\lambda \in [0,1]$, all the terms are positive, except for $\ln(1-\lambda)$ that is negative. Thus $\frac{\partial(\frac{aN}{L})}{\partial T} < 0$ which directly implies $\frac{\partial \bar{p}(\frac{\bar{d}}{2})}{\partial T} < 0$.

Welfare Analysis

We will show that $\frac{\partial SC}{\partial T} < 0$. The SC function (64) reads :

$$SC = aNf + L\frac{t\bar{d}}{2} + \lambda L\frac{s}{2\bar{d}V} \quad \#$$

To proceed with the comparative statics exercise, we need to write the social cost function in terms of the structural parameters of the model. We substitute aN (63), $\bar{d}$ (49) and $V \equiv \frac{aN-(1-\lambda)L}{aN}$. For readability reasons we will let V in the SC function, but keeping in mind that V is a function of T through aN. (75) then reads :

$$SC = \frac{(1-\lambda)L}{1-V}f + \frac{L}{2}\sqrt{\lambda st}\,V^{-\frac{1}{2}}\left[(1-(1-\lambda)^T)^{-\frac{1}{2}} + (1-(1-\lambda)^T)^{\frac{1}{2}}\right] \quad \#$$

Deriving (76) with respect to T yields, after simplification :

$$\frac{\partial SC}{\partial T} = \frac{(1-\lambda)Lf}{(1-V)^2}\frac{\partial V}{\partial T} - \frac{L}{4}\sqrt{\lambda st}\,V^{-\frac{3}{2}} * \left\{\frac{\partial V}{\partial T}\left[(1-(1-\lambda)^T)^{-\frac{1}{2}} + (1-(1-\lambda)^T)^{\frac{1}{2}}\right] + V(1-\lambda)^T\ln(1-\lambda)\left[(1-(1-\lambda)^T)^{-\frac{1}{2}} - (1-(1-\lambda)^T)^{-\frac{3}{2}}\right]\right\} \quad \#$$

To sign this expression we need to determine two elements : (a) the sign of $\frac{\partial V}{\partial T}$ and (b) the sign of the expression in the brackets $\{\quad\}$.

(a) $V \equiv \frac{aN-(1-\lambda)L}{aN} = 1 - \frac{(1-\lambda)L}{aN} = 1-(1-\lambda)(\frac{aN}{L})^{-1}$. We already know from the Appendix (Section 8.2) that $\frac{\partial(\frac{aN}{L})}{\partial T} < 0$. Using this result it directly follows that

$$\frac{\partial V}{\partial T} = (1-\lambda)(\frac{aN}{L})^{-2}\frac{\partial(\frac{aN}{L})}{\partial T} < 0 \quad \#$$

As λ is a probability, the first term in (77) is negative. We now need to sign the second term of (77).

(b) The first term in the brackets $\{\quad\}$ is negative whereas the second one is positive. After substitution of V (and of $\frac{aN}{L}$ from (63) in V) and simplification, the term in the brackets $\{\quad\}$ can be rewritten as follows :

$$\frac{\frac{\lambda st}{f^2}(1-\lambda)^{2T}(\frac{(2-(1-\lambda)^T)^2}{(1-(1-\lambda)^T)^3})\ln(1-\lambda)\left[\frac{1-\lambda}{\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}} - 1\right]}{(1-(1-\lambda)^T)^{-\frac{1}{2}}\left[\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}} + (1-\lambda)^2\right]^2} \quad \#$$

This expression is in fact easily signable. The denominator is always positive as λ is a probability. The numerator consists of a product of two terms. The first term is $\frac{\lambda st}{f^2}(1-\lambda)^{2T}(\frac{(2-(1-\lambda)^T)^2}{(1-(1-\lambda)^T)^3})\ln(1-\lambda)$ and is always negative since $\ln(1-\lambda) < 0$. In the second term $\left[\frac{1-\lambda}{\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}} - 1\right]$ the numerator is always smaller than the denominator ; subtracting 1 then yields a negative number. The product of these two negative terms is positive. Thus the term in brackets $\{\quad\}$ is positive.

We now can sign the whole $\frac{\partial SC}{\partial T}$ expression : it is a sum of two negative terms. Thus $\frac{\partial SC}{\partial T} < 0$.

Comparative Statics : p_1 and $p_2(d)$

We will first show that $\frac{\partial p_1}{\partial T} > 0$ and then that $\frac{\partial p_2(d)}{\partial T} < 0$.

$$p_1 = \sqrt{\frac{\lambda st}{V}(1-(1-\lambda)^T)} - \frac{(1-\lambda)^T\lambda}{1-(1-\lambda)^T}F \quad \#$$

For presentational ease we will define $x = (1-\lambda)^T$. (80) thus rewrites

$$p_1 = \sqrt{\lambda st}\,(\frac{1-x}{V})^{\frac{1}{2}} - \lambda F\frac{x}{1-x} \quad \#$$

Deriving (81) with respect to T yields :

$$\frac{\partial p_1}{\partial T} = \sqrt{\lambda st}\,\frac{1}{2}(\frac{1-x}{V})^{-\frac{1}{2}}\frac{-\frac{\partial x}{\partial T}V-(1-x)\frac{\partial V}{\partial T}}{V^2} - \lambda F\frac{\frac{\partial x}{\partial T}(1-x)-x(-\frac{\partial x}{\partial T})}{(1-x)^2} \quad \#$$

This derivative is easy to sign. As $x = (1-\lambda)^T$, $\frac{\partial x}{\partial T} = (1-\lambda)^T\ln(1-\lambda) < 0$ since λ is a probability. We already showed in Section 8.3 that $\frac{\partial V}{\partial T} < 0$. We thus have all the elements to unambiguously sign (82) : $\frac{\partial p_1}{\partial T} > 0$.

We now turn to the determination of the sign of $\frac{\partial p_2(d)}{\partial T}$.

$$p_2(d) = \sqrt{\frac{\lambda st}{V}}\,[\frac{1}{\sqrt{1-(1-\lambda)^T}} + \sqrt{1-(1-\lambda)^T}\,] + \lambda F - td \quad \#$$

Similarly here, we rewrite $p_2(d)$ with $x = (1-\lambda)^T$.

$$p_2(d) = \sqrt{\lambda st}\, V^{-\frac{1}{2}}[(1-x)^{-\frac{1}{2}} + (1-x)^{\frac{1}{2}}] + \lambda F - td \qquad \#$$

Deriving this expression with respect to T :

$$\frac{\partial p_2(d)}{\partial T} = \sqrt{\lambda st}\,(-\frac{1}{2})V^{-\frac{3}{2}}\frac{\partial V}{\partial T}[(1-x)^{-\frac{1}{2}} + (1-x)^{\frac{1}{2}}] + \qquad \#$$

$$\sqrt{\lambda st}\, V^{-\frac{1}{2}}[-\frac{1}{2}(1-x)^{-\frac{3}{2}}(-\frac{\partial x}{\partial T}) + \frac{1}{2}(1-x)^{-\frac{1}{2}}(-\frac{\partial x}{\partial T})] =$$

$$= \sqrt{\lambda st}\,(-\frac{1}{2})V^{-\frac{3}{2}}\left\{\frac{\partial V}{\partial T}[(1-x)^{-\frac{1}{2}} + (1-x)^{\frac{1}{2}}] +\right.$$

$$\left. V\frac{\partial x}{\partial T}[-\frac{1}{2}(1-x)^{-\frac{3}{2}} + \frac{1}{2}(1-x)^{-\frac{1}{2}}]\right\}$$

The problem is to sign the term in the brackets $\{\quad\}$. We have already determined the sign of this expression in Section 8.3 ; it is positive. Multiplying it by the negative term $\sqrt{\lambda st}\,(-\frac{1}{2})V^{-\frac{3}{2}}$ directly yields the result $\frac{\partial p_2(d)}{\partial T} < 0$.

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