A Note on Fat Cats and Puppy Dogs

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Abstract

This paper studies under what circumstances an incumbent has an incentive to over-invest in a "commitment variable" such as advertising or R&D expenditures. It is sometimes argued that the answer crucially depends on the question, whether the "second stage variables" are strategic complements or substitutes. We show that in the derivation of this result the authors implicitly make the (very) restrictive assumption that the first stage "commitment variable" has no direct effect on the competitor's reaction function. Once this assumption is relaxed the clear cut distinction along the lines "strategic complements", "strategic substitutes" no longer holds.

1 Introduction

As a result of the wide-spread use of game theoretic tools in the study of industrial organisation, the analysis of strategic interactions plays a prominent role in many theoretical analyses. An early example is the "animal terminology" introduced by Fudenberg and Tirole (1984) which attempts to classify an incumbent's behaviour when threatened by an entrant whom she might have to either accommodate or deter. One of the prominent features of this analysis is the central role played by the slope of the firms' reaction functions; it is claimed that the decision by an accommodating incumbent to over- or under-invest in the first stage commitment variable is determined by the question, whether the second stage reaction functions have a positive or negative slope (the variables are strategic complements or substitutes). This classification has achieved wide-spread attention, since it has been taken up in the Tirole (1989) textbook and is thus presumably taught in post-graduate I.O. courses throughout the world.

The purpose of this note is to point out that the emphasis placed on the slope of the second stage reaction functions may be misplaced. Indeed, the whole analysis is based on a very restrictive implicit assumption. The assumption is as follows: While the author(s) explicitly allow the incumbent's first stage choice of commitment variable to have a direct effect on the entrant's profit function, they also implicitly assume that the commitment variable can have no effect on the entrant's reaction function. The reaction function is, of course, the locus of the maxima of the profit function. The analysis is thus de facto limited to those situations, where the incumbent's commitment variable leads to parallel shifts of the profit function.

Once this (implicit) assumption is relaxed and one allows the incumbent's choice of commitment variable to have a direct impact on the entrant's reaction function, it is no longer possible to determine whether the incumbent should over- or under-invest simply by looking at the slope of the reaction function. Indeed any "indirect strategic effect" due to this slope may be more than compensated by the "direct strategic effect", i.e. the fact that the incumbent's choice of commitment variable leads to a shift of the entrant's reaction function.

The rest of this paper is organised as follows: Section 2 reproduces the analysis of Tirole (1989), shows where the implicit assumption is introduced and discusses how relaxing it would modify the results. Section 3 illustrates this point using a simple Hotelling model where one of the firms (the incumbent) can increase the perceived quality of her product by investing into advertising. Section 4 ends with some concluding remarks.

2 The textbook analysis.

Tirole (1989) discusses an example where firm 1 (the incumbent) can choose the level of some commitment variable K_1 in the first stage of the game. In the second stage the two competitors simultaneously choose their strategy variable x_1 and x_2. The equilibrium concept used is the perfect Nash equilibrium. The purpose of the analysis is to study how the incumbent should determine the level of K_1, given that the choice of K_1 can influence (have a strategic effect on) the entrant's choice of x_2. Both the case of entry deterrence and accommodation are discussed.

2.1 Entry deterrence.

Firm 2's profit function can be written.

$$\pi^2 = \pi^2(K_1, x_1(K_1), x_2(K_1)) \tag{1}$$

The effect of K_1 on π^2 is:

$$\frac{d\pi^2}{dK_1} = \frac{\partial \pi^2}{\partial K_1} + \frac{\partial \pi^2}{\partial x_1}\frac{dx_1^*}{dK_1} \tag{2}$$

The first term on the RHS of 2 is the direct effect the choice of K_1 may have on the entrant's (expected) profit. The second term is the indirect (strategic) effect, due to the fact that the choice of K_1 can influence the second stage choice of x_1 and this again will have some effect on the entrant's profits.

This formulation thus clearly allows for the incumbent's choice of K_1 to have a direct effect on the entrant's profit π^2.

2.2 Accommodation

If the incumbent decides to accommodate entry, her motivation changes. Rather than choosing K_1 with the aim of reducing the entrant's profits, she now has to choose K_1 with the aim of maximising her own post-entry profits. Her profit function can be written:

$$\pi^1 = \pi^1(K_1, x_2(K_1), x_1(K_1)) \tag{3}$$

and the first order condition reads:

$$\frac{d\pi^1}{dK_1} = \frac{\partial \pi^1}{\partial K_1} + \frac{\partial \pi^1}{\partial x_2}\frac{dx_2^*}{dK_1} \tag{4}$$

Tirole (1989) then goes to on to rewrite the second term on the RHS of (4) as:

$$\frac{\partial \pi^1}{\partial x_2}\frac{dx_2^*}{dK_1} = \frac{\partial \pi^1}{\partial x_2^*}\frac{dx_2^*}{dx_1^*}\frac{dx_1^*}{dK_1} \tag{5}$$

This, unfortunately, is not in general correct. Indeed, it implies that the only influence of K_1 on x_2 is an indirect effect through its influence on x_1. The formulation implies that the choice of K_1 has an influence on the entrant's choice of x_2 only to the extent that it influences the incumbent's second stage choice of x_1 or, alternatively formulated, that the entrant's reaction function is invariant with respect to K_1.

It is, of course, true that there is a whole class of models (i.e. the excess capacity models) where the entrant's profit function is invariant with respect to K_1, and for all those models it will also be the case that the choice of K_1 will have no impact on the entrant's reaction function. If, however, one wishes to have a model which allows for a direct effect of K_1 on π^2 then one needs to choose very specific functional forms if one wishes to avoid a direct impact of K_1 on the entrant's reaction function and thus his choice of x_2. The more general formulation should thus read:

$$\frac{\partial \pi^1}{\partial x_2}\frac{dx_2^*}{dK_1} = \frac{\partial \pi^1}{\partial x_2^*}\frac{dx_2^*}{dx_1^*}\frac{dx_1^*}{dK_1} + \frac{\partial \pi^1}{\partial x_2^*}\frac{\partial x_2^*}{\partial K_1} \tag{6}$$

The first term on the RHS of 6 might be called the "indirect strategic effect" (i.e. the effect via the adjustment of x_1) whereas the second term can be thought of as the "direct strategic effect", i.e. the shift of the entrant's reaction function in response to a change in K_1.

The sign of the "indirect strategic effect" is determined by the slope of the reaction function (inter alia), the sign of the "direct strategic effect" is determined by the shift of the reaction function.

The "direct strategic effect" is, of course, the result of the fact that a change in K_1 which directly influences the entrant's profit function π^2, will also in general affect the slope of this profit function. There is no a priori reason why the "direct strategic effect" should have the same sign as the "indirect strategic effect", or that it should be smaller (or larger). As a result one cannot in general say anything about the optimal accommodating strategy of the incumbent by looking simply at the slope of the entrant's reaction function (the indirect strategic effect), while ignoring the shift of the entrant's reaction function (direct strategic effect).

One potential explanation for the lack of transparency in the analysis in Tirole (1989) is the fact that Fudenberg and Tirole in their original

(1984) article chose an example, where the incumbent's choice of K_1 had the effect of rotating the demand curve faced by the entrant around its intercept with the price axis. This (combined with constant marginal costs) is, of course, one of the few ways of modifying the entrant's profit function while leaving his reaction functions unchanged. It was perhaps subsequently forgotten that the example chosen was in fact based on this restrictive assumption.

To illustrate what can happen in a more general setting, we shall in the next section discuss an example of a Hotelling model, where one of the producer's can influence the perceived quality of his brand by investing into advertising. This will have the effect of shifting both the competitor's profit function and his reaction function. The example chosen is such that the "direct strategic effect" goes in the opposite direction to the "indirect strategic effect" and is also larger (in absolute terms). The implications of this model are thus exactly the opposite ones of those one would obtain by simply applying the "strategic substitutes" "strategic complements" dichotomy.

3 A Hotelling Example.

Consumers are uniformly distributed (with density one) along a line of length L. The two producers are located at both ends of the line. For the sake of simplicity, marginal production costs are assumed to be zero. In the absence of advertising, consumers derive a utility U^* from their most preferred brand. Consumers living at a distance d from firm 1 will buy from her iff.

$$U^* - td - p_1 \geq U^* - t(L - d) - p_2 \tag{7}$$

The demand function facing producer 1 is thus given by:

$$x_1 = \frac{L}{2} + \frac{p_2 - p_1}{2t} \tag{8}$$

Advertising will be modelled in the same spirit as in Sutton (1991). Firm 1 can increase the perceived quality of her brand by A_1, if she invests an amount $C(A_1)$ into advertising (with $C' > 0$, and $C'' > 0$). Consumes will then buy from firm 1 iff:

$$(U^* + A_1) - td - p_1 \geq U^* - t(L - d) - p_2 \tag{9}$$

The demand function facing firm 1 is now :

$$x_1 = \frac{L + A_1}{2} + \frac{p_2 - p_1}{2t} \tag{10}$$

Similarly for firm 2 we have:

$$x_2 = \frac{L - A_1}{2} + \frac{p_1 - p_2}{2t} \tag{11}$$

3.1 The Benchmark

In order to understand, whether firm 1 will over- or under-invest in advertising for strategic reasons, one needs a benchmark. For this purpose, let us first evaluate firm 1's optimal choice of A_1 under the assumption, that firm 2 cannot observe the choice of A_1, i.e. when p_2 is exogenous with respect to A_1.

Firm 1's profit maximisation problem can then be written:

$$\max \pi^1 = p_1 \left(\frac{L + A_1}{2} + \frac{\overline{p}_2 - p_1}{2t} \right) - C(A_1) \tag{12}$$

Deriving with respect to p_1 we obtain:

$$p_1 = \frac{t(L + A_1)}{2} + \frac{\overline{p}_2}{2} \tag{13}$$

The first order condition with respect to A_1 yields:

$$\frac{t(L + A_1)}{4} + \frac{\overline{p}_2}{4} = C'(A_1) \tag{14}$$

The LHS of 14 gives us firm 1's incentive to invest into advertising in the absence of any strategic effects. All we have to do is insert the appropriate value of $\overline{p}_2$and compare the resulting value with the corresponding expression obtained for the case when p_2 adjusts to the level of A_1.

3.2 The equilibrium level of advertising

In this section we compute the equilibrium level of advertising chosen by the incumbent. To do so we first solve the second stage game where the two firms set their equilibrium prices for any given level of advertising expenditures.

The two firm's profit functions are:

$$\pi^1 = p_1 \left(\frac{L + A_1}{2} + \frac{p_2 - p_1}{2t} \right) - C(A_1) \tag{15}$$

and

$$\pi^2 = p_2 \left(\frac{L - A_1}{2} + \frac{p_1 - p_2}{2t} \right) \tag{16}$$

The FOC's with respect to p_1 and p_2 yield the reaction functions

$$p_1 = \frac{t(L + A_1)}{2} + \frac{p_2}{2} \tag{17}$$

and

$$p_2 = \frac{t(L - A_1)}{2} + \frac{p_1}{2} \tag{18}$$

The equilibrium second stage prices are thus

$$p_1^* = tL + \frac{tA_1}{3} \tag{19}$$

and

$$p_2^* = tL - \frac{tA_1}{3} \tag{20}$$

Substituting the equilibrium prices into firm 1's profit function yields

$$\pi^1 = \frac{t}{2}\left(L + \frac{A_1}{3}\right)^2 - C(A_1) \tag{21}$$

The first stage choice of A_1 is obtained by deriving 21 with respect to A_1. The solution is:

$$\frac{t}{3}\left(L + \frac{A_1}{3}\right) = C'(A_1) \tag{22}$$

The LHS of 22 gives us firm 1's incentive to invest into advertising when she takes into account the strategic effects this has on firm 2. Denote the solution to 22 by A_S.

3.3 Comparison

To see whether the incentives to advertise hare greater or less in the presence of strategic effects one has to compare equations 14 and 22. However one has to be careful to do so at the same level of prices. The solution is thus to substitute the equilibrium price for firm 2 given by 20 into equation 14. This yields:

$$\frac{t}{2}\left(L + \frac{A_1}{3}\right) = C'(A_1) \tag{23}$$

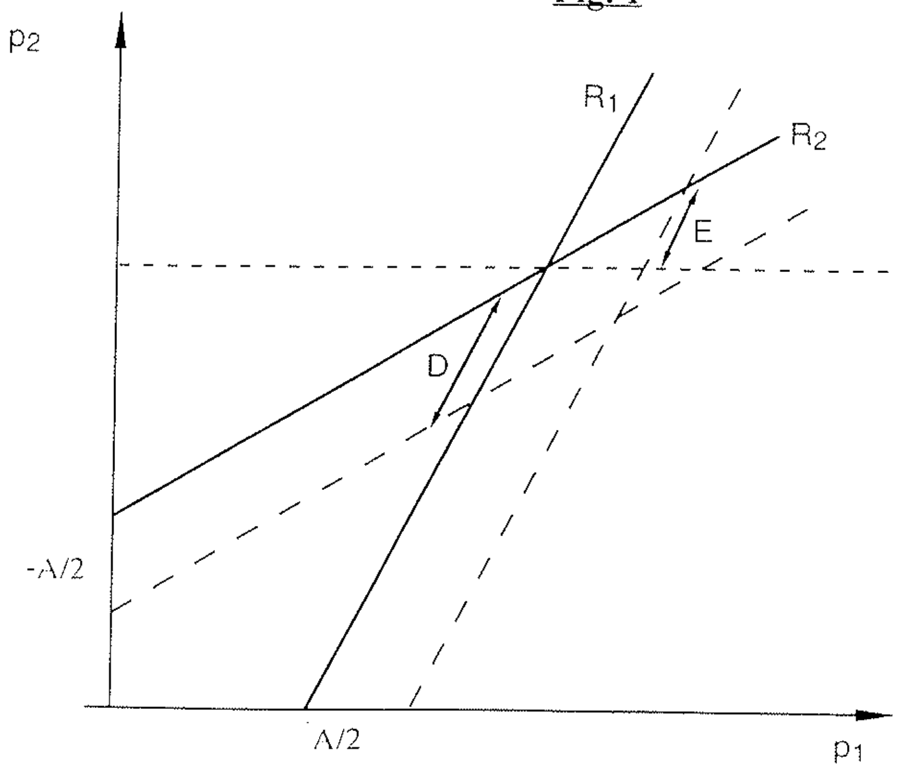
Denote the solution to 23 by A_B. A_S is obviously and unambiguously lower than A_B (the LHS of 22 is lower than the LHS of 23). When the incumbent takes into account the strategic effect of her advertising expenditures, she will reduce her expenditures as compared to the benchmark case. This can be explained by the following trade-off.

a) The "indirect strategic effect": When firm 1 increases A_1 this will lead to an increase in p_1. Since p_1 and p_2 are strategic complements, the increase in p_1 will lead to an increase in p_2, further increasing firm 1's profits. As a result firm 1 has an incentive to over-invest into advertising.

b) The "direct strategic effect". When firm 1 increases A_1, this will shift the demand curve facing firm 2 downward. He will react by decreasing her price. This reduces firm 1's profits. As result firm 1 has an incentive to under-invest into advertising.

The situation is diagrammatically represented in Fig. 1. As one can see from both the diagram and the algebra, in the present model the "direct strategic effect" unambiguously overcompensates the "indirect strategic effect". Firm 1 should unambiguously under-invest in advertising, in spite of the fact that prices are strategic complements.

Fig. 1



D= Direct Strategic Effect
I= Indirect Strategic Effect

4 Conclusion

The purpose of this note was to point out that one cannot simply determine an accommodating incumbent's optimal strategy by studying whether the second stage variables are strategic complements or substitutes. In all those situations, where the incumbent's commitment decision has a direct impact on the entrant's profit function, the direct strategic effect due to the shift of the incumbent's reaction function may play at least as important a role.

There are of course situations, in which there is no such direct effect. Well known examples are the models of capacity commitment. However, when one studies the strategic incentives to invest into advertising or R&D with spillovers, one probably obtains more realistic result, if one explicitly allows for a non-zero "direct strategic effect".

5 References

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