UNIL | Université de Lausanne
HEC Lausanne
Institut CREA

CREA Working Paper No. 2

The Swiss Economy in Real Time

Mathieu Grobéty and Richard Schmidt

December 2021

UNIL | Université de Lausanne
HEC Lausanne

HEC Lausanne
CREA Swiss Institute of Applied Economics
Bâtiment Internef | CH-1015 Lausanne
Phone +41 21 692 33 53 | crea@unil.ch
www.unil.ch/crea

The Swiss Economy in Real Time

Mathieu Grobéty*[1] and Richard Schmidt†[2]

[1]CREA Swiss Institute of Applied Economics, University of Lausanne
[2]University of Lausanne

This draft: December 2021
First draft: June 2021

Abstract

In this paper, we construct a novel business cycle index that aims at monitoring the Swiss economy in real-time. Our indicator based on a restricted dataset comprising mixed- and high-frequency data can be computed at the daily and monthly frequency. The methodology mainly drawn from Aruoba, Diebold, and Scotti (2009) is based on a dynamic factor framework and a Kalman filtering algorithm, which is well-suited to handle missing observations. The dimensionality problem posed by the maximum likelihood estimation entailed by this method is addressed by using the first principal components and the Harvey accumulator. We conduct an assessment of the newly computed monthly business cycle index by performing in-sample and out-of-sample evaluations.

JEL Classification: C32, C38, C53, E32, E37

Keywords: Business Cycle Index, Dynamic Factor Model, Kalman Filter, Mixed Frequency, Principal Components, Switzerland

*Mathieu Grobéty. Tel.: +41-21-692-36-70; E-mail: mathieu.grobety@unil.ch
†Richard Schmidt. Tel.: +41-58-631-26-48; E-mail: richard.schmidt@unil.ch

1 Introduction

The recent COVID Crisis highlighted once more the importance of having timely and precise information concerning the constant evolution of the economy. In times of rapid economic changes, decision-makers need economic indicators to develop sound economic policies and make proper business decisions. However, most economic indicators are released only at specific frequencies with noticeable delays, often making the information about the business conditions outdated for decision-making. As an illustration of this issue, the Swiss economic downturn during the Spring of 2020 due to the lockdown measures enacted by the Federal Council demonstrates perfectly that the economic situation can shift abruptly. However, economic agents were still required to wait a non-negligible amount of time to grasp the impact of the restrictions on the Swiss economy because of delayed indicators. We propose a novel monthly Business Cycle Index (BCI) for Switzerland and adapt it to the daily setting, thereby enabling us to monitor the Swiss economy in real time. The framework used in this paper builds on the work of Aruoba, Diebold, and Scotti (2009) and has three main characteristics. Firstly, a dynamic factor model is developed to treat business conditions as a latent variable related to observable economic measures. Secondly, the methodology allows to incorporate various indicators available at different frequencies (namely quarterly, monthly and daily). Thirdly, we use the Kalman filter to handle the great number of missing values entailed by mixed-frequency data.

We extend the Aruoba-Diebold-Scotti (ADS) framework by tackling the dimensionality problem that arises during the maximum likelihood estimation (MLE) of the model. In a first step, the first principal components are used to aggregate indicators by frequency (quarterly, monthly, and daily) and type (flow and stock), allowing a more generous number of variables to be included without forfeiting an excessive amount of information. In a second step, the Harvey accumulator is implemented to reduce the dimension of the matrices in the state equation. Those two adjustments significantly reduce the number of parameters to be estimated as well as shorten computation time. Since the final purpose of this papter to produce a high-frequency estimator updated on a regular basis, it is imperative for the BCI to be estimated in a reasonable amount of time.

We depart from the existing literature related to the estimation of BCI for the Swiss economy in two ways. We base our BCI on a relatively small amount of underlying indicators. This modeling choice aims at carefully selecting the variables that should best capture the state of the Swiss economy and cover its most relevant segments ranging from ouptut, consumption and exports to labor and financial markets. The indexes that have been developed so far for Switzerland are traditionally based on large datasets. Noticeable examples are the ones constructed by the Swiss National Bank (Galli, 2018) and the KOF (Abberger, Graff, Siliverstovs, and Sturm, 2014).

Our second contribution is to use a methodology that allows us to extract a BCI at a high frequency using mixed-frequency data. Burri and Kaufmann (2020) developed an real time indicator by considering only daily variables (see also WegmÃ¼ller, Glocker and Guggia, 2021). One of the drawback of their approach is that they must rely on forward-looking financial and news-based variables which are known to lead the cycle (citation). By contrast, our index is constructed from data that are linked to the current state of the economy and generally not available a high frequency. We show that our BCI based on small amount of mixed-frequency data generates better nowcasting performance than traditional monthly indexes computed by the SNB and the KOF as well as recent high-frequency indexes calculated by Burri and Kaufmann (2020) and WegmÃ¼ller et al. (2021)

This paper contributes to the extensive literature on business cycle indexes. For the Euro area, there are three leading indices. The EuroSTING Index (Camacho and Perez-Quiros, 2010) aims at producing short-term forecasts for the Euro area GDP growth. The EuroCOIN Index (Altissimo, Bassanetti, Cristadoro, Forni, Lippi, Reichlin and Veronese, 2001) is designed to keep track of the Euro area GDP growth and the EuroMIND Index (Frale, Marcellino, Mazzi, and Prioetti, 2010) provides an indicator of the business cycle for the Euro area as well as for its most prominent members. All these indicators are computed at the monthly frequency.

In the case of the U.S, the daily ADS Business Conditions Index based on the methodology developed by Aruoba, Diebold, and Scotti (2009) grants economic agents with a high-frequency indicator of the U.S business cycle. The monthly Federal Reserve Bank of Philadelphia State Coincident Indices (Crone and Clayton-Matthews, 2005) are business cycle indices published for the fifty different states of the U.S. Finally, the Conference Board Coincident Economic Index (The Conference Board, 2001) is intended at signaling peaks and troughs in the U.S business cycle.

In Switzerland, there are mainly two business cycle indices used by economic agents. The first one is from the Swiss National Bank (Galli, 2018), and the second one is the KOF Barometer (Abberger, Graff, Siliverstovs, and Sturm, 2014). Both of them are released monthly and will be the benchmarks against which the monthly Business Cycle Index developed in this paper will be compared. The lesser-known but most recent index for Switzerland is the "fever curve" developed by Burri and Kaufmann (2020). This index is released at a daily frequency with a lag of one day making this it convenient for economic agents in need of the most recent available information concerning the Swiss business cycle. However, we are the first to compute a high-frequency BCI for Switzerland based on mixed-frequency data.

The remainder of the paper is organized as follows. In Section 2, the dataset and variables used are described. Section 3 presents the dynamic-factor model framework as well as the model implementation and estimation. In section 4, the results of the signal extraction for the monthly and daily Business Cycle Indices are reported. We also assess the efficiency of the monthly Business Cycle Index to nowcast macroeconomic variables with a focus on GDP growth. Section 5 concludes.

2 Data

In contrast to the two most relevant indices for Switzerland (SNB and KOF) that both employ a large dataset (hundreds of variables), ours is based on a relatively small amount of underlying indicators. Instead of going through a thorough selection process combining statistical information (cross-correlation) and expert judgments, we select the most relevant variables for the business cycle based on economic intuition. Small datasets has substantial advantages over large datasets.

Firstly, a dataset comprising too many variables could introduce too much noise, and thus, the Business Cycle Index would not be properly estimated. Boivin and Ng (2006) showed that factor models using small dataset can effectively outperform those using broader dataset. In particular, the methodology used in this paper involves the use of the first principal components, which entails a loss of information when squeezing the variables in the said first principal components. Therefore, introducing new indicators induces a trade-off between the information brought by the new indicators and the loss of information contained in the previously selected variables, which become less influential.

Secondly, a small dataset requires less "maintenance" regarding which variable should be excluded or kept in the dataset over time because the variables we select should comove with the business cycle in every period and every country. It requires also less "maintenance" regarding the data download. This is a key feature because one of the objective of the paper is to produce a high-frequency estimator updated on a regular basis.

As a final argument in favor of small datasets, the framework described in this paper could be used not only to produce an estimate of the Swiss business cycle but also for the various Cantons' business cycles. Therefore, in order to make a comparison across the Cantons, all the business cycle indices need to be estimated with the same set of underlying indicators. This represents a serious issue with a large dataset as Cantons do not necessarily keep track of all the same variables. Hence, by carefully selecting a relevant but restricted number of indicators common to all Cantons, it is possible to alleviate this problem.

As a result, we decided to base the Business Cycle Index on a relatively small amount of underlying indicators, comprising: real GDP (quarterly 1980-2021, SECO), retail sales (monthly 2000-2021, SFSO), consumer confidence index (quarterly 1972-2021, SECO), exports of goods (monthly 1997-2021, AFD), unemployment rate (monthly 1975-2021, SECO), employment (quarterly 1991-2021, SFSO), vacancies (quarterly 1992-2021, SFSO) as well as industrial production (monthly 2010-2021, SFSO) for the monthly index. The term premium (daily 1988-2021, SNB) defined as the difference between the spot interest rates on Swiss Confederation bonds at 20 and 2 years maturity, the liquidity premium (daily 2001-2021, SNB) specified as the difference in yields of Swiss Government bonds and other AAA-rated bonds at maturity of 8 years, and the daily consumption (daily 2019-2021, Monitoring Consumption Switzerland, SIX and Worldline) are added to the dataset for the daily index.

To bypass the trend estimation by MLE, all monthly and quarterly variables are de-trended with the Hodrick-Prescott filter (Hodrick and Prescott, 1997) with the corresponding smoothing parameter for each frequency ($\lambda_q = 1600$, $\lambda_m = 14400$) as proposed by Ravn and Uhlig (2002). As the HP filter is not well suited to de-trend daily variables, the STL procedure employing the loess smoother is used for daily indicators (Cleveland, Cleveland McRae, and Terpenning, 1990). Moreover,

All the variables are seasonally-, calendar- and outlier-adjusted using the X-13ARIMA-SEATS. Moreover, to ensure that every time-series are stationary, the first-difference of all variables is taken, and a Dickey-Fuller test is used as a confirmation for each variable[1]. An additional transformation is applied explicitly to daily variables to reduce the volatility of the data by taking the rolling average with an interval of 7 days.

3 Methodology

As mentioned, it is assumed that the state of the economy changes continuously over time, and hence the goal is to produce an index at the highest frequency possible to monitor the business cycle with the maximum accuracy and minimum lag. As a result, two indices are computed at different frequencies (monthly and daily). In the present section, the model and the methodology are primarily explained through the scope of a monthly frequency index. Nonetheless, differences in methodology entailed by a daily frequency index are pointed out throughout the developments.

3.1 The Dynamic Factor Model

Let b_t denotes the Business Cycle Index capturing the dynamic of the Swiss economy at the highest frequency available in the dataset (i.e., monthly or daily). This variable follows an auto-regressive process of order p :

$$b_t = \phi_1 b_{t-1} + ... + \phi_p b_{t-p} + \epsilon_t \tag{1}$$

where ϵ_t is assumed to be a white noise innovation with variance $\sigma_e^2 = 1 - \sum_{j=1}^{p} \phi_j^2$
Let y_t^i denotes the i-th economic variable at month (day) t, which depends linearly on b_t and lags of y^i.

$$y_t^i = c^i + \lambda_i b_t + \rho_{i1} y_{t-D_i}^i + ... + \rho_{in} y_{t-nD_i}^i + u_t^i \tag{2}$$

Where u_t^i are contemporaneously and serially uncorrelated white noises with variance σ_i^2. D_i is the number of months (days) per observational period. For instance, if y^i is collected every quarter, then D_i will be equal to 3 in the case of a monthly Business Cycle Index and to $\{90, 91, 92\}$ for an index at the daily frequency. Hence, notice that D_i is time-varying in the case of a daily index for monthly and quarterly collected data[1]. To simplify notations, it is assumed that D_i is fixed (as for an index at the monthly frequency), but in the state-space representation, it will be treated as time-varying. As in Aruoba, Diebold, and Scotti (2009), lags of y_t^i in multiple of D_i are also introduced in equation (2) since assuming persistence only at the monthly (daily) frequency would not be appropriate as it would disappear prematurely for variables at a lower frequency.
Even though most variables evolve daily, only a limited number can actually be observed and collected at a monthly frequency, let alone on a daily basis. For instance, most quarterly variables are only collected at the end of the observational period (i.e., the last day of the quarter). Thankfully, the Kalman filter is well suited to handle the significant number of missing values in the dataset.

[1]The null hypothesis being non-stationary, all the tests resulted in the rejection of the null hypothesis with a p-value smaller than 0.01.

[1]$D_i \in \{90, 91, 92\}$ for quarterly variables and $D_i \in \{28, 29, 30, 31\}$ for monthly variables

However, while the missing observations do not pose any major issue to estimate the Business Cycle Index, on the other hand, the differentiation between flow and stock variables is crucial for signal extraction.

Let $\tilde{y}_t^i$ denote the i-th variable collected at a given frequency. The relationship between $\tilde{y}_t^i$ and y_t^i will heavily depend on whether it is a stock or flow variable. If it is a stock variable then the treatment is straightforward since a stock variable will have a value at one point in time. Hence, on the last month (day) of the observational period, $\tilde{y}_t^i = y_t^i$ and $\tilde{y}_t^i$ will be a missing value denoted by NA otherwise. Therefore, the measurement equation (2) for stock variables becomes :

$$\tilde{y}_t^i = \begin{cases} c^i + \lambda_i b_t + \rho_{i1}\tilde{y}_{t-D_i}^i + ... + \rho_{in}\tilde{y}_{t-nD_i}^i + u_t^i, & \text{On the last month (day) of the observational period} \\ NA, & \text{Otherwise} \end{cases} \tag{3}$$

Where $\tilde{y}_{t-D_i}^i = y_{t-D_i}^i, ..., \tilde{y}_{t-nD_i}^i = y_{t-nD_i}^i$ since it is observed at an interval of D_i months (days). In the case of a flow variable, $\tilde{y}_t^i$ is simply the sum of all the unobserved values over the observational period.

$$\tilde{y}_t^i = \begin{cases} \sum_{j=0}^{D_i-1} y_{t-j}^i, & \text{If } y_t^i \text{ is observed} \\ NA, & \text{Otherwise} \end{cases} \tag{4}$$

Using equation (2) and (4), the measurement equation for a flow variable is :

$$\tilde{y}_t^i = \begin{cases} c^{i*} + \lambda_i \sum_{j=0}^{D_i-1} b_{t-j} + \rho_{i1}\tilde{y}_{t-D_i}^i + ... + \rho_{in}\tilde{y}_{t-nD_i}^i + u_t^{i*}, & \text{On the last month (day) of the observational period} \\ NA, & \text{Otherwise} \end{cases} \tag{5}$$

where, by definition, $\tilde{y}_{t-D_i}^i = y_{t-D_i}^i, ..., \tilde{y}_{t-nD_i}^i = y_{t-nD_i}^i$, $c^{i*} = D_i c^i$ and $u_t^{i*} = \sum_{j=0}^{D_i-1} u_{t-j}^i$. Even though u_t^{i*} follows a MA process of order $D_i - 1$ with variance $\sigma_i^{*2} = D_i \sigma_i^2$, it can still be considered as a white noise since it is serially uncorrelated at the observational frequency.

3.2 State Space Representation

The dynamic factor model represented by equations (1), (3), and (5) can be formulated in a state-space form as follows. Let y_t denotes a $(n \times 1)$ vector of variables observed at date t. The dynamic that governs y_t can be described in terms of latent variables in a vector ξ $(r \times 1)$ called the state vector. Hence, the state-space representation of the dynamic of y is given by the following system of equations :

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{6}$$
$$y_t = A'x_t + H'\xi_t + w_t \tag{7}$$

Where F, A' and H' are matrices of parameters of dimension $(r\times r)$, $(n\times k)$ and $(n\times r)$, respectively, and x_t is a $(k \times 1)$ vector of exogenous variables. Equation 6 is known as the State Equation, and equation 7 is known as the Observation Equation.
The vector v_t $(r \times 1)$ and the vector w_t $(n \times 1)$ are considered white noises :

$$E(v_t v'_\tau) = \begin{cases} Q & \text{, For } t = \tau \\ 0 & \text{, Otherwise} \end{cases} \tag{8}$$
$$E(w_t w'_\tau) = \begin{cases} R & \text{, For } t = \tau \\ 0 & \text{, Otherwise} \end{cases} \tag{9}$$

Where Q and R are $(r \times r)$ and $(n \times n)$ matrices, respectively. The noises v_t and w_t are assumed to be uncorrelated across all periods :

$$E(v_t w'_\tau) = 0 \text{ , for all } t \text{ and } \tau \tag{10}$$

The fact that x_t is exogenous implies that x_t provides no information about ξ_{t+s} or w_{t+s} for any positive s beyond the information already contained in $y_{t-1}, y_{t-2}, ..., y_1$.

3.3 Model implementation

The estimation problem faced to compute the Business Cycle Index is substantial. A significant number of observations is involved (especially for the daily Business Cycle Index), and a constraint regarding the number of coefficients that can be estimated with MLE requires to think the process through. One simplifying assumption is made to reduce the number of parameters that need to be estimated by maximum likelihood. It is assumed that the Business Cycle Index and the various observed variables follow first-order dynamics ($p = 1$). Furthermore, we make two adjustments in the state space system to ease computation for signal extraction and parameter estimation.

Firstly, the Harvey Accumulator (Harvey, 1990), C_t^k, is implemented to reduce the size of the state vector by summarizing the necessary information required to construct observed flow variables. It is defined as follows :

$$C_t^k = \zeta_t^k C_{t-1}^k + b_t \tag{11}$$
$$= \zeta_t^k C_{t-1}^k + \phi_1 b_{t-1} + ... + +\phi_p b_{t-p} + \epsilon_t \tag{12}$$

Where ζ_t^k is defined as :

$$\zeta_t^k = \begin{cases} 0, & \text{If t is the last day of the observational period} \\ 1, & \text{Otherwise} \end{cases} \tag{13}$$

One of the main concern regarding the method used in this paper is the size of the estimation problem. Each new variable introduced in the dataset imply new parameters to be computed by maximum likelihood estimation. However, there is a soft cap to the number of coefficients [1] that can be efficiently estimated by the implemented algorithms. Given this constraint, principal components analysis is introduced to reduce the dimensionality of the problem. The principal components analysis aims at reducing the number of variables of a dataset while preserving as much information as possible. To do so, the first principal component for each frequency (quarterly, monthly, and daily) and each type (flow and stock) is extracted. Notice that there is no distinction between flow and stock variables at the frequency of the Business Cycle Index. For instance, at the monthly frequency instead of having 8 variables and 33 coefficients to estimate making the MLE cumbersome, the variables are "bundled" in 3 different new variables (i.e, quarterly stock, quarterly flow and monthly) effectively reducing at 13 the number of coefficients to be estimated.

First, each variable is standardized by subtracting to each observation the mean of the variable and then by dividing it with the variable's standard deviation. As a result, all variables will contribute equally to the analysis. This step is crucial as principal components analysis is quite sensible to the variance of the variables. Then, the covariance matrix is computed for each group of indicators.
The next step is to compute the eigenvectors and associated eigenvalues of the covariance matrix to determine the principal components. Principal components are new variables build as linear combinations of the preliminary indicators. There are as many principal components as there are initial variables. The resulting principal components are uncorrelated, and the most significant part of the information within the initial variables is squeezed into the first component and then the remaining information into the second one and so on. That is why the first principal component is chosen as the future variable of interest, as it will contain most of the information of the initial variables. To determine the first principal component, the eigenvalues need to be ordered. The reason is that the eigenvector is the direction in which there is most of the variance, while the eigenvalue indicates how much variance is present in that direction. Hence, the eigenvector with the highest eigenvalue is the first principal component. The last step consists of multiplying the first principal component by the standardized dataset. The result will be the sought variable.
The two biggest drawbacks of principal components analysis are the obvious loss of information through the compression of the variables into a single one, and that principle components in themselves do not have a meaning or interpretation. While the first one is regrettable, even though necessary, the second one is not of great importance in this study as principal components are only

[1] Approximately 20 even though estimating more coefficients is possible, it is a lot more time consuming

used to estimate the Business Cycle Index.

3.3.1 Monthly Business Cycle Index

The equations that defines the monthly model are the state equation :

$$\underbrace{\begin{bmatrix} b_{t+1} \\ c^m_{t+1} \end{bmatrix}}_{=\,\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi & 0 \\ \phi & \zeta^m_t \end{bmatrix}}_{=\,\text{F}} \underbrace{\begin{bmatrix} b_t \\ c^m_t \end{bmatrix}}_{=\,\xi_t} + \underbrace{\begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}}_{=\,v_{t+1}}, \tag{14}$$

and the observation equation :

$$\underbrace{\begin{bmatrix} \tilde{y}_{1t} \\ \tilde{y}_{2t} \\ \tilde{y}_{3t} \\ \tilde{y}_{4t} \end{bmatrix}}_{=\,y_t} = \underbrace{\begin{bmatrix} c_1 & \rho_1 & 0 & 0 & 0 \\ c_2 & 0 & \rho_2 & 0 & 0 \\ c_3 & 0 & 0 & \rho_3 & 0 \\ c^*_4 & 0 & 0 & 0 & \rho_4 \end{bmatrix}}_{=\,\text{A'}} \underbrace{\begin{bmatrix} 1 \\ \tilde{y}_{1t-3} \\ \tilde{y}_{2t-1} \\ \tilde{y}_{3t-1} \\ \tilde{y}_{4t-3} \end{bmatrix}}_{=\,x_t} + \underbrace{\begin{bmatrix} \lambda^q_s & 0 \\ \lambda^m_s & 0 \\ \lambda^m_f & 0 \\ 0 & \lambda^q_f \end{bmatrix}}_{=\,\text{H'}} \underbrace{\begin{bmatrix} b_t \\ c^m_t \end{bmatrix}}_{=\,\xi_t} + \underbrace{\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u^*_{4t} \end{bmatrix}}_{=\,w_t}, \tag{15}$$

with :

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_{2x1} \\ 0_{4x1} \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R_t \end{bmatrix} \right), \quad Q = \begin{bmatrix} 1-\phi^2 & 0 \\ 0 & 1-\phi^2 \end{bmatrix}, \quad R = \begin{bmatrix} \sigma^2_1 & 0 & 0 & 0 \\ 0 & \sigma^2_2 & 0 & 0 \\ 0 & 0 & \sigma^2_3 & 0 \\ 0 & 0 & 0 & \sigma^{*2}_4 \end{bmatrix}$$

Note that even though there is no difference between monthly flow and stock variables, we decided to separate them during the extraction of the first principal components to preserve as much information as possible since the monthly framework do not pose any particular challenge regarding the MLE.

3.3.2 Daily Business Cycle Index

The equations that defines the daily model are the state equation :

$$\underbrace{\begin{bmatrix} b_{t+1} \\ c^m_{t+1} \\ c^q_{t+1} \end{bmatrix}}_{=\,\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi & 0 & 0 \\ \phi & \zeta^m_t & 0 \\ \phi & 0 & \zeta^q_t \end{bmatrix}}_{=\,\text{F}} \underbrace{\begin{bmatrix} b_t \\ c^m_t \\ c^q_t \end{bmatrix}}_{=\,\xi_t} + \underbrace{\begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}}_{=\,v_{t+1}}, \tag{16}$$

and the observation equation :

$$\underbrace{\begin{bmatrix}\tilde{y}_{1t}\\ \tilde{y}_{2t}\\ \tilde{y}_{3t}\\ \tilde{y}_{4t}\\ \tilde{y}_{5t}\end{bmatrix}}_{=\,y_t} = \underbrace{\begin{bmatrix}c_1 & \rho_1 & 0 & 0 & 0 & 0\\ c_2 & 0 & \rho_2 & 0 & 0 & 0\\ c_3^* & 0 & 0 & \rho_3 & 0 & 0\\ c_4^* & 0 & 0 & 0 & \rho_4 & 0\\ c_5 & 0 & 0 & 0 & 0 & \rho_5\end{bmatrix}}_{=\,\text{A'}} \underbrace{\begin{bmatrix}1\\ \tilde{y}_{1t-D_q}\\ \tilde{y}_{2t-D_m}\\ \tilde{y}_{3t-D_m}\\ \tilde{y}_{4t-D_q}\\ \tilde{y}_{5t-1}\end{bmatrix}}_{=\,x_t} + \underbrace{\begin{bmatrix}\lambda_s^q & 0 & 0\\ \lambda_s^m & 0 & 0\\ 0 & \lambda_f^m & 0\\ 0 & 0 & \lambda_f^q\\ \lambda^d & 0 & 0\end{bmatrix}}_{=\,\text{H'}} \underbrace{\begin{bmatrix}b_t\\ c_t^m\\ c_t^q\end{bmatrix}}_{=\,\xi_t} + \underbrace{\begin{bmatrix}u_{1t}\\ u_{2t}\\ u_{3t}^*\\ u_{4t}^*\\ u_{5t}\end{bmatrix}}_{=\,w_t}, \tag{17}$$

with :

$$\begin{bmatrix}v_{t+1}\\ w_t\end{bmatrix} \sim N\left(\begin{bmatrix}0_{3x1}\\ 0_{5x1}\end{bmatrix}, \begin{bmatrix}Q & 0\\ 0 & R_t\end{bmatrix}\right),$$

$$Q = \begin{bmatrix}1-\phi^2 & 0 & 0\\ 0 & 1-\phi^2 & 0\\ 0 & 0 & 1-\phi^2\end{bmatrix}, \quad R = \begin{bmatrix}\sigma_1^2 & 0 & 0 & 0 & 0\\ 0 & \sigma_2^2 & 0 & 0 & 0\\ 0 & 0 & \sigma_3^{*2} & 0 & 0\\ 0 & 0 & 0 & \sigma_4^{*2} & 0\\ 0 & 0 & 0 & 0 & \sigma_5^2\end{bmatrix}$$

Note that the incorporation of weekly variables in the model would not pose any major difficulties.

3.4 Model estimation

Using homemade R codes and the ADS methodology, the estimation process is described below. First, start-up coefficients' values for the stock variables and flow variables at the frequency of the Business Cycle Index are estimated by running the Kalman filter once without using flow variables at a lower frequency than the index (see Appendix A.1 and A.2 for a description of the Kalman filter algorithm and model estimation by MLE).
The first step, which was just presented, provides the first estimate for all the coefficients of stock variables and flow variables at the frequency of the Business Cycle Index in the observation equation and all the coefficients of the state equation. Moreover, a first estimation of the business cycle b_t^* is extracted.
Then, to obtain initial values for the coefficients associated with the remaining flow variables in the observation equation, a simple OLS regression of the flow variables on the first estimate of the business cycle and the lag of the said variables is run.

$$y_t^i = \lambda^i b_t^* + \rho^i y_{t-1}^i + \epsilon_t^i \tag{18}$$

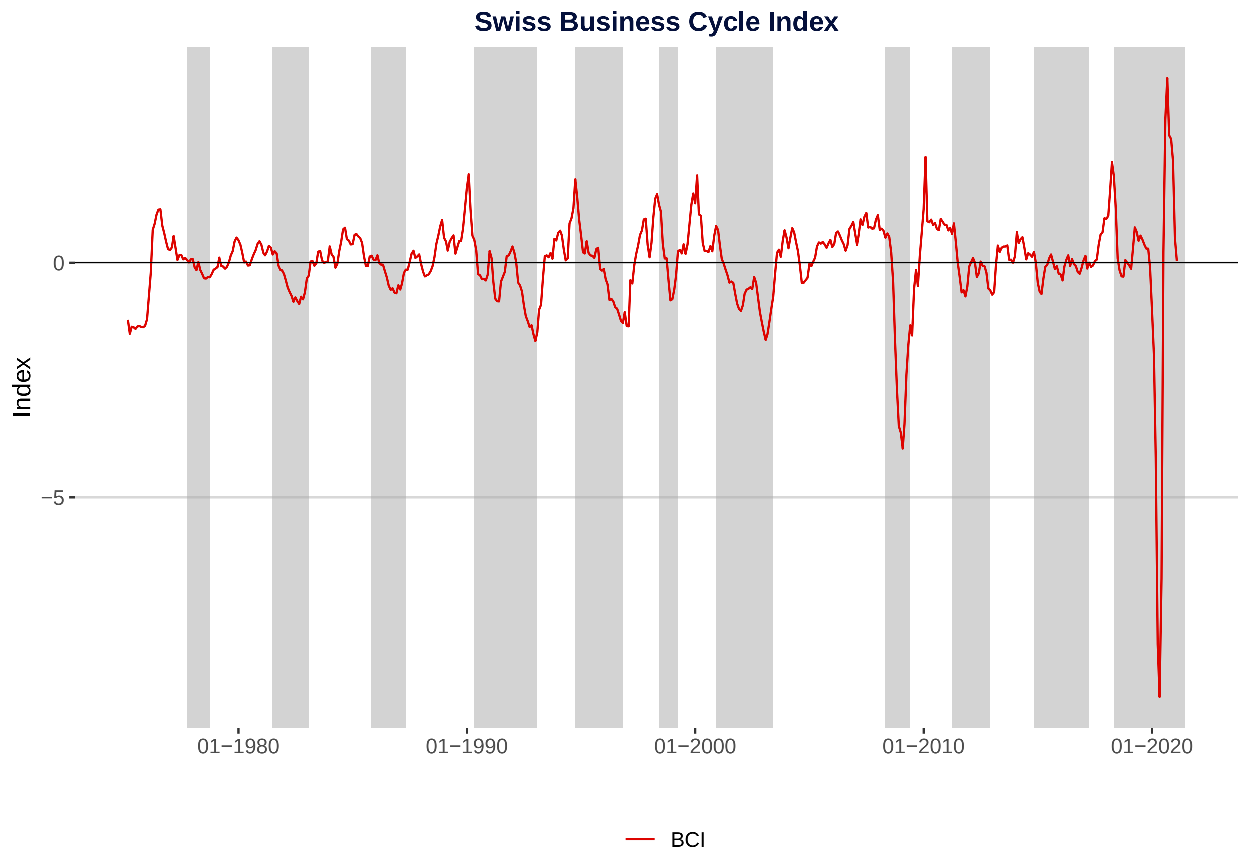
At the end of this second stage, initial values for all the parameters of the model are available. Finally, an estimation using all the model's coefficients jointly is made before extracting the final smoothed estimate of the Business Cycle Index.
It is worth noting that to perform the maximum likelihood estimation, two optimization algorithms are used alternatively to increase the chances of reaching a global maximum rather than a local one. This is due to algorithms tending to identify a path in which they will be locked until convergence. Thus, using two algorithms that do not use the same method alleviate this problem. The two algorithms used are part of the optimx R package and are named "nlminb" and "Nelder-Mead".

4 Results

As expressed previously, the main focus will be on the monthly Business Cycle Index results as they are the most promising ones. Therefore, that is also the index with which the evaluations are made. In addition, however, findings regarding the daily Business Cycle Index are presented as well as directions in which further improvements and additions could be made to improve it.

4.1 Monthly Business Cycle Index

4.1.1 Basic Results



Note: The grey areas represent the Swiss recessions as reported by the OECD Indicator. The Business Cycle Index is standardized.

Figure 1: Monthly Swiss Business Cycle Index

Figure 1 shows the focal point of this paper. It shows the Business Cycle Index (BCI) extracted at the monthly frequency. The Swiss recessions estimated by the OECD are also represented in grey. Observations concerning the validity of the BCI can already be made. First, it is pretty encouraging to see that the Business Cycle Index seems to follow the periods of recessions estimated by the OECD across the whole sample closely. Naturally, the OECD indications regarding recession periods are not necessarily the best comparison available, but it is nevertheless reassuring that the two are coherent with each other. By looking chronologically at the evolution of the Swiss economy over the period spanning from the 1980s to 2021, a descriptive assessment of the Business Cycle Index can be made.

In the first place, the recessions induced by the oil crises due to a shortage of oil supply which caused a hike in oil prices in the global markets after the Iranian Revolution and the Iran-Iraq War at the end of the 1970s and the beginning of the 1980s are well captured by the BCI. For the rest of

the decade, the Swiss business cycle only experienced a sluggish recovery which was even impaired by a slight economic slump in 1986. It is only at the end of the 1980s that the Swiss economy soared again.
Then, the end of the housing boom, as well as the global recession partly due to tight monetary policies at the beginning of the 1990s, are clearly visible. During this period, Switzerland had one of the weakest growth of the Western world. The Business Cycle Index captures those movements quite well by remaining mainly in negative territory. From the second half of the 1990s, the Swiss economy rose up again until 1999, when it faced a slight economic downturn.
The expansion of the Swiss economy reached new heights by mid-2000 until the end of the dot-com bubble and the subsequent global recession, which lasted until mid-2003. Throughout this period, the BCI remained negative. Thereafter, the economy picked up again and saw a phase of prolonged expansion until mid-2008 and the onset of the Great Recession. The BCI appears to be able to follow this trend pretty well. The Great Recession caused a sharp but rather short dip, and the economy started to recover as soon as the end of 2009.
However, the expansion was only short-lived this time as the main trading partners of Switzerland, the members of the European Union, entered a new recession because of the Euro Sovereign Debt Crisis. The rise in uncertainty and demand for safe assets drove a growing number of investors to massively invest in the Swiss Franc, which led to an appreciation of the Swiss national currency. It entailed a recession that lasted until early-2012. One reason for the end of the recession is the implementation of the minimum exchange rate against the Euro by the Swiss National Bank in September 2011, which effectively ended the Swiss franc appreciation. Furthermore, various QE programs were introduced by the European Central Bank, one in November 2011, to take back control of the Euro Area.
The economy and the BCI reentered positive ground after 2012, even though it never reached the same level as before the Great Recession. This momentum lasted for three years until the Swiss National Bank decided to let go of the floor in January 2015. Following this decision, the Swiss economy experienced a slow-down in growth and saw next to zero growth until the end of 2018. The Business Cycle Index shows that effectively by oscillating around zero for the whole period. Thenceforth, the Swiss economy went through an expansion phase adequately captured by the BCI, but it was once again short-lived due to the Covid crisis, which plunged the economy into a deep recession.
Overall, the Business Cycle Index is able to accurately track the different phases of the Swiss economy between 1980 and 2021. It is also worth noting that the magnitude also seems in line with reality as the recessions of 2008 (Subprime Crisis) and 2020 (Covid Crisis) are the most prominent ones as they should be. Besides, it is possible as well to distinguish the economic rebound in the summer of 2020 when Switzerland reduced the different measures against the Covid.

By looking at the different estimated coefficients of the monthly model, there are also encouraging signals concerning the validity of the BCI. The most interesting coefficients are the λ as they are the easiest to understand and predict. As a matter of fact, λ represents the magnitude of co-movements between the Business Cycle Index and the different variables. Hence, depending on the observable, it is possible to predict the most likely sign of λ.

Table 1 reports the different coefficients. For example, Quarterly Stock (Employment, Vacancies and Consumer Confidence Index), Monthly Flow (Industrial Production), and Quarterly Flow (Real GDP) are all pro-cyclical variables. Therefore, if the business conditions are improving, then it is only natural to expect a positive coefficient for those variables. It is effectively the case by looking at the corresponding values of λ, which are all positive. On the other hand, Monthly Stock (Unemployment) is an anti-cyclical variable. Therefore, when the business conditions improve, there should be opposed movements between the BCI and the monthly stock variables. Once again, it is verified with a negative λ.

Table 1: Monthly Swiss Business Cycle Coefficients

Variable	ϕ	ρ	λ	**c**	σ^2
Quarterly Stock (1)	0.801	0.118	0.425	-0.005	0.842
Monthly Stock (2)	0.801	0.344	-0.335	0.003	0.456
Monthly Flow (3)	0.801	-0.400	0.207	0.005	1.029
Quarterly Flow (4)	0.801	-0.324	0.566	0.014	0.352

Finally, by looking at the auto-regressive coefficient of the Business Cycle Index ϕ, it is apparent that the business cycle seems quite persistent given the high value of the coefficient.[2] Hence, it is plausible to assume that business conditions are not to move abruptly from one period to the other without a substantial shock to the economy.

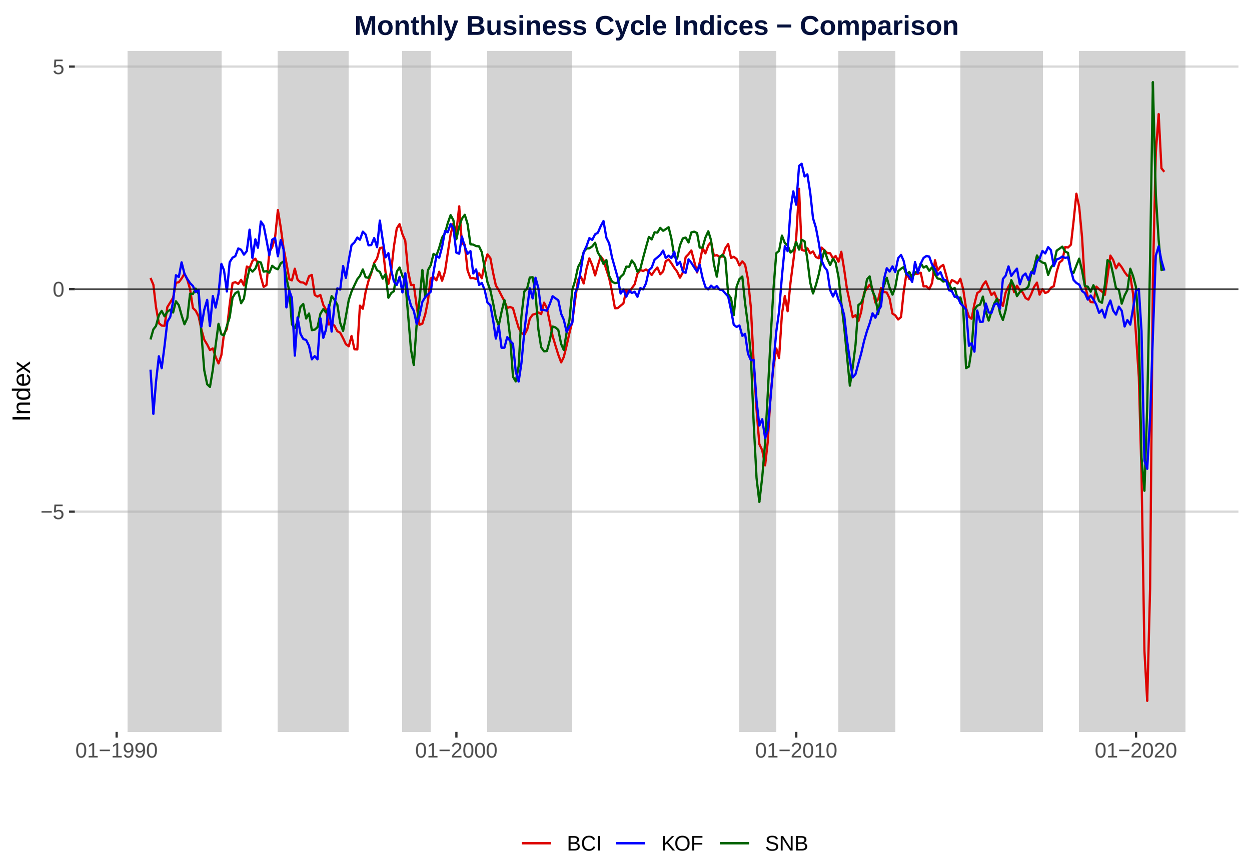
I think we should make more clear throughout the paper and here that we seek to develop a coincident BCI and not a leading BCI. We are interested in nowcasting real acivity indicators and not forecating them. By doing so, we strengthen our contribution in my opinion. What do you think? Figure 2 shows the BCI with the other two most prominent Swiss indices (the KOF Barometer and the SNB Index). Predominantly, the three indices follow the same path over the whole sample, which is a comforting observation concerning the validity of the Business Cycle Index. As a confirmation of this examination of Figure 2, a correlation of 0.657 is found between the BCI and the SNB Index while the correlation between the BCI and the KOF Barometer is 0.594.

Nevertheless, a few recurring differences can be noticed. It appears that for most recessions, the BCI tends to decrease at a later date while the KOF and SNB Indicators lead the recessions. It is especially apparent for the dot-com bubble, the Great Recession, and the Euro Sovereign Debt Crisis. Furthermore, the same observation can be made concerning the expansion phases. Once again, the BCI seems to recover only after the two other indices. A plausible explanation for that pattern is that the KOF Barometer and the SNB Indicator are using more leading variables in their

[2]See Equation 1 in Section 3.

dataset.

The magnitude also differs substantially during specific periods. It is especially conspicuous during the COVID crisis, where the BCI shows a more remarkable economic downturn than the KOF and SNB indices. In addition, at the onset of the COVID crisis, the BCI implies a healthier economy while the other two suggest a mild expansion phase.

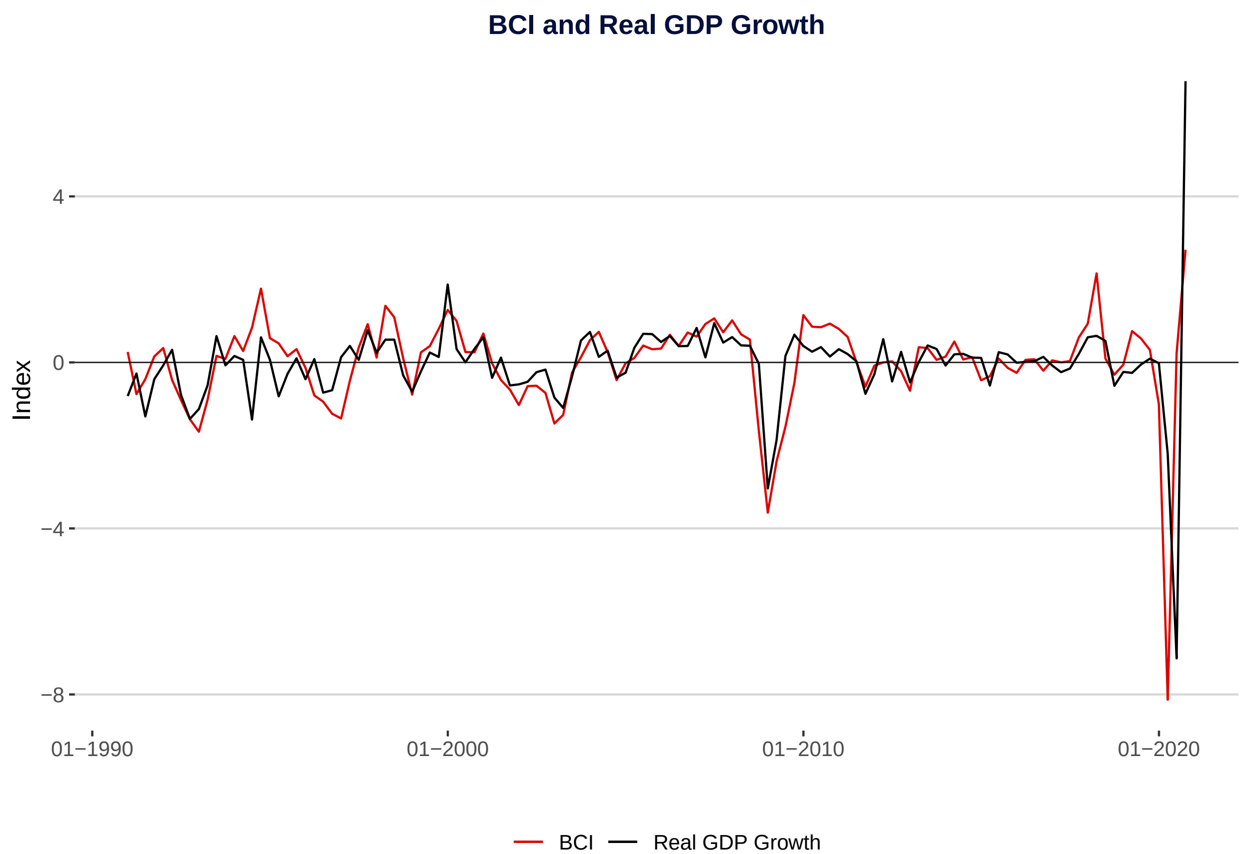


Note: The grey areas represent the Swiss recessions as reported by the OECD Indicator. All time-series are standardized for comparison purposes.

Figure 2: Monthly Business Cycle Indices - Comparison

In Figure 3, the real GDP growth and the BCI are featured to determine whether the BCI effectively follows the real economy's trend. The focus is on the comparison of the BCI with the real GDP growth since it is arguably the best proxy for the economic conditions at a given period. As it can be seen, the BCI is able to track the movements of the real GDP quite precisely, especially during periods of economic downturns, hence making it a viable indicator for the state of the economy in troubled times.

The main concern is the tendency of the BCI to over-estimate the rebound of the business cycle after an economic deterioration, as it can be seen after the Great Recession and the Swiss housing boom. Apart from the period just before the Covid Crisis where the Business Cycle Index over-estimates the real GDP Growth, the BCI tracks the real GDP growth accurately from the Euro Sovereign Debt Crisis to 2019 thanks to a greater number of variables used to estimate it. It is particularly visible when comparing the 1990s and the 2010s, where the difference in accuracy for the BCI is noteworthy.

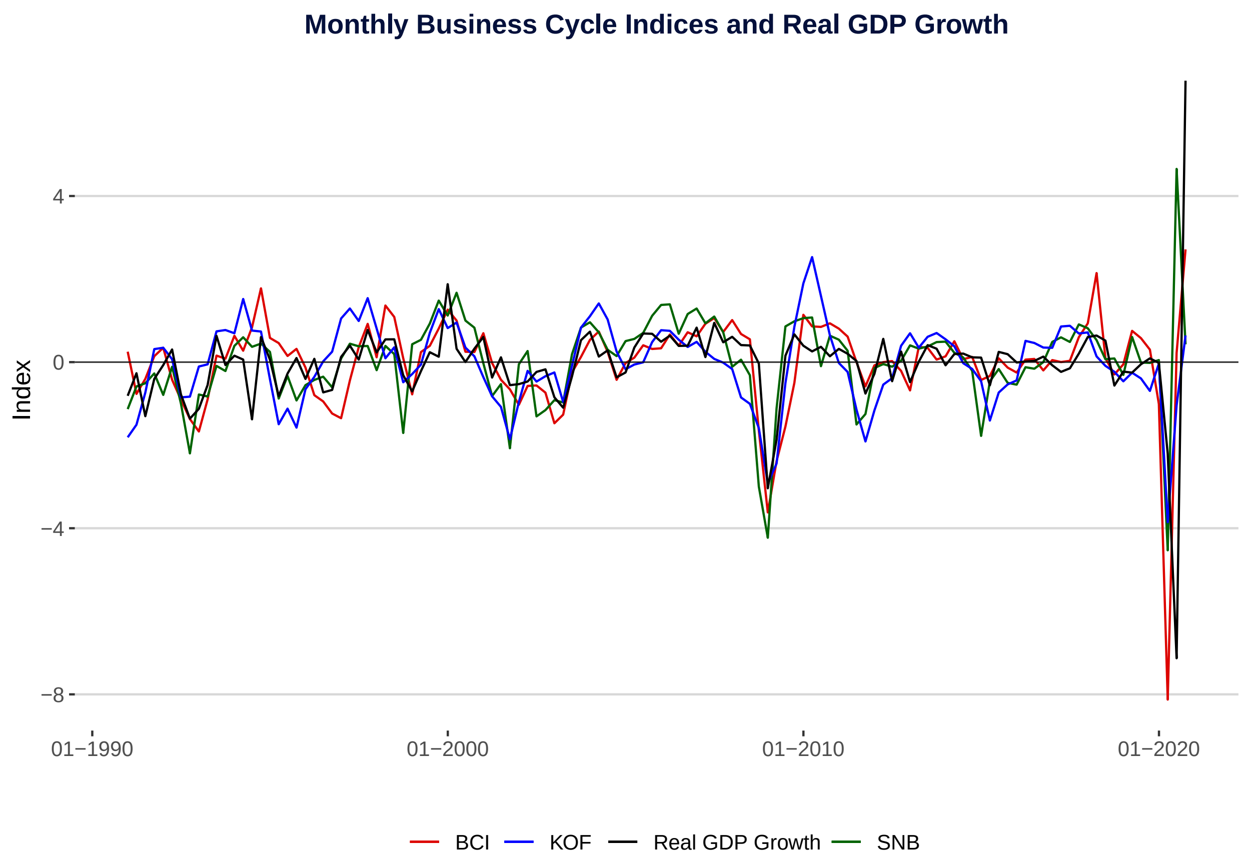


Note: All time-series are standardized for comparison purposes.

Figure 3: Monthly Business Cycle Index - Real GDP Growth

Finally, Figure 4 displays the three indices with the real GDP growth. Similar graphs can be found for the other variables in the appendix. It is pretty discernible that the three indicators and the real GDP growth co-move closely as well. It appears that the BCI and the SNB Indicator are moving quite closely with each other except for the COVID crisis, where both the KOF and SNB indices fail to capture the depth of the recession. Analog to the BCI, the KOF Barometer also seems to overshoot after an economic deterioration. Moreover, it is once again apparent that the KOF and the SNB indicators lead the business cycle while the BCI is always a bit tardy in indicating changes in the economy.

While the results are encouraging for real GDP growth, the same observation does not apply to all the other variables. For instance, Figure F9 of the appendix, which shows the three indices and industrial production growth, demonstrates that the BCI, as well as the other two indices, fail to capture the movements of industrial production. However, by looking at Figure F6 and F7, the results for the consumer confidence index and the employment growth are somewhat more appealing.



Note: All time-series are standardized for comparison purposes.

Figure 4: Monthly Business Cycle Indices - Real GDP Growth

Further arguments for the validity of the Business Cycle Index can be found in Table 2 with the different correlations between the observed variables and the three indices. The correlations between real GDP growth and the different indicators are positive and especially high for the BCI. This could be explained by the fact that the KOF Barometer and the SNB Index employ a much broader dataset than the BCI, which could entail that the real GDP has a much more considerable influence in the BCI than in the two other indices. Thus, making it harder for the two other indicators to keep track of real GDP growth. By moving away from real GDP growth and looking at the correlations with the other variables, it is possible to see notable differences where sometimes the other two indices are better than the BCI (Retail Sales Growth) while being worse on others (Unemployment Growth) or fairly equal (CCI).

An additional observation can be made to further establish the validity of the BCI. By looking at the signs of the correlation's values, it is evident that the three indices are coherent with the pro-cyclical and anti-cyclical variables' movements.

Table 2: Correlations

Variables	**BCI**	**SNB**	**KOF**
Real GDP Growth	0.511	0.153	0.441
Employment Growth	0.430	0.241	0.144
Vacancies Growth	0.6	0.482	0.548
CCI	0.527	0.543	0.464
Industrial Production Growth	0.215	0.188	0.177
Retail Sales Growth	0.014	0.175	0.027
Export Growth	0.218	0.211	0.167
Unemployment Growth	-0.647	-0.480	-0.426

4.1.2 In-sample Assessment

To determine whether the Business Cycle Index has nowcasting power, in-sample and out-of-sample assessments are used to confront the BCI with the SNB and the KOF indices for the former assessment and an AR(1) benchmark for the latter. As Diebold (2020) stated, the best assessment would be to use vintage data for the out-of-sample evaluation. However, given the difficulties entailed with gathering all the necessary vintages, revised data are used for the analysis. Out of "fairness", different benchmarks are applied for the in-sample and out-of-sample assessments. Using vintages of the KOF index for an out-of-sample comparison would be misleading because the Business Cycle Index uses revised data. Hence, the KOF index would be at a disadvantage because, thanks to revised data, future information is already incorporated in the data used to extract the Business Cycle Index. Therefore, the in-sample strategy will compare the BCI with the SNB and the KOF indices while, on the other hand, an AR(1) process is simulated for the out-of-sample assessment and is opposed against the BCI. Nowcasting real GDP growth is especially of interest, and thus, a quarterly frequency for the different business cycle indicators is used. To do so, the mean of the different indices' values within a given quarter is computed.

The strategy for the in-sample assessment is straightforward. The previously presented Kalman filter algorithm is run to compute the Business Cycle Index for the whole sample $t = 1, ..., T$. For each quarter $t = R, ..., T$ in the evaluation sample and each monthly BCI i under evaluation, we then generate the following nowcast for real GDP growth $\hat{y}_{i,t}$:

$$\hat{y}_{i,t} = \hat{\alpha}_i + \hat{\beta}_i \overline{BCI}_{i,t}, \qquad t = R, ..., T \tag{19}$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are coefficients estimated by OLS, and $\overline{BCI}_{i,t}$ is the mean value of the monthly BCI i over the quarter t.

In Figure 5, real GDP fitted values computed with the BCI are displayed with real GDP growth and 95% confidence intervals. The results are promising given the ability of the BCI nowcasts to track real GDP growth with a satisfactory accuracy throughout the evaluation period with a correlation of 0.83. However, it fails at correctly estimating the real GDP growth by a substantial margin during the recent Covid crisis.

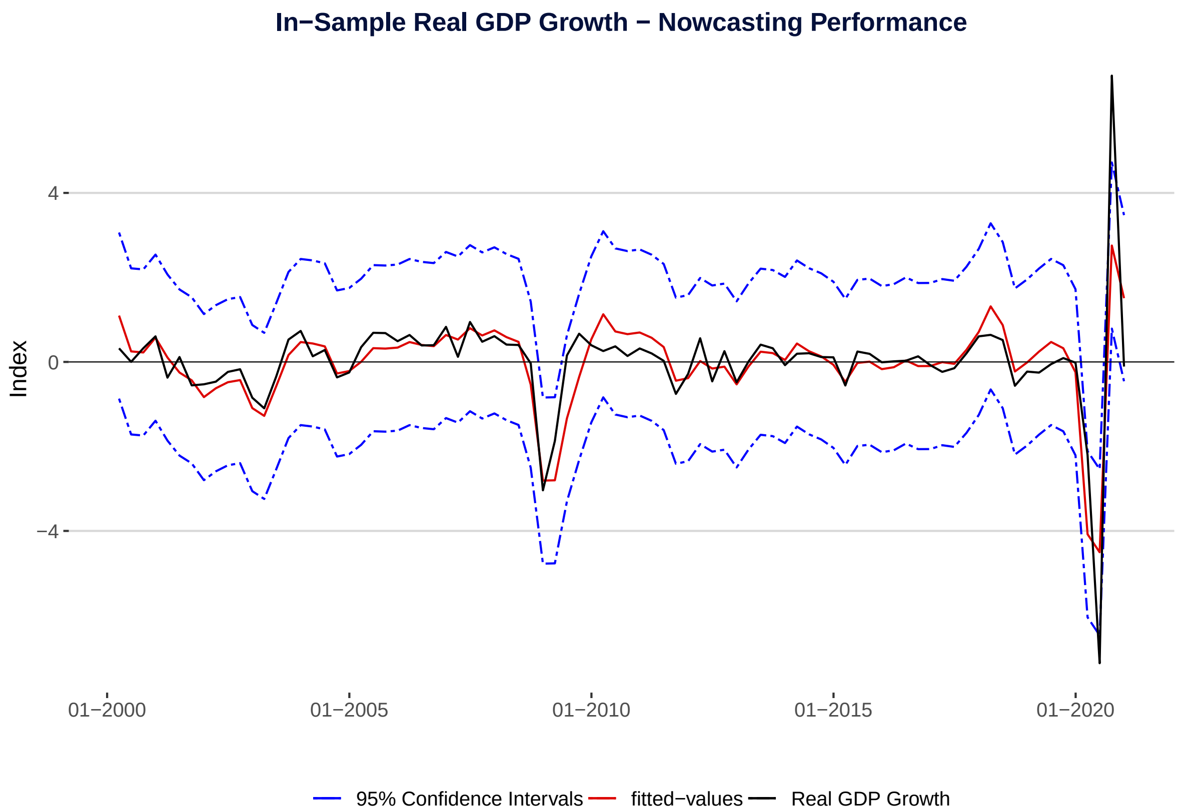


Figure 5: In-Sample - Real GDP Growth Nowcasting Performance

We now compare the nowcasting performance of our BCI to those of monthly indices computed by the SNB and the KOF for the evaluation sample running from Q1 2000 to Q4 2020. To do so, nowcasts are generated for KOF and SNB indices as well. We then compute the root mean square nowcasting error of each of three indexes. Table 3 shows the results of the in-sample assessment for real GDP growth. It is directly striking that the BCI is quantitatively better than the KOF or the SNB Index by approximately 40%. By looking at Figures 6 and 7, where the nowcasts using the different indicators are plotted with their associated nowcasting errors, it is possible to find the reason for such a big difference in root-mean-square nowcasting errors.

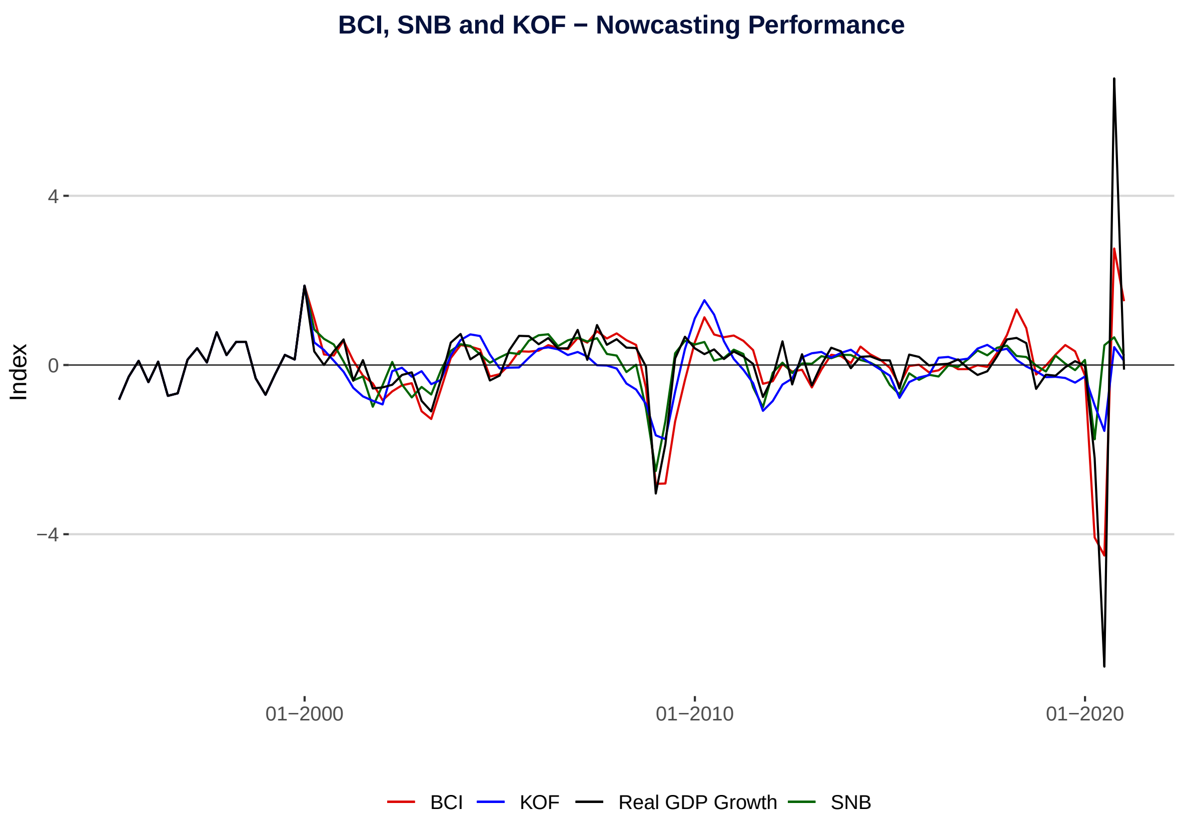
Throughout the evaluation period (Q1 2000-Q4 2020), all indices seem to follow the real GDP growth path quite closely except for the Covid crisis period, where the BCI is better fitted to nowcast real GDP growth even though it still falls short of capturing the real magnitude of the decline and rise of the real GDP. As mentioned previously, a possible explanation could be that the underlying dataset used to estimate the BCI is better suited to capture the changes in real GDP growth during an economic crisis. To support this argument, the next focus will obviously be the Great Recession of 2008. Once again, the BCI follows the real GDP growth path more closely at the onset of the crisis. Moreover, it is also able to capture precisely the depth of the recession. Therefore, it could be, once again, argued that the BCI has better nowcasting abilities for real GDP during periods of economic distress.

On the other hand, the statistical tests' results are not as auspicious. The only significant result is the second test which indicates that the BCI is significantly more accurate at 10% than the KOF Barometer in nowcasting real GDP. Table T2 of the appendix reports nowcasting performances for other variables. The accuracy of the BCI does not seem higher or lower than the other two indicators except for employment growth, for which the BCI is quantitatively better and significantly more accurate than the KOF Barometer (at 5%) and the SNB Index (at 10%).

Table 3: In-Sample Assessment - Real GDP Growth

	Ratio	Test 1	Test 2
Index	η	**p-value (DM test)**	**p-value (DM test)**
SNB	0.618	0.24	0.12
KOF	0.654	0.12	0.06*

Nonetheless, the previously reported results are still quite encouraging given the fact that the model used in this paper is rather basic. Many upgrades could be implemented to increase the accuracy of the BCI further by, for instance, increasing the number of variables used, including open-economy aspects to the model by introducing a world Business Cycle Index or moving to higher frequency data to exploit financial variables as well.



Note: All time-series are standardized for comparison purposes. The evaluation starts on the 1st January 2000.

Figure 6: In-Sample Assessment - BCI, SNB and KOF

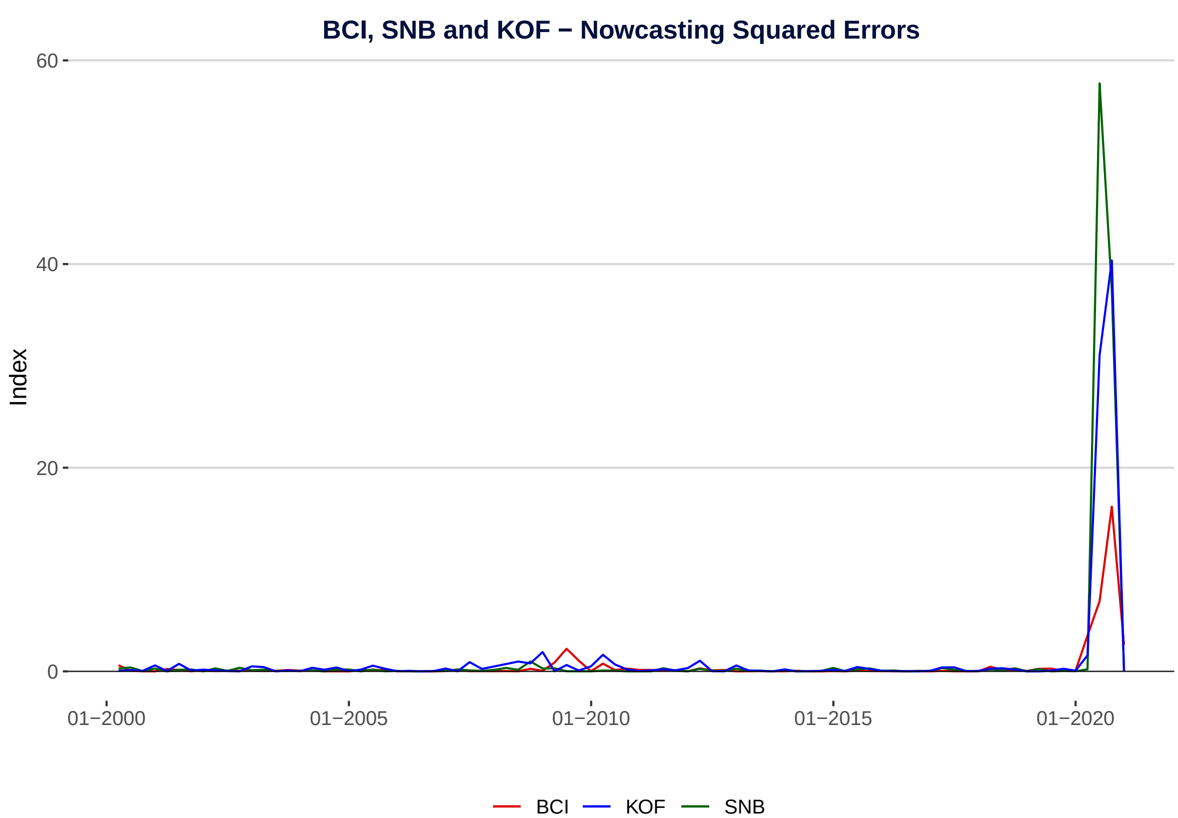


Figure 7: In-Sample Assessment - Nowcasting Errors

4.1.3 Out-of-Sample Assessment

The out-of-sample strategy is similar in many aspects to the in-sample assessment with a few additional steps. Let it be assumed once again that the sample runs from date $t = 1$ to T. Quarter R will be the starting date for the assessment. As first step, the Kalman filter algorithm is run to compute the BCI for the sample running from $t = 1$ to R. The second step consists in producing the nowcast (19) using this sub-sample to obtain an OLS estimate α_i and β_i. The same process is run all over again for the periods $R+1, R+2, ..., T$ in order to generate a sequence series of out-of-sample nowcasting errors. Finally, the root mean square nowcasting error of the BCI is calculated. We choose the AR(1) process as a benchmark and compute its root mean square nowcasting error using the same out-of-sample strategy.

Table 4: Out-of-Sample Assessment - BCI and AR(1)

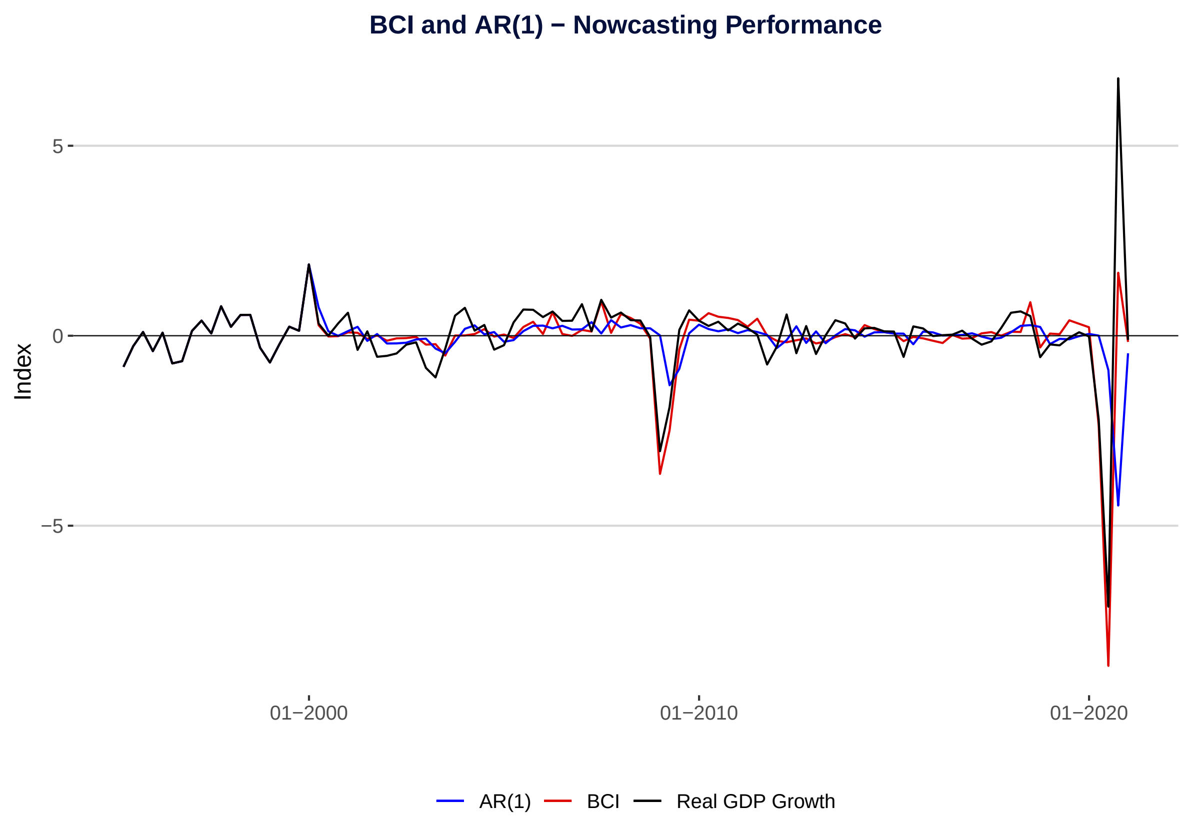
	Ratio	Test 1	Test 2
Index	η	**p-value (DM test)**	**p-value (DM test)**
AR(1)	0.45	0.154	0.08*

Table 4 shows the quantitative and statistical results of the out-of-sample assessment, while Figure 8 and 9 display the nowcasting estimates for the evaluation period (2000-2021) and the nowcasting errors, respectively. The results are again encouraging regarding the nowcasting abilities of the Business Cycle Index. It is significantly more accurate than an AR(1) at 10% and is unmistakably quantitatively better with a ratio of 0.45. Furthermore, a correlation of 0.843 is found between the BCI and real GDP growth.

Looking at Figures 8 and 9, a similar explanation to the in-sample assessment can be found as to why the BCI outperforms the AR(1) process by such a large margin. Figure 8 provides the first part of the argument by demonstrating how the fitted values produced with the Business Cycle Index are tracking the real GDP growth path much more closely than the fitted values estimated with the AR(1) process. While not necessarily evident during periods of stagnant economic growth, the Business Cycle Index nowcasting abilities exhibit themselves during the Great Recession and the Covid Crisis, where the AR(1) unmistakably fails to follow the growth path of real GDP accurately.

Another observation can be made regarding the timing and the magnitude with which the two different fitted values time-series show the economic crises cited above. Although the BCI nowcast estimates describe the depth of the recessions with accuracy and at the right time, the same cannot be said about the AR(1) nowcast estimates which are tardy to indicate an economic deterioration as well as unsuccessful to show the magnitude of the recessions.

Figure 9 confirms those observations by revealing that the BCI is making much fewer nowcasting errors during periods of economic downturns than the AR(1). All in all, the out-of-sample exercise presented in this subsection further reinforced the argument that the newly developed Business Cycle Index could be used to monitor and nowcast real GDP growth during periods of sudden economic change. Further analysis has been conducted to determine whether that performance is only limited to real GDP growth or if it could be generalized to all the variables of the underlying dataset used to estimate the Business Cycle Index. The results are displayed in Figure F20-26 and Table T2 of the appendix. By looking at Table T3, it is evident that the nowcasting power of the BCI is limited regarding the rest of the variables and by focusing on the different figures, only the CCI, unemployment growth and vacancies growth give somewhat satisfying outcomes.



Note: All time-series are standardized for comparison purposes. The evaluation starts on the 1st January 2000.

Figure 8: Out-of-Sample Assessment - BCI and AR(1)

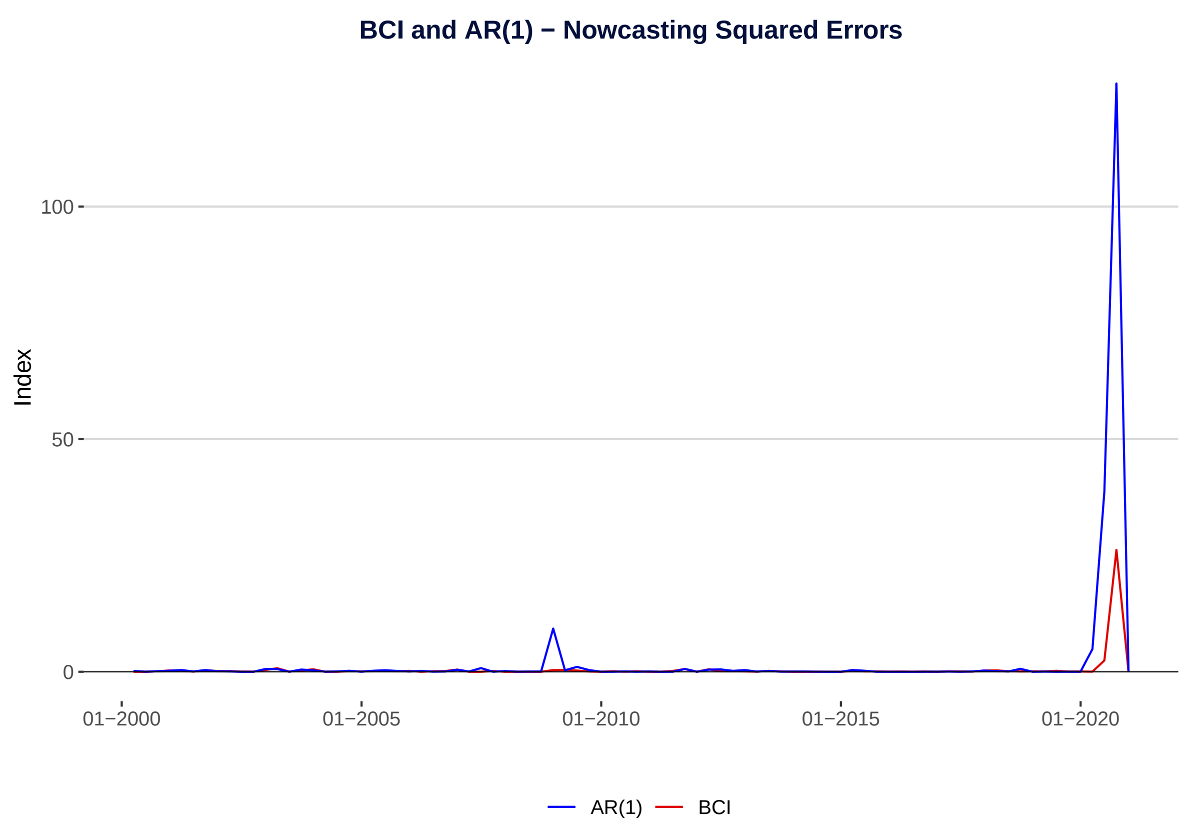
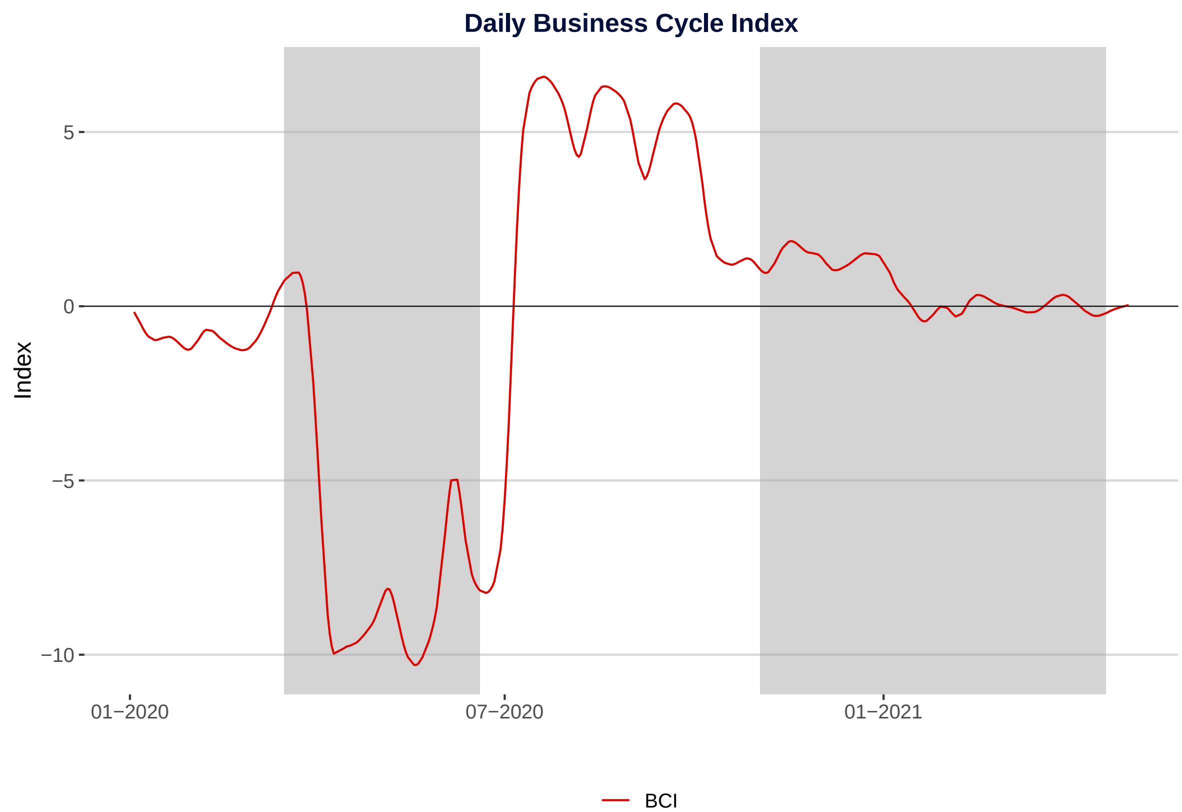


Figure 9: Out-of-Sample Assessment - Nowcasting Errors

4.2 Daily Business Cycle Index

This subsection presents an attempt at moving the BCI at a higher frequency, namely the daily. The period of interest will be the COVID Crisis for two main reasons. The second reason concerns mostly the real goal of computing a daily Business Cycle Index which to be able to monitor the economy on real time. Hence, by focusing on the recent crisis, it allows to demonstrate what could be achieved with such a high frequency index. Knowing that lots of indicators used by economic agents are reported with a substantial lag, this could hamper their decision-making capabilities in times of rapid economic changes. Thus, having a high frequency index could alleviate such problem by giving an indication on which direction the business cycle is going as well as daily estimation of macroeconomic variables.



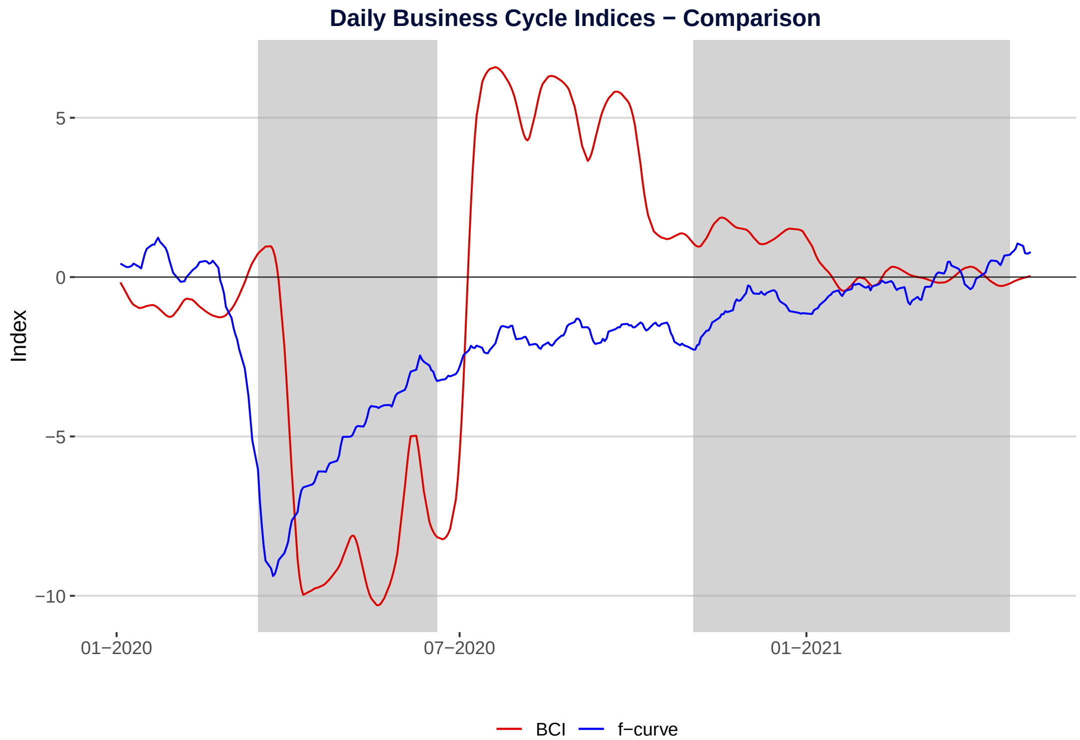
Note: The grey areas represent the periods of lockdown measures in Switzerland. The BCI is smoothed with a rolling average of 7 days.

Figure 10: Daily Swiss Business Cycle Index

Figure 10 shows the Business Cycle Index at the daily frequency for the period between 2020 and 2021. The grey areas represent periods in which Switzerland was under relatively strict lockdown measures. The first one went from the 16th March 2020 to 19th June 2020, during which Switzerland was in a state of health emergency declared by the Federal Council. For that duration, home-working became mandatory, and all non-necessary businesses such as restaurants and bars were closed. Moreover, festivals and gatherings of large groups of people were prohibited. Those measures greatly impacted the consumption of the Swiss population, which refrained from spending their income due to their inability to do so and possibly as precautionary savings due to the great uncertainty brought by the Covid pandemic.

The second period started on 28th October 2020 and lasted until 19th April 2021. The second lockdown was more or less a repetition of the first one in terms of restrictions while still being more lenient towards non-necessary businesses initially. For instance, restaurants and bars were only required to close down at 11 pm. However, many Cantons (In particular the french-speaking Cantons, which were the hardest hit by the second wave.) went further on 26th December and decided to completely close those businesses as it was done during the first lockdown.

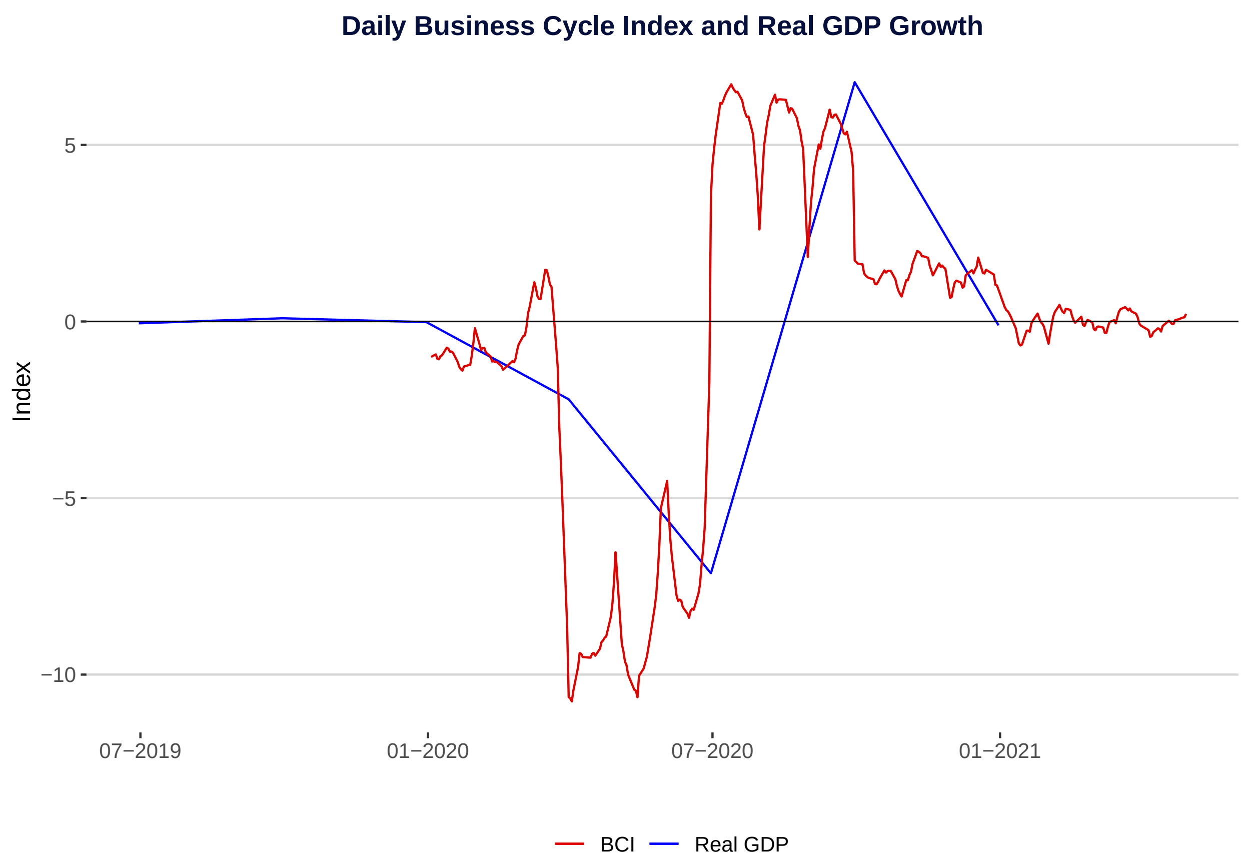
A few points can be raised immediately by looking at the movements of the Business Cycle Index in Figure 10. First, as expected, the Swiss economy crumbled instantly even though for a short period once the state of emergency was announced and enacted by the Swiss government. The decrease in economic activity is undoubtedly linked to the fall in consumption, industrial production, and overall uncertainty. Once the measures were lifted in June, the economy rose sharply, and the business cycle remained positive for the summer period due to the accumulated savings and delayed consumption of previous months. With the reopening of non-necessary businesses as well as the possibility for the tourism industry to make up for their losses of the Spring season, the Business Cycle Index depicts a rather positive outlook for the Swiss economy. After that, however, with the threat of a second wave and new restrictive measures, the economic situation deteriorated again in autumn. The slowdown in economic activity was, however, much less remarkable than during the spring lockdown, possibly due to the positive outlook of a vaccine coming up in 2021 and somewhat milder measures.



Note: The grey areas represent the periods of lockdown measures in Switzerland. The BCI is smoothed with a rolling average of 7 days.

Figure 11: Daily Swiss Business Cycle Index - Comparison

A comparison is made between the BCI and the fever curve developed by Burri and Kaufmann (2020) in Figure 11. As for the monthly index, it appears that the BCI has delayed reaction to economic changes while the fever curve leads the decrease in economic activity. Given the emphasis on financial and news data, such variables may be incorporating more agent's expectations about the future economic situation and therefore are leading variables. Nonetheless, both indices seem to evaluate the depth of the economic downturn equally for Spring 2020. The main difference resides in Summer 2020, where the fever curve remains negative while the BCI rises sharply. Once more, the explanation could be coming from the underlying variables used to estimate the indices. The BCI using daily consumption, which experienced a hike once the measures were lifted, could be primarily influenced by this economic variable while the fever curve only sluggishly rose again as economic agents readjusted their future expectations positively. Finally, both indicators oscillate around 0 for the period covering the second lockdown, which could be indicating that neither economic nor financial variables are displaying any signs of fast recovery for the time being.



Note: All time-series are standardized for comparison purposes.

Figure 12: Daily Swiss Business Cycle Index - Real GDP Growth

Finally, a basic comparison between real GDP growth and the daily Business Cycle Index is made in Figure 12. Overall, the movements of real GDP growth are followed by the BCI which gives weight to the fact that even though the daily BCI does not track economic activity the same way as the fever curve, it may still be relevant to describe the business cycle. Especially, looking at the magnitude and the timing of the decline and expansion of the real GDP, the BCI seems to be relatively well aligned with real GDP growth.

Given the limited amount of data used to estimate the daily Business Cycle Index, such results are encouraging for the future. Nonetheless, this descriptive assessment needs to be complemented with in-sample and out-of-sample assessments once data spanning a more prolonged period are available. Only then, the validity of the daily index can be thoroughly tested.

5 Conclusion

Building on Aruoba, Diebold, and Scotti (2009), I develop a monthly and daily Business Cycle Index for Switzerland, which capture the economy's underlying business conditions with a dataset combining mixed- and high-frequency variables. The dimensionality problem is addressed by implementing the Harvey accumulator and the first principal components. In addition, the Kalman filtering algorithm used is well equipped to deal with a large number of missing observations and allows for the optimal extraction of the latent variable: the Business Cycle Index.

The newly estimated Business Cycle Index goes through a series of tests to assess its nowcasting power for real GDP growth relatively to other similar Swiss indices. The results found are encouraging with the BCI quantitatively beating the SNB and KOF indices in the in-sample evaluation and being significantly more accurate than the KOF index. The out-of-sample assessment in which the BCI had to outperform an AR(1) also showed promising outcomes. Therefore, I was able to demonstrate the possibility of tracking the business cycle with a relatively small amount of variables. Furthermore, the model proposed in this paper is fairly basic, and thus, there is much room for improvements.

So far, considering the variables used in the Business Cycle Index, Switzerland could be considered as a closed economy since the only variable taking into account the rest of the world is exports. However, Switzerland is heavily dependent on the world economy. Hence, it would be of interest to include the world business cycle conditions in the model. To do so, one could do the same exercise as done in this paper and compute a Business Cycle Index for the main trade partners of Switzerland and construct a "World Business Cycle Index" by weighting Swiss trade partners with their importance in trade with Switzerland.

References

[1] K. Abberger, M. Graff, B. Siliverstovs, and J.-E. Sturm. The kof economic barometer, version 2014. KOF Working papers 14-353, KOF Swiss Economic Institute, ETH Zurich, 2014.

[2] F. Altissimo, A. Bassanetti, R. Cristadoro, M. Forni, M. Lippi, L. Reichlin, and G. Veronese. Eurocoin: A real time coincident indicator of the euro area business cycle. *European Economics eJournal*, 2001.

[3] S. B. Aruoba, F. X. Diebold, and C. Scotti. Real-time measurement of business conditions. *Journal of Business Economic Statistics*, 27(4):417–427, 2009.

[4] J. Boivin and S. Ng. Are more data always better for factor analysis? *Journal of Econometrics*, 132(1):169–194, 2006.

[5] A. F. Burns and W. C. Mitchell. Measuring business cycles. *National Bureau of Economic Research*, 1946.

[6] M. Burri and D. Kaufmann. A daily fever curve for the swiss economy. *Swiss Journal of Economics and Statistics*, 156(1):1–11, 2020.

[7] M. Camacho and G. Perez-Quiros. Introducing the euro-sting: Short-term indicator of euro area growth. *Journal of Applied Econometrics*, 25(4):663–694, 2010.

[8] M. Chauvet. An econometric characterization of business cycle dynamics with factor structure and regime switching. *International Economic Review*, 39(4):969–996, 1998.

[9] R. B. Cleveland, W. S. Cleveland, J. E. McRae, and I. Terpenning. Stl: A seasonal-trend decomposition procedure based on loess (with discussion). *Journal of Official Statistics*, 6:3–73, 1990.

[10] F. X. Diebold. Real-time real economic activity: Exiting the great recession and entering the pandemic recession. Working Paper 27482, National Bureau of Economic Research, July 2020.

[11] F. X. Diebold and R. S. Mariano. Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 20(1):134–144, 2002.

[12] F. X. Diebold and G. D. Rudebusch. Measuring business cycles: A modern perspective. *The Review of Economics and Statistics*, 78(1):67–77, 1996.

[13] J. Durbin and S. J. Koopman. *Time Series Analysis by State Space Methods.* Oxford University Press, 2 edition, 2012.

[14] M. D. D. Evans. Where Are We Now? Real-Time Estimates of the Macroeconomy. *International Journal of Central Banking*, 1(2), September 2005.

[15] M. Forni, M. Hallin, M. Lippi, and L. Reichlin. The generalized dynamic-factor model: Identification and estimation. *The Review of Economics and Statistics*, 82(4):540–554, 2000.

[16] A. Galli. Which Indicators Matter? Analyzing the Swiss Business Cycle Using a Large-Scale Mixed-Frequency Dynamic Factor Model. *Journal of Business Cycle Research*, 14(2):179–218, November 2018.

[17] J. D. Hamilton. *Time Series Analysis*. Princeton University Press, 1 edition, Jan. 1994.

[18] A. C. Harvey. *Forecasting, structural time series models, and the Kalman filter / Andrew Harvey*. Cambridge University Press Cambridge ; New York, 1990.

[19] R. Hodrick and E. Prescott. Postwar u.s business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, 29(1):1–16, Feb. 1997.

[20] R. Mariano and Y. Murasawa. A coincident index, common factors, and monthly real gdp. *Oxford Bulletin of Economics and Statistics*, 72:27–46, 02 2010.

[21] R. S. Mariano and Y. Murasawa. A new coincident index of business cycles based on monthly and quarterly series. *Journal of Applied Econometrics*, 18(4):427–443, 2003.

[22] T. Proietti and F. Moauro. Dynamic factor analysis with non-linear temporal aggregation constraints. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 55(2):281–300, 2006.

[23] M. Ravn and H. Uhlig. On adjusting the hodrick-prescott filter for the frequency of observations. *The Review of Economics and Statistics*, 84(2):371–375, 2002.

[24] J. Stock and M. Watson. New indexes of coincident and leading economic indicators. In *NBER Macroeconomics Annual 1989, Volume 4*, pages 351–409. National Bureau of Economic Research, Inc, 1989.

[25] J. Stock and M. W. Watson. *A Probability Model of the Coincident Economic Indicators*, pages 63–90. Cambridge University Press, 1991.

[26] J. H. Stock and M. W. Watson. Macroeconomic forecasting using diffusion indexes. *Journal of Business & Economic Statistics*, 20(2):147–162, 2002.

Appendix

A.1 Kalman Filter and Signal Extraction

The Kalman filter is an algorithm providing estimates of latent variables using observed measurements over time by sequentially updating a linear projection for a dynamic system. It allows the computation of accurate finite-sample forecasts, the exact likelihood function and VAR estimation with parameters that change over time. Based on the observations contained in the vectors y_t and x_t, the components' values of the state vector are estimated through date t.

$$\xi_{t+1|t} \equiv E[\xi_{t+1}|Y_t] \tag{20}$$

where

$$Y_t = (y_t, y_{t-1}, \cdots, y_1, x_t, x_{t-1}, \cdots, x_1)' \tag{21}$$

and $E[\xi_{t+1}|Y_t]$ denotes the linear projections of ξ_{t+1} on Y_t and a constant. The Kalman filter computes these forecasts recursively , generating a serie of state vectors $\xi_{1|0}$, $\xi_{2|1}$, $\cdots$, $\xi_{T|T-1}$. Associated with each of these forecasts is a mean squared error (MSE) matrix, represented by the following $(r \times r)$ matrix :

$$P_{t+1|t} \equiv E[(\xi_{t+1} - \xi_{t+1|t})(\xi_{t+1} - \xi_{t+1|t})'] \tag{22}$$

The recursion starts with $\xi_{1|0}$, which denotes a forecast of ξ_1 based on no observations of y or x. Thus, this is simply the unconditional mean of ξ_1,

$$\xi_{1|0} = E(\xi_1) \tag{23}$$

with the associated MSE :

$$P_{1|0} = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\} \tag{24}$$

Following Hamilton (1994), the subsequent starting values are used to initiate the recursion :

$$\xi_{1|0} = 0$$

$P_{1|0}$ the $(r \times r)$ matrix whose elements are expressed as a column vector is given by :

$$vec(P_{1|0}) = [I_{r^2} - (F \otimes F)]^{-1} vec(Q)$$

Given the starting values $\xi_{1|0}$ and $P_{1|0}$, the next step is to compute $\hat{y}_{1|0}$ and $Var(\hat{y}_{1|0})$.

$$\hat{y}_{1|0} = A'x_1 + H'\xi_{1|0} \tag{25}$$

$$Var(\hat{y}_{1|0}) = V_{y_{1|0}} = H'P_{1|0}H + R \tag{26}$$

For the remainder of the paper, ψ_t represents the surprise between the real value of the process (y_t) and the Kalman filter estimation ($\hat{y}_{t|t-1}$). This surprise could either come from measurement noises w_t or from the movement of $\xi_{t|t-1}$ to $\xi_{t|t}$ during the updating step of the recursion.
K_t represents the gain matrix which will convert the surprise esperance of the movement of ξ_t. For instance, using an extreme case where there is no measurement noises ($w_t = 0$) then the gain matrix K_t will completely allocate the surprise to the movement of ξ_t and thus the filter will be able to retrieve perfectly the path of ξ_t as a result of perfect observations. The next stage is, therefore, the computation of the surprise ψ_1 and the gain matrix K_1.

$$\psi_1 = (y_1 - A'x_1 - H'\xi_{1|0}) \tag{27}$$

$$K_1 = P_{1|0}H(H'P_{1|0}H + R)^{-1} \tag{28}$$

Finally, the estimate of the state vector $\xi_{1|1}$ can be computed with the associated MSE $P_{1|1}$.

$$\xi_{1|1} = \xi_{1|0} + K_1\psi_1 \tag{29}$$

$$P_{1|1} = P_{1|0} - K_1H'P_{1|0} \tag{30}$$

The calculations for t = 2,3,..., T all have the same steps so that it will be described in general terms for step t.

$$\begin{aligned}
&\text{Prediction stage}\\
\xi_{t|t-1} &= F\xi_{t-1|t-1}\\
P_{t|t-1} &= FP_{t-1|t-1}F' + Q\\
\hat{y}_{t|t-1} &= A'x_t + H'\xi_{t|t-1}\\
Var(\hat{y}_{t|t-1}) = V_{y_{t|t-1}} &= H'P_{t|t-1}H + R\\
&\text{Updating stage}\\
\psi_t &= (y_t - A'x_t - H'\xi_{t|t-1})\\
K_t &= P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}\\
\xi_{t|t} &= \xi_{t|t-1} + K_t\psi_t\\
P_{t|t} &= P_{t|t-1} - K_tH'P_{t|t-1}
\end{aligned}$$

Given the significant number of missing observations in the dataset, it is crucial to consider it in the recursion. Following Aruoba, Diebold, and Scotti (2009), when there are no observations in y_t, updating does not take place, and thus the algorithm becomes :

$$\xi_{t|t} = F\xi_{t-1|t-1} \tag{31}$$

$$P_{t|t} = FP_{t-1|t-1}F' + Q \tag{32}$$

Furthermore, let's assume that only a number n^* $(n > n^* > 0)$ of observations are available in y_t. Then, the measurement equation needs to be adapted as follows :

$$y_t^* = A'x_t^* + H^{*\prime}\xi_t + w_t^* \tag{33}$$

Where y_t^*, x_t^*, H^* and w_t^* are matrices and vectors with missing rows or columns corresponding to the missing observations. Thus, the transformation made in equation 24 corresponds to creating a matrix Γ_t which contains the n^* rows of I_n associated with those of y_t with an observed value such that :

$$\begin{aligned} y_t^* &= \Gamma_t y_t \\ x_t^* &= \Gamma_t x_t \\ H^* &= \Gamma_t H \\ w_t^* &= \Gamma_t w_t \\ R_t^* &= \Gamma_t R_t \Gamma_t' \end{aligned}$$

By performing those transformations, only the non-missing entries of y_t are kept and used in the updating stage of the algorithm. In the implementation, the three matrices in need of corrections due to missing observations are R_t $(n \times n)$, ψ_t $(n \times 1)$ and H_t $(r \times n)$. For instance, let's assume that at time t, the observation for the second observed measurement is missing $(n^* = n - 1)$. Then, using the transformation described above, the corresponding rows and columns of R_t, ψ_t and H_t are removed. Hence, the new dimensions of the aforementioned matrices are : R_t $(n-1 \times n-1)$, ψ_t $(n-1 \times 1)$ and H_t $(r \times n-1)$. The procedure is analog when dealing with more than one missing observation.

The value of ξ_t is of interest given its structural interpretation: the Business Cycle Index. Therefore, the Kalman smoothing algorithm is used to obtain a better inference of the value of ξ_t based on the full set of data collected $\mathcal{Y}_T = (y_t, y_{t+1}, ..., y_T, x_t, x_{t+1}, ..., x_T)'$.
Such inference is called the "smoothed" estimate of ξ_t written as :

$$\xi_{t|T} \equiv E[\xi_t|\mathcal{Y}_T] \tag{34}$$

with MSE :

$$P_{t|T} \equiv E[(\xi_t - \xi_{t|T})(\xi_t - \xi_{t|T})'] \tag{35}$$

Once again, the methodology proposed by Hamilton (1994) is used to compute the smoothed estimates. As in any recursion, a starting value is needed, and for this purpose the Kalman filter is run first in order to acquire the sequences $\{\xi_{t|t}\}_{t=1}^{T}$, $\{\xi_{t+1|t}\}_{t=0}^{T-1}$, $\{P_{t|t}\}_{t=1}^{T}$ and $\{P_{t+1|t}\}_{t=0}^{T-1}$. Then, as a starting value for the algorithm, $\xi_{T|T}$ as the last entry in $\{\xi_{t|t}\}_{t=1}^{T}$ is chosen. The next stage consists of computing the sequence $\{J_t\}_{t=1}^{T-1}$ using :

$$J_t = P_{t|t} F' P_{t+1|t}^{-1} \tag{36}$$

Finally, the smoothed estimates are computed as follows :

$$\xi_{t|T} = \xi_{t|t} + J_t(\xi_{t+1|T} - \xi_{t+1|t}) \tag{37}$$

$$P_{t|T} = P_{t|t} + J_t(P_{t+1|T} - P_{t+1|t})J_t' \tag{38}$$

Proceeding backward through the sample in this fashion allows for the calculation of the complete set of smoothed estimates : $\{\xi_{t|T}\}_{t=1}^{T}$.

A.2 Maximum Likelihood Estimation

Thus far, it was assumed that the different matrices of parameters were known, which is evidently not appropriate for the purpose of this paper. Thankfully, the Kalman filter is perfectly equipped to address this issue. As a matter of fact, the computation of the log-likelihood is quite straightforward given the multivariate Gaussian density function :

$$f_Y(y_{1_i}, \cdots, y_{n_i}) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(y_i-\mu)'\Sigma^{-1}(y_i-\mu)}$$

$$\ln(\prod_{i=1}^{T} f_Y(y_{1_i}, \cdots, y_{n_i})) = -\frac{Tn}{2}\ln(2\pi) - \frac{T}{2}\ln(|\Sigma|) - \frac{1}{2}\sum_{i=1}^{T}(y_i-\mu)'\Sigma^{-1}(y_i-\mu)$$

Then, the log-likelihood can be computed as (taking into consideration that n may not be constant across periods because of missing observations) :

$$\ln(\prod_{i=1}^{T} f_{Y_t|X_t\mathcal{Y}_{t-1}}(y_t|x_t, \mathcal{Y}_{t-1})) = -\frac{1}{2}\sum_{i=1}^{T}(n_i * \ln(2\pi) + \ln(\det(H'P_{t|t-1}H + R))$$
$$+ (y_t - A'x_t - H'\hat{\xi}_{t|t-1})'(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1})$$

Or using previous notations :

$$\ln(\prod_{i=1}^{T} f_{Y_t|X_t\mathcal{Y}_{t-1}}(y_t|x_t,\mathcal{Y}_{t-1})) = -\frac{1}{2}\sum_{i=1}^{T} n_i \ln(2\pi) + \ln(\det(V_{iy_{t|t-1}})) + \psi_t'(V_{iy_{t|t-1}})^{-1}\psi_t \quad (39)$$

Note that if all observations of y_t are missing then the contribution of period t to the log-likelihood will be equal to zero.

A.3 Tables and Figures

Table T1: Data

Variables	**Frequency**	**Period**	**Delay**	**Source**
Real GDP	Quarterly	1980-2021	9 weeks	SECO
Consumer Confidence Index	Quarterly	1972-2021	4 weeks	SECO
Employment	Quarterly	1991-2021	8 weeks	SFSO
Vacancies	Quarterly	1992-2021	8 weeks	SFSO
Unemployment	Monthly	1975-2021	1 weeks	SECO
Retail Sales	Monthly	2000-2021	4 weeks	SFSO
Exports	Monthly	1997-2021	3 weeks	AFD
Industrial Production	Monthly	2010-2021	7 weeks	SFSO
Term Premium	Daily	1988-2021	1 day	SNB
Liquidity Premium	Daily	2001-2021	1 day	SNB
Daily Consumption Switzerland, SIX, Worldline	Daily	2019-2021	1 week	Monitoring Consumption

Note: All delays are only approximate.

Table T2: In-Sample Assessments

		Ratio	Test 1	Test 2
Variables	**Index**	η	**p-value (DM test)**	**p-value (DM test)**
Consumer Confidence Index	SNB	1.042	0.71	0.65
	KOF	1.013	0.91	0.55
Employment Growth	SNB	0.846	0.15	0.07*
	KOF	0.849	0.05*	0.02*
Vacancies Growth	SNB	1.084	0.19	0.91
	KOF	0.971	0.64	0.32
Industrial Production Growth	SNB	0.986	0.89	0.44
	KOF	0.997	0.89	0.45
Retail Sales Growth	SNB	1.008	0.15	0.93
	KOF	1	0.71	0.84
Exports Growth	SNB	1.004	0.77	0.38
	KOF	0.967	0.44	0.22
Unemployment Growth	SNB	0.925	0.63	0.32
	KOF	0.998	0.92	0.46

Table T3: Out-of-Sample Assessments

		Ratio	Test 1	Test 2
Variables	**Index**	η	**p-value (DM test)**	**p-value (DM test)**
Consumer Confidence Index	AR(1)	1.556	0.099	0.95
Employment Growth	AR(1)	1.245	0.082	0.96
Vacancies Growth	AR(1)	1.077	0.73	0.63
Industrial Production Growth	AR(1)	1.058	0.278	0.14
Retail Sales Growth	AR(1)	0.859	0.164	0.08*
Exports Growth	AR(1)	1.045	0.409	0.2
Unemployment Growth	AR(1)	0.874	0.021*	0.99

Table T4: Daily Swiss Business Cycle Coefficients

Variable	ϕ	ρ	λ	**c**	σ^2
Quarterly Stock (1)	0.763	0.27	0.101	0.01	1.242
Monthly Stock (2)	0.763	0.605	-0.477	-0.004	0.203
Monthly Flow (3)	0.763	-0.349	0.037	0.004	1.125
Quarterly Flow (4)	0.763	-0.175	0.156	-0.001	0.924
Daily (5)	0.763	0.746	-0.123	-0.007	0.38

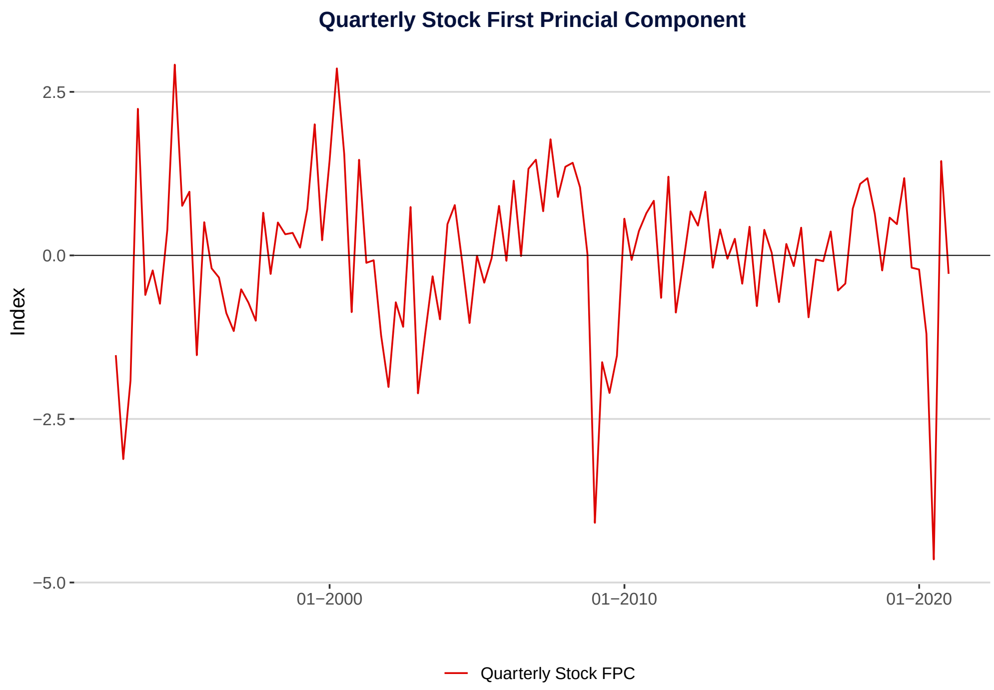


Figure F1: Quarterly Stock First Principal Component

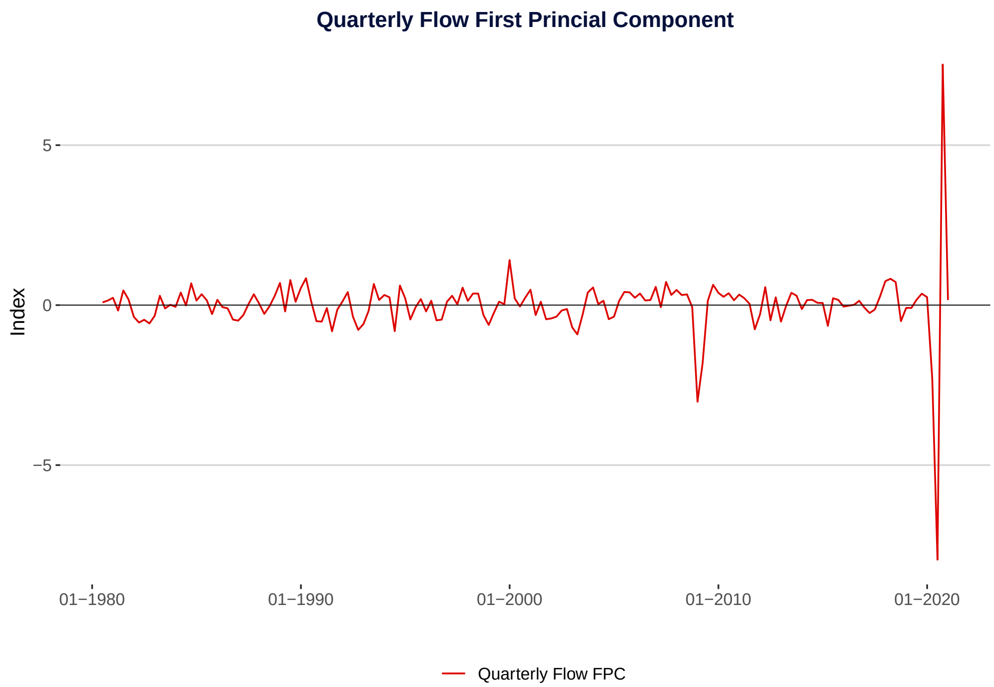


Figure F2: Quarterly Flow First Principal Component

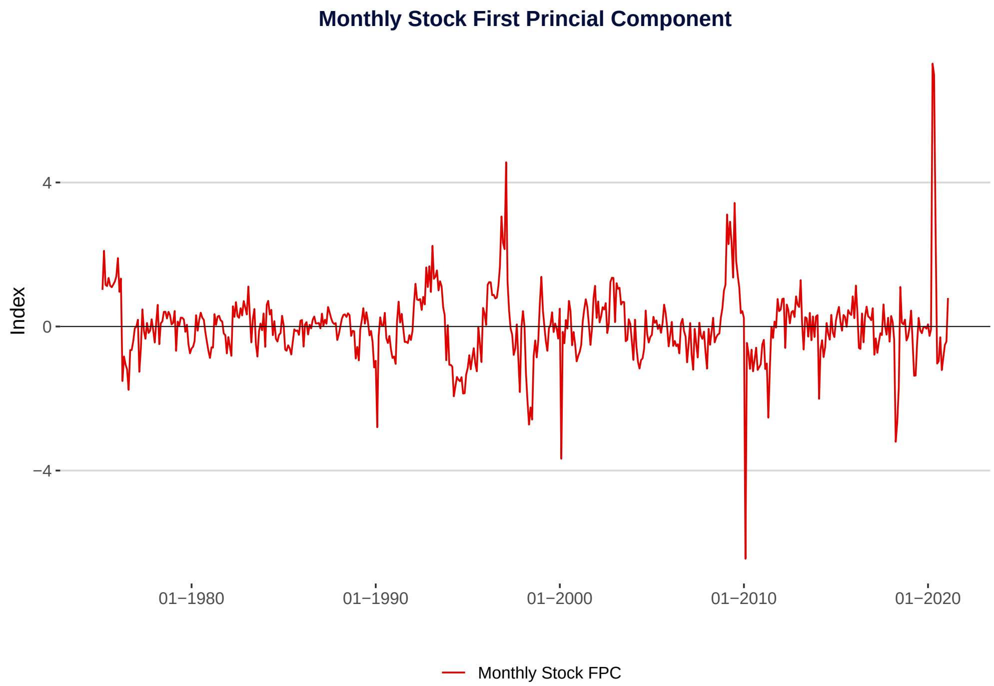


Figure F3: Monthly Stock First Principal Component

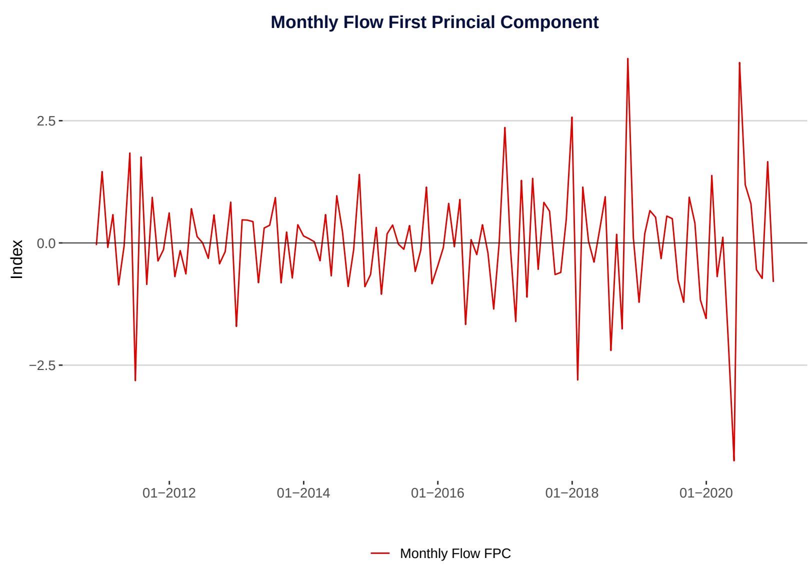


Figure F4: Monthly Flow First Principal Component

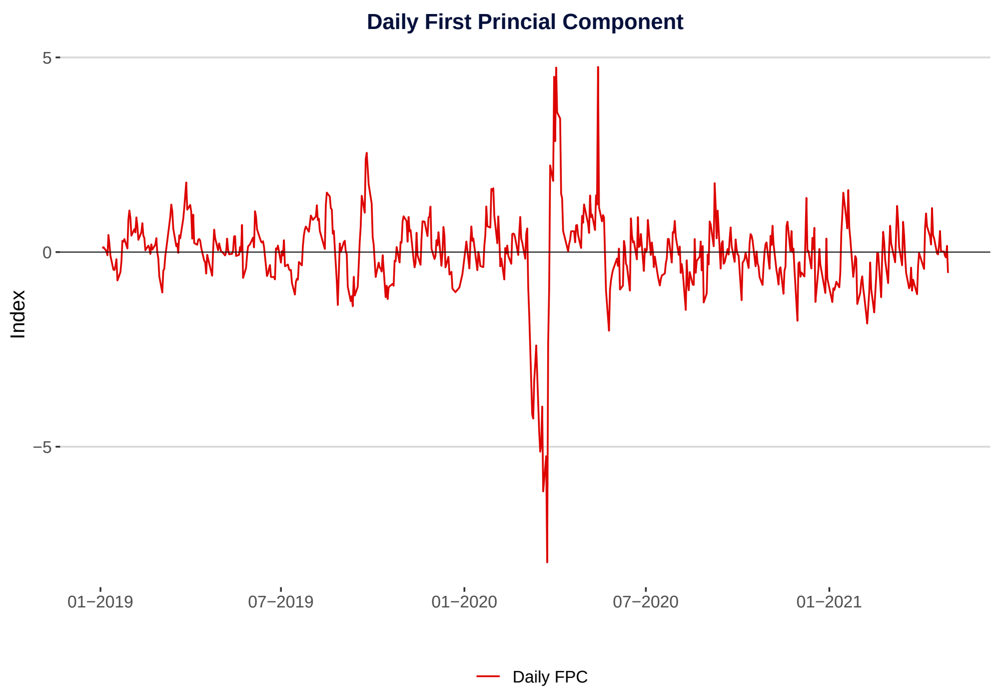


Figure F5: Daily First Principal Component

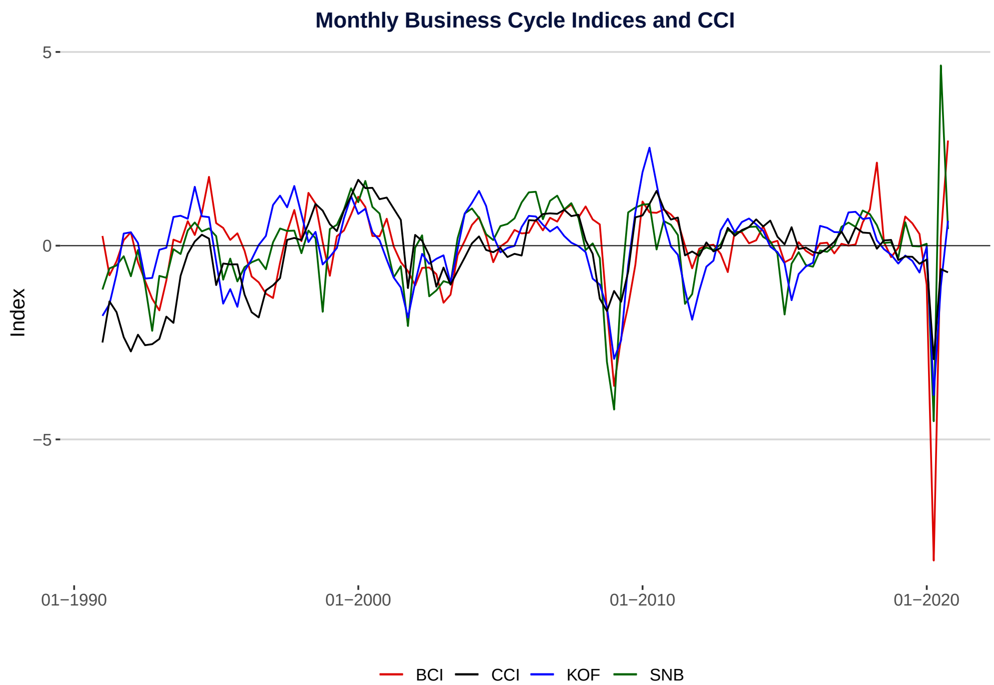


Figure F6: Indices & CCI

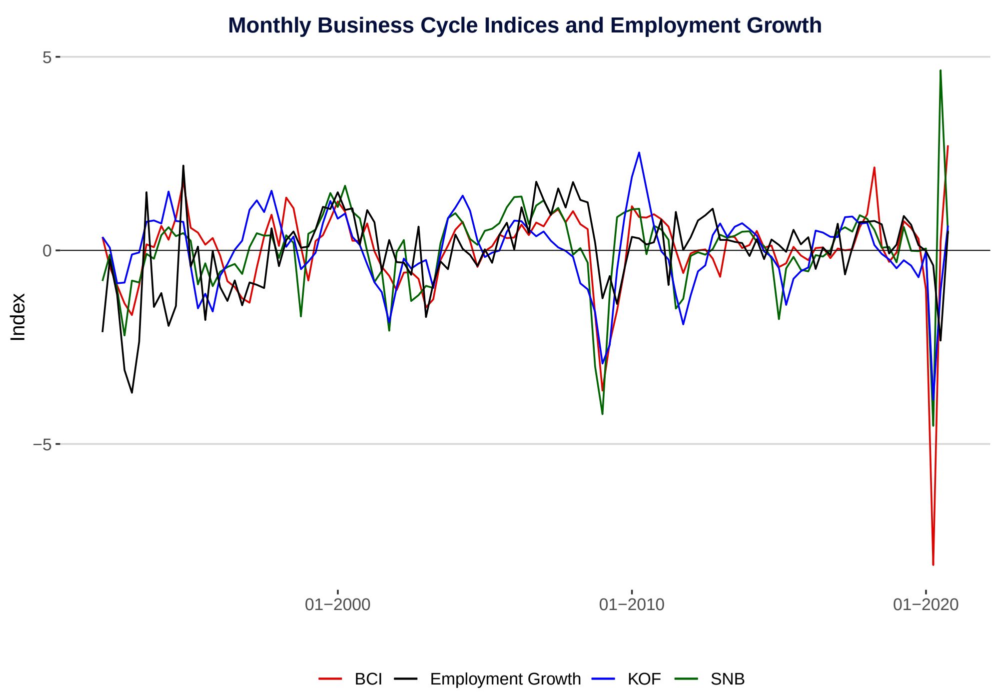


Figure F7: Indices & Employment Growth

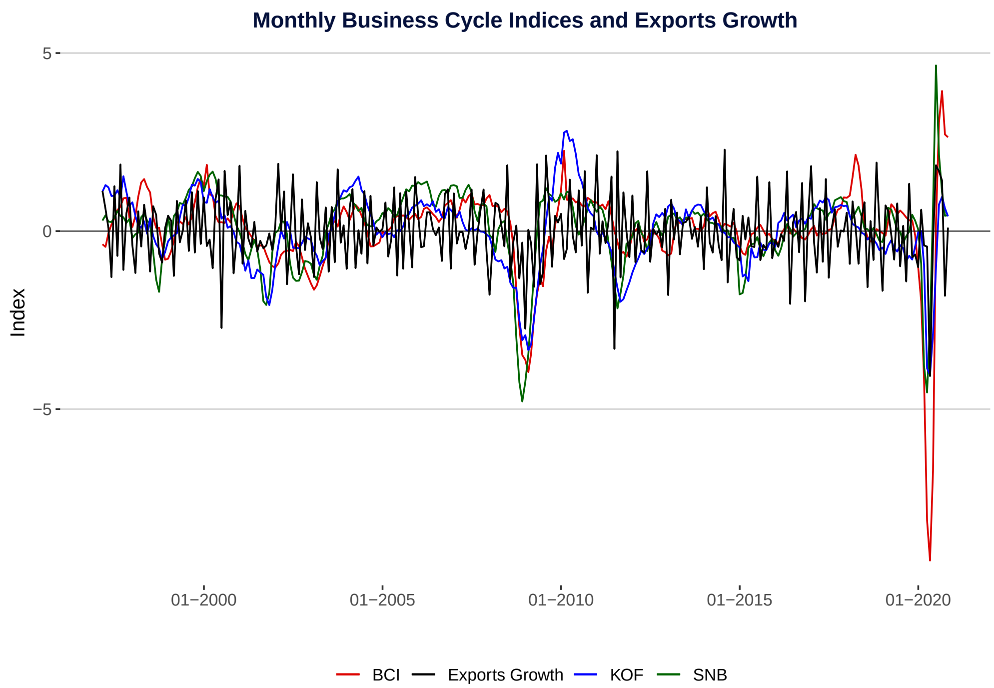


Figure F8: Indices & Exports Growth

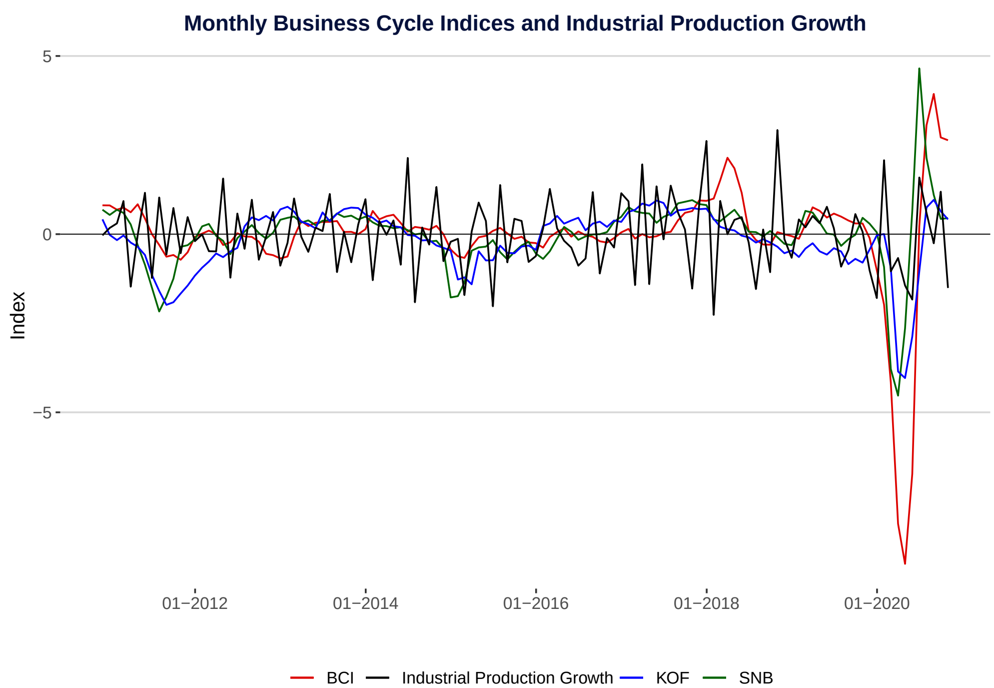


Figure F9: Indices & Industrial Production Growth

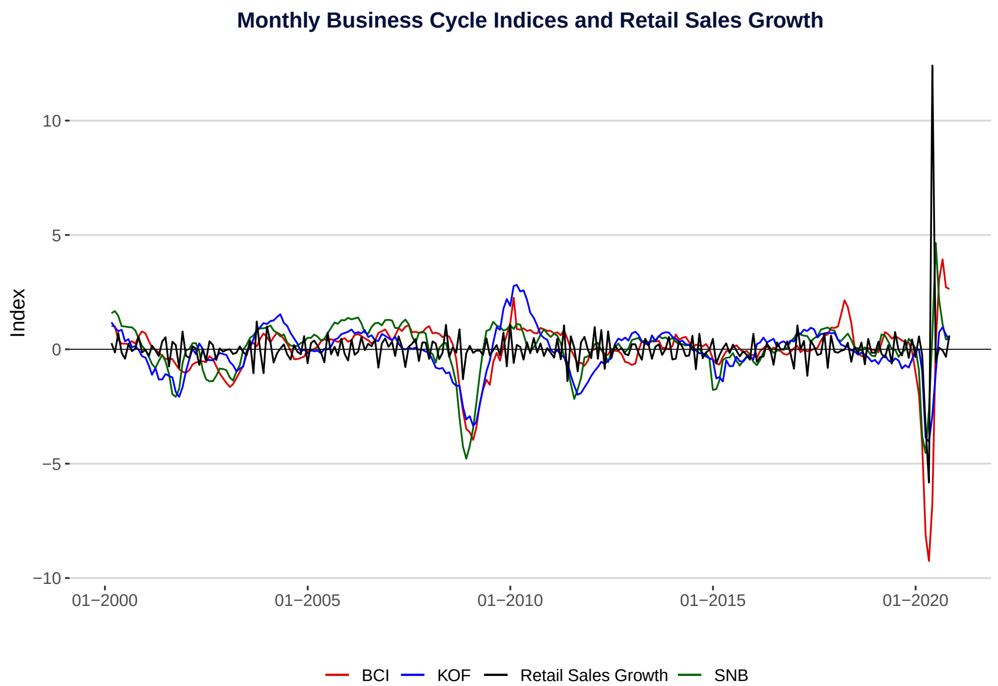


Figure F10: Indices & Retail Sales Growth

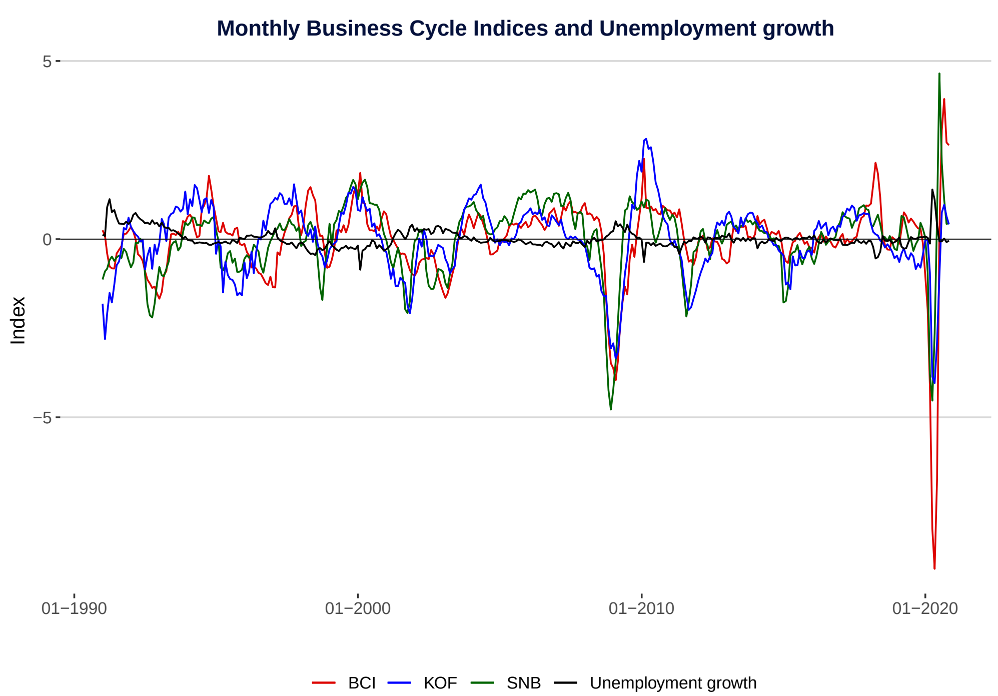


Figure F11: Indices & Unemployment Growth

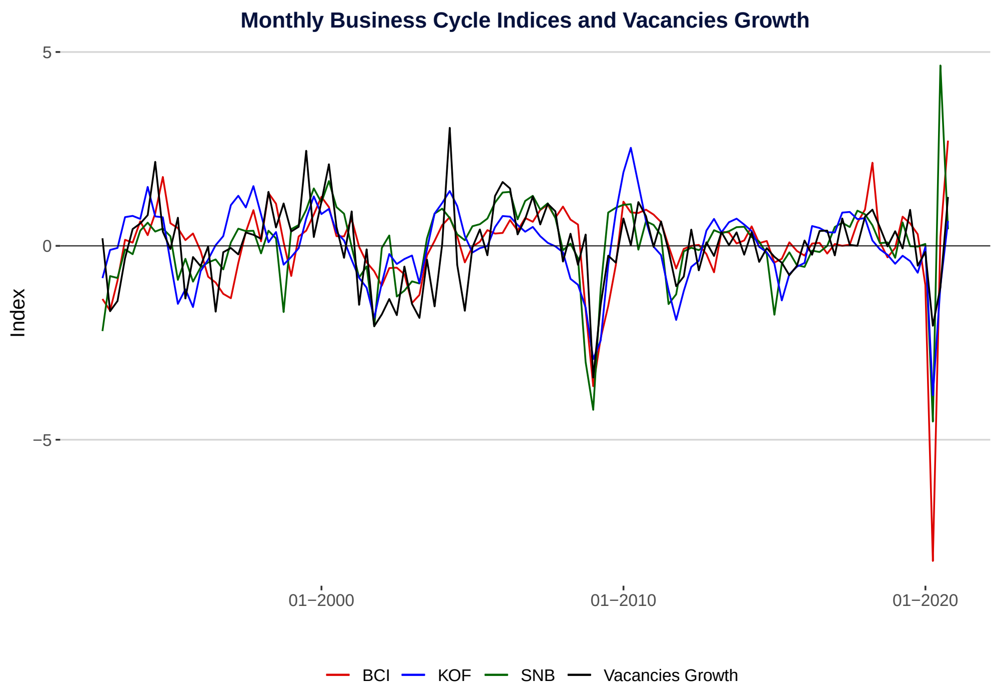


Figure F12: Indices & Vacancies Growth

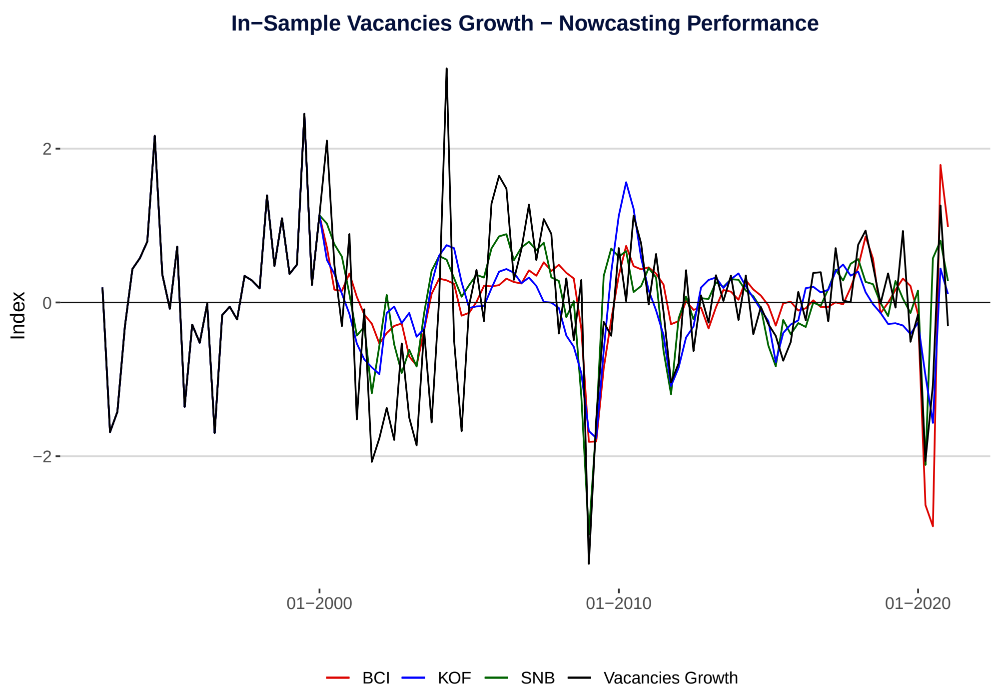


Figure F13: In-Sample - Vacancies Growth Nowcasting Performance

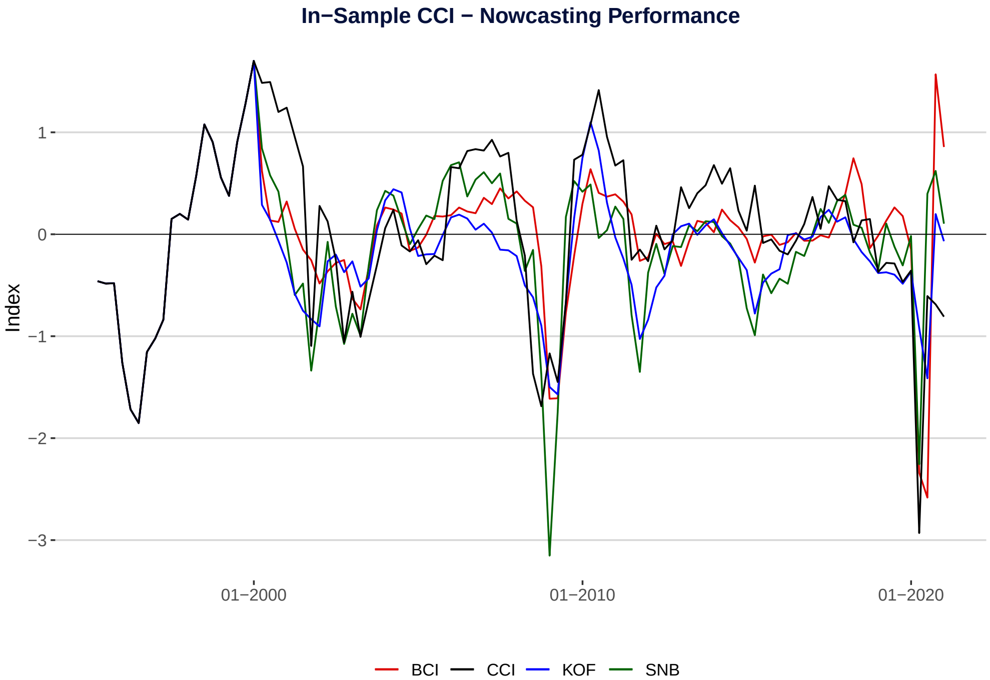


Figure F14: In-Sample - CCI Nowcasting Performance

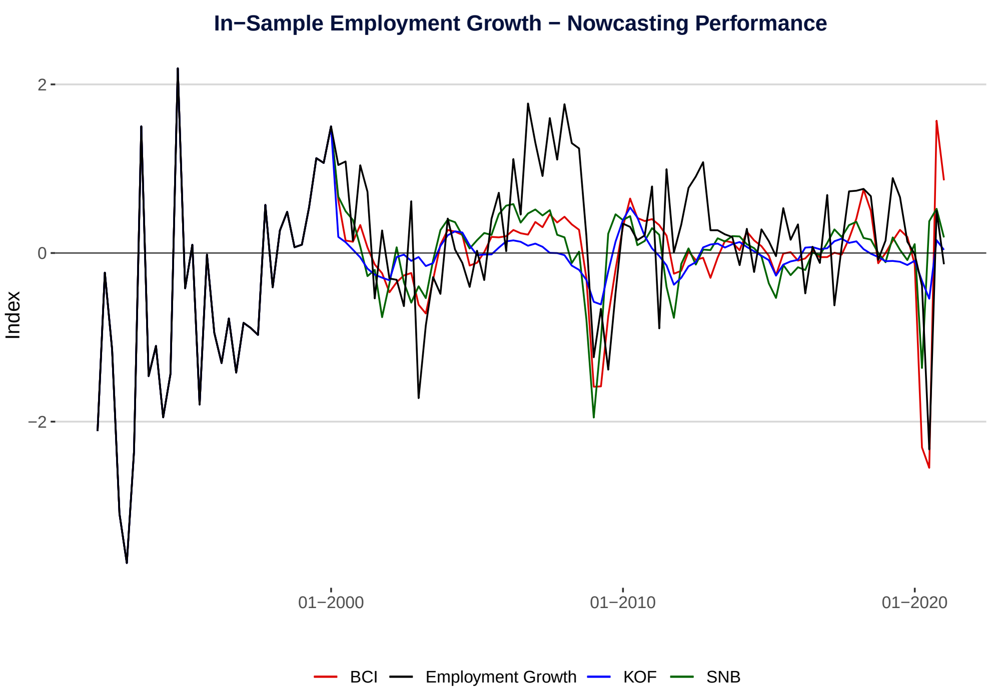


Figure F15: In-Sample - Employment Growth Nowcasting Performance

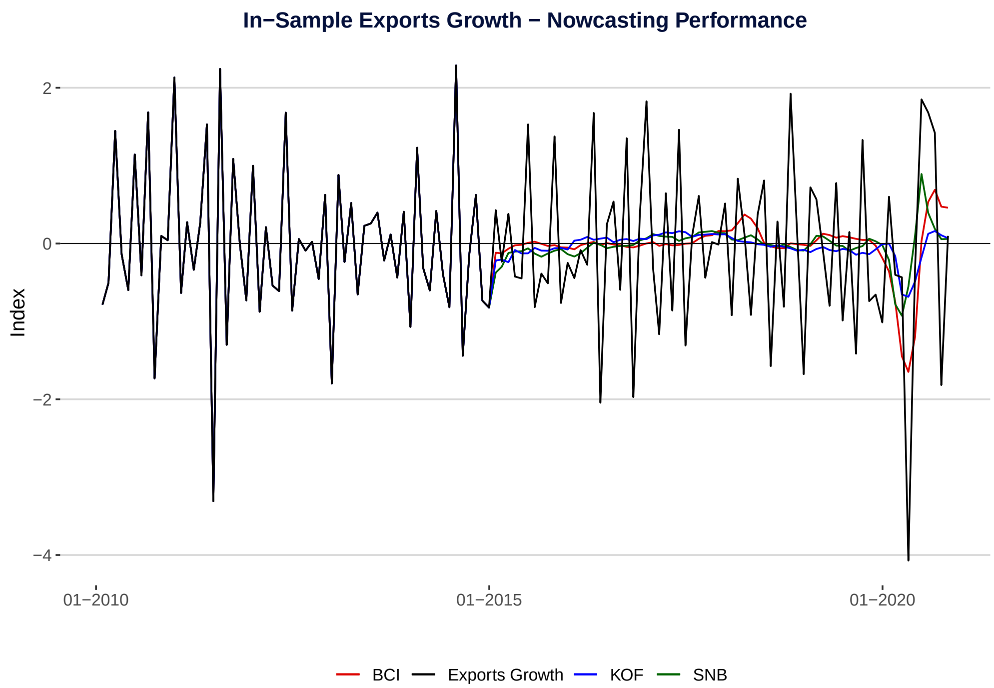


Figure F16: In-Sample - Exports Growth Nowcasting Performance

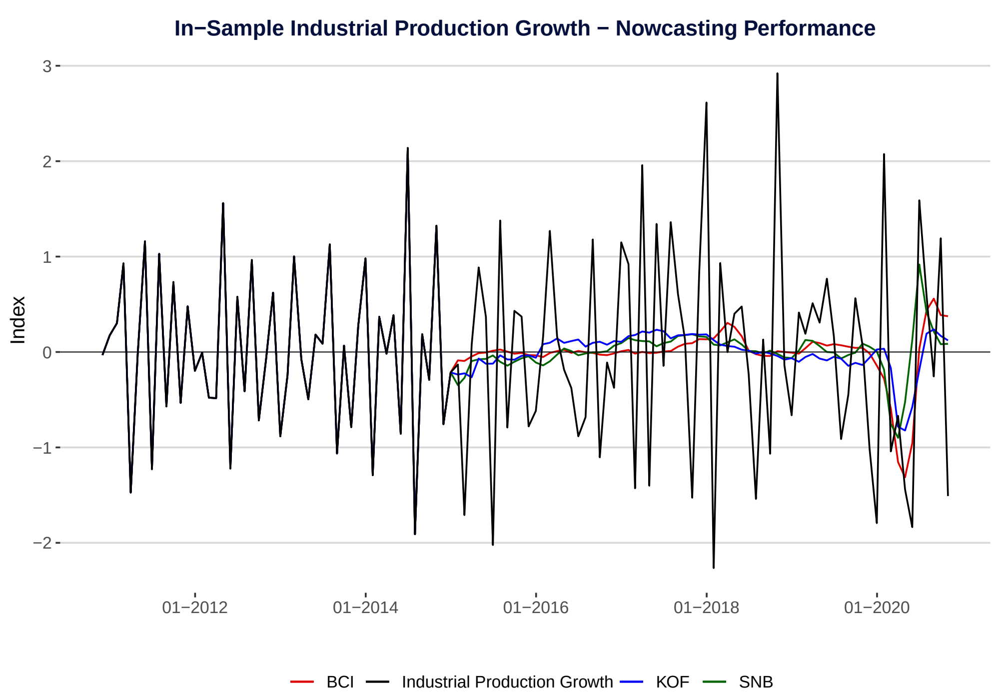


Figure F17: In-Sample - Industrial Production Growth Nowcasting Performance

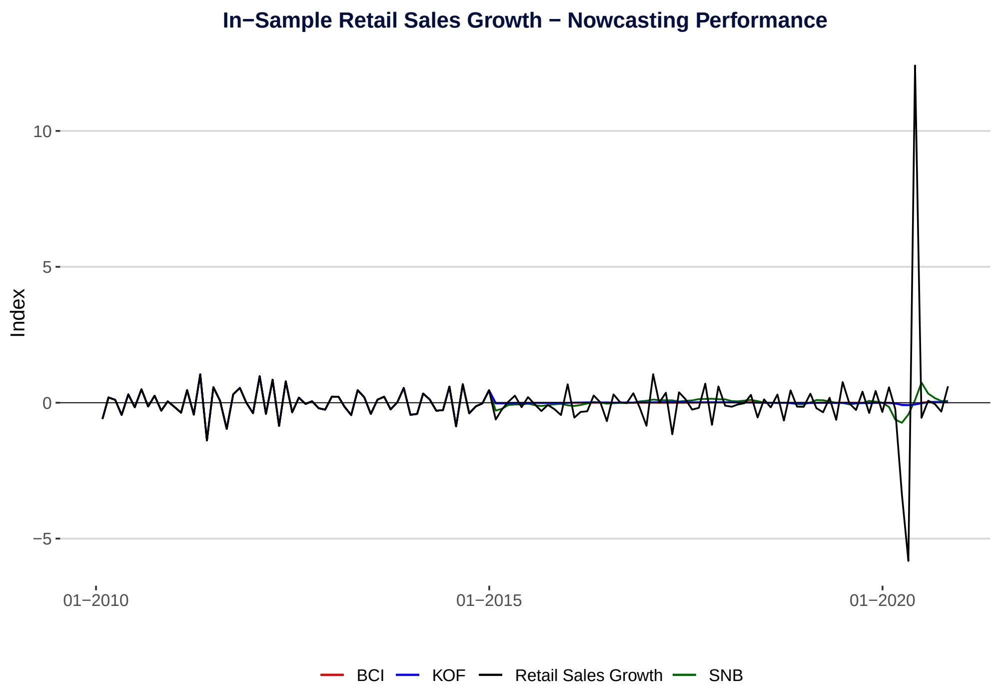


Figure F18: In-Sample - Retail Sales Growth Nowcasting Performance

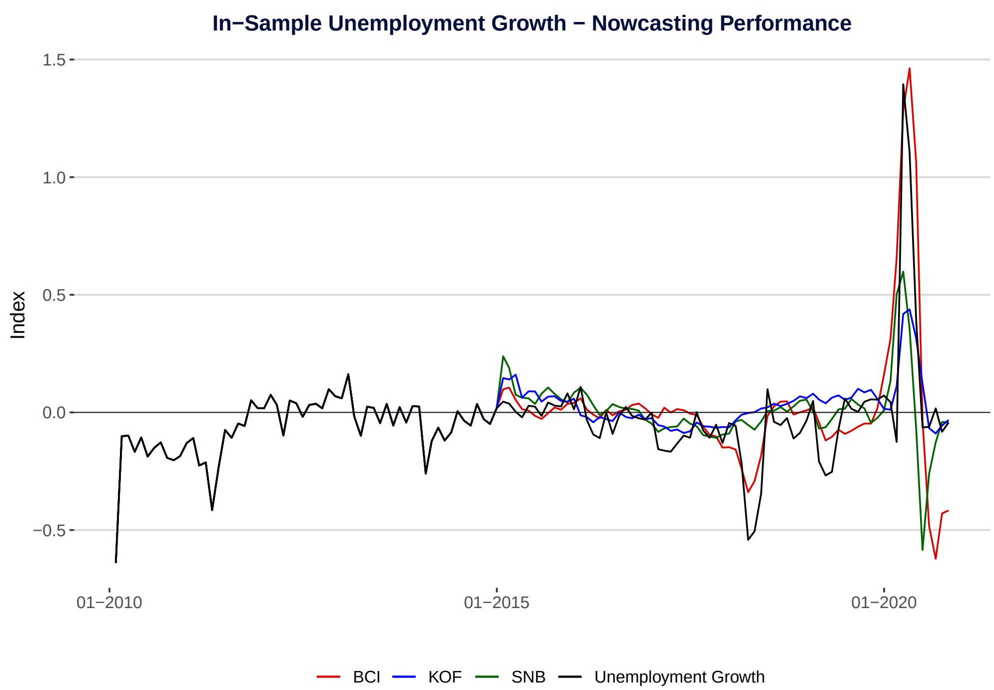


Figure F19: In-Sample - Unemployment Growth Nowcasting Performance

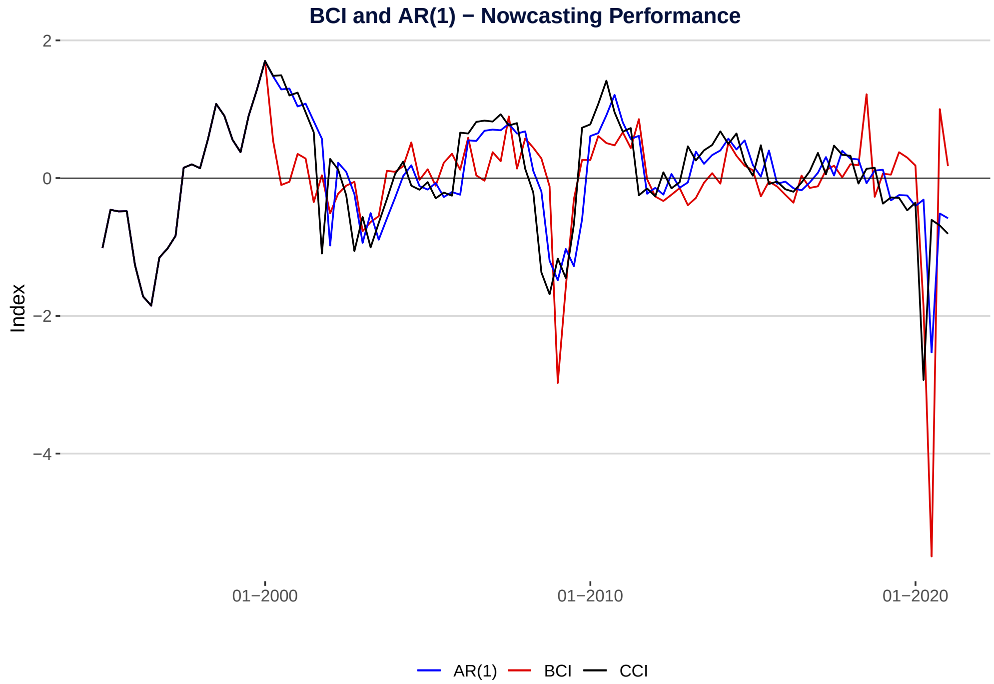


Figure F20: Out-of-Sample - CCI Nowcasting Performance

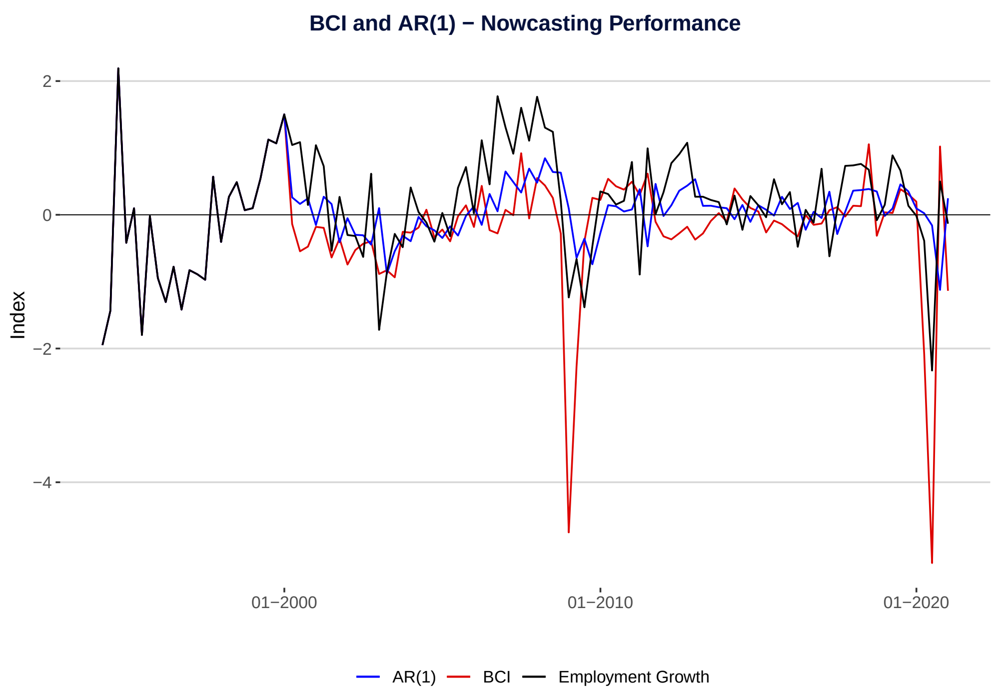


Figure F21: Out-of-Sample - Employment Growth Nowcasting Performance

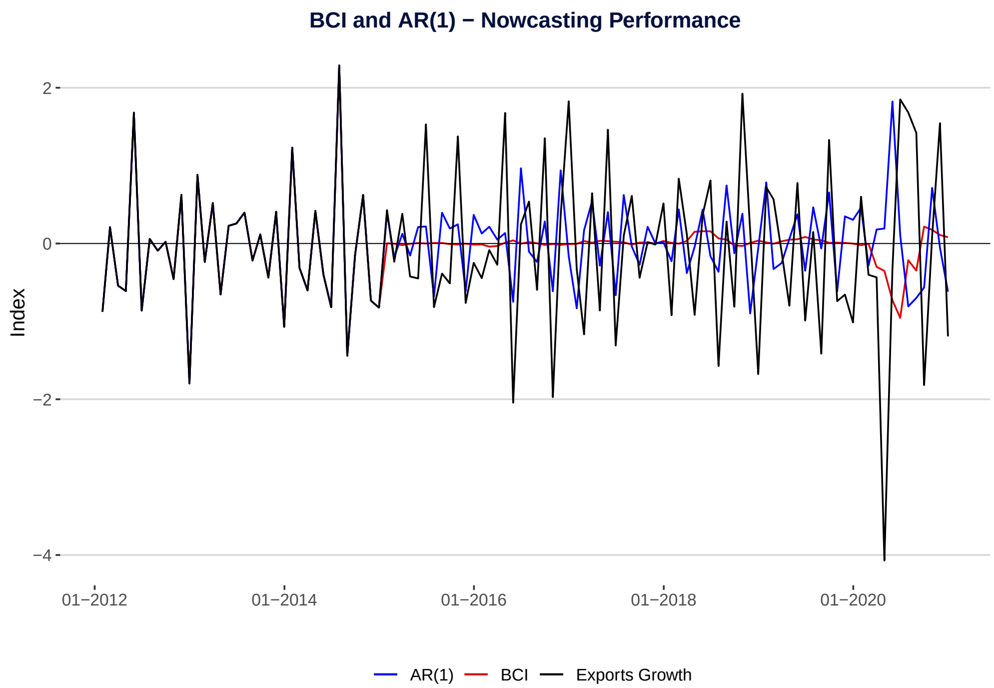


Figure F22: Out-of-Sample - Exports Growth Nowcasting Performance

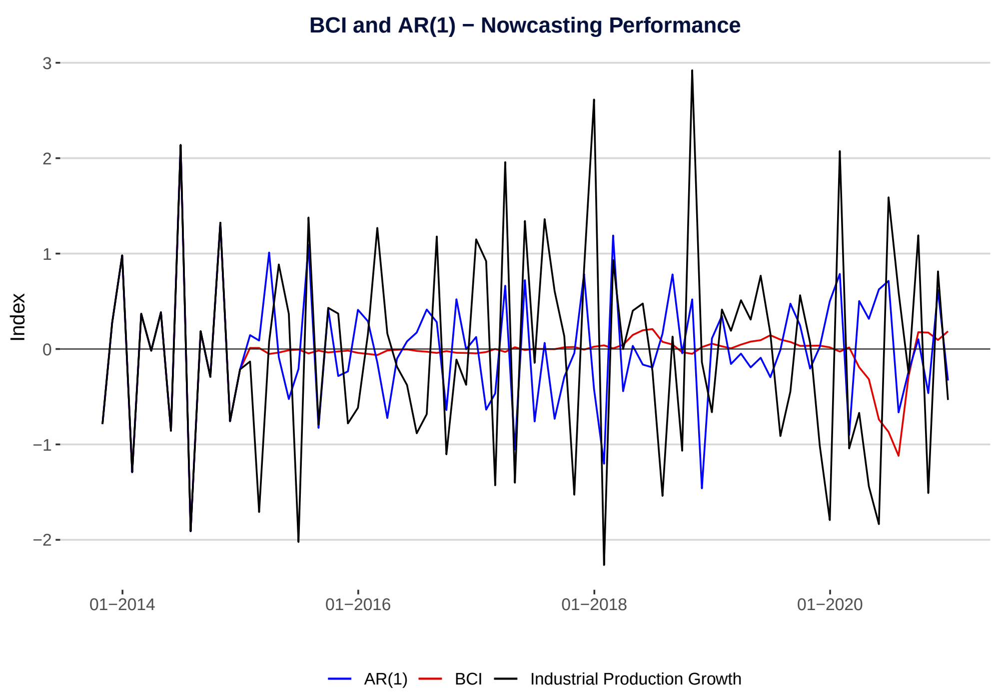


Figure F23: Out-of-Sample - Industrial Production Growth Nowcasting Performance

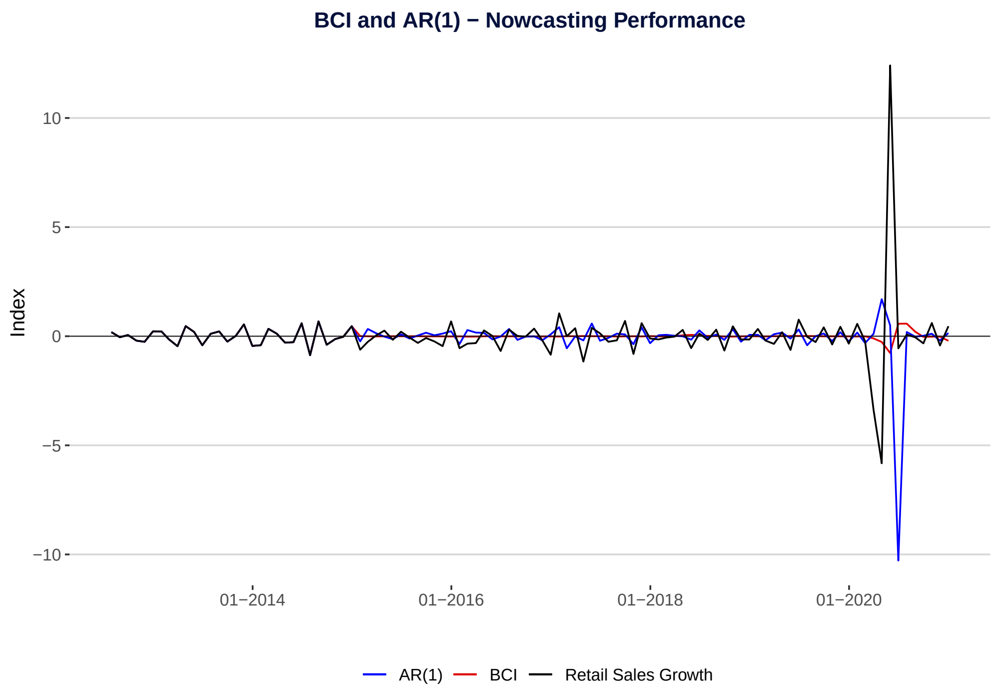


Figure F24: Out-of-Sample - Retail Sales Growth Nowcasting Performance

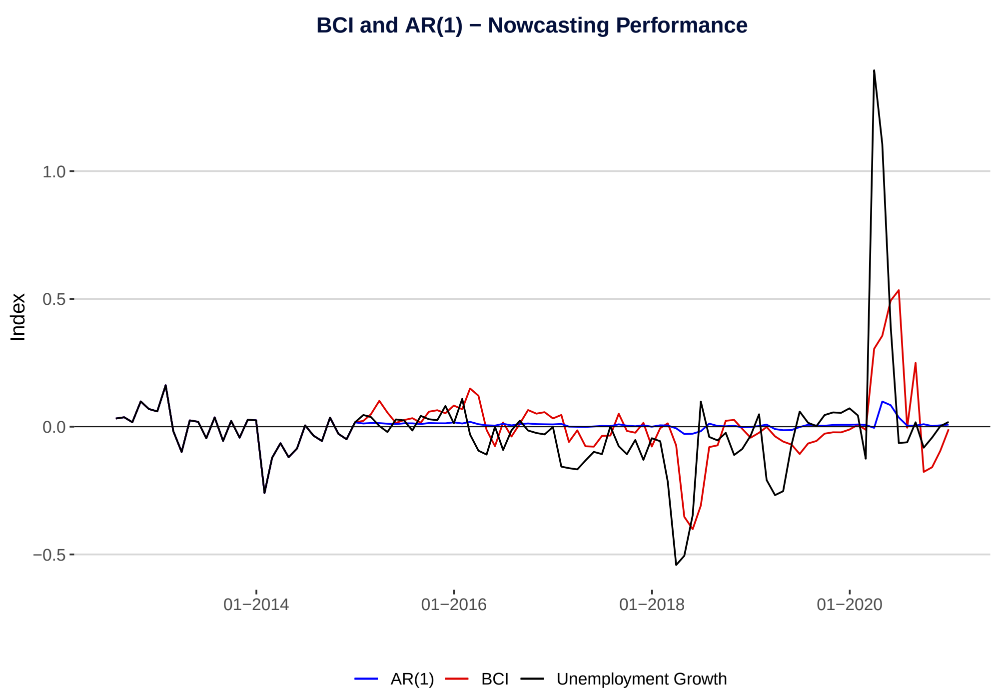


Figure F25: Out-of-Sample - Unemployment Growth Nowcasting Performance

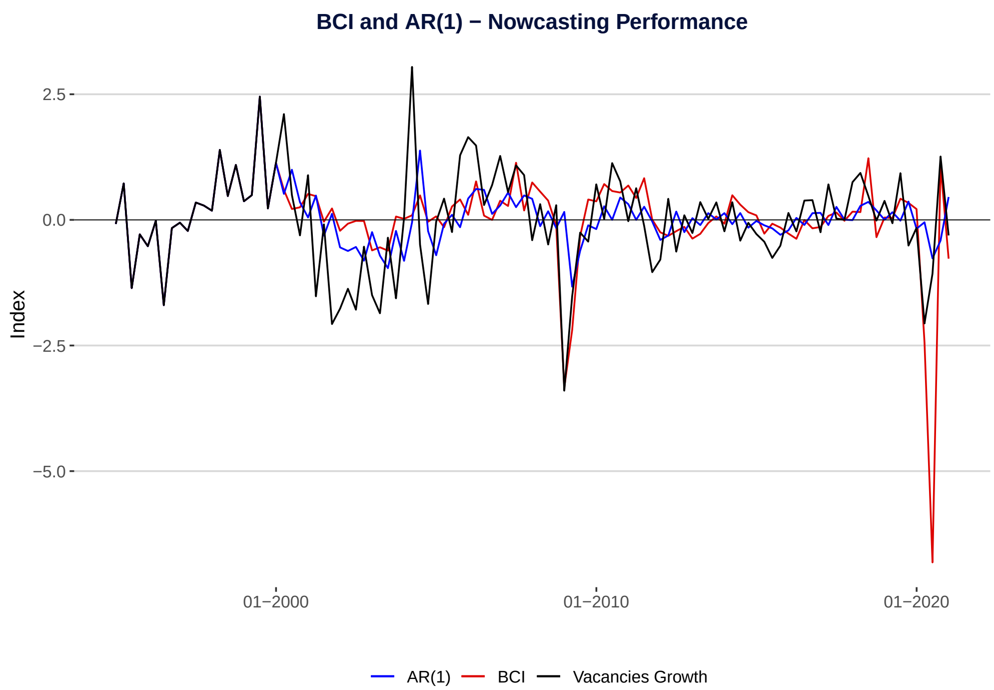


Figure F26: Out-of-Sample - Vacancies Growth Nowcasting Performance