Remarks on particles, fields and waves

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'I have a problem. I am an intellectual,
but at the same time I am not very clever'
(from *The Secret Diary of Adrian Mole, aged* $13\frac{3}{4}$, by Sue Townsend)

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Lausanne, 1 May 2014

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Introduction

We expect the world picture of quantum theory to be a mix of that for classical particles (due to the quantum particle concept) with that for classical continua and classical fields (due to the wave-function).

Much of the expected mix *does* hold good, despite ongoing debate about the mystery of quantization, the measurement problem, non-locality.

I aim to make the philosophy more precise by tying it to the formalism. Examples will include:

i) haecceitism as: permutations of particle labels change the physical state;

ii) counterfactual conditionals in terms of the state-space;

iii) whether forces are additional to matter and motion, as: forces do not supervene on the state of matter;

iv) disambiguating ‘intrinsic’ as: no implications of facts about elsewhere; or at least, none apart from what continuity of a field requires.

v) connecting Lewis’ conception of law as best (simplest-strongest) description of particular facts with Newton’s deduction from the phenomena, and his *hypotheses non fingo*.

I like to think that the philosophical aspects of the various theories have heuristic value for physics. And it is appropriate to discuss the ontology of an emergent theory. This is so, even if you accept the theory is sick. For the effective ontology you see in it may well survive the demise of orthodox quantum theory. But ... two philosophical warnings!

Philosophical categories and distinctions are themselves questionable e.g. intrinsic vs. extrinsic, dispositional vs. categorical.

Maybe the quantum measurement problem is not ripe for solution. So how seriously should we take metaphysical suggestions coming from rivals to quantum theory, aimed at solving that problem?

The philosophical themes will be:

1. Identity of matter: haecceitism, persistence over time
2. Mass and other quantities: essential? intrinsic?
3. Force: determined by the state of matter?
4. Composition: in kinematics and dynamics
5. Modality: in terms of state spaces
6. Law: does the minimal covering-law conception fit?

and how should causation be understood?

and can laws, e.g. forces be derived from the motions of matter?

Classical particles

N point particles in $\mathbb{R}^3$, under say Newtonian gravity; a force-formula in terms of masses and distances; action-at-a-distance!

Three salient state spaces: configuration space $\mathbb{R}^{3N}$; the Lagrangian and Hamiltonian state spaces.

The force-formula defines an acceleration vector field on $\mathbb{R}^3$: how a test-particle would accelerate.

Conundrums include: What about two point particles coinciding? Colliding? What about their infinite density? What about action-at-a-distance ?

Philosophical themes

1. Identity

(i) Haecceitism in that a permutation of particles gives a different configuration (even if the masses are same).

(ii) Identity over time: use trajectories as individuators.

2. Mass: It is normal to construe the property of mass as:
i) essential (in another configuration this particle would have same mass)
ii) intrinsic (ie no implications of facts about elsewhere);
iii) not individuating, since could be shared with another particle;
iv) quidditist: an "electrostatic world" with same spatial configuration, but of massless positively charged particles, attracting each other by $F = ca$, would be a different world.

3. Force:
Force contingently supervenes on matter in that it is is an explicit function of the dynamical variables, say $q, \dot{q}$ (usually just q). This underpins treating the Lagrangian and Hamiltonian as scalars.

4. Composition

(i) kinematics: by cartesian product. Non-holism in that state $s \in \mathbb{R}^{3N}$ (or Lagrangian or Hamiltonian phase space) is fully defined by its projections in the particle factor state-spaces.

(ii) dynamics: a potential energy $V(q_1 - q_2)$ is a property of the whole; but non-holism in that two-body forces suffice. No need for configurational forces *a la* Broad. And the composition of forces is easy viz. by vector addition.

5. Modality:

Modal involvement due to state-spaces!

We usually fix the state-space and the forces (L or H) and consider only variation in the initial conditions.

Counterfactuals in terms of nearest possible worlds can be made precise, naturally enough, in terms of Euclidean distance in each factor space $\mathbb{R}^3$ or $\mathbb{R}^6$.

Roughly: 'if q_i were different, all the other coordinates q_j ($j \neq i$) and all the p_j would be as they actually are'.

6.Law etc

(i) The minimal Humean covering-law conception is:
Newton's laws and the force-formula describe and so explain phenomena. There is no need to ask for how one mass attracts another, or does so at a distance, or responds to force from another, or why force vectors add.

Cf. Hacking (*Emergence of Probability*): 'the causes are the signs'.

(ii) Causation?
The action-at-a-distance means there are no hyperbolic PDEs with which to analyse the propagation of disturbance. So the causation seems only:

a) The global determinism of Laplace, with the cause = a prior universal state.

But if you hanker after more, you could . . .

b) Appeal to counterfactuals, as in: 'If the Sun were placed differently ...'. But: Is this more than a verbal expression of the functional dependence seen in the force-formula?

(iii) Philosophers naturally ask about the prospects for a force-formula being best (simplest-strongest) description of motions, *a la* Lewis' views (with motions of particles being part of the mosaic for Humean supervenience).

Under some restrictions (e.g. two-body forces count as simpler; actual configurations suitably non-symmetric, and one sees a sufficient variety of them), this is promising project.

Cf. with Newton's deduction from the phenomena, and his *hypotheses non fingo*. This echoes the minimal conception in (i) above.

Classical continua

1. Infinitely many infinitesimal "elements" in $\mathbb{R}^3$, obeying, say, Euler's or Navier-Stokes equation.

2. The dynamical variables include more than $q, \dot{q}, p$, and mass/density: viz. their kin (pressure, shear, strain, stress, viscosity .. .) ; and also their not-so-kin e.g. temperature.

3. Infinite dimensional state spaces:
E.g. For an incompressible fluid in a container C, $\mathcal{Q} :=$ Diffm(C).
NB: This state-space has no natural factorization as the Cartesian product of state-spaces of infinitesimal elements.
An element is instead specified by e.g. (i) its location in the fiducial configuration given by $id \in$ Diffm(C), and (ii) where it is placed by all the $d \in \mathcal{Q}$.

4. The speed of propagation of disturbances (sound) is finite. A happier home for Laplacian determinism.

Technical subtleties include:
Given classical atomism, the equations are effective; so how to deduce them as effective equations?

Conundrums include:
(1) Leibniz's principle of the identity of indiscernibles is flagrantly false, in that there are infinitely many intrinsically indiscernible infinitesimal fluid/solid elements.

(2) Issues in Hilbert's sixth problem: e.g. we must accept surface (traction) forces, at arbitrary internal surfaces of the body. Recall the long struggle leading to the stress tensor.

Philosophical themes

1. Identity

(i) Haecceitism in that a permutation of fluid elements—a different placement of them in physical space—gives a different configuration.

(ii) Identity over time: primitive in that trajectories have no real observational content.

2. Mass

The mass of a body of fluid is as for classical particles.
The density of an infinitesimal element is, in two respects, like classical particles: viz. not individuating, and quidditist.
But for a compressible fluid, it is in two other respects *unlike* classical particles:

not essential, because indeed changeable;

not intrinsic, since derived from a measure (though it is local in the usual mathematical sense).

3. Force

(i) Force contingently supervenes on matter in a more liberal sense than for classical particles: the new variables, like pressure, stress, viscosity, temperature, are related to the basic $q, \dot{q}, p$ and represented by geometric fields on the volume C.
(iii) There are fields with momentum and energy, e.g. the momentum-density field. So fields are more real than the gravitational potential field of classical particles.

4. Composition

(i) kinematics: the state-space has no natural factorization as a cartesian product. And there is no natural expression of separability as the independent specifiability of states of different elements/spatial regions.
But there is a residual non-holism: a configuration is given by an infinite conjunction of where each infinitesimal element is placed.

(ii) dynamics: non-holism in that no configurational forces *a la* Broad. But major generalizations of two-body forces, e.g. surface forces.

5. Modality:

The broad situation seems to be as it was for classical particles.

6. Law

(i) Like for classical particles: The minimal Humean covering law conception applies: Euler's equation etc describe and so explain phenomena. 'The causes are the signs'.

(ii) Causation?

The finite speed of propagation means hyperbolic PDEs can analyse the propagation of disturbance, and so maybe causation.

(iii) Philosophers naturally ask about the prospects for the specification of the forces being the best (simplest-strongest) description of the motions, *a la* Lewis' views.

Again: under some restrictions, this is a promising project. Indeed: the mosaic (the basis for Humean supervenience) is much richer than it is for classical particles—though we must not require all basis properties to be intrinsic to spatial or spacetime points.

Classical fields

We expect this to be like classical continua in most regards, since both cases have a smooth infinite-dimensional system.

But there is a major disanalogy: our focus is now the electromagnetic field *in vacuo* rather than ponderable matter. This makes for some more specific disanalogies:

(i) non-haecceitism about infinitesimal field elements;

(ii) classical atomism does not render Maxwell equations merely effective; rather, it just lands us in the late nineteenth-century dualism of field (filling the void) and point-particle.

1. The E and B fields (or if you prefer A) in $\mathbb{R}^3$, obeying Maxwell equations; and q, p of point particles: NB: The field contains energy and momentum, even in the void, where there is no ponderable mass. So fields are more real than was the gravitational potential field of classical particles.

2. Infinite-dimensional state spaces:
But there are no infinitesimal "field-elements": rotating a Coulomb field of a charge produces no change—it is gauge.
So separability has an easier expression than for classical continua. There, we had no natural factorization of the state-space by either infinitesimal elements, or by a partition of the containing region C in to spatial regions. But here, the state of a field is, roughly, the conjunction of its states on the spatial subregions (the partition cells).

3. The speed of propagation of disturbances (light) is finite. A happier home for Laplacian determinism.

Philosophical themes

To avoid repetition, I set aside ponderable matter.

1. Identity
Non-haecceitism about infinitesimal field elements: rotating a Coulomb field of a charge yields no change! It is gauge. So there is no issue about identity over time.

2. Charge of a point-particle is (like the mass of a classical particle):
a state-independent property, and normally construed as:
not individuating, since it could be shared with another particle; and
quidditist; intrinsic; essential, and unchangeable.

The new dynamical variables E, B are construed similarly, for the first two out of the four aspects. That is, they are: not individuating; quidditist. But they are not intrinsic and not essential (to the spacetime point that is their instance).

3. Force
(i) Force contingently supervenes on the configuration of radiation:
(ii) Again, we can treat the Lagrangian and Hamiltonian as scalars on the infinite dimensional state-spaces. E.g. Hamiltonian density is $E^2 + B^2$.

4. Composition
(i) kinematics: the non-haecceitism about field values at a point makes the configuration space, and so the Lagrangian and Hamiltonian formalisms, easier to specify: a natural factorization as a cartesian product of factors describing regions.
(ii) dynamics: non-holism in that there are no configurational forces *a la* Broad. Forces still combine in an easy fashion, though again higher-rank geometric objects eg the Faraday tensor are indispensable.

5. Modality:
1. The broad situation seems to be as it was for classical particles, and for classical continua.

6. Law
(i) Like for classical particles and classical continua: The minimal Humean covering-law conception applies. 'The causes are the signs'.
(ii) Causation?
Like for classical continua: the finite speed of propagation promises us a handle on causation.
(iii) Like for classical particles and classical continua. Here, the time-evolution of E and B are part of the mosaic for Humean supervenience. Again: under some restrictions, this is a promising project, partly because the basis is much richer than for classical particles: in particular with less intrinsicality to a spatial or spacetime point. .

Orthodox "minimal" quantum mechanics

Beware the dreaded quantum mysteries . . . thus Simone Weil:

'The proper method of philosophy consists in clearly conceiving the insoluble problems in all their insolubility and then in simply contemplating them, fixedly and tirelessly, year after year, without any hope, patiently waiting.'

We expect the quantum world picture to be a mix of that for classical particles (due to the quantum particle concept) with that for classical continua and classical fields (due to the wave function). Much of the expected mix *does* hold good.

Agreed: we could see this as a sign that the categories deployed in previous Sections do not cut ice about quantum mysteries—at least if we limit ourselves to this orthodox/minimal theory.

Anyway, we *do* see some major disanalogies with the foregoing: especially about composition.

The formalism is centred around :

$$L^2(\mathbb{R}^3)^{\otimes N} = L^2(\mathbb{R}^{3N}) =: \mathcal{H}^N \tag{1}$$

For indistinguishable particles, the symmetric/antisymmetric part of $\mathcal{H}^N$.
So: an infinite-dimensional state-space for a finite dimensional system.

With a unitary evolution U induced by Hamiltonian $H : \mathcal{H}^N \to \mathcal{H}^N$:
Laplacian determinism (in particular solutions have infinite lifetime)—apart from perhaps in measurement processes.

Conundrums include (apart from measurement problem and non-locality!):
Studying decoherence: e.g. to deduce effective hydrodynamic equations

Philosophical themes

A matter of judgment what are the problems to analyze; and what to make of rival solutions.

An example, less obvious than the decades of debate about measurement and non-locality:–

Some suggest deducing physical 3-space as "a shadow" from ψ defined on configuration space.
But the project has no motivation once we see orthodox quantum mechanics as emergent from quantum field theory for which physical space is built in already (Myrvold 2014)).

1. Identity
(i) Non-haecceitism about infinitesimal ψ-value elements: rotating a spherically symmetric wave-function produces no change. (And the value at point x is also not intrinsic to x, since it is derived from a probability measure.) So the issue is about particle labels —

Permutation of particle labels produces no change: it acts as identity on states.

This is unlike the first two classical cases; and unlike the third, taken with spatial regions acting like labels.

(ii) Identity over time: the particle labels on factor Hilbert spaces makes this primitive; not by following trajectories.

2. Mass:
At first one says: this is like classical particles: mass is conserved: and is normally taken as:
Essential, intrinsic (both because state-independent), quidditist and non-individuating.

But there is more to say:
(i) 'intrinsic' is questionable for any quantity because of the measurement problem.
(ii) The wave-function is equally representative of matter. There seems to be 'mass in the ψ' in that the COW experiment shows that ψ feels the gravitational potential energy (naively *mgh*).

3. Force

Force contingently supervenes on the configuration of matter, in that H is an explicit function of the dynamical variables q, p.

4. Composition

(i) kinematics: compositionality of state spaces (natural factorization as tensor product though not a cartesian product) ...
But the tensor product yields: *entanglement*.
A much-discussed threat to Lewis' Humean supervenience.
But NB: a kindred idea survives: ψ assigns a complex number to an N-tuple of points of physical space.

(ii) dynamics: non-holism in that there are no configurational forces *a la* Broad, and only two-body forces.

5. Modality:
The broad situation seems to be as it was for all three classical frameworks. But here there is the extra interest about ...

Counterfactuals in terms of nearest possible worlds:
We can understand these in terms of Lueders rule, or the Sasaki hook; on analogy with the Stalnaker conditional.

6.Law etc
Once we set aside measurement problem!, the situation is like for classical particles (not: classical continua and classical fields, since we here return to action-at-a-distance):

(i) The minimal Humean covering-law conception applies: Schroedinger equation describes and so explains phenomena.

(ii) Causation?
The action-at-a-distance means no hyperbolic PDEs with which to analyse propagation of disturbance . . .

(iii) Philosophers naturally ask about the Schroedinger equation, or the H in it, being the best (simplest-strongest) description of he Humean mosaic (including the evolution of the actual ψ). Under some restrictions similar to those given earlier, this is a promising project—and merits comparison with process tomography . . .

de Broglie-Bohm mechanics

(1) We expect this to be a mix of:

(a) what was said in Section 5 about ψ as a field; with

(b) what was said about classical particles—due to the heterodox Bohmian corpuscles.

I think it is this disunity between:

(a) the treatment of ψ like in orthodox quantum mechanics, and

(b) the treatment of corpuscles like classical particles,

that makes people uncomfortable with pilot-wave theory.

(2) It is amazing how a precise physical picture, with continuity, determinism etc. of a kind that the founding fathers alleged was forbidden, *can* be maintained, with 'just' the addition of corpuscles following the *orthodox* probability current.
I say 'just' because it seems the simplest/oldest (1687 or 1734!) way to add a well-defined ontology in spacetime to the orthodox ψ.

As in Section 5, the formalism is centred round

$$L^2(\mathrm{I\!R}^3)^{\otimes N} = L^2(\mathrm{I\!R}^{3N}) =: \mathcal{H}^N \tag{2}$$

Or for indistinguishable particles, the symmetric or antisymmetric part of $\mathcal{H}^N$: with a unitary evolution U induced by Hamiltonian H.

The guidance equation, coupled to the Schroedinger equation, is deterministic (and equivariant). So we have: Laplacian determinism—and here, no weasel-words *caveat* 'apart from perhaps in measurement processes'!
For we add corpuscles following the *orthodox* probability current.

I think that in what follows, nothing turns on the contrast:
second-order formalism vs. first-order formalism: or
'pilot-wave' / 'causal interpretation' (HJ theory, quantum potential) of Bohm, Hiley, Holland, vs. 'Bohmian mechanics' of Durr, Goldstein, Zanghi et al.

Philosophical themes

1. Identity

(i) Non-haecceitism about infinitesimal ψ-value elements. So the issue is about:

(a) particle labels (factor spaces): permutation acts as identity of physical states: unlike the classical cases

(b) permutation of the corpuscles: permutation does not act as identity. But thanks to the corpuscles being indistinguishable, a permutation makes no difference to any experiment's statistics.

(ii) Identity over time: again we need to consider separately (a) and (b):–

(a) the labelling of factor spaces makes it a primitive; but

(b) use trajectories as individuators (in theory not practice!)

2. Mass and other quantities

Mass: At first one says: the mass of a corpuscle is conceptually like the mass of a classical particle. Viz.: mass is conserved: and taken as: essential, intrinsic (both because state-independent), quidditist and non-individuating.

But there is more to it. Recall from Section 5 that the wave-function is equally representative of matter. This slogan is now surprising: for the 'mass in the ψ' shown by the COW experiment means that the mass is not where you would expect, viz. in the corpuscle.

Quantities in general: extrinsicality again, but differently than in Section 5. There we said that all state dependent quantities are extrinsic because orthodoxy requires a conceptual connection to measurement result.

But here the extrinsicality is much more detailed, thanks to the de Broglie-Bohm reconstruction of the orthodox quantum formalism e.g. the proof that the trace formula encodes measurement probabilities.

3. Force

Recall that I set aside the contrast: second-order vs. first-order formalism. So 'force' just refers to whatever is added to the initial conditions of the equation of motion to get the next derivative of position and thus by integration the future motion.

(a) Within the Schroedinger equation, the situation is as for orthodoxy. Force contingently supervenes on the configuration of matter, in that H is an explicit function of the dynamical variables q, p.

(b) Similarly for the guidance formula. The gradient of the phase S of ψ, evaluated at the actual location of all the corpuscles, drives the motion: $p = \nabla S$. So again, forces are determined in a simple way by the dynamical variables.

But cf. 6. below for discussion of the idea that ψ is itself a law, not a dynamical variable.

4. Composition

(i) kinematics: compositionality of state spaces (for ψ, use the tensor product; for the possessed positions, use the *N*-fold cartesian product of physical space eg $\mathbb{R}^3$).

For the orthodox quantum state ψ, there is a holism: viz. entanglement. But a state assigns a complex number to an *N*-tuple of points of physical space.

(ii) dynamics: again we split into (a) the orthodox quantum aspect, and (b) the corpuscle aspect:

(a) non-holism in that there are no configurational forces *a la* Broad, and only two-body forces. But

(b): the guidance of corpuscles shows configurational forces *galore*. The 'force-formula' (the generator of motion) is not fixed by the configuration but by the configuration together with the phase field S.

Again, a disunited treatment: maybe worrying—and a reason for wanting ψ to be a law as in 6. below.

5. Modality:
The broad situation seems to be as we said for orthodox quantum mechanics, and for all three classical frameworks. But here there is extra interest since ...

It is natural to consider counterfactually altering corpuscles' positions while ψ is fixed: hence the idea of quantum dis-equilibrium, ie of breaking $\rho = |\psi|^2$.

6.Law etc
Once we set aside the difference from orthodoxy about the measurement problem: (orthodoxy ignored it, de Broglie-Bohm's pilot-waves solve it!): the situation is mostly like for classical particles and for orthodox quantum mechanics . . .
except that here also we see the option mentioned at the end of 4. above: that ψ is a law of motion for the corpuscles, *not* a dynamical variable.

(i) The minimal Humean covering-law conception applies:
The Schroedinger equation, and guidance equation, describe and so explain phenomena.

(ii) Causation?
The action-at-a-distance (of Newtonian gravity and of the guidance equation) means there are no hyperbolic PDEs with which to analyse propagation of disturbance. So the precise expression of causation seems to be only:

(a) the global determinism of Laplace: the cause = a prior universal state; and if wanted:

(b) an appeal to counterfactuals, as in: for gravity: ‘if the Sun were placed differently, . . .’;
for the guidance equation: ‘if this corpuscle were placed differently, . . .’
Is this more than a verbal expression of functional dependence seen in the force-formula?

(iii) Philosophers naturally ask about the prospects for the equation of motion being the best description of motions, *a la* Lewis' views. Again this breaks down in to

(a) the question about the Schroedinger equation, or the H in it, posed in Section 5;

(b) the corresponding question about the guidance equation and the corpuscles:

Is the guidance equation the best (simplest-strongest) description of motions? This leads to

(iv) The idea of ψ as a law of motion for the corpuscles, not itself a dynamical variable.

So now the natural Humean question is:

Is ψ itself recoverable from the totality of motions?

The answer is no, for two reasons:

1) The totality of motions does not determine the velocity field on the configuration space that ψ defines; (cf. the elementary point that a test-particle does not explore whole field!)

And

2) ψ is not recoverable from the velocity field on the configuration space that ψ defines (Cf. Belot (2012): different stationary states can have the same velocity field, e.g. 0).

Of course, one can then try to revise the theory so as to overcome 2). But there is surely no hope of overcoming 1).

References