The Emergence of a Classical World in Bohmian Mechanics

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  - Wavefunction: the state-vector expressed in the coordinate representation

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- Physical state of a quantum system:
  - (normalized) vector in Hilbert space: state-vector;
  - Wavefunction: the state-vector expressed in the coordinate representation;
- Physical properties of the system (Observables):
  - Observable ↔ hermitian operator acting on the state-vector or the wavefunction;
  - Given a certain operator A, the quantum system has a definite physical property respect to A iff it is an eigenstate of A;
  - The only possible measurement's results of the physical property corresponding to the operator A are the eigenvalues of A;
  - Why hermitian operators? They have a real spectrum of eigenvalues → the result of a measurement can be interpreted as a physical value.

Structure of QM

- In the general case, the state-vector of a system is expressed as a linear combination of the eigenstates of a generic operator that acts upon it:

$$A|\Psi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle$$

- When we perform a measure of A on the system $|\Psi\rangle$, the latter randomly collapses or in the state $|\Psi_1\rangle$ with probability $|a|^2$ or in the state $|\Psi_2\rangle$ with probability $|b|^2$.

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$$\Downarrow$$

- SQM seems a theory that speaks more about the results of a measurement then the actual quantum physical processes.

Common Ontology of CM and BM

- Classical Mechanics

Fundamental entities: Particles moving along definite and deterministic trajectories in 3D physical space;

- Bohmian Mechanics

Fundamental entities: Particles moving along definite and deterministic trajectories in 3D physical space;

- ➢ BM is a quantum theory with a classical ontology and a highly non-classical dynamics (non locality of the velocity field)
- ➢ Ontological continuity between the quantum and the classical level (same basic physical entities)

Classical Limit in BM

Two problems related to the issue of the classical limit in BM:

- Physical problem (the emergence of the classical trajectories):

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- Philosophical problem (passage from a non-local world to a local one):

  - BM is a holistic theory: the universal wavefunction relates all the particles of the universe in a whole within a non-local dynamical structure.
  - CM is a "local" theory: the dynamical behavior of a single particle is (FAPP) autonomous and independent from that of the other particles.

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Note: here I use the term *local* for CM only in opposition to the holistic feature of the Bohmian non-locality (also in CM, in fact, there are non-local effects, due to the gravitational and electormagnetic interactions).

Measurement Process in BM

- What is a “Classical World”?

Natural answer: the Classical World is composed by open systems, i.e., physical systems which continuously interact each others.

$$\Downarrow$$

Classical or macroscopic world = huge-number-of-interactions world.

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- In order to show the emergence of a classical world within a Bohmian framework, we shall analyze how BM describes the interactions among quantum systems.

- Since in BM there is no conceptual difference between “quantum systems” and “classical apparata”, we can naturally use the measurement process as a general example of an interaction between two Bohmian physical systems.

Measurement Process in BM

- The main aim of this analysis:

We want to show that Bohmian Mechanics, by introducing a primitive ontology of particles, is able to:

1) Solve the measurement problem

2) Give an account of the local behavior of the classical systems through the notion of the effective wavefunction

3) Suggest a new interpretation about the meaning of the decoherence

(as the natural consequence of the interactions among particles)

Measurement Process in BM

- Quantum system: $\psi(x)$
- Classical apparatus: $\phi(y)$

- Total system:

$$\Psi(x, y) = \psi(x)\phi(y)$$

$\Psi(x, y)$ is the wave function of the total system (it can be expressed as a product of two separate wave functions for the system and the apparatus since, before the interaction, the two systems are physically decoupled each other)

- x = degrees of freedom of the system
- y = degrees of freedom of the environment

Measurement Process in BM

- One particle Bohmian system: $\psi(x)$
- Wavefunction of the system: $\psi(x) = \alpha\psi_1(x) + \beta\psi_2(x)$

$$\downarrow$$

linear superposition of two eigenfunctions

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The system interacts with an apparatus with particle coordinates Y;
Initial wf of the apparatus: $\phi_0(y)$, where $Y \in supp\ \phi_0 = 0$
When the interaction takes place, only two possible wf for the apparatus are allowed:

- $\phi_1(y)$, where $Y \in supp\ \phi_1 = 1 \Rightarrow$ pointer points to the outcome “1”;
- $\phi_2(y)$, where $Y \in supp\ \phi_2 = 2 \Rightarrow$ pointer points to the outcome “2”;

- Note: the wf of two distinct pointer states have disjoint supports in the configuration space of the apparatus $\Rightarrow supp\ \phi_1 \cap supp\ \phi_2 \cong 0$.

Effective Collapse

- Physical chain of the measurement process (eq. 1):

$$\Psi(x,y) = (\alpha\psi_1(x) + \beta\psi_2(x))\phi_0(y) \xrightarrow{t\to T} \alpha\psi_1(x)\phi_1(y) + \beta\psi_2(x)\phi_2(y)$$

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If we deal only with the wave function, this chain leads directly to the measurement problem;

- The actual particle of the system is described by just one of the two eigenfunctions;

Suppose the particle is described by $\psi_1(x)$;

- the actual configuration of the particles of the apparatus willl collapse in the state $\phi_1(y) \rightarrow$ effective collapse of the total system
- we call $\psi_1(x)$ the effective wave function of the system

  (this is the wf describing the dynamical behavior of the particle)
- we call $\psi_2(x)$ the empty wave function (it can FAPP be ignored after the measure)

Effective Collapse

- BM: the measurement process ends with a definite result
- (same empirical prediction of SQM ↔ quantum equilibrium hypothesis)

- Effective wave function = “localised” wave function
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- Interaction between two particles → the final quantum (Bohmian) systems are described by localised or effective wf;

- And if the classical world = huge-number-of-interactions-among-particles world

- Then the classical physical systems may be described by “continuously collapsed” wavefunctions → this (partially) recovers their local behavior

- Note: this idea seems to be not so much heretical, in fact it is not so far from the decoherence conceptual picture according to which classicality emerges from the continuous entanglement among quantum systems

(Main idea: in BM, entanglement = interactions between particles)

Decoherence

- Given the state-vector that describes the physical state of a quantum system $|\psi\rangle$
  We can obtain the density matrix of the quantum system:

$$\rho = |\psi\rangle\langle\psi|$$

- We can write the density matrix of the total quantum system considered before:

$$\rho = (\alpha|\psi_1\rangle|\phi_1\rangle + \beta|\psi_2\rangle|\phi_2\rangle)(\alpha^*\langle\psi_1|\langle\phi_1| + \beta^*\langle\psi_2|\langle\phi_2|)$$

- This formula corresponds to a matrix whose diagonal terms represent the probabilities for the two definite outomes of the pointer, and the non-diagonal ones represent the interference terms between two different quantum states;

Decoherence

- Starting from the density matrix of the total system, and by tracing over the degrees of freedom of the environment/apparatus, we obtain the reduced density matrix for the system alone:

$$\rho^{red} = Tr(\phi_i)\rho = \sum_i |\phi_i\rangle|\psi\rangle\langle\psi|$$

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- If we suppose that environment's states are orthogonal (the condition of the disjointness of the supports satisfies this condition), then the reduced density matrix becomes diagonal, since the non-diagonal terms become negligible and are FAPP destroyed (eq. 2):

$$\rho^{red} = |\alpha|^2|\psi_1\rangle\langle\psi_1|+|\beta|^2|\psi_2\rangle\langle\psi_2| \quad iff \quad \langle\phi_i|\phi_j\rangle = \delta_{ij} \;\; or \;\; supp(Y_i) \cap supp(Y_j) \cong 0$$

Decoherence

- What is the meaning of $\rho^{red} = |\alpha|^2|\psi_1\rangle\langle\psi_1|+|\beta|^2|\psi_2\rangle\langle\psi_2|$?

- The global state of the system (system + apparatus) is a pure state yet.

The reduced density matrix (RDM) is an improper mixture of states, so it cannot be interpreted as an epistemic ignorance about a definite and unique state of the system.

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The reduced density matrix (RDM) is an improper mixture of states, so it cannot be interpreted as an epistemic ignorance about a definite and unique state of the system.

- Meaning of equation (2):
- After the measurement-interaction, two definite outcomes are possible: namely, or the pointer collapses in the state $|\phi_1\rangle$ with probability $|\alpha|^2$ or in the state $|\phi_2\rangle$ with probability $|\beta|^2$.
- The RDM does not represent a statistical mixture of definite results, but a superposition of two definite macroscopically distinct outcomes (improper mixture).

Decoherence

These conclusions have been already pointed out in literature.

“How are we to interpret density operators arising as reduced states of entangled systems? Certainly not as proper mixtures!

Indeed, if a composite quantum system is in a pure entangled state, this state cannot be further decomposed as a weighted average of other quantum states, so cannot be interpreted in terms of ignorance. But then, neither can the states of the subsystems be interpreted in terms of ignorance, despite the fact that the subsystems are necessarily described by density operators.

[…]A mixed state arising as the reduced state of a subsystem, where the total system is in a pure state, is generally referred to as an improper mixture”

(Guido Bacciagaluppi (2011): *Measurement and Classical Regime in Quantum Mechanics*, p. 14)

Decoherence

According to Dürr and Teufel, the formula which describes the RDM "easily ranks among the most severely misinterpreted results in science":

"Why is this result often misunderstood? Suppose we do not know the wave function of a system, but our ignorance about it can be expressed in terms of probabilities, viz., with probability $|\alpha|^2$ the wave function is $|\psi_1\rangle$ and with probability $|\beta|^2$ it is $|\psi_2\rangle$. Then the corresponding density matrix would be exactly the right hand side of [eq. 2]. Therefore one may easily be trapped into thinking that the left-hand side of [eq. 2], which approximately equals the right-hand side, also submits to the ignorance interpretation.

In short, decoherence seems to turn "and" into "or". But [eq. 2] is nothing but a rephrasing of [eq. 1]. The only difference is the mathematical formulation.

[…]Only with Bohmian Mechanics can [eq. 2] be interpreted as a mixture"

(Detlef Dürr, Stefan Teufel (2009): *Bohmian Mechanics. The Physics and Mathematics of Quantum Theory*, p. 195-196).

Decoherence

- Question to the audience:

Does the mathematical expression of the reduced density matrix of a subsystem

$$\rho^{red} \cong |\alpha|^2 |\psi_1\rangle\langle\psi_1| + |\beta|^2 |\psi_2\rangle\langle\psi_2|$$

have the same physical meaning of the final step of a measurement-like interaction between a quantum system and a classical apparatus?

$$(\alpha\psi_1(x) + \beta\psi_2(x))\phi_0(y) \xrightarrow{t\to T} \alpha\psi_1(x)\phi_1(y) + \beta\psi_2(x)\phi_2(y)$$

- Note: even the final step of a measurement-like interaction does not contain the interference terms: the cat is entirely dead or entirely alive!

Provisional Conclusions

Decoherence in the framework of SQM:

1) Cannot solve the measurement problem;

2) Cannot eliminate the superpositions between macroscopically distinct physical states (improper mixture of states);

→ To eliminate these superpositions is a necessary step in order to explain the classical world, composed by not-superposed and definite physical systems (relationship between the measurement problem and the classical regime of Quantum Theory)

BM and Decoherence

Proposal for the classical limit of quantum theory:

- Decoherence + primitive ontology of particles (BM).

- We introduce *a priori* the basic physical entities of the theory: point- particles that move in 3D physical space

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Proposal for the classical limit of quantum theory:

- Decoherence + primitive ontology of particles (BM).

- We introduce *a priori* the basic physical entities of the theory: point- particles that move in 3D physical space;

- Since in BM we do not have superpositions of physical states, this approach may permit to eliminate superpositions of states within the framework of decoherence and single out definite classical physical states.

- Passage from improper mixtures to proper ones?
- Emergence of unique and classical physical states?

New Interpretation for Decoherence?

Meaning of the decoherence within BM = interactions among particles

- Within the Bohmian framework, the global entanglement between the quantum states of a system and those of its environment means simply a continuous interaction among particles
- Each particle of the universe “measures” the position of all the other particles which it interacts with
- This naturally leads to the creation of localised/effective WF for the interacting-physical systems
- It is natural to think that at the classical level each particle interacts with many other ones at all time, so that the classical objects can always be described by effective WF (Classical World → emergence of a local dynamical behavior)

Open Questions

- Can the Bohmian interpretation of the decoherence explain all the theorectical and experimental results of decoherence?

- Can the decoherence be useful to Bohmian Mechanics in order to derive the classical trajectories starting from the Bohmian ones?

- Does Bohmian Mechanics need decoherence to explain the classical dynamical regime of the classical physical systems?

- Can the local behavior of the classical physical systems be explained only by effective wave functions?

References

- Valia Allori, Detlef Dürr, Sheldon Goldstein, Nino Zanghì: *Seven Steps towards the Classical World*, Journal of Optics, 2002.
- Guido Bacciagaluppi: *Measurement and Classical Regime in Quantum Mechanics*, 2011, in Batterman (ed.): *The Oxford Handbook of Philosophy of Physics*, Oxford, 2013.
- Detlef Dürr, Stefan Teufel: *Bohmian Mechanics*. *The Physics and Mathematics of Quantum Theory*, Springer, 2008.
- Detlef Dürr, Sheldon Goldstein, Nino Zanghì: *Quantum Equilibrium and the Origin of Absolute Uncertainty*, Journal of Statistical Physics, 1992.
- Peter Holland: *Quantum Theory of Motion. An Account of the De Broglie-Bohm Causal Interpretation of Quantum Mechanics*, Cambridge, 1995.