What is a Quantum-Gravitational State?

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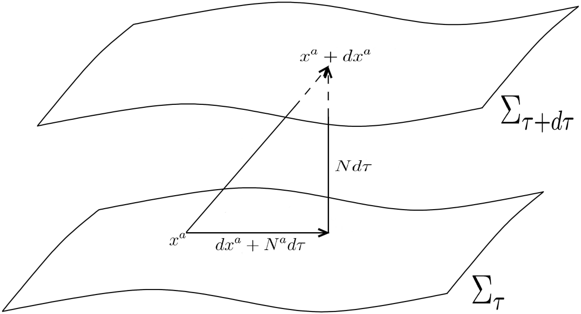
Hamiltonian General Relativity



$$ds^2 = h_{ab}(dx^a + N^a d\tau)(dx^b + N^b d\tau) - (N d\tau)^2$$

$$g_{ij} = \begin{pmatrix} N_c N^c - N^2 & N_b \\ N_a & h_{ab} \end{pmatrix}$$

Hamiltonian General Relativity

- The equations of motion are

$$\dot{h}_{ab} \approx \{h_{ab}, H\}, \quad \dot{p}^{cd} \approx \{p^{cd}, H\};$$

- Where the total Hamiltonian reads

$$H = \int d^3x (N\mathfrak{X}_\perp + N^a \mathfrak{X}_a);$$

- Together with the constraints

$$\mathfrak{X}_\perp = 16\pi G_{abcd} p^{ab} p^{cd} - \frac{\sqrt{det||h_{ab}||}\mathfrak{R}}{16\pi} \approx 0,$$

$$\mathfrak{X}_a = -2D_b {p_a}^b \approx 0.$$

Canonical Quantum General Relativity

Five steps towards canonical quantum general relativity:

1. Put the hat on the dynamical variables.
2. Translate Poisson brackets into commutators.
3. Implement the constraints (if not solved already at the classical level).
4. Find the observables.
5. Construct the (Hilbert) space of observables.

Canonical Quantum General Relativity

Formally (for quantum geometrodynamics):

$$h_{ab} \to \hat{h}_{ab}\Psi[h_{ab}] = h_{ab}\Psi[h_{ab}]$$

$$p^{cd} \to \hat{p}^{cd}\Psi[h_{ab}] = -i\hbar\frac{\delta}{\delta h_{cd}}\Psi[h_{ab}]$$

$$\Downarrow$$

$$\{h_{ab}, p^{cd}\} = \delta^{c}_{(a}\delta^{d}_{b)}\delta(x,y) \to [\hat{h}_{ab}, \hat{p}^{cd}] = i\hbar\delta^{c}_{(a}\delta^{d}_{b)}\delta(x,y)$$

$$\Downarrow$$

$$\mathfrak{X}_\mu \approx 0 \to \hat{\mathfrak{X}}_\mu\Psi[h_{ab}] = 0$$

$$\Downarrow$$

$$\{\mathcal{O}, \mathfrak{X}_\mu\} \approx 0 \to [\hat{\mathcal{O}}, \hat{\mathfrak{X}}_\mu]\Psi[h_{ab}] = 0$$

Canonical Quantum General Relativity

Other possible choices:

- $SU(2)$-connection A_a^i plus densitized triad E_j^b.
- Holonomy $U_{\mathfrak{a}}(A) \sim e^{\int_{\mathfrak{a}} A_a^i}$ of the connection A_a^i along a loop $\mathfrak{a}$ plus flux $E \sim \int_S E_j^b$ of the densitized triad through a 2-surface S.

Whatever the choice of variables, the constraints generate the dynamics of the theory:

Wheeler-DeWitt Equation(s)

$$\hat{\mathfrak{X}}_\mu \Psi = 0$$

Canonical Quantum General Relativity

Question

What is a quantum-gravitational state?

Tentative Answer 1

It represents the physical information regarding the (quantum) spatial geometry.

Canonical Quantum General Relativity

Problems:

- Canonical variables do not commute. Given that they encode information on the relation between 3-geometries and 4-spacetime, the classical spacetime picture fades away.
- If each physically relevant quantity has to commute with all the constraints, then their physical interpretation is problematic.
- If we allow physically relevant quantities not commuting with all the constraints, i.e. $[\hat{\mathcal{O}}, \hat{\mathcal{X}}_\mu]\Psi \neq 0$, then $\hat{\mathcal{X}}_\mu(\hat{\mathcal{O}}\Psi) \neq 0$.
- The latter problem is faced in particular by whatever local geometrical observable (areas, volumes, lengths) independently of its spectrum.
- There is no guarantee that the space on which "orthodox" observables act as operators is the Hilbert space of states annihilated by the constraints. Indeed, it is highly unlikely that they can be represented by self-adjoint operators.

Canonical Quantum General Relativity

Conclusions:

- If there is any information about quantum geometry carried by a quantum-gravitational state, it is so well hidden that we do not know how to extract it.
- For sure a Copenhagen-style interpretation of the quantum formalism leads to disastrous consequences.
- It seems that alternative formulations (e.g. path integral-like approaches) are less hard to deal with.

Canonical Quantum Gravity and Primitive Ontology

Question

What is a quantum-gravitational state?

Tentative Answer 2

It is a law-like object that describes the behavior of the primitive ontology.

Canonical Quantum Gravity and Primitive Ontology

In general, a primitive ontology approach involves a dual structure $(\mathscr{X}, \Psi)$, where:

- $\mathscr{X}$ is the primitive ontology properly said, i.e. a "decoration of spacetime";
- Ψ generates the dynamics for the primitive ontology.

Two (inter-related) problems with this definition:

1. Unsuitable for the pure gravitational case.
2. Not straightforwardly addressing the background-independent case.

- The first problem can be solved simply by making clear the commitment (if any) to non-material structures in $\mathscr{X}$.
- The second problem can be defused by giving a concrete example of a background-independent model involving a primitive ontology.

Canonical Quantum Gravity and Primitive Ontology

A primitive ontology approach to canonical quantum gravity consists of two steps:

1. Postulate a primitive ontology.
2. Specify a dynamical law for the primitive ontology.

Such an implementation may exploit two mainstream approaches:

- Dynamical collapse models à la GRW.
- "Causal" models à la de Broglie-Bohm.

Canonical Quantum Gravity and Primitive Ontology

Chellenges to GRW implementations (dynamics):

Collapse operator in the standard formulation

$$\Lambda_i(x) = (2\pi\sigma^2)^{-\frac{3}{2}} exp\left[-\frac{(\hat{Q}_i - x)^2}{2\sigma^2} \right]$$

- Is it possible to define a correspondent operator in the background-independent case?
- Problems in defining a Hilbert space of quantum-gravitational states.
- Difficulties in making sense of a Schrödinger-like evolution (need for some kind of approximation?).

Canonical Quantum Gravity and Primitive Ontology

Chellenges to GRW implementations (ontology):

Matter density

$$m(x,t) = \int_{\mathbb{Q}} d\mathbf{q} |\Psi_t|^2 \sum_i m_i \delta(q_i - x)$$

- Is it possible to define a pure gravitational background-independent analogue of a matter density field (e.g. a "metrical density" field)?
- Alternatively, how to make sense of the flash ontology? (A unique big flash *of* spacetime? A series of flashes of space weaving up time?).

Canonical Quantum Gravity and Primitive Ontology

Chellenges to Bohmian implementations:

Guiding equation in the standard theory

$$\dot{Q} \propto \frac{Im(\Psi, \nabla\Psi)}{(\Psi, \Psi)} = \frac{j^{\Psi}}{\rho^{\Psi}}$$

- Construction of a background-independent guiding equation (“spacetime rigidity” issues, no substantive general covariance).
- Making sense of a pure gravitational background-independent primitive ontology (in particular, in case of theories with discrete space at the Planck scale).
- Giving a precise meaning to the quantum equilibrium hypothesis (in general, no distinguished equilibrium distribution).

Concluding Remarks

- For sure, the two answers presented are not the only options on the table.
- However, among the two answers presented, the latter one seems to make conceptually more sense (*modulo* technical difficulties).
- Considering just primitive ontology approaches, the Bohmian one seems *prima facie* more promising than the dynamical collapse one.
- But, are we *really* sure that gravity should be quantized?