In *Geometric Possibility*, Gordon Belot proposes a careful philosophical investigation on how relationalism can account for the geometric structure of space; more precisely, he considers the question of the nature of the modal tools that are available to the relationalist. To this aim Belot discusses the application of standard accounts of laws and modality to the notion of possibility in the geometric context, namely to possible geometric facts. His discussion sometimes relies on rather technical examples from the theory of metric spaces, which makes the reading fascinating but which might be a little bit hard to follow for some readers. Nevertheless, Belot’s book will be of great interest to anyone interested in the philosophical foundations of relationalism about space.

Belot very carefully distinguishes the debate between substantivalism and relationalism about space from the debate between realism and anti-realism about space (the book focuses on space rather than spacetime). He rightly underlines that relationalism about space should not be confused with anti-realism about space: the relationalist can accept space in her ontology and consider space to consist of geometric relations among material bodies while denying the existence of underlying spatial regions. In this context, she should be able to make claims about the geometric features of space – e.g. about its dimension, about whether it is finite or infinite. Belot is interested in how the relationalist can account for such claims and what their truth conditions could be.

There are two main relationalist strategies, which are discussed in chapter 2. Conservative relationalism makes sense of claims about the geometric structure of space only in terms of the actual geometric relations among material bodies – what Belot calls ‘material geometry’. For instance, on this account, the extent of space is finite if and only if the actual distances between material bodies do not exceed some finite value. Despite its rather intuitive appeal, Belot argues that this conservative relationalist account is not satisfactory in cases where spatial geometry and material geometry cannot be straightforwardly identified. In particular he considers the example of a boundlessly expanding sphere of matter: in this case, should the extent of space be considered finite, indefinite but not infinite or infinite? Belot argues that each possibility involves undesirable consequences within the framework of conservative relationalism. Such difficulties belong to the main motivations for the investigation of the modal strategy for relationalism.

Modal relationalism makes sense of claims about the geometric structure of space in terms of possible geometric relations among material bodies, i.e. in terms of possible material configurations (with respect to the considered world). On this account, the extent of space is finite if and only if there is an upper bound such that it is geometrically impossible for material bodies to be located at some distance from one another greater than this upper bound. Belot’s main aim is to investigate the peculiar nature of geometric possibility involved in modal relationalism; it can be neither logical nor metaphysical possibility and Belot further argues that it is not physical possibility either, providing examples where physical possibility and geometrical possibility do not coincide.

Belot suggests three desirable but incompatible features that one might want any account of geometric possibility to possess. Such an account is grounded if all ‘the facts about geometric possibility at a world supervene on the intrinsic properties at that world’ (52). It is metric if ‘the basic geometric facts at a world [are] facts about the distances between bits of matter at that world (and whatever facts follow from these)’ (52). It is ambitious if for ‘every substantivalist world w with material configuration C, there is a relationalist world w* whose
material configuration $C^*$ mirrors the geometry of $C$ such that the facts about geometric possibility at $w^*$ mirror those at $w$ (5). Each of the three key chapters (ch. 3-5) considers in turn an account of geometric possibility that satisfies two of these conditions while rejecting the third one.

Before turning to these accounts of geometric modality, a word on metric spaces. Belot argues in the first chapter that metric spaces – basically sets equipped with a distance function – are good candidates to represent possible spatial geometries, mainly on the ground that the notion of distance is central to (almost) any spatial geometry. This representational choice provides him with the rigorous framework of the mathematical theory of metric spaces within which precise examples are discussed along the book. As acknowledged in the preface (vii), not all the discussion and examples about metric spaces are directly relevant for the main theses of the book, but they provide the interested reader with a few glimpses of the wonderful theory of metric spaces (two of the five appendices deal with technical issues related to metric spaces).

In chapter 3, Belot considers the application of David Lewis’ best-system analysis of laws of nature to the case of geometric possibility. This approach is clearly grounded and metric, but not ambitious: ‘facts about geometric possibility supervene on facts about distances between material points’ (59), but obviously not all substantival possibilities can be accounted for – one-particle worlds, which are often invoked along the book, cannot. Similarly to the context of laws, the best-system analysis of geometric possibility involves the notions of strength, simplicity and balance between the two. There are well-known worries with these notions; Belot argues that these worries are even more serious in the geometric context. It seems difficult to assess the global simplicity of specific geometries or metric spaces. Lewis himself conceded that the balance between strength and simplicity may well depend on species-related considerations linked to cognitive capacities. In the geometric case, Belot argues that such a dependence has ‘ideal’ aspects that are unwelcome for the regularity account: ‘the traditional name of the doctrine that the structure of space depends in part on our constitution and might well be different for beings of a different cognitive constitution is […] ‘transcendental idealism.’ […] But it is not the sort of thing we are led to expect when we are first told that the regularity view of laws can be fixed up to take care of well-known problems’ (72).

An obvious answer to these difficulties is to take geometric possibility to be a primitive (i.e. ungrounded) notion, irreducible to geometric facts – in much the same way as laws of nature can be considered as primitive. The prospects of ungrounded but ambitious and metric approaches to geometric modality are discussed in chapter 4. Characterizing the ambition condition and then developing a primitive modal relationalist account that satisfies it are tricky issues – the main reason being that in the general case specifying the distance relations between material points fails to determine the geometric features of the region occupied by matter. However, Belot proposes several sets of postulates and principles that encode various primitivist approaches to modal relationalism; they are ambitious in the sense that they match the substantivalist notion(s) of geometric possibility.

In the last chapter, Belot considers ambitious and grounded approaches to geometric modality. Again, the sought after account is inspired by a specific approach of laws of nature: according to nomic necessitarianism, ‘the identity of a fundamental property is so tightly bound up with the nomic role of that property that two worlds can differ in their laws only if they differ as to the fundamental properties instantiated’ (106). This necessitarian view about laws clearly puts strong constraints on how things can be recombined in different worlds. Belot exploits these constraints, using ‘compatibility properties’ – the property of being compatible with the instantiation or absence of instantiation of a given property – in order to ground geometric modality. The price to pay for this grounding it to accept these
‘compatibility properties’ as genuine fundamental geometric properties; this move makes the approach non-metric. The resulting account is a kind of necessitarian view about geometric possibility in the sense that two worlds can disagree about what is geometrically possible only if they differ as to the compatibility properties and the pattern of distance relations that are instantiated; it is not the case in the best-system approach for instance.

The ambition of the primitivist and necessitarian approaches to geometric modality is one of the reasons for Belot to favour them over the best-system account; the difficulties he sees with the latter constitute another reason. The principal benefit of ambition that Belot mentions in the examples he considers seems that it allows the ambitious modal relationalist to account for one-particle worlds differing as to the structure of space. One could however wonder to what extent it is reasonable to require the relationalist to provide an account for geometric features of one-particle worlds – or worse, of matter-free worlds, which are discussed within the primitivist approach.

Even for those who think that the debate between substantivalism and relationalism is best discussed within a given physical theory, Belot’s book constitutes a useful rigorous investigation into the philosophical foundations of relationalism. Moreover, the book ends with five appendices that are worthwhile on their own – including a presentation of some aspects of the mathematical notion of Gromov-Hausdorff distance between metric spaces, a review of the philosophical debate over the nature of space before the seventeenth century and an argument in favour of understanding Leibniz as a modal (rather than conservative) relationalist.

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