The No-Alternatives Argument

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Motivation

- Scientists argue for their theories and models.
- We ask: (i) which argument types do they use, and (ii) how can we evaluate them?
- Question (i) is an empirical question. We have to study how scientists argue and take it from there. One finds that besides deductive and inductive reasoning, scientists use argument types such as inference to the best explanation (IBE).

  → In this presentation, we will identify another argument type.
- Question (ii) requires us to check (a) the truth of the premisses and (b) the step from the premisses to the conclusion.

  → In this presentation, we will work in a confirmation-theoretical framework to address (b).

The No-Alternatives Argument

Scientists often argue like this:

1. Hypothesis H satisfies several desirable conditions. For example, it incorporates various principles, coheres with other theories,. . .
2. Despite a lot of effort, the scientific community has not yet found an alternative to H.
3. Hence, we have one reason in support of H.

We ask:

- How good are No-Alternatives Arguments?
- Under what conditions, if any, do they work?

Overview

1. Motivation
2. NAAs in Science and Philosophy
3. A Bayesian Analysis
4. Applications
5. Outlook

II. NAAs in Science and Philosophy

Examples from Fundamental Physics

There are many examples of NAAs in fundamental physics, mainly because discriminating empirical evidence is hard to come by.

1. String Theory
   This theory cannot (yet) be tested empirically. What speaks in its favor are (mostly unproven) coherence arguments and the NAA.
2. Cosmic Inflation
   This theory enjoys a very limited degree of empirical confirmation. Trust in the theory crucially relies on the NAA.

   This is a nice example which can be used to study how empirical and non-empirical (NAA) confirmation can work together.

Examples from the Special Sciences

The NAA also plays a role in sciences where contingent historical developments are reconstructed based on scarce evidence.

1. Palaeontology
   Conjectures about animal species and their behavior are based on scarce fossil evidence.
2. Early History
   Conjectures about past events are often established based on the argument that no other satisfactory interpretation of sources and archeological findings exists.

Examples from Philosophy

A similar argumentative strategy is also popular in philosophy. It proceeds as follows. Start with a certain account, examine a number of possible alternatives to it, find out that none of them works, and conclude (or be more certain) that the original proposal is right.

Here are two examples.

1. The design argument (Paley's watch. . .)
2. Williamson on knowledge

III. A Bayesian Analysis

Bayesianism

To formally analyze NAAs, we first choose a theoretical framework. The situation here is analogous to the situation in physics where one chooses, say, classical mechanics or quantum mechanics as a framework and then constructs models in this framework. We choose Bayesianism.

Bayesianism has a static and a dynamical aspect:

1. Statics: Bayesianism starts with the folk-psychological truism that we believe propositions with a certain strength (at a certain point in time). Belief comes in degrees, and these degrees of belief are, or so the Bayesians argue, probabilities.
2. Dynamics: At some later point in time we may learn some new information. This prompts a rational Bayesian agent to update her beliefs according to Bayes' Theorem.

An Illustration

- A Bayesian agent entertains the following hypothesis:
- H: The sun will shine tomorrow in Saig.
- She assigns a prior probability of $P(\mathrm{H}) = .7$ to it.
- Next, she listens to the radio and hears the weatherman saying that the sun will shine tomorrow in Freiburg. Hence, she takes as evidence for H the following proposition:
- E: The weatherman says that the sun will shine tomorrow in Freiburg.
- She then updates her beliefs according to

Bayes Theorem

$P'(\mathrm{H}) = P(\mathrm{H}|\mathrm{E})$.

- $P'(\mathrm{H})$ is the posterior probability of H.

Confirmation

- To calculate the posterior probability, it is useful to recall the mathematical identity:

Bayes Rule

$P(\mathrm{H}|\mathrm{E}) = \frac{P(\mathrm{E}|\mathrm{H}) \cdot P(\mathrm{H})}{P(\mathrm{E})}$

- Let $P'(\mathrm{H}) = .9$. Then $P'(\mathrm{H}) > P(\mathrm{H})$ and E confirms H.
- The

Difference Measure

$d(\mathrm{H}, \mathrm{E}) := P(\mathrm{H}|\mathrm{E}) - P(\mathrm{T})$

measures how strongly E confirms H. (There are other measures.)
- If E confirms H, then E is a reason in support of H.

Formalizing NAAs

- Let $\mathcal{C}$ be a set of constraints and $\mathcal{D}$ be a set of data. The community aims at a theory that fulfills $\mathcal{C}$ and explains $\mathcal{D}$.
- The hypothesis H fulfills $\mathcal{C}$ and explains $\mathcal{D}$.
- So far, no alternative hypothesis has been found that fulfills $\mathcal{C}$ and explains $\mathcal{D}$.
- We ask: To what extend, if at all, does this observation about the scientific community confirm H?

To address this question, we introduce two propositional variables:

1. T has two values, viz. T: The hypothesis H is empirically adequate, and ¬T: The hypothesis H is not empirically adequate.
2. F also has two values, viz. F: The scientific community has not yet found an alternative to H that fulfills $\mathcal{C}$ and explains $\mathcal{D}$, and ¬F: The scientific community has found an alternative to H that fulfills $\mathcal{C}$ and explains $\mathcal{D}$.

A First Bayesian Attempt

Goal: Show that F confirms T, i.e. that

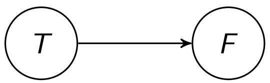
$$d(\mathrm{T},\mathrm{F}) := P(\mathrm{T}|\mathrm{F}) - P(\mathrm{T}) > 0.$$

$d(\mathrm{T},\mathrm{F})$ is again the difference measure of confirmation. $P(\mathrm{T})$ is the prior probability of T, and $P(\mathrm{T}|\mathrm{F})$ is the posterior probability of T, i.e. the probability of T after having learned the evidence F.

- How can this be done?
- Wave your hands, and claim that F is *obviously* positively relevant for T.
- But why should we accept this? And even if we do so, the Bayesian machinery does not add much or anything here.

Another Worry

Here is a standard Bayesian Network representation that is used to model the relation between a hypothesis and a piece of evidence:



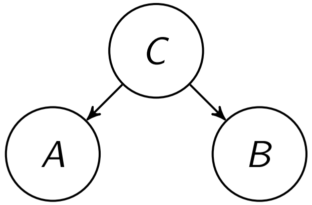
- On this account, there is a direct influence of the hypothesis on the evidence. The hypothesis is deductively or inductively implied by the evidence.(Example: T: All ravens are black. F: This raven is black.)
- But no such relation holds in the present case. F is at best some kind of indirect evidence for T. F is an example of what we call non-empirical evidence.

A Conjecture

- Recall Reichenbach's Common Cause Principle: Simultaneous correlated events have a prior common cause that screens off the correlation.
- Example: A = Paul has yellow fingers, B = Paul has a heart decease

A Conjecture

- Recall Reichenbach's Common Cause Principle: Simultaneous correlated events have a prior common cause that screens off the correlation.
- Example: A = Paul has yellow fingers, B = Paul has a heart decease, and C = Paul smokes.



- Hence our conjecture: There is a common cause variable Y that screens off the variables F and T.
- But what could Y be?

Introducing Y

We introduce a third variable.

3. Y has N values, viz. Y_i: There are exactly i hypotheses which fulfill $\mathcal{C}$ and explain $\mathcal{D}$. (H is one of them.)

Note that N can be ∞.

- The alternative theories make different predictions, and can therefore be individuated. Theories that make exactly the same (or almost the same) empirical predictions, are considered to be identical. Scientists have a good sense of what counts as a different theory, and what not.
- We claim that scientists have beliefs (supported by arguments) about the distribution of the Y_i.

Relations between F, T and Y

Repetition:

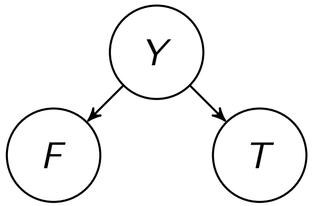
1. T has two values, viz. T: The hypothesis H is empirically adequate, and ¬T: The hypothesis H is not empirically adequate.
2. F also has two values, viz. F: The scientific community has not yet found an alternative to H that fulfills $\mathcal{C}$ and explains $\mathcal{D}$, and ¬F: The scientific community has found an alternative to H that fulfills $\mathcal{C}$ and explains $\mathcal{D}$.
3. Y has N values, viz. Y_i: There are exactly i hypotheses which fulfill $\mathcal{C}$ and explain $\mathcal{D}$. (H is one of them.)

A Bayesian Network Representation

T screens off F and T. Put differently, Y is conditionally independent of F given Y:

Conditional Independence

$$T \perp\!\!\!\perp F | Y \Leftrightarrow P(\mathrm{T},\mathrm{F}|\mathrm{Y}) = P(\mathrm{T}|\mathrm{Y})\, P(\mathrm{F}|\mathrm{Y})$$



Once we know the value of Y, we won't learn anything new about the truth value of T if we learn the truth value of F.

The Prior Probabilities

To complete the Bayesian Network, we have to fix the values of $P(\mathrm{Y_i})$, $P(\mathrm{T}|\mathrm{Y_i})$ and $P(\mathrm{F}|\mathrm{Y_i})$ for all i.

1. The Priors y_i

$P(\mathrm{Y_i}) =: y_i$, with $0 \leq y_i < 1$.

This assignment reflects the fact that we do not know the number of alternative theories a priori.

2. The Conditional Probabilities t_i

The Conditional Probabilities

$P(\mathrm{T}|\mathrm{Y_i}) =: t_i$ are monotonically decreasing in i (and $t_1 = 1$).

This is plausible: An increase in the more alternative theories does not make it more likely that scientists have already identified an empirically adequate theory.

3. The Conditional Probabilities f_i

The Conditional Probabilities

$P(\mathrm{F}|\mathrm{Y_i}) =: f_i$ are monotonically decreasing in i (and $f_1 = 1$).

This is plausible: A higher number of alternatives does not increase the likelihood of finding an alternative to H.

Calculating the Difference Measure $d(\mathrm{T}, \mathrm{F})$

Given our "common-cause" Bayesian Network, the following holds:

Lemma

$$d(\mathrm{T}, \mathrm{F}) = \frac{1}{2\,P(\mathrm{F})} \cdot \sum_{i \neq j = 1}^{N} (f_i - f_j)\,(t_i - t_j)\,y_i\,y_j$$

Hence,

Theorem

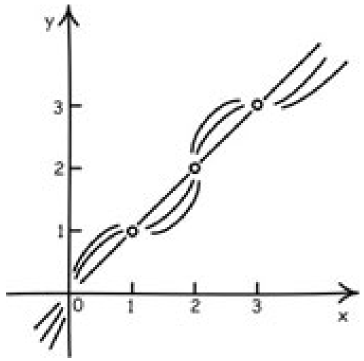
If f_i and t_i are monotonically decreasing in i and if there is at least one pair (i, j) with $j > i$ for which (i) $y_i\,y_j > 0$, (ii) $f_i > f_j$ and (iii) $t_i > t_j$, then $d(\mathrm{T}, \mathrm{F}) > 0$.

Discussion 1: Underdetermination

- Note that the assumptions of the theorem are rather weak. If an agent assigns degrees of belief that satisfy them, then she will be rational to make the NAA.
- However, if someone is certain that the number of alternatives has a fixed value, then F does not confirm T and the NAA has no pull.
- One could, for example, argue that the number of alternatives is infinite (i.e. that $y_\infty = 1$).
- Note, though, that scientists are often convinced that the number of alternative theories is rather small (without knowing the precise value). They are impressed by the difficulty to construct them. And this explains their conviction (supported by our analysis) that F confirms T.
- But is this line of thought convincing?

Discussion 1: Underdetermination

- Enter philosophy of science.
- According to the Underdetermination Thesis, there is always an infinite number of alternative theories that is consistent with a given (finite) set of data ($\mathcal{D}$).



Discussion 1: Underdetermination

So should a rational agent set $y_\infty = 1$?

The NAA-er has two options to respond:

1. Argue that the constraints $\mathcal{C}$ reduce the number of alternatives to a finite number. – It is not quite clear how to make this case.
2. Argue that the Underdetermination Thesis only shows that $0 < y_\infty < 1$ and, of course, that there is at least one (finite) pair (i, j) for which the assumptions of the theorem hold. It would be dogmatic to be certain that $y_\infty = 1$, which is clearly not a logical truth.

Discussion 2: Difficulty of the Problem

- One might also want to include the difficulty of the scientific problem at hand in our model, because the difficulty of the problem may also explain why no alternative has been found yet.
- This can be done straightforwardly (see the paper).
- The present results hold for a fixed value of the difficulty of the problem. The parameters f_i and t_i are fixed given a specific degree of difficulty of the problem.
- If there is uncertainty about the difficulty, then our more general results in the paper apply.

Discussion 3: Challenging $\mathcal{C}$ and $\mathcal{D}$

- Often different scientists disagree upon (i) which theoretical constraints $\mathcal{C}$ the theory should fulfill and (ii) what the data $\mathcal{D}$ are the theory should explain.
- Clearly, a no-alternatives argument is only effective if everybody agrees on $\mathcal{C}$ and $\mathcal{D}$.

IV. Applications

1. String Theory

Despite extensive research, no empirically distinguishable alternatives to String Theory have been found.

1. Alternative approaches to achieve a unification of General Theory of Relativity and Quantum Field Theory have not led to new promising ideas and were eventually given up.
2. The scope of the program of Canonical Quantum Gravity (Carlo Rovelli, Lee Smolin et al.) is more limited.
3. Different versions of String Theory are equivalent.

Hence, the NAA applies but: (i) reasons have to be provided that $y_\infty \neq 1$ and (ii) the set of conditions $\mathcal{C}$ have to be accepted.

2. Scientific Realism

- We have defined the propositional variable T as follows:

  T has two values, viz. T: The hypothesis H is empirically adequate, and ¬T: The hypothesis H is not empirically adequate.
- That is, we (only) want to infer to the truth of what H says about observable things.
- A scientific realist wants more, i.e. truth *tout cours*. Can we model this as well? And will the corresponding hypothesis also be confirmed?

2. Scientific Realism (Cont'd)

- Replace T by T'. T' has two values, viz. T': The hypothesis H is true, and ¬T': The hypothesis H is not true.
- F and Y remain as before.
- The independence assumption (i.e. $T' \perp\!\!\!\perp F|Y$) holds and our argument goes through.
- We ask: Which of the two propositions T and T' is better confirmed? Note that T' → T (but not T → T'), i.e. $P(\mathrm{T}) > P(\mathrm{T'})$ and $P(\mathrm{T}|\mathrm{F}) > P(\mathrm{T'}|\mathrm{F})$.
- Hence it is not clear whether $d(\mathrm{T}, \mathrm{F}) > d(\mathrm{T'}, \mathrm{F})$. It needs to be explored under which conditions which proposition is better confirmed.

3. Inference to the Best Explanation (IBE)

- Under which conditions is an IBE justified?
- Replace F by F'. F' has two values, viz. F: The scientific community has not yet found a better explanation than H, and ¬F: The scientific community has found a better explanation than H.
- T (or T') and Y remain as before.
- The independence assumption (i.e. $T \perp\!\!\!\perp F'|Y$ or $T' \perp\!\!\!\perp F'|Y$) holds and the argument goes through.
- Question: Can this kind of reasoning be used to respond to van Fraassen's "bad lot" argument?
- Again, the answer depends on how good our reasons are that the true or empirically adequate theory is amongst a finite number of considered alternative theories. (Note the possible difference between ordinary reasoning and scientific reasoning.)

3. The Existence of God

- Recall N. R. Hansson's argument against the existence of God: So far we have no good reason for the existence of God, which provides a reason against the existence of God.
- Three propositions:
  1. T'' has two values, viz. T": God exists, and ¬T": God does not exist.
  2. F'' also has two values, viz. F": No one has yet found a good reason for the existence of God, and ¬F": Someone has found a good reason for the existence of God.
  3. Y'' has N values, viz. Y_i": There are exactly i good reasons for the existence of God.
- The independence assumption (i.e. $T'' \perp\!\!\!\perp F''|Y''$) holds and the parameters f_i'', t_i'' and y_i'' are defined as before.

3. The Existence of God (Cont'd)

The Conditional Probabilities

$P(\mathrm{T''}|\mathrm{Y_i''}) =: t_i''$ are monotonically increasing in i.

Remember our (suitably adapted) lemma:

Lemma

$d(\mathrm{T''}, \mathrm{F''}) = \frac{1}{2\,\mathrm{P(F'')}} \cdot \sum_{\mathrm{i \neq j = 1}}^{\mathrm{N}} (\mathrm{f_i''} - \mathrm{f_j''})\,(\mathrm{t_i''} - \mathrm{t_j''})\,\mathrm{y_i''}\,\mathrm{y_j''}$

Theorem

If f_i'' are monotonically decreasing in i and t_i'' are monotonically increasing in i and if there is at least one pair (i, j) with $j > i$ for which (i) $y_i''\,y_j'' > 0$, (ii) $f_i'' < f_j''$ and (iii) $t_i'' > t_j''$, then $d(\mathrm{T''}, \mathrm{F''}) < 0$.

V. Outlook

Outlook

1. We have provided a Bayesian account of NAAs and analyzed under which conditions this argument-type is powerful.
2. Given that various assumptions have to be fulfilled, the acceptance of a proposed NAA will often be controversial.
3. Most crucial is the assumption that a non-zero probability can be assigned to the proposition that the number of alternative theories (including the true or empirically adequate one) is not infinite. The argument for this will depend on the specific application in question.
4. Open questions: Detailed case studies from science (e.g. string theory), NAAs in philosophy (e.g. the design argument), and (I am sure) more.

Thanks for your attention!

The talk is based on joint work with Richard Dawid (Vienna) and Jan Sprenger (Tilburg).

The paper is forthcoming in *The British Journal for the Philosophy of Science.* A popular discussion of the paper can be found in http://tinyurl.com/NAA-Guardian.