BOHMIAN MECHANICS: METHODOLOGY AND ONTOLOGY

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Standard Quantum Mechanics

- **Physical states** → unit vectors on Hilbert space $\mathcal{H}$, whose properties are described by operators acting on these vectors;
- **Operators** → mathematical operation which modify vectors in a particular way
- The dimension $D$ of $\mathcal{H}$ depends on the state which we want to describe; $D$ could be infinite.

“The kind of Hilbert space used to represent a quantum state depends on the kind of system in which we are interested. Hilbert spaces used to represent states of position, momentum, or energy contain dimensions corresponding to allowable values of position, momentum, or energy that the particular system may have” (A. Ney, 2013).
Standard Quantum Mechanics

- or...

- \(N\)-point particles systems \(\Rightarrow\) wave functions in configuration space \(\mathbb{R}^{3N}\).

- Quantum state is *completely* described by a wave function \(\psi:\)

\[
\psi = \psi(q) = \psi(q_1, \ldots, q_n)
\]

- \(q\) represents variables

\(\psi\) contains all the possible physical information accessible to us
Standard Quantum Mechanics

- **Dynamics:**
  - The evolution of (an isolated) system is given by the *Schrödinger equation* (*linear and deterministic* equation):

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi}{\partial t} = H\psi
\]

- with Schrödinger Hamiltonian

\[
H = -\sum_{k=1}^{N} \frac{\hbar^2}{2m} \nabla^2_k + V
\]
Standard Quantum Mechanics

- **Linearity** ⇒ if \( \psi_1 \) and \( \psi_2 \) are both solution of the same Schrödinger equation, then a linear combination of the two gives a new solution:

\[
\psi = c_1 \psi_1 + c_2 \psi_2
\]

- **Consequence of linearity** ⇒ superposition of the states:

  if a physical state could be described by \( \psi_1 \) and \( \psi_2 \), then we can obtain a new description for such state combining the previous wfs:

- **Determinism** ⇒ given the state of a system at a certain time \( t \) (initial conditions), it is possible to predict with certainty what state of that system will be at some future time \( t_n \).
The properties (measurable quantities \( \approx \) observables \( O \)) of a certain system are represented by operators \( A \).

Measurement: an interaction between a quantum system and a classical apparatus.

Measuring an observable, the only possible outcomes are the eigenvalues of the measured operator (which form its spectrum).

After a measurement we have probability 1 to obtain one of these values, but...we do not know which one!
Born’s Statistical Interpretation of the wave function:

$$|\psi(x, y, z, t)|^2$$

represents the probability to find a particle of position \((x, y, z)\) at the time \(t\) if a measurement is performed. This quantity gives the probability density of an outcome of a position measurement.

\(\psi\) acquires its physical meaning through the notion of probability amplitude: the wf is a summary of the probabilities of certain outcomes \(\Rightarrow\) for a position measurement, the wf gives information about where is more or less likely that one will find the particle. (A. Ney, 2013).
Standard Quantum Mechanics

- After a measurement the system is left in an eigenstate of $O$.
- NB: In SQM it is not conceivable that a system has properties before measurement processes.
- Collapse of the wave function:

\[
\psi = \frac{1}{\sqrt{2}} (|\text{no gas released}\rangle_{device}|\text{alive}\rangle_{cat} + |\text{gas released}\rangle_{device}|\text{dead}\rangle_{cat})
\]

- here $\psi$ is in a superposition of eigenstates

When we operate a measurement the superposition of the wave packet *collapses* instantaneously in one of the possible states that the wf admits
Probabilities emerge naturally from this formalism: the observation processes cause a discontinuity in the evolution of the wave function → and the Schrödinger dynamics is suppressed.

Probabilities in QM are *objective* and non-epistemic.
Problems with SQM

- Incoherence between Schrödinger equation and the collapse rule;
- Generally, in QM it is not possible to attribute properties to a system before measurement processes;
- Active role of the observers;
- Pragmatic distinction between the quantum world and the classic world;
Questions..

- Where could the limit be established between the “quantum” and the “classic” world?

- What could justify this distinction?

- How could the classical behaviour of macro-objects emerge?

- What is exactly a quantum system (e.g. a particle) before measurement processes?

- What is entailed by the observation processes?
Methodology

- How theory meets the world?
- A theory starts with a phenomenology

↓

The domain of physical facts that a theory should be able to explain;

→ several aspects are related: mathematics, physics and experience

Manifest/Scientific Image of the world

Reality how we perceive it

World’s description given by a certain theory
Methodology

Physics

Primitive Ontology – Physical laws

Geometric/Algebraic Structures

Macroscopic Ontology

Mathematics

Experience

Manifest Image of the world

↑

Subjective Experiences

↓
Methodology

- A physical theory could be:
  - Informationally completeness → a description provided by a physical theory is informationally complete if every physical fact about a certain situation under observation is recovered by this description;
  - NB: a theory could be informationally complete even if does not provide a description of what is (supposed to be) physically real!
  - Physics can develop accurate and powerful models without any ontological commitment regarding the entities presented in the mathematical framework of the theory!
Methodology

- A mathematical representation of a certain theory could have different forms: e.g. Heisenberg Matrix Mechanics and Schrödinger Wave Mechanics;

- In Classical Electromagnetic Theory we can describe a certain physical situation in term of the field \( \mathbf{E} \) and \( \mathbf{H} \), but we could describe the same situation with the potential \( \mathbf{A} \) and \( \phi \rightarrow \) and with different potentials!

Keep the distinction between mathematical and physical entities as sharp as possible!
Local Beables

- **Observable**: it is a (mathematically) well defined notion, *but physically*? ⇒ Which processes are observations?

- **Beable**: what the theory is about ⇒ *entities which are assigned to finite spatiotemporal regions (Local)*;

- **Local Beables**: introduce a division between Physical and non-Physical entities;

- Bell’s example: in classical electromagnetism the fields $E$ and $H$ are real, physical entities, while the potential $A$ and $\phi$ are non-physical;

Local Beables ⇒ Local Observables
Local Observables ≠ Local Beables
Primitive Ontology

- *Primitive variables* $\rightarrow$ formal counterparts whose referents are real entities in the world (according to the theory);

- NB: these primitive variables are those from which our manifest image of the world is dependent on;

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Primitive Ontology (PO): it is a physical assumption regarding the fundamental objects of the world: e.g. particles evolving in 3D space or in spacetime. Methodological role: PO coordinates the construction of a theory introducing certain features/constraints;
Primitive Ontology

- **Explanatory function** of the primitive variables: they are *primitive* because every physical object or phenomenon must be connected or explained in terms of PO;

- A physical theory with PO should explain a set of phenomena in terms of these P-Variables

- PO → Logical clarity in derivations of empirical predictions:
  
  - $\mathcal{E}(\text{experiment}) \rightarrow Z(\text{outcome})$, where $Z = \xi(PO)$,
  
  - Z is a function of the PO!
Primitive Ontology gives constraints in theory construction

E.g. PO and Symmetries:

The solutions of dynamical equations yield the possible configurations (evolution) of the PO

Symmetries: the “possible histories” allowed dynamically are still possible solutions when transformed by a certain symmetry;

Example: P-variables: geometric entities in physical space

Spacetime symmetries apply to the PO, e.g. transforming trajectories: the new trajectories are still solutions of dynamical equations.
What does SQM describe?

W. Heisenberg: Quantum state ↔ physically measurable quantities

It seems that axioms and physical laws concerns our epistemic access to the quantum world instead of a realistic description of quantum phenomena

N. Bohr
Epistemological principle: it is in principle impossible to separate sharply between the behaviour of atomic objects and the interaction with measuring devices
Quantum Theory without Observers

It seems that SQM concerns primarily results of measurement processes → SQM as phenomenological algorithm; a set of rules for computing the probability of measurements’ outcomes.

But SQM should be concerned with Electrons, Protons, Neutrons ...

Where are they in the quantum description? (assuming that these particles are different objects with respect to the wave function)
Quantum Theory without Observers

What do we need?

→ Ontological Clarity: a theory should claim clearly its fundamental entities, which are supposed to be the basic objects from which we could recover our manifest image of the world;

→ Observers Independence: the notion of “observer” is rather vague and should not play any crucial role in a fundamental physical theory;

→ Non-vagueness: it should be clear the domain in which the theory is valid;
Bohmian Mechanics

- QTWO as foundation of SQM

The aim of the project QTWO is to recover the phenomenology of SQM providing a clear account of what there is in the world.
Bohmian Mechanics

Question:

Is BM a new theory or an interpretation of SQM?
Bohmian Mechanics

- BM is a non-relativistic theory of particles moving in the ordinary physical space following world lines (trajectories);
- Why hidden variables? → in BM $\psi$ does not describe entirely a physical system: something more is needed

particles’ positions

- In BM it is supposed that particles have a defined position at any given time $t$;
- Common misunderstanding: the “hidden” variables are the real entities from which the theory provides a realistic description of nature; the wave function is never measured, rigorously speaking, it should be the hidden parameter! We should say additional variables.
Bohmian Mechanics

- In BM a physical system is described by the pair \((\psi, Q)\), where the former is the usual wave function and the latter is the actual configuration of particles in space;

- The theory has two dynamical equations:
  
  i. The usual Schrödinger evolution for the wave function;
  
  ii. A new equation for the motion of the particles, the so called guiding law → it expresses the velocities of the particles, which is dependent on the wave function;
Bohmian Mechanics

- Consider a $N$-particle system:

  $Q_i(t)$ describes the position in $\mathbb{R}^3$ of the $i$-th particle at time $t$:

  
  
i. $i\hbar \frac{\partial \psi}{\partial t} = H\psi$

  
  
  ii. $\frac{dQ_i}{dt} = \nu \psi_i(Q_1, \ldots, Q_N) = \frac{\hbar}{m_i} \psi^* \nabla_i \psi(Q_1, \ldots, Q_N)$

  i. + ii. specify *completely* the deterministic dynamics of the theory $\rightarrow$ *no observer is needed*!
Bohmian Mechanics

- Quantum randomness: BM is empirically equivalent to SQM, it describes all the physical phenomena in the SQM domain;
- BM is *equivariant*: the equations are compatible with the usual Born statistical $|\psi|^2$ distribution:

  If a configuration $Q(t_0)$ has distribution $|\psi_0(q)|^2$ at time $t_0$, then at any later time $t$, the configuration $Q(t)$ will have distribution $|\psi_t(q)|^2$. This distribution is called the *Quantum Equilibrium Distribution*.

- For detail see DGZ (2013) Chap. 2;
while in orthodox quantum theory the collapse is merely superimposed upon the unitary evolution of the wave function, without a precise specification of the circumstances under which it may legitimately be invoked - and this ambiguity is nothing but another facet of the measurement problem - Bohmian mechanics consistently embodies both the unitarity evolution and the collapse of the wave function as appropriate. Concerning the evolution of the wave function, Bohmian mechanics is indeed formulated in terms of Schrödinger's equation alone. However, since observation implies interaction, a system under observation cannot be a closed system but rather must be a subsystem of a larger system that is closed, e.g., the entire universe. And there is no reason a priori why a subsystem of a Bohmian universe should itself be a Bohmian system, even if the subsystem happens to be “closed”. Indeed, it is not even clear a priori what should be meant by the wave function of a subsystem of a Bohmian universe. DGTZ (2005), Bohmian Mechanics.
Bohmian Mechanics

- Subsystems: consider the configuration $Q$ of the Universe; if we want to consider subsystems of the universe, then this configuration naturally splits into $X$, the configuration of a sub system and $Y$ the configuration of its environment;

Consider the wave function of the universe $\Psi = \Psi(q) = \Psi(x,y)$, then the wave function of the sub system is obtained isolating this system from its environment, in this way we obtain the conditional wave function $\psi(x) = \Psi(x,Y)$.

- The procedure is the following: it is sufficient to plug in the WF of the universe the \textit{actual} configuration of the environment $\rightarrow$ this cwf obeys to the Schrödinger equation and it collapses under the usual rules according to SQM, namely when it occurs an interaction between subsystem and environment;
Bohmian Mechanics

- Macroscopic World and Measurements

- Recall that in BM the wave function has a double role: it guides the particles’ motion and it governs the statistical distribution of the positions;

- *In BM all what we measure are positions:*

  In Bohmian Mechanics the physical quantity we can effectively measure is the *position*, every other physical quantity could be explained in terms of positions.
Bohmian Mechanics

Measurement experiment: the system $\psi$ is correlated with the pointer wf $\rightarrow$ a macroscopic wf; Different pointer positions occupy different regions in space (Fig. 1), they are described by different macroscopic disjoint (in configuration space) wf, each with a different support.

Support of the wf: the domain in which $\psi \neq 0$

These macroscopic wf entail random superpositions of many one-particle-wf;
Bohmian Mechanics

\( X = \) system coordinates in \( m \)-dimensional configuration space;
\( Y = \) apparatus coordinates in \( n \)-dimensional configuration space;
Suppose the system is in a superposition of two wfs: \( \psi_1(x) + \psi_2(x) \).

Possible wfs of the apparatus
\[
\begin{cases}
\text{null orientation } \varphi_0(y), (Y \in \text{supp } \varphi_0 \cong 0) \\
1 - \text{orientation } \varphi_1(y), (Y \in \text{supp } \varphi_1 \cong 1) \\
2 - \text{orientation } \varphi_2(y), (Y \in \text{supp } \varphi_2 \cong 2)
\end{cases}
\]

→ A measurement is defined as an interaction between the system and the apparatus.
Bohmian Mechanics

- The Schrödinger evolution for this interaction is such that

\[
\psi_i \varphi_0 \xrightarrow{t \rightarrow T} \psi_i \varphi_i \quad (1)
\]

Where \( t \rightarrow T \) is the time evolution Schrödinger evolution

Pointer positions are correlated with the system wfs
Bohmian Mechanics

- When we start the measurement system and apparatus are independent, formally

\[ \Psi(q) = \Psi(x, y) = \psi_i(x)\varphi_0(y), \text{with } i = 1, 2 \]

- The Measurement problem is a consequence of (1) + the linearity of the Schrödinger equation;
Consider the case: \( \psi(x) = \alpha \psi_1(x) + \beta \psi_2(x) \), where \(|\alpha|^2 + |\beta|^2 = 1\); by linearity equation (1) yields:

\[
\psi \varphi_0 = \sum_{i=1,2} \alpha_i \psi_i \varphi_0 \xrightarrow{t \to T} = \sum_{i=1,2} \alpha_i \psi_i \varphi_i
\]

The Measurement problem arises if one has only the wave function!
Bohmian Mechanics

Fig. 2

\[ \text{supp } \Phi_1 \]

\[ \text{supp } \Phi_0 \]

\[ \text{supp } \Phi_2 \]

\[ \text{supp } \psi \]

\[ X \]

\[ Y \]

\[ (X_T, Y_T) \]

\[ (X_0, Y_0) \]

Bohmian trajectory
Bohmian Mechanics

- In BM we have another ingredient: the evolution of the particles with definite position via the guiding law;
- We are interested in the pointer positions after the measurements $\Rightarrow$ configuration of the pointer particles $Y(T)$;
- The pointer will *necessarily* point either in 1-direction or in 2-direction in virtue of:
  
  i. Initial distribution of particles $(X_0, Y_0)$
  ii. Deterministic laws for the evolution of $(\psi, Q)$
  iii. Quantum Equilibrium Distribution

- In this way embarrassing macroscopic superpositions are avoided!
References

- Slide 4 is a simplification of “Cos’è la Filosofia della Fisica”, N. Zanghì’s lecture available on his personal webpage [http://www.ge.infn.it/~zanghi/filo/fdf2012.html](http://www.ge.infn.it/~zanghi/filo/fdf2012.html)
Figures

- Fig. 1: Dürr D., Teufel S., 2009, *Bohmian Mechanics*, Spinger, p. 174 (fig. 9.1 in the original);
- Fig. 2: Dürr D., Teufel S., 2009, *Bohmian Mechanics*, Spinger, p. 176 (fig. 9.2 in the original);