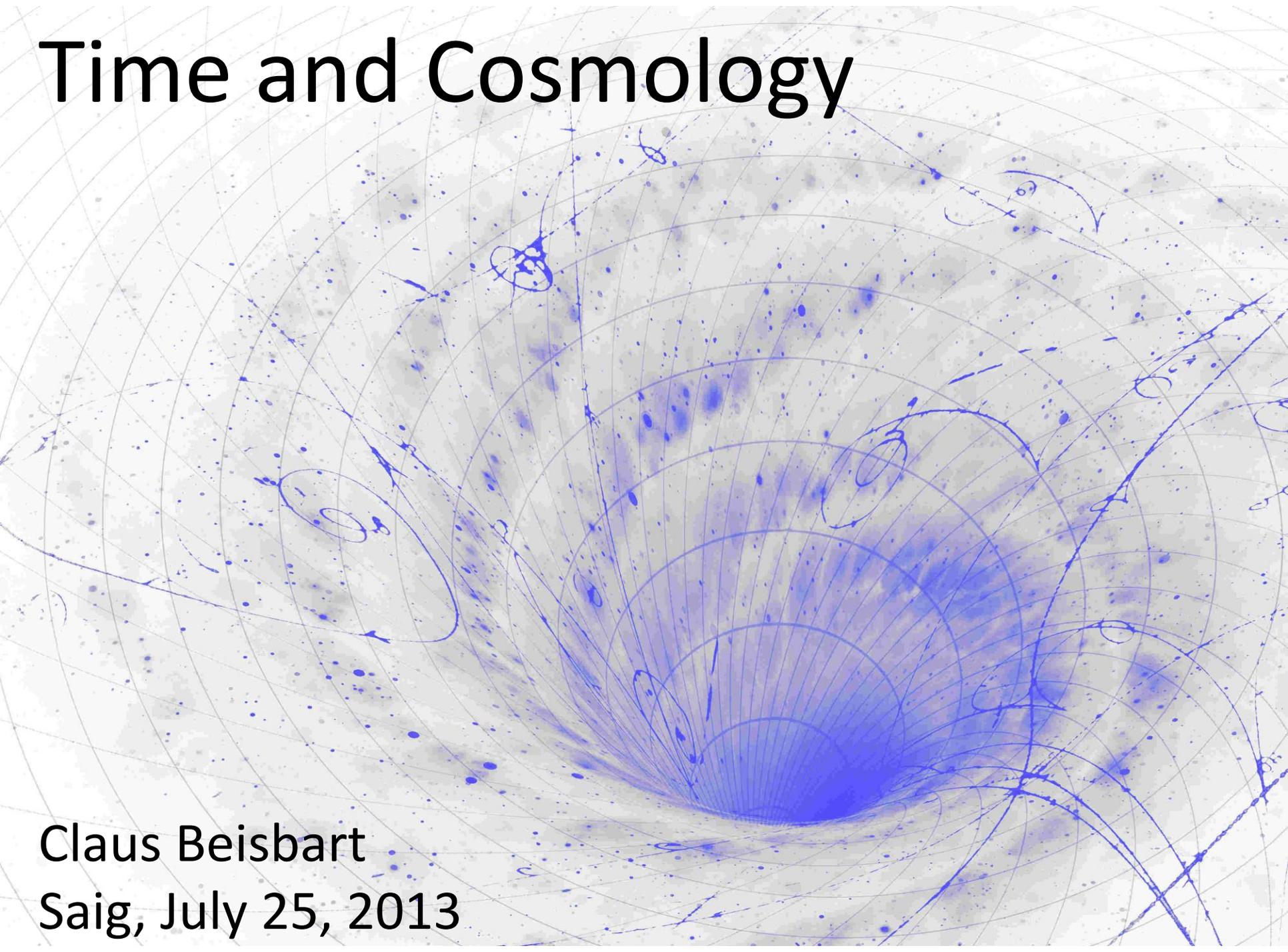


Time and Cosmology

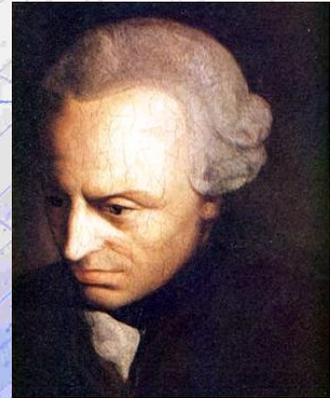


Claus Beisbart

Saig, July 25, 2013

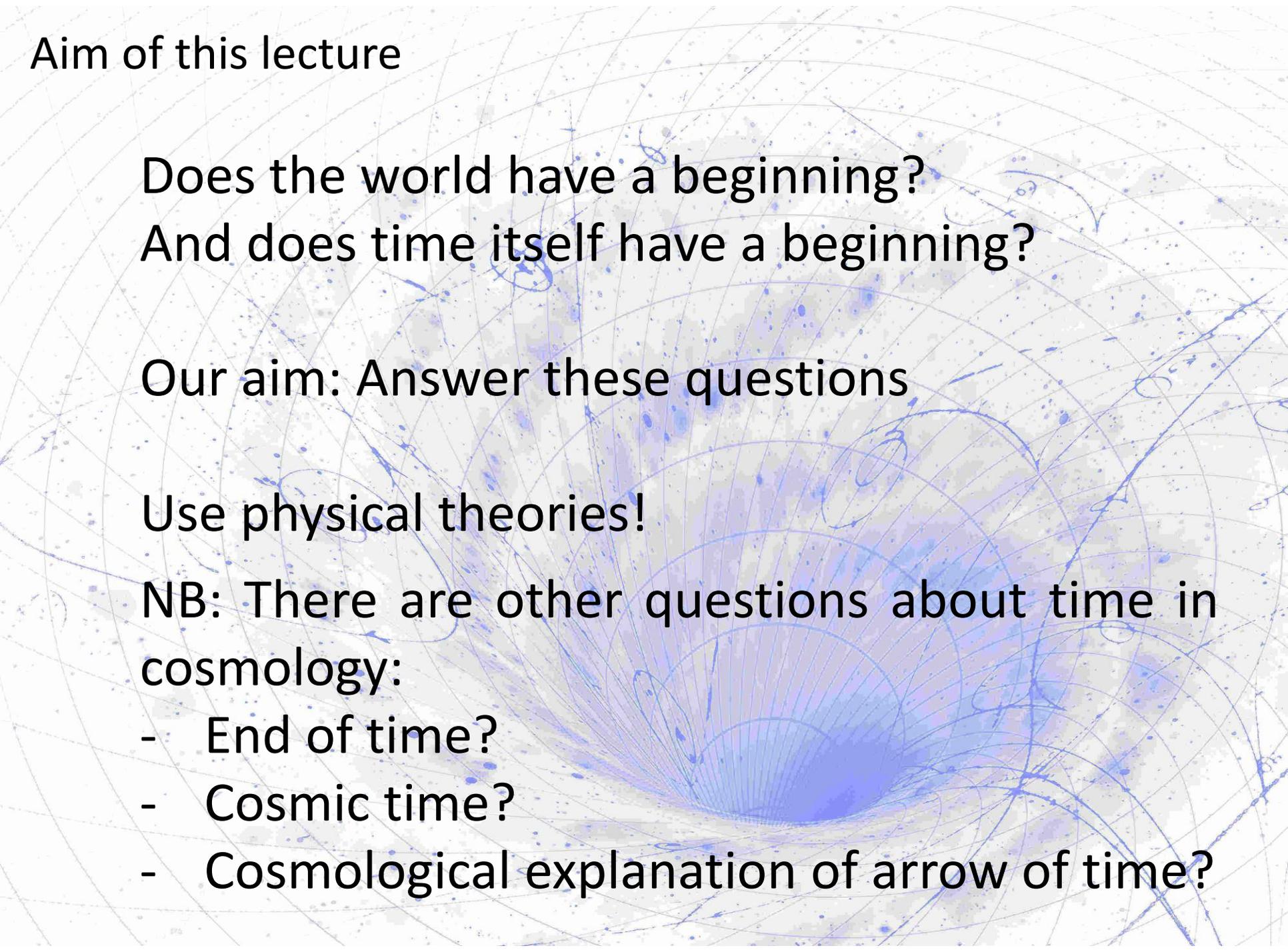
Questions

Does the world have a beginning?
And does time itself have a beginning?



Kant's view:

Reason is caught in contradiction if it tries to answer the first question (antinomy of pure reason)



Aim of this lecture

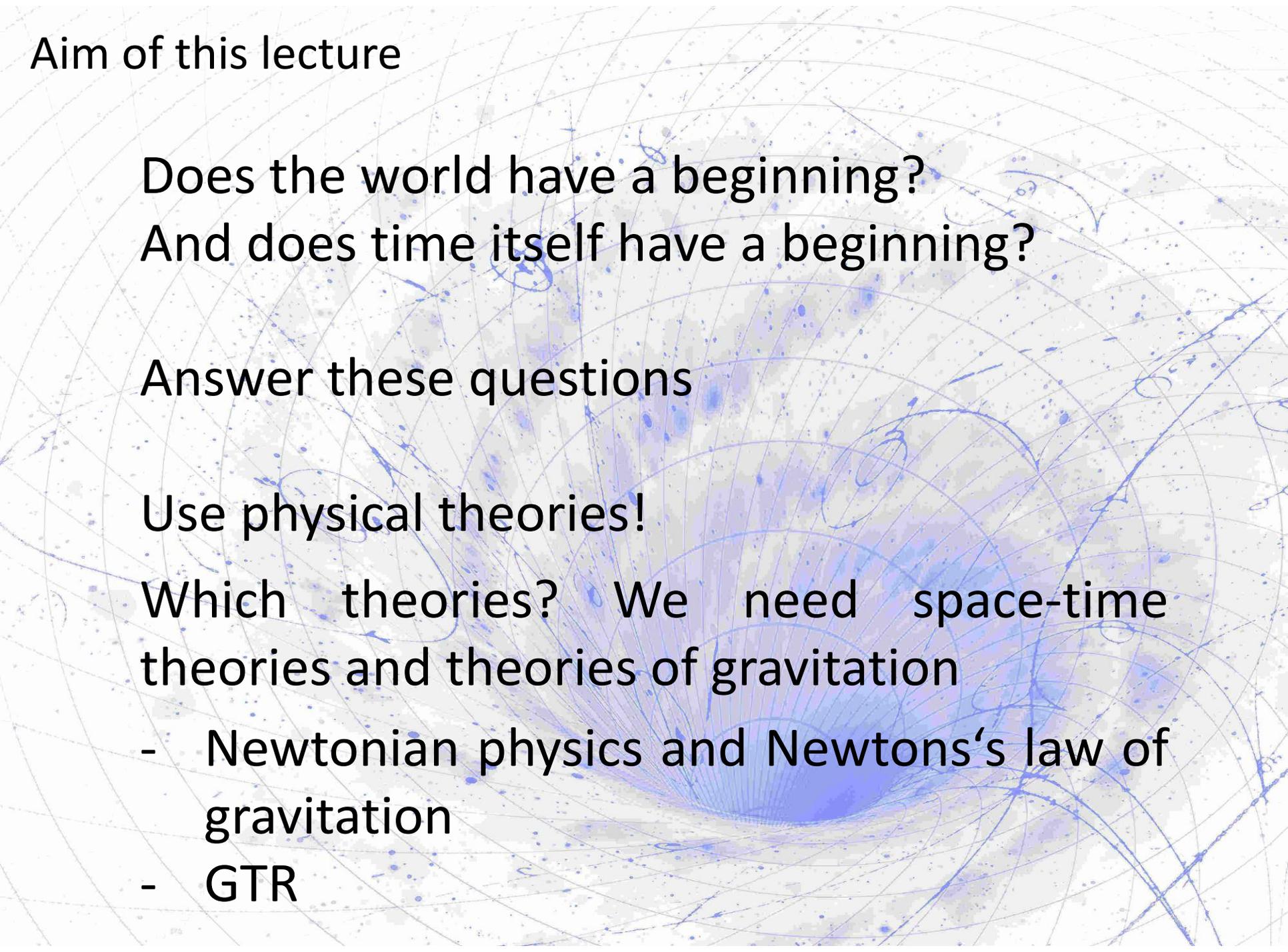
Does the world have a beginning?
And does time itself have a beginning?

Our aim: Answer these questions

Use physical theories!

NB: There are other questions about time in cosmology:

- End of time?
- Cosmic time?
- Cosmological explanation of arrow of time?



Aim of this lecture

Does the world have a beginning?
And does time itself have a beginning?

Answer these questions

Use physical theories!

Which theories? We need space-time theories and theories of gravitation

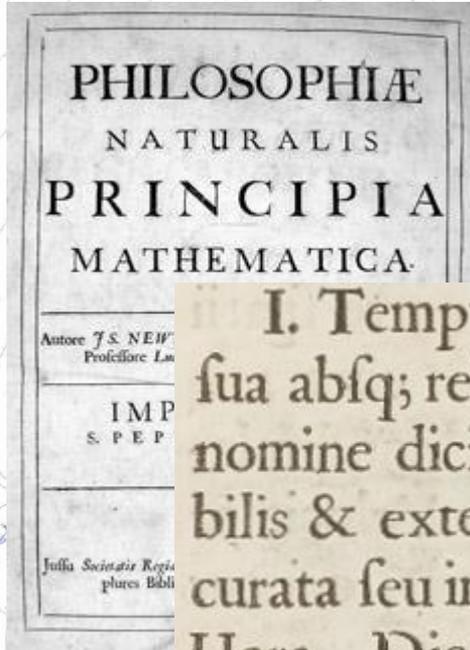
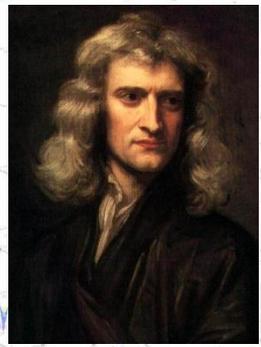
- Newtonian physics and Newton's law of gravitation
- GTR

Cf. Determinism

„no precise definition can be fashioned without making substantive assumptions about the nature of physical reality, but as we move from classical to relativistic [...] physics these assumptions vary and the definition [...] must, to some degree, covary with them.“

J. Earman 1986, p. 4

1. Newtonian physics

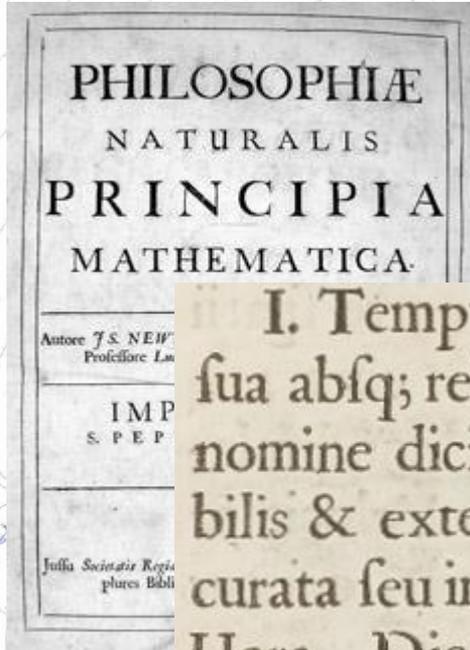
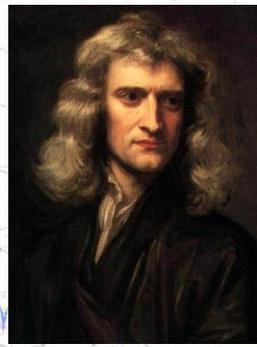


I. Tempus absolutum verum & Mathematicum, in se & natura sua absq; relatione ad externum quodvis, æquabiliter fluit, alioq; nomine dicitur Duratio; relativum apparens & vulgare est sensibilis & externa quævis Durationis per motum mensura, (seu accurata seu inæquabilis) qua vulgus vice veri temporis utitur; ut Hora, Dies, Mensis, Annus.

Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an Hour, a Day, a Month, a Year.

(transl. A. Motte, image: G. Kneller)

1. Newtonian physics



I. Tempus absolutum verum & Mathematicum, in se & natura sua absq; relatione ad externum quodvis, æquabiliter fluit, alioq; nomine dicitur Duratio; relativum apparens & vulgare est sensibilis & externa quævis Durationis per motum mensura, (seu accurata seu inæquabilis) qua vulgus vice veri temporis utitur; ut Hora, Dies, Mensis, Annus.

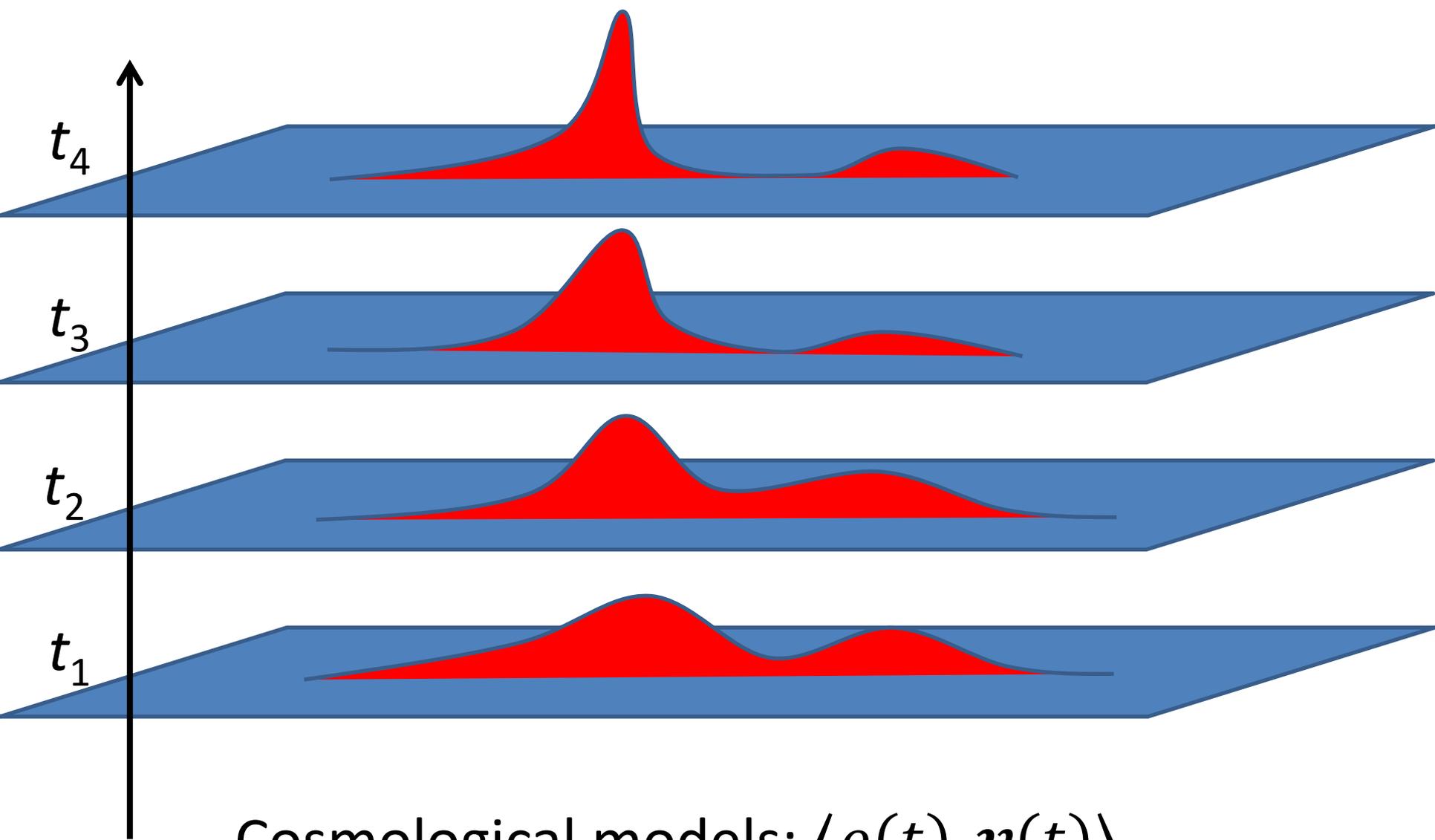
General scholium: God is eternal, so time can't have an end or a beginning.

Space-time in Newtonian physics



No beginning of time (2nd question answered)

Cosmological models



Cosmological models: $\langle \rho(t), \mathbf{v}(t) \rangle$

A beginning of the world?

Question: Does the world have a beginning?



Thesis: Yes

„The world has a beginning in time“

Antithesis: No

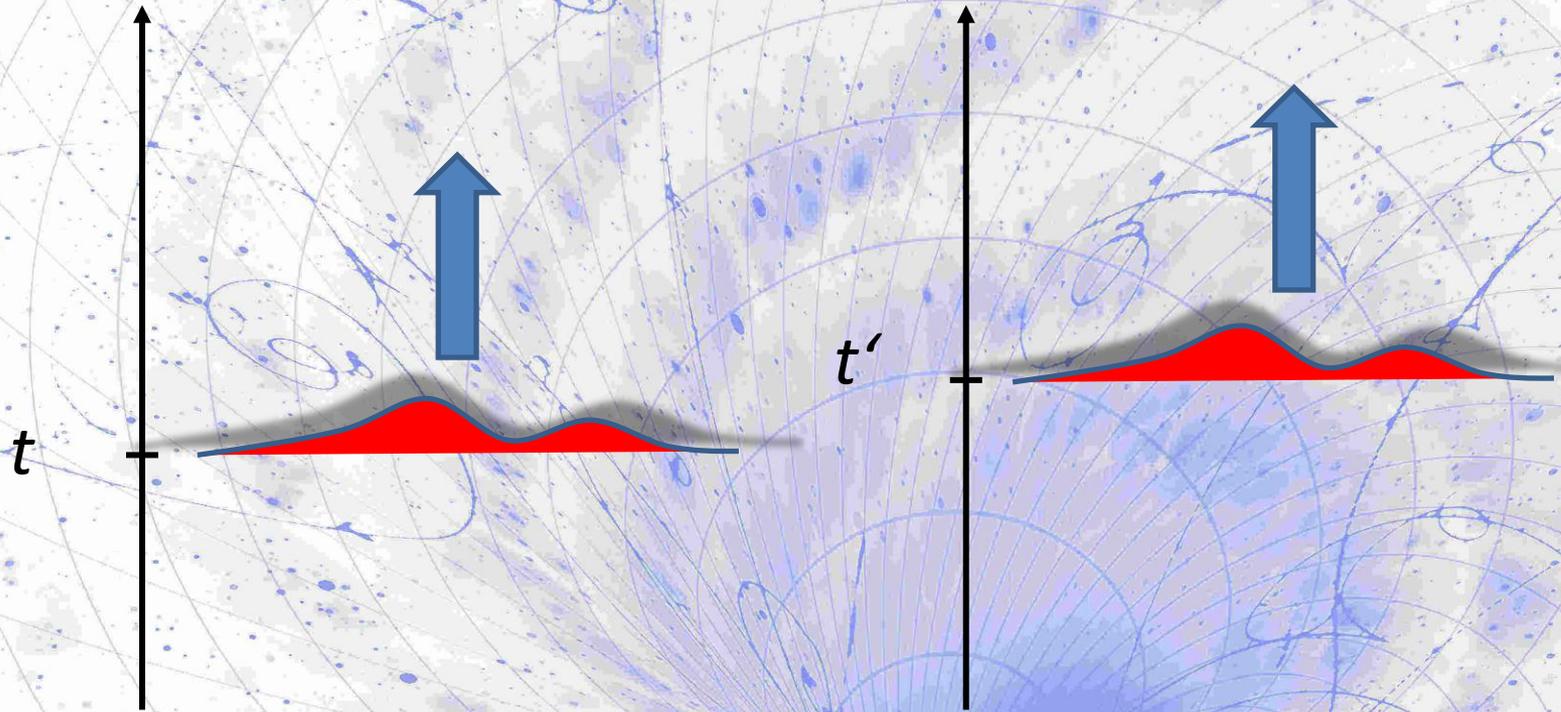
„The world has no beginning in time [...] and is [...] infinite concerning time“

Critique of Pure Reason, A426-7/B454-5

Translation CB

Kant on the beginning

If there is a beginning , ...

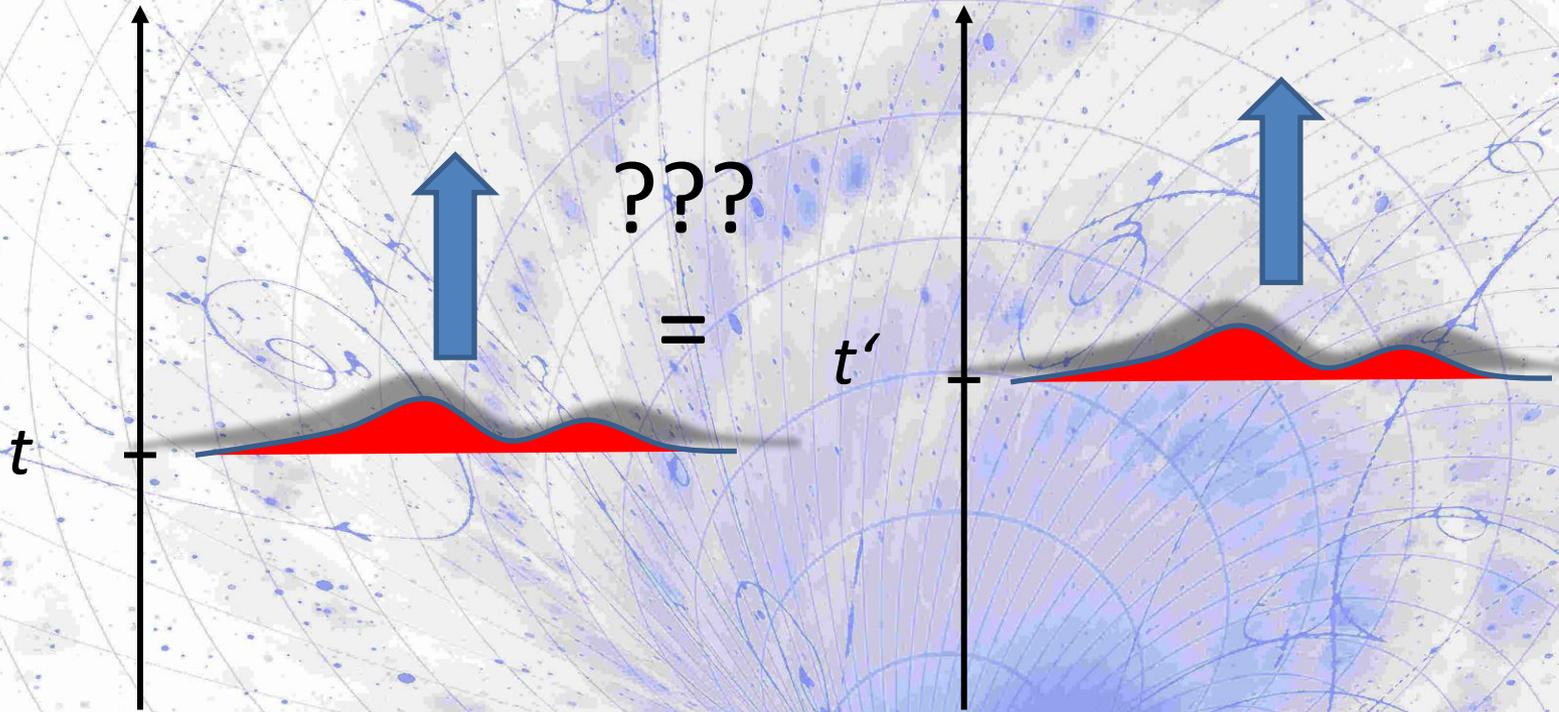


Principle of Sufficient Reason is violated.

Why not?

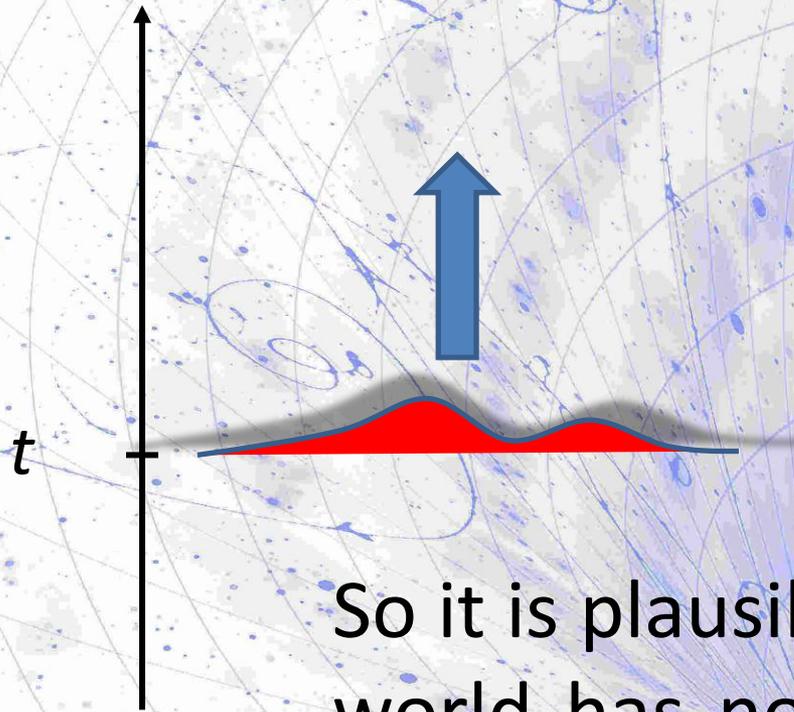
A presumption

The argument works only ...



... if substantivalism about time is assumed.

A more worrisome problem



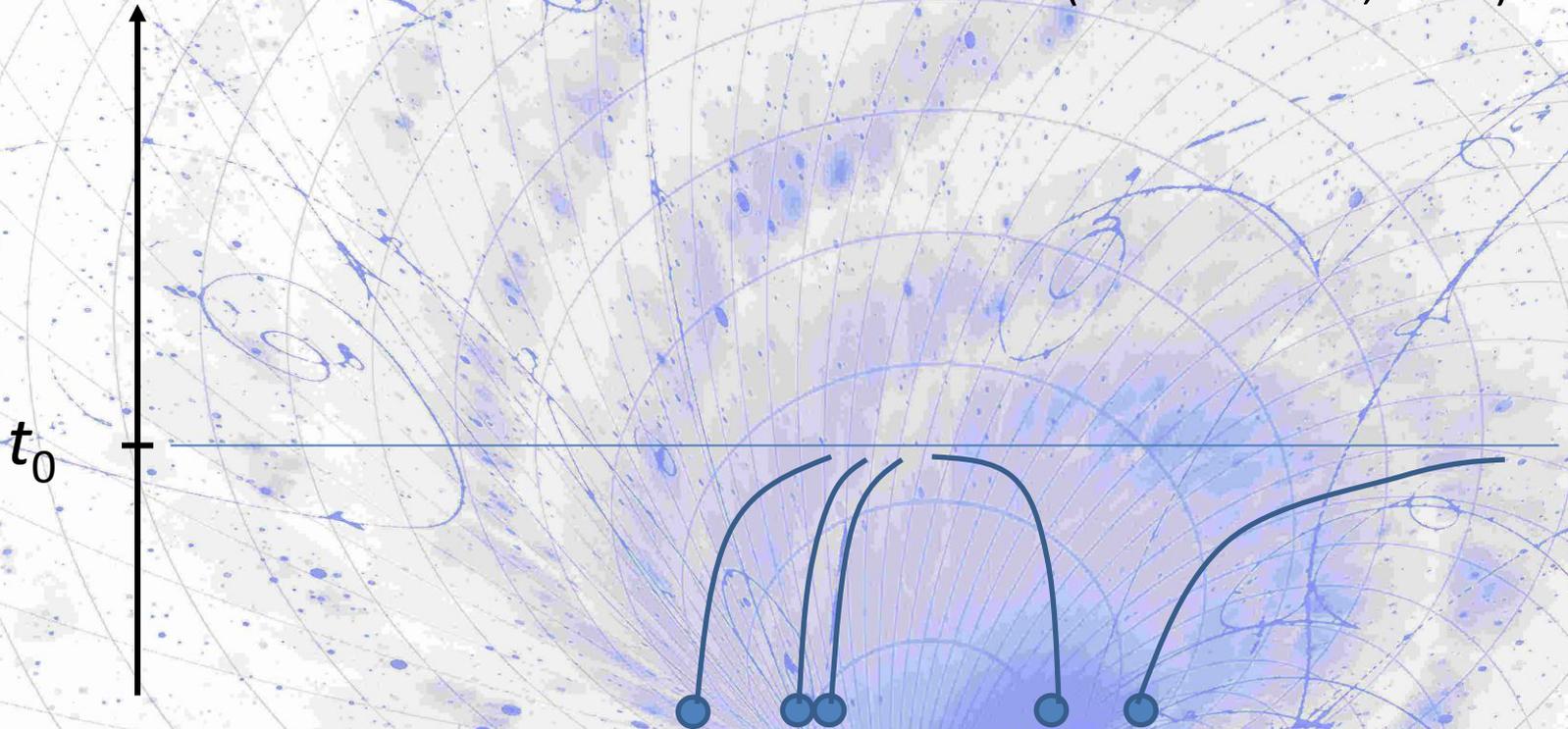
The usual conservation laws are violated.

So it is plausible to assume that the world has no beginning in time in Newtonian physics.

But

There are „solutions“ in which particles can arise from nothing

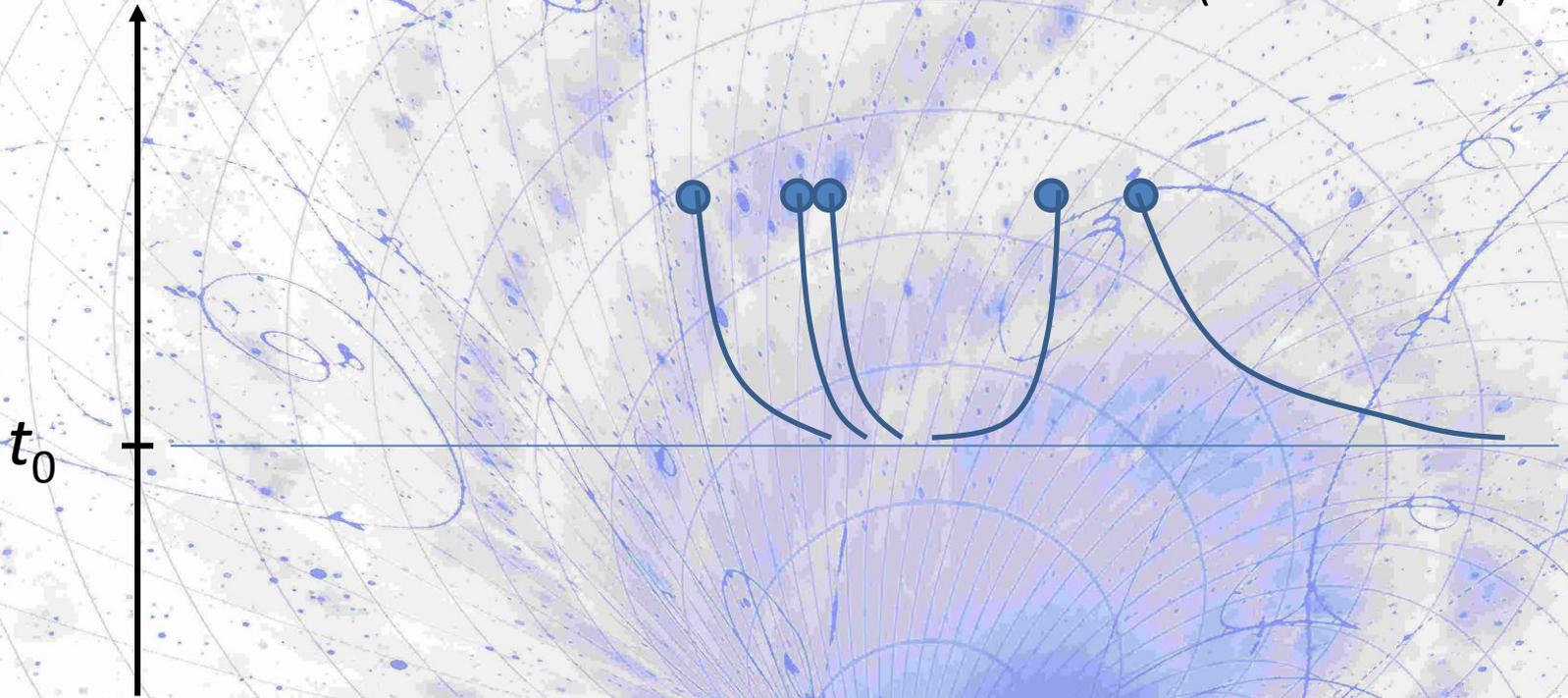
(Earman 1986, Ch. 3)



But

There are „solutions“ in which particles
can arise from nothing

(Earman 1986)



No first time of the world, but it's pretty close.

But

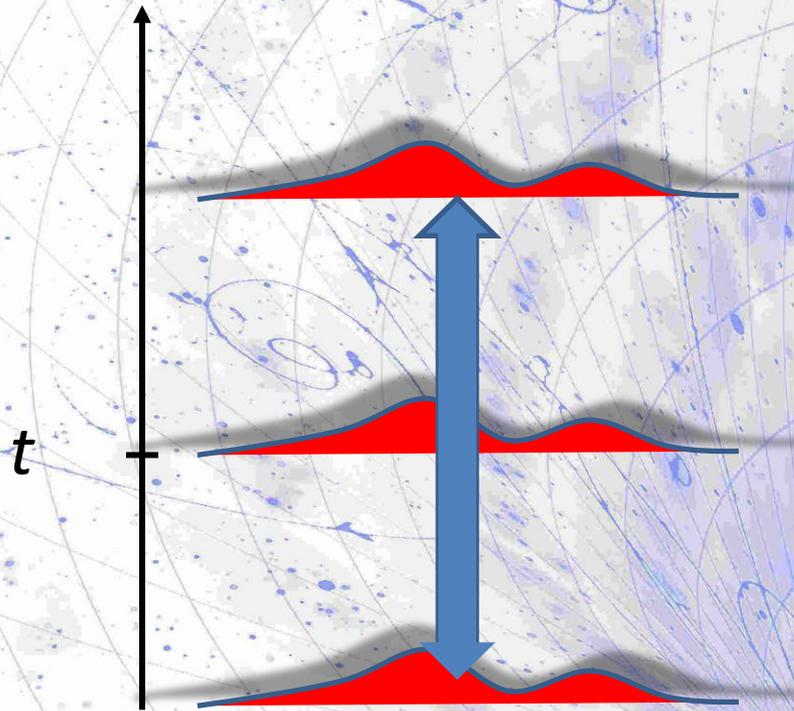
There are some reasons to discard these solutions from the outset:

- Conservation laws are violated.
- There is a singularity at t_0 , technically, we don't have a full solution at t_0

Conclusion: In Newtonian physics, there cannot plausibly be a beginning of the world.

Kant's worries about the antithesis

Suppose, the world does not have a beginning in time, ...



... then it would have existed for an infinity of moments,

which is impossible because an infinite series of states cannot yet have finished.

What is Kant's argument???

Explaining Kant's worries

Grounding:

- Earlier states of the world are the basis for later states.
- If there is no first state, then all states are ungrounded.

If this is the correct understanding of the argument, then the argument is not very convincing.

Newtonian cosmology

Kant: There are problems with the spatial distribution of stuff as well.

We face a theoretical dilemma:

alternative 1: matter is confined in space



a. Principle of sufficient reason violated

b. The solutions are not stationary

c. Many solutions evolve into singularities

NB: Newtonian cosmology

Problems with the spatial distribution of stuff as well

alternative 2: matter is not so confined¹



¹More analysis is needed to establish a theoretical dilemma.

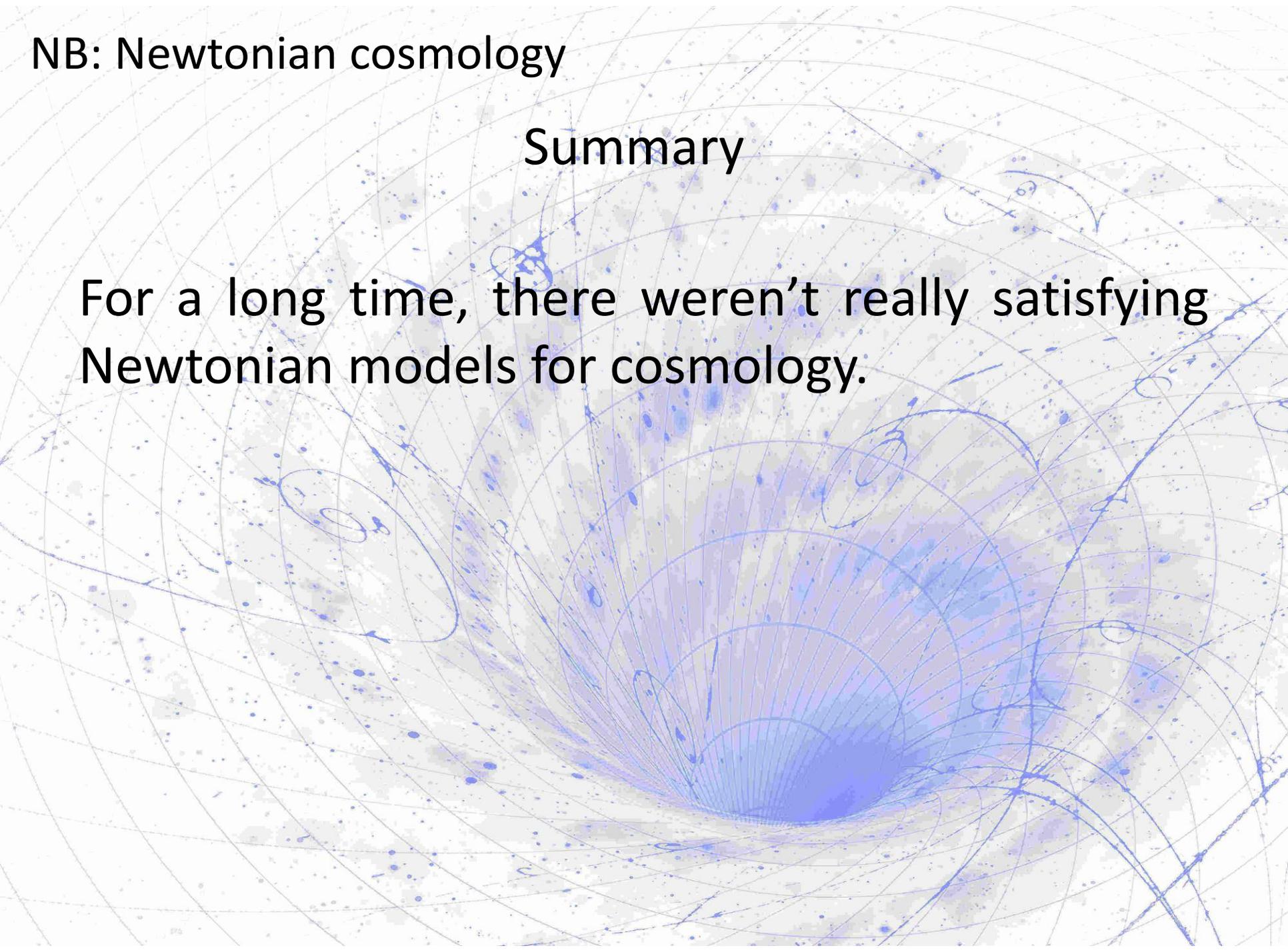
Expressions for forces diverge

see e.g. Norton 2000

NB: Newtonian cosmology

Summary

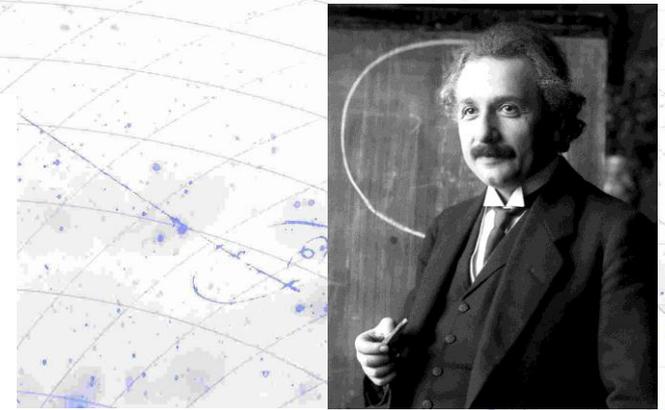
For a long time, there weren't really satisfying Newtonian models for cosmology.



2. General relativity

Zur allgemeinen Relativitätstheorie.

VON A. EINSTEIN.



Die formale Grundlage der allgemeinen Relativitätstheorie.

VON A. EINSTEIN.

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation

VON A. EINSTEIN.

1916.

№ 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage
der allgemeinen Relativitätstheorie;
von A. Einstein.*

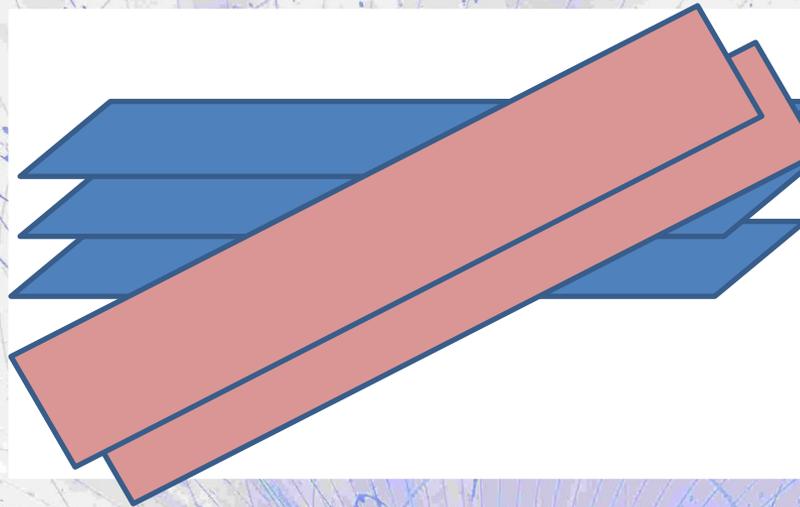
General relativity

Consequences for time and cosmology

- space-time not fixed a priori; a great variety of space-time manifolds are possible depending on the matter distribution. Thus, cosmological models characterize space-time and the matter distribution: $\langle M, g, T \rangle$ with M manifold, g metric and T energy momentum tensor.

General relativity

- In general, a space-time can be foliated in a number of different ways into space-like hypersurfaces.



- There are a lot of complicated space-times, so the question is what a beginning of time/the world is supposed to mean.
In the following focus on beginning of time.

Cosmological models

To learn more about time in cosmology, consult cosmological models.

Constraints on cosmological models?

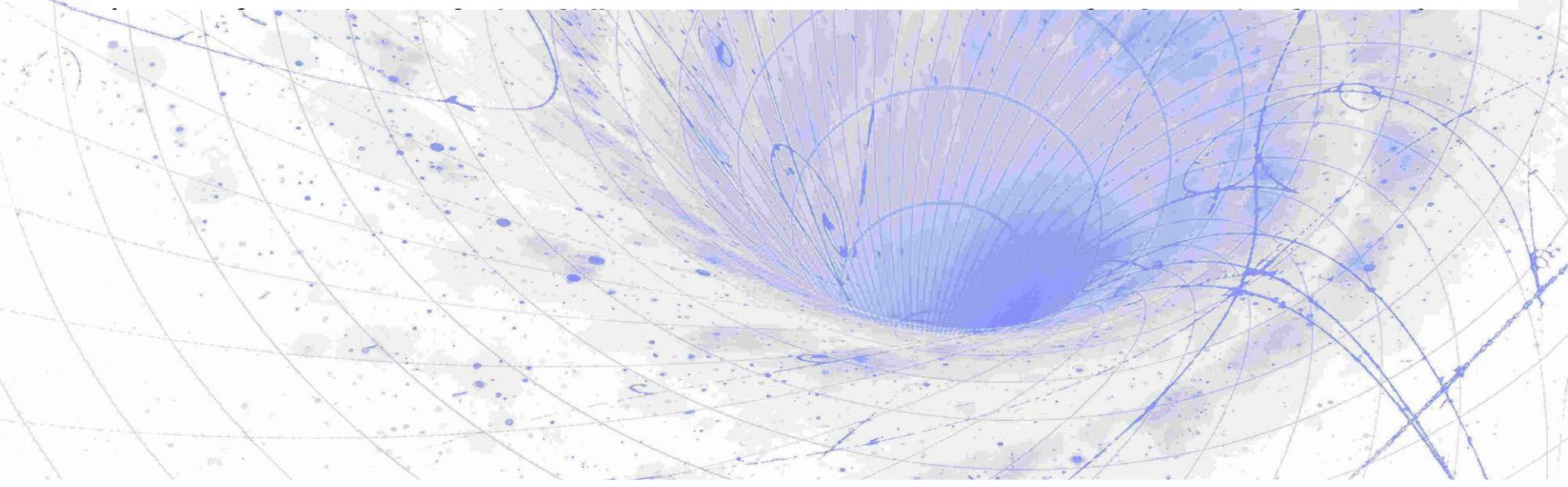
- accordance with evidence about large-scale structure of space-time and matter distribution

But evidence is systematically restricted, underdetermines choice of model

Cosmological models

Cosmologist G. Ellis (1991, p. 553)

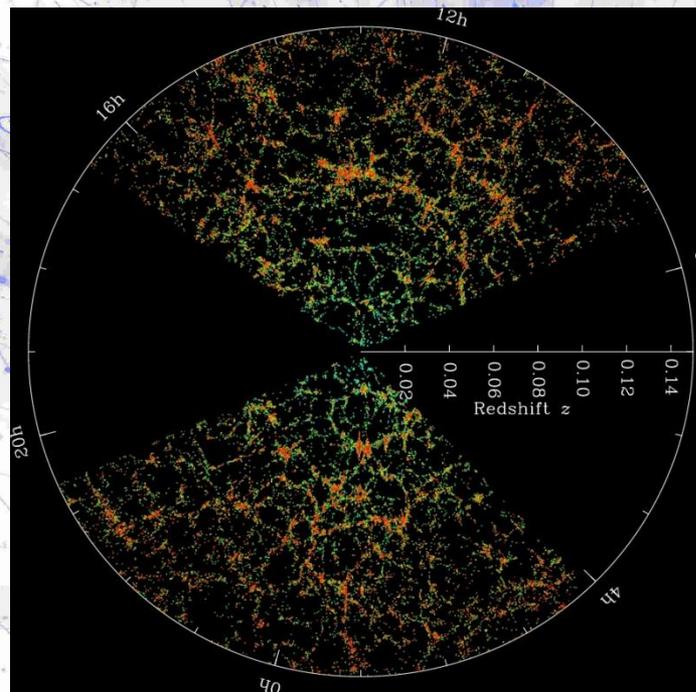
However there are substantially different approaches to the subject, based on different philosophical understandings; and the degree to which models of the universe are satisfactory depends to a considerable degree on the approach taken (indeed even the concept of what is a “satisfactory model” depends on the philosophical stance adopted).



The Cosmological Principle

Rough idea: The Universe is spatially homogeneous and isotropic about every point.

Motivation (1): The distribution of galaxies looks homogeneous and isotropic to us.

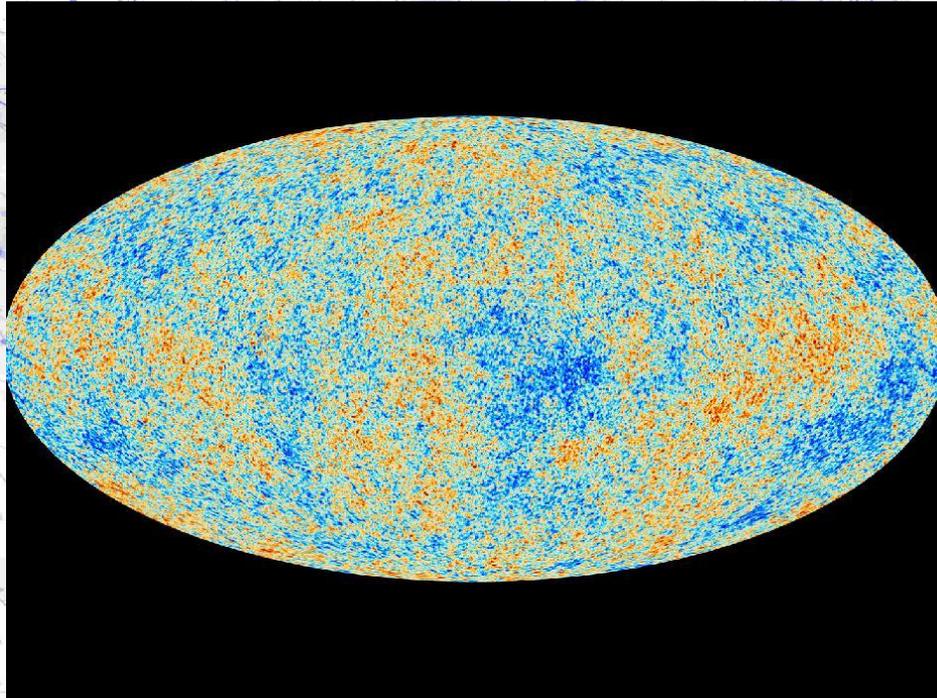


Data: SDSS

The Cosmological Principle

Rough idea: The Universe is spatially homogeneous and isotropic about every point.

Motivation (2): The distribution of light from the Cosmic Microwave Background looks isotropic to us.



Data: Planck

The Cosmological Principle

Rough idea: The Universe is **spatially homogeneous** and isotropic about every point.

Why only *spatial* homogeneity (cf. the perfect cosmological principle, cf. Einstein whose first cosmological model was static)?

Answer: the spectra of other galaxies mostly appear redshifted, which means that there is a dynamical evolution (expansion).

Hubble's result

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

BY EDWIN HUBBLE

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Communicated January 17, 1929

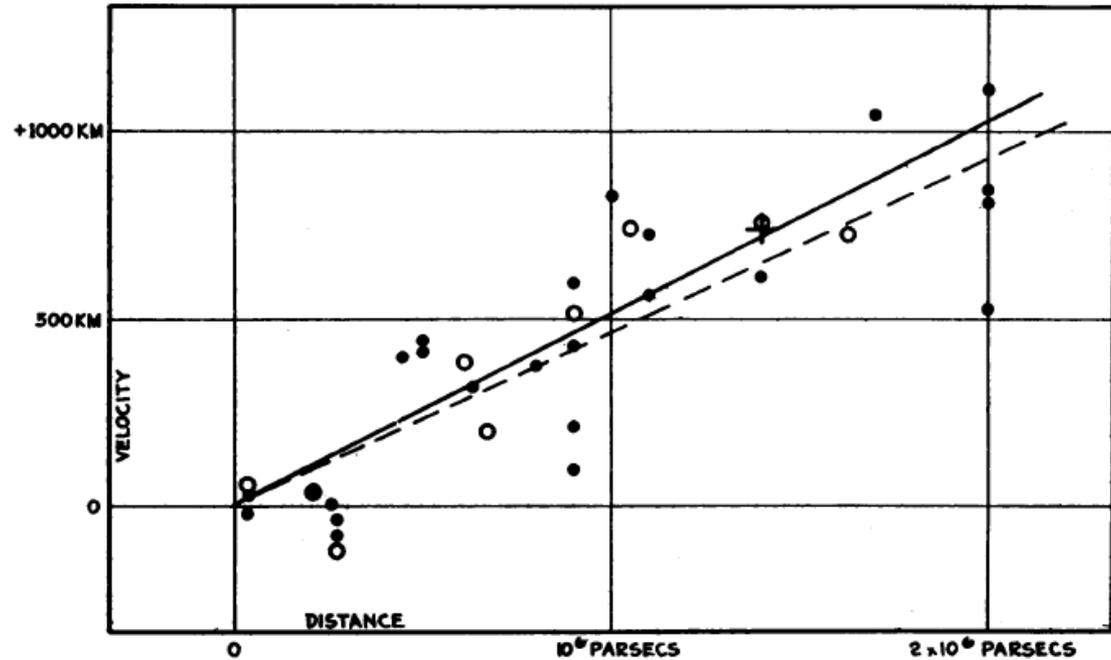


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

The Cosmological Principle

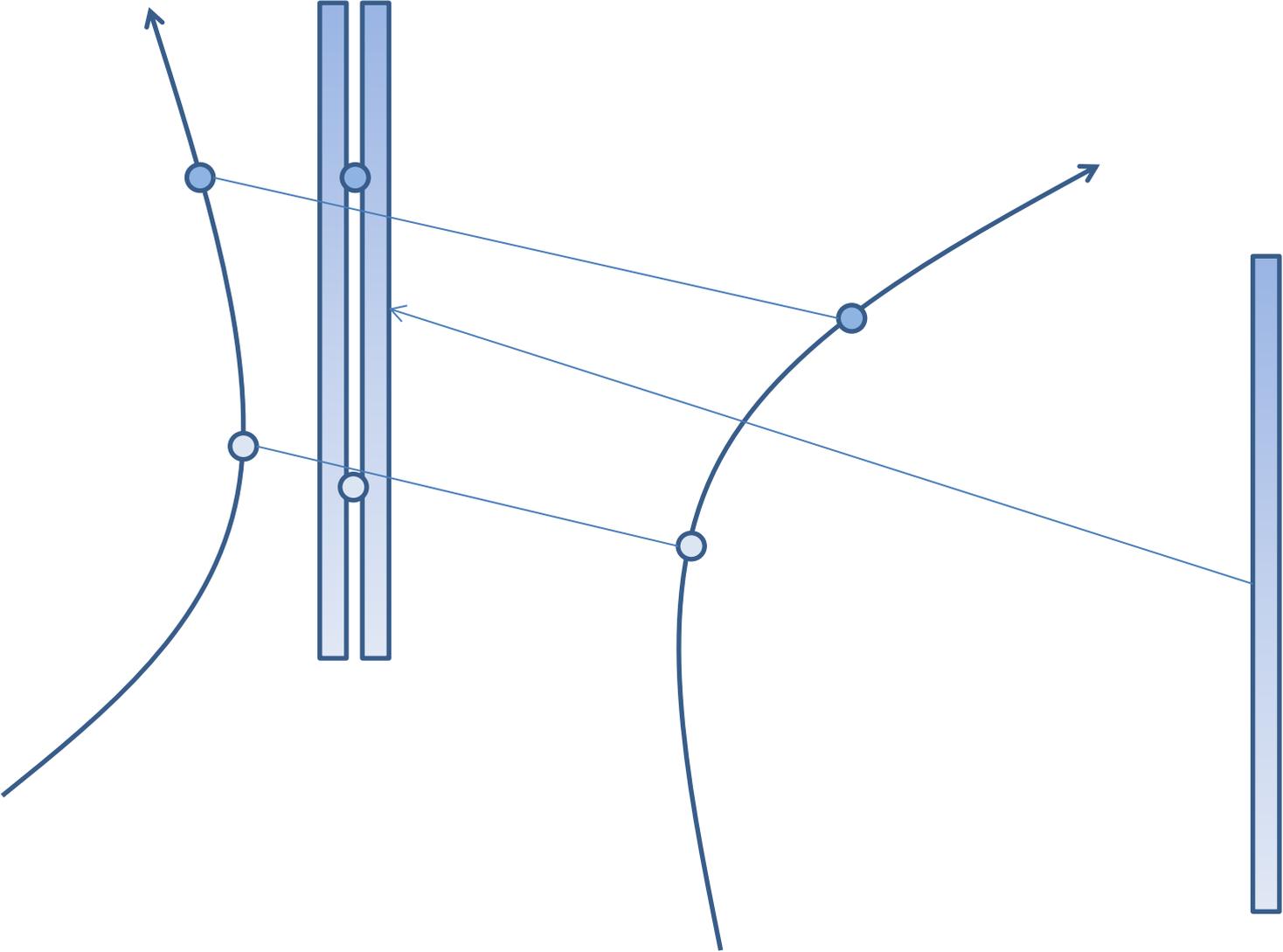
Rough idea: The Universe is **spatially homogeneous** and isotropic about every point.

What is spatial homogeneity?

Answer: Assume, the Universe is filled with a cosmic one-component fluid. We assume dust (no pressure), thus fluid elements take geodesics. Let fundamental observers move with the cosmic fluid.

The idea: The histories of observations that fundamental observers take can be mapped onto each other

Illustration



The Cosmological Principle

Consequence: There is a foliation of space-time into space-like homogeneous hypersurfaces.

(i.e., each point in the hypersurface can be mapped to each other using an isometry, which leaves metric structure and matter distribution invariant)



Wald 1984, pp. 92 – 93

Consequences for time

In coordinates in which t measures proper time of fundamental observers, we have

$$ds^2 = dt^2 - g_{ab}(t) dx^a dx^b$$

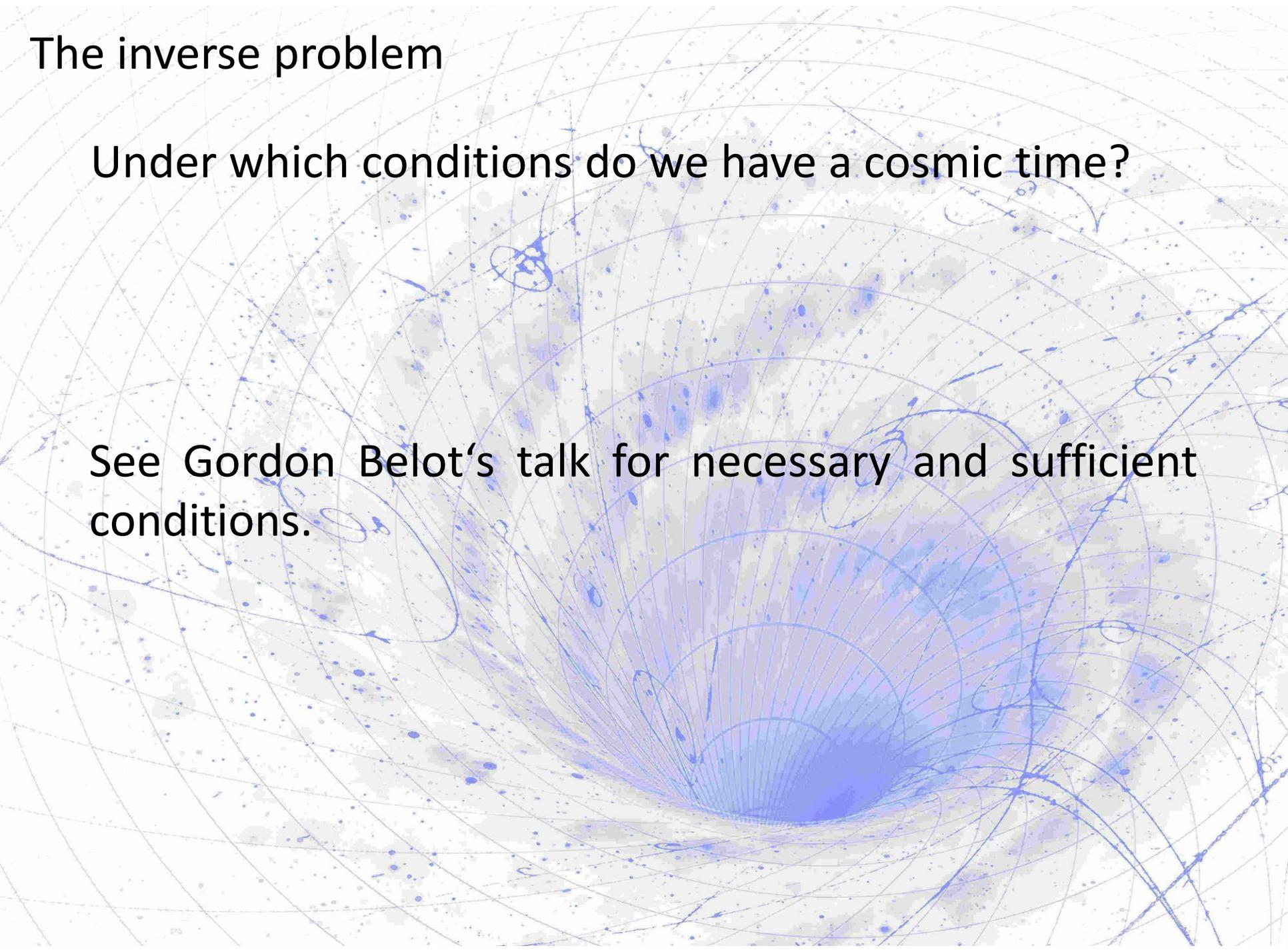
This means that we have a privileged time coordinate: time as measured by the fundamental observers is “intersubjective”: cosmic time.

NB: Our cosmic time is both „dust time“ and „time from symmetry“ (cf. Belot’s handout).

The inverse problem

Under which conditions do we have a cosmic time?

See Gordon Belot's talk for necessary and sufficient conditions.



The Cosmological Principle

Rough idea: The Universe is spatially homogeneous and isotropic about every point.

What is spatial isotropy?

Answer: Every observer cannot distinguish between different directions in terms of her observations.

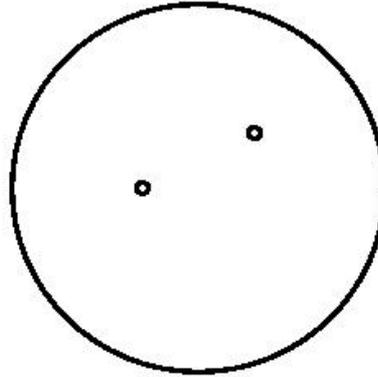
Homogeneity plus isotropy:

The space-like hypersurfaces have constant curvature, line element:

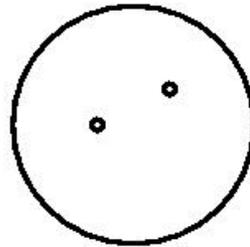
$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

The scale factor

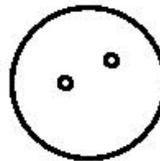
$$a(t_3) = 3.5$$



$$a(t_2) = 2$$



$$a(t_1) = 1$$



$a(t)$ is the scale factor.

The evolution of the scale factor

Einstein's field equations yield for the scale factor
Raychaudhuri's equation:

$$3 \frac{\ddot{a}(t)}{a(t)} = -4\pi G c^2 \rho + \Lambda$$

Friedman's equation:

$$3 \frac{\dot{a}^2(t)}{a^2(t)} = 8\pi G \rho - \frac{3kc^2}{a^2(t)} + \Lambda c^2$$

Solutions depend on the values of

- curvature k
- Density ρ
- Cosmological constant Λ

Beware topology

Friedman's equation derives from a partial differential equation, for a full solution we need boundary conditions or topological constraints on the universe.

Concerning „cosmic space“ a lot of things can be done. For instance, in a flat universe ($k = 0$), one can impose periodic boundary conditions for space.

Nevertheless, for some choices of the parameters, Friedman's equation has non-trivial consequences.

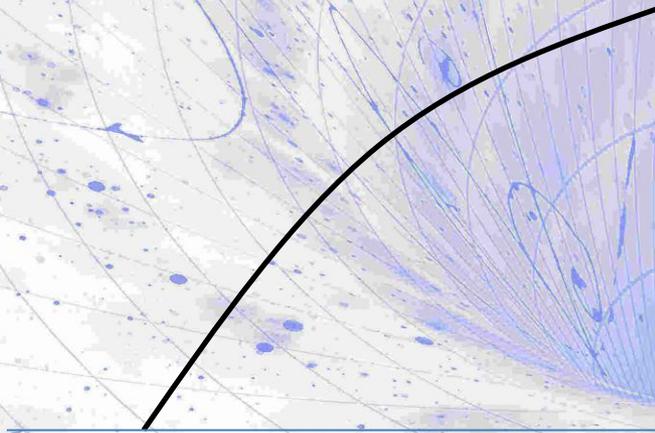
So let's go back in time.

Back in time

Friedman-Lemaître universe singularity theorem
(Ehlers, Ellis)

$$3 \frac{\ddot{a}(t)}{a(t)} = -4\pi G c^2 \left(\rho + p/c^2 \right) + \Lambda$$

For „normal“ matter and a negative Λ we obtain a concave function



Scale factor $a \rightarrow 0$, thus $\rho \rightarrow \infty$: singularity

Some solutions

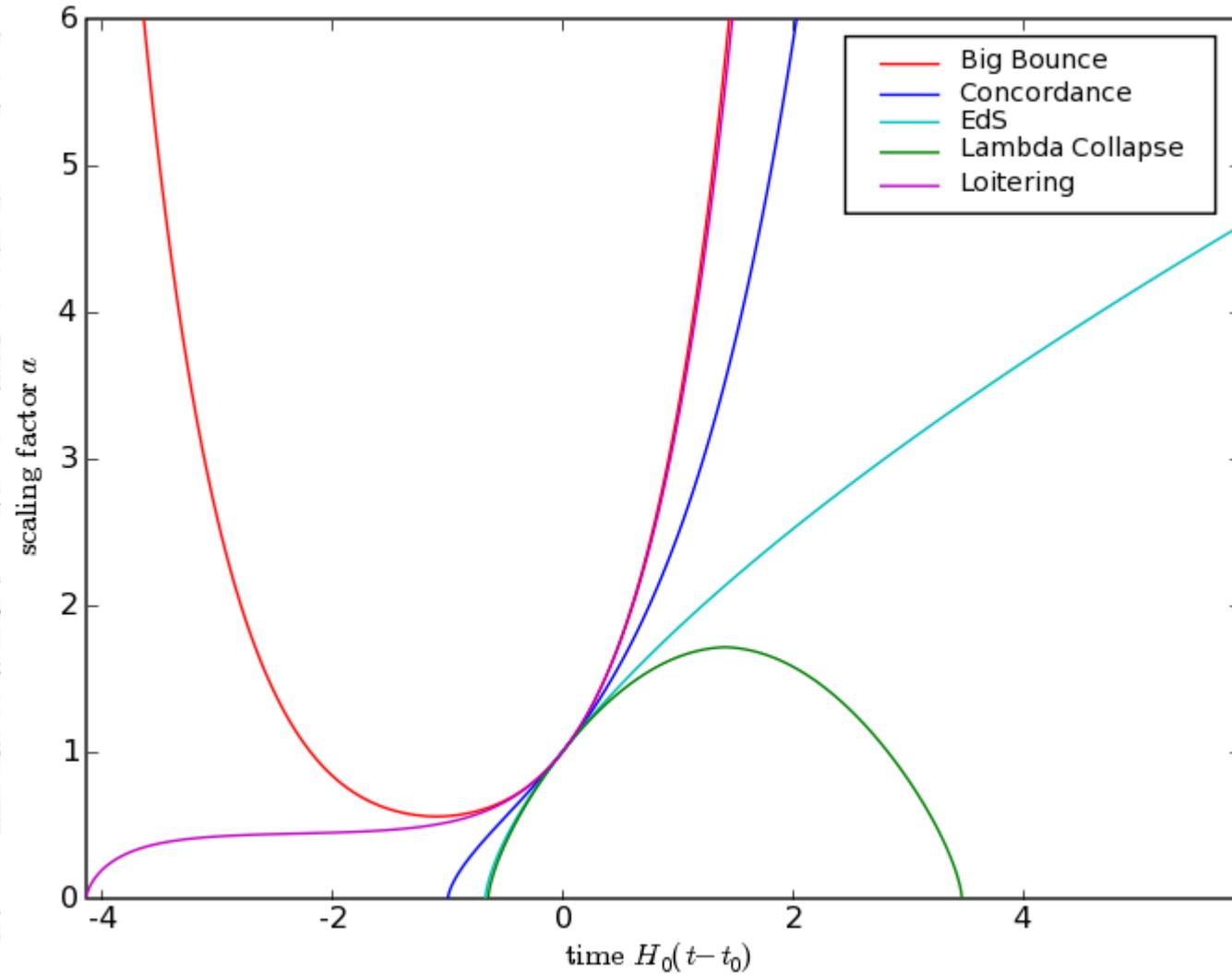


Image: J. Hidding

The scale factor

$$3 \frac{\dot{a}^2(t)}{a^2(t)} = 8\pi G\rho - \frac{3kc^2}{a^2(t)} + \Lambda c^2$$

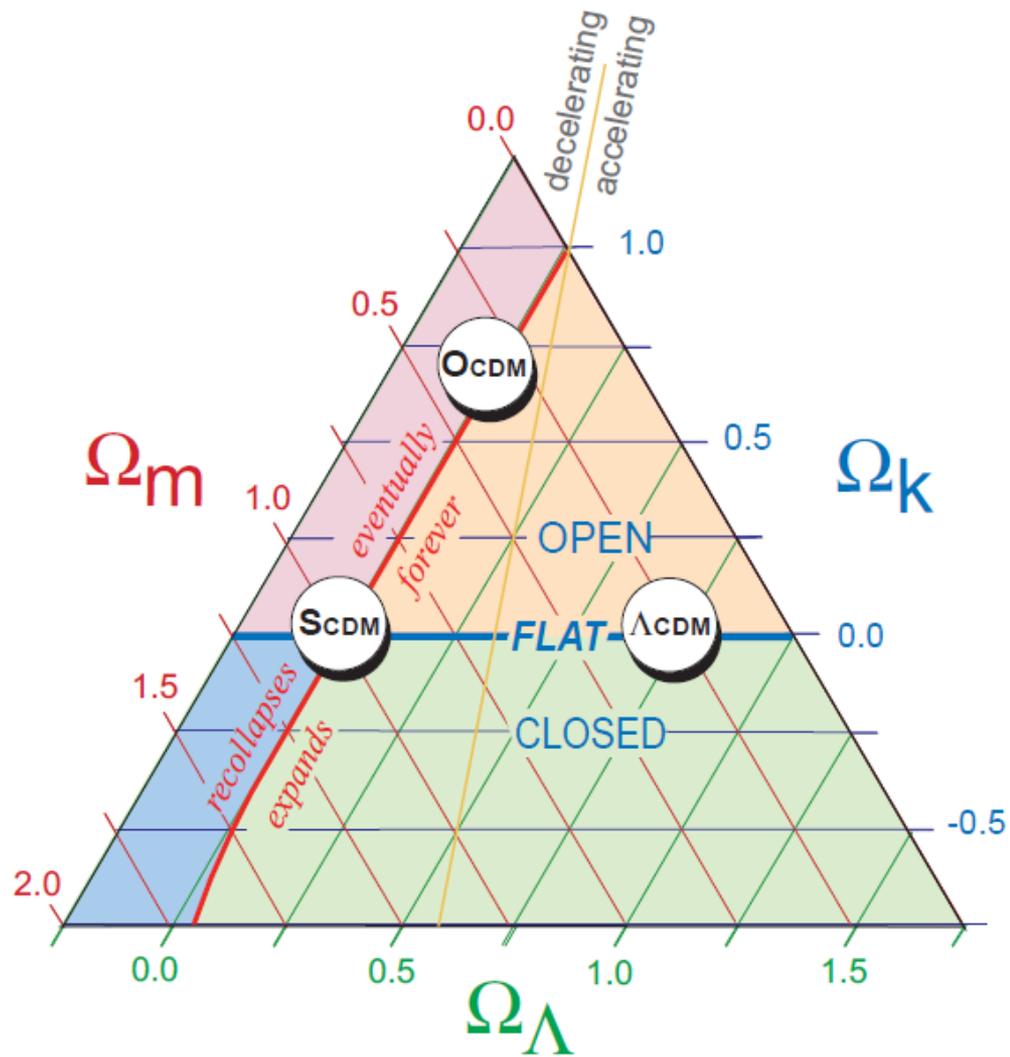
Convenient parametrization for dust:

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

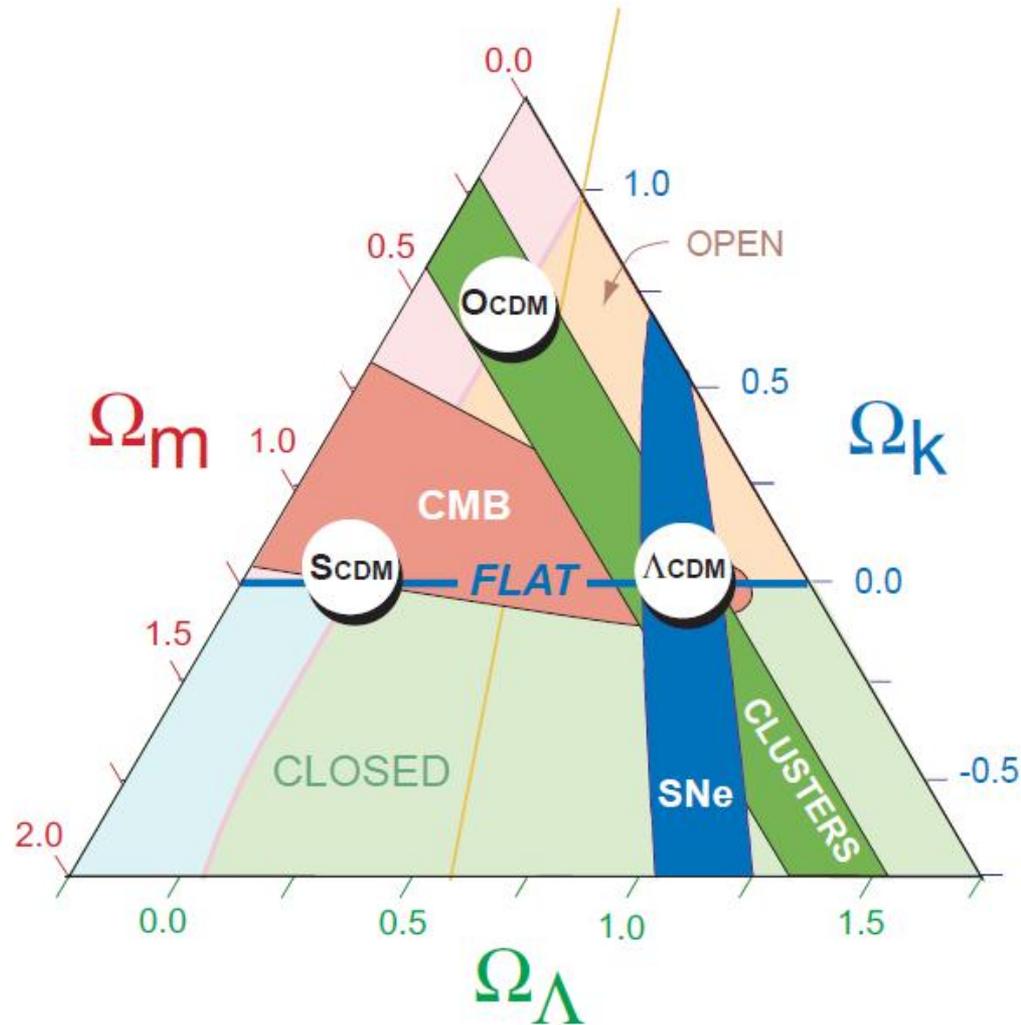
$$1 = \Omega_m(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

Cosmological parameters

Phase diagrams of solutions



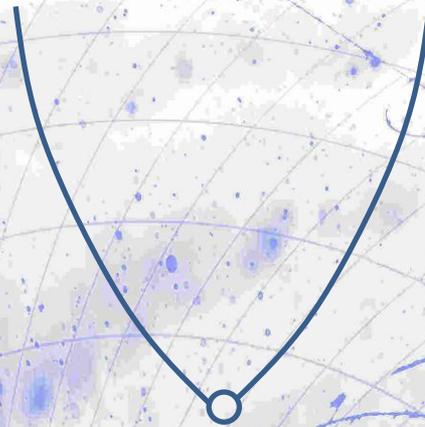
Phase diagrams of solutions plus constraints from data



Initial singularity

Notes:

- There is no point in the manifold at which the equations of GTR don't hold.
- There is no first cosmic time.
- If fundamental observers move back in time, their geodesics cannot be extended after some finite amount of time (geodesic incompleteness)



Initial singularity

Can we take this seriously?

Cf. Smeenk 2013



A singularity of this sort is unphysical.



Well, there is not point in space-time at which density etc. diverges. Is there a general prohibition on geodesic incompleteness?

Initial singularity

Can we take this seriously?



The initial singularity arises only in highly symmetric solutions to the Einstein equations.



This is not true, a singularity theorem due to Hawking and Penrose proves that there are incomplete geodesics under quite general assumptions. Cosmologists argue that the conditions for the theorem hold and that they point to an initial singularity. Ellis and Hawking 1973, Chs. 8 and 10

Initial singularity

Can we take this seriously?

Near the initial singularity, the models don't tell us anything about time because the notion of time isn't applicable any more. There is no suitable physical basis for clocks and measurement of time in the early Universe. Rugh and Zinkernagel 2009



Well, this sounds very operationalist and thus not very convincing. Cosmologists use well-established relationships between time and other characteristics to define time.

Initial singularity

Can we take this seriously?



For times smaller than the so-called Planck time, GTR breaks down.



Well, this may be true, but we don't yet have a theory of quantum gravity.
See the lecture by Chris Wüthrich.

Particle horizons

Question: from what part of the universe can we have obtained information thus far?

Light: null geodesics

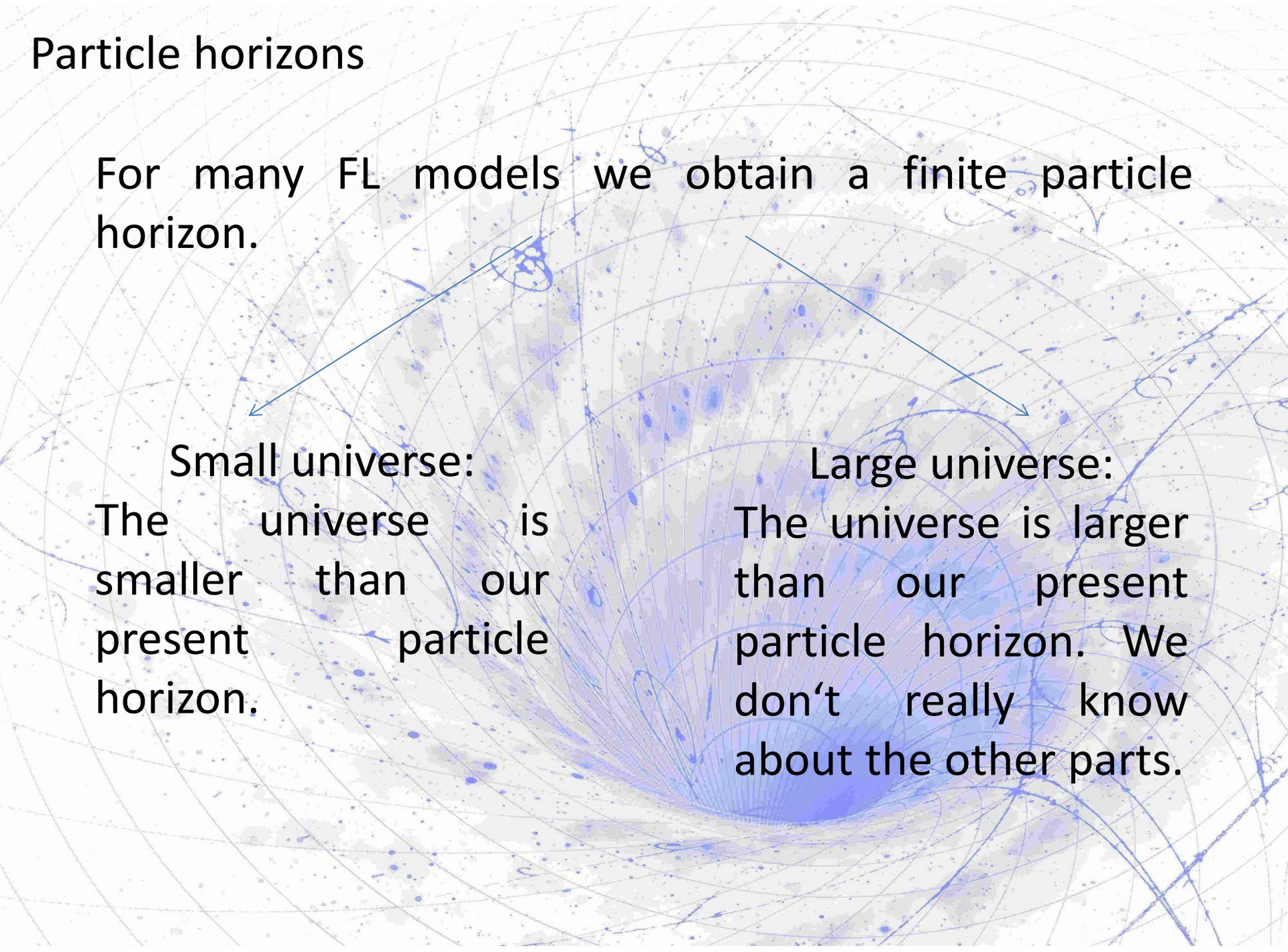
$$ds = 0$$

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) = 0$$

$$\frac{dt}{a(t)} = \frac{dr}{\sqrt{1 - kr^2}}$$

Integration from initial singularity to now gives present particle horizon.

Particle horizons



For many FL models we obtain a finite particle horizon.

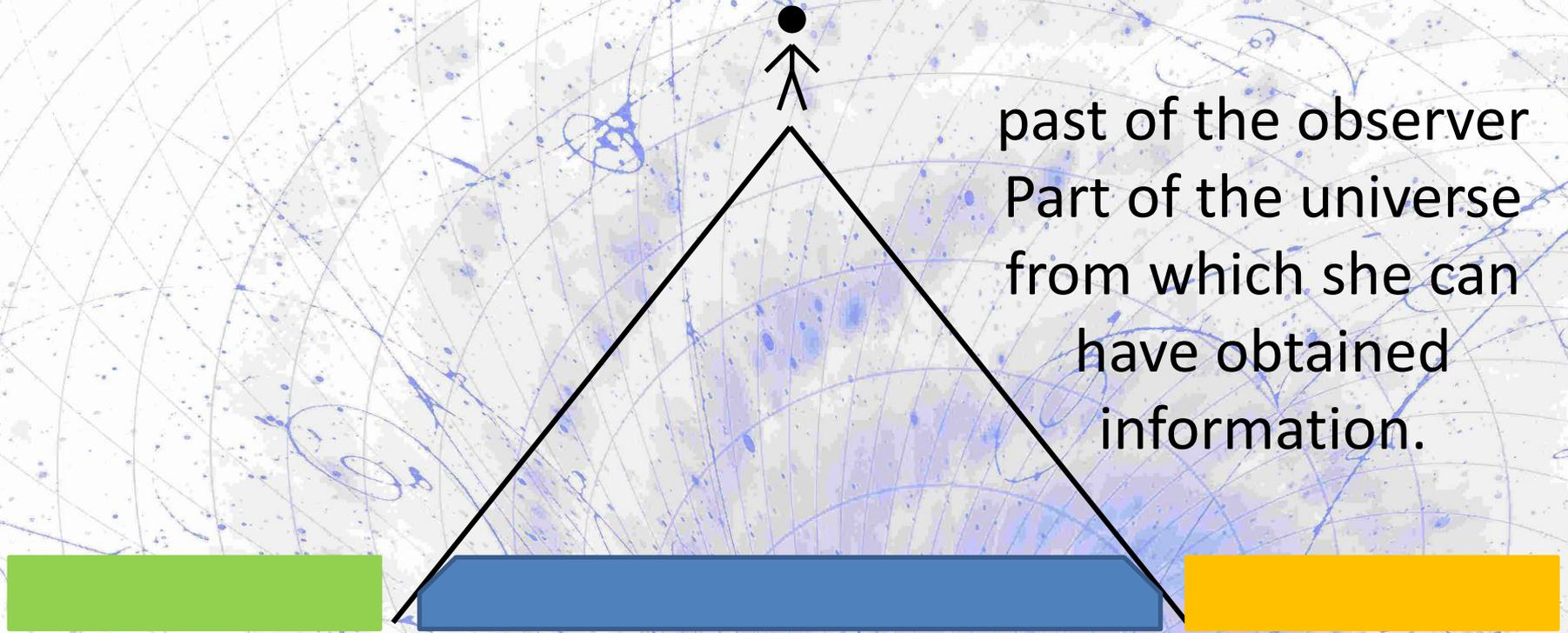
Small universe:

The universe is smaller than our present particle horizon.

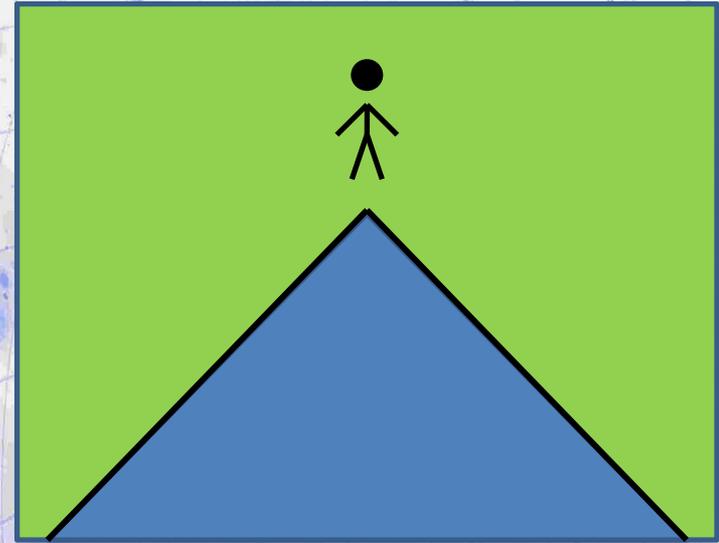
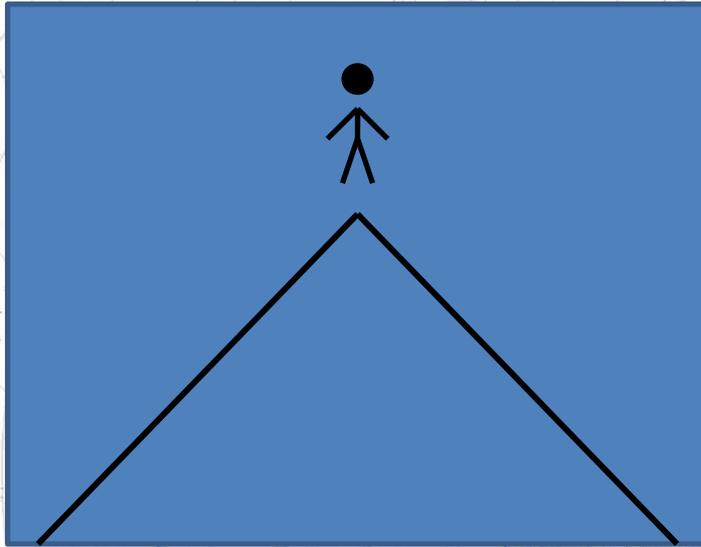
Large universe:

The universe is larger than our present particle horizon. We don't really know about the other parts.

Particle horizons

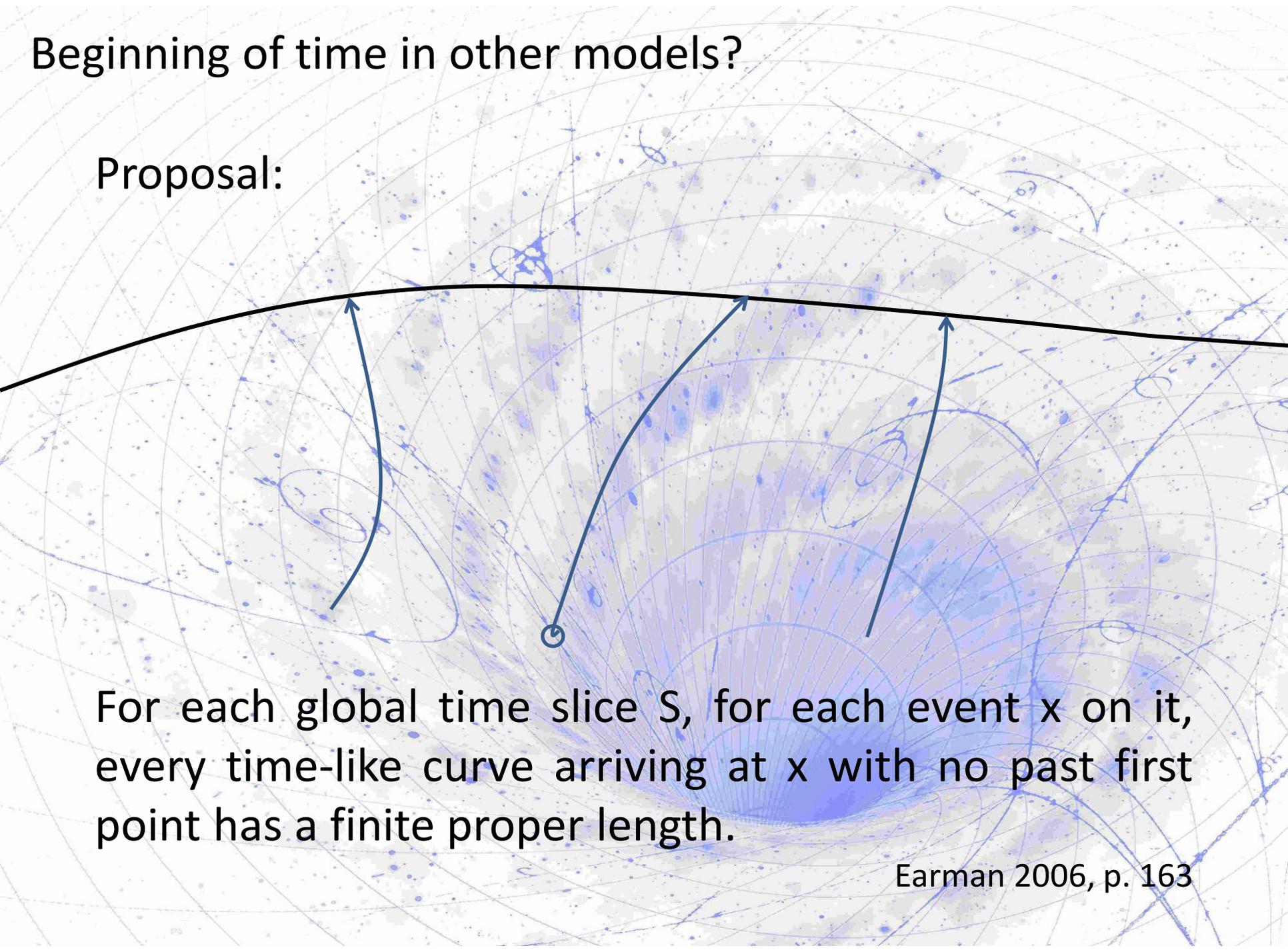


Observational indistinguishability



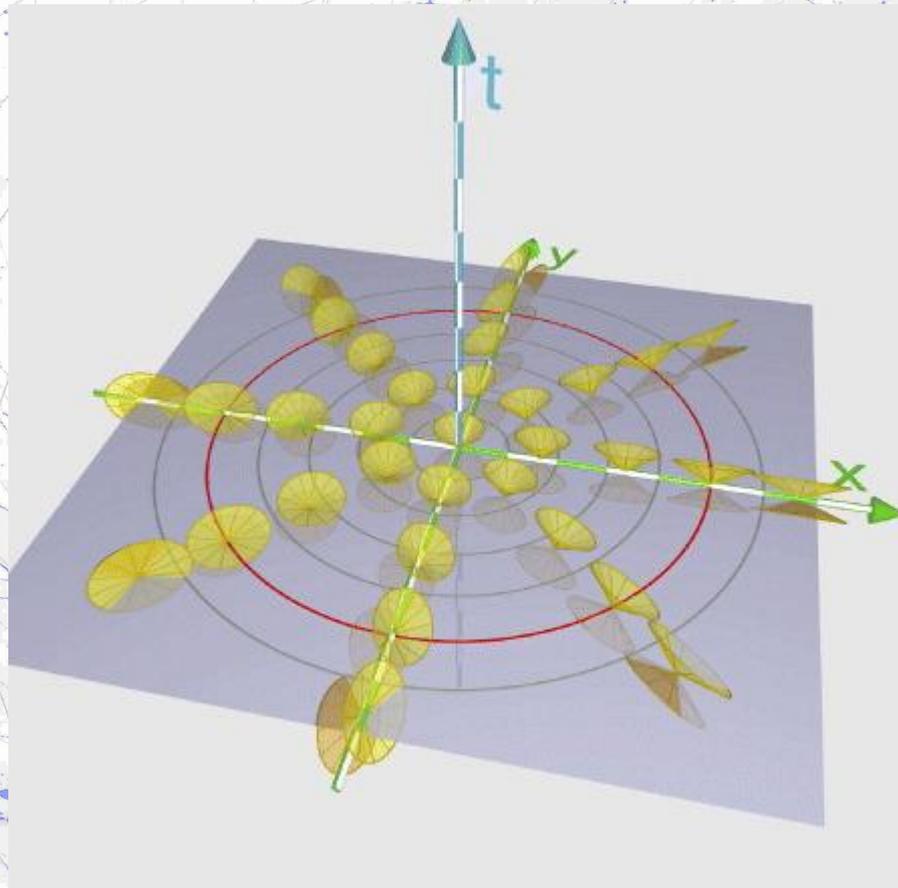
Beginning of time in other models?

Proposal:



For each global time slice S , for each event x on it, every time-like curve arriving at x with no past first point has a finite proper length.

Problem 1: Gödel Universe

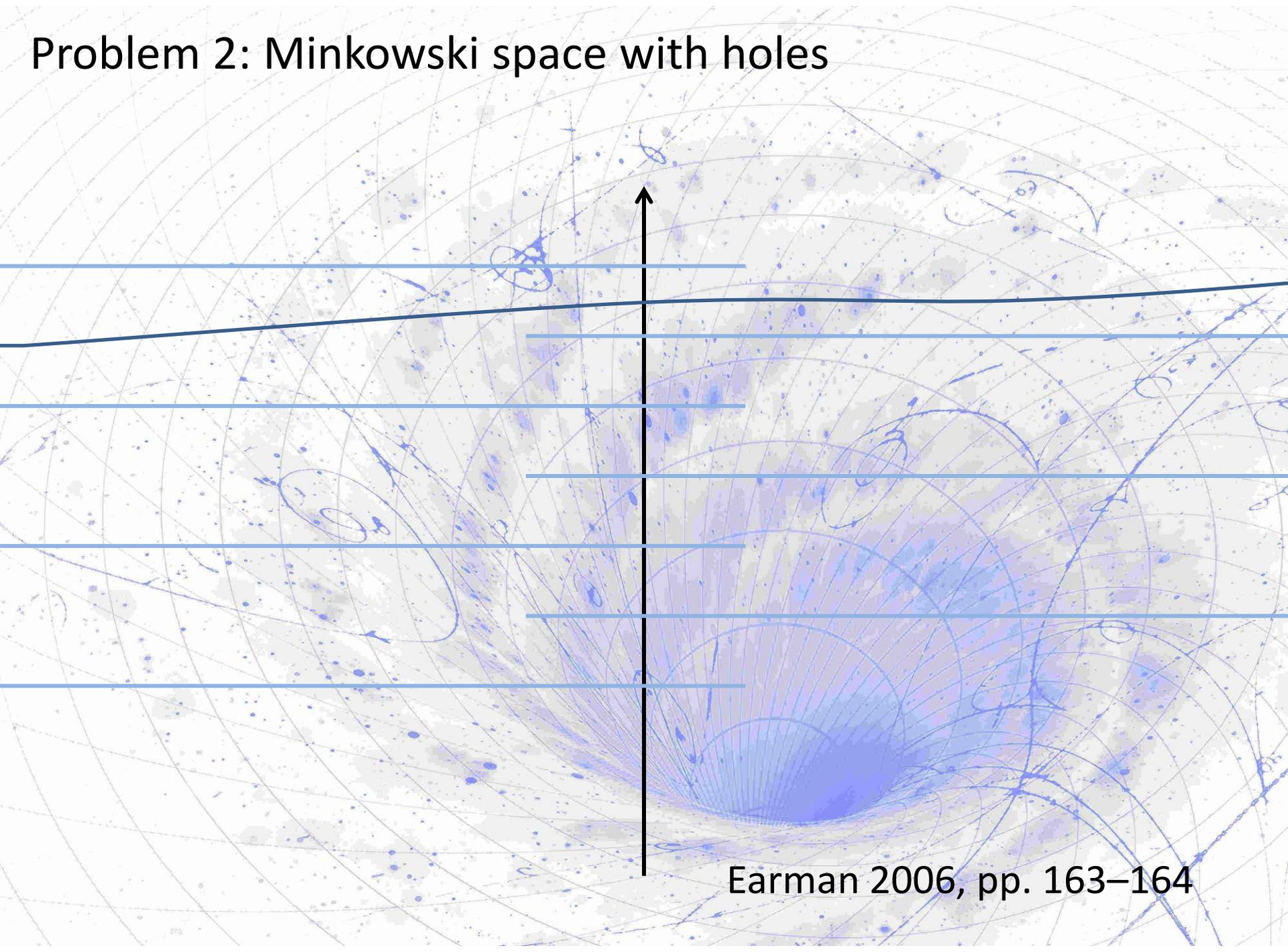


There are no global space-like curves in this model

Earman 2006

Image: Buser et al. 2013

Problem 2: Minkowski space with holes

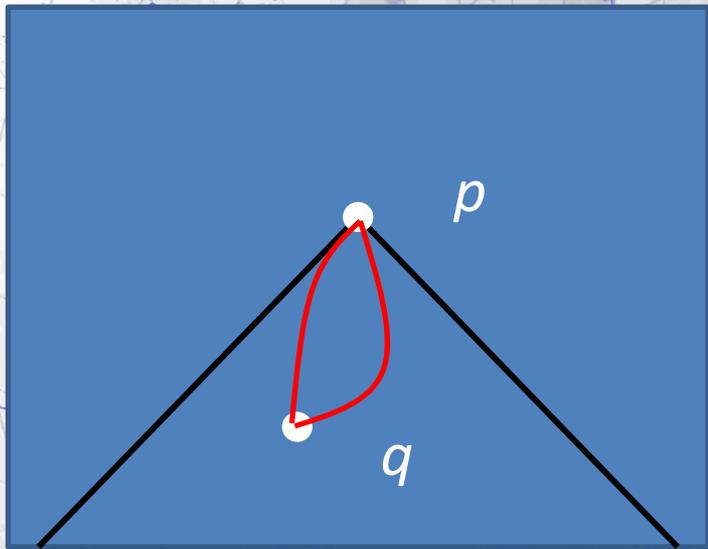


Earman 2006, pp. 163–164

The benefits of a beginning

Andersson et al. 1998:

Time function:



Temporal distance between p and q : maximal length of time-like curve

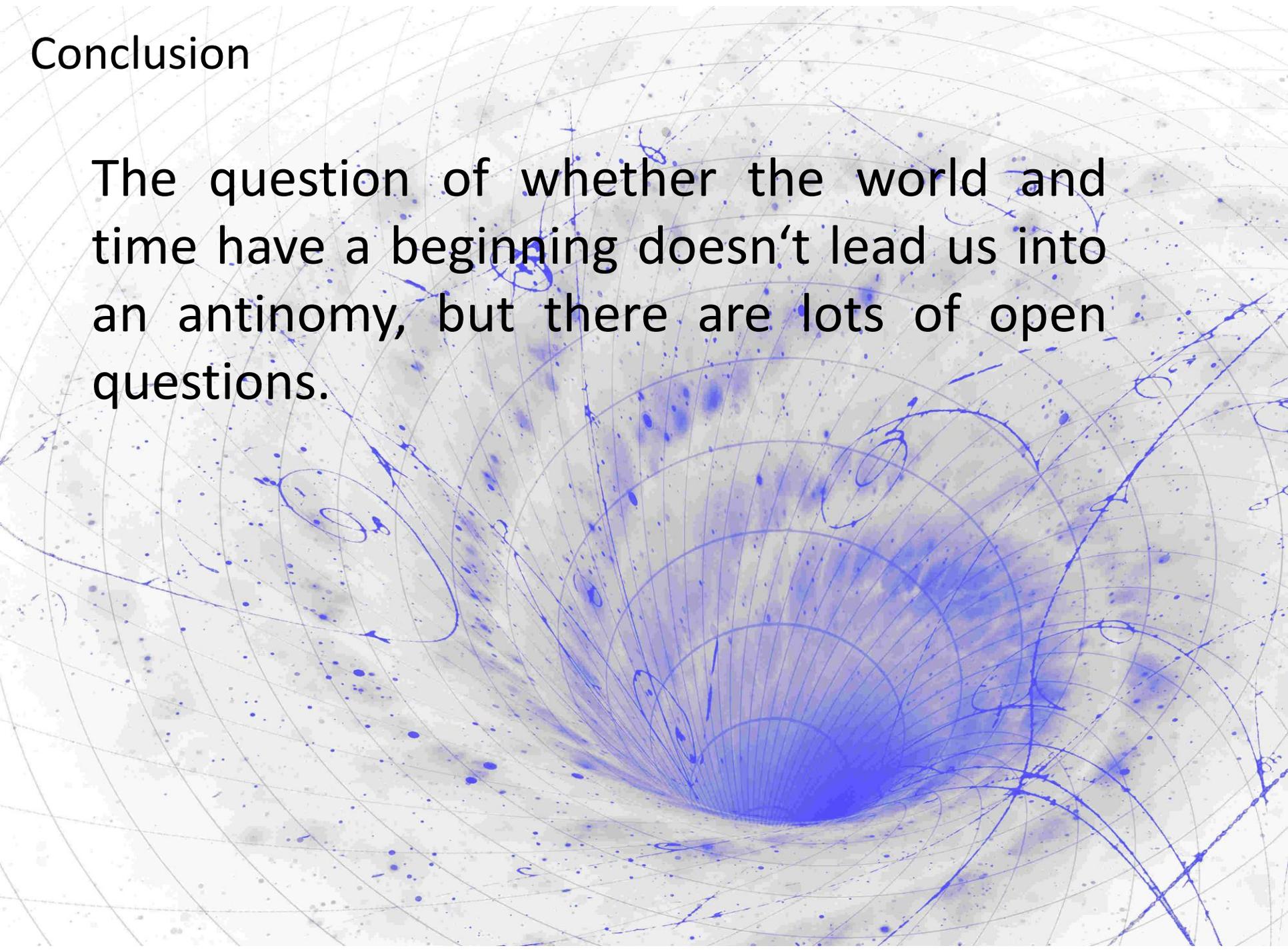
The benefits of a beginning

Andersson et al. 1998:

If the time function is a real number everywhere and if it goes to zero for inextendible causal curves under some technical conditions, then the space-time is globally hyperbolic and the time function defines a cosmic time.

Conclusion

The question of whether the world and time have a beginning doesn't lead us into an antinomy, but there are lots of open questions.



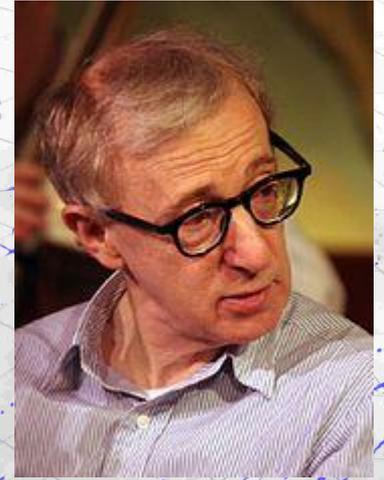
Open questions

1. Can we exclude some space-time manifolds as unphysical or unreasonable from the outset?

cf. Kant about grounding vs. extendibility

2. Can we define what we mean by a beginning of time in such space-times?

3. Can we know whether our own universe has such a beginning of time?



“Can we actually “know” the universe? My God, it's hard enough to find your way around in Chinatown.”

W. Allen (1991, p. 170)

Image: C. Swan

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